

$$\frac{\partial \vec{F}^o}{\partial t} + \frac{\partial \vec{F}^L}{\partial x^i} = 0$$

$$\frac{\partial \vec{F}^o}{\partial t} + \frac{\partial \vec{F}^k}{\partial x^i}$$

$$\frac{\partial \vec{F}^o}{\partial \vec{p}} \cdot \frac{\partial \vec{p}}{\partial t} + \frac{\partial \vec{F}^k}{\partial \vec{p}} \cdot \frac{\partial \vec{p}}{\partial x} = 0$$

$$\mathcal{F}^0 \partial_t \hat{\rho} + \mathcal{F}^k \partial_i \hat{\rho} = 0$$

$$\partial_t \tilde{p} + (F^o)^{..} F^k \partial_i \tilde{p} = 0$$

\vec{e} is stand in components of right eigen vector

$$\vec{e} = (e^0, e^j, e^4, \hat{e}^j)^T$$

$$(f^o)^{-1} f^k \vec{e} = \lambda^k \vec{e}$$

$$f^k_{\tilde{c}} = \lambda^k F^0_{\theta^k}$$

$$\left(\zeta^k - \lambda^k \zeta^0 \right)_{\bar{e}} = 0$$

$$\chi = \frac{\partial P}{\partial p}$$

$$\gamma = \frac{\partial h_c}{\partial p_0} = 1 + \varepsilon + \chi$$

$$X = \frac{\partial P}{\partial E}$$

Second
speed

$$h_e C_s^2 = P_0 X + \frac{P}{P_0} X$$

$$Q = h_{e_i} W^z + B^z$$

$$\frac{F^o}{\sqrt{h}} = \begin{pmatrix} w & w^3 P_0 v_j & 0 & 0 \\ w^2 v_{r_i} & h_{ij} Q + 2h_e W^4 v_r v_j - B_i B_j & (P_0 + \lambda) W^2 v_r & 2v_r B_j - B_i v_j - (B_r) h_{ij} \\ r W^2 - w X & (2h_e W^4 + B^2 - W^3 P_0) v_j & -(B_r) B_j & (P_0 + \lambda) W^2 - X \\ 0 & 0 & 0 & (z - w^2) B_j - (B_r) B_j - (B_r) v_j \end{pmatrix}$$

$$\frac{F^k}{\alpha \sqrt{h}} = \begin{pmatrix} W_r^k & W_{P_0}(W_{v_j}^z \bar{v}^k + h_j^k) & 0^k & O_j^k \\ W^2 \gamma v_i \bar{v}^k + h_i^k \chi & A_{ij}^k & (\rho_0 + \lambda) W_{v_i}^z \bar{v}^k + h_i^k \lambda & B_{ij}^k \\ (W^z \gamma - W - \chi) \bar{v}^k & C_{ij}^k & (\rho_0 + \lambda) W^z \bar{v}^k - \lambda (\bar{v}^k - v^k) & D_{ij}^k \\ O^{ik} & B^i h_j^k - B^k h_j^i & O^{ik} & h_i^k \bar{v}^k - h_j^k \bar{v}^i \end{pmatrix}$$

$$A_{ij}^k = (h_{ij} Q + 2h_e W_{v_i v_j}^z - B_i B_j) \bar{v}^k + (Q_{v_i} - (B_v) B_i) h_j^k + h_i^k ((B_v) B_j - B^z v_j) - (h_{ij} (B_v) + v_i B_j - 2B_i v_j) B^k$$

$$B_{ij}^k = (2v_i B_j - B_i v_j - (B_v) h_{ij}) \bar{v}^k - \left(\frac{h_{ij}}{w^2} + v_i v_j \right) B^k + h_i^k ((B_v) v_j + \frac{B_j}{w^2}) - h_j^k \left(\frac{B_i}{w^2} + (B_v) v_i \right)$$

$$C_{ij}^k = (2h_e W_{v_j}^z - (B_v) B_j + B^z v_j - \rho_0 W_{v_j}^3) \bar{v}^k + (Q - W_{P_0}) h_j^k + ((B_v) B_j - B^z v_j) v^k$$

$$D_{ij}^k = (2B_j - (B_v) v_j - \frac{B_j}{w^2}) \bar{v}^k + ((B_v) v_j + \frac{B_j}{w^2}) v^k - v_j B^k - (B_v) h_j^k - B_j B^k$$

$$\det |F^k - \lambda^k F^0| = (\alpha \sqrt{h})^8 .$$

$$\begin{vmatrix} W(\bar{v}^k - \lambda^k) & W_{P_0}^3 v_j (\bar{v}^k - \lambda^k) + W_{P_0} h_j^k & 0^k & O_j^k \\ W^2 \gamma v_i (\bar{v}^k - \lambda^k) + h_i^k \chi & A_{ij}^k & (\rho_0 + \lambda) W^2 v_i (\bar{v}^k - \lambda^k) + h_i^k \chi & B_{ij}^k \\ (W^z \gamma - W - \chi) (\bar{v}^k - \lambda^k) + \chi v^k & C_{ij}^k & [(\rho_0 + \lambda) W^z \chi] (\bar{v}^k - \lambda^k) + \chi v^k & D_{ij}^k \\ O^{ik} & B^i h_j^k - B^k h_j^i & O^{ik} & h_i^k (\bar{v}^k - \lambda^k) - h_j^k \bar{v}^i \end{vmatrix}$$

$$\tilde{A}_{ij}^k = (h_{ij} Q + 2h_e W_{v_i v_j}^z - B_i B_j) (\bar{v}^k - \lambda^k) + (Q_{v_i} - (B_v) B_i) h_j^k + h_i^k ((B_v) B_j - B^z v_j) - (h_{ij} (B_v) + v_i B_j - 2B_i v_j) B^k$$

$$\tilde{B}_{ij}^k = (2v_i B_j - B_i v_j - (B_v) h_{ij}) (\bar{v}^k - \lambda^k) - \left(\frac{h_{ij}}{w^2} + v_i v_j \right) B^k + h_i^k ((B_v) v_j + \frac{B_j}{w^2}) - h_j^k \left(\frac{B_i}{w^2} + (B_v) v_i \right)$$

$$\tilde{C}_{ij}^k = (2h_e W^z + B^z - \rho_0 W^3) v_j (\bar{v}^k - \lambda^k) - (B_v) B_j (\bar{v}^k - \lambda^k) + (Q - W_{P_0}) h_j^k + ((B_v) B_j - B^z v_j) v^k - B_j B^k$$

$$\tilde{D}_{ij}^k = (2B_j - (B_v) v_j - \frac{B_j}{w^2}) (\bar{v}^k - \lambda^k) + ((B_v) v_j + \frac{B_j}{w^2}) v^k - v_j B^k - (B_v) h_j^k$$

$$a^k = v^k - \lambda^k$$

$$\Delta^{kk} = (a^k)^2 Q - 2(B_r) a^k B^k - \frac{B^k B^k}{w^2}$$

W_a^k	$W_{\beta r_j}^3 a^k + W_{\beta h_j^k}$	0^k	D_j^k
$W^2 \gamma r_i a^k + h_i^k x$	\tilde{A}_{ij}^k	$(p_0 + \lambda) W^2 r_i a^k + h_i^k x$	\tilde{B}_{ij}^k
$(W^2 \gamma - W - \lambda) a^k + x r^k$	\tilde{C}_j^k	$[(p_0 + \lambda) W^2 x] a^k + x r^k$	\tilde{D}_j^k
0^{ik}	$B_{h_j^k}^i - B^k h_j^i$	0^{ik}	$h_j^i a^k - h_j^k r^i$

Add row 1 to row 3

W_a^k	$W_{\beta r_j}^3 a^k + W_{\beta h_j^k}$	0^k	D_j^k
$W^2 \gamma r_i a^k + h_i^k x$	\tilde{A}_{ij}^k	$(p_0 + \lambda) W^2 r_i a^k + h_i^k x$	\tilde{B}_{ij}^k
$(W^2 \gamma - \lambda) a^k + x r^k$	$\tilde{C}_j^k + W_{\beta r_j}^3 a^k + W_{\beta h_j^k}$	$[(p_0 + \lambda) W^2 x] a^k + x r^k$	\tilde{D}_j^k
0^{ik}	$B_{h_j^k}^i - B^k h_j^i$	0^{ik}	$h_j^i a^k - h_j^k r^i$

$$R_2^+ = \cdot v_i \circ R_3$$

W_a^k	$W_{\beta r_j}^3 a^k + W_{\beta h_j^k}$	0^k	D_j^k
$\chi [h_i^k + r_i (a^k - r^k)]$	\tilde{C}_{ij}^k	$\chi [h_i^k + r_i (a^k - r^k)]$	$\tilde{B}_{ij}^k - v_i \tilde{D}_j^k$
$(W^2 \gamma - \lambda) a^k + x r^k$	$\tilde{C}_j^k + W_{\beta r_j}^3 a^k + W_{\beta h_j^k}$	$[(p_0 + \lambda) W^2 x] a^k + x r^k$	\tilde{D}_j^k
0^{ik}	$B_{h_j^k}^i - B^k h_j^i$	0^{ik}	$h_j^i a^k - h_j^k r^i$

$$\begin{aligned}
 & W^2 \gamma r_i a^k + h_i^k x & \tilde{A}_{ij}^k & \chi (p_0 + \lambda) W^2 r_i a^k + h_i^k x & \tilde{B}_{ij}^k \\
 & -v_i [(W^2 \gamma - \lambda) a^k + x r^k] & -v_i (\tilde{C}_j^k + W_{\beta r_j}^3 a^k + W_{\beta h_j^k}) & -v_i [(p_0 + \lambda) W^2 x] a^k + x r^k & -v_i \tilde{D}_j^k \\
 & \chi [h_i^k + r_i (a^k - r^k)] & \chi_{ij}^k & h_i^k \chi + r_i \chi (a^k - r^k) r^k & \\
 & & & \chi [h_i^k + r_i (a^k - r^k)] &
 \end{aligned}$$

$$C_1 \doteq -\frac{\chi}{2} C_3$$

W_a^k	$W_{\beta v_j}^3 a^k + W_p h_j^k$	O^k	D_j^k
D_i^k	ζ_{ij}^k	$\chi [h_i^k + v_i(a^k - r^k)]$	$\tilde{B}_{ij}^k - v_i \tilde{D}_j^k$
$W_a^k [\gamma - \chi (1 + \frac{P_0}{2})]$	$\zeta_j^k + W_{\beta v_j}^3 a^k + W_p h_j^k$	$[(\rho_0 + \chi) W^3 - \chi] a^k + \chi r^k$	\tilde{D}_j^k
O^{ik}	$B^i h_j^k - B^k h_j^i$	O^{ik}	$h_j^i a^k - h_j^k a^i$

$$(W^3 \gamma - \chi) a^k + \chi r^k - \frac{\chi}{2} \{ [(\rho_0 + \chi) W^3 - \chi] a^k + \chi r^k \}$$

$$W^3 \gamma a^k - \cancel{W^3 a^k + \chi r^k} - \frac{\chi}{2} \rho_0 W^3 a^k - \chi W^3 a^k + \cancel{\chi r^k} - \cancel{\chi r^k}$$

$$W^3 a^k [\gamma - \chi (1 + \frac{P_0}{2})]$$

$$R_3 \doteq -W[\gamma - \chi (1 + \frac{P_0}{2})] R_1$$

W_a^k	$W_{\beta v_j}^3 a^k + W_p h_j^k$	O^k	D_j^k
D_i^k	ζ_{ij}^k	$\chi [h_i^k + v_i(a^k - r^k)]$	$\tilde{B}_{ij}^k - v_i \tilde{D}_j^k$
O^k	ζ_j^k	$[(\rho_0 + \chi) W^3 - \chi] a^k + \chi r^k$	\tilde{D}_j^k
O^{ik}	$B^i h_j^k - B^k h_j^i$	O^{ik}	$h_j^i a^k - h_j^k a^i$

$$\zeta_j^k + W_{\beta v_j}^3 a^k + W_p h_j^k - W[\gamma - \chi (1 + \frac{P_0}{2})] [W_{\rho_0}^3 v_j a^k + W_{\rho_0} h_j^k]$$

$$\xi_j^k = \zeta_j^k + (W_{\rho_0}^3 v_j a^k + W_{\rho_0} h_j^k) (1 - W[\gamma - \chi (1 + \frac{P_0}{2})])$$

$$\gamma - \chi (1 + \frac{P_0}{2}) = \frac{h_e}{\rho_0 \lambda} (\lambda - \rho_0 c_s^2)$$

$$\frac{\mathcal{F}^k}{\alpha\sqrt{h}} = \begin{pmatrix} W\bar{v}^k & W\rho_0(W^2v_j\bar{v}^k + h_j^k) & 0^k & 0_j^k \\ W^2\gamma v_i\bar{v}^k + h_i^k\chi & A_{ij}^k & (\rho_0 + \kappa)W^2v_i\bar{v}^k + h_i^k\kappa & B_{ij}^k \\ (W^2\gamma - W - \chi)\bar{v}^k & C_j^k & (\rho_0 + \kappa)W^2\bar{v}^k - \kappa(\bar{v}^k - v^k) & D_j^k \\ 0^k & B^ih_j^k - B^kh_j^i & 0^k & h_j^ih_j^k - h_j^kh_j^i \end{pmatrix}$$

$$D_j^k = (2B_j - (Bv)v_j - B_j/W^2)\bar{v}^k + ((Bv)v_j + B_j/W^2)v^k - v_jB^k - (Bv)h_j^k$$

$$\frac{\mathcal{F}^0}{\sqrt{h}} = \begin{pmatrix} W & W^2\rho_0v_j & 0 & 0_j \\ W^2\gamma v_i & h_{ij}Q + 2h_iW^4v_iv_j - B_iB_j & (\rho_0 + \kappa)W^2v_i & 2v_iB_j - B_iv_j - (Bv)h_{ij} \\ \gamma W^2 - W - \chi & (2h_iW^4 + B^2 - W^3\rho_0)v_j - (Bv)B_j & (\rho_0 + \kappa)W^2 - \kappa(2 - W^{-2})B_j - (Bv)B_j - (Bv)v_j \\ 0^i & 0_j^i & 0^i & h_j^i \end{pmatrix}$$

$$(\mathcal{F}^k - \lambda^k \mathcal{F}^0),$$

$$\tilde{D}_j^k = \lambda^k \left[(2 - \omega^{-2})B_j - (B_{vr})\tilde{B}_j - (B_{vr})v_j \right]$$

$$(2B_j - (B_{vr})v_j - \frac{B_j}{\omega^2})\bar{v}^k + ((B_{vr})v_j + \frac{B_j}{\omega^2})v^k - v_jB^k - (B_{vr})h_j^k = \lambda^k \left[(2 - \omega^{-2})B_j - (B_{vr})\tilde{B}_j - (B_{vr})v_j \right]$$

$$(2B_j - (B_{vr})v_j - \frac{B_j}{\omega^2})\bar{v}^k + ((B_{vr})v_j + \frac{B_j}{\omega^2})v^k - v_jB^k - (B_{vr})h_j^k = \lambda^k \left[(2 - \omega^{-2})B_j - (B_{vr})\tilde{B}_j - (B_{vr})v_j \right]$$

$$(2B_j - (B_{vr})v_j - \frac{B_j}{\omega^2})\bar{v}^k + ((B_{vr})v_j + \frac{B_j}{\omega^2})v^k - v_jB^k - (B_{vr})h_j^k = \lambda^k \left[2B_j - \frac{B_j}{\omega^2} - (B_{vr})B_j - (B_{vr})v_j \right]$$

$$(2B_j - (B_{vr})v_j - \frac{B_j}{\omega^2})\bar{v}^k + ((B_{vr})v_j + \frac{B_j}{\omega^2})v^k - v_jB^k - (B_{vr})h_j^k = \lambda^k \left[2B_j - (B_{vr})v_j - \frac{B_j}{\omega^2} - (B_{vr})B_j \right]$$

$$(2B_j - (B_{vr})v_j - \frac{B_j}{\omega^2})\bar{v}^k + ((B_{vr})v_j + \frac{B_j}{\omega^2})v^k - v_jB^k - (B_{vr})h_j^k = \lambda^k (B_{vr})B_j$$

$$\det |\mathcal{F}^k - \lambda^k \mathcal{F}^0| = (\alpha\sqrt{h})^8 \begin{vmatrix} W(v^k - \lambda^k) & W^3\rho_0v_j(v^k - \lambda^k) + W\rho_0h_j^k \\ W^2\gamma v_i(v^k - \lambda^k) + h_i^k\chi & \bar{A}_{ij}^k \\ (W^2\gamma - W - \chi)(v^k - \lambda^k) + \chi v^k & C_j^k \\ 0^k & B^ih_j^k - B^kh_j^i \\ & 0^k & 0^k \\ & (\rho_0 + \kappa)W^2v_i(v^k - \lambda^k) + h_i^k\kappa & \bar{B}_{ij}^k \\ & [(\rho_0 + \kappa)W^2 - \kappa](v^k - \lambda^k) + \kappa v^k & D_j^k \\ & 0^k & h_j^i(v^k - \lambda^k) - h_j^k v^i \end{vmatrix}$$

$$\tilde{D}_j^k = (2B_j - (Bv)v_j - B_j/W^2)(\bar{v}^k - \lambda^k) + ((Bv)v_j + B_j/W^2)v^k - v_jB^k - (Bv)h_j^k$$

$$\tilde{D}_j^k = \tilde{D}_j^k = \lambda^k \left[(2 - \omega^{-2})B_j - (B_{vr})\tilde{B}_j - (B_{vr})v_j \right]$$

$$\tilde{D}_j^k = \left(2B_j - (B_v)v_j - \frac{B_j}{v^2}\right)(\bar{v}^k - \lambda^k) + ((B_v)v_j + \frac{B_j}{v^2})v^k - v_j B^k - (B_v)v_j^k$$

$$= \left(2B_j - (B_v)v_j - \frac{B_j}{v^2}\right)(\bar{v}^k - \lambda^k) + ((B_v)v_j + \frac{B_j}{v^2})v^k - v_j B^k - v_j B^k - (B_v)v_j^k - \lambda^k(B_v)B_j$$

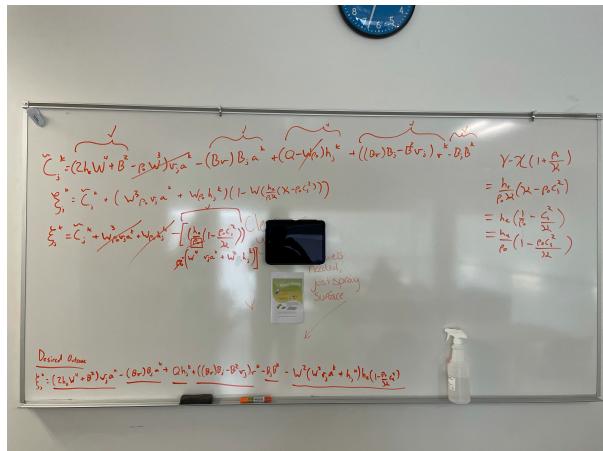
$$\xi_j^k = (2h_e W^y + B^z)v_j a^k - (B_v)B_j a^k + Q h_j^k + ((B_v)B_j - B^z v_j)v^k - B_j B^k$$

$$- W(W v_j a^k + h_j^k)h_e (1 - \frac{\rho_0}{\rho} C_s^2)$$

$$\tilde{C}_j^k = (2h_e W^y + B^z - \rho_0 W^3)v_j(\bar{v}^k - \lambda^k) - (B_v)B_j(\bar{v}^k - \lambda^k) + (Q - W\rho_0)h_j^k + ((B_v)B_j - B^z v_j)v^k - B_j B^k$$

$$= (2h_e W^y + B^z - \rho_0 W^3)v_j a^k - (B_v)B_j a^k + (Q - W\rho_0)h_j^k + ((B_v)B_j - B^z v_j)v^k - B_j B^k$$

$$\xi_j^k = \tilde{C}_j^k + (W\rho_0 v_j a^k + W\rho_0 h_j^k)(1 - W(\gamma - \chi(\frac{\rho_0}{\rho})))$$



$$1 - \frac{h_e W}{\rho_0 \gamma} (\gamma - \rho_0 C_s^2)$$

$$\chi_{ij}^k = (h_{ij}(Q - B_i B_j)a^k - (B_v)B_i h_j^k - (h_{ij}(B_v) - 2B_i v_j)B^k$$

$$+ ((B_v)B_j - B^z v_j)(h_i^k + v_i(a^k - v^k))$$

$\tilde{A}_{ij} = (h_{ij}Q - B_i B_j) \alpha^k + (B_r B_j) h_{ij}^k + h_{ij}^k ((B_r B_j - B_i B_r) - (h_{ij} B_r) \alpha^k) \alpha^k$
 $\tilde{C}_{ij} = (2h_i \alpha^k + B_r B_j \alpha^k) v_{ij}^k - (B_r v_{ij})^k + (Q - 2h_i \alpha^k) v_{ij}^k + ((B_r B_j - B_i B_r) v_{ij}^k - B_i B_r)$
 $\tilde{x}_{ij} = \tilde{A}_{ij} - (\tilde{C}_{ij} + 2h_i \alpha^k + B_r B_j \alpha^k) v_{ij}^k$
 Cleaner for your area
 - No paper towels needed
 just spray surface

$$c_2 \doteq -((B_r)B_j - B_{rj}) \beta_k \cdot c_3$$

w_{α}^k	$w_{\beta v_j}^k \alpha^k + w_{\beta j}^k$	0^k	0_j^k
D_i^k	\tilde{x}_{ij}^k	$\times [h_i^k + v_i(\alpha^k - v^k)]$	$B_{ij}^k - v_i D_j^k$
O^k	\tilde{y}_{ij}^k	$[(\rho_0 + \lambda) w^k - \lambda] \alpha^k + \lambda v^k$	D_j^k
O^k	$B_i^k h_j^k - B^k h_j^i$	0^k	$h_j^i \alpha^k - h_j^k v^i$

$$\tilde{\alpha}_{ij}^k = (h_{ij}(Q - B_i B_j) \alpha^k - (B_r) B_i h_j^k - (h_{ij}(B_r) - 2B_i v_j) B^k)$$

$$\tilde{\xi}_j^k = (2h_i w^k + \epsilon B^k) v_j^k + Q h_j^k - B_j B^k - W(w_{v_j}^k \alpha^k + h_j^k) h_e \left(1 - \frac{\rho_0}{2} \zeta^2\right)$$

$$\begin{aligned}
 \tilde{\alpha}_{ij}^k &= (h_{ij}(Q - B_i B_j) \alpha^k - (B_r) B_i h_j^k - (h_{ij}(B_r) - 2B_i v_j) B^k \\
 &\quad + ((B_r) B_j - B_{rj}^k) (h_i^k + v_i(\alpha^k - v^k)))
 \end{aligned}$$

$$\tilde{\alpha}_{ij}^k = \alpha_{ij}^k - \underbrace{\lambda [h_i^k + v_i(\alpha^k - v^k)] \cdot ((B_r) B_j - B_{rj}^k)}$$

$$\tilde{\kappa}_{ij}^k = (h_{ij}Q - B_i B_j) \alpha^k - (B_r) B_i h_j^k - (h_{ij} B_r - B_{ir} h_j) B^k$$

$$\xi_j^k = \frac{(2h_e W^z + B^z) v_j \alpha^k - (B_r) B_j \alpha^k + Q h_j^k + ((B_r) B_j - B^z v_j) v^k - B_j B^k}{-W^z (W^z v_j \alpha^k + h_j^k) h_e (1 - \frac{P_0}{2} c_s^2)}$$

$$\tilde{\xi}_j^k = \xi_j^k - \frac{((B_r) B_j - B^z v_j) \left[[(\rho_0 + \chi) W^z - \chi] \alpha^k + \chi r^k \right]}{2} - ((B_r) B_j - B^z v_j) \left[\left[\left(\frac{\rho_0}{2} + 1 \right) W^z - 1 \right] \alpha^k + r^k \right]$$

$$\tilde{\xi}_j^k = \underbrace{(2h_e W^z + B^z) v_j \alpha^k + Q h_j^k - B_j B^k}_{-W^z \alpha^k \left(1 + \frac{\rho_0}{2} \right) ((B_r) B_j - B^z v_j)} - W^z (W^z v_j \alpha^k + h_j^k) h_e \left(1 - \frac{\rho_0}{2} c_s^2 \right)$$

$+ \epsilon B^2$

$- B^z v_j \alpha^k ?$

This term shouldn't
be here; for me, $\epsilon = 0$

$$C_4 \doteq -((Bv)_{v_j} + B_j/\omega^2) \frac{1}{\lambda} \cdot C_3$$

W_a^k	$\tilde{W}_{Bv_j} a^k + W_B h_j^k$	O^k	D_j^k
D_i^k	\tilde{X}_{ij}^k	$X[h_i^k + v_i(a^k - v^k)]$	B_{ij}^k
O^k	\tilde{Y}_j^k	$[(A + X)W^2 - X]a^k + Xv^k$	δ_j^k
O^{ik}	$B^k h_j^k - B^k h_j^i$	O^{ik}	$h_j^i a^k - h_j^k v^i$

$$\beta_{ij}^k = -(B_{ij}v_j + (Bv)h_{ij})(a^k) - (h_{ij}B^k + h_j^k B_i) \frac{1}{\omega^2} +$$

$$\in ((Bv)B_j - B^2 v_j)(h_i^k + v_i(a^k - v^k))$$

$$S_j^k = 2B_j a^k - v_j B^k - (Bv)h_j^k - W_a^k ((Bv)v_j + B_j \frac{1}{\omega^2}) \left(1 + \frac{A}{\lambda} \right)$$

$$\begin{aligned} \beta_{ij}^k &= \tilde{B}_{ij}^k - v_i \tilde{D}_j^k - ((Bv)_{v_j} + B_j/\omega^2) \frac{1}{\lambda} \cdot X[h_i^k + v_i(a^k - v^k)] \\ \beta_{ij}^k &= \tilde{B}_{ij}^k - v_i \tilde{D}_j^k - ((Bv)_{v_j} + B_j/\omega^2) \frac{1}{\lambda} \cdot X[h_i^k + v_i(a^k - v^k)] \end{aligned}$$

$$\begin{aligned} \tilde{B}_{ij}^k &= (2v_i B_j - B_i v_j - (Bv)h_{ij})(a^k) - \cancel{(h_{ij} + v_i v_j)B^k} + h_i^k ((Bv)v_j + \frac{B}{\omega^2}) - h_j^k (\frac{B_i}{\omega^2} + (Bv)v_i) \\ - v_i \tilde{D}_j^k &= - \left[(2B_j - (Bv)v_j - \frac{B}{\omega^2})(a^k) + ((Bv)_{v_j} + \frac{B}{\omega^2})v^k - h_j^k B^k - (Bv)h_j^k \right] v_i \\ &\quad - (Bv)_{v_j} h_i^k - B_j \cancel{\frac{1}{\omega^2}} h_i^k - (Bv)_{v_j} v_i (a^k - v^k) - B_j v_i v_j (a^k - v^k) \end{aligned}$$

$$\beta_{ij}^k = - \underbrace{(B_{ij}v_j + (B_v)h_{ij})}_{\text{Red}} a^k - \underbrace{(h_{ij}B^k + h_j^k B_i)}_{\text{Red}} \frac{1}{\omega^2} +$$

$\underbrace{((B_v)B_j - B^2 v_j)(h_i^k + v_i(a^k - v^k))}_{\text{Purple}}$

$$(B_v)B_j h_i^k + (B_v)B_j v_i(a^k - v^k) - B^2 v_j h_i^k - B^2 v_i(a^k - v^k)$$

These terms shouldn't be here; add ϵ and for me, $\epsilon=0$

$$- (B_{ij}v_j + B_v h_{ij}) a^k$$

$$S_j^k = \tilde{D}_j^k - \cancel{\left((B_v)v_j + B_j/\omega^2 \right)} \circ \left[[(p_0 + \lambda)\omega^2 - \lambda] a^k + k r^k \right]$$

$$S_j^k = \tilde{D}_j^k - \cancel{\left((B_v)v_j + B_j/\omega^2 \right)} \circ \left[\left[\left(\frac{p_0}{\omega} + 1 \right) \omega^2 - 1 \right] a^k + v^k \right]$$

$$\tilde{D}_j^k = \cancel{\left(2B_j - (B_v)v_j - \frac{B_j}{\omega^2} \right)} (a^k) + \cancel{\left((B_v)v_j + \frac{B_j}{\omega^2} \right)} v^k - \cancel{v_j B^k} - \cancel{(B_v)h_j^k}$$

$$- \cancel{(B_v)v_j} \left[\cancel{\left(\frac{p_0}{\omega} + 1 \right) \omega^2 - 1} \right] a^k - \cancel{(B_v)v_j^k} - \cancel{\frac{B_j}{\omega^2}} \left[\cancel{\left(\frac{p_0}{\omega} + 1 \right) \omega^2 - 1} \right] a^k$$

$$-\frac{B_j}{\omega} \cancel{\omega^k}$$

$$\zeta_j^k = \underline{2B_j a^k} - \underline{v_j B^k} - \underline{(B_r) h_j^k} - \underline{\sqrt{a^k} ((B_r)v_j + B_j \bar{\omega}^2) \left(1 + \frac{a^k}{2} \right)}$$

Factor out $\left(\frac{1}{a^k}\right)^3$, $C_2 \doteq B^k C_4$

$$\left(\frac{1}{a^k} \right)^3 \begin{array}{l|l} \begin{array}{c} Wa^k \\ D_i^k \\ O^k \\ O^{ik} \end{array} & \begin{array}{c} (W_B v_j a^k + W_B h_j^k) a^k \\ a^k \tilde{\alpha}_{ij}^k + B^k \beta_{ij}^k \\ a^k \tilde{\delta}_j^k + B^k \delta_j^k \\ B^k h_j^k a^k - B^k h_j^k \tilde{r}^i \end{array} \\ \hline \begin{array}{c} O^k \\ X[h_i^k + v_i(a^k - r^k)] \\ [(a^k + X)W^k - X]a^k + Xr^k \\ O^{ik} \end{array} & \begin{array}{c} D_j^k \\ B_{ij}^k \\ \delta_j^k \\ h_j^k a^k - h_j^k \tilde{r}^i \end{array} \end{array}$$

$$C_2 \doteq -B^j C_4 h_j^k$$

$$\left(\frac{1}{a^k} \right)^3 \begin{array}{l|l} \begin{array}{c} Wa^k \\ D_i^k \\ O^k \\ O^{ik} \end{array} & \begin{array}{c} (W_B v_j a^k + W_B h_j^k) a^k \\ a^k \tilde{\alpha}_{ij}^k + B^k \beta_{ij}^k - B^i \beta_{il}^k h_j^k \\ a^k \tilde{\delta}_j^k + B^k \delta_j^k - B^i \delta_{il}^k h_j^k \\ D_j^{ik} \end{array} \\ \hline \begin{array}{c} O^k \\ X[h_i^k + v_i(a^k - r^k)] \\ [(a^k + X)W^k - X]a^k + Xr^k \\ O^{ik} \end{array} & \begin{array}{c} D_j^k \\ B_{ij}^k \\ \delta_j^k \\ h_j^k a^k - h_j^k \tilde{r}^i \end{array} \end{array}$$

$$a^k \tilde{\alpha}_{ij}^k + B^k \beta_{ij}^k - B^i \beta_{il}^k h_j^k = h_{ij} \left[(a^k)^2 Q - 2(B_r) a^k B^k - \frac{1}{\omega^2} B^k B^k \right] - B_i \left[(a^k)^2 B_j - a^k ((B_r) h_j^k + v_j B^k) - \frac{1}{\omega^2} h_j^k B^k \right]$$

$$= h_{ij} \Delta^{kk} - B_i E_j^{kk}$$

$$= A_{ij}^{kk}$$

$$\alpha^k \tilde{\alpha}_{ij}^k = \left[(h_{ij}(Q - B_i B_j))_a^k - (B_r) B_i h_j^k - (h_{ij}(B_r) - 2B_i v_j) B^k \right] a^k$$

$$+ B^k B_{ij}^k = \left[-(B_i v_j + (B_r) h_{ij}) a^k - (h_{ij} B^k + h_j^k B_i) \frac{1}{\omega^2} + \right.$$

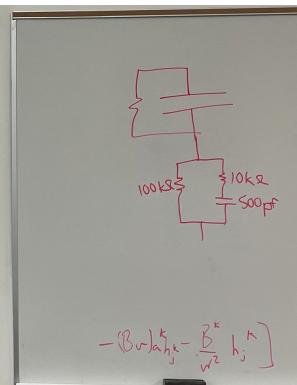
$$\left. \epsilon ((B_r) B_j - B^z v_j)(h_i^k + v_i(a^k - v^k)) \right] B^k$$

$$- B^j B_{ij}^k h_j^k = \left[-(B_i v_j + (B_r) h_{ij}) a^k - (h_{ij} B^k + h_j^k B_i) \frac{1}{\omega^2} + \right.$$

$$\left. \epsilon ((B_r) B_j - B^z v_j)(h_i^k + v_i(a^k - v^k)) \right] B^j h_j^k$$

$$\begin{aligned} \alpha^k \tilde{\alpha}_{ij}^k - B^j B_{ij}^k h_j^k &= h_{ij} \left[(\alpha^k)^2 Q - 2(B_r) \alpha^k B^k - \frac{1}{\omega^2} B^k B^k \right] \\ &\quad - B_i \left[(\alpha^k)^2 B_i - \alpha^k ((B_r) h_j^k + v_j B^k) - \frac{1}{\omega^2} h_j^k B^k \right] \\ &= h_{ij} \Delta^{kk} - B_i E_j^{kk} = A_{ij}^{kk} \end{aligned}$$

$$\begin{aligned} \alpha^k \tilde{\alpha}_{ij}^k &= \left[h_{ij}(Q - B_i B_j)_a^k - (B_r) B_i h_j^k - (h_{ij}(B_r) - 2B_i v_j) B^k \right] a^k \\ &+ B^k B_{ij}^k = B^k \left[-(B_i v_j + (B_r) h_{ij}) a^k - (h_{ij} B^k + h_j^k B_i) \frac{1}{\omega^2} + ((B_r) B_j - B^z v_j)(h_i^k + v_i(a^k - v^k)) \right] \\ &- B^j B_{ij}^k h_j^k = -B^k \left[-(B_i v_j + (B_r) h_{ij}) a^k - (h_{ij} B^k + h_j^k B_i) \frac{1}{\omega^2} + ((B_r) B_j - B^z v_j)(h_i^k + v_i(a^k - v^k)) \right] h_j^k \\ &= h_{ij} \left[Q(\alpha^k)^2 - 2B_r B^k a^k - \frac{(B^k)^2}{\omega^2} \right] - B_i \left[B_j a^k + v_j B^k + v_i (a^k - v^k) \right] h_j^k \end{aligned}$$



$$\begin{aligned} \alpha^k \tilde{\xi}_j^k + B^k S_j^k - B^l S_l^k h_j^k &= \alpha^k \alpha^k \left[W^2 (W v_j - (B_r) B_j) + \frac{p_0}{2} W^2 (h_e W^2 C_s^2 v_j + B^z v_j - (B_r) B_j) \right] \\ &\quad + \alpha^k \left[h_j^k W^2 (B_r)^2 - W^2 (B_r) v_j B^k \right. \\ &\quad \left. + \frac{p_0}{2} \left((B^z + W^2 (B_r))^2 + h_e W^2 C_s^2 \right) h_j^k - B^k (W^2 (B_r) v_j + B_j) \right] \\ &\quad - B^k \left[h_j^k (B_r) - v_j B^k \right] \\ &= C_j^{kk} \end{aligned}$$

$$\begin{aligned}
a^k \zeta_j^k &= a^k \left[(2h_e W^u + \epsilon B^2) v_j a^k + Q h_j^k - B_j B^k - W^2 (h_j^k a^k + h_j^k) h_e \left(1 - \frac{P_0}{2} c_s^2\right) \right. \\
&\quad \left. - W^2 a^k \left(1 + \frac{P_0}{2}\right) ((Bv) B_j - B^2 v_j) \right] \\
+B^k \zeta_j^k &= B^k \left[2B_j a^k - v_j B^k - (Bv) h_j^k - W^2 a^k ((Bv) v_j + B_j \frac{1}{2} c_s^2) \right] \\
-B^k \zeta_e h_j^k &= -B^k h_j^k \left[2B_k a^k - v_k B^k - (Bv) h_k^k - W^2 a^k ((Bv) v_k + B_k \frac{1}{2} c_s^2) \right]
\end{aligned}$$

$$\begin{aligned}
&= a^k a^k \left[\cancel{2h_e W^u v_j} + \cancel{\epsilon B^2 v_j} - \cancel{W^u v_j h_e} + \cancel{W^u v_j h_e \frac{P_0}{2} c_s^2} - \cancel{W^2 (Bv) B_j} \right. \\
&\quad \left. + \cancel{W^2 B^2 v_j} - \cancel{W^2 \frac{P_0}{2} (Bv) B_j} + \cancel{W^2 \frac{P_0}{2} B^2 v_j} \right] \text{Mine}
\end{aligned}$$

$$\begin{aligned}
&= a^k a^k \left[W^2 (W v_j - (Bv) B_j) + \cancel{\frac{P_0}{2} W^2 (h_e W^2 c_s^2 v_j + B^2 v_j - (Bv) B_j)} \right] \text{Thesis} \\
&= a^k \left[\cancel{Q h_j^k} - \cancel{B_j B^k} - \cancel{W^2 h_j^k h_e} + \cancel{W^2 h_j^k h_e \frac{P_0}{2} c_s^2} + \cancel{2B_j B^k} \right. \\
&\quad \left. - \cancel{W^2 B(Bv) v_j} - \cancel{W^2 B(Bv) v_j \frac{P_0}{2}} - \cancel{W^2 B_j B^k - B_j B^k \frac{P_0}{2}} \right. \\
&\quad \left. - \cancel{W^2 B^2 h_j^k} + \cancel{W^2 (Bv)^2 h_j^k} + \cancel{W^2 (Bv)^2 h_j^k \frac{P_0}{2}} \right. \\
&\quad \left. + \cancel{B^2 h_j^k} + \cancel{B^2 h_j^k \frac{P_0}{2}} \right] \text{Mine}
\end{aligned}$$

$$Q = h_e W + B^2$$

$$= \alpha^k \left[h_j^k W^2 (B_v)^2 - \underline{W^2 (B_v) v_j} B^k + \frac{\rho_0}{2} \left(\underline{(B^2 + W^2 (B_v))^2} + \underline{h_e W^2 C_s^2} \right) h_j^k - B^k \underline{(W^2 (B_v) v_j + B_j)} \right] \text{Thesis}$$

$$= -B^k (v_j B^k + (B_v) h_j^k - (B_v) h_j^k)$$

$$= -B^k (v_j B^k - (B_v) h_j^k)$$

$$(C_j^{kk}) = \alpha^k \alpha^k \left[W^2 (W_{v,j} + (h_e W_{v,j}^2 + B_{v,j}^2) - (B_v) B_j) + \frac{\rho_0}{2} W^2 (h_e W^2 C_s^2 v_j + B^2 v_j - (B_v) B_j) \right] + \alpha^k \left[h_j^k W^2 (B_v)^2 - W^2 (B_v) v_j B^k + \frac{\rho_0}{2} \left((B^2 + W^2 (B_v))^2 + h_e W^2 C_s^2 \right) h_j^k - B^k (W^2 (B_v) v_j + B_j) \right]$$

$$- B^k [h_j^k (B_v) - v_j B^k]$$

$$\left(\begin{matrix} 1 \\ \alpha^k \end{matrix} \right)^3 \left| \begin{array}{cccc} W_a^k & (W^2 \rho v_j a^k + W \rho h_j^k) a^k & 0^k & D_j^k \\ 0_i^k & A_{ij}^{kk} & x [h_i^k + v_i (\alpha^k - v^k)] & B_{ij}^k \\ 0^k & C_j^{kk} & W^2 a^k (\rho_0 + \lambda) - 2 \lambda (\alpha^k - v^k) & \delta_j^k \\ 0^{ik} & D_j^{ik} & 0^{ik} & h_j^k \alpha^k - h_j^k v^i \end{array} \right.$$

$$W_a^k (\alpha^2 h)^4 \left(\begin{matrix} 1 \\ \alpha^k \end{matrix} \right)^3 \left| \begin{array}{cccc} A_{ij}^{kk} & x [h_i^k + v_i (\alpha^k - v^k)] & B_{ij}^k & \\ C_j^{kk} & W^2 a^k (\rho_0 + \lambda) - 2 \lambda (\alpha^k - v^k) & \delta_j^k & \\ D_j^{ik} & 0^{ik} & h_j^k \alpha^k - h_j^k v^i & \end{array} \right.$$

$$\left[\begin{array}{ccc} \alpha^k & D_j^k & -\bar{v}^1 \\ 0^k & \alpha^k & -\bar{v}^2 \\ 0 & 0 & -\lambda^k \end{array} \right]$$

$$\left(\alpha^k h \right)^4 \left(\frac{1}{\alpha^k} \right)^3 w_a^k (-\lambda^k) (\alpha^k)^2 \quad \left| \begin{array}{cc} A_{ij}^{kk} & \lambda(h_i^k + v_i(\alpha^k - v^k)) \\ C_j^{kk} & w_a^k (\rho_0 + \lambda) - \lambda(\alpha^k - v^k) \end{array} \right|$$

$$\begin{vmatrix} a_{ij} & v_j \\ c_j & x \end{vmatrix} = \det(a_{ij}) \cdot \left[x - \vec{c}^T (a_{ij})^{-1} \vec{v} \right]$$

$$A_{ij}^{kk} = \begin{bmatrix} h_{11}\Delta - B_1 E_1 & h_{12}\Delta - B_1 E_2 & h_{13}\Delta - B_1 E_3 \\ h_{21}\Delta - B_2 E_1 & h_{22}\Delta - B_2 E_2 & h_{23}\Delta - B_2 E_3 \\ h_{31}\Delta - B_3 E_1 & h_{32}\Delta - B_3 E_2 & h_{33}\Delta - B_3 E_3 \end{bmatrix}$$

```
In[5]:= a = {{h11*d - b1*e1, h12*d - b1*e2, h13*d - b1*e3},
           {h21*d - b2*e1, h22*d - b2*e2, h23*d - b2*e3},
           {h31*d - b3*e1, h32*d - b3*e2, h33*d - b3*e3}};
a // MatrixForm
Out[6]//MatrixForm=
\begin{pmatrix} -b1 e1 + d h11 & -b1 e2 + d h12 & -b1 e3 + d h13 \\ -b2 e1 + d h21 & -b2 e2 + d h22 & -b2 e3 + d h23 \\ -b3 e1 + d h31 & -b3 e2 + d h32 & -b3 e3 + d h33 \end{pmatrix}

In[8]:= Det[a] // Simplify
Out[8]= d^2 (b3 (e3 h12 h21 - e2 h13 h21 - e3 h11 h22 + e1 h13 h22 + e2 h11 h23 - e1 h12 h23) +
           b1 e3 h22 h31 - d h13 h22 h31 - b1 e2 h23 h31 + d h12 h23 h31 - b1 e3 h21 h32 + d h13 h21 h32 +
           b1 e1 h23 h32 - d h11 h23 h32 + b1 e2 h21 h33 - d h12 h21 h33 - b1 e1 h22 h33 + d h11 h22 h33 +
           b2 (-e3 h12 h31 + e2 h13 h31 + e3 h11 h32 - e1 h13 h32 - e2 h11 h33 + e1 h12 h33))
```

$$\begin{aligned} & \Delta^3 (h_{11}(h_{22}h_{33} - h_{23}h_{32}) + h_{12}(h_{23}h_{31} - h_{21}h_{33}) + h_{13}(h_{21}h_{32} - h_{22}h_{31})) \\ &= h \Delta^{kk} \\ &+ [B_1 E_1 (h_{23}h_{32} - h_{22}h_{33}) + B_1 E_2 (h_{21}h_{33} - h_{23}h_{31}) + B_1 E_3 (h_{22}h_{31} - h_{21}h_{32}) \\ &+ B_2 E_1 (h_{12}h_{33} - h_{13}h_{32}) + B_2 E_2 (h_{13}h_{31} - h_{11}h_{33}) + B_2 E_3 (h_{11}h_{32} - h_{12}h_{31})] \end{aligned}$$

$$+ B_3 E_1 (h_{13} h_{22} - h_{12} h_{23}) + B_3 E_2 (h_{11} h_{23} - h_{13} h_{21}) + B_3 E_3 (h_{12} h_{21} - h_{11} h_{22}) \Big] \Delta^2$$

$C^{ij} B_i E_j$

$$C^{ij} = \begin{bmatrix} h_{23} h_{32} - h_{22} h_{33} & h_{21} h_{33} - h_{23} h_{31} & h_{22} h_{31} - h_{21} h_{32} \\ h_{12} h_{33} - h_{13} h_{32} & h_{13} h_{31} - h_{11} h_{33} & h_{11} h_{32} - h_{12} h_{31} \\ h_{13} h_{22} - h_{12} h_{23} & h_{11} h_{23} - h_{13} h_{21} & h_{12} h_{21} - h_{11} h_{22} \end{bmatrix}$$

$$h = h_{ij} C^{ij}$$

$$C^{ij} = h^i h^{ij}$$

$$\left| A_{ij}^{kk} \right| = h \cdot (\Delta^{kk})^3 - h B^j E_j^{kk} (\Delta^{kk})^2$$

$$= h \cdot (\Delta^{kk})^2 (\Delta^{kk} - B^j E_j^{kk})$$

$$\Delta^{kk} - B^j E_j^{kk} = A_{jj}^{kk} = (\alpha^k)^2 h_c \omega^2$$

Q = h_c \omega^2 + B^2

$$\Delta^{kk} = (\alpha^k)^2 Q - 2(B\omega) \alpha^k B^k - \frac{1}{\omega^2} B^k B^k$$

$$E_j^{kk} = (\alpha^k)^2 B_j - \alpha^k ((B\omega) h_j^k + \omega_j B^k) - \frac{1}{\omega^2} h_j^k B^k$$

$$\begin{aligned}
 & (\alpha^k)^2 h_e w^2 + (\alpha^k)^2 B - 2(B\alpha^k)_{ik} B^k - \frac{1}{w^2} B^k B^k \\
 & - [(B\alpha^k)^2 - \alpha^k (B_{ik} B^k + B_i B^k) - B^{kk}] \\
 & = (\alpha^k)^2 h_e w^2
 \end{aligned}$$

$$|A_{ij}^{kk}| = h(\Delta^{kk})^2 h_e w^2 (\alpha^k)^2$$

$$(A^{-1})^{ij} = X h^{ij} + Y B^i E^j$$

$$\begin{aligned}
 A_{ij} (A^{-1})^{ij} &= (h_{ij} \Delta - B_{ij} E_j) (X h^{ij} + Y B^i E^j) \\
 &= X(\Delta - (BE)) + Y(BE)(\Delta - BE)
 \end{aligned}$$

In order to equal unity, $X = \frac{1}{\Delta}$, $Y = \frac{1}{\Delta(\Delta - BE)}$

$$(A_{ij}^{kk})^{-1} = \frac{1}{\Delta^{kk}} \left(h^{ij} + \frac{B^i E^j}{h_e w^2 (\alpha^k)^2} \right)$$

$$|\mathcal{F}^k - \lambda^k \mathcal{F}^0| = \\ (\alpha^2 h)^3 \left(\frac{1}{\alpha^k} \right)^3 W^k (-\lambda^k) (\alpha^k)^2 \quad \begin{vmatrix} A_{ij}^{kk} & \lambda(h_i^k + v_i(\alpha^k - v^k)) \\ C_j^{kk} & W^k (\rho_0 + \lambda) - \lambda(\alpha^k - v^k) \end{vmatrix}$$

$$\begin{vmatrix} a_{ij} & v_j \\ c_j & x \end{vmatrix} = \det(a_{ij}) \cdot \left[x - \bar{c}^T (a_{ij})^{-1} \bar{v} \right]$$

$$h(\Delta^{kk})^2 h_e W^k (\alpha^k)^2 \cdot \left[W^k (\rho_0 + \lambda) - \lambda(\alpha^k - v^k) - C_i^{kk} (A_{ij}^{kk})^{-1} \right. \\ \left. \cdot \lambda(h_i^k + v_i(\alpha^k - v^k)) \right]$$

$$|\mathcal{F}^k - \lambda^k \mathcal{F}^0| = \text{Pulling into main phrase} \\ - \alpha^2 h^5 \lambda^k h_e W^3 (\alpha^k)^2 (\Delta^{kk}) \left\{ (a^k)^2 (h_e W^2 + B^k) - 2(B_v) a^k B^k - \frac{1}{w^2} B^k B^k \right\} \cdot \\ \left[(W^k (\rho_0 + \lambda) - \lambda) a^k + \lambda v^k \right] - C_i^{kk} \left[h_{ij}^{ij} + \frac{B^i E^j}{h_e W^2 (\alpha^k)^2} \right] \cdot \\ \lambda \left(h_j^k + v_j (\alpha^k - v^k) \right) \\ \text{Should be } i's?$$

(2)

$$\left[(a^k)^2 (h_e W^2 + B^k) - 2(B_v) a^k B^k - \frac{1}{w^2} B^k B^k \right] \cdot \left[(W^k (\rho_0 + \lambda) - \lambda) a^k + \lambda v^k \right] \\ \cdot \left[W^k (\rho_0 + \lambda) - \lambda(\alpha^k - v^k) \right]$$

Add $(\alpha^k)^2$ to all these terms
 \downarrow

$$(a^k)^2(h_e W + B^k) \tilde{W}^k a^k (\rho_0 + \lambda) - (a^k)^2(h_e W + B^k) \lambda (a^k - v^k)$$

$$-2(B_v)(a^k)^2 B^k \tilde{W}^k (\rho_0 + \lambda) + 2(B_v)a^k B^k \lambda (a^k - v^k)$$

$$-\frac{1}{\omega^2} B^k B^k \tilde{W}^k a^k (\rho_0 + \lambda) + \frac{1}{\omega^2} B^k B^k \lambda (a^k - v^k)$$

Try $v^2 = 1 - \frac{1}{\omega^2}$

(B)

Attempt 1



$$C_j^{kk} = a^k a^k \left[W^2 \left(W_{v,j} + \left(h_e W_{v,j}^2 + B_{v,j}^2 \right) \mu - (B_v) B_j \right) + \frac{\rho_0}{2} W^2 \left(h_e W_{c,j}^2 v_j^2 + B_{v,j}^2 - (B_v) B_j \right) \right]$$

$$+ a^k \left[h_j^k W^2 (B_v)^2 - W^2 (B_v) v_j B^k + \frac{\rho_0}{2} \left((B^2 + W^2 (B_v)^2 + h_e W^2 c_j^2) h_j^k - B^k (W^2 (B_v) v_j + B_j) \right) \right]$$

$$- B^k \left[h_j^k (B_v) - v_j B^k \right] \zeta$$

$$E^{j k l c} = \left[(a^k)^2 B^j - a^k \left((B_v) h^{j k c} + v^j B^k \right) - \frac{1}{\omega^2} h^{j k} B^k \right]$$

$$\det \begin{bmatrix} \tilde{J}^k - \lambda^k \tilde{J}^0 \\ \vdots \end{bmatrix} = -\alpha^8 h^5 \lambda^k h_e W^3 (a^k)^2 \Delta^{kk}$$

$$\left\{ \begin{array}{l} \left[(a^k) (h_e W^2 + B^2) - 2 a^k (B_v) B^k - \frac{B^k}{\omega^2} \right] \left[(W^2 (\rho_0 + \lambda) - \lambda) a^k + \lambda v^k \right] \\ - C_j^{kk} \left[h^{jk} + \frac{B^j E^k}{h_e W^2 (a^k)^2} \right] \left(h_i^k + \nu_i^k (a^k - v^k) \right) \end{array} \right\}$$

$$= C_j^{kk} \left[h^{jk} + v^j (a^k - v^k) + \frac{1}{h_e W^2 (a^k)^2} \left(B^k + B_v \cdot (a^k - v^k) \right) \right]$$

$$= \lambda \cdot C_j^{kk} \left(h^{jk} + v^j (a^k - v^k) \right) + \underbrace{\frac{\lambda}{h_e W^2 (a^k)^2} \left(C_j^{kk} E^k \right) \left(B^k + B_v \cdot (a^k - v^k) \right)}_{= 0}$$

$$= \cancel{h} \cdot \underbrace{\left(h^{jk} + v^i (a^k - v^k) \right)}_{\substack{(1) \\ h^{jk}}} + \cancel{\frac{v}{h_e w^2(a^i)^2}} \left(C_j^{kk} E^i \right) \left(B^k + B v (a^k - v^k) \right)$$

① $h^{jk} \left\{ \begin{array}{l} \cancel{h^{jk} \left[\cancel{w^2 (W_r e + (h_e w^2 v + B v)^2)} - (B v) B^k \right]} \\ \cancel{+ a^k \left[h^k w^2 (B v)^2 - w^2 (B v) v^k B^k + \frac{P_0}{h_e} w^2 (h_e w^2 c_s^2 v_s + B^2 v_s - (B v) B_s) \right]} \\ \cancel{- B^k \left[h^k (B v) - v^k B^k \right]} \end{array} \right\}_{\substack{h^{jk} (B v) - v^k B^k}}$

$$\begin{aligned} & (1) \left\{ \begin{array}{l} h^{jk} \left[w^2 (W_r e + (h_e w^2 v + B v)^2) \mu - (B v) B^k + \frac{P_0}{h_e} w^2 (h_e w^2 c_s^2 v + B^2 v - (B v) B^k) \right] \\ + a^k \left[h^{ik} w^2 (B v)^2 - w^2 (B v) v^k B^k + \frac{P_0}{h_e} ((B^2 + w^2 (B v)^2 + h_e w^2 c_s^2) h^k - B^k (w^2 (B v) v_s + B_s)) \right] \\ - B^k \left[h^k (B v) - v^k B^k \right] \end{array} \right\} \end{aligned}$$

Multiply full second half by -1?

Note: # of clusters per power of a^k EXCLUDING a^{k+j}

- DO NOT forget $(a^k - v^k)$ in terms outside of this

$$\begin{aligned} & \begin{array}{l} E^{jk} = E_{jk} h^{jk} = B^k ((a^k)^2 B^j - a^k ((B v) h^{jk} + v^k B^j) - \frac{1}{2} h^{jk} B^m) \\ \text{With } (a^k - v^k) \\ (a^k)^2 \rightarrow (1, 2, 3, 1) I + II \\ (a^k)^3 \rightarrow (1, 2, 3, 1) I + II, (1, 2, 3, 2, 1) I + II \\ (a^k)^4 \rightarrow (1, 2, 3, 1) I, (1, 2, 3, 2, 1) I + II \\ (a^k)^5 \rightarrow (2, 1, 1, 1, 1) I + II, (2, 1, 1, 1, 1) II \\ (a^k)^6 \rightarrow (2, 1, 1, 1, 1, 1) I + II \\ (a^k)^7 \rightarrow (2, 1, 1, 1, 1, 1, 1) I + II \\ a^k \rightarrow (2, 1, 1, 1, 1, 1, 1) \end{array} \\ & \begin{array}{l} (1) \\ h^{jk} \left(h^{jk} + v^i (a^k - v^k) \right) + \cancel{\frac{v}{h_e w^2(a^i)^2}} \left(C_j^{kk} E^i \right) \left(B^k + B v (a^k - v^k) \right) \end{array} \\ & (2) \left\{ \begin{array}{l} \cancel{h^{jk} \left[w^2 (h_e w^2 v^2 - B^2 v^2) - (B v)^2 \right]} \\ \cancel{+ a^k \left[h^k w^2 (B v)^2 - w^2 (B v) v^k B^k + \frac{P_0}{h_e} ((B^2 + w^2 (B v)^2 + h_e w^2 c_s^2) h^k - B^k (w^2 (B v) v_s + B_s)) \right]} \\ \cancel{- B^k \left[h^k (B v) - v^k B^k \right]} \end{array} \right\}_{\substack{(2) I \\ \cancel{h^{jk} (B v) - v^k B^k}}} \end{aligned}$$

Not Same

Don't cancel, $\rightarrow v^k (B v)$

(2) I

(3) I $\left\{ \begin{array}{l} w^2 (B v) h^{jk} B^j - B^j (B v) h^{jk} B^j \\ + a^k \left[h^k w^2 (B v)^2 - w^2 (B v) v^k B^k + \frac{P_0}{h_e} ((B^2 + w^2 (B v)^2 + h_e w^2 c_s^2) h^k - B^k (w^2 (B v) v_s + B_s)) \right] \\ - B^k \left[h^k (B v) - v^k B^k \right] \end{array} \right\}_{\substack{(3) II \\ B^k (B v) - B^k B^k}}$

(3) II

$\square \frac{h^{jk}}{h_e w^2} \left[-v^k (B^k)^2 + (1^{**} - (a^k - v^k)^2) B^k B^k + (B v)^2 (a^k - v^k)^2 h^{jk} \right]$

$$\cancel{h} (1) + (2) + (3.1) I + (3.1) II + (1.1) I + (1.1) II + (2.1) I + (2.1) II + (1.2) I + (1.2) II$$

②

$$(a^k - v^k) \{ a^k \left[w^2 \left(W_{Br}^2 + (h_e w^2 Br^2 + B^2 Br^2) \mu - (Br)^2 \right) \right. \right. \\ \left. \left. + \frac{P_0}{2} w^4 c_s^2 h_e v^2 \right] \right. \\ \left. + a^k \left[v^k w^2 (Br)^2 + \frac{P_0}{2} ((B^2 + w^2 (Br)^2 + h_e w^2 c_s^2) v^k - B^k Br) \right] \right. \\ \left. - B^k [v^k (Br) - v^2 B^k] \right\}$$

③.1 I

$$B^k (a^k)^2 \left\{ (a^k)^2 \left[w^2 \left(W(Br)_E + (h_e w^2 (Br) + B^2 (Br)) \mu - (Br)^2 \right) \right. \right. \\ \left. \left. + \frac{P_0}{2} w^4 h_e c_s^2 (Br) \right] \right. \\ \left. + a^k \left[\frac{P_0}{2} h_e w^2 c_s^2 B^k \right] \right\}$$

③.1 II

$$Br(a^k - v^k) (a^k)^2 \left\{ (a^k)^2 \left[w^2 \left(W(Br)_E + (h_e w^2 (Br) + B^2 (Br)) \mu - (Br)^2 \right) \right. \right. \\ \left. \left. + \frac{P_0}{2} w^4 h_e c_s^2 (Br) \right] \right. \\ \left. + a^k \left[\frac{P_0}{2} h_e w^2 c_s^2 B^k \right] \right\}$$

(Σ.1) I

$$\begin{aligned} & - \alpha^k \left(B^k \right)^2 \left\{ \alpha^k \alpha^k \left[\omega^2 \left(W_{re}^2 + (h_e W_r^2 + B_r^2) \right) \mu - (B_r)^2 \right] \right. \\ & \quad \left. + \frac{\rho_0}{2} w^4 C_s^2 h_e v^2 \right\} \\ & + \alpha^k \left[v^k w^2 (B_r)^2 + \frac{\rho_0}{2} ((B^2 + w^2 (B_r)^2 + h_e w^2 C_s^2) v^k - B^k B_r) \right] \\ & - B^k \left[v^k (B_r) - v^2 B^k \right] \zeta \end{aligned}$$

(Σ.1) II

$$\begin{aligned} & - B_r (\alpha^k - v^k) \alpha^k B^k \left\{ \alpha^k \alpha^k \left[\omega^2 \left(W_{re}^2 + (h_e W_r^2 + B_r^2) \right) \mu - (B_r)^2 \right] \right. \\ & \quad \left. + \frac{\rho_0}{2} w^4 C_s^2 h_e v^2 \right\} \\ & + \alpha^k \left[v^k w^2 (B_r)^2 + \frac{\rho_0}{2} ((B^2 + w^2 (B_r)^2 + h_e w^2 C_s^2) v^k - B^k B_r) \right] \\ & - B^k \left[v^k (B_r) - v^2 B^k \right] \zeta \end{aligned}$$

(1.2) I

$$\begin{aligned} & - \frac{(B^k)^2}{w^2} \alpha^k \alpha^k \left\{ \omega^2 \left(W_{re}^k + (h_e W_r^k + B_r^k) \right) \mu - (B_r) B^k + \frac{\rho_0}{2} w^2 (h_e W_s^2 v^k + B_r^k - (B_r) B^k) \right\} \\ & + \alpha^k \left[h^{kk} w^2 (B_r)^2 - W^2 (B_r) v^k B^k + \frac{\rho_0}{2} ((B^2 + w^2 (B_r)^2 + h_e w^2 C_s^2) h^{kk} - B^k (W^2 (B_r) v^k + B^k)) \right] \\ & - B^{kk} \left[h^{kk} (B_r) - v^k B^k \right] \zeta \end{aligned}$$

(1.2) II

$$\begin{aligned}
 & - (B_r)_{(a^k - v^k)} \frac{B^k}{w^2} \left\{ a^{kc} a^{lc} \left[w^2 (W_{vr}^k + (h_e W_{vr}^{z^k} + B_{vr}^{z^k}) \mu - (B_r) B^k + \frac{P_0}{g} w (h_e W_{zs}^{zz^k} + B_{zs}^{zz^k} - (B_r) B^k) \right] \right. \\
 & \quad + a^{lc} \left[h^{lck} w^2 (B_r)^2 - W^2 (B_r) v^k B^k + \frac{P_0}{g} ((B^k + w^2 (B_r)^2 + h_e W_{zs}^{zz^2}) h^{kk} - B^k (W^2 (B_r) v^k + B^k)) \right] \\
 & \quad \left. - B^{lc} [h^{kk} (B_r) - v^k B^k] \right\}
 \end{aligned}$$

(1.1) I

$$\begin{aligned}
 & - a^{lc} B_r B^k \left\{ a^{kc} a^{lc} \left[w^2 (W_{vr}^k + (h_e W_{vr}^{z^k} + B_{vr}^{z^k}) \mu - (B_r) B^k + \frac{P_0}{g} w (h_e W_{zs}^{zz^k} + B_{zs}^{zz^k} - (B_r) B^k) \right] \right. \\
 & \quad + a^{lc} \left[h^{lck} w^2 (B_r)^2 - W^2 (B_r) v^k B^k + \frac{P_0}{g} ((B^k + w^2 (B_r)^2 + h_e W_{zs}^{zz^2}) h^{kk} - B^k (W^2 (B_r) v^k + B^k)) \right] \\
 & \quad \left. - B^{lc} [h^{kk} (B_r) - v^k B^k] \right\}
 \end{aligned}$$

(1.1) II

$$\begin{aligned}
 & -(B_r)^2_{(a^k - v^k)} a^{lk} \left\{ a^{kc} a^{lc} \left[w^2 (W_{vr}^k + (h_e W_{vr}^{z^k} + B_{vr}^{z^k}) \mu - (B_r) B^k + \frac{P_0}{g} w (h_e W_{zs}^{zz^k} + B_{zs}^{zz^k} - (B_r) B^k) \right] \right. \\
 & \quad + a^{lc} \left[h^{lck} w^2 (B_r)^2 - W^2 (B_r) v^k B^k + \frac{P_0}{g} ((B^k + w^2 (B_r)^2 + h_e W_{zs}^{zz^2}) h^{kk} - B^k (W^2 (B_r) v^k + B^k)) \right] \\
 & \quad \left. - B^{lc} [h^{kk} (B_r) - v^k B^k] \right\}
 \end{aligned}$$

$$- C_i^{kk} \left[h^{ij} + \frac{B^i E^j}{h_e w^2 (a^k)^2} \right] \cdot \lambda \left(h_j^k + v_j^k (a^k - v^k) \right)$$

$$- \left[C_i^{kk} h^{ij} + \frac{C_i^{kk} B^i E^j}{h_e w^2 (a^k)^2} \right] \cdot \lambda \left(h_j^k + v_j^k (a^k - v^k) \right)$$

$$- \lambda \left[C_i^{kk} h^{ik} + C_i^{kk} v_i^k (a^k - v^k) + \frac{C_i^{kk} B^i}{h_e w^2 (a^k)^2} [E^k + E^j v_j^k (a^k - v^k)] \right]$$

$$\begin{aligned} C_i^{kk} &= a^k a^k \left[w^2 (w_{v_i E} + (h_e w_{v_i}^2 + B_{v_i}) B_i) + \frac{p_0}{2} w^2 (h_e w_{c_s}^2 v_i^k + B_{v_i}^2 - (B_{v_i}) B_i) \right] \\ &\quad + a^k \left[h_i^k w^2 (B_{v_i})^2 - w^2 (B_{v_i}) v_i^k B^k + \frac{p_0}{2} ((B^2 + w^2 (B_{v_i}))^2 + h_e w^2 c_s^2) h_i^k - B^k (w^2 (B_{v_i}) v_i^k + B_i) \right] \end{aligned}$$

$$- B^{ik} [h_i^k (B_{v_i}) - v_i^k B^k] \zeta$$

$$E^{jkic} = \left[(a^k)^2 B^j - a^k ((B_{v_i}) h^{jic} + v^j B^k) - \frac{1}{w^2} h^{jic} B^k \right]$$

$$C_i^{kk} h^{ik}$$

$$\begin{aligned} &= a^k a^k \left[w^2 (w_{v_i E} + (h_e w_{v_i}^2 + B_{v_i}) B^k) + \frac{p_0}{2} w^2 (h_e w_{c_s}^2 v_i^k + B_{v_i}^2 - (B_{v_i}) B^k) \right] \\ &\quad + a^k \left[h^{kk} w^2 (B_{v_i})^2 - w^2 (B_{v_i}) v_i^k B^k + \frac{p_0}{2} ((B^2 + w^2 (B_{v_i}))^2 + h_e w^2 c_s^2) h^{kk} - B^k (w^2 (B_{v_i}) v_i^k + B^k) \right] \end{aligned}$$

$$- B^{ik} [h^{kk} (B_{v_i}) - v_i^k B^k] \zeta$$

A

$$C_i^{(k)} =$$

$$\begin{aligned} & v^i \left\{ a^k a^k \left[w^2 (w_{v_i t} + (h_e w_{v_i}^2 + B_{v_i}) \mu - (B_{v_i}) B_i) + \frac{p_0}{2} w^2 (h_e w_{c_s}^2 v_i + B_{v_i}^2 - (B_{v_i}) B_i) \right] \right. \\ & + a^k \left[h_i^k w^2 (B_{v_i})^2 - w^2 (B_{v_i}) v_i B^k + \frac{p_0}{2} ((B^2 + w^2 (B_{v_i})^2 + h_e w^2 c_s^2) h_i^k - B^k (w^2 (B_{v_i}) v_i + B_i)) \right] \\ & \left. - B^{jk} [h_i^k (B_{v_i}) - v_i B^k] \right\} \end{aligned}$$

$$\begin{aligned} & = a^k a^k \left[w^2 (w_{v_i t}^2 + (h_e w_{v_i}^2 + B_{v_i}) \mu - (B_{v_i})^2) + \frac{p_0}{2} w^2 (h_e w_{c_s}^2 v_i^2 + B_{v_i}^2 - (B_{v_i})^2) \right] \\ & + a^k \left[v^k w^2 (B_{v_i})^2 - w^2 (B_{v_i}) v_i^2 B^k + \frac{p_0}{2} ((B^2 + w^2 (B_{v_i})^2 + h_e w^2 c_s^2) v_i^k - B^k (w^2 (B_{v_i}) v_i^2 + B_i)) \right] \\ & - B^{jk} [v^k (B_{v_i}) - v_i^2 B^k] \end{aligned}$$

$$\begin{aligned} & w^2 v^2 = w^2 - \\ & - B^k ((B_{v_i}) w^2 - (B_i) + (B_i)) \\ & - B^k (B_{v_i}) w^2 \end{aligned}$$

$$③ C_i^{(k)k} B_i =$$

$$\begin{aligned} & B^i \left\{ a^k a^k \left[w^2 (w_{v_i t} + (h_e w_{v_i}^2 + B_{v_i}) \mu - (B_{v_i}) B_i) + \frac{p_0}{2} w^2 (h_e w_{c_s}^2 v_i + B_{v_i}^2 - (B_{v_i}) B_i) \right] \right. \\ & + a^k \left[h_i^k w^2 (B_{v_i})^2 - w^2 (B_{v_i}) v_i B^k + \frac{p_0}{2} ((B^2 + w^2 (B_{v_i})^2 + h_e w^2 c_s^2) h_i^k - B^k (w^2 (B_{v_i}) v_i + B_i)) \right] \\ & \left. - B^{jk} [h_i^k (B_{v_i}) - v_i B^k] \right\} \end{aligned}$$

$$\begin{aligned} & = a^k a^k \left[w^2 (w_{v_i t} + (h_e w_{v_i}^2 + B_{v_i}) \mu - (B_{v_i}) B^2) + \frac{p_0}{2} w^2 (h_e w_{c_s}^2 B_i + B_{v_i}^2 - (B_{v_i}) B^2) \right] \\ & + a^k \left[\cancel{B^k w^2 (B_{v_i})^2} - \cancel{w^2 (B_{v_i})^2} B^k + \frac{p_0}{2} ((\cancel{B^2} + w^2 (B_{v_i})^2 + h_e w^2 c_s^2) B^k - B^k (w^2 (B_{v_i})^2 + \cancel{B^2})) \right] \\ & - B^{jk} [\cancel{B^k (B_{v_i})} - (\cancel{B} \cancel{B})^k] \end{aligned}$$

$$= \alpha^k \left[W^2 (W(B_r) \epsilon + (h_e W^2 B_r + B^2)) - (B_{rr}) B^2 \right] + \frac{\rho_0}{2} W^2 (h_e W^2 C_s^2 B_r + B^2(B_r) - (B_{rr}) B^2) \\ + \alpha^k \left[\frac{\rho_0}{2} h_e W^2 C_s^2 B^2 \right]$$

See Mathematica Notebook

Ryan's Characteristic Equation:

$$-\alpha^8 h^5 \rho_0 h_e W^3 \lambda^k a^k \Delta^{kk} \left\{ h_e W^4 (1 - C_s^2) (a^k)^4 \right. \\ \left. + \left[(a^k)^2 (h_e W^2 + B^2 + W^2 (B_r)^2) - C_s^2 \left(a^k W(B_r) + \frac{B^k}{W^2} \right)^2 \right] \cdot [(a^k - v^k)^2 - h^{kk}] \right\}$$

Our Characteristic Equation

$$-\alpha^8 h^5 \rho_0 h_e W^3 \lambda^k a^k \Delta^{kk} \left\{ h_e W^4 (a^k)^4 \right. \\ \left. + \left[(a^k)^2 (B^2 + W^2 (B_r)^2) - C_s^2 \left(a^k W(B_r) + \frac{B^k}{W^2} \right)^2 \right] \cdot [(a^k - v^k)^2 - h^{kk}] \right\} \\ - C_s^2 \left((a^k)^2 W^2 h_e ((W^2 - 1)(a^k)^2 + 2a^k v^k + h^{kk} - (v^k)^2) \right) \\ - C_s^2 (a^k)^2 h_e (W^2 (a^k)^2 - a^k v^k + 2a^k v^k - (v^k)^2 + h^{kk})$$

$$\begin{aligned}
 & - (\omega^k)^2 - 2\alpha k \omega^k + (\omega^k)^2 \\
 & - \alpha \left(\omega^2(a^k)^2 - (a^k - v^k)^2 + h^{kk} \right) \\
 & - \alpha \omega^2(a^k)^2 + \alpha ((a^k - v^k)^2 - h^{kk}) \\
 & - \underbrace{\zeta_s^2 (a^k)^4}_{c_s^2} \omega^4 h_e + h_e \omega^2 (a^k)^2 c_s^2
 \end{aligned}$$

Our Final Characteristic Equation

$$\begin{aligned}
 & -\alpha^8 h^5 \rho h_e \omega^3 \lambda^k a^k \Delta^{kk} \left\{ h_e \omega^4 (a^k)^4 (1 - c_s^2) \right. \\
 & \left. + \left[(a^k)^2 \left(h_e \omega^2 \zeta_s^2 + \beta^2 + \omega^2 (\beta v)^2 \right) - \zeta_s^2 \left(a^k \omega (\beta v) + \frac{\beta^k}{\omega} \right)^2 \right] \cdot \left[(a^k - v^k)^2 - h^{kk} \right] \right\}
 \end{aligned}$$

Interlude Pg. 27

$$(4.1) \quad e^{\sigma} \omega_{a^k} + (\nu e) \nabla_{P_0}^3 a^k + e^k w_{P_0} = 0$$

$$\begin{aligned}
 (4.3) \quad 0 &= e^{\sigma} [(\omega^z \gamma - \omega \chi) a^k + \chi v^k] \\
 &+ \left\{ (\nu e) [(2h_e \omega^4 + B^2 - \omega_{P_0}^3) a^k - B^2 v^k] - (B_e) [(Bv)(a^k - v^k) + B^k] \right. \\
 &+ e^k (Q - w_{P_0}) \} \\
 &+ e^4 [(P_0 + \lambda) \omega^z a^k + \lambda (v^k - a^k)] \\
 &+ \left\{ - (v \hat{e}) [(Bv)(a^k - v^k) + B^k] + (B \hat{e}) \left[(2 - \frac{1}{\omega^2}) a^k + \frac{1}{\omega^2} v^k \right] \right. \\
 &\left. - \hat{e}^k (Bv) \right\}
 \end{aligned}$$

4.1 + 4.3

$$\begin{aligned}
 (4.7) \quad 0 &= e^{\sigma} [(\omega^z \gamma - \chi) a^k + \chi v^k] \\
 &+ \left\{ (\nu e) [(2h_e \omega^4 + B^2) a^k - B^2 v^k] - (B_e) [(Bv)(a^k - v^k) + B^k] \right. \\
 &+ e^k Q \} \\
 &+ e^4 [(P_0 + \lambda) \omega^z a^k + \lambda (v^k - a^k)]
 \end{aligned}$$

$$+ \left\{ -(\nu \hat{e}) [(\bar{B}_v)(a^k v^k) + B^k] + (\bar{B} \hat{e}) \left[\left(z - \frac{1}{\omega^2} \right)^k + \frac{1}{\omega^2} v^k \right] \right. \\ \left. - \hat{e}^k (\bar{B}_v) \right\}$$

$$-\sqrt{i} \cdot 4.7 + 4.2$$

$$(4.2)$$

$$0 = e^0 \left[\cancel{\omega^2} \cancel{v_i a^k} + \chi h_i^k \right] \\ + \left\{ e^i \left[Q a^k - (\bar{B}_v) B^k \right] + (\nu e) \left[2 h_e \cancel{\omega^4} \cancel{v_i a^k} - h_i^k \cancel{B^2} + 2 B_i B^k \right] \right. \\ + (\bar{B} \hat{e}) \left[-B_i a^k + h_i^k (\bar{B}_v) - \cancel{v_i B^k} \right] + \hat{e}^k \left[\cancel{Q} \cancel{v_i} - (\bar{B}_v) B_i \right] \} \\ + e^4 \left[\cancel{(P_0 + 2)} \cancel{\omega^2} \cancel{v_i a^k} + h_i^k \cancel{\chi} \right] \\ + \left\{ \hat{e}_i \left[-(\bar{B}_v) a^k - \frac{1}{\omega^2} B^k \right] + (\nu \hat{e}) \left[-B_i a^k - \cancel{v_i B^k} + h_i^k (\bar{B}_v) \right] \right. \\ \left. + (\bar{B} \hat{e}) \left[\cancel{z} \cancel{a^k} + \frac{1}{\omega^2} h_i^k \right] - \hat{e}^k \left[\cancel{(\bar{B}_v) v_i} + \frac{1}{\omega^2} B_i \right] \right\}$$

$$0 = -\nu \left[e^0 \left[\cancel{\omega^2} \cancel{-\chi} a^k + \chi v^k \right] \right. \\ \left. + \left\{ (\nu e) \left[\cancel{2 h_e \omega^4} + \cancel{B^2} \right] a^k - \cancel{B^2} v^k \right] - (\bar{B} \hat{e}) \left[(\bar{B}_v)(a^k - v^k) + \cancel{B^k} \right] \right. \\ \left. + \cancel{e^k Q} \right\} \\ + e^4 \left[\cancel{(P_0 + 2)} \cancel{\omega^2} \cancel{a^k} + \cancel{\chi} (v^k - a^k) \right] \\ + \left\{ -(\nu \hat{e}) \left[(\bar{B}_v)(a^k - v^k) + \cancel{B^k} \right] + (\bar{B} \hat{e}) \left[\cancel{z} - \frac{1}{\omega^2} a^k + \frac{1}{\omega^2} v^k \right] \right\}$$

$$- \hat{e}^k / (B_r) \}$$

$$\begin{aligned}
0 = & e^0 \left[\chi_{h_i^k} \right] \\
& + \left\{ e^i \left[Q a^k - (B_r) B^k \right] + (\nu e) \left[-h_i^k B^2 + 2 B_i B^k \right] \right. \\
& + (B_e) \left[-B_i a^k + h_i^k (B_r) \right] - e^k \\
& + e^4 \left[h_i^k \chi \right] \\
& + \left\{ \hat{e}_i \left[-(B_r) a^k - \frac{1}{\omega^2} B^k \right] + (\nu \hat{e}) \left[-B_i a^k + h_i^k (B_r) \right] \right. \\
& \left. + (B \hat{e}) \left[-\frac{1}{\omega^2} h_i^k \right] - \hat{e}^k \left[\frac{1}{\omega^2} B_i \right] \right\}
\end{aligned}$$

$$\begin{aligned}
0 = & -v_i \left[e^0 \left[(-\chi) a^k + \chi v^k \right] \right. \\
& + \left\{ (\nu e) \left[(+ B^2) a^k - B^2 v^k \right] - (B_e) \left[(B_r)(a^k - v^k) \right] \right. \\
& \left. \left. \right\} \right. \\
& + e^4 \left[\chi (v^k - a^k) \right] \\
& + \left\{ -(\nu \hat{e}) \left[(B_r)(a^k - v^k) \right] + (B \hat{e}) \left[-\frac{1}{\omega^2} a^k + \frac{1}{\omega^2} v^k \right] \right.
\end{aligned}$$

$$\begin{aligned}
 (4.8) \quad O &= e^0 \left[-\chi ((v^k - a^k)_{v_i} - h_i^k) \right] \\
 &+ \left\{ e^i \left[Q_a^k - (B_v)B^k \right] + (v_c) \left[2B_i B^k + B^2 ((v^k - a^k)_{v_i} - h_i^k) \right] \right. \\
 &+ (B_e) \left[-B_i a^k + (B_v) ((a^k - v^k)_{v_i} + h_i^k) \right] - e^k (B_v) B_i \Big\} \\
 &+ e^4 \left[-\chi ((v^k - a^k)_{v_i} - h_i^k) \right] \\
 &+ \left\{ \hat{e}_i \left[-(B_v)_a^k - \frac{1}{\omega^2} B^k \right] + (v_c^i) \left[-(B_v) ((v^k - a^k)_{v_i} - h_i^k) - \cancel{B_i a^k} \right] \right. \\
 &+ (B_e^i) \left[-\frac{1}{\omega^2} ((v^k - a^k)_{v_i} - h_i^k) \right] - \cancel{\frac{\omega^2}{\omega^2} B_i} \Big\}
 \end{aligned}$$

4.4 contract w/ B_i

$$\begin{aligned}
 (4.9) \quad O &= (B^i e^k - B^k e^i + \hat{e}_i a^k) B_i \\
 O &= B^k e^k - (B_e) B^k + (B_e^i) a^k
 \end{aligned}$$

4.4 contract w/ v_i

$$\begin{aligned}
 (4.10) \quad O &= (B^i e^k - B^k e^i + \hat{e}_i a^k) v_i \\
 O &= (B_v)_c^k - (v_e) B^k + (v_c^i) a^k
 \end{aligned}$$

Use above to simplify

$$\begin{aligned}
D &= (\epsilon^0 \chi + c^4) \alpha \left((a^k - v^k)_{v_i} + h_i^k \right) \\
&+ \left\{ \hat{e}_i^i \left[Q a^k - (B_v) B^k \right] + (v_c) \left[\cancel{\alpha} B_i B^k + B^2 ((v^k - a^k)_{v_i} - h_i^k) \right] \right. \\
&+ (B_e) \left[-B_i a^k + (B_v) ((a^k - v^k)_{v_i} + h_i^k) \right] - \cancel{e^k} (B_v) B_i^k \} \\
&+ \left\{ \hat{e}_i^i \left[-(B_v) a^k - \frac{1}{\omega^2} B^k \right] + (v_c) \left[-(B_v) ((v^k - a^k)_{v_i} - h_i^k) - \cancel{B a^k} \right] \right. \\
&+ (B_e) \left[-\frac{1}{\omega^2} ((v^k - a^k)_{v_i} - h_i^k) \right] \}
\end{aligned}$$

$$D = (B_v)_c^k - (v_e) B^k + (v_c)_a^k$$

$$\begin{aligned}
(B_v)_c^k &= (v_e) B^k - (v_c)_a^k \\
-e^k (B_v) B_i &= - (v_e) B^k B_i + (v_c)_a^k B_i
\end{aligned}$$

$$\begin{aligned}
^{(4.11)} D &= (\epsilon^0 \chi + c^4) \alpha \left((a^k - v^k)_{v_i} + h_i^k \right) \\
&+ \left\{ \hat{e}_i^i \left[Q a^k - (B_v) B^k \right] + (v_c) \left[B_i B^k + B^2 ((v^k - a^k)_{v_i} - h_i^k) \right] \right. \\
&+ (B_e) \left[-B_i a^k + (B_v) ((a^k - v^k)_{v_i} + h_i^k) \right] \} \\
&+ \left\{ \hat{e}_i^i \left[-(B_v) a^k - \frac{1}{\omega^2} B^k \right] + (v_c) \left[-(B_v) ((v^k - a^k)_{v_i} - h_i^k) \right] \right. \\
&+ (B_e) \left[-\frac{1}{\omega^2} ((v^k - a^k)_{v_i} - h_i^k) \right] \}
\end{aligned}$$

$$\begin{aligned}
 (4.7) \quad 0 &= e^{\omega} \left[(\omega^2 - \chi) a^k + \chi v^k \right] \\
 &+ \left\{ (\nu e) \left[(2h_e \omega^4 + \beta^2) a^k - \beta^2 v^k \right] - (\beta e) \left[(\beta v) (a^k - v^k) + \cancel{\beta^2} \right] \right. \\
 &\quad \left. + e^k (h_e \omega^2 + \cancel{\beta^2}) \right\} \\
 &+ e^4 \left[(\rho_0 + \lambda) \omega^2 a^k + \lambda (v^k - a^k) \right] \\
 &+ \left\{ -(\nu e) \left[(\beta v) (a^k - v^k) + \beta^2 \right] + (\beta e) \left[\left(2 - \frac{1}{\omega^2} \right) a^k + \frac{1}{\omega^2} v^k \right] \right. \\
 &\quad \left. - \cancel{\lambda^2} (\beta v) \right\} \\
 &\quad a^k - \frac{a^k}{\omega^2} + \frac{1}{\omega^2} v^k \\
 &\quad \frac{(\beta e)}{\omega^2} \left[\omega^2 a^k - a^k + v^k \right] \\
 0 &= \beta^2 e^k - (\beta e) \beta^k + (\beta e) a^k
 \end{aligned}$$

$$\begin{aligned}
 (4.12) \quad 0 &= e^{\omega} \left[(\omega^2 - \chi) a^k + \chi v^k \right] \\
 &+ \left\{ (\nu e) \left[(2h_e \omega^4 + \beta^2) a^k - \beta^2 v^k \right] + (\beta e) \left[(\beta v) (v^k - a^k) \right] \right\} \\
 &\quad + e^k (h_e \omega^2) \} \\
 &+ e^4 \left[(\rho_0 + \lambda) \omega^2 a^k + \lambda (v^k - a^k) \right] \\
 &+ \left\{ (\nu e) \left[(\beta v) (v^k - a^k) - \beta^2 \right] + \frac{(\beta e)}{\omega^2} \left[\omega^2 a^k - a^k + v^k \right] \right\}
 \end{aligned}$$

4.11 Contract w/ $v_i \cdot \omega^2$

$$1 - \frac{1}{\omega^2}$$

$$k - k \propto v^k \dots k$$

$$(4.11) \quad \dot{O} = v_i \left[(e^o \chi + e^u \lambda) \left((a^k - v^k) \underbrace{v_i^z + h_i^k}_{1 - \frac{1}{\omega^2}} \right) \right] \xrightarrow{\begin{array}{l} a^r - \frac{a^i}{\omega^2} - x^r + \frac{v^i}{\omega^2} = 0 \\ \frac{1}{\omega^2} (w^z a^k - a^k + v^k) \end{array}}$$

$$\begin{aligned} &+ \left\{ \hat{e}_i^{(v_e)} \left[(h_e v^z + B^z) \underbrace{a^k}_{\cancel{*}} - (B_v) B^k \right] + (v_c) \left[\underbrace{B_i^z B^k}_{1 - \frac{1}{\omega^2}} + B^z ((v^k - a^k) \underbrace{v_i^z}_{1 - \frac{1}{\omega^2}} - \underbrace{h_i^k}_{\cancel{*}}) \right] \right. \\ &+ (B_e) \left[- \underbrace{B_i^z a^k}_{\cancel{*}} + (B_v) \left((a^k - v^k) \underbrace{v_i^z}_{1 - \frac{1}{\omega^2}} + \underbrace{h_i^k}_{\cancel{*}} \right) \right] \left. \right\} \\ &+ \left\{ \hat{e}_i^{(v_e)} \left[- (B_v) \underbrace{a^k}_{\cancel{*}} - \frac{1}{\omega^2} B^k \right] + (v_c) \left[- (B_v) ((v^k - a^k) \underbrace{v_i^z}_{1 - \frac{1}{\omega^2}} - \underbrace{h_i^k}_{\cancel{*}}) \right] \right. \\ &+ (B_e) \left[- \frac{1}{\omega^2} ((v^k - a^k) \underbrace{v_i^z}_{1 - \frac{1}{\omega^2}} - \underbrace{h_i^k}_{\cancel{*}}) \right] \left. \right\} \\ &- \left[(v^k - a^k) - \frac{1}{\omega^2} (v^k - a^k) - \cancel{v^k} \right] \end{aligned}$$

$$a^k \omega^2 + v^k - a^k$$

$$(4.13) \quad \dot{O} = (e^o \chi + e^u \lambda) (v^k - a^k + w^z a^k)$$

$$+ (v_e) [h_e w^z a^k - B^z (v^k - a^k)] + (B_e) [(B_v) (v^k - a^k)]$$

$$+ (v_e) [(B_v) (v^k - a^k) - \cancel{B^k}] + (B_e) \frac{1}{\omega^2} (v^k - a^k + w^z a^k)$$

$$4.11 \text{ contract w/ } B^i$$

$$(4.11) \quad O = (e^0 \chi + e^4 \varphi) ((a^k - v^k)_{v_i}^{(B_v)} + h_i^k)$$

$$\begin{aligned}
& + \left\{ e_i \left[Q_a^k - \underbrace{(B_v) B^k}_{(B_e)} \right] + (v_c) \left[\underbrace{\frac{B^k}{B_i}}_{(B_e)} + B^2 ((v^k - a^k)_{v_i} - h_i^k) \right] \right. \\
& + (B_e) \left[- \underbrace{B_i a^k}_{(B_e)} + (B_v) ((a^k - v^k)_{v_i} + h_i^k) \right] \} \\
& + \left\{ \hat{e}_i \left[-(B_v) a^k - \underbrace{\frac{1}{\omega^2} B^k}_{(B_e)} \right] + (v_c) \left[-(B_v) ((v^k - a^k)_{v_i} \cdot h_i^k) \right] \right. \\
& \left. + (B_e) \left[- \underbrace{\frac{1}{\omega^2} ((v^k - a^k)_{v_i} - h_i^k)}_{(B_e)} \right] \right\}
\end{aligned}$$

$$\begin{aligned}
(4.14) \quad O &= (e^0 \chi + e^4 \varphi) [B^k - (B_v)(v^k - a^k)] \\
& + (B_e) [h_c \omega^2 a^k - (B_v)^2 (v^k - a^k)] + (v_c) [B^2 (B_v)(v^k - a^k)] \\
& - (B_e) \frac{(B_v)}{\omega^2} [v^k - a^k + \omega^2 a^k] + (v_c) [(B_v) (B^k - (B_v)(v^k - a^k))]
\end{aligned}$$

4.14 + (B_v) 4.13

$$(4.14)$$

$$0 = (e^0 \chi + e^4 \lambda) [B^k - \underbrace{(Bv)(v^k - a^k)}_{\text{Red}}]$$

$$+ (Be) [h_e w^z a^k - \underbrace{(Bv)^2 (v^k - a^k)}_{\text{Red}}] + (ve) [\underbrace{\bar{B}^z (Bv)(v^k - a^k)}_{\text{Red}}]$$

$$- \underbrace{(Be) \frac{(Bv)}{w^z} [v^k - a^k + w^z a^k]}_{\text{Red}} + (ve) [(Bv) (B^k - \underbrace{(Bv)(v^k - a^k)}_{\text{Red}})]$$

$$+ B_v [(e^0 \chi + e^4 \lambda) (\underbrace{v^k - a^k + w^z a^k}_{\text{Red}})]$$

$$+ (ve) [h_e w^4 a^k - \underbrace{\bar{B}^z (v^k - a^k)}_{\text{Red}}] + (Be) [\underbrace{(Bv)(v^k - a^k)}_{\text{Red}}]$$

$$+ (ve) [\underbrace{(Bv)(v^k - a^k) - \bar{B}^k}_{\text{Red}}] + \underbrace{(Be) \frac{1}{w^z} (v^k - a^k + w^z a^k)}_{\text{Red}}$$

$$(4.15)$$

$$0 = (e^0 \chi + e^4 \lambda) (B^k + w^z a^k (Bv)) + (ve) h_e w^4 a^k (Bv) + (Be) h_e w^z a^k$$

4.12 - 4.13

$$0 = e^0 [(w^z \gamma - \chi) a^k + \cancel{\chi v^k}]$$

$$+ \left\{ (ve) \left[\underbrace{(2h_e w^4 + \bar{B}^z)}_{\text{Red}} \right] a^k - \cancel{\bar{B}^z v^k} \right\} + \underbrace{(Be) [(Bv)(v^k - a^k)]}_{\text{Red}}$$

$$+ e^k (h_e w^z) \}$$

$$+ e^4 \left[(P_0 + \lambda) \cancel{w^z a^k} + \cancel{\lambda (v^k - a^k)} \right]$$

$$\begin{aligned}
& + \left\{ \left(\hat{v}e \right) \left[\underbrace{(Bv)(v^k - a^k)}_{\text{red}} - B^k \right] + \frac{(Be)}{\omega^2} \left[\underbrace{w_a^k - a^k}_{\text{red}} + v^k \right] \right. \\
& \quad \left. - \left[(e^0 \chi + e^4 \lambda) (v^k - a^k + w_a^k) \right] \right. \\
& + (ve) \left[\underbrace{h_e w_a^k}_{\text{red}} - \underbrace{B^k (v^k - a^k)}_{\text{red}} \right] + \underbrace{(Be)}_{\text{red}} \left[(Bv)(v^k - a^k) \right] \\
& \left. + \left(\hat{v}e \right) \left[\underbrace{(Bv)(v^k - a^k)}_{\text{red}} - B^k \right] + (Be) \frac{1}{\omega^2} \left[\underbrace{v^k - a^k}_{\text{red}} + \underbrace{w_a^k}_{\text{red}} \right] \right]
\end{aligned}$$

$$\chi \cancel{e^0(v^k - a^k + w_a^k)} \quad e^4 \cancel{\lambda(v^k - a^k + w_a^k)}$$

$$D = e^0 \omega^2 a^k (\gamma - \chi) + e^4 w_a^k A + (ve) h_e w_a^k + e^k h_e w^2$$

$$\gamma = 1 + \epsilon + \chi$$

$$h_e = P_0 (1 + \epsilon) + P$$

$$D = e^0 \omega^2 a^k \left(\frac{h_e - P}{P_0} \right) + e^4 w_a^k A + (ve) h_e w_a^k + e^k h_e w^2$$

$$\text{This } \uparrow - \frac{h_e \omega}{P_0}, 4.1$$

$$\begin{aligned}
D &= e^0 \omega^2 a^k \left(\cancel{\frac{h_e - P}{P_0}} \right) + e^4 w_a^k A + (ve) \cancel{h_e w_a^k} + e^k \cancel{h_e w^2} \\
&\quad - \frac{h_e \omega}{P_0} \left(e^0 \cancel{a^k} + (ve) \cancel{\frac{P}{P_0} a^k} + e^k \cancel{w_a^k} \right)
\end{aligned}$$

$$O = \frac{e^0 \chi}{P_0} + e^4 \sqrt{\alpha P_0}$$

$$\frac{e^0 P}{P_0} = e^4 P_0$$

$$(4.16) \quad e^4 = \frac{e^0 P}{P_0^2}$$

$$\begin{aligned}
 O &= (e^0 \chi + e^4 \chi) (B^k + W^2 \alpha^k (B_{kr})) + (ve) h_e W^4 \alpha^k (B_{kr}) + (\beta_e) h_e W^2 \alpha^k \\
 &= \left(e^0 \chi + \frac{e^0 P \chi}{P_0^2} \right) \parallel \\
 &= e^0 \left(\chi + \frac{P \chi}{A^2} \right) \parallel \\
 &= \frac{e^0}{P_0^2} (\chi P_0^2 + P \chi) \parallel
 \end{aligned}$$

$$(4.17) \quad O = \frac{e^0}{P_0} h_e c_s^2 (B^k + W^2 \alpha^k (B_{kr})) + (ve) h_e W^4 \alpha^k (B_{kr}) + (\beta_e) h_e W^2 \alpha^k$$

$$\begin{aligned}
 (4.18) \quad O &= (e^0 \chi + e^4 \chi) (v^k - \alpha^k + W^2 \alpha^k) \\
 &+ (ve) [h_e W^4 \alpha^k - B^2 (v^k - \alpha^k)] + (\beta_e) [(B_{kr})(v^k - \alpha^k)] \\
 &+ (\hat{ve}) [(B_{kr})(v^k - \alpha^k) - B^k] + (\hat{\beta_e}) \frac{1}{\omega} (v^k - \alpha^k + W^2 \alpha^k)
 \end{aligned}$$

$$= \left(e^0 \chi + \frac{e^0 P \chi}{P_0^2} \right) \parallel$$

$$= e^0 \left(\chi + \frac{P \chi}{P_0^2} \right) \parallel$$

$$= \frac{e^0}{P_0^2} (\chi P_0^2 + P \chi) \parallel$$

$$\begin{aligned} O &= \frac{e^0}{P_0} h_e c_s^2 (v^k - a^k + \tilde{w}^k a^k) \\ &+ (v_e) \left[h_e w^k a^k - B^k (v^k - a^k) \right] + (B_e) \left[(B v) (v^k - a^k) \right] \\ &+ (v_e) \left[(B v) (v^k - a^k) - B^k \right] + (B_e) \frac{1}{\omega^2} (v^k - a^k + \tilde{w}^k a^k) \end{aligned}$$

$$\begin{aligned} &(v^k - a^k + \tilde{w}^k a^k) \\ &(B^k - (B v) (v^k - a^k)) \\ &(B^k + \tilde{w}^k a^k (B v)) \end{aligned}$$

$$(v^k - a^k + \tilde{w}^k a^k)$$

Help needed for

"Utilize 4.13 & 4.14"

$$0 = e \frac{0 h_e C_s^2}{B} (v^k - a^k + w_a^2 a^k) + (ve)(h_e w^4 a^k - c^k) \\ + (Be) D^k - e^k \underbrace{\frac{E^k}{a^k}}$$

$$c^k = (v^k - a^k) \left[B^2 - (Br) \frac{B^k}{a^k} \right] + \frac{B^k B^k}{a^k}$$

$$D^k = (v^k - a^k) \left[(Br) + \frac{B^k}{w^2 a^k} \right] + B^k$$

$$E^k = (v^k - a^k) \left[\frac{B^2}{w^2} + (Br)^2 \right] + B^2 a^k - (Br) B^k$$

This ↑ + $\frac{E^k}{P_0 a k w} \cdot 4.1$

w was not in Ryan's thesis; typo

$$0 = e \frac{0 h_e C_s^2}{B} (v^k - a^k + w_a^2 a^k) + (ve)(h_e w^4 a^k - c^k)$$

$$+ (Be) D^k - e^k \underbrace{\frac{E^k}{a^k}}$$

$$+ \frac{E^k}{w p_0 a^k} \left[c^0 w_{a^k} + (\nabla e) \sqrt{3} p_0 a^k + \underbrace{e^k w_{p_0}}_{\text{in blue}} \right]$$

$$\begin{aligned} 4.14 \\ 0 &= \frac{e^0}{p_0} \left(h_e c_s^2 (v^k - a^k) + w^2 a^k \right) + E^k \\ &+ (\nabla e) \left(h_e w^k a^k - c^k + w^2 E^k \right) \\ &+ (\beta e) D^k \end{aligned}$$

Alfvén Waves

$$(4.17) \quad \partial = \frac{e^0}{\rho_0} h_e c_s^2 \left(B + \omega_a^k (B_v) \right) + (\nu_e) h_e \omega_a^k (B_v) + (\beta_e) h_e \omega_a^k$$

$$-(\nu_e) h_e \omega_a^k (B_v) = (\beta_e) h_e \omega_a^k$$

$$4.19 \quad -(\nu_e)(B_v) \omega_a^k = \beta_e$$

$$4.20 \quad -(\nu_e) \omega_a^k = e^k$$

$$(4.9) \quad \partial = \tilde{B}_e^k - (\beta_e) B^k + (\beta_e)_a^k$$

$$(\beta_e) = -\underbrace{(\nu_e)(B_v)}_{a^k} \tilde{B}^k - \tilde{B}(-\nu_e) \omega_a^k$$

$$(4.22) \quad (\beta_e) = (\nu_e) \omega_a^k \left[\tilde{B}^k + \text{S} (B_v) \frac{B^k}{a^k} \right]$$

$$4.10 \quad \partial = (B_v)_e^k - (\nu_e) B^k + (\nu_e)_a^k$$

$$(\nu_e)_a^k = (B_v)(\nu_e) \omega_a^k + (\nu_e) B^k$$

$$(4.21) \quad (\nu_e) = (\nu_e) \left[\omega_a^k (B_v) + \frac{B^k}{a^k} \right]$$

$$(4.4) \quad 0 = B^i e^k - B^k e^i + \hat{e}^i \hat{a}^k$$

$$(4.20) \quad -(\omega_e) \bar{\omega}_a^k = e^k$$

$$0 = B^i (-(\omega_e) \bar{\omega}_a^k) - B^k e^i + \hat{e}^i \hat{a}^k$$

$$(4.23) \quad \begin{aligned} B^k e^i &= B^i (-(\omega_e) \bar{\omega}_a^k) + \hat{e}^i \hat{a}^k \\ e^i &= \frac{a^k}{B^k} \left[\hat{e}^i - (\omega_e) \bar{\omega}_a^k B^i \right] \end{aligned}$$

$$\begin{aligned} e^i &= (e_1^1, e_1^2, e_1^k) & \hat{e}^i &= (\hat{e}_1^1, \hat{e}_1^2, \hat{e}_1^k) \\ (\nu \hat{e}) &= \nu_1 \hat{e}_1^1 + \nu_2 \hat{e}_1^2 & (B \hat{e}) &= B_1 \hat{e}_1^1 + B_2 \hat{e}_1^2 \end{aligned}$$

$$(\nu \hat{e}) \cdot \left(-\frac{B_2}{\nu_2} \right) + (B \hat{e})$$

$$\begin{aligned} \left(\nu_1 \hat{e}_1^1 + \nu_2 \hat{e}_1^2 \right) \cdot -\frac{B_2}{\nu_2} + B_1 \hat{e}_1^1 + B_2 \hat{e}_1^2 &= (\nu \hat{e}) \cdot \left(-\frac{B_2}{\nu_2} \right) + (B \hat{e}) \\ -B_2 \frac{\nu_1 \hat{e}_1^1}{\nu_2} - \cancel{B_2 \frac{\nu_2 \hat{e}_1^2}{\nu_2}} + B_1 \hat{e}_1^1 + \cancel{B_2 \frac{\nu_2 \hat{e}_1^2}{\nu_2}} &\quad || \\ \hat{e}_1^1 \left(B_1 - \frac{B_2 \nu_1}{\nu_2} \right) &= || \end{aligned}$$

$$\hat{e}_\perp = \frac{1}{B_1 - B_2 \frac{v_1}{v_2}} (v_c^\perp) \cdot \left(-\frac{B_2}{v_2} \right) + (B \hat{e})$$

$$= \frac{1}{B_1 - B_2 \frac{v_1}{v_2}} \left(B_2 (v_c^\perp) - v_2 (B_c^\perp) \right)$$

$$\hat{e}_\perp^1 = \frac{(v_c)}{v_1 B_2 - v_2 B_1} \left[B_2 \left(\omega^2 (B_v) + \frac{B^k}{\alpha^k} \right) - v_2 \omega^2 \left(B - (B_v) \frac{B^k}{\alpha^k} \right) \right]$$

$$(v_c^\perp) \cdot \left(-\frac{B_1}{v_1} \right) + (B \hat{e})$$

$$(v_1 \hat{e}_\perp^1 + v_2 \hat{e}_\perp^2) \cdot -\frac{B_1}{v_1} + B_1 \hat{e}_\perp^1 + B_2 \hat{e}_\perp^2 = (v_c^\perp) \cdot \left(-\frac{B_1}{v_1} \right) + (B \hat{e})$$

$$-\cancel{B_1 \frac{v_2 \hat{e}_\perp^2}{v_1}} - \cancel{B_1 \hat{e}_\perp^1} + \cancel{B_2 \hat{e}_\perp^2} + \cancel{B_2 \hat{e}_\perp^1} \quad ||$$

$$\hat{e}_\perp^2 \left[B_2 - \frac{B_1 v_2}{v_1} \right] = ||$$

$$\hat{e}_\perp^2 = \frac{1}{B_2 - B_1 \frac{v_2}{v_1}} (v_c^\perp) \cdot \left(-\frac{B_1}{v_1} \right) + (B \hat{e})$$

$$= \frac{1}{B_2 - B_1 \frac{v_2}{v_1}} (-B (v_c^\perp) + v_r (B_c^\perp))$$

$$\bar{B}_2 - \frac{B_1 v_2}{v_1}$$

$$\hat{e}_\perp^z = \frac{(ve)}{v_1 B_2 - v_2 B_1} \left[B_1 \left(\omega^2 (Bv) + \frac{B^k}{a^k} \right) + v_1 \tilde{W} \left(B^2 - (Bv) \frac{B^k}{a^k} \right) \right]$$

All components of Alfvén wave:

$$e_\perp^0 = 0$$

$$e_\perp^1 = \frac{(ve)}{v_1 B_2 - v_2 B_1} \left[\frac{a^k}{B^k} \left[B_2 \left(\omega^2 (Bv) + \frac{B^k}{a^k} \right) - v_2 \tilde{W} \left(B^2 - (Bv) \frac{B^k}{a^k} \right) \right] - (ve) \tilde{W} \frac{a^k}{B^k} \right]$$

$$e_\perp^2 = \frac{(ve)}{v_1 B_2 - v_2 B_1} \left[\frac{a^k}{B^k} \left[B_1 \left(\omega^2 (Bv) + \frac{B^k}{a^k} \right) + v_1 \tilde{W} \left(B^2 - (Bv) \frac{B^k}{a^k} \right) \right] - (ve) \tilde{W} \frac{a^k}{B^k} \right]$$

$$e^k = -(ve) \tilde{W} \frac{a^k}{B^k}$$

$$e^4 = 0$$

$$\hat{e}_\perp^4 = \frac{(ve)}{v_1 B_2 - v_2 B_1} \left[B_2 \left(\omega^2 (Bv) + \frac{B^k}{a^k} \right) - v_2 \tilde{W} \left(B^2 - (Bv) \frac{B^k}{a^k} \right) \right]$$

$$\hat{e}_\perp^2 = \frac{(ve)}{v_1 B_2 - v_2 B_1} \left[B_1 \left(\omega^2 (Bv) + \frac{B^k}{a^k} \right) + v_1 \tilde{W} \left(B^2 - (Bv) \frac{B^k}{a^k} \right) \right]$$

$$\hat{e}^k = 0$$

Magneto sonic Waves

$$\begin{pmatrix} h_e \omega_a^k - c^k + \omega^z E^k & D^k \\ h_e \omega^y (B_r) a^k & h_e \omega_a^{2k} \end{pmatrix} \begin{pmatrix} V_e \\ B_e \end{pmatrix} = \frac{-e}{\rho_0} \begin{pmatrix} h_e c_s^2 (v^k - a^k + \omega_a^z) + E^k \\ h_e c_s^2 (B^k + \omega_a^2 (B_r)) \end{pmatrix}$$

$$c^k = (v^k - a^k) \left[B^2 - (B_r) \frac{B^k}{a^k} \right] + \frac{B^k B^k}{a^k}$$

$$D^k = (v^k - a^k) \left[(B_r) + \frac{B^k}{\omega_a^2 a^k} \right] + B^k$$

$$E^k = (v^k - a^k) \left[\frac{B^2}{\omega^2} + (B_r)^2 \right] + B^2 a^k - (B_r) B^k$$

$$\Delta^{kk} = (a^k)^2 (h_e \omega^z + B^2) - 2(B_r) a^k B^k - \frac{(B^k)^2}{\omega^2}$$

$$\begin{pmatrix} V_e \\ B_e \end{pmatrix} = \frac{1}{ad - bc} \begin{pmatrix} h_e \omega_a^{2k} & -D^k \\ -h_e \omega^y (B_r) a^k & h_e \omega_a^k - c^k + \omega^z E^k \end{pmatrix} \begin{pmatrix} h_e c_s^2 (v^k - a^k + \omega_a^z) + E^k \\ h_e c_s^2 (B^k + \omega_a^2 (B_r)) \end{pmatrix}$$

1

ad - bc

$$h_e \omega_a^z k \left(h_e \omega_a^k - c^k + \omega^z E^k \right) - D^k h_e \omega^y (B_r) a^k$$

$$\begin{aligned}
 & h_e \bar{w}^2 \left\{ h_e \bar{w}^{4k} - (\bar{w}^k - a^k) \left[\frac{\bar{B}^2}{\bar{w}^2} - (B_v) \frac{\bar{B}^k}{a^k} \right] - \frac{\bar{B}^k \bar{B}^k}{a^k} \right. \\
 & + \bar{w}^2 \left[(\bar{w}^k - a^k) \left[\frac{\bar{B}^2}{\bar{w}^2} + (B_v)^2 \right] + \bar{B}^2 a^k - (B_v) \bar{B}^k \right] \Big\} \\
 & - h_e \bar{w}^4 (B_v) a^k \left[(\bar{w}^k - a^k) \left[(B_v) + \frac{\bar{B}^k}{\bar{w}^2 a^k} \right] + \bar{B}^k \right]
 \end{aligned}$$

$$h_e \bar{w}^2 \bar{w}^6 (a^k)^2 - h_e \bar{w}^2 (\bar{B}^k)^2 + h_e \bar{w}^4 (a^k)^2 \bar{B}^2 - 2 h_e \bar{w}^4 (B_v) \bar{B}^k$$

$$h_e \bar{w}^4 \left(h_e \bar{w}^2 (a^k)^2 - \frac{(\bar{B}^k)^2}{\bar{w}^2} + (a^k)^2 \bar{B}^2 - 2 a^k (B_v) \bar{B}^k \right)$$

$$h_e \bar{w}^4 \Delta^{kk}$$

$$\begin{pmatrix} V_e \\ B_e \end{pmatrix} = \frac{-e^o}{\rho_0 h_e \bar{w}^4 \Delta^{kk}} \begin{pmatrix} h_e \bar{w}^2 a^k & -D^k \\ -h_e \bar{w}^4 (B_v) a^k & h_e \bar{w}^4 a^k - C^k + \bar{w}^2 E^k \end{pmatrix} \begin{pmatrix} h_e C_s^2 (V^k - a^k + \bar{w}^2 a^k) + E^k \\ h_e C_s^2 (B^k + \bar{w}^2 a^k (B_v)) \end{pmatrix}$$

$$(\nabla \ell) =$$

$$(\beta_c) \approx$$

$$(4.1) \quad e^{\sigma} W_{\alpha^k} + (\nabla e) \sqrt[3]{\rho_0} \alpha^k + e^k W_{\rho_0} = 0$$

$$e^{\sigma} W_{\alpha^k} + (\nabla e) \sqrt[3]{\rho_0} \alpha^k + e^k W_{\rho_0} = 0$$

$$\underline{e^k} = - \frac{(e^{\sigma} W_{\alpha^k} + (\nabla e) \sqrt[3]{\rho_0} \alpha^k)}{\sqrt[3]{\rho_0}}$$

$$= - \frac{\alpha^k}{\rho_0} (e^{\sigma} + (\nabla e) \sqrt[3]{\rho_0})$$

$$\dot{e}^k = -\frac{\alpha^k}{\rho_0} \left(e^0 + (\nu e) \bar{W}_{\rho_0}^z \right)$$

$$(4.9) \quad \dot{O} = \bar{B}^z e^k - (\bar{B} e)^k + (\bar{B} \hat{e})_{\alpha}^k$$

$$(4.10) \quad \dot{O} = (\bar{B} \nu)_c^k - (\nu e)^k + (\nu \hat{e})_{\alpha}^k$$

$$(\bar{B} \hat{e}) = \frac{(\bar{B} e)^k - \bar{B}^z e^k}{\alpha^k}$$

$$(\nu \hat{e}) = \frac{(\nu e)^k - (\bar{B} \nu)_c^k}{\alpha^k}$$

$$(\bar{B} \hat{e}) =$$

Left Eigen Vectors

$$\tilde{A}_{ij}^k = \left(h_{ij} Q + 2h_e W_{v_i v_j} - B_j B_i \right) (\bar{v}^k - \lambda^k) + \left(Q_{v_i} - (B_v) B_i \right) h_j^k + h_i^k \left((B_v) B_j - B^2 v_j \right) - \left(h_{ij} (B_v) + v_i B_j - 2B_i v_j \right) B^k$$

$$\tilde{B}_{ij}^k = \left(2v_i B_j - B_i v_j - (B_v) h_{ij} \right) (\bar{v}^k - \lambda^k) - \left(\frac{h_{ij}}{w^2} + v_i v_j \right) B^k + h_i^k \left((B_v) v_j + \frac{B_j}{w^2} \right) - h_j^k \left(\frac{B_i}{w^2} + (B_v) v_i \right)$$

$$\tilde{C}_j^k = \left(2h_e W^4 + B^2 - W_{P_0}^3 \right) v_j (\bar{v}^k - \lambda^k) - (B_v) B_j (\bar{v}^k - \lambda^k) + \left(Q - W_{P_0} \right) h_j^k + \left((B_v) B_j - B^2 v_j \right) v^k - B_j B^k$$

$$\tilde{D}_j^k = \left(2B_j - (B_v) v_j - \frac{B_j}{w^2} \right) (\bar{v}^k - \lambda^k) + \left((B_v) v_j + \frac{B_j}{w^2} \right) v^k - v_j B^k - (B_v) h_j^k$$

$$\begin{bmatrix} W_a^k & W_{P_0 v_j} a^k + W_{P_0} h_j^k & 0^k & D_j^k \\ W^2 v_i a^k + h_i^k \chi & \tilde{A}_{ij}^k & (P_0 + \chi) W^2 v_i a^k + h_i^k \chi & \tilde{B}_{ij}^k \\ (W^2 \gamma - W - \chi) a^k + \chi v^k & \tilde{C}_j^k & [(P_0 + \chi) W^2 \chi] a^k + \chi v^k & D_j^k \\ 0^k & B^2 h_j^k - B^2 v_j & 0^k & h_j^k a^k - h_j^k \bar{v}^k \end{bmatrix}$$

$$(4.26) \quad D = e^0 W_a^k + (ve) W^2 \gamma a^k + e^k \chi + e^4 \left[(W^2 \gamma - W - \chi) a^k + \chi v^k \right]$$

$$(4.27) \quad D = e^0 \left[W_{P_0 v_j} a^k + W_{P_0} h_j^k \right] + \left\{ e_{ij} \left[Q a^k - (B_v) B^k \right] \right. \\ + (ve) \left[2h_e W^4 v_j a^k + Q h_j^k - B_j B^k \right] \\ + (Be) \left[-B_j a^k - (B_v) h_j^k + Z v_j B^k \right] + e^k \left[(B_v) B_j - B^2 v_j \right] \} \\ + e^4 \left[(2h_e W^4 + B^2 - W_{P_0}^3) v_j a^k - (B_v) B_j a^k + (Q - W_{P_0}) h_j^k \right. \\ \left. + ((B_v) B_j - B^2 v_j) v^k - B_j B^k \right]$$

$$(4.28) \quad + (\beta \hat{e}) h_j^k - \hat{e}_j^k \beta^k$$

$$D = (ve) \left[(\rho_0 + 2\lambda) w^2 a^k \right] + e^k \cancel{\lambda} + e^4 \left[(\rho_0 + 2\lambda) w^2 a^k + 2(v^k - a^k) \right]$$

$$(4.29) \quad O = \left\{ -e_j \left[(\beta v)_a^k + \frac{\beta^k}{w^2} \right] + (ve) \left[2\beta_j a^k - v_j \beta^k - (\beta v) h_j^k \right] \right.$$

$$- (\beta e) \left[v_j a^k + \frac{h_j^k}{w^2} \right] + e^k \left[(\beta v) v_j + \frac{\beta_j}{w^2} \right] \}$$

$$+ e^4 \left[(z - \frac{1}{w}) \beta_j a^k - (\beta v) v_j a^k + ((\beta v) v_j + \frac{\beta_j}{w^2}) v^k - v_j \beta^k - (\beta v) h_j^k \right]$$

$$+ \hat{e}_j^k a^k - (\bar{\tau} \hat{e}) h_j^k$$

$\chi/2 \cdot 4.28 + 4.26$

$$O = e^0 w_a^k + (ve) w^2 \gamma_a^k + \cancel{e^k \chi} + e^4 \left[(w^2 \gamma - v - \chi)_a^k + \cancel{\lambda} v^k \right]$$

$$\cancel{\frac{\chi}{2}} \left[(ve) \left[(\rho_0 + 2\lambda) w^2 a^k \right] + \cancel{e^k \lambda} + e^4 \left[(\rho_0 + 2\lambda) w^2 a^k + 2(v^k - a^k) \right] \right]$$

$$O = e^0 w_a^k + (ve) \left[w^2 \gamma_a^k - \frac{\chi}{2} (\rho_0 + 2\lambda) w^2 a^k \right]$$

$$+ e^4 \left[(w^2 \gamma - v - \chi)_a^k + \cancel{\lambda} v^k - \frac{\chi}{2} \left[(\rho_0 + 2\lambda) w^2 a^k + 2(v^k - a^k) \right] \right]$$

$$= e^0 + (ve) \left[v \gamma - \frac{\chi}{2} (\rho_0 + 2\lambda) w \right]$$

$$+ e^4 \left[v \gamma - 1 - \frac{\chi}{2} (\rho_0 + 2\lambda) w \right]$$

$$= e^0 - e^4 + w \left[\gamma - \chi \left(1 + \frac{P_0}{\rho_0} \right) \right] (e^4 + ve)$$

$$\gamma - \chi \left(1 - \frac{P_0}{\rho_0} \right) = \frac{h_e}{\rho_0 c_s^2} (Q - P_0 c_s^2)$$

$$(4.30) \quad O = e^0 - e^4 + \frac{h_e w}{P_0} \left(1 - \frac{P_0 c_s^2}{\rho_0} \right) (ve + e^4)$$

4.27. B^j

$$B^j \left\{ e^0 \left[W_{P_0 v_j}^3 a^k + W_{B^j}^k \right] + e_j \left[Q a^k - (B_v) B^k \right] \right.$$

\downarrow
 $W_{P_0}^3 (B_v)_a^k$ \downarrow
 $W_{P_0} B^k$ \downarrow
 (B_e) \downarrow
 $(h_e w^2 + B^2)_a^k$
 \equiv

$$+ (ve) \left[2 h_e w^4 v_j a^k + Q h_j^k - B_j B^k \right]$$

\downarrow
 $2 h_e w^4 (B_v)_a^k$ \downarrow
 $Q B^k$ \downarrow
 $- B^2 B^k$

$$+ (B_e) \left[- B_j a^k - (B_v) h_j^k + Z v_j B^k \right] + e^k \left[(B_v) B_j - B^2 v_j \right]$$

\downarrow
 $- B^2 a^k$ \downarrow
 $- (B_v) B^k$ \downarrow
 $Z (B_v) B^k$ \downarrow
 $(B_v) B^2 - B^2 (B_v)$

$$+ e^4 \left[(2 h_e w^4 + B^2 - W_{P_0}^3) v_j a^k - (B_v) B_j a^k + (Q - W_{P_0}) h_j^k \right]$$

\downarrow
 $(B_v)_a^k$ \downarrow
 $- (B_v) B^2 a^k$ \downarrow
 $h_e w^2 + B^2$ \downarrow
 B^k

$$+ ((B_v) B_j - B^2 v_j) v^k - B_j B^k \right] + (B_e) h_j^k - e_j B^k$$

\downarrow
 $(B_v) B^2$ \downarrow
 $- B^2 (B_v)$ \downarrow
 $- B^2 B^k$ \downarrow
 $(B_v) B^k$ \downarrow
 $- (B^2) B^k$

$$0 = e^0 [w^2 (B_v)_{\alpha}^k + B^k] + (Be) [h_e w^2 \alpha^k]$$

$$+ (ve) [2h_e w^4 (B_v)_{\alpha}^k + h_e w^2 B^k + B^2 B^k - B^2 B^k]$$

$$+ e^4 [(2h_e w^4 - w^3 p_0) (B_v)_{\alpha}^k + h_e w^2 B^k - w_p B^k]$$

$$(4.31) \quad 0 = w p_0 (w^2 (B_v)_{\alpha}^k + B^k) (e^0 - e^4)$$

$$+ h_e w^2 (2w^2 (B_v)_{\alpha}^k + B^k) (ve + e^4) + (Be) h_e w^2 \alpha^k$$

4.27. \sqrt{j}

$$\begin{aligned} & v^j \left\{ e^0 \left[w^3 p_v j_{\alpha}^k + w_p h_j^k \right] + e_j \left[Q \alpha^k - (B_v) B^k \right] \right. \\ & \quad \left. \begin{array}{c} \downarrow \\ w^3 p_v v^2 \alpha^k \end{array} \quad \begin{array}{c} \downarrow \\ w_p v^k \end{array} \quad \begin{array}{c} \downarrow \\ (ve) \end{array} \quad \begin{array}{c} \downarrow \\ (h_e w^2 + B^2) \alpha^k \end{array} \right. \\ & + (ve) \left[2h_e w^4 v_j^k \alpha^k + Q h_j^k - B_j B^k \right] \\ & \quad \left. \begin{array}{c} \downarrow \\ 2h_e w^4 v^2 \alpha^k \end{array} \quad \begin{array}{c} \downarrow \\ (h_e w^2 + B^2) v^k \end{array} \quad \begin{array}{c} \downarrow \\ - (B_v) B^k \end{array} \right. \\ & + (Be) \left[- B_j \alpha^k - (B_v) h_j^k + Z v_j B^k \right] + e^k \left\{ (B_v) B_j - B^2 v_j \right\} \\ & \quad \left. \begin{array}{c} \downarrow \\ - (B_v) \alpha^k \end{array} \quad \begin{array}{c} \downarrow \\ - (B_v) v^k \end{array} \quad \begin{array}{c} \downarrow \\ Z v^2 B^k \end{array} \quad \begin{array}{c} \downarrow \\ (B_v)^2 - B^2 v^2 \end{array} \right. \\ & + e^4 \left[(2h_e w^4 + B^2 - w^3 p_0) v_j \alpha^k - (B_v) B_j \alpha^k + (Q - w_p) h_j^k \right. \\ & \quad \left. \begin{array}{c} \downarrow \\ v^2 \alpha^k \end{array} \quad \begin{array}{c} \downarrow \\ - (B_v)^2 \alpha^k \end{array} \quad \begin{array}{c} \downarrow \\ h_e w^2 + B^2 \end{array} \quad \begin{array}{c} \downarrow \\ v^k \end{array} \right. \\ & + ((B_v) B_j - B^2 v_j) v^k - B_j B^k \left. \right\} + (Be) h_j^k - e_j B^k \end{aligned}$$

$$\omega^3 \omega^2 = \omega^3(1 - \frac{1}{\omega^2}) \\ = \omega^3 - \omega$$

$$0 = W_{P_0} (v^k - a^k + \omega^2 a^k) (e^0 - e^4) \\ + (\nu e) \left[2 h_e \omega^4 a^k \left(1 - \frac{1}{\omega^2} \right) - 2(Bv) B^k + (h_e \omega^2 + B^2) (v^k + a^k) \right] \\ + e^4 \left[(2 h_e \omega^4 a^k \left(1 - \frac{1}{\omega^2} \right) + ((Bv)^2 - B^2 \left(1 - \frac{1}{\omega^2} \right))) (v^k - a^k) \right. \\ \left. + (h_e \omega^2 + B^2) v^k - (Bv) B^k \right] - (Be) \left[(Bv) (v^k + a^k - 2B^k e) \right. \\ \left. - 2B^k \left(1 - \frac{1}{\omega^2} \right) \mu \right] + e^k \left[(Bv)^2 - B^2 \left(1 - \frac{1}{\omega^2} \right) \right] \\ + (B\hat{e}) v^k - (\omega\hat{e}) B^k$$

$$\uparrow - \frac{1}{2}((Bv)^2 - B^2 \left(1 - \frac{1}{\omega^2} \right)) \cdot 4.28$$

$$0 = W_{P_0} (v^k - a^k + \omega^2 a^k) (e^0 - e^4) \\ + (\nu e) \left[2 h_e \omega^4 a^k \left(1 - \frac{1}{\omega^2} \right) - 2(Bv) B^k + (h_e \omega^2 + B^2) (v^k + a^k) \right] \\ + e^4 \left[(2 h_e \omega^4 a^k \left(1 - \frac{1}{\omega^2} \right) + ((Bv)^2 - B^2 \left(1 - \frac{1}{\omega^2} \right))) (v^k - a^k) \right. \\ \left. + (h_e \omega^2 + B^2) v^k - (Bv) B^k \right] - (Be) \left[(Bv) (v^k + a^k - 2B^k e) \right. \\ \left. - 2B^k \left(1 - \frac{1}{\omega^2} \right) \mu \right] + e^k \left[(Bv)^2 - B^2 \left(1 - \frac{1}{\omega^2} \right) \right] \\ + (B\hat{e}) v^k - (\omega\hat{e}) B^k$$

$$-\frac{1}{\omega^2} \left((\beta v)^2 - \beta^2 \left(1 - \frac{1}{\omega^2} \right) \right).$$

$$\left\{ (re) \left[(p_0 + 2) w^2 a^k \right] + e^k 2r + e^4 \left[(p_0 + 2) w^2 a^k + 2(r^k - a^k) \right] \right\}$$

$$= (\beta r)^2 + \beta^2 \left(1 - \frac{1}{\omega^2}\right)$$

$$\left\{ (ve) \left[\frac{(\rho_0 + v^k)}{2} w_a^k \right] + \underbrace{e^k}_{\text{---}} + e^4 \left[\frac{(\rho_0 + v^k)}{2} w_a^k + \underbrace{(v^k - a^k)}_{\text{---}} \right] \right\}$$

$$(4.32) \quad O = W_p (v^k - a^k + W_a^k) (e^0 - e^4)$$

$$+ \left[2h_e W^4 \alpha^k \left(1 - \frac{1}{\gamma^2} \right) - (Bv) B^k + (h_e W^2 + B^2) v^k \right]$$

$$-\omega_a^2 k \left(1 + \frac{P_0}{\omega} \right) \left[(Br)^2 - B^2 \left(1 - \frac{1}{r^2} \right) \right] (ve + e^4)$$

$$(v_e) \left[- (B_v) B^k + (h_e \sqrt{z} + B^z) a^k \right] - (B_e) [(B_v)(v^k + a^k - z B^k)]$$

$$-2B^k \left(1 - \frac{1}{\omega^2}\right) \mu \Big] + (B\hat{\epsilon})^k - (\omega\hat{\epsilon}) B^k$$

4.29. B^j

$$B^j \left(\underbrace{\{ -e_j \left[(Br)_a^k + \frac{B^k}{w^2} \right] + (ve) \left[2B_j a^k - v_j B^k - (Br) h_j^k \right] }_{\downarrow \uparrow} \right)$$

$$-(Be) \left[\sqrt{j} a^k + \frac{h_j^k}{\omega} \right] -$$

$$2B^2 a - (Bu)B^k - (Bu)B^k \nearrow$$

$$\left[(B_{rj})_{Vj} + \frac{B_j}{V^2} \right] \}$$

$$+ e^4 \left[\left(z - \frac{1}{\omega^2} \right) B_j a^k - (B_v) v_j a^k + ((B_v) v_j + \frac{B_j}{\omega^2}) v^k - v_j B^k - (B_v) h_j^k \right]$$

\downarrow \downarrow \downarrow \downarrow \downarrow
 B_a^k $-(B_v)^2 a^k$ $(B_v)^z$ $\frac{B^z}{\omega^2}$ $-(B_v) B^k$
 $+ \hat{e}_j \cdot a^k - (\bar{v} \hat{e}) h_j^k \right)$

\downarrow \downarrow
 $(B \hat{e}) a^k$ $-(\bar{v} \hat{e}) B^k$

$$(4.33) \quad D = 2 \left[B_a^k - (B_v) B^k \right] (v_e + e^4) + \left[(B_v)^2 + \frac{B^2}{\omega^2} \right] (e^4 v^k + e^k)$$

$$- 2(B_e) \left[(B_v) a^k + \frac{B^k}{\omega^2} \right] - e^4 \left[(B_v)^2 a^k - \frac{B^2}{\omega^2} a^k \mu \right] + (B \hat{e}) a^k - (\bar{v} \hat{e}) B^k$$

$$v^i \left(\left\{ -e_j \left[(B_v)_a^k + \frac{B^k}{\omega^2} \right] + (v_e) \left[2B_j a^k - v_j B^k - (B_v) h_j^k \right] \right.$$

\downarrow \downarrow \downarrow
 $-(v_e)$ $(B_v)_a^k$ $-\frac{(v^2) B^k}{B^k + \frac{B^k}{\omega^2}} - (B_v) v^k$
 $- (B_e) \left[v_j a^k + \frac{h_j^k}{\omega^2} \right] + e^k \left[(B_v) v_j + \frac{B_j}{\omega^2} \right] \right\}$

\downarrow \downarrow \downarrow \downarrow
 $(v^2) a^k$ $\frac{v^k}{\omega^2}$ $(B_v)_v^z$ $\frac{(B_v)}{\omega^2}$
 $a^k - \frac{a^k}{\omega^2}$ v^k $(B_v) - \frac{B^k}{\omega^2}$ $\cancel{\omega^2}$
 $+ e^4 \left[\left(z - \frac{1}{\omega^2} \right) B_j a^k - (B_v) v_j a^k + ((B_v) v_j + \frac{B_j}{\omega^2}) v^k - v_j B^k - (B_v) h_j^k \right]$

\downarrow \downarrow \downarrow \downarrow \downarrow
 $(B_v)_a^k$ $-(B_v)^2 a^k$ $(B_v)_v^z$ $\frac{B^z}{\omega^2}$ $-(v^2) B^k$
 $\frac{2(B_v)_a^k - (B_{vv})_a^k}{\omega^2}$ $-(B_v)_a^k + \frac{(B_v)_a^k}{\omega^2}$ $(B_v) - \frac{B^k}{\omega^2}$ $-B^k + \frac{B^k}{\omega^2}$
 $+ \hat{e}_j \cdot a^k - (\bar{v} \hat{e}) h_j^k \right) =$

\downarrow \downarrow
 $(v \hat{e})_a^k$ $-(\bar{v} \hat{e}) v^k$

$$(4.34) \quad (\nu e) \left[(Bv)_{\alpha}^k - v^k \right] - B^k - \beta e \left(\alpha^k + (v^x - \alpha^x) \frac{v^z}{\omega^2} \right) \\ + e^k \cdot (Bv) + e^4 \left[(Bv)_{\alpha}^k - B^k + \frac{B^k}{\omega^2} \right] + (\nu \hat{e})_{\alpha}^k - (\bar{\nu} \hat{e}) v^k$$

(4.34)

$$0 = \left[(Bv)_{\alpha}^k - B^k \right] (\nu e + e^4) - (\nu e) (Bv)_{\alpha}^k + e^4 \frac{B^k}{\omega^2} \\ - (\beta e) \frac{1}{\omega^2} (v^k - \alpha^k + \omega^2 \alpha^k) + e^k (Bv) + (\nu \hat{e})_{\alpha}^k - (\bar{\nu} \hat{e}) v^k$$

The Entropy Wave (Left Eigenvector)

$$(4.31) \quad 0 = \cancel{\rho_0} \left(\cancel{\omega} (Bv)_{\alpha}^k + B^k \right) (e^0 - e^4) \\ + h_e \cancel{\omega^4} \cancel{(2\omega^2 (Bv)_{\alpha}^k + B^k)} \cancel{(\nu e + e^4)} + (\beta e) h_e \cancel{\omega^2} \cancel{\alpha^k}$$

$$0 = \rho_0 (e^0 - e^4) + h_e \omega (\nu e + e^4)$$

$$e^0 - e^4 = - \frac{h_e \omega}{\rho_0} (\nu e + e^4)$$

$$(4.30) \quad 0 = e^0 - e^4 + \frac{h_e \omega}{\rho_0} \left(1 - \frac{\rho_0 c_s^2}{\Sigma} \right) (\nu e + e^4)$$

$$O = \frac{h_e \omega}{\rho_0} \left(\frac{\rho_0 C_s^2}{2} \right) (v_e + e^4)$$

$$(4.35) \quad O = (v_e + e^4)$$

$$(4.36) \quad O = (e^0 - e^4)$$

$$(4.26) \quad O = e^0 \cancel{w_a}^0 + (v_e) \cancel{y_a}^0 + e^k \chi + e^4 \left[\cancel{w_y}^0 - \cancel{v_x}^0 + \chi v^k \right]$$

$$O = e^k + e^4 v^k$$

$$e^k = -e^4 v^k$$

$$\begin{aligned} (4.33) \quad O &= 2 \left[B^2 a^k - (Bv) B^k \right] (v \cancel{x}^0 + e^4) + \left[(Bv)^2 + \frac{B^2}{w^2} \right] (e^4 \cancel{v}^k + e^k) \\ &\quad - 2(Be) \left[(B \cancel{v}) a^k + \frac{B^k}{w^2} \right] - e^4 \left[(B \cancel{v}) a^k - \frac{B^2}{w^2} \cancel{a}^k \mu \right] + (B \cancel{v}) a^k - (\cancel{v}^k) B^k \end{aligned}$$

$$O = -2(Be) \cancel{\frac{B^k}{w^2}} - (\cancel{v}^k) \cancel{B^k}$$

$$(Be) = -\frac{1}{2} w^2 (\cancel{v}^k)$$

$$\begin{aligned} O &= \left[(B \cancel{v}) a^k - B^k \right] (v \cancel{e}^4) - (v_e) (Bv) v^k + e^4 \frac{B^k}{w^2} \\ &\quad - (Be) \frac{1}{w^2} (v^k \cancel{a}^k + \cancel{v}^k \cancel{a}^k) + e^k (Bv) + (\cancel{v}^k) a^k - (\cancel{v}^k) v^k \end{aligned}$$

$$\begin{aligned}
 0 &= e^0 (Bv) v^k + e^0 \frac{B^k}{w^2} - (Be) \frac{v^k}{w^2} - e^0 v^k (Bv) - (\bar{v}e) v^k \\
 &= e^0 [(Bv) v^k + \frac{B^k}{w^2} - v^k (Bv)] - (Be) \frac{v^k}{w^2} - (\bar{v}e) v^k \\
 &= \quad \quad \quad || \quad \quad \quad + \frac{1}{2} (\bar{v}e) v^k - (\bar{v}e) v^k \\
 &= \quad \quad \quad || \quad \quad \quad - \frac{1}{2} (\bar{v}e) v^k
 \end{aligned}$$

$$(4.36) \quad (\bar{v}_e^k) = \frac{2}{\sqrt{2}} \left(\frac{\beta^k}{v^k} \right)_e^o$$

$$= e^{\int \frac{B^k}{w^2} - \left(B_e \right) \frac{v^k}{w^2} - \left(\hat{v} \hat{e} \right) v^k}$$

$$(4.3g) \quad (\text{Be}) = - \frac{e^2 B^k}{m}$$

$$\begin{aligned}
 (4.27) \quad D &= e^0 \left[w_{\beta_0}^3 a^k + w_{\beta_0} h_j^k \right] + \{ e_j [Q a^k - (B_r) B^k] \} \\
 &+ (v e) \left[\cancel{Z h_e} \cancel{w^4 v_j a^k} + Q h_j^k - B_j B^k \right] \\
 &+ (B e) \left[- \cancel{B_j a^k} - (B_r) h_j^k + Z v_j B^k \right] + e^k \{ (B_r) \cancel{B_j} - \cancel{B^2 v_j} \} \\
 &+ e^4 \left[\cancel{(Z h_e w^4 + B^2 - w^3)} \cancel{v_j a^k} - (B_r) \cancel{B_j a^k} + (Q - w_{\beta_0}) h_j^k \right. \\
 &\quad \left. + ((B_r) \cancel{B_j} - \cancel{B^2 v_j}) v^k - B_j B^k \right] + (B e) h_j^k - e_j B^k
 \end{aligned}$$

$$0 = \underbrace{e^0 W_{p_0} h_j^k}_{-\frac{\beta^k}{v^k} e^0 \left[-(B_r) h_j^k + 2 v_j B^k \right]} + \left\{ -e_j (B_r) B^k - e^0 \left[Q h_j^k - \underline{B_j B^k} \right] \right. \\ \left. - \frac{\beta^k}{v^k} e^0 \left[(B_r) B_j - \underline{B_{v_j}^k} \right] + e^0 \left[(B_r) B_j - \underline{B_{v_j}^k} \right] \right\} \\ + e^0 \left[(Q - \underline{W_{p_0}}) h_j^k + ((B_r) B_j - \underline{B_{v_j}^k}) v^k - \underline{B_j B^k} \right] \\ + (\beta \hat{e}) h_j^k - \hat{e}_j B^k$$

$$e_j (B_r) B^k + \hat{e}_j B^k =$$

$$- \frac{\beta^k}{v^k} e^0 \left[-(B_r) h_j^k + 2 v_j B^k \right] - \cancel{v^k e^0 \left[(B_r) B_j \right]} \\ + \cancel{e^0 \left[(B_r) B_j \right]} + (\beta \hat{e}) h_j^k$$

$$e_j (B_r) + \hat{e}_j = \frac{2 \beta^k}{v^k} v_j e^0$$

(4.7a)

$$0 = \left\{ -e_j \left[(B_r) a^k + \frac{B^k}{v^2} \right] + (v e) \left[2 B_j a^k - v_j B^k - (B_r) h_j^k \right] \right. \\ \left. - (\beta e) \left[v_j a^k + \frac{h_j^k}{v^2} \right] + e^k \left[(B_r) v_j + \frac{B_j}{v^2} \right] \right\} \\ + e^k \left[(z - \frac{1}{v^2}) B_j a^k - (B_r) v_j a^k + ((B_r) v_j + \frac{B_j}{v^2}) v^k - v_j B^k - (B_r) h_j^k \right] \\ + \hat{e}_j a^k - (\bar{v} \hat{e}) h_j^k$$

$$0 = -e_j \left[\frac{B^k}{\omega^2} \right] - e^0 \left[-v_j B^k - \underline{(Br) h_j^k} \right]$$

$$+ \cancel{e^0 \frac{B^k}{\omega^2} \frac{h_{j,k}}{\omega^2}} - v^k e^0 \cancel{[(Br)v_j + \frac{B_j}{\omega^2}]} + e^0 \cancel{[(Br)v_j + \frac{B_j}{\omega^2}]} v^k - \cancel{v_j B^k} - \cancel{(Br) h_j^k}$$

$$- \cancel{\frac{2}{\omega^2} \left(\frac{B^k}{\omega^2} \right)_0 h_j^k}$$

$$e_j \left[\frac{B^k}{\omega^2} \right] = -e^0 \frac{1}{\omega^2} \frac{B^k}{\omega^2} h_j^k$$

$$e_j = -e^0 \frac{h_j^k}{\omega^2}$$

$$\underset{j \neq k}{e_j = 0}$$

(4.40)

$$\hat{e}_j = -v_j \frac{B^k}{\omega^2} e^0$$

$$(\bar{v} \hat{e}) = \bar{v}^j \hat{e}_j = \frac{2}{\omega^2} \left(\frac{B^k}{\omega^2} \right)^0 e = \bar{v}^1 \hat{e}_1 + \bar{v}^2 \hat{e}_2 + \bar{v}^k \hat{e}_k$$

$$\frac{2}{\omega^2} \left(\frac{B^k}{\omega^2} \right)^0 e = \bar{v}^1 \hat{e}_1 + \bar{v}^2 \hat{e}_2 + \bar{v}^k \hat{e}_k$$

$$\frac{2}{\omega^2} \left(\frac{B^k}{\omega^2} \right)^0 e - \bar{v}^1 \hat{e}_1 - \bar{v}^2 \hat{e}_2 = \bar{v}^k \hat{e}_k$$

$$\frac{2}{\omega^2} \left(\frac{B^k}{\omega^2} \right)^0 e + 2\bar{v}^1 v_1 \frac{B^k}{\omega^2} e^0 + 2\bar{v}^2 v_2 \frac{B^k}{\omega^2} e^0 = \bar{v}^k \hat{e}_k$$

(4.41)

$$\hat{e}_k = 2 \frac{B^k}{\omega^2 \bar{v}^k} \left[\frac{1}{\omega^2} + \bar{v}^1 v_1 + \bar{v}^2 v_2 \right] e^0$$

$$\begin{aligned}
 \vec{e}^k &= e^0 \\
 e^i &= 0^i \\
 e^4 &= e^0 \\
 \hat{e}^1 &= \left[-Zv_1 \frac{B^k}{\omega^k} \right] e^0 \\
 \hat{e}^2 &= \left[-Zv_2 \frac{B^k}{\omega^k} \right] e^0 \\
 \hat{e}^k &= \left\{ Z \frac{B^k}{\omega^k} \left[\frac{1}{\omega^i} + \tilde{v}^i v_i + \tilde{\omega}^2 v_2 \right] \right\} e^0
 \end{aligned}$$

Allén Waves (Left Eigenvectors)

$$(4.42) \quad O = e^0 - e^4 + \frac{h_e \omega}{\rho_0} \left(1 - \frac{\rho_0 c_s^2}{\omega^2} \right) (v_e + e^4)$$

$$\begin{aligned}
 O &= (e^0 - e^4) \nabla \rho_0 (\omega^2 v_j \alpha^k + h_j^k) + e_j (h_e \omega^2 + \tilde{B}^2) \alpha^k \\
 &\quad - (\tilde{B} v) (e_j \tilde{B}^k + (B_e) h_j^k) + (v_e + e^4) [2 h_e \omega^2 v_j \alpha^k + (h_e \omega^2 + \tilde{B}) h_j^k] \\
 &\quad - \tilde{B}_j \tilde{B}^k] + (\tilde{B} e) (2 v_j \tilde{B}^k - \tilde{B}_j \alpha^k) + (e^k - e^4 (\alpha^k - v^k)) \cdot \\
 &\quad ((\tilde{B} v) \tilde{B}_j - \tilde{B}^2 v_j) + (\tilde{B}_e) h_j^k - \hat{e}_j \tilde{B}^k
 \end{aligned}$$

$$(4.44) \quad O = (v_e + e^4) \left[\left(1 + \frac{\rho_0}{\omega^2} \right) \omega^2 \alpha^k \right] + e^k - e^4 (\alpha^k - v^k)$$

$$\begin{aligned}
 (4.45) \quad O &= (v_e + e^4) \left[2 \tilde{B}_j \alpha^k - v_j \tilde{B}^k - (\tilde{B} v) h_j^k \right] - \alpha^k [e_j (\tilde{B} v) \\
 &\quad + (\tilde{B} e) v_j] - \frac{1}{\omega^2} (e_j \tilde{B}^k + (B_e) h_j^k) + (e^k - e^4 (\alpha^k - v^k)) \cdot \\
 &\quad ((\tilde{B} v) v_j + \frac{1}{\omega^2} \tilde{B}_j) + \hat{e}_j \alpha^k - (\tilde{v} \hat{e}) h_j^k
 \end{aligned}$$

$$(4.44) \quad e^k - e^4 = - \left[\frac{h_e \omega}{P_0} \left(1 - \frac{P_0 C_s^2}{\Sigma} \right) (v_e + e^4) \right]$$

$$e^k - e^4 (a^k - v^k) = -(v_e + e^4) \left[\left(1 + \frac{P_0}{\Sigma} \right) \omega_{ak}^2 \right]$$

(4.43)

$$\begin{aligned} 0 &= - \left[\frac{h_e \omega}{\cancel{\Sigma}} \left(1 - \frac{P_0 C_s^2}{\cancel{\Sigma}} \right) (v_e + e^4) \right] \cancel{\omega_{ak}} (\omega_{vj,a}^2 + h_j^k) + e_j (h_e \omega^2 + B^2)_a^k \\ &\quad - \left[h_e \omega^2 \left(\cancel{\omega_{vj,a}^2 + h_j^k} \right) - \frac{h_e \omega P_0 C_s^2}{\cancel{\Sigma}} \left(\cancel{\omega_{vj,a}^2 + h_j^k} \right) \right] \\ &\quad - (B_v)(e_j B^k + (B_e)h_j^k) + (v_e + e^4) \left[h_e \omega^2 v_{j,a}^k + (h_e \omega^2 + \hat{B}) h_j^k \right. \\ &\quad \left. - B_j B^k \right] + (B_e) (2v_j B^k - B_j a^k) - (v_e + e^4) \left[\left(1 + \frac{P_0}{\Sigma} \right) \omega_{ak}^2 \right] \\ &\quad ((B_v)B_j - \hat{B}_{vj,j}) + (B_e) h_j^k - \hat{e}_j \hat{B}^k \end{aligned}$$

(4.46)

$$\begin{aligned} &(v_e + e^4) \left[h_e \omega^2 (\omega_{vj,a}^2 + \frac{P_0 C_s^2}{\Sigma} (\omega_{vj,a}^2 + h_j^k)) \right] \\ &- \omega_{ak}^2 \left(1 + \frac{P_0}{\Sigma} \right) ((B_v)B_j - \hat{B}_{vj,j}) - B^k h_j^k - B_j B^k \Big] \\ &+ e_j (h_e \omega^2 + B^2)_a^k - (B_v)(e_j \cdot B^k + (B_e)h_j^k) + (B_e) (2v_j B^k - B_j a^k) \\ &+ (\hat{B}_e) h_j^k - \hat{e}_j \hat{B}^k \end{aligned}$$

$$(4.45) \quad O = (v_e + e^4) [2B_j a^k - v_j B^k - (Bv) h_j^k] - a^k [e_j (Bv) + (Be)v_j] - \frac{1}{\omega^2} (e_j B^k + (Be) h_j^k) - (v_e + e^4) \left[(1 + \frac{P_0}{\omega}) \omega^2 a^k \right] .$$

$$((Bv)v_j + \frac{1}{\omega^2} B_j) + \hat{e}_j a^k - (\bar{v} \hat{e}) h_j^k$$

$$(v_e + e^4) [2B_j a^k - v_j B^k - (Bv) h_j^k - (1 + \frac{P_0}{\omega}) \omega^2 (Bv) v_j - \frac{\omega^2 a^k}{\omega^2} B_j - \frac{P_0}{\omega} \underbrace{\omega^2 a^k}_{\cancel{\omega^2}} B_j]$$

$$2B_j a^k - a^k B_j (1 + \frac{P_0}{\omega})$$

$$B_j a^k (1 - \frac{P_0}{\omega})$$

$$(4.47) \quad (v_e + e^4) \left(B_j a^k \left(1 - \frac{P_0}{\omega} \right) - (1 + \frac{P_0}{\omega}) \omega^2 (Bv) v_j - v_j B^k - (Bv) h_j^k \right)$$

$$- a^k [e_j (Bv) + (Be) v_j] - \frac{1}{\omega^2} (e_j B^k + (Be) h_j^k) + \hat{e}_j a^k - (\bar{v} \hat{e}) h_j^k$$

$$a^k \cdot 4.46 + B^k \cdot 4.47$$

$$O = a^k \cdot \left\{ (v_e + e^4) \left[h_e \bar{v} \left(\bar{w} v_j a^k + \frac{P_0 C_s^2}{\omega} (w v_j a^k + h_j^k) \right) - \bar{w} a^k \left(1 + \frac{P_0}{\omega} \right) ((Bv) B_j - \bar{B} v_j) - \bar{B} h_j^k - \underline{B_j B^k} \right] + e_j (h_e \bar{w} + \bar{B}) a^k - \underbrace{(Bv)}_{\cancel{(e_j \cdot B^k + (Be) h_j^k)}} + (Be) \underline{(2 v_j B^k - B_j a^k)} + (Be) h_j^k - \underline{\hat{e}_j B^k} \right\} + B^k \left\{ (v_e + e^4) \left(B_j a^k \left(1 - \frac{P_0}{\omega} \right) - (1 + \frac{P_0}{\omega}) \omega^2 (Bv) v_j - v_j B^k - (Bv) h_j^k \right) \right\}$$

$$-\alpha^k \left[\underbrace{e_j [B_v]}_{\text{wavy}} + \underbrace{[B_e] v_j}_{\text{wavy}} \right] - \frac{1}{2} (e_j \cdot B^k + (B_e) h_j^k) + \hat{e}_j^k \{ \bar{v}_e \} h_j^k \}$$

$$\begin{aligned} O = & \alpha^k \left\{ (v_e + e^4) \left[h_e \nabla^2 (W v_j \alpha^k + \frac{\rho_0 c_s^2}{2} (W v_j \alpha^k + h_j^k)) \right. \right. \\ & - W \alpha^k \left(1 + \frac{\rho_0}{2} \right) ((B_v) B_j - B^2 v_j) - B^2 h_j^k \left. \right] \\ & + e_j (h_e W^2 + B^2) \alpha^k - (B_v) \underbrace{(e_j \cdot B^k + (B_e) h_j^k)}_{\text{wavy}} + (B_e) \underbrace{(2 v_j B^k - B_j \alpha^k)}_{\text{wavy}} \\ & \left. \left. + (B_e) h_j^k - \hat{e}_j^k B^k \right\} \right. \\ & + B^k \left\{ (v_e + e^4) \left(B_j \alpha^k \left[-\frac{\rho_0}{2} \right] - (1 + \frac{\rho_0}{2}) W \alpha^k (B_v) v_j - v_j B^k - (B_v) h_j^k \right) \right. \\ & \left. \left. - \alpha^k \left[\underbrace{e_j [B_v]}_{\text{wavy}} + \underbrace{[B_e] v_j}_{\text{wavy}} \right] - \frac{1}{2} (e_j \cdot B^k + (B_e) h_j^k) + \hat{e}_j^k \{ \bar{v}_e \} h_j^k \right\} \right. \end{aligned}$$

$$\begin{aligned} O = & \left\{ (v_e + e^4) \left[\alpha^k \left\{ h_e \nabla^2 (W v_j \alpha^k + \frac{\rho_0 c_s^2}{2} (W v_j \alpha^k + h_j^k)) \right. \right. \right. \right. \\ & - W \alpha^k \left(1 + \frac{\rho_0}{2} \right) ((B_v) B_j - B^2 v_j) - B^2 h_j^k \left. \right\} + B^k \left\{ B_j \alpha^k \left[-\frac{\rho_0}{2} \right] \right. \\ & \left. \left. - (1 + \frac{\rho_0}{2}) W \alpha^k (B_v) v_j - v_j B^k - (B_v) h_j^k \right] \right\} \\ & + e_j (h_e W^2 + B^2) \alpha^k - 2(B_v) (e_j \cdot B^k + (B_e) h_j^k) \alpha^k + (B_e) (v_j B^k - B_j \alpha^k) \alpha^k \\ & + (B_e) h_j^k \alpha^k - \frac{1}{2} (e_j \cdot B^k + (B_e) h_j^k) B^k - \{ \bar{v}_e \} h_j^k B^k \\ & \rightarrow \Delta^{kk} e_j \quad \Delta^{kk} = (\alpha^k)^2 Q - 2(B_v) \alpha^k B^k - \frac{(B^k)^2}{W^2} \end{aligned}$$

$$\begin{aligned}
O = & \left\{ (v_r e + e^u) \left[a^k \left\{ h_c w^2 (w^2 v_j a^k + \frac{\rho_0 c_s}{2} (w^2 v_j a^k + h_j^k)) \right\} \right. \right. \\
& - w^2 a^k \left(1 + \frac{\rho_0}{2} \right) ((B_r) B_j - B_j v_j) - B_j h_j^k \left. \right\} + B^k \left\{ B_j a^k \left[-\frac{\rho_0}{2} \right] \right. \\
& - \left. \left. \left(1 + \frac{\rho_0}{2} \right) w^2 a^k (B_r) \right] v_j - v_j B^k - (B_r) h_j^k \right\} \\
& - 2(B_r) ((B_e) h_j^k)_a + (B_e) (v_j B^k - B_j a^k)_a \\
& + (B_e) h_j^k a^k - \frac{1}{2} ((B_e) h_j^k) B^k - \{ \bar{v}_e \} h_j^k B^k + \Delta^{kk} e_j
\end{aligned}$$

These factored by
 a^k in thesis; should be B^k

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$$\begin{aligned}
O = & (v_r e + e^u) \left[w^2 (a^k)^2 (Q_{v_j} - (B_r) (B_j + v_j)) \right. \\
& + \frac{\rho_0}{2} w^2 a^k \left(h_c c_s^2 w^2 v_j a^k + h_j^k \right) + a^k ((B_r) (B_j - v_j) \\
& - B^2 v_j) - (v_j B^k + (B_r) h_j^k) a^k - (B^2 h_j^k) \\
& + B_j B^k \frac{\rho_0}{2} a^k \left. \right] + e_j \Delta^{kk} + (B_e) \left[(v_j B^k - (B_r) h_j^k) a^k \right. \\
& - B_j (a^k)^2 - \frac{1}{2} B^k h_j^k \left. \right] + (B_e) h_j^k a^k - (\bar{v}_e) h_j^k B^k
\end{aligned}$$

$$\begin{aligned}
^{(4.418)} O = & (v_r e + e^u) \left[w^2 (a^k)^2 (Q_{v_j} - (B_r) (B_j + v_j)) \right. \\
& + \frac{\rho_0}{2} w^2 a^k \left(h_c c_s^2 w^2 v_j a^k + h_j^k \right) + a^k ((B_r) (B_j - v_j) \\
& - B^2 v_j) - (v_j B^k + (B_r) h_j^k) a^k - (B^2 h_j^k) \\
& + B_j B^k \frac{\rho_0}{2} a^k \left. \right] + e_j \Delta^{kk} + (B_e) \left[(v_j B^k - (B_r) h_j^k) a^k \right. \\
& - B_j (a^k)^2 - \frac{1}{2} B^k h_j^k \left. \right]
\end{aligned}$$

$$-\beta_j(a^k)^2 - \frac{1}{\omega^2} B^k h_j^{ik}] + (\beta e) h_j^{ik} a^k - (\bar{\nu} e) h_j^{ik} B^k$$