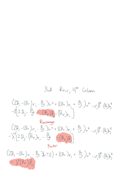
$$\frac{\partial \vec{F}^0}{\partial t} + \frac{\partial \vec{F}^i}{\partial x^i} = 0$$

$$\frac{\partial \vec{F}^0}{\partial t} + \frac{\partial \vec{F}^k}{\partial x^i}$$

$$\frac{\partial \vec{F}^0}{\partial \vec{p}}\frac{\partial \vec{p}}{\partial t} + \frac{\partial \vec{F}^k}{\partial \vec{p}}\frac{\partial \vec{p}}{\partial x^i} = 0$$



α — lapse $\qquad$ β_i - shift vector field

$$h_{ab} = g_{ab} + n_a n_b ; \quad n_a = (-\alpha, 0, 0, 0)$$

$$\bar{v}^k = v^k - \frac{\beta^k}{\alpha}$$

3:24 Nash

3:43

$$\mathcal{F}^0 \partial_t \vec{p} + \mathcal{F}^k \partial_i \vec{p} = 0$$

$$\partial_t \vec{p} + (\mathcal{F}^0)^{-1} \mathcal{F}^k \partial_i \vec{p} = 0$$

$\vec{e}$ is stand in components of right eigen vector

$$\vec{e} = (e^0, e^j, e^4, \hat{e}^j)^T$$

$$(\mathcal{F}^0)^{-1} \mathcal{F}^k \vec{e} = \lambda^k \vec{e}$$

$$\mathcal{F}^k \vec{e} = \lambda^k \mathcal{F}^0 \vec{e}$$

$$(\mathcal{F}^k - \lambda^k \mathcal{F}^0)\vec{e} = 0$$

$$\chi = \frac{\partial P}{\partial \rho_0} \qquad \gamma = \frac{\partial h_e}{\partial \rho_0} = 1 + \varepsilon + \chi$$

$$\kappa = \frac{\partial P}{\partial \varepsilon}$$

Sound speed

$$h_e c_s^2 = \rho_0 \chi + \frac{P}{\rho_0}\kappa \qquad Q = h_e W^2 + B^2$$

$$\frac{\mathcal{F}^0}{\sqrt{h}} = \begin{pmatrix} W & W^3 \rho_0 v_j & 0 & 0_j \\ W^2 \gamma v_i & h_{ij} Q + 2 h_e W^4 v_i v_j - B_i B_j & (\rho_0 + \kappa) W^2 v_i & 2 v_i B_j - B_i v_j - (Bv) h_{ij} \\ \gamma W^2 - W \chi & (2 h_e W^4 + B^2 - W^3 \rho_0) v_j - (Bv) B_j & (\rho_0 + \kappa) W^2 - \kappa & (2 - W^{-2}) B_j - (Bv) B_j - (Bv) v_j \\ 0^i & 0^i_j & 0^i & h^i_j \end{pmatrix}$$

$$
\frac{\mathcal{F}^k}{\alpha\sqrt{h}} = \begin{bmatrix}
W\bar{v}^k & W\rho_0(W^2 v_j \bar{v}^k + h_j{}^k) & 0^k & 0_j{}^k \\
W^2\gamma v_i \bar{v}^k + h_i{}^k \chi & A^k{}_{ij} & (\rho_0+\chi)W^2 v_i \bar{v}^k + h_i{}^k \chi & B^k{}_{ij} \\
(W^2\gamma - W - \chi)\bar{v}^k & C^k{}_j & (\rho_0+\chi)W^2\bar{v}^k - \chi(\bar{v}^k - v^k) & D^k{}_j \\
0^{ik} & B^i h_j{}^k - B^k h_j{}^i & 0^{ik} & h^i{}_j \bar{v}^k - h^k{}_j \bar{v}^i
\end{bmatrix}
$$

$$
A^k{}_{ij} = (h_{ij}Q + 2h_e W^4 v_i v_j - B_i B_j)\bar{v}^k + (Q v_i - (Bv)B_i)h_j{}^k + h_i{}^k((Bv)B_j - B^2 v_j) - (h_{ij}(Bv) + v_i B_j - 2B_i v_j)B^k
$$

$$
B^k{}_{ij} = (2v_i B_j - B_i v_j - (Bv)h_{ij})\bar{v}^k - \left(\frac{h_{ij}}{W^2} + v_i v_j\right)B^k + h_i{}^k\left((Bv)v_j + \frac{B_j}{W^2}\right) - h_j{}^k\left(\frac{B_i}{W^2} + (Bv)v_i\right)
$$

$$
C^k{}_j = (2h_e W^4 v_j - (Bv)B_j + B^2 v_j - \rho_0 W^3 v_j)\bar{v}^k + (Q - W\rho_0)h_j{}^k + ((Bv)B_j - B^2 v_j)v^k - B_j B^k
$$

$$
D^k{}_j = \left(2B_j - (Bv)v_j - \frac{B_j}{W^2}\right)\bar{v}^k + \left((Bv)v_j + \frac{B_j}{W^2}\right)v^k - v_j B^k - (Bv)h_j{}^k
$$

$$
\det|\mathcal{F}^k - \lambda^k \mathcal{F}^0| = (\alpha\sqrt{h})^8 \cdot
$$

$$
\begin{vmatrix}
W(\bar{v}^k - \lambda^k) & W^3\rho_0 v_j(\bar{v}^k - \lambda^k) + W\rho_0 h_j{}^k & 0^k & 0_j{}^k \\
W^2\gamma v_i(\bar{v}^k - \lambda^k) + h_i{}^k\chi & \breve{A}^k{}_{ij} & (\rho_0+\chi)W^2 v_i(\bar{v}^k - \lambda^k) + h_i{}^k\chi & \breve{B}^k{}_{ij} \\
(W^2\gamma - W - \chi)(\bar{v}^k - \lambda^k) + \chi v^k & \breve{C}^k{}_j & [(\rho_0+\chi)W^2 - \chi](\bar{v}^k - \lambda^k) + \chi v^k & \breve{D}^k{}_j \\
0^{ik} & B^i h_j{}^k - B^k h_j{}^i & 0^{ik} & h^i{}_j(\bar{v}^k - \lambda^k) - h^k{}_j \bar{v}^i
\end{vmatrix}
$$

$$
\breve{A}^k{}_{ij} = (h_{ij}Q + 2h_e W^4 v_i v_j - B_i B_j)(\bar{v}^k - \lambda^k) + (Q v_i - (Bv)B_i)h_j{}^k + h_i{}^k((Bv)B_j - B^2 v_j) - (h_{ij}(Bv) + v_i B_j - 2B_i v_j)B^k
$$

$$
\breve{B}^k{}_{ij} = (2v_i B_j - B_i v_j - (Bv)h_{ij})(\bar{v}^k - \lambda^k) - \left(\frac{h_{ij}}{W^2} + v_i v_j\right)B^k + h_i{}^k\left((Bv)v_j + \frac{B_j}{W^2}\right) - h_j{}^k\left(\frac{B_i}{W^2} + (Bv)v_i\right)
$$

$$
\breve{C}^k{}_j = (2h_e W^4 + B^2 - \rho_0 W^3)v_j(\bar{v}^k - \lambda^k) - (Bv)B_j(\bar{v}^k - \lambda^k) + (Q - W\rho_0)h_j{}^k + ((Bv)B_j - B^2 v_j)v^k - B_j B^k
$$

$$
\breve{D}^k{}_j = \left(2B_j - (Bv)v_j - \frac{B_j}{W^2}\right)(\bar{v}^k - \lambda^k) + \left((Bv)v_j + \frac{B_j}{W^2}\right)v^k - v_j B^k - (Bv)h_j{}^k
$$

$$a^k = \bar{r}^k - \lambda^k \qquad \Delta^{kk} = (a^k)^2 Q - 2(Br)_a{}^k B^k - \frac{B^k B^k}{w^2}$$

$$\begin{vmatrix} W a^k & W^3_{\beta} r_j a^k + W_\beta h_j{}^k & 0^k & 0_j{}^k \\ W^2 \gamma r_i a^k + h_i{}^k \chi & \breve{A}^k{}_{ij} & (\rho_0 + \chi) W^2 r_i a^k + h_i{}^k \chi & \breve{B}^k{}_{ij} \\ (W^2\gamma - W - \chi) a^k + \chi r^k & \breve{C}^k{}_j & [(\rho_0+\chi)W^2 - \chi] a^k + \chi r^k & \breve{D}^k{}_j \\ 0^{ik} & B^i h_j{}^k - B^k h_j{}^i & 0^{ik} & h^i{}_j a^k - h^k{}_j \bar{r}^i \end{vmatrix}$$

Add row 1 to row 3

$$\begin{vmatrix} W a^k & W^3_{\beta} r_j a^k + W_\beta h_j{}^k & 0^k & 0_j{}^k \\ W^2 \gamma r_i a^k + h_i{}^k \chi & \breve{A}^k{}_{ij} & (\rho_0 + \chi) W^2 r_i a^k + h_i{}^k \chi & \breve{B}^k{}_{ij} \\ (W^2\gamma - \chi) a^k + \chi r^k & \breve{C}^k{}_j + W^3_\beta r_j a^k + W_\beta h_j{}^k & [(\rho_0+\chi)W^2 - \chi] a^k + \chi r^k & \breve{D}^k{}_j \\ 0^{ik} & B^i h_j{}^k - B^k h_j{}^i & 0^{ik} & h^i{}_j a^k - h^k{}_j \bar{r}^i \end{vmatrix}$$

$R_2 \mathrel{+}= -v_i \cdot R_3$

$$\begin{vmatrix} W a^k & W^3_{\beta} r_j a^k + W_\beta h_j{}^k & 0^k & 0_j{}^k \\ \chi[h_i{}^k + v_i(a^k - v^k)] & \alpha^k{}_{ij} & \chi[h_i{}^k + v_i(a^k - v^k)] & \breve{B}^k{}_{ij} - v_i \breve{D}^k{}_j \\ (W^2\gamma - \chi) a^k + \chi r^k & \breve{C}^k{}_j + W^3_\beta r_j a^k + W_\beta h_j{}^k & [(\rho_0+\chi)W^2 - \chi] a^k + \chi r^k & \breve{D}^k{}_j \\ 0^{ik} & B^i h_j{}^k - B^k h_j{}^i & 0^{ik} & h^i{}_j a^k - h^k{}_j \bar{r}^i \end{vmatrix}$$

$$\begin{array}{cccc} W^2 \cancel{\gamma r_i} a^k + h_i{}^k \chi & \breve{A}^k{}_{ij} & \cancel{(\rho_0 + \chi) W^2 v_i a^k} + h_i{}^k \chi & \breve{B}^k{}_{ij} \\ -v_i[(W^2\cancel{\gamma} - \chi) a^k + \chi r^k] & -v_i(\breve{C}^k{}_j + W^3_\beta r_j a^k + W_\beta h_j{}^k) & -v_i[[\cancel{(\rho_0+\chi)W^2} - \chi] a^k + \chi r^k] & -v_i \breve{D}^k{}_j \\ \hline \chi[h_i{}^k + v_i(a^k - v^k)] & \alpha^k{}_{ij} & h_i{}^k \chi + v_i \chi a^k - v_i \chi r^k & \\ & & \chi[h_i{}^k + v_i(a^k - v^k)] & \end{array}$$

$$C_1 \doteq -\frac{\chi}{\mathcal{K}} C_3$$

$$\begin{vmatrix}
W a^k & W^3_{\rho_0 v_j} a^k + W_{\rho_0} h_j^{\,k} & 0^k & 0_j^{\,k} \\
0_i^{\,k} & \alpha^k{}_{ij} & \chi\left[h_i^{\,k} + v_i(a^k - r^k)\right] & \tilde{B}^k{}_{ij} - v_i \tilde{D}^k{}_j \\
W^2 a^k\left[\gamma - \chi\left(1+\frac{\rho_0}{\mathcal{K}}\right)\right] & \tilde{C}^k_j + W^3_{\rho_0 v_j} a^k + W_{\rho_0} h_j^{\,k} & \left[(\rho_0+\mathcal{K})W^2 - \mathcal{K}\right]a^k + \mathcal{K} r^k & \tilde{D}^k{}_j \\
0^{ik} & B^i h_j^{\,k} - B^k h_j^{\,i} & 0^{ik} & h^i{}_j a^k - h^k_j r^i
\end{vmatrix}$$

$$(W^2\gamma - \chi)a^k + \chi r^k - \frac{\chi}{\mathcal{K}}\left\{\left[(\rho_0+\mathcal{K})W^2 - \mathcal{K}\right]a^k + \mathcal{K} r^k\right\}$$

$$W^2\gamma a^k \cancel{- \chi a^k} \cancel{+ \chi r^k} - \frac{\chi}{\mathcal{K}}\rho_0 W^2 a^k - \chi W^2 a^k \cancel{+ \chi a^k} \cancel{- \chi r^k}$$

$$W^2 a^k\left[\gamma - \chi\left(1+\frac{\rho_0}{\mathcal{K}}\right)\right]$$

$$R_3 \doteq -W\left[\gamma - \chi\left(1+\frac{\rho_0}{\mathcal{K}}\right)\right] R_1$$

$$\begin{vmatrix}
W a^k & W^3_{\rho_0 v_j} a^k + W_{\rho_0} h_j^{\,k} & 0^k & 0_j^{\,k} \\
0_i^{\,k} & \alpha^k{}_{ij} & \chi\left[h_i^{\,k} + v_i(a^k - r^k)\right] & \tilde{B}^k{}_{ij} - v_i \tilde{D}^k{}_j \\
0^k & \xi^k_j & \left[(\rho_0+\mathcal{K})W^2 - \mathcal{K}\right]a^k + \mathcal{K} r^k & \tilde{D}^k{}_j \\
0^{ik} & B^i h_j^{\,k} - B^k h_j^{\,i} & 0^{ik} & h^i{}_j a^k - h^k_j r^i
\end{vmatrix}$$

$$\tilde{C}^k_j + W^3_{\rho_0 v_j} a^k + W_{\rho_0} h_j^{\,k} - W\left[\gamma - \chi\left(1+\frac{\rho_0}{\mathcal{K}}\right)\right]\left[W^3_{\rho_0} v_j a^k + W_{\rho_0} h_j^{\,k}\right]$$

$$\xi_j^{\,k} = \tilde{C}_j^{\,k} + \left(W^3_{\rho_0} v_j a^k + W_{\rho_0} h_j^{\,k}\right)\left(1 - W\left(\gamma - \chi\left(1+\frac{\rho_0}{\mathcal{K}}\right)\right)\right)$$

$$\gamma - \chi\left(1+\frac{\rho_0}{\mathcal{K}}\right) = \frac{h_e}{\rho_0 \mathcal{K}}\left(\mathcal{K} - \rho_0 c_s^2\right)$$

$$\frac{\mathscr{F}^k}{\alpha\sqrt{h}} = \begin{pmatrix} W\bar{v}^k & W\rho_0(W^2 v_j \bar{v}^k + h_j{}^k) & 0^k & 0_j{}^k \\ W^2\gamma v_i \bar{v}^k + h_i{}^k\chi & A^k{}_{ij} & (\rho_0+\kappa)W^2 v_i \bar{v}^k + h_i{}^k\kappa & B^k{}_{ij} \\ (W^2\gamma - W - \chi)\bar{v}^k & C^k{}_j & (\rho_0+\kappa)W^2\bar{v}^k - \kappa(\bar{v}^k - v^k) & D^k{}_j \\ 0^{ik} & B^i h_j{}^k - B^k h_j{}^i & 0^{ik} & h^i{}_j \bar{v}^k - h^k{}_j \bar{v}^i \end{pmatrix}$$

$$D^k{}_j = (2B_j - (Bv)v_j - B_j/W^2)\bar{v}^k + ((Bv)v_j + B_j/W^2)v^k - v_j B^k - (Bv)h_j{}^k$$

$$\frac{\mathscr{F}^0}{\sqrt{h}} = \begin{pmatrix} W & W^3\rho_0 v_j & 0 & 0_j \\ W^2\gamma v_i & h_{ij}Q + 2h_e W^4 v_i v_j - B_i B_j & (\rho_0+\kappa)W^2 v_i & 2v_i B_j - B_i v_j - (Bv)h_{ij} \\ \gamma W^2 - W - \chi & (2h_e W^4 + B^2 - W^3\rho_0)v_j - (Bv)B_j & (\rho_0+\kappa)W^2 - \kappa & (2 - W^{-2})B_j - (Bv)B_j - (Bv)v_j \\ 0^i & 0^i{}_j & 0^i & h^i{}_j \end{pmatrix}$$

$$(\mathscr{F}^k - \lambda^k \mathscr{F}^0),$$

$$D^k_j - \lambda^k\left[(2-W^{-2})B_j - (Bv)B_j - (Bv)v_j\right]$$

$$\left(2B_j - (Bv)v_j - \frac{B_j}{W^2}\right)\bar{v}^k + \left((Bv)v_j + \frac{B_j}{W^2}\right)v^k - v_j B^k - (Bv)h_j^k - \lambda^k\left[(2-W^{-2})B_j - (Bv)B_j - (Bv)v_j\right]$$

$$\left(2B_j - (Bv)v_j - \frac{B_j}{W^2}\right)\bar{v}^k + \left((Bv)v_j + \frac{B_j}{W^2}\right)v^k - v_j B^k - (Bv)h_j^k - \lambda^k\left[(2-W^{-2})B_j - (Bv)B_j - (Bv)v_j\right]$$

$$\left(2B_j - (Bv)v_j - \frac{B_j}{W^2}\right)\bar{v}^k + \left((Bv)v_j + \frac{B_j}{W^2}\right)v^k - v_j B^k - (Bv)h_j^k - \lambda^k\left[2B_j - \frac{B_j}{W^2} - (Bv)B_j - (Bv)v_j\right]$$

$$\left(2B_j - (Bv)v_j - \frac{B_j}{W^2}\right)\bar{v}^k + \left((Bv)v_j + \frac{B_j}{W^2}\right)v^k - v_j B^k - (Bv)h_j^k - \lambda^k\left[2B_j - (Bv)v_j - \frac{B_j}{W^2} - (Bv)B_j\right]$$

$$\left(2B_j - (Bv)v_j - \frac{B_j}{W^2}\right)(\bar{v}^k - \lambda^k) + \left((Bv)v_j + \frac{B_j}{W^2}\right)v^k - v_j B^k - (Bv)h_j^k - \lambda^k(Bv)B_j$$

$$det \mid \mathscr{F}^k - \lambda^k\mathscr{F}^0 \mid = (\alpha\sqrt{h})^8 \begin{vmatrix} W(\bar{v}^k - \lambda^k) & W^3\rho_0 v_j(\bar{v}^k - \lambda^k) + W\rho_0 h_j{}^k & 0^k & 0_j{}^k \\ W^2\gamma v_i(\bar{v}^k - \lambda^k) + h_i{}^k\chi & \tilde{A}^k{}_{ij} & (\rho_0+\kappa)W^2 v_i(\bar{v}^k - \lambda^k) + h_i{}^k\kappa & \tilde{B}^k{}_{ij} \\ (W^2\gamma - W - \chi)(\bar{v}^k - \lambda^k) + \chi v^k & \tilde{C}^k{}_j & [(\rho_0+\kappa)W^2 - \kappa](\bar{v}^k - \lambda^k) + \kappa v^k & \tilde{D}^k{}_j \\ 0^{ik} & B^i h_j{}^k - B^k h_j{}^i & 0^{ik} & h^i{}_j(\bar{v}^k - \lambda^k) - h^k{}_j \bar{v}^i \end{vmatrix}$$

$$\tilde{D}^k{}_j = (2B_j - (Bv)v_j - B_j/W^2)(\bar{v}^k - \lambda^k) + ((Bv)v_j + B_j/W^2)v^k - v_j B^k - (Bv)h_j{}^k$$

$$\tilde{D}^k_j = D^k_j - \lambda^k\left[(2-W^{-2})B_j - (Bv)B_j - (Bv)v_j\right]$$

$$\tilde{D}^k_j = \left(2B_j - (B_v)v_j - \frac{B_j}{W^2}\right)(v^k - \lambda^k) + \left((B_v)v_j + \frac{B_j}{W^2}\right)v^k - v_j B^k - (B_v)h_j^k$$

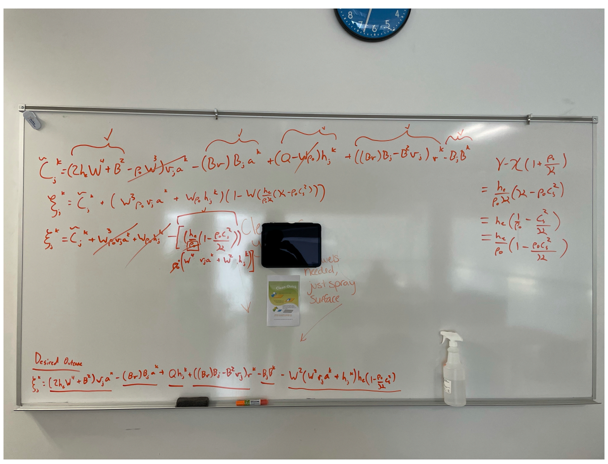
$$= \left(2B_j - (B_v)v_j - \frac{B_j}{W^2}\right)(v^k - \lambda^k) + \left((B_v)v_j + \frac{B_j}{W^2}\right)v^k - v_j B^k - (B_v)h_j^k - \lambda^k (B_v) B_j$$

$$\xi_j^k = (2h_e W^4 + B^2) v_j a^k - (B_v)B_j a^k + Q h_j^k + ((B_v)B_j - B^2 v_j) v^k - B_j B^k$$
$$- W^2(W^2 v_j a^k + h_j^k) h_e \left(1 - \frac{\rho_0}{\chi} c_s^2\right)$$

$$\tilde{\zeta}_j^k = (2h_e W^4 + B^2 - \rho_0 W^3) v_j (v^k - \lambda^k) - (B_v)B_j(v^k - \lambda^k) + (Q - W\rho_0) h_j^k + ((B_v)B_j - B^2 v_j)v^k - B_j B^k$$

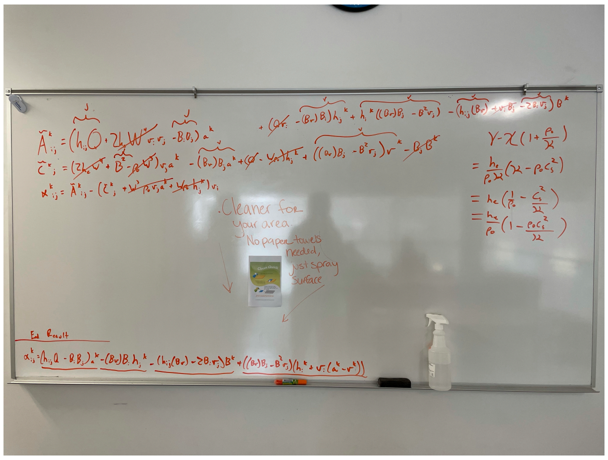
$$= (2h_e W^4 + B^2 - \rho_0 W^3) v_j a^k - (B_v)B_j a^k + (Q - W\rho_0) h_j^k + ((B_v)B_j - B^2 v_j)v^k - B_j B^k$$

$$\xi_j^k = \tilde{\zeta}_j^k + (W^3 \rho_0 v_j a^k + W\rho_0 h_j^k)\left(1 - W\left(\gamma - \chi\left(1 + \frac{\rho_0}{\chi}\right)\right)\right)$$



$$1 - \frac{h_e W}{\rho_0 \chi}(\chi - \rho_0 c_s^2)$$

$$\alpha^k_{ij} = (h_{ij} Q - B_i B_j) a^k - (B_v) B_i h_j^k - (h_{ij}(B_v) - 2B_i v_j) B^k$$
$$+ ((B_v)B_j - B^2 v_j)(h_i^k + v_i(a^k - v^k))$$



$$C_2 \stackrel{+}{=} -\left((Bv)B_j - B^2 v_j\right)/\varkappa \cdot C_3$$

$$\begin{vmatrix} W a^k & W^3_{\beta_0 v_j} a^k + W_{\beta_0} h_j^k & 0^k & 0_j^k \\ 0_i^k & \tilde{\alpha}^k_{ij} & \varkappa[h_i^k + v_i(a^k - v^k)] & \tilde{B}^k_{ij} - v_i \tilde{D}^k_j \\ 0^k & \tilde{\xi}^k_{j} & [(\beta_0+\varkappa)W^2 - \varkappa]a^k + \varkappa v^k & \tilde{D}^k_j \\ 0^{ik} & B^i h_j^k - B^k h_j^i & 0^{ik} & h^i_j a^k - h^k_j v^i \end{vmatrix}$$

$$\tilde{\alpha}^k_{ij} = (h_{ij}Q - B_i B_j)a^k - (Bv)B_i h_j^k - (h_{ij}(Bv) - 2B_i v_j)B^k$$

$$\tilde{\xi}_j^k = (2h_e W^4 + \epsilon B^2) v_j a^k + Q h_j^k - B_j B^k - W^2(W^2 v_j a^k + h_j^k) h_e \left(1 - \frac{\beta_0}{\varkappa} c_s^2\right)$$

$$\alpha^k_{ij} = (h_{ij}Q - B_i B_j)a^k - (Bv)B_i h_j^k - (h_{ij}(Bv) - 2B_i v_j)B^k$$
$$+ \cancel{((Bv)B_j - B^2 v_j)(h_i^k + v_i(a^k - v^k))}$$

$$\tilde{\alpha}^k_{ij} = \alpha^k_{ij} - \cancel{\frac{\varkappa[h_i^k + v_i(a^k - v^k)]\cdot((Bv)B_j - B^2 v_j)}{\varkappa}}$$

$$\tilde{\alpha}^k_{ij} = (h_{ij}Q - B_i B_j)a^k - (B v) B_i h^k_j - (h_{ij}(B v) - 2 B_i v_j) B^k$$

$$\xi^k_j = \underline{(2h_e W^4 + B^2) v_j a^k} - (B v) B_j a^k + \underline{Q h^k_j} + \underline{\cancel{((B v) B_j - B^2 v_j)}} v^k \underline{- B_j B^k}$$
$$\underline{- W^2 (W^2 v_j a^k + h^k_j) h_e \left(1 - \frac{\rho_0}{\chi} c_s^2\right)}$$

$$\tilde{\xi}^k_j = \xi^k_j - \frac{((B v) B_j - B^2 v_j)}{\chi} \left[[(\rho_0 + \chi) W^2 - \chi] a^k + \chi v^k \right]$$
$$\underline{- ((B v) B_j - B^2 v_j) \left[\left[\left(\frac{\rho_0}{\chi} + 1\right) W^2 - 1 \right] a^k \cancel{+ v^k} \right]}$$

$$\tilde{\xi}^k_j = \underline{(2h_e W^4 + B^2) v_j a^k} + \underline{Q h^k_j} \underline{- B_j B^k} \underline{- W^2 (W^2 v_j a^k + h^k_j) h_e \left(1 - \frac{\rho_0}{\chi} c_s^2\right)}$$
$$\underline{- W^2 a^k \left(1 + \frac{\rho_0}{\chi}\right) ((B v) B_j - B^2 v_j)}$$

$$- B^2 v_j a^k \, ?$$

$$+ \epsilon B^2$$

This term shouldn't be here; for me, $\epsilon = 0$

$$C_4 \stackrel{+}{=} -\left((Bv)_{v_j} + B_j/W^2\right)/\varkappa \cdot C_3$$

$$\begin{vmatrix} W a^k & W^3_{\beta_0 v_j} a^k + W_{\beta_0} h_j^k & 0^k & 0_j^k \\ 0_i^k & \tilde{\alpha}^k_{ij} & \varkappa\left[h_i^k + v_i(a^k - v^k)\right] & B^k_{ij} \\ 0^k & \tilde{\xi}^k_j & \left[(\beta_0+\varkappa)W^2 - \varkappa\right]a^k + \varkappa v^k & \delta^k_j \\ 0^{ik} & B^i h_j^k - B^k h_j^i & 0^{ik} & h^i_j a^k - h^k_j v^i \end{vmatrix}$$

$$\beta^k_{ij} = -\left(B_i v_j + (Bv) h_{ij}\right) a^k - \left(h_{ij} B^k + h_j^k B_i\right)\frac{1}{W^2} +$$

$$\epsilon\left((Bv)B_j - B^2 v_j\right)\left(h_i^k + v_i(a^k - v^k)\right)$$

$$\delta^k_j = 2B_j a^k - v_j B^k - (Bv)h_j^k - W^2 a^k\left((Bv)_{v_j} + B_j \frac{1}{W^2}\right)\left(1 + \frac{\beta_0}{\varkappa}\right)$$

$$B^k_{ij} = \tilde{B}^k_{ij} - v_i \tilde{D}^k_j - \left((Bv)_{v_j} + B_j/W^2\right)\cancel{/\varkappa} \cdot \cancel{\varkappa}\left[h_i^k + v_i(a^k - v^k)\right]$$

$$B^k_{ij} = \tilde{B}^k_{ij} - v_i \tilde{D}^k_j - \left((Bv)_{v_j} + B_j/W^2\right) \cdot \left[h_i^k + v_i(a^k - v^k)\right]$$

$$\tilde{B}^k_{ij} = \left(\cancel{2v_i B_j} - B_i v_j - (Bv)h_{ij}\right)\left(a^k\right) - \left(\frac{h_{ij}}{W^2} + \cancel{v_i v_j}\right)B^k + h_i^k\left(\cancel{(Bv)_{v_j}} + \cancel{\frac{B_j}{W^2}}\right) - h_j^k\left(\frac{B_i}{W^2} + \cancel{(Bv)v_i}\right)$$

$$-v_i \tilde{D}^k_j = -\left[\left(\cancel{2B_j} - \cancel{(Bv)_{v_j}} - \cancel{\frac{B_j}{W^2}}\right)\left(a^k\right) + \left(\cancel{(Bv)_{v_j} + \frac{B_j}{W^2}}\right)v^k - \cancel{v_j B^k} - \cancel{(Bv)h_j^k}\right]v_i$$

$$-\cancel{(Bv)_{v_j}} h_i^k - \cancel{B_j/W^2}\, h_i^k - \cancel{(Bv)_{v_j} v_i (a^k - v^k)} - B_j/W^2\, v_i\cancel{(a^k - v^k)}$$

$$\beta^k_{;ij} = -\left(\underline{B_{;i} v_{;j}} + \underline{(B_v) h_{;ij}}\right) a^k - \left(\underline{h_{;ij}\, B^k} + \underline{h_{;j}{}^k B_{;i}}\right)\frac{1}{w^2} + \underbrace{\left((B_v) B_{;j} - B^2 v_{;j}\right)\left(h_{;i}{}^k + v_{;i}\left(a^k - v^k\right)\right)}$$

$$(B_v) B_{;j} h_{;i}{}^k + (B_v) B_{;j} v_{;i}\left(a^k - v^k\right) - B^2 v_{;j} h_{;i}{}^k - B^2 v_{;i}\left(a^k - v^k\right)$$

These terms shouldn't be here; add ϵ and for me, $\epsilon = 0$

$$-\left(B_{;i} v_{;j} + B_v h_{;ij}\right) a^k$$

$$\delta_j{}^k = \tilde{D}_j{}^k - \left((B_v) v_{;j} + B_{;j}/w^2\right) / \cancel{\mathcal{K}} \circ \left[\left[(\rho_0 + \mathcal{K}) w^2 - \mathcal{K}\right] a^k + \mathcal{K} v^k\right]$$

$$\downarrow$$

$$\delta_j{}^k = \tilde{D}_j{}^k - \left((B_v) v_{;j} + B_{;j}/w^2\right) \cancel{\mathcal{K}} \left[\left[\left(\frac{\rho_0}{\mathcal{K}} + 1\right) w^2 - 1\right] a^k + v^k\right]$$

$$\tilde{D}_j{}^k = \left(\underline{2B_{;j}} - \cancel{(B_v) v_{;j}} - \cancel{\frac{B_{;j}}{w^2}}\right)\left(a^k\right) + \left(\cancel{(B_v) v_{;j}} + \cancel{\frac{B_{;j}}{w^2}}\right) v^k \underline{- v_{;j} B^k} - \underline{(B_v) h_{;j}{}^k}$$

$$-(B_v) v_{;j}\left[\underline{\left(\frac{\rho_0}{\mathcal{K}} + 1\right)} w^2 - \cancel{1}\right] a^k - \cancel{(B_v) v_{;j} v^k} - \frac{B_{;j}}{w^2}\left[\underline{\left(\frac{\rho_0}{\mathcal{K}} + 1\right)} w^2 - \cancel{1}\right] a^k$$

$$\cancel{-B_j/\omega^2 \cdot v^k}$$

$$\delta_j^{\ k} = \underline{2B_j a^k} - \underline{v_j B^k} - \underline{(Bv) h_j^{\ k}} - \underline{W^2 a^k \left((Bv) v_j + B_j \frac{1}{W^2}\right)\left(1+\frac{\beta}{\chi}\right)}$$

Factor out $\left(\frac{1}{a^k}\right)^3$, $C_2 \mathrel{+}= B^k C_4$

$$\left(\frac{1}{a^k}\right)^3 \begin{vmatrix} W a^k & \left(W^3_{\beta} v_j a^k + W_\beta h_j^{\ k}\right) a^k & 0^k & 0_j^{\ k} \\ 0_i^{\ k} & a^k \tilde{\alpha}^k_{;ij} + B^k \beta^k_{;ij} & \chi\left[h_i^{\ k} + v_i(a^k - v^k)\right] & B^k_{;j} \\ 0^k & a^k \tilde{\xi}^{\ k}_{ij} + B^k \delta^k_{\ j} & \left[(\beta+\chi)W^2 - \chi\right]a^k + \chi v^k & \delta^k_{\ j} \\ 0^{ik} & B^i h_j^{\ k} a^k - B^k h_j^{\ k} v^i & 0^{ik} & h^i_{\ j} a^k - h^k_{\ j} v^i \end{vmatrix}$$

$$C_2 \pm= -B^j C_4 \quad h_j^{\ k}$$

$$\left(\frac{1}{a^k}\right)^3 \begin{vmatrix} W a^k & \left(W^3_{\beta} v_j a^k + W_\beta h_j^{\ k}\right) a^k & 0^k & 0_j^{\ k} \\ 0_i^{\ k} & a^k \tilde{\alpha}^k_{;ij} + B^k \beta^k_{;ij} - B^l \beta^k_{;il} h_j^{\ k} & \chi\left[h_i^{\ k} + v_i(a^k - v^k)\right] & B^k_{;j} \\ 0^k & a^k \tilde{\xi}^{\ k}_{ij} + B^k \delta^k_{\ j} - B^l \delta^k_{\ l} h_j^{\ k} & \left[(\beta+\chi)W^2 - \chi\right]a^k + \chi v^k & \delta^k_{\ j} \\ 0^{ik} & 0_j^{\ ik} & 0^{ik} & h^i_{\ j} a^k - h^k_{\ j} v^i \end{vmatrix}$$

$$a^k \tilde{\alpha}^k_{;ij} + B^k \beta^k_{;ij} - B^l \beta^k_{;il} h_j^{\ k} = h_{ij}\left[(a^k)^2 Q - 2(Bv)a^k B^k - \frac{1}{W^2} B^k B^k\right]$$
$$- B_i\left[(a^k)^2 B_j - a^k\left((Bv)h_j^{\ k} + v_j B^k\right) - \frac{1}{W^2} h_j^{\ k} B^k\right]$$

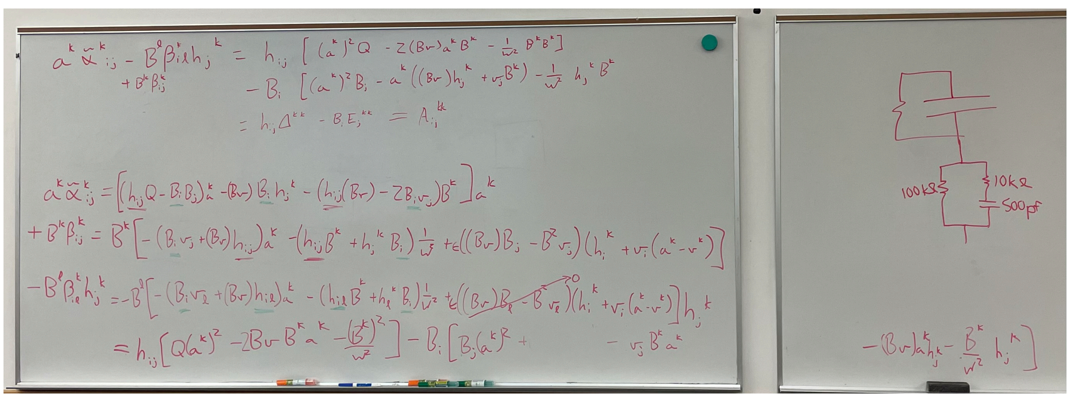
$$= h_{;ij}\,\Delta^{kk} - B_i\, E_j^{kk}$$

$$= A_{;j}^{kk}$$

$$a^k \tilde{\alpha}^k_{ij} = \left[(h_{ij} Q - B_i B_j)_a^k - (Bv) B_i h_j^k - \left(h_{ij}(Bv) - 2B_i v_j\right) B^k \right] a^k$$

$$+ B^k \beta^k_{ij} = \left[-\left(B_i v_j + (Bv) h_{ij}\right) a^k - \left(h_{ij} B^k + h_j^k B_i\right)\frac{1}{W^2} + \epsilon\left((Bv)B_j - B^2 v_j\right)\left(h_i^{\,k} + v_i\left(a^k - v^k\right)\right)\right] B^k$$

$$-B^j \beta^k_{ij} h_j^k = -\left[-\left(B_i v_j + (Bv) h_{ij}\right) a^k - \left(h_{ij} B^k + h_j^k B_i\right)\frac{1}{W^2} + \epsilon\left((Bv)B_j - B^2 v_j\right)\left(h_i^{\,k} + v_i\left(a^k - v^k\right)\right)\right] B^j h_j^k$$



$$a^k \tilde{\xi}_j^k + B^k \delta_j^k - B^l \delta^k_l h_j^k = a^k a^k \left[W^2\left(W v_j - (Bv)B_j\right) + \frac{\rho_0}{\mathcal{H}} W^2\left(h_e W^2 c_s^2 v_j + B^2 v_j - (Bv)B_j\right)\right]$$

$$+ a^k \Big[h_j^k W^2 (Bv)^2 - W^2 (Bv) v_j B^k$$

$$+ \frac{\rho_0}{\mathcal{H}}\left(\left(B^2 + W^2 (Bv)^2 + h_e W^2 c_s^2\right) h_j^k - B^k\left(W^2 (Bv) v_j + B_j\right)\right)\Big]$$

$$- B^k \left[h_j^k (Bv) - v_j B^k \right]$$

$$= C_j^{kk}$$

$$a^k \tilde{\xi}^k_j = a^k \Big[(2h_e W^4 + \epsilon B^2) v_j a^k + Q h_j^k - B_j B^k - W^2 (W v_j a^k + h_j^k) h_e \left(1 - \frac{\rho_0}{\chi} c_s^2\right) - W^2 a^k \left(1 + \frac{\rho_0}{\chi}\right) \left((Bv) B_j - B^2 v_j\right) \Big]$$

$$+ B^k \zeta^k_j = B^k \Big[2 B_j a^k - v_j B^k - (Bv) h_j^k - W^2 a^k \left((Bv) v_j + B_j \frac{1}{W^2}\right)\left(1 + \frac{\rho_0}{\chi}\right) \Big]$$

$$- B^l \zeta^k_l h_j^k = - B^l h_j^k \Big[2 B_l a^k - v_l B^k - (Bv) h_l^k - W^2 a^k \left((Bv) v_l + B_l \frac{1}{W^2}\right)\left(1 + \frac{\rho_0}{\chi}\right) \Big]$$

$$= a^k a^k \Big[\cancel{2} h_e W^4 v_j + \epsilon B^2 v_j - \cancel{W^4 v_j h_e} + W^4 v_j h_e \frac{\rho_0}{\chi} c_s^2 - W^2 (Bv) B_j + W^2 B^2 v_j - W^2 \frac{\rho_0}{\chi} (Bv) B_j + W^2 \frac{\rho_0}{\chi} B^2 v_j \Big]$$

Mine

$$= a^k a^k \Big[W^2 \left(W v_j - (Bv) B_j\right) + \frac{\rho_0}{\chi} W^2 \left(h_e W^2 c_s^2 v_j + B^2 v_j - (Bv) B_j\right) \Big]$$

Thesis

$$h_e \cancel{W^2} h_j^k + \cancel{B^2 h_j^k}$$

$$= a^k \Big[\cancel{Q h_j^k} - \cancel{B_j B^k} - \cancel{W^2 h_j^k} h_e + W^2 h_j^k h_e \frac{\rho_0}{\chi} c_s^2 + \cancel{2 B_j B^k}$$

$$- W^2 B^k (Bv) v_j - W^2 B^k (Bv) v_j \frac{\rho_0}{\chi} - \cancel{W^2 \frac{1}{W^2}} B_j B^k - B_j B^k \frac{\rho_0}{\chi}$$

$$- \cancel{2 B^2 h_j^k} + W^2 (Bv)^2 h_j^k + W^2 (Bv)^2 h_j^k \frac{\rho_0}{\chi}$$

$$+ \cancel{B^2 h_j^k} + B^2 h_j^k \frac{\rho_0}{\chi} \Big]$$

Mine

$$Q = h_e W^2 + B^2$$

$$= a^k \Big[\underline{h_j{}^k W^2 (Bv)^2} - \underline{W^2 (Bv) v_j B^k}$$
$$+ \frac{\rho_0}{\chi}\Big(\big(\underline{B^2} + \underline{W^2 (Bv)^2} + \underline{h_e W^2 c_s{}^2}\big) h_j{}^k - B^k\big(\underline{W^2 (Bv) v_j} + \underline{B_j}\big)\Big)\Big]$$ Thesis

$$= -B^k \left(v_j B^k + (Bv) h_j{}^k \cancel{-(Bv) h_j{}^k} \cancel{-(Bv) h_j{}^k} \right)$$
$$= -B^k \left(v_j B^k - (Bv) h_j{}^k \right)$$

$$C_j{}^{kk} = a^k a^k \Big[W^2\big(W_{v_j \epsilon} + (h_e W^2_{v_j} + B^2_{v_j})_\mu - (Bv) B_j \big) + \frac{\rho_0}{\chi} W^2\big(h_e W c_s{}^2{}_{v_j} + B^2_{v_j} - (Bv) B_j \big) \Big]$$
$$+ a^k \Big[h_j{}^k W^2 (Bv)^2 - W^2 (Bv) v_j B^k + \frac{\rho_0}{\chi}\Big(\big(B^2 + W^2 (Bv)^2 + h_e W^2 c_s{}^2\big) h_j{}^k - B^k\big(W^2 (Bv) v_j + B_j\big)\Big)\Big]$$
$$- B^k \left[h_j{}^k (Bv) - v_j B^k \right] \zeta$$

$$\left(\frac{1}{a^k}\right)^3 \begin{vmatrix} W a^k & (W^3_{\rho_0 v_j} a^k + W_{\rho_0} h_j{}^k) a^k & 0^k & 0_j{}^k \\ 0_i{}^k & A_{ij}{}^{kk} & \chi\left[h_i{}^k + v_i(a^k - v^k)\right] & B^k_{ij} \\ 0^k & C_j{}^{kk} & W^2 a^k(\rho_0 + \chi) - \chi(a^k - v^k) & \delta^k_j \\ 0^{ik} & 0_j{}^{ik} & 0^{ik} & h^i_j a^k - h^k_j \bar{v}^i \end{vmatrix}$$

$$W a^k (\alpha^2 h)^4 \left(\frac{1}{a^k}\right)^3 \begin{vmatrix} A_{ij}{}^{kk} & \chi\left[h_i{}^k + v_i(a^k - v^k)\right] & B^k_{ij} \\ C_j{}^{kk} & W^2 a^k(\rho_0 + \chi) - \chi(a^k - v^k) & \delta^k_j \\ 0_j{}^{ik} & 0^{ik} & h^i_j a^k - h^k_j \bar{v}^i \end{vmatrix}$$

$$\begin{bmatrix} a^k & 0 & -\bar{v}^1 \\ 0 & a^k & -\bar{v}^2 \\ 0 & 0 & -\lambda^k \end{bmatrix}$$

$$\left(\alpha^2 h\right)^4 \left(\frac{1}{a^k}\right)^3 W_a^k\left(-\lambda^k\right)\left(a^k\right)^2 \begin{vmatrix} A_{ij}^{kk} & \mathcal{K}\left(h_i^k + v_i\left(a^k - v^k\right)\right) \\ c_j^{kk} & W_a^{2k}\left(\rho_0+\lambda\right) - \mathcal{K}\left(a^k - v^k\right) \end{vmatrix}$$

$$\begin{vmatrix} a_{ij} & v_j \\ c_j & x \end{vmatrix} = \det\left(a_{ij}\right) \cdot \left[x - \vec{c}^{\,T}\left(a_{ij}\right)^{-1}\vec{v}\right]$$

$$A_{ij}^{kk} = \begin{bmatrix} h_{11}\Delta - B_1E_1 & h_{12}\Delta - B_1E_2 & h_{13}\Delta - B_1E_3 \\ h_{21}\Delta - B_2E_1 & h_{22}\Delta - B_2E_2 & h_{23}\Delta - B_2E_3 \\ h_{31}\Delta - B_3E_1 & h_{32}\Delta - B_3E_2 & h_{33}\Delta - B_3E_3 \end{bmatrix}$$

```
In[5]:= a = {{h11 * d - b1 * e1, h12 * d - b1 * e2, h13 * d - b1 * e3},
         {h21 * d - b2 * e1, h22 * d - b2 * e2, h23 * d - b2 * e3},
         {h31 * d - b3 * e1, h32 * d - b3 * e2, h33 * d - b3 * e3}};
        a // MatrixForm
```

Out[6]//MatrixForm=

$$\begin{pmatrix} -b1\,e1 + d\,h11 & -b1\,e2 + d\,h12 & -b1\,e3 + d\,h13 \\ -b2\,e1 + d\,h21 & -b2\,e2 + d\,h22 & -b2\,e3 + d\,h23 \\ -b3\,e1 + d\,h31 & -b3\,e2 + d\,h32 & -b3\,e3 + d\,h33 \end{pmatrix}$$

```
In[8]:= Det[a] // Simplify
```

Out[8]= d^2 (b3 (e3 h12 h21 − e2 h13 h21 − e3 h11 h22 + e1 h13 h22 + e2 h11 h23 − e1 h12 h23) +
b1 e3 h22 h31 − d h13 h22 h31 − b1 e2 h23 h31 + d h12 h23 h31 − b1 e3 h21 h32 + d h13 h21 h32 +
b1 e1 h23 h32 − d h11 h23 h32 + b1 e2 h21 h33 − d h12 h21 h33 − b1 e1 h22 h33 + d h11 h22 h33 +
b2 (− e3 h12 h31 + e2 h13 h31 + e3 h11 h32 − e1 h13 h32 − e2 h11 h33 + e1 h12 h33))

$$\Delta^3\left(h_{11}\left(h_{22}h_{33} - h_{23}h_{32}\right) + h_{12}\left(h_{23}h_{31} - h_{21}h_{33}\right) + h_{13}\left(h_{21}h_{32} - h_{22}h_{31}\right)\right)$$
$$= h\,\Delta^{kk}$$

$$+\left[B_1E_1\left(h_{23}h_{32} - h_{22}h_{33}\right) + B_1E_2\left(h_{21}h_{33} - h_{23}h_{31}\right) + B_1E_3\left(h_{22}h_{31} - h_{21}h_{32}\right)\right.$$
$$+ B_2E_1\left(h_{12}h_{33} - h_{13}h_{32}\right) + B_2E_2\left(h_{13}h_{31} - h_{11}h_{33}\right) + B_2E_3\left(h_{11}h_{32} - h_{12}h_{31}\right)$$

$$+\underbrace{B_3E_1(h_{13}h_{22}-h_{12}h_{23}) + B_3E_2(h_{11}h_{23}-h_{13}h_{21}) + B_3E_3(h_{12}h_{21}-h_{11}h_{22})}_{C^{ij}B_iE_j}\Big]\Delta^2$$

$$C^{ij} = \begin{bmatrix} h_{23}h_{32}-h_{22}h_{33} & h_{21}h_{33}-h_{23}h_{31} & h_{22}h_{31}-h_{21}h_{32} \\ h_{12}h_{33}-h_{13}h_{32} & h_{13}h_{31}-h_{11}h_{33} & h_{11}h_{32}-h_{12}h_{31} \\ h_{13}h_{22}-h_{12}h_{23} & h_{11}h_{23}-h_{13}h_{21} & h_{12}h_{21}-h_{11}h_{22} \end{bmatrix}$$

$$h = h_{ij}C^{ij}$$
$$C^{ij} = h\cdot h^{ij}$$

$$|A_{ij}{}^{kk}| = h\cdot(\Delta^{kk})^3 - hB^jE_j{}^{kk}(\Delta^{kk})^2$$
$$= h\cdot(\Delta^{kk})^2(\Delta^{kk}-B^jE_j{}^{kk})$$

$$\Delta^{kk} - B^jE_j{}^{kk} = A_j{}^{jkk} = (a^k)^2 h_e W^2$$

$$Q = h_e W^2 + B^2$$

$$\Delta^{kk} = (a^k)^2 Q - 2(B_v)_a{}^k B^k - \frac{1}{W^2}B^kB^k$$

$$E_j{}^{kk} = (a^k)^2 B_j - a^k\left((B_v)h_j{}^k + v_j B^k\right) - \frac{1}{W^2}h_j{}^kB^k$$

$$(a^k)^2 h_e W^2 + \cancel{(a^k)^2 B^2} - \cancel{2(B v) a^k B^k} - \cancel{\frac{1}{W^2} B^k B^k}$$

$$- \left[\cancel{(a^k)^2 B^2} - \cancel{a^k \left((B v) B^k + (B v) B^k \right)} - \cancel{\frac{1}{W^2} B^{kk}} \right]$$

$$= (a^k)^2 h_e W^2$$

$$|A_{ij}^{kk}| = h (\Delta^{kk})^2 h_e W^2 (a^k)^2$$

$$(A^{-1})^{ij} = X h^{ij} + Y B^i E^j$$

$$A_{ij} (A^{-1})^{ij} = (h_{ij} \Delta - B_i E_j)(X h^{ij} + Y B^i E^j)$$
$$= X(\Delta - (BE)) + Y(BE)(\Delta - BE)$$

In order to equal unity, $X = \frac{1}{\Delta}$, $Y = \frac{1}{\Delta(\Delta - BE)}$

$$(A_{ij}^{kk})^{-1} = \frac{1}{\Delta^{kk}} \left(h^{ij} + \frac{B^i E^j}{h_e W^2 (a^k)^2} \right)$$

$$|\mathcal{F}^k - \lambda^k \mathcal{F}^0| =$$

$$(\alpha^2 h)^4 \left(\frac{1}{a^k}\right)^3 W_a^k (-\lambda^k)(a^k)^2 \begin{vmatrix} A_{ij}^{kk} & \mathcal{K}(h_i^k + v_i(a^k - v^k)) \\ C_j^{kk} & W^2 a^k(\rho_0 + \mathcal{K}) - \mathcal{K}(a^k - v^k) \end{vmatrix}$$

$$\begin{vmatrix} a_{ij} & v_j \\ c_j & x \end{vmatrix} = \det(a_{ij}) \cdot \left[x - \vec{c}^{\,T}(a_{ij})^{-1}\vec{v}\right]$$

$$h(\Delta^{kk})^2 h_e W^2 (a^k)^2 \cdot \left[W^2 a^k(\rho_0 + \mathcal{K}) - \mathcal{K}(a^k - v^k) - C_i^{kk}(A_{ij}^{kk})^{-1} \cdot \mathcal{K}(h_i^k + v_i(a^k - v^k))\right]$$

$$|\mathcal{F}^k - \lambda^k \mathcal{F}^0| =$$

Pulling into main phrase

$$-\alpha^8 h^5 \lambda^k h_e W^3 (a^k)^2 (\Delta^{kk}) \Big\{ \left[(a^k)^2(h_e W^2 + B^2) - 2(B_v) a^k B^k - \frac{1}{W^2} B^k B^k\right] \cdot$$

$$\left[(W^2(\rho_0 + \mathcal{K}) - \mathcal{K}) a^k + \mathcal{K} v^k\right] - C_i^{kk}\left[h^{ij} + \frac{B^i E^j}{h_e W^2 (a^k)^2}\right] \cdot$$

$$\mathcal{K}(h_j^k + v_j(a^k - v^k)) \Big\}$$

Should be i's ?

$$\textcircled{\alpha}\ \left[(a^k)^2(h_e W^2 + B^2) - 2(B_v) a^k B^k - \frac{1}{W^2} B^k B^k\right] \cdot \left[(W^2(\rho_0 + \mathcal{K}) - \mathcal{K}) a^k + \mathcal{K} v^k\right]$$

$$'' \qquad \cdot \left[W^2 a^k(\rho_0 + \mathcal{K}) - \mathcal{K}(a^k - v^k)\right]$$

Add $(a^k)^2$ to all these terms

$\downarrow$

$$(a^k)^2(h_eW^2+B^2)\,W^2a^k(\rho_0+\mathcal{K}) - (a^k)^2(h_eW^2+B^2)\,\mathcal{K}(a^k-v^k)$$

$$-2(Bv)(a^k)^2B^kW^2(\rho_0+\mathcal{K}) + 2(Bv)a^kB^k\mathcal{K}(a^k-v^k)$$

$$-\frac{1}{\cancel{W^2}}B^kB^k\cancel{W^2}a^k(\rho_0+\mathcal{K}) + \frac{1}{W^2}B^kB^k\mathcal{K}(a^k-v^k)$$

Try $v^2 = 1-\frac{1}{W^2}$

(B)

Attempt 1

$$C_j^{\;kk} = a^ka^k\Big[W^2\big(Wv_j\epsilon + (h_eW^2v_j + B^2v_j)\mu - (Bv)B_j\big) + \frac{\rho_0}{\mathcal{K}}W^2\big(h_eW^2c_s^2v_j + B^2v_j - (Bv)B_j\big)\Big]$$

$$+\,a^k\Big[h_j^{\;k}W^2(Bv)^2 - W^2(Bv)v_jB^k + \frac{\rho_0}{\mathcal{K}}\Big(\big(B^2+W^2(Bv)^2+h_eW^2c_s^2\big)h_j^{\;k} - B^k\big(W^2(Bv)v_j + B_j\big)\Big)\Big]$$

$$-\,B^k\big[h_j^{\;k}(Bv) - v_jB^k\big]\zeta$$

$$E^{jkk} = \Big[(a^k)^2B^j - a^k\big((Bv)h^{jk} + v^jB^k\big) - \frac{1}{W^2}h^{jk}B^k\Big]$$

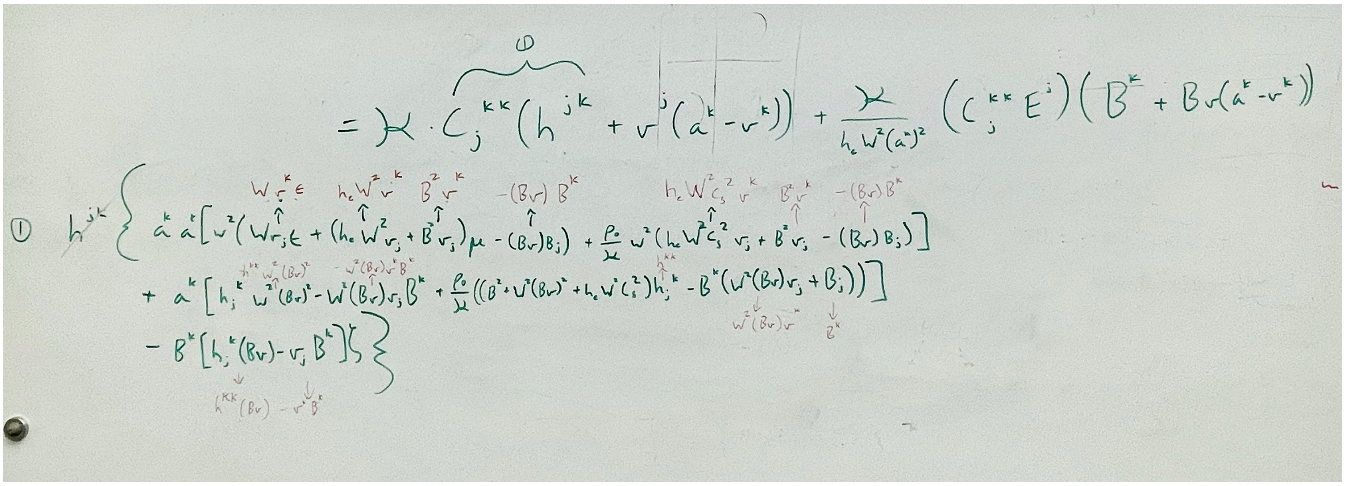
$$\det\big[\tilde{\mathcal{J}}^k - \lambda^k\tilde{\mathcal{J}}^0\big] = -\alpha^8h^5\lambda^kh_eW^3(a^k)^2\Delta^{kk}$$

$$\Big\{\Big[(a^k)^2(h_eW^2+B^2) - 2a^k(Bv)B^k - \frac{B^{k\,2}}{W^2}\Big]\Big[\big(W^2(\rho_0+\mathcal{K})-\mathcal{K}\big)a^k + \mathcal{K}v^k\Big]$$

$$-\,C_j^{\;kk}\Big[h^{ij} + \frac{B^iE^j}{h_eW^2(a^k)^2}\Big]\mathcal{K}\big(h_i^{\;k} + v_i(a^k-v^k)\big)\Big\}$$

$$= C_j^{\;kk}\mathcal{K}\Big[h^{jk} + v^j(a^k-v^k) + \frac{1}{h_eW^2(a^k)^2}\big(B^k + Bv\,(a^k-v^k)\big)E^j\Big]$$

$$= \mathcal{K}\cdot C_j^{\;kk}\big(h^{jk}+v^j(a^k-v^k)\big) + \frac{\mathcal{K}}{h_eW^2(a^k)^2}\underline{\big(C_j^{\;kk}E^j\big)}\big(B^k+Bv\,(a^k-v^k)\big)$$

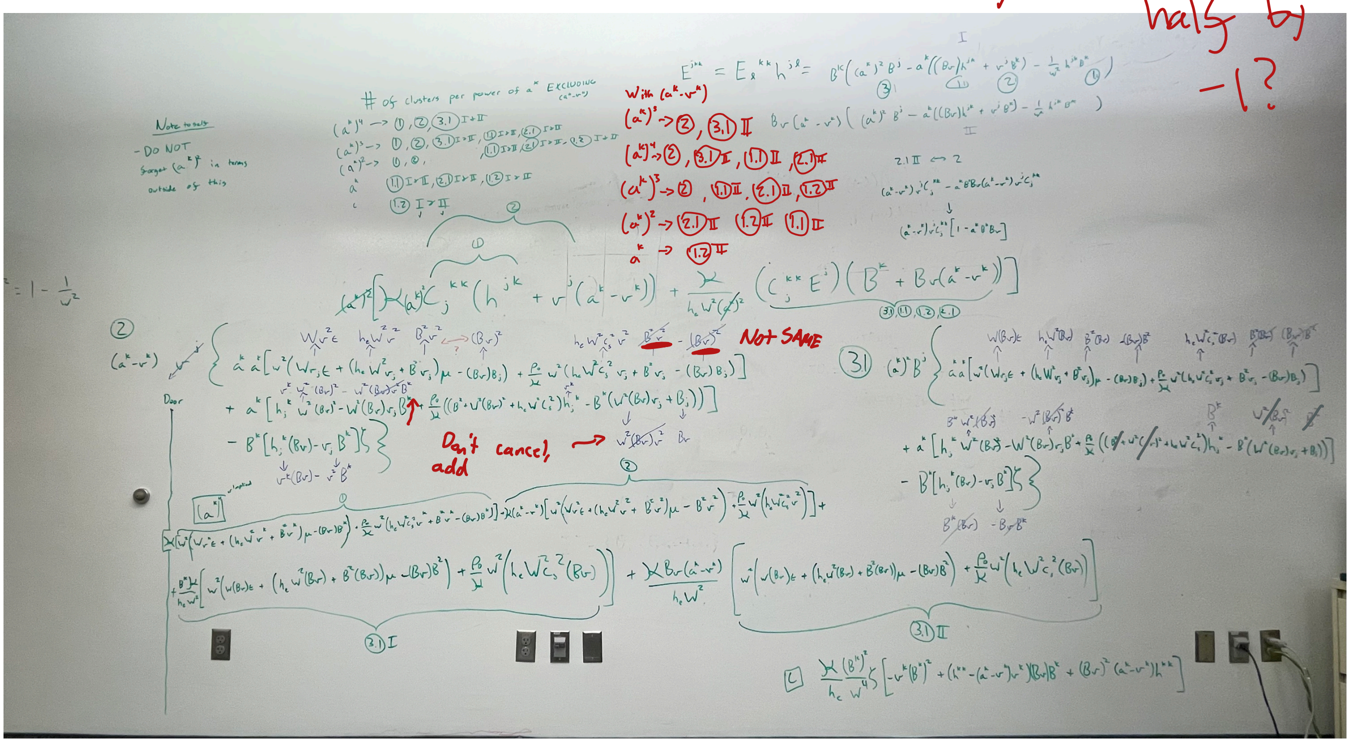


$$①\ (a^k)^2\Big\{ a^k a^k \Big[w^2\big(W v^k \epsilon + (h_c W^2 v^k + B^2 v^k)\mu - (Bv)B^k \big) + \frac{P_0}{\varkappa} w^2 \big(h_c W^2 C_s^2 v^k + B^2 v^k - (Bv)B^k \big) \Big]$$

$$+\ a^k \Big[h^{kk} W^2 (Bv)^2 - W^2 (Bv) v^k B^k + \frac{P_0}{\varkappa}\big((B^2 + w^2 (Bv)^2 + h_c W^2 C_s^2) h^{kk} - B^k (W^2 (Bv) v^k + B^k) \big) \Big]$$

$$-\ B^k \big[h^{kk}(Bv) - v^k B^k \big] \Big\} \Big\}$$

Multiply full second half by -1?



$$\varkappa(① + ②) + \left(3.1_{I} + 3.1_{II} + 1.1_{I} + 1.1_{II} + 2.1_{I} + 2.1_{II} + 1.2_{I} + 1.2_{II} \right)\frac{\varkappa}{h_c W^2}$$

2

$$(a^k - v^k)(a^k)^2 \Big\{ a^k a^k \Big[W^2 \Big(W v^2 \epsilon + \big(h_e W^2 v^2 + B^2 v^2\big)\mu - (Bv)^2 \Big) + \frac{\rho_0}{\chi} W^4 c_s^2 h_e v^2 \Big]$$
$$+ a^k \Big[v^k W^2 (Bv)^2 + \frac{\rho_0}{\chi}\Big(\big(B^2 + W^2 (Bv)^2 + h_e W^2 c_s^2 \big) v^k - B^k Bv \Big) \Big]$$
$$- B^k \Big[v^k (Bv) - v^2 B^k \Big] \zeta \Big\}$$

3.1 I

$$B^k (a^k)^2 \Big\{ (a^k)^2 \Big[W^2 \Big(W (Bv)\epsilon + \big(h_e W^2 (Bv) + B^2 (Bv) \big)\mu - (Bv) B^2 \Big) + \frac{\rho_0}{\chi} W^4 h_e c_s^2 (Bv) \Big]$$
$$+ a^k \Big[\frac{\rho_0}{\chi} h_e W^2 c_s^2 B^k \Big] \Big\}$$

3.1 II

$$Bv (a^k - v^k)(a^k)^2 \Big\{ (a^k)^2 \Big[W^2 \Big(W (Bv)\epsilon + \big(h_e W^2 (Bv) + B^2 (Bv) \big)\mu - (Bv) B^2 \Big) + \frac{\rho_0}{\chi} W^4 h_e c_s^2 (Bv) \Big]$$
$$+ a^k \Big[\frac{\rho_0}{\chi} h_e W^2 c_s^2 B^k \Big] \Big\}$$

(2.1) I

$$-a^k (B^k)^2 \Big\{ a^k a^k \Big[W^2 \Big(W v^2 \epsilon + \big(h_e W^2 v^2 + B^2 v^2\big)\mu - (Bv)^2 \Big) + \frac{\rho_0}{2\alpha} W^4 c_s^2 h_e v^2 \Big]$$

$$+ a^k \Big[v^k W^2 (Bv)^2 + \frac{\rho_0}{2\alpha}\Big(\big(B^2 + W^2 (Bv)^2 + h_e W^2 c_s^2\big) v^k - B^k Bv\Big)\Big]$$

$$- B^k \big[v^k (Bv) - v^2 B^k \big] \zeta \Big\}$$

(2.1) II

$$-Bv(a^k - v^k) a^k B^k \Big\{ a^k a^k \Big[W^2 \Big(W v^2 \epsilon + \big(h_e W^2 v^2 + B^2 v^2\big)\mu - (Bv)^2 \Big) + \frac{\rho_0}{2\alpha} W^4 c_s^2 h_e v^2 \Big]$$

$$+ a^k \Big[v^k W^2 (Bv)^2 + \frac{\rho_0}{2\alpha}\Big(\big(B^2 + W^2 (Bv)^2 + h_e W^2 c_s^2\big) v^k - B^k Bv\Big)\Big]$$

$$- B^k \big[v^k (Bv) - v^2 B^k \big] \zeta \Big\}$$

(1.2) I

$$-\frac{(B^k)^2}{W^2} a^k a^k \Big\{ \Big[W^2 \Big(W v^k \epsilon + \big(h_e W^2 v^k + B^2 v^k\big)\mu - (Bv) B^k + \frac{\rho_0}{2\alpha} W^2 \big(h_e W^2 c_s^2 v^k + B^2 v^k - (Bv) B^k\big) \Big]$$

$$+ a^k \Big[h^{kk} W^2 (Bv)^2 - W^2 (Bv) v^k B^k + \frac{\rho_0}{2\alpha}\Big(\big(B^2 + W^2 (Bv)^2 + h_e W^2 c_s^2\big) h^{kk} - B^k\big(W^2 (Bv) v^k + B^k\big)\Big)\Big]$$

$$- B^k \big[h^{kk} (Bv) - v^k B^k \big] \zeta \Big\}$$

(1.2) II

$$
-(Bv)(a^k-v^k)\frac{B^k}{W^2}\Big(\Big\{a^k a^k\Big[W^2\Big(Wv^k\epsilon+(h_eW^2v^k+B^2v^k)\mu-(Bv)B^k+\frac{\rho_0}{\chi}W^2\big(h_eW^2c_s^2v^k+B^2v^k-(Bv)B^k\big)\Big)\Big]
$$
$$
+a^k\Big[h^{kk}W^2(Bv)^2-W^2(Bv)v^kB^k+\frac{\rho_0}{\chi}\Big(\big(B^2+W^2(Bv)^2+h_eW^2c_s^2\big)h^{kk}-B^k\big(W^2(Bv)v^k+B^k\big)\Big)\Big]
$$
$$
-B^k\Big[h^{kk}(Bv)-v^kB^k\Big]\Big\}\Big\}
$$

(1.1) I

$$
-a^kB_vB^k\Big\{\Big\{a^k a^k\Big[W^2\Big(Wv^k\epsilon+(h_eW^2v^k+B^2v^k)\mu-(Bv)B^k+\frac{\rho_0}{\chi}W^2\big(h_eW^2c_s^2v^k+B^2v^k-(Bv)B^k\big)\Big)\Big]
$$
$$
+a^k\Big[h^{kk}W^2(Bv)^2-W^2(Bv)v^kB^k+\frac{\rho_0}{\chi}\Big(\big(B^2+W^2(Bv)^2+h_eW^2c_s^2\big)h^{kk}-B^k\big(W^2(Bv)v^k+B^k\big)\Big)\Big]
$$
$$
-B^k\Big[h^{kk}(Bv)-v^kB^k\Big]\Big\}\Big\}
$$

(1.1) II

$$
-(Bv)^2(a^k-v^k)\,a^k\Big\{\Big\{a^k a^k\Big[W^2\Big(Wv^k\epsilon+(h_eW^2v^k+B^2v^k)\mu-(Bv)B^k+\frac{\rho_0}{\chi}W^2\big(h_eW^2c_s^2v^k+B^2v^k-(Bv)B^k\big)\Big)\Big]
$$
$$
+a^k\Big[h^{kk}W^2(Bv)^2-W^2(Bv)v^kB^k+\frac{\rho_0}{\chi}\Big(\big(B^2+W^2(Bv)^2+h_eW^2c_s^2\big)h^{kk}-B^k\big(W^2(Bv)v^k+B^k\big)\Big)\Big]
$$
$$
-B^k\Big[h^{kk}(Bv)-v^kB^k\Big]\Big\}\Big\}
$$

$$- C_i{}^{kk}\left[h^{ij} + \frac{B^i E^j}{h_e W^2 (a^k)^2}\right] \cdot \mathcal{H}\left(h_j{}^k + v_j(a^k - v^k)\right)$$

$$- \left[C_i{}^{kk} h^{ij} + \frac{C_i{}^{kk} B^i E^j}{h_e W^2 (a^k)^2}\right] \cdot \mathcal{H}\left(h_j{}^k + v_j(a^k - v^k)\right)$$

$$- \mathcal{H}\left[\overset{①}{C_i{}^{kk} h^{ik}} + \overset{②}{C_i{}^{kk} v_i (a^k - v^k)} + \overset{③}{\frac{C_i{}^{kk} B^i}{h_e W^2 (a^k)^2}} \left[E^{k} + E^j v_j (a^k - v^k)\right]\right]$$

$$\begin{aligned} C_i{}^{kk} = {} & a^k a^k \left[W^2\left(W v_i \epsilon + (h_e W^2 v_i + B^2 v_i)\mu - (Bv)B_i\right) + \frac{\rho_0}{\mathcal{H}} W^2\left(h_e W^2 c_s^2 v_i + B^2 v_i - (Bv)B_i\right)\right] \\ & + a^k \left[h_i{}^k W^2 (Bv)^2 - W^2 (Bv) v_i B^k + \frac{\rho_0}{\mathcal{H}}\left(\left(B^2 + W^2 (Bv)^2 + h_e W^2 c_s^2\right) h_i{}^k - B^k\left(W^2 (Bv) v_i + B_i\right)\right)\right] \\ & - B^k\left[h_i{}^k (Bv) - v_i B^k\right] \zeta \end{aligned}$$

$$E^{jkk} = \left[(a^k)^2 B^j - a^k\left((Bv) h^{jk} + v^j B^k\right) - \frac{1}{W^2} h^{jk} B^k\right]$$

①

$$C_i{}^{kk} h^{ik}$$

$$\begin{aligned} = {} & a^k a^k \left[W^2\left(W v^k \epsilon + (h_e W^2 v^k + B^2 v^k)\mu - (Bv)B^k\right) + \frac{\rho_0}{\mathcal{H}} W^2\left(h_e W^2 c_s^2 v^k + B^2 v^k - (Bv)B^k\right)\right] \\ & + a^k \left[h^{kk} W^2 (Bv)^2 - W^2 (Bv) v^k B^k + \frac{\rho_0}{\mathcal{H}}\left(\left(B^2 + W^2 (Bv)^2 + h_e W^2 c_s^2\right) h^{kk} - B^k\left(W^2 (Bv) v^k + B^k\right)\right)\right] \\ & - B^k\left[h^{kk} (Bv) - v^k B^k\right] \zeta \end{aligned}$$

2

$$C_i^{\;kk} v^i =$$

$$v^i \Big\{ a^k a^k \Big[W^2\big(W v_i \epsilon + (h_e W^2 v_i + B^2 v_i)\mu - (Bv) B_i \big) + \frac{\rho_0}{\mathcal{H}} W^2 \big(h_e W^2 c_s^2 v_i + B^2 v_i - (Bv) B_i \big) \Big]$$

$$+ a^k \Big[h_i^{\;k} W^2 (Bv)^2 - W^2 (Bv) v_i B^k + \frac{\rho_0}{\mathcal{H}} \Big(\big(B^2 + W^2 (Bv)^2 + h_e W^2 c_s^2 \big) h_i^{\;k} - B^k \big(W^2 (Bv) v_i + B_i \big) \Big) \Big]$$

$$- B^k \Big[h_i^{\;k} (Bv) - v_i B^k \Big] \zeta \Big\}$$

$$= a^k a^k \Big[W^2\big(W v^2 \epsilon + (h_e W^2 v^2 + B^2 v^2)\mu - (Bv)^2 \big) + \frac{\rho_0}{\mathcal{H}} W^2 \big(h_e W^2 c_s^2 v^2 + B^2 v^2 - (Bv)^2 \big) \Big]$$

$$+ a^k \Big[v^k W^2 (Bv)^2 - W^2 (Bv) v^2 B^k + \frac{\rho_0}{\mathcal{H}} \Big(\big(B^2 + W^2 (Bv)^2 + h_e W^2 c_s^2 \big) v^k - B^k \big(W^2 (Bv) v^2 + Bv \big) \Big) \Big]$$

$$- B^k \Big[v^k (Bv) - v^2 B^k \Big] \zeta \Big\}$$

$$W^2 v^2 = W^2 - 1$$

$$- B^k \big((Bv) W^2 - \cancel{(Bv)} + \cancel{(Bv)} \big)$$

$$- B^k (Bv) W^2 \Big]$$

3

$$C_i^{\;kk} B_i =$$

$$B^i \Big\{ a^k a^k \Big[W^2\big(W v_i \epsilon + (h_e W^2 v_i + B^2 v_i)\mu - (Bv) B_i \big) + \frac{\rho_0}{\mathcal{H}} W^2 \big(h_e W^2 c_s^2 v_i + B^2 v_i - (Bv) B_i \big) \Big]$$

$$+ a^k \Big[h_i^{\;k} W^2 (Bv)^2 - W^2 (Bv) v_i B^k + \frac{\rho_0}{\mathcal{H}} \Big(\big(B^2 + W^2 (Bv)^2 + h_e W^2 c_s^2 \big) h_i^{\;k} - B^k \big(W^2 (Bv) v_i + B_i \big) \Big) \Big]$$

$$- B^k \Big[h_i^{\;k} (Bv) - v_i B^k \Big] \zeta \Big\}$$

$$= a^k a^k \Big[W^2\big(W (Bv) \epsilon + (h_e W^2 (Bv) + B^2 (Bv))\mu - (Bv) B^2 \big) + \frac{\rho_0}{\mathcal{H}} W^2 \big(h_e W^2 c_s^2 Bv + B^2 (Bv) - (Bv) B^2 \big) \Big]$$

$$+ a^k \Big[\cancel{B^k W^2 (Bv)^2} - \cancel{W^2 (Bv)^2 B^k} + \frac{\rho_0}{\mathcal{H}} \Big(\big(\cancel{B^2} + \cancel{W^2 (Bv)^2} + h_e W^2 c_s^2 \big) B^k - B^k \big(\cancel{W^2 (Bv)^2} + \cancel{B^2} \big) \Big) \Big]$$

$$- B^k \Big[\cancel{B^k (Bv)} - \cancel{(Bv) B^k} \Big] \zeta$$

$$= a^k a^k \left[W^2 \left(W(B_v)_\epsilon + \left(h_e W^2 (B_v) + B^2 (B_v) \right)_\mu - (B_v) B^2 \right) + \frac{\rho_0}{\lambda} W^2 \left(h_e W^2 c_s^2 B_v + \cancel{B^2 (B_v) - (B_v) B^2} \right) \right]$$

$$+ a^k \left[\frac{\rho_0}{\lambda} h_e W^2 c_s^2 \; B^k \right]$$

See Mathematica Notebook

Ryan's Characteristic Equation:

$$-\alpha^8 h^5 \rho_0 h_e W^3 \lambda^k a^k \Delta^{kk} \Big\{ h_e W^4 (1 - c_s^2) (a^k)^4$$

$$+ \left[(a^k)^2 \left(h_e W^2 + B^2 + W^2 (B_v)^2 \right) - c_s^2 \left(a^k W (B_v) + \frac{B^k}{W^2} \right)^2 \right] \cdot \left[(a^k - v^k)^2 - h^{kk} \right] \Big\}$$

Our Characteristic Equation

$$-\alpha^8 h^5 \rho_0 h_e W^3 \lambda^k a^k \Delta^{kk} \Big\{ h_e W^4 (a^k)^4$$

$$+ \left[(a^k)^2 \left(B^2 + W^2 (B_v)^2 \right) - c_s^2 \left(a^k W (B_v) + \frac{B^k}{W} \right)^2 \right] \cdot \left[(a^k - v^k)^2 - h^{kk} \right]$$

$$- c_s^2 \left((a^k)^2 W^2 h_e \left((W^2 - 1)(a^k)^2 + 2 a^k v^k + h^{kk} - (v^k)^2 \right) \right)$$

$$- c_s^2 (a^k)^2 W^2 h \left(W^2 (a^k)^2 - a^{k\,2} + 2 a^k v^k - (v^k)^2 + h^{kk} \right)$$

$$- \left((a^k)^2 - 2 a^k v^k + (v^k)^2 \right)$$

$$- \alpha \left(W^2 (a^k)^2 - (a^k - v^k)^2 + h^{kk} \right)$$

$$- \alpha W^2 (a^k)^2 + \alpha \left((a^k - v^k)^2 - h^{kk} \right)$$

$$- \underbrace{c_s^2 (a^k)^4 W^4 h_e} + h_e W^2 (a^k)^2 c_s^2$$

Our Final Characteristic Equation

$$-\alpha^8 h^5 \rho_0 h_e W^3 \lambda^k a^k \Delta^{kk} \Big\{ h_e W^4 (a^k)^4 (1 - c_s^2)$$

$$+ \left[(a^k)^2 \left(h_e W^2 c_s^2 + B^2 + W^2 (Bv)^2 \right) - c_s^2 \left(a^k W (Bv) + \frac{B^k}{W} \right)^2 \right] \cdot \left[(a^k - v^k)^2 - h^{kk} \right]$$

Interlude pg. 27

$$(4.1) \quad e^0 W a^k + (ve) W^3_{p_0} a^k + e^k W_{p_0} = 0$$

$$(4.3) \quad 0 = e^0\left[(W^2\gamma - W - \chi)a^k + \chi v^k\right]$$
$$+ \left\{(ve)\left[(2h_e W^4 + B^2 - W^3_{p_0})a^k - B^2 v^k\right] - (B_e)\left[(Bv)(a^k - v^k) + B^k\right]\right.$$
$$\left. + e^k(Q - W_{p_0})\right\}$$
$$+ e^4\left[(p_0 + \mathcal{K}) W^2 a^k + \mathcal{K}(v^k - a^k)\right]$$
$$+ \left\{-(v\hat{e})\left[(Bv)(a^k - v^k) + B^k\right] + (B\hat{e})\left[\left(2 - \frac{1}{W^2}\right)a^k + \frac{1}{W^2} v^k\right]\right.$$
$$\left. - \hat{e}^k (Bv)\right\}$$

4.1 + 4.3

$$(4.7) \quad 0 = e^0\left[(W^2\gamma - \chi)a^k + \chi v^k\right]$$
$$+ \left\{(ve)\left[(2h_e W^4 + B^2)a^k - B^2 v^k\right] - (B_e)\left[(Bv)(a^k - v^k) + B^k\right]\right.$$
$$\left. + e^k Q\right\}$$
$$+ e^4\left[(p_0 + \mathcal{K}) W^2 a^k + \mathcal{K}(v^k - a^k)\right]$$

$$+\left\{-(v\hat{e})\left[(Bv)(a^k - v^k) + B^k\right] + (B\hat{e})\left[\left(2 - \frac{1}{W^2}\right)a^k + \frac{1}{W^2}v^k\right] - \hat{e}^k (Bv)\right\}$$

$-v_i \cdot 4.7 + 4.2$

(4.2)

$$0 = e^0\left[\cancel{W^2\gamma v_i a^k} + \chi h_i^{\;k}\right]$$

$$+\left\{e^i\left[Q a^k - (Bv)B^k\right] + (ve)\left[2h_e \cancel{W^4} \cancel{v_i a^k} - h_i^{\;k} B^2 + 2B_i B^k\right]\right.$$

$$\left. + (B_e)\left[-B_i a^k + h_i^{\;k}(Bv) - \cancel{v_i} B^k\right] + e^k\left[\cancel{Q v_i} - (Bv)B_i\right]\right\}$$

$$+ e^4\left[\cancel{(\rho_0 + \chi)W^2 v_i a^k} + h_i^{\;k}\chi\right]$$

$$+\left\{\hat{e}_i\left[-(Bv)a^k - \frac{1}{W^2}B^k\right] + (v\hat{e})\left[-B_i a^k - \cancel{v_i B^k} + h_i^{\;k}(Bv)\right]\right.$$

$$\left. + (B\hat{e})\left[\cancel{2 v_i a^k} + \frac{1}{W^2}h_i^{\;k}\right] - \hat{e}^k\left[\cancel{(Bv) v_i} + \frac{1}{W^2}B_i\right]\right\}$$

$$0 = -v_i\left[e^0\left[(\cancel{W^2\gamma} - \chi)a^k + \chi v^k\right]\right.$$

$$+\left\{(ve)\left[(\cancel{2h_e W^4} + B^2)a^k - B^2 v^k\right] - (B_e)\left[(Bv)(a^k - v^k) + \cancel{B^k}\right]\right.$$

$$\left. + \cancel{e^k Q}\right\}$$

$$+ e^4\left[\cancel{(\rho_0 + \chi)W^2 a^k} + \chi(v^k - a^k)\right]$$

$$+\left\{-(v\hat{e})\left[(Bv)(a^k - v^k) + \cancel{B^k}\right] + (B\hat{e})\left[\left(\cancel{2} - \frac{1}{W^2}\right)a^k + \frac{1}{W^2}v^k\right]\right.$$

$$- \cancel{\hat{e}^k (B v)} \}]$$

$$0 = e^0 \left[\qquad \chi h_i^k \right]$$

$$+ \left\{ e^i \left[Q a^k - (Bv) B^k \right] + (ve) \left[\qquad -h_i^k B^2 + 2 B_i B^k \right] \right.$$

$$\left. + (Be) \left[-B_i a^k + h_i^k (Bv) \qquad \right] - e^k \qquad (Bv) B_i \right\}$$

$$+ e^4 \left[\qquad h_i^k \chi \right]$$

$$+ \left\{ \hat{e}_i \left[-(Bv) a^k - \frac{1}{W^2} B^k \right] + (v\hat{e}) \left[-B_i a^k \qquad + h_i^k (Bv) \right] \right.$$

$$\left. + (B\hat{e}) \left[\qquad \frac{1}{W^2} h_i^k \right] - \hat{e}^k \left[\qquad \frac{1}{W^2} B_i \right] \right\}$$

$$0 = -_{v_i} \left[e^0 \left[(\qquad - \chi) a^k + \chi v^k \right] \right.$$

$$+ \left\{ (ve) \left[(\qquad + B^2) a^k - B^2 v^k \right] - (Be) \left[(Bv)(a^k - v^k) \qquad \right] \right.$$

$$\left. \right\}$$

$$+ e^4 \left[\qquad + \chi (v^k - a^k) \right]$$

$$+ \left\{ -(v\hat{e}) \left[(Bv)(a^k - v^k) + \qquad \right] + (B\hat{e}) \left[(- \frac{1}{W^2}) a^k + \frac{1}{W^2} v^k \right] \right.$$

(4.8)
$$0 = e^0 \left[-\chi\left((v^k - a^k) v_i - h_i^{\ k} \right) \right]$$
$$+ \Big\{ e^i \left[Q a^k - (Bv) B^k \right] + (ve) \left[2 B_i B^k + B^2 \left((v^k - a^k) v_i - h_i^{\ k} \right) \right]$$
$$+ (Be) \left[-B_i a^k + (Bv) \left((a^k - v^k) v_i + h_i^{\ k} \right] - e^k (Bv) B_i \Big\}$$
$$+ e^4 \left[-\lambda \left((v^k - a^k) v_i - h_i^{\ k} \right) \right]$$
$$+ \Big\{ \hat{e}_i \left[-(Bv) a^k - \frac{1}{w^2} B^k \right] + (v\hat{e}) \left[-(Bv)\left((v^k - a^k) v_i - h_i^{\ k} \right) - B_i a^k \right]$$
$$+ (B\hat{e}) \left[-\frac{1}{w^2} \left((v^k - a^k) v_i - h_i^{\ k} \right) \right] - \cancel{\hat{e}^k \frac{B_i}{w^2}} \Big\}$$

4.4 contract w/ B_i

$$0 = \left(B^i e^k - B^k e^i + \hat{e}^i a^k \right) B_i$$
(4.9)
$$0 = B^2 e^k - (Be) B^k + (B\hat{e}) a^k$$

4.4 contract w/ v_i

$$0 = \left(B^i e^k - B^k e^i + \hat{e}^i a^k \right) v_i$$
(4.10)
$$0 = (Bv) e^k - (ve) B^k + (v\hat{e}) a^k$$

Use above to simplify

$$0 = (e^0 \chi + e^4 \lambda)\left((a^k - v^k)_{v_i} + h_i{}^k\right)$$
$$+ \Big\{ e^i \left[Q a^k - (Bv) B^k \right] + (ve)\left[\cancel{2} B_i B^k + B^2\left((v^k - a^k)_{v_i} - h_i{}^k\right)\right]$$
$$+ (Be)\left[-B_i a^k + (Bv)\left((a^k - v^k)_{v_i} + h_i{}^k\right)\right] \cancel{- e^k (Bv) B_i} \Big\}$$
$$+ \Big\{ \hat{e}_i \left[-(Bv) a^k - \frac{1}{w^2} B^k \right] + (v\hat{e})\left[-(Bv)\left((v^k - a^k)_{v_i} - h_i{}^k\right) \cancel{- B_i a^k} \right]$$
$$+ (B\hat{e})\left[-\frac{1}{w^2}\left((v^k - a^k)_{v_i} - h_i{}^k\right)\right] \Big\}$$

$$0 = (Bv) e^k - (ve) B^k + (v\hat{e}) a^k$$
$$(Bv) e^k = (ve) B^k - (v\hat{e}) a^k$$
$$-e^k (Bv) B_i = -(ve) B^k B_i + (v\hat{e}) a^k B_i$$

(4.11)
$$0 = (e^0 \chi + e^4 \lambda)\left((a^k - v^k)_{v_i} + h_i{}^k\right)$$
$$+ \Big\{ e^i \left[Q a^k - (Bv) B^k \right] + (ve)\left[B_i B^k + B^2\left((v^k - a^k)_{v_i} - h_i{}^k\right)\right]$$
$$+ (Be)\left[-B_i a^k + (Bv)\left((a^k - v^k)_{v_i} + h_i{}^k\right)\right] \Big\}$$
$$+ \Big\{ \hat{e}_i \left[-(Bv) a^k - \frac{1}{w^2} B^k \right] + (v\hat{e})\left[-(Bv)\left((v^k - a^k)_{v_i} - h_i{}^k\right) \right]$$
$$+ (B\hat{e})\left[-\frac{1}{w^2}\left((v^k - a^k)_{v_i} - h_i{}^k\right)\right] \Big\}$$

(4.7) $0 = e^0\left[(W^2\Upsilon - \chi)a^k + \chi v^k\right]$

$+ \left\{(ve)\left[(2h_c W^4 + B^2)a^k - B^2 v^k\right] - (Be)\left[(Bv)(a^k - v^k) + \cancel{B^k}\right]\right.$

$\left. + e^k(h_c W^2 + \cancel{B^2})\right\}$

$+ e^4\left[(p_0 + \mathcal{K})W^2 a^k + \mathcal{K}(v^k - a^k)\right]$

$+ \left\{-(v\hat{e})\left[(Bv)(a^k - v^k) + B^k\right] + (B\hat{e})\left[(\cancel{2}^{\,1} - \frac{1}{W^2})a^k + \frac{1}{W^2}v^k\right]\right.$

$\left. - \cancel{e^k}^{\,0}(Bv)\right\}$

$a^k - \frac{a^k}{W^2} + \frac{1}{W^2}v^k$

$\frac{(B\hat{e})}{W^2}\left[W^2 a^k - a^k + v^k\right]$

$0 = B^2 e^k - (Be)B^k + (B\hat{e})a^k$

(4.12) $0 = e^0\left[(W^2\Upsilon - \chi)a^k + \chi v^k\right]$

$+ \left\{(ve)\left[(2h_c W^4 + B^2)a^k - B^2 v^k\right] + (Be)\left[(Bv)(v^k - a^k)\right]\right.$

$\left. + e^k(h_c W^2)\right\}$

$+ e^4\left[(p_0 + \mathcal{K})W^2 a^k + \mathcal{K}(v^k - a^k)\right]$

$+ \left\{(v\hat{e})\left[(Bv)(v^k - a^k) - B^k\right] + \frac{(B\hat{e})}{W^2}\left[W^2 a^k - a^k + v^k\right]\right.$

4.11 contract w/ v_i $\cdot W^2$

$1 - \frac{1}{W^2}$

(4.11) $0 = v_i\Big[\big(e^0\chi + e^4\mathcal{K}\big)\Big(\underbrace{(a^k - v^k)v_i + h_i{}^k}_{a^k - \frac{a^k}{W^2} - v^k + \frac{v^k}{W^2} + v^k \;=\; \frac{1}{W^2}(W^2a^k - a^k + v^k)}\Big)$

$v^2 + v^k$

$+ \Big\{\hat{e}^i\big[(h_eW^2 + B^2)a^k - (B_v)B^k\big] + (v_e)\big[B_iB^k + B^2((v^k - a^k)v_i - h_i{}^k)\big]$

(ve), (Bv), v^2, v^k, $1 - \frac{1}{W^2}$

$(v^k - a^k) - \frac{1}{W^2}(v^k - a^k)$

$+ (B_e)\big[-B_ia^k + (B_v)((a^k - v^k)v_i + h_i{}^k)\big]\Big\}$

(Bv), v^2, v^k, $1 - \frac{1}{W^2}$

$(a^k - v^k) - \frac{1}{W^2}(a^k - v^k)$

$+ \Big\{\hat{e}_i\big[-\cancel{(B_v)a^k} - \frac{1}{W^2}B^k\big] + (v\hat{e})\big[-(B_v)((v^k - a^k)v_i - h_i{}^k)\big]$

($v\hat{e}$), v^2, v^k, $1 - \frac{1}{W^2}$

$-B_v\big[(v^k - a^k) - \frac{1}{W^2}(v^k - a^k)\big]$

$+ (B\hat{e})\big[-\frac{1}{W^2}((v^k - a^k)v_i - h_i{}^k)\big]\Big\}\Big]$

v^2, v^k, $1 - \frac{1}{W^2}$

$-\big[\cancel{(v^k} - a^k) - \frac{1}{W^2}(v^k - a^k) - \cancel{v^k}\big]$

$a^kW^2 + v^k - a^k$

(4.13) $0 = (e^0\chi + e^4\mathcal{K})(v^k - a^k + W^2a^k)$

$+ (v_e)\big[h_eW^4a^k - B^2(v^k - a^k)\big] + (B_e)\big[(B_v)(v^k - a^k)\big]$

$+ (v\hat{e})\big[(B_v)(v^k - a^k) - B^k\big] + (B\hat{e})\frac{1}{W^2}(v^k - a^k + W^2a^k$

4.11 contract w/ B^i

(4.11)
$$0 = (e^0 \chi + e^4)\mathcal{R})\left((a^k - v^k)\overset{(Bv)}{\overset{\uparrow}{v_i}} + \overset{B^k}{\overset{\uparrow}{h_i{}^k}}\right)$$

$$+ \left\{ \overset{(Be)}{\overset{\uparrow}{e_i}}\left[\overset{(h_e w^2 + B^2)}{\overset{\uparrow}{Q}}\, a^k - (Bv)B^k\right] + (ve)\left[\overset{B^2}{\overset{\uparrow}{B_i}} B^k + B^2\left((v^k - a^k)\overset{(Bv)}{\overset{\uparrow}{v_i}} - \overset{B^k}{\overset{\uparrow}{h_i{}^k}}\right)\right]\right.$$

$$\left. + (Be)\left[-\overset{(B^2)}{\overset{\uparrow}{B_i}}\, a^k + (Bv)\left((a^k - v^k)\overset{Bv}{\overset{\uparrow}{v_i}} + \overset{B^k}{\overset{\uparrow}{h_i{}^k}}\right)\right]\right\}$$

$$+ \left\{ \overset{(B\hat{e})}{\overset{\uparrow}{\hat{e}_i}}\left[-(Bv)a^k - \frac{1}{w^2}B^k\right] + (v\hat{e})\left[-(Bv)\left((v^k - a^k)\overset{(Bv)}{\overset{\uparrow}{v_i}} - \overset{B^k}{\overset{\uparrow}{h_i{}^k}}\right)\right]\right.$$

$$\left. + (B\hat{e})\left[-\frac{1}{w^2}\left((v^k - a^k)\overset{Bv}{\overset{\uparrow}{v_i}} - \overset{B^k}{\overset{\uparrow}{h_i{}^k}}\right)\right]\right\}$$

(4.14)
$$0 = (e^0 \chi + e^4)\mathcal{R})\left[B^k - (Bv)(v^k - a^k)\right]$$

$$+ (Be)\left[h_e w^2 a^k - (Bv)^2(v^k - a^k)\right] + (ve)\left[B^2 (Bv)(v^k - a^k)\right]$$

$$- (B\hat{e})\frac{(Bv)}{w^2}\left[v^k - a^k + w^2 a^k\right] + (v\hat{e})\left[(Bv)\left(B^k - (Bv)(v^k - a^k)\right)\right]$$

4.14 + (Bv) 4.13

(4.14)

$$0=(e^0\chi+e^4\varkappa)\left[B^k-(Bv)(v^k-a^k)\right]$$
$$+(Be)\left[h_e W^2 a^k-(Bv)^2(v^k-a^k)\right]+(ve)\left[B^2(Bv)(v^k-a^k)\right]$$
$$-(B\hat{e})\frac{(Bv)}{W^2}\left[v^k-a^k+W^2a^k\right]+(v\hat{e})\left[(Bv)\left(B^k-(Bv)(v^k-a^k)\right)\right]$$

$$+\;Bv\Big[(e^0\chi+e^4\varkappa)(v^k-a^k+W^2a^k)$$
$$+(ve)\left[h_e W^4 a^k-B^2(v^k-a^k)\right]+(Be)\left[(Bv)(v^k-a^k)\right]$$
$$+(v\hat{e})\left[(Bv)(v^k-a^k)-B^k\right]+(B\hat{e})\frac{1}{W^2}(v^k-a^k+W^2a^k)\Big]$$

(4.15)

$$0=(e^0\chi+e^4\varkappa)\left(B^k+W^2a^k(Bv)\right)+(ve)\,h_e W^4 a^k(Bv)+(Be)h_e W^2 a^k$$

4.12 - 4.13

$$0=\;e^0\left[(W^2\gamma-\chi)a^k+\chi v^k\right]$$
$$+\Big\{(ve)\left[(2h_eW^4+B^2)a^k-B^2v^k\right]+(Be)\left[(Bv)(v^k-a^k)\right]$$
$$+\;e^k(h_eW^2)\Big\}$$
$$+\;e^4\left[(p_0+\varkappa)W^2a^k+\varkappa(v^k-a^k)\right]$$

$$+\Big\{ (v\hat{e})\left[(Bv)(v^k - a^k) - B^k\right] + \frac{(B\hat{e})}{W^2}\left[W^2 a^k - a^k + v^k\right)$$

$$-\left[(e^0 \chi + e^4)\lambda\right)(v^k - a^k + W^2 a^k)$$

$$+ (ve)\left[h_e W^4 a^k - B^2(v^k - a^k)\right] + (Be)\left[(Bv)(v^k - a^k)\right]$$

$$+ (v\hat{e})\left[(Bv)(v^k - a^k) - B^k\right] + (B\hat{e})\frac{1}{W^2}(v^k - a^k + W^2 a^k)\Big]$$

$$\chi e^0 (v^k - a^k + W^2 a^k) \qquad e^4 \lambda (v^k - a^k + W^2 a^k)$$

$$0 = e^0 W^2 a^k (\gamma - \chi) + e^4 W^2 a^k \rho_0 + (ve) h_e W^4 a^k + e^k h_e W^2$$

$$\gamma = 1 + \varepsilon + \chi$$

$$h_e = \rho_0 (1 + \varepsilon) + P$$

$$0 = e^0 W^2 a^k \left(\frac{h_e - P}{\rho_0}\right) + e^4 W^2 a^k \rho_0 + (ve) h_e W^4 a^k + e^k h_e W^2$$

This $\uparrow$ $-\frac{h_e W}{\rho_0} \cdot 4.1$

$$0 = e^0 W^2 a^k \left(\frac{\cancel{h_e} - P}{\rho_0}\right) + e^4 W^2 a^k \rho_0 + (ve) \cancel{h_e} W^4 a^k + e^k \cancel{h_e W}^2$$

$$-\frac{h_e W}{\rho_0}\left(e^0 \cancel{W} a^k + (ve) \cancel{W^3 \rho_0} a^k + e^k \cancel{W \rho_0} \right)$$

$$0 = \frac{-e^0 \cancel{W^2} \cancel{a^k} P}{\rho_0} + e^4 \cancel{W^2} \cancel{a^k} \rho_0$$

$$\frac{e^0 P}{\rho_0} = e^4 \rho_0$$

(4.16) $$e^4 = \frac{e^0 P}{\rho_0^2}$$

$$0 = (e^0 \chi + e^4 \kappa)(B^k + W^2 a^k (B v)) + (ve)\, h_e W^4 a^k (B v) + (Be) h_e W^2 a^k$$

$$= \left(e^0 \chi + \frac{e^0 P \kappa}{\rho_0^2}\right) ''$$

$$= e^0 \left(\chi + \frac{P\kappa}{\rho_0^2}\right) ''$$

$$= \frac{e^0}{\rho_0^2} (\chi \rho_0^2 + P\kappa) ''$$

(4.17) $$0 = \frac{e^0}{\rho_0} h_e c_s^2 (B^k + W^2 a^k (B v)) + (ve)\, h_e W^4 a^k (B v) + (Be) h_e W^2 a^k$$

(4.13) $$0 = (e^0 \chi + e^4 \kappa)(v^k - a^k + W^2 a^k)$$

$$+ (ve)\left[h_e W^4 a^k - B^2 (v^k - a^k)\right] + (Be)\left[(Bv)(v^k - a^k)\right]$$

$$+ (v\hat{e})\left[(Bv)(v^k - a^k) - B^k\right] + (B\hat{e}) \frac{1}{W^2} (v^k - a^k + W^2 a^k)$$

$$= \left(e^0 \chi + \frac{e^0 P \mathcal{X}}{\rho_0^2}\right) \parallel$$

$$= e^0 \left(\chi + \frac{P \mathcal{X}}{\rho_0^2}\right) \parallel$$

$$= \frac{e^0}{\rho_0^2} \left(\chi \rho_0^2 + P \mathcal{X}\right) \parallel$$

$$0 = \frac{e^0}{\rho_0} h_\varepsilon c_s^2 \left(v^k - a^k + W^2 a^k\right)$$

$$+ (ve)\left[h_e W^4 a^k - B^2 (v^k - a^k)\right] + (Be)\left[(Bv)(v^k - a^k)\right]$$

$$+ (v\hat{e})\left[(Bv)(v^k - a^k) - B^k\right] + (B\hat{e}) \frac{1}{W^2}\left(v^k - a^k + W^2 a^k\right)$$

$$\left(v^k - a^k + W^2 a^k\right)$$

$$\left(B^k - (Bv)(v^k - a^k)\right)$$

$$\left(B^k + W^2 a^k (Bv)\right)$$

$$\left(v^k - a^k + W^2 a^k\right)$$

Help needed for

"utilize 4.13 & 4.14"

$$0 = e\,\frac{{}^{0}h_e c_s^2}{\rho_0}\left(v^k - a^k + W^2 a^k\right) + (ve)\left(h_e W^4 a^k - C^k\right)$$
$$+ (Be) D^k - e^k \frac{E^k}{a^k}$$

$$C^k = \left(v^k - a^k\right)\left[B^2 - (Bv)\frac{B^k}{a^k}\right] + \frac{B^k B^k}{a^k}$$
$$D^k = \left(v^k - a^k\right)\left[(Bv) + \frac{B^k}{W^2 a^k}\right] + B^k$$
$$E^k = \left(v^k - a^k\right)\left[\frac{B^2}{W^2} + (Bv)^2\right] + B^2 a^k - (Bv) B^k$$

This ↑ $+ \dfrac{E^k}{\rho_0 a^k W} \cdot$ 4.1

W was not in Ryan's thesis; typo

$$0 = e\,\frac{{}^{0}h_e c_s^2}{\rho_0}\left(v^k - a^k + W^2 a^k\right) + (ve)\left(h_e W^4 a^k - C^k\right)$$
$$+ (Be) D^k - e^k \frac{E^k}{a^k}$$

$$+\frac{E^k}{W\rho_0 a^k}\left[e^0 W a^k + (ve) W^3 \rho_0 a^k + \underbrace{e^k W \rho_0}\right]$$

4.18

$$0 = \frac{e^0}{\rho_0}\left(h_e c_s^2 \left(v^k - a^k + W^2 a^k\right) + E^k\right)$$

$$+ (v_e)\left(h_e W^4 a^k - C^k + W^2 E^k\right)$$

$$+ (Be) D^k$$

Alfvén Waves

(4.17)
$$0 = \frac{e^0}{\rho_0} h_z c_s^2 \cancel{(B^k + W^2 a^k (B\cdot v))} + (v\cdot e) h_e W^4 a^k (B\cdot v) + (B\cdot e) h_e W^2 a^k$$

$$-(v\cdot e)\cancel{h_e} \cancel{W^4} \cancel{a^k} (B\cdot v) = (B\cdot e)\cancel{h_e}\cancel{W^2}\cancel{a^k}$$

4.19
$$-(v\cdot e)(B\cdot v) W^2 = B\cdot e$$

4.20
$$-(v\cdot e) W^2 a^k = e^k$$

(4.9)
$$0 = B^2 e^k - (B\cdot e) B^k + (B\cdot \hat{e}) a^k$$

$$(B\cdot\hat{e}) = \frac{-(v\cdot e)(B\cdot v) W^2 B^k}{a^k} - B^2(-v\cdot e) W^2$$

(4.22)
$$(B\cdot\hat{e}) = (v\cdot e) W^2 \left[B^2 + \mathcal{S}(B\cdot v)\frac{B^k}{a^k} \right]$$

4.10
$$0 = (B\cdot v) e^k - (v\cdot e) B^k + (v\cdot\hat{e}) a^k$$

$$(v\cdot\hat{e}) a^k = (B\cdot v)(v\cdot e) W^2 a^k + (v\cdot e) B^k$$

(4.21)
$$(v\cdot\hat{e}) = (v\cdot e)\left[W^2 (B\cdot v) + \frac{B^k}{a^k} \right]$$

(4.4) $0 = B^i e^k - B^k e^i + \hat{e}^i a^k$

(4.20) $-(ve) W^2 a^k = e^k$

$$0 = B^i\left(-(ve) W^2 a^k\right) - B^k e^i + \hat{e}^i a^k$$

$$B^k e^i = B^i\left(-(ve) W^2 a^k\right) + \hat{e}^i a^k$$

(4.23) $e^i = \frac{a^k}{B^k}\left[\hat{e}^i - (ve) W^2 B^i\right]$

$e^i = (e_\perp^1, e_\perp^2, e^k)$ $\quad$ $\hat{e}^i = (\hat{e}_\perp^1, \hat{e}_\perp^2, \hat{e}^k)$

$(v\hat{e}) = v_1 \hat{e}_\perp^1 + v_2 \hat{e}_\perp^2$ $\quad$ $(B\hat{e}) = B_1 \hat{e}_\perp^1 + B_2 \hat{e}_\perp^2$

$$(v\hat{e}) \cdot \left(\frac{-B_2}{v_2}\right) + (B\hat{e})$$

$$\left(v_1 \hat{e}_\perp^1 + v_2 \hat{e}_\perp^2\right) \cdot \frac{-B_2}{v_2} + B_1 \hat{e}_\perp^1 + B_2 \hat{e}_\perp^2 = (v\hat{e}) \cdot \left(\frac{-B_2}{v_2}\right) + (B\hat{e})$$

$$-\frac{B_2 v_1 \hat{e}_\perp^1}{v_2} - \cancel{B_2 \hat{e}_\perp^2} + B_1 \hat{e}_\perp^1 + \cancel{B_2 \hat{e}_\perp^2} \qquad \parallel$$

$$\hat{e}_\perp^1 \left(B_1 - \frac{B_2 v_1}{v_2}\right) = \parallel$$

$$\hat{e}_\perp^1 = \frac{1}{B_1 - \frac{B_2 v_1}{v_2}} (v\hat{e}) \cdot \left(-\frac{B_2}{v_2}\right) + (B\hat{e})$$

$$= \frac{1}{B_1 - \frac{B_2 v_1}{v_2}} \frac{-1}{v_2} \left(B_2 (v\hat{e}) - v_2 (B\hat{e}) \right)$$

$$\hat{e}_\perp^1 = \frac{(ve)}{v_1 B_2 - v_2 B_1} \left[B_2 \left(W^2 (Bv) + \frac{B^k}{a^k} \right) - v_2 W^2 \left(B^2 - (Bv) \frac{B^k}{a^k} \right) \right]$$

$$(v\hat{e}) \cdot \left(-\frac{B_1}{v_1}\right) + (B\hat{e})$$

$$(v_1 \hat{e}_\perp^1 + v_2 \hat{e}_\perp^2) \cdot \frac{-B_1}{v_1} + B_1 \hat{e}_\perp^1 + B_2 \hat{e}_\perp^2 = (v\hat{e}) \cdot \left(-\frac{B_1}{v_1}\right) + (B\hat{e})$$

$$-\frac{B_1 v_2 \hat{e}_\perp^2}{v_1} - \cancel{B_1 \hat{e}_\perp^1} + B_2 \hat{e}_\perp^2 + \cancel{B_1 \hat{e}_\perp^1} \qquad \|$$

$$\hat{e}_\perp^2 \left(B_2 - \frac{B_1 v_2}{v_1} \right) = \|$$

$$\hat{e}_\perp^2 = \frac{1}{B_2 - \frac{B_1 v_2}{v_1}} (v\hat{e}) \cdot \left(-\frac{B_1}{v_1}\right) + (B\hat{e})$$

$$= \frac{1}{} \frac{1}{} \left(-B (v\hat{e}) + v (B\hat{e}) \right)$$

$$v_1 \qquad \frac{B_2 - B_1 v_2}{v_1}$$

$$\hat{e}^2_\perp = \frac{(ve)}{v_1 B_2 - v_2 B_1}\left[-B_1\left(W^2(Bv) + \frac{B^k}{a^k}\right) + v_1 W^2\left(B^2 - (Bv)\frac{B^k}{a^k}\right)\right]$$

All components of Alfvén wave:

$$e^0 = 0$$

$$e^1_\perp = \frac{(ve)}{v_1 B_2 - v_2 B_1}\left(\frac{a^k}{B^k}\right)\left[B_2\left(W^2(Bv) + \frac{B^k}{a^k}\right) - v_2 W^2\left(B^2 - (Bv)\frac{B^k}{a^k}\right)\right] - (ve)W^2 B^1 \frac{a^k}{B^k}$$

$$e^2_\perp = \frac{(ve)}{v_1 B_2 - v_2 B_1}\left(\frac{a^k}{B^k}\right)\left[-B_1\left(W^2(Bv) + \frac{B^k}{a^k}\right) + v_1 W^2\left(B^2 - (Bv)\frac{B^k}{a^k}\right)\right] - (ve)W^2 B^2 \frac{a^k}{B^k}$$

$$e^k = -(ve)W^2 a^k$$

$$e^4 = 0$$

$$\hat{e}^1_\perp = \frac{(ve)}{v_1 B_2 - v_2 B_1}\left[B_2\left(W^2(Bv) + \frac{B^k}{a^k}\right) - v_2 W^2\left(B^2 - (Bv)\frac{B^k}{a^k}\right)\right]$$

$$\hat{e}^2_\perp = \frac{(ve)}{v_1 B_2 - v_2 B_1}\left[-B_1\left(W^2(Bv) + \frac{B^k}{a^k}\right) + v_1 W^2\left(B^2 - (Bv)\frac{B^k}{a^k}\right)\right]$$

$$\hat{e}^k = 0$$

Magneto sonic Waves

$$\begin{pmatrix} h_e W^4 a^k - C^k + W^2 E^k & D^k \\ h_e W^4 (Bv) a^k & h_e W^2 a^k \end{pmatrix} \begin{pmatrix} v_e \\ B_e \end{pmatrix} = \frac{-e^0}{\rho_0} \begin{pmatrix} h_e c_s^2 \left(v^k - a^k + W^2 a^k\right) + E^k \\ h_e c_s^2 \left(B^k + W^2 a^k (Bv)\right) \end{pmatrix}$$

$$C^k = (v^k - a^k)\left[B^2 - (Bv)\frac{B^k}{a^k}\right] + \frac{B^k B^k}{a^k}$$

$$D^k = (v^k - a^k)\left[(Bv) + \frac{B^k}{W^2 a^k}\right] + B^k$$

$$E^k = (v^k - a^k)\left[\frac{B^2}{W^2} + (Bv)^2\right] + B^2 a^k - (Bv) B^k$$

$$\Delta^{kk} = (a^k)^2 (h_e W^2 + B^2) - 2(Bv) a^k B^k - \frac{(B^k)^2}{W^2}$$

$$\begin{pmatrix} v_e \\ B_e \end{pmatrix} = \frac{1}{\cdots} \begin{pmatrix} h_e W^2 a^k & -D^k \\ -h_e W^4 (Bv) a^k & h_e W^4 a^k - C^k + W^2 E^k \end{pmatrix} \begin{pmatrix} h_e c_s^2 \left(v^k - a^k + W^2 a^k\right) + E^k \\ h_e c_s^2 \left(B^k + W^2 a^k (Bv)\right) \end{pmatrix}$$

$$\frac{1}{ad - bc}$$

$$h_e W^2 a^k \left(h_e W^4 a^k - C^k + W^2 E^k\right) - D^k h_e W^4 (Bv) a^k$$

$$h_e W^2 a^k \Big\{ h_e W^4 a^k - \cancel{(v^k - a^k)\left[B^2 - (Bv)\frac{B^k}{a^k}\right]} - \frac{B^k B^k}{a^k}$$

$$+ W^2 \left[\cancel{(v^k - a^k)\left[\frac{B^2}{W^2} + (Bv)^2\right]} + B^2 a^k - (Bv) B^k \right] \Big\}$$

$$- h_e W^4 (Bv) a^k \left[\cancel{(v^k - a^k)\left[(Bv) + \frac{B^k}{W^2 a^k}\right]} + B^k \right]$$

$$h_e^2 W^6 (a^k)^2 - h_e W^2 (B^k)^2 + h_e W^4 (a^k)^2 B^2 - 2 h_e W^4 a^k (Bv) B^k$$

$$h_e W^4 \left(h_e W^2 (a^k)^2 - \frac{(B^k)^2}{W^2} + (a^k)^2 B^2 - 2 a^k (Bv) B^k \right)$$

$$h_e W^4 \Delta^{kk}$$

$$\begin{pmatrix} v_e \\ B_e \end{pmatrix} = \frac{-e^0}{\rho_0 h_e W^4 \Delta^{kk}} \begin{pmatrix} h_e W^2 a^k & -D^k \\ -h_e W^4 (Bv) a^k & h_e W^4 a^k - C^k + W^2 E^k \end{pmatrix} \begin{pmatrix} h_e c_s^2 (v^k - a^k + W^2 a^k) + E^k \\ h_e c_s^2 (B^k + W^2 a^k (Bv)) \end{pmatrix}$$

$(Ve) =$

$(Be) =$

(4.1) $e^0 W_a^k + (ve) W^3_{\rho_0} a^k + e^k W_{\rho_0} = 0$

$$e^0 W_a^k + (ve) W^3_{\rho_0} a^k + e^k W_{\rho_0} = 0$$

$$e^k = -\frac{\left(e^0 \cancel{W}_a^k + (ve) W^3_{\rho_0} a^k\right)}{\cancel{W}_{\rho_0}}$$

$$= -\frac{a^k}{\rho_0}\left(e^0 + (ve) W^2_{\rho_0}\right)$$

$$e^k = -\frac{a^k}{\rho_0}\left(e^0 + (ve)W^2_{\rho_0}\right)$$

(4.9)
$$0 = B^2 e^k - (Be)B^k + (B\hat{e})a^k$$

(4.10)
$$0 = (Bv)e^k - (ve)B^k + (v\hat{e})a^k$$

$$(B\hat{e}) = \frac{(Be)B^k - B^2 e^k}{a^k}$$

$$(v\hat{e}) = \frac{(ve)B^k - (Bv)e^k}{a^k}$$

$$(B\hat{e}) =$$

Left Eigenvectors

$$\tilde{A}^k_{ij} = (h_{ij}Q + 2h_e W^4 v_i v_j - B_i B_j)(\bar{v}^k - \lambda^k) + (Q v_i - (Bv)B_i)h_j^k + h_i^k((Bv)B_j - B^2 v_j) - (h_{ij}(Bv) + v_i B_j - 2B_i v_j)B^k$$

$$\tilde{B}^k_{ij} = (2v_i B_j - B_i v_j - (Bv)h_{ij})(\bar{v}^k - \lambda^k) - \left(\frac{h_{ij}}{W^2} + v_i v_j\right)B^k + h_i^k\left((Bv)v_j + \frac{B_j}{W^2}\right) - h_j^k\left(\frac{B_i}{W^2} + (Bv)v_i\right)$$

$$\tilde{C}^k_j = (2h_e W^4 + B^2 - \rho_0 W^3)v_j(\bar{v}^k - \lambda^k) - (Bv)B_j(\bar{v}^k - \lambda^k) + (Q - W\rho_0)h_j^k + ((Bv)B_j - B^2 v_j)v^k - B_j B^k$$

$$\tilde{D}^k_j = \left(2B_j - (Bv)v_j - \frac{B_j}{W^2}\right)(\bar{v}^k - \lambda^k) + \left((Bv)v_j + \frac{B_j}{W^2}\right)v^k - v_j B^k - (Bv)h_j^k$$

$$\begin{bmatrix} Wa^k & W^3\rho_0 v_j a^k + W\rho_0 h_j^k & 0^k & 0_j^k \\ W^2\gamma v_i a^k + h_i^k \chi & \tilde{A}^k_{ij} & (\rho_0 + \chi)W^2 v_i a^k + h_i^k \chi & \tilde{B}^k_{ij} \\ (W^2\gamma - W - \chi)a^k + \chi v^k & \tilde{C}^k_j & [(\rho_0 + \chi)W^2 - \chi]a^k + \chi v^k & \tilde{D}^k_j \\ 0^{ik} & B^i h_j^k - B^k h_j^i & 0^{ik} & h^i_j a^k - h^k_j v^i \end{bmatrix}$$

(4.26)
$$0 = e^0 W a^k + (ve)W^2\gamma a^k + e^k \chi + e^4\left[(W^2\gamma - W - \chi)a^k + \chi v^k\right]$$

(4.27)
$$0 = e^0\left[W^3_{\rho_0} v_j a^k + W_{\rho_0} h_j^k\right] + \Big\{e_j\left[Q a^k - (Bv)B^k\right]$$
$$+ (ve)\left[2h_e W^4 v_j a^k + Q h_j^k - B_j B^k\right]$$
$$+ (Be)\left[-B_j a^k - (Bv)h_j^k + 2v_j B^k\right] + e^k\left[(Bv)B_j - B^2 v_j\right]\Big\}$$
$$+ e^4\Big[(2h_e W^4 + B^2 - W^3\rho_0)v_j a^k - (Bv)B_j a^k + (Q - W\rho_0)h_j^k$$
$$+ ((Bv)B_j - B^2 v_j)v^k - B_j B^k\Big]$$

$$+ (B\hat{e})h_j^k - \hat{e}_j B^k$$

(4.28)
$$0 = (ve)\left[(\rho_0 + \mathcal{H})W^2 a^k\right] + e^k \mathcal{H} + e^4\left[(\rho_0 + \mathcal{H})W^2 a^k + \mathcal{H}(v^k - a^k)\right]$$

(4.29)
$$0 = \Big\{ -e_j\left[(Bv)a^k + \frac{B^k}{W^2}\right] + (ve)\left[2B_j a^k - v_j B^k - (Bv)h_j^k\right]$$
$$-(Be)\left[v_j a^k + \frac{h_j^k}{W^2}\right] + e^k\left[(Bv)v_j + \frac{B_j}{W^2}\right]\Big\}$$
$$+ e^4\left[\left(2 - \frac{1}{W^2}\right)B_j a^k - (Bv)v_j a^k + \left((Bv)v_j + \frac{B_j}{W^2}\right)v^k - v_j B^k - (Bv)h_j^k\right]$$
$$+ \hat{e}_j \cdot a^k - (\bar{v}\hat{e})h_j^k$$

$$-\chi/\mathcal{H} \cdot 4.28 + 4.26$$

$$0 = e^0 W a^k + (ve)W^2\gamma a^k + \cancel{e^k}\chi + e^4\left[(W^2\gamma - W - \chi)a^k + \chi v^k\right]$$
$$\frac{-\chi}{\mathcal{H}}\left[(ve)\left[(\rho_0 + \mathcal{H})W^2 a^k\right] + \cancel{e^k}\mathcal{H} + e^4\left[(\rho_0 + \mathcal{H})W^2 a^k + \mathcal{H}(v^k - a^k)\right]\right]$$

$$0 = e^0 W a^k + (ve)\left[W^2\gamma a^k - \frac{\chi}{\mathcal{H}}(\rho_0 + \mathcal{H})W^2 a^k\right]$$
$$+ e^4\left[(W^2\gamma - W - \chi)a^k + \chi v^k - \frac{\chi}{\mathcal{H}}\left[(\rho_0 + \mathcal{H})W^2 a^k + \mathcal{H}(v^k - a^k)\right]\right]$$

$$= e^0 + (ve)\left[W\gamma - \frac{\chi}{\mathcal{H}}(\rho_0 + \mathcal{H})W\right]$$
$$+ e^4\left[W\gamma - 1 - \frac{\chi}{\mathcal{H}}(\rho_0 + \mathcal{H})W\right]$$

$$= e^0 - e^4 + W\left[\gamma - \chi\left(1 + \frac{p_0}{\Omega}\right)\right](e^4 + (ve))$$

$$\gamma - \chi\left(1 - \frac{p_0}{\Omega}\right) = \frac{h_e}{\rho_0 \Omega}\left(\Omega - \rho_0 c_s^2\right)$$

(4.30)

$$0 = e^0 - e^4 + \frac{h_e W}{\rho_0}\left(1 - \frac{\rho_0 c_s^2}{\Omega}\right)(ve + e^4)$$

4.27 · B^j

$$B^j \Big\{ e^0 \left[W^3_{\rho_0} v_j a^k + W_{\rho_0} h_j^k \right] + e_j \left[Q a^k - (Bv) B^k \right]$$

$$\downarrow\ W^3 \rho_0 (Bv) a^k \qquad \downarrow\ W\rho_0 B^k \qquad \downarrow\ (Be) \qquad \downarrow\ (h_e W^2 + B^2) a^k$$

$$+ (ve)\left[2 h_e W^4 v_j a^k + Q h_j^k - B_j B^k \right]$$

$$\downarrow\ 2 h_e W^4 (Bv) a^k \qquad \downarrow\ Q B^k \qquad \downarrow\ - B^2 B^k$$

$$+ (Be)\left[- B_j a^k - (Bv) h_j^k + 2 v_j B^k \right] + e^k \left[(Bv) B_j - B^2 v_j \right]$$

$$\downarrow\ - B^2 a^k \qquad \downarrow\ -(Bv) B^k \qquad \downarrow\ 2(Bv) B^k \qquad \downarrow\ (Bv) B^2 - B^2 (Bv)$$

$$+ e^4 \left[(2 h_e W^4 + \cancel{B^2} - W^3 \rho_0) v_j a^k - (Bv) B_j a^k + (Q - W\rho_0) h_j^k \right.$$

$$\downarrow\ (Bv) a^k \qquad \downarrow\ -(Bv) B^2 a^k \qquad \downarrow\ h_e W^2 + \cancel{B^2} \qquad \downarrow\ B^k$$

$$\left. + ((Bv) B_j - B^2 v_j) v^k - B_j B^k \right] + (B\hat{e}) h_j^k - \hat{e}_j B^k \Big\}$$

$$\downarrow\ \cancel{(Bv)} B^2 \qquad \downarrow\ - \cancel{B^2 (Bv)} \qquad \downarrow\ - B^2 B^k \qquad \downarrow\ \cancel{(B\hat{e}) B^k} \qquad \downarrow\ - \cancel{(B\hat{e})} B^k$$

$$0 = e^0 W\rho_0\left[W^2 (Bv)a^k + B^k\right] + (Be)\left[h_e W^2 a^k\right]$$
$$+ (ve)\left[2h_e W^4 (Bv)a^k + h_e W^2 B^k + \cancel{B^2 B^k} - \cancel{B^2 B^k}\right]$$
$$+ e^4\left[\left[2h_e W^4 - W^3\rho_0\right](Bv)a^k + h_e W^2 B^k - W\rho_0 B^k\right]$$

$$(4.31)\quad 0 = W\rho_0\left(W^2 (Bv)a^k + B^k\right)(e^0 - e^4)$$
$$+ h_e W^2\left(2W^2 (Bv)a^k + B^k\right)(ve + e^4) + (Be)h_e W^2 a^k$$

4.27 $\cdot\, v^j$

$$v^j\Big\{ e^0\Big[\underbrace{W^3\rho_0 v_j a^k}_{W^3\rho_0 v^2 a^k \,\to\, W^3\rho_0 a^k - W\rho_0 a^k} + \underbrace{W\rho_0 h_j^k}_{W\rho_0 v^k}\Big] + \Big\{\underbrace{e_j}_{(ve)}\Big[\underbrace{Q a^k}_{(h_e W^2 + B^2)a^k} - (Bv)B^k\Big]$$

$$+ (ve)\Big[\underbrace{2h_e W^4 v_j a^k}_{2h_e W^4 v^2 a^k} + \underbrace{Q h_j^k}_{(h_e W^2 + B^2)v^k} - \underbrace{B_j B^k}_{-(Bv)B^k}\Big]$$

$$+ (Be)\Big[\underbrace{-B_j a^k}_{-(Bv)a^k} \underbrace{- (Bv)h_j^k}_{-(Bv)v^k} + \underbrace{2v_j B^k}_{2v^2 B^k}\Big] + e^k\Big[\underbrace{(Bv)B_j}_{(Bv)^2} - \underbrace{B^2 v_j}_{-B^2 v^2}\Big]\Big\}$$

$$+ e^4\Big[(2h_e W^4 + B^2 - W^3\rho_0)\underbrace{v_j a^k}_{v^2 a^k} - \underbrace{(Bv)B_j a^k}_{-(Bv)^2 a^k} + (\underbrace{Q}_{h_e W^2 + B^2} - W\rho_0)\underbrace{h_j^k}_{v^k}$$

$$+ \Big(\underbrace{(Bv)B_j}_{(Bv)^2} - \underbrace{B^2 v_j}_{-B^2 v^2}\Big)v^k - \underbrace{B_j B^k}_{-(Bv)B^k}\Big] + \underbrace{(B\hat{e})h_j^k}_{(B\hat{e})v^k} - \underbrace{\hat{e}_j B^k}_{-(v\hat{e})B^k}\Big\}$$

$$W^3 v^2 = W^3\left(1-\frac{1}{W^2}\right)$$
$$= W^3 - W$$

$$0 = W\rho_0\left(v^k - a^k + W^2 a^k\right)\left(e^0 - e^4\right)$$
$$+ (ve)\left[2h_e W^4 a^k\left(1-\frac{1}{W^2}\right) - 2(Bv)B^k + (h_e W^2 + B^2)(v^k + a^k)\right]$$
$$+ e^4\left[\left(2h_e W^4 a^k\left(1-\frac{1}{W^2}\right) + \left((Bv)^2 - B^2\left(1-\frac{1}{W^2}\right)\right)\right)(v^k - a^k)\right.$$
$$\left. + (h_e W^2 + B^2)v^k - (Bv)B^k\right] - (Be)\left[(Bv)(v^k + a^k - 2B^k\epsilon)\right.$$
$$\left. - 2B^k\left(1-\frac{1}{W^2}\right)\mu\right] + e^k\left[(Bv)^2 - B^2\left(1-\frac{1}{W^2}\right)\right]$$
$$+ (B\hat{e})v^k - (v\hat{e})B^k$$

$$\uparrow \quad -\frac{1}{\sqrt{2}}\left((Bv)^2 - B^2\left(1-\frac{1}{W^2}\right)\right) \cdot 4.28$$

$$0 = W\rho_0\left(v^k - a^k + W^2 a^k\right)\left(e^0 - e^4\right)$$
$$+ (ve)\left[2h_e W^4 a^k\left(1-\frac{1}{W^2}\right) - 2(Bv)B^k + (h_e W^2 + B^2)(v^k + a^k)\right]$$
$$+ e^4\left[\left(2h_e W^4 a^k\left(1-\frac{1}{W^2}\right) + \left((Bv)^2 - B^2\left(1-\frac{1}{W^2}\right)\right)\right)(v^k - a^k)\right.$$
$$\left. + (h_e W^2 + B^2)v^k - (Bv)B^k\right] - (Be)\left[(Bv)(v^k + a^k - 2B^k\epsilon)\right.$$
$$\left. - 2B^k\left(1-\frac{1}{W^2}\right)\mu\right] + e^k\left[(Bv)^2 - B^2\left(1-\frac{1}{W^2}\right)\right]$$
$$+ (B\hat{e})v^k - (v\hat{e})B^k$$

$$-\frac{1}{\Omega}\left((Bv)^2 - B^2\left(1-\frac{1}{W^2}\right)\right)\cdot$$
$$\left\{(ve)\left[(\rho_0+\Omega)W^2 a^k\right] + e^k\Omega + e^4\left[(\rho_0+\Omega)W^2 a^k + \Omega(v^k - a^k)\right]\right\}$$

$$-(Bv)^2 + B^2\left(1-\frac{1}{W^2}\right)\cdot$$
$$\left\{(ve)\left[\left(\frac{\rho_0}{\Omega}+1\right)W^2 a^k\right] + e^k + e^4\left[\left(\frac{\rho_0}{\Omega}+1\right)W^2 a^k + (v^k - a^k)\right]\right\}$$

$$(4.32)\quad 0 = W\rho_0\left(v^k - a^k + W^2 a^k\right)(e^0 - e^4)$$
$$+\Big[2h_e W^4 a^k\left(1-\frac{1}{W^2}\right) - (Bv)B^k + (h_e W^2 + B^2)v^k$$
$$-W^2 a^k\left(1+\frac{\rho_0}{\Omega}\right)\left((Bv)^2 - B^2\left(1-\frac{1}{W^2}\right)\right)\Big](ve + e^4)$$
$$(ve)\Big[-(Bv)B^k + (h_e W^2 + B^2)a^k \Big] \quad -(Be)\left[(Bv)(v^k + a^k - 2B^k\epsilon)\right]$$
$$-2B^k\left(1-\frac{1}{W^2}\right)\mu\Big] \quad + (B\hat{e})v^k - (v\hat{e})B^k$$

4.29 $\cdot B^j$

$$B^j\Big(\Big\{-e_j\left[(Bv)a^k + \frac{B^k}{W^2}\right] + (ve)\left[2B_j a^k - v_j B^k - (Bv)h_j{}^k\right]$$
$$\qquad -(Be) \qquad\qquad\qquad 2B^2 a^k \quad -(Bv)B^k \quad - \cancel{(Bv)B^k}\ 2$$
$$-(Be)\left[\cancel{v_j a^k} + \frac{h_j{}^k}{W^2}\right] + e^k\left[(Bv)v_j + \frac{B_j}{W^2}\right]\Big\}$$
$$\qquad (Bv)a^k \qquad \frac{B^k}{W^2} \qquad\qquad (Bv)^2 \qquad \frac{B^2}{W^2}$$

$$+e^4\left[(2-\tfrac{1}{W^2})B_j a^k - (Bv)v_j a^k + \left((Bv)v_j + \tfrac{B_j}{W^2}\right)v^k - v_j B^k - (Bv)h_j{}^k\right]$$

$$B^2 a^k \qquad -(Bv)^2 a^k \qquad (Bv)^2 \qquad \frac{B^2}{W^2} \qquad -(Bv)B^k \qquad -\cancel{(Bv)B^k}\ 2$$

$$+\hat{e}_j\cdot a^k - (\bar{v}\hat{e})h_j{}^k \Big)$$

$$(B\hat{e})a^k \qquad -(\bar{v}\hat{e})B^k$$

(4.33)

$$0 = 2\{\left[B^2 a^k - (Bv)B^k\right](v_e + e^4) + \left[(Bv)^2 + \frac{B^2}{W^2}\right](e^4 v^k + e^k)$$

$$-2(Be)\left[(Bv)a^k + \frac{B^k}{W^2}\right] - e^4\left[(Bv)a^k - \frac{B^2}{W^2}a^k\mu\right] + (B\hat{e})a^k - (\bar{v}\hat{e})B^k$$

$$v^j\Big(\Big\{-e_j\left[\cancel{(Bv)}a^k + \frac{B^k}{W^2}\right] + (ve)\left[2B_j a^k - v_j B^k - (Bv)h_j{}^k\right]$$

$$-(ve) \qquad 2(Bv)a^k \qquad -(v^2)B^k \to -B^k + \frac{B^k}{W^2} \qquad -(Bv)v^k$$

$$-(Be)\left[v_j a^k + \frac{h_j{}^k}{W^2}\right] + e^k\left[(Bv)v_j + \frac{B_j}{W^2}\right]\Big\}$$

$$(v^2)a^k \to a^k - \frac{a^k}{W^2} \qquad \frac{v^k}{W^2} \qquad (Bv)v^2 \to (Bv) - \frac{Bv}{W^2} \qquad \frac{(Bv)}{W^2}$$

$$+e^4\left[(2-\tfrac{1}{W^2})B_j a^k - (Bv)v_j a^k + \left((Bv)v_j + \tfrac{B_j}{W^2}\right)v^k - v_j B^k - (Bv)h_j{}^k\right]$$

$$(Bv)a^k \to 2(Bv)a^k - \frac{(Bv)a^k}{W^2} \qquad -(Bv)v^2 a^k \to -(Bv)a^k + \frac{(Bv)a^k}{W^2}$$

$$(Bv)v^2 \to \cancel{Bv} - \cancel{\frac{Bv}{W^2}} \qquad \cancel{\frac{(Bv)}{W^2}} \qquad -(v^2)B^k \to -B^k + \frac{B^k}{W^2} \qquad \cancel{-(Bv)v^k}$$

$$+\hat{e}_j\cdot a^k - (\bar{v}\hat{e})h_j{}^k \Big)$$

$$(v\hat{e})a^k \qquad -(\bar{v}\hat{e})v^k$$

$$(ve)\left[(Bv)(a^k - v^k) - B^k\right] - Be\left(a^k + (v^k - a^k)\frac{1}{W^2}\right)$$
$$+ e^k \cdot (Bv) + e^4\left[(Bv)a^k - B^k + \frac{B^k}{W^2}\right] + (v\hat{e})a^k - (\bar{v}\hat{e})v^k$$

(4.34)

$$0 = \left[(Bv)a^k - B^k\right](ve + e^4) - (ve)(Bv)v^k + e^4\frac{B^k}{W^2}$$
$$-(Be)\frac{1}{W^2}(v^k - a^k + W^2 a^k) + e^k(Bv) + (v\hat{e})a^k - (\bar{v}\hat{e})v^k$$

The Entropy Wave (Left Eigenvector)

(4.31) $$0 = \cancel{W}\rho_0\left(\cancelto{0}{W^2(Bv)a^k} + \cancel{B^k}\right)(e^0 - e^4)$$

Factored out

$$+ h_e W^4\left(\cancelto{0}{2W^2(Bv)a^k} + \cancel{B^k}\right)(ve + e^4) + (Be)h_e \cancelto{0}{W^2 a^k}$$

Factored out

$$0 = \rho_0(e^0 - e^4) + h_e W(ve + e^4)$$

$$e^0 - e^4 = -\frac{h_e W}{\rho_0}(ve + e^4)$$

(4.30)

$$0 = e^0 - e^4 + \frac{h_e W}{\rho_0}\left(1 - \frac{\rho_0 c_s^2}{\mathcal{X}}\right)(ve + e^4)$$

$$0 = \frac{h_e W}{\rho_0}\left(\frac{\rho_0 c_s^2}{\partial \lambda}\right)(ve + e^4)$$

(4.35) $$0 = (ve + e^4)$$

(4.36) $$0 = (e^0 - e^4)$$

(4.26)
$$0 = e^0 \cancelto{0}{W a^k} + (ve)\cancelto{0}{W^2 \gamma a^k} + e^k \chi + e^4\left[\cancelto{0}{(W^2\gamma - W - \chi)a^k} + \chi v^k\right]$$

$$0 = e^k + e^4 v^k$$

$$e^k = -e^4 v^k$$

(4.33)
$$0 = 2\left\{\left[B^2 a^k - (Bv)B^k\right]\cancelto{0}{(ve + e^4)} + \left[(Bv)^2 + \frac{B^2}{W^2}\right]\cancelto{0}{(e^4 v^k + e^k)}\right.$$
$$- 2(Be)\left[\cancelto{0}{(Bv)}a^k + \frac{B^k}{W^2}\right] - e^4\left[\cancelto{0}{(Bv)^2 a^k - \frac{B^2}{W^2}a^k \mu}\right] + \cancelto{0}{(Be)}a^k - (\bar{v}\hat{e})B^k$$

$$0 = -2(Be)\frac{\cancel{B^k}}{W^2} - (\bar{v}\hat{e})\cancel{B^k}$$

$$(Be) = -\frac{1}{2}W^2(\bar{v}\hat{e})$$

$$0 = \left[\cancelto{0}{(Bv)}a^k - B^k\right]\cancelto{0}{(ve + e^4)} - (ve)(Bv)v^k + e^4\frac{B^k}{W^2}$$
$$- (Be)\frac{1}{W^2}\left(v^k - \cancelto{0}{a^k} + \cancelto{0}{W^2 a^k}\right) + e^k(Bv) + \cancelto{0}{(v\hat{e})a^k} - (\bar{v}\hat{e})v^k$$

$$0 = e^0 (Bv) v^k + e^0 \frac{B^k}{W^2} - (Be)\frac{v^k}{W^2} - e^0 v^k (Bv) - (\bar{v}\hat{e}) v^k$$

$$= e^0 \left[\cancel{(Bv)} v^k + \frac{B^k}{W^2} - v^k \cancel{(Bv)} \right] - (Be)\frac{v^k}{W^2} - (\bar{v}\hat{e}) v^k$$

$$= \quad '' \quad + \frac{1}{2} (\bar{v}\hat{e}) v^k - (\bar{v}\hat{e}) v^k$$

$$= \quad '' \quad - \frac{1}{2} (\bar{v}\hat{e}) v^k$$

(4.38) $$(\bar{v}\hat{e}) = \frac{2}{W^2}\left(\frac{B^k}{v^k}\right) e^0$$

$$= e^0 \frac{B^k}{W^2} - (Be)\frac{v^k}{W^2} - (\bar{v}\hat{e}) v^k$$

$$\quad '' \quad + \frac{2(Be)}{W^2} v^k$$

(4.39) $$(Be) = -\frac{e^0 B^k}{v^k}$$

(4.27)
$$0 = e^0 \left[\cancel{W^3_{\rho_0 v_j} a^k}^{\to 0} + W_{\rho_0} h_j^k \right] + \Big\{ e_j \left[\cancel{Q a^k}^{\to 0} - (Bv) B^k \right]$$
$$+ (ve) \left[\cancel{2 h_e W^4 v_j a^k}^{k \to 0} + Q h_j^k - B_j B^k \right]$$
$$+ (Be) \left[- \cancel{B_j a^k}^{k \to 0} - (Bv) h_j^k + 2 v_j B^k \right] + e^k \left[(Bv) B_j - B^2 v_j \right] \Big\}$$
$$+ e^4 \left[\cancel{(2 h_e W^4 + B^2 - W^3_{\rho_0}) v_j a^k}^{k \to 0} - \cancel{(Bv) B_j a^k}^{k \to 0} + (Q - W_{\rho_0}) h_j^k \right.$$
$$\left. + ((Bv) B_j - B^2 v_j) v^k - B_j B^k \right] + (B\hat{e}) h_j^k - \hat{e}_j B^k$$

$$0 = e^0 W_{\rho_0} h_j^{\,k} + \Big\{ -e_j (Bv) B^k - e^0 \left[Q h_j^{\,k} - B_j B^k \right]$$

$$- \frac{B^k}{v^k} e^0 \left[-(Bv) h_j^{\,k} + 2 v_j B^k \right] - v^k e^0 \left[(Bv) B_j - B^2 v_j \right] \Big\}$$

$$+ e^0 \left[(Q - W_{\rho_0}) h_j^{\,k} + \left((Bv) B_j - B^2 v_j \right) v^k - B_j B^k \right]$$

$$+ (B\hat{e}) h_j^{\,k} - \hat{e}_j B^k$$

$$e_j (Bv) B^k + \hat{e}_j B^k =$$

$$- \frac{B^k}{v^k} e^0 \left[-(Bv) \cancelto{0}{h_j^{\,k}} + 2 v_j B^k \right] - \cancelto{0}{v^k e^0 \left[(Bv) B_j \right]}$$

$$+ \cancelto{0}{e^0 \left[(B_v) B_j v^k \right]} + (B\hat{e}) \cancelto{0}{h_j^{\,k}}$$

$$e_j (B_v) + \hat{e}_j = \frac{2B^k}{v^k} v_j e^0$$

(4.29)

$$0 = \Big\{ -e_j \left[(B_v) \cancelto{0}{a^k} + \frac{B^k}{W^2} \right] + (ve) \left[2 B_j \cancelto{0}{a^k} - v_j B^k - (B_v) h_j^{\,k} \right]$$

$$- (Be) \left[v_j \cancelto{0}{a^k} + \frac{h_j^{\,k}}{W^2} \right] + e^k \left[(Bv) v_j + \frac{B_j}{W^2} \right] \Big\}$$

$$+ e^4 \left[\left(2 - \frac{1}{W^2}\right) \cancelto{0}{B_j a^k} - \cancelto{0}{(Bv) v_j a^k} + \left((Bv) v_j + \frac{B_j}{W^2} \right) v^k - v_j B^k - (Bv) h_j^{\,k} \right]$$

$$+ \hat{e}_j \cancelto{0}{a^k} - (v \hat{e}) h_j^{\,k}$$

$$0 = -e_j\left[\frac{B^k}{W^2}\right] - e^0\left[-v_j B^k - (Bv)h_j^{\ k}\right]$$

$$+ \cancel{\frac{e^0 B^k}{v^k}}\frac{h_j^{\ k}}{W^2} - v^k e^0\left[(Bv)v_j + \frac{B_j}{W^2}\right] + e^0\left[((Bv)v_j + \frac{B_j}{W^2})v^k - v_j B^k - (Bv)h_j^{\ k}\right]$$

$$- \frac{2}{W^2}\left(\frac{B^k}{v^k}\right)e^0 h_j^{\ k}$$

$$e_j\left[\frac{B^k}{\cancel{W^2}}\right] = -e^0\frac{1}{\cancel{W^2}}\frac{B^k}{v^k}h_j^{\ k}$$

$$e_j = -e^0\frac{h_j^{\ k}}{v^k}$$

$j \neq k$

$e_j = 0$

(4.40)

$$\hat{e}_j = -2v_j\frac{B^k}{v^k}e^0$$

$$(\bar{v}\hat{e}) = \bar{v}^j\hat{e}_j = \frac{2}{W^2}\left(\frac{B^k}{v^k}\right)e^0 = \bar{v}^1\hat{e}_1 + \bar{v}^2\hat{e}_2 + \bar{v}^k\hat{e}_k$$

$$\frac{2}{W^2}\left(\frac{B^k}{v^k}\right)e^0 = \bar{v}^1\hat{e}_1 + \bar{v}^2\hat{e}_2 + \bar{v}^k\hat{e}_k$$

$$\frac{2}{W^2}\left(\frac{B^k}{v^k}\right)e^0 - \bar{v}^1\hat{e}_1 - \bar{v}^2\hat{e}_2 = \bar{v}^k\hat{e}_k$$

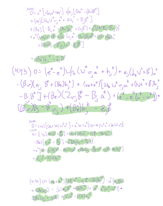
$$\frac{2}{W^2}\left(\frac{B^k}{v^k}\right)e^0 + 2\bar{v}^1 v_1\frac{B^k}{v^k}e^0 + 2\bar{v}^2 v_2\frac{B^k}{v^k}e^0 = \bar{v}^k\hat{e}_k$$

(4.41)

$$\hat{e}_k = 2\frac{B^k}{v^k\bar{v}^k}\left[\frac{1}{W^2} + \bar{v}^1 v_1 + \bar{v}^2 v_2\right]e^0$$

$$\vec{e}^{\,k} \to \begin{cases} e^0 \\ e^i \\ e^4 \\ \hat{e}^i \end{cases}$$

$$e^0 = e^0$$
$$e^i = 0^i$$
$$e^4 = e^0$$
$$\hat{e}^1 = \left[-2v_1 \frac{B^k}{v^k}\right] e^0$$
$$\hat{e}^2 = \left[-2v_2 \frac{B^k}{v^k}\right] e^0$$
$$\hat{e}^k = \left\{ 2\frac{B^k}{v^k v^k}\left[\frac{1}{W^2} + v^1 v_1 + v^2 v_2\right]\right\} e^0$$



Alfvén Waves (Left Eigenvectors)

(4.30 + 4.42)

$$0 = e^0 - e^4 + \frac{h_e W}{\rho_0}\left(1 - \frac{\rho_0 c_s^2}{\Omega}\right)(v_e + e^4)$$

(4.27 + 4.43)

$$0 = (e^0 - e^4) W/\rho_0 \left(W^2 v_j a^k + h_j^{\ k}\right) + e_j\left(h_e W^2 + B^2\right) a^k$$
$$- (Bv)\left(e_j B^k + (Be) h_j^{\ k}\right) + (v_e + e^4)\left[2h_e W^4 v_j a^k + (h_e W^2 + B^2) h_j^{\ k}\right.$$
$$\left. - B_j B^k\right] + (Be)\left(2v_j B^k - B_j a^k\right) + \left(e^k - e^4(a^k - v^k)\right) \cdot$$
$$\left((Bv)B_j - B^2 v_j\right) + (B\hat{e}) h_j^{\ k} - \hat{e}_j B^k$$

(4.44 + 4.28)

$$0 = (v_e + e^4)\left[\left(1 + \frac{\rho_0}{\Omega}\right) W^2 a^k\right] + e^k - e^4(a^k - v^k)$$

(4.45 + 4.29)

$$0 = (v_e + e^4)\left[2B_j a^k - v_j B^k - (Bv) h_j^{\ k}\right] - a^k\left[e_j (Bv)\right.$$
$$\left. + (Be) v_j\right] - \frac{1}{W^2}\left(e_j B^k + (Be) h_j^{\ k}\right) + \left(e^k - e^4(a^k - v^k)\right) \cdot$$
$$\left((Bv) v_j + \frac{1}{W^2} B_j\right) + \hat{e}_j a^k - (v\hat{e}) h_j^{\ k}$$

$$e^{0} \overset{(4.42)}{-} e^{4} = -\left[\frac{h_e W}{\rho_0}\left(1-\frac{\rho_0 c_s^2}{\mathcal{H}}\right)(v_e + e^4)\right]$$

(4.44)

$$e^k - e^4(a^k - v^k) = -(v_e + e^4)\left[\left(1+\frac{\rho_0}{\mathcal{H}}\right)W^2 a^k\right]$$

(4.43)

$$0 = -\left[\frac{h_e W}{\cancel{\rho_0}}\left(1-\frac{\rho_0 c_s^2}{\mathcal{H}}\right)(v_e+e^4)\right] W \cancel{\rho} \left(W^2 v_j a^k + h_j^k\right) + e_j\left(h_e W^2 + B^2\right) a^k$$

$(v_e + e^4)$

$$-\left[h_e W^2\left(W^2 v_j a^k + h_j^k\right) - \frac{h_e W^2 \rho_0 c_s^2}{\mathcal{H}}\left(W^2 v_j a^k + h_j^k\right)\right]$$

$$-(Bv)\left(e_j B^k + (Be)h_j^k\right) + (v_e+e^4)\left[\cancel{2} h_e W^4 v_j a^k + (h_e W^2 + B^2) h_j^k\right.$$

$$\left. - B_j B^k\right] + (Be)\left(2 v_j B^k - B_j a^k\right) - (v_e + e^4)\left[\left(1+\frac{\rho_0}{\mathcal{H}}\right)W^2 a^k\right] \cdot$$

$$\left((Bv)B_j - B^2 v_j\right) + (B\hat{e})h_j^k - \hat{e}_j B^k$$

(4.46)

$$(v_e + e^4)\left[h_e W^2\left(W^2 v_j a^k + \frac{\rho_0 c_s^2}{\mathcal{H}}\left(W^2 v_j a^k + h_j^k\right)\right)\right.$$

$$\left. - W^2 a^k\left(1+\frac{\rho_0}{\mathcal{H}}\right)\left((Bv)B_j - B^2 v_j\right) - B^2 h_j^k - B_j B^k\right]$$

$$+ e_j\left(h_e W^2 + B^2\right)a^k - (Bv)\left(e_j \cdot B^k + (Be)h_j^k\right) + (Be)\left(2 v_j B^k - B_j a^k\right)$$

$$+ (B\hat{e})h_j^k - \hat{e}_j B^k$$

(4.45) $0 = (v_e + e^4)\left[2B_j a^k - v_j B^k - (Bv) h_j^k\right] - a^k\left[e_j (Bv) + (Be) v_j\right] - \frac{1}{W^2}\left(e_j B^k + (Be) h_j^k\right) - (v_e + e^4)\left[\left(1+\frac{\rho_0}{\Omega}\right) W^2 a^k\right] \cdot \left((Bv) v_j + \frac{1}{W^2} B_j\right) + \hat{e}_j a^k - (\bar{v}\hat{e}) h_j^k$

$(v_e + t e^4)\left[2B_j a^k - v_j B^k - (Bv) h_j^k - \left(1+\frac{\rho_0}{\Omega}\right) W^2 a^k (Bv) v_j - \frac{W^2 a^k}{\cancel{W^2}} B_j - \frac{\rho_0}{\Omega}\frac{W^2 a^k}{\cancel{W^2}} B_j\right]$

$2B_j a^k - a^k B_j \left(1 + \frac{\rho_0}{\Omega}\right)$

$B_j a^k \left(1 - \frac{\rho_0}{\Omega}\right)$

(4.47)
$(v_e + e^4)\left(B_j a^k \left(1 - \frac{\rho_0}{\Omega}\right) - \left(1+\frac{\rho_0}{\Omega}\right) W^2 a^k (Bv) v_j - v_j B^k - (Bv) h_j^k\right) - a^k\left[e_j (Bv) + (Be) v_j\right] - \frac{1}{W^2}\left(e_j B^k + (Be) h_j^k\right) + \hat{e}_j a^k - (\bar{v}\hat{e}) h_j^k$

$a^k \cdot 4.46 + B^k \cdot 4.47$

$0 = a^k \cdot \Big\{ (v_e + e^4)\Big[h_e W^2 \left(W^2 v_j a^k + \frac{\rho_0 c_s^2}{\Omega}\left(W^2 v_j a^k + h_j^k\right)\right) - W^2 a^k \left(1+\frac{\rho_0}{\Omega}\right)\left((Bv) B_j - B^2 v_j\right) - B^2 h_j^k - \underline{B_j B^k} \Big]$

$+ e_j (h_e W^2 + B^2) a^k - (Bv)\left(\cancel{e_j} \cdot B^k\right)^2 + (Be) h_j^k) + (Be)\left(2 v_j B^k - B_j a^k\right)$

$+ (B\hat{e}) h_j^k - \hat{e}_j B^k \Big\}$

$+ B^k \Big\{ (v_e + e^4)\left(B_j a^k \left(1 - \frac{\rho_0}{\Omega}\right) - \left(1+\frac{\rho_0}{\Omega}\right) W^2 a^k (Bv) v_j - v_j B^k - (Bv) h_j^k\right)$

$$-a^k\left[e_j(Bv) + (Be)v_j\right] - \frac{1}{\mathcal{J}^2}\left(e_j B^k + (Be)h_j^k\right) + \hat{e}_j a^k - (\bar{v}\hat{e})h_j^k \Big\}$$

$$0 = a^k \cdot \Big\{(ve+e^4)\Big[h_e W^2\Big(W^2 v_j a^k + \frac{\rho_0 c_s^2}{\mathcal{X}}(W^2 v_j a^k + h_j^k)\Big)$$

$$- W^2 a^k \left(1+\frac{\rho_0}{\mathcal{X}}\right)\left((Bv)B_j - B^2 v_j\right) - B^2 h_j^k \Big]$$

$$+ e_j(h_e W^2 + B^2)a^k - (Bv)\,2\,(e_j\cdot B^k + (Be)h_j^k) + (Be)(2v_j B^k - B_j a^k)$$

$$+ (B\hat{e})h_j^k - \hat{e}_j B^k \Big\}$$

$$+ B^k\Big\{(ve+e^4)\Big(B_j a^k\left(-\frac{\rho_0}{\mathcal{X}}\right) - \left(1+\frac{\rho_0}{\mathcal{X}}\right)W^2 a^k (Bv)v_j - v_j B^k - (Bv)h_j^k\Big)$$

$$-a^k\left[e_j(Bv) + (Be)v_j\right] - \frac{1}{\mathcal{J}^2}\left(e_j B^k + (Be)h_j^k\right) + \hat{e}_j a^k - (\bar{v}\hat{e})h_j^k \Big\}$$

$$0 = \Big\{(ve+e^4)\Big[a^k\Big\{h_e W^2\Big(W^2 v_j a^k + \frac{\rho_0 c_s^2}{\mathcal{X}}(W^2 v_j a^k + h_j^k)\Big)$$

$$- W^2 a^k\left(1+\frac{\rho_0}{\mathcal{X}}\right)\left((Bv)B_j - B^2 v_j\right) - B^2 h_j^k\Big\} + B^k\Big\{B_j a^k\left(-\frac{\rho_0}{\mathcal{X}}\right)$$

$$- \left(1+\frac{\rho_0}{\mathcal{X}}\right)W^2 a^k (Bv)v_j - v_j B^k - (Bv)h_j^k\Big\}\Big]$$

$$+ e_j(h_e W^2 + B^2)(a^k)^2 - 2(Bv)(e_j\cdot B^k + (Be)h_j^k)a^k + (Be)(v_j B^k - B_j a^k)a^k$$

$$+ (B\hat{e})h_j^k a^k \quad - \frac{1}{\mathcal{J}^2}\left(e_j B^k + (Be)h_j^k\right)B^k \quad - (\bar{v}\hat{e})h_j^k B^k$$

$$\to \Delta^{kk} e_j \qquad \Delta^{kk} = (a^k)^2 Q - 2(Bv)a^k B^k - \frac{(B^k)^2}{W^2}$$

Us

$$0 = \Big\{(ve + e^4)\Big[a^k\Big\{h_e W^2\Big(W^2 v_j a^k + \frac{\rho_0 c_s^2}{\chi}\big(W^2 v_j a^k + h_j^k\big)\Big)$$

Missing c_s^2 in thesis

$$- W^2 a^k\Big(1 + \frac{\rho_0}{\chi}\Big)\big((Bv)B_j - B^2 v_j\big) - B^2 h_j^k\Big\} + B^k\Big\{B_j a^k\Big(-\frac{\rho_0}{\chi}\Big)$$

$$-\Big(1+\frac{\rho_0}{\chi}\Big)W^2 a^k (Bv) v_j - v_j B^k - (Bv) h_j^k\Big\}\Big]$$

These factored by a^k in thesis; should be B^k

$$-2(Bv)\big((Be)h_j^k\big)_a^k + (Be)\big(v_j B^k - B_j a^k\big)_a^k$$

$$+(B\hat{e})h_{j\,a}^{k\,k} \quad -\frac{1}{2}\big((Be)h_j^k\big)B^k \quad +(\bar{v}\hat{e})h_j^k B^k + \Delta^{kk} e_j$$

Ryan's Thesis

$$0 = (ve + e^4)\Big[W^2(a^k)^2\big(Q v_j - (Bv)(B_j + v_j)\big)$$

$$+ \frac{\rho_0}{\chi}W^2 a^k\Big(h_e c_s^2 W^2 v_j a^k + \overset{h_e c_s^2}{h_j^k} + a^k\big((Bv)(\overset{-a^k}{B_j} - \overset{B^k}{v_j})$$

$$- B^2 v_j\big)\Big) - \big(v_j B^k + (Bv)h_j^k\big)\overset{B^k}{a^k} - \Big(B^2 h_j^k$$

$$+ B_j B^k \frac{\rho_0}{\chi}\Big) a^k\Big] + e_j \Delta^{kk} + (Be)\Big[\big(v_j B^k - 2(Bv)h_j^k\big)_a^k$$

$$- B_j(a^k)^2 - \frac{1}{W^2}B^k h_j^k\Big] + (B\hat{e})h_{j\,a}^{k\,k} - (\bar{v}\hat{e})h_j^k B^k$$

(4.48)

$$0 = (ve + e^4)\Big[W^2(a^k)^2\big(Q v_j - (Bv)(B_j + v_j)\big)$$

$$+ \frac{\rho_0}{\chi}W^2 a^k\Big(h_e c_s^2 W^2 v_j a^k + \overset{h_e c_s^2}{h_j^k} + a^k\big((Bv)(\overset{-a^k}{B_j} - \overset{B^k}{v_j})$$

$$- B^2 v_j\big)\Big) - \big(v_j B^k + (Bv)h_j^k\big)\overset{B^k}{a^k} - \Big(B^2 h_j^k$$

$$+ B_j B^k \frac{\rho_0}{\chi}\Big) a^k\Big] + e_j \Delta^{kk} + (Be)\Big[\big(v_j B^k - 2(Bv)h_j^k\big)_a^k$$

$$-\beta_j (a^k)^2 - \frac{1}{\omega^2} \beta^k h_j^{\,k} \Big] + (\beta \hat{e}) h_j^{\,k} a^k - (\bar{v} \hat{e}) h_j^k \beta^k$$