Pg 10

$$\frac{\tilde{\mathcal{F}}^0}{\sqrt{h}} = \begin{bmatrix} W & \cdots & 0 \\ \gamma W^2 v_i & \cdots & 0^i \\ \gamma W^2 - W - \chi & \cdots & 0 \\ 0^i & \cdots & 0^i \\ 0 & \cdots & 1 \end{bmatrix}$$

D accounts for other parts of literature

$$\partial_t(\;) + \partial_j(B^i) = \partial_i$$

Conserved variables

$$\begin{pmatrix} D \\ S_i \\ \tau \\ B^i \\ \Phi \end{pmatrix}$$

$$\frac{\tilde{\mathcal{F}}^k}{\alpha\sqrt{h}} \begin{bmatrix} \vdots & \vdots & \vdots & 0_j{}^k & 0^k \\ \vdots & \vdots & \vdots & B^k{}_{ij} & 0^{ik} \\ \vdots & \vdots & \vdots & D^k{}_j & 0^k \\ 0^{ik} & B^i h_j{}^k - B^k h_j{}^i & 0^{ik} & h^i_j \bar{v}^k - h^k_j \bar{v}^i - h^k_j \eta \frac{\beta^i}{\alpha} & h^{ik} \\ 0^k & 0^k_j & 0^k & h^k_j & -\frac{\beta^k}{\alpha} \end{bmatrix}$$

$$\det\left|\tilde{\mathcal{J}}^k - \lambda^k \tilde{\mathcal{J}}^0\right| = \left(\alpha\sqrt{h}\right)^9 .$$

$$\begin{vmatrix}
W(\bar{v}^k-\lambda^k) & W^3\rho_0 v_j(\bar{v}^k-\lambda^k)+W\rho_0 h_j^k & 0^k & 0_j^{\ k} & 0^k \\
W^2\gamma v_i(\bar{v}^k-\lambda^k)+h_i^k\chi & \tilde{A}^k_{\ ij} & (\rho_0+\chi)W^2 v_i(\bar{v}^k-\lambda^k)+h_i^k\chi & \tilde{B}^k_{\ ij} & 0^{ik} \\
(W^2\gamma - W-\chi)(\bar{v}^k-\lambda^k)+\chi v^k & \tilde{C}^k_{\ j} & [(\rho_0+\chi)W^2-\chi](\bar{v}^k-\lambda^k)+\chi v^k & \tilde{D}^k_{\ j} & 0^k \\
0^{ik} & B^i h_j^k - B^k h_j^i & 0^{ik} & h^i_j(\bar{v}^k-\lambda^k)-h_j^k\bar{v}^i-h_j^k\gamma\frac{\beta^i}{\alpha} & h^{ik} \\
0^k & 0^k_j & 0^k & h_j^k & -\frac{\beta^k}{\alpha}-\lambda^k
\end{vmatrix}$$

$$\tilde{A}^k_{ij} = \left(h_{ij}Q + 2h_e W^4 v_i v_j - B_i B_j\right)(\bar{v}^k-\lambda^k) + \left(Qv_i - (Bv)B_i\right)h_j^k + h_i^k\left((Bv)B_j - B^2 v_j\right) - \left(h_{ij}(Bv) + v_i B_j - 2B_i v_j\right)B^k$$

$$\tilde{B}^k_{ij} = \left(2v_i B_j - B_i v_j - (Bv)h_{ij}\right)(\bar{v}^k-\lambda^k) - \left(\frac{h_{ij}}{W^2} + v_i v_j\right)B^k + h_i^k\left((Bv)v_j + \frac{B_j}{W^2}\right) - h_j^k\left(\frac{B_i}{W^2} + (Bv)v_i\right)$$

$$\tilde{C}^k_j = \left(2h_e W^4 + B^2 - \rho_0 W^3\right)v_j(\bar{v}^k-\lambda^k) - (Bv)B_j(\bar{v}^k-\lambda^k) + (Q - W\rho_0)h_j^k + \left((Bv)B_j - B^2 v_j\right)v^k - B_j B^k$$

$$\tilde{D}^k_j = \left(2B_j - (Bv)v_j - \frac{B_j}{W^2}\right)(\bar{v}^k-\lambda^k) + \left((Bv)v_j + \frac{B_j}{W^2}\right)v^k - v_j B^k - (Bv)h_j^k$$

$$\frac{\beta^k}{\alpha} = v^k - \bar{v}^k \qquad J\frac{\beta^i}{\alpha} = \psi^i$$

$$-\frac{\beta^k}{\alpha} = -v^k + \bar{v}^k$$

$$-\frac{\beta^k}{\alpha} - \lambda^k = -v^k + \bar{v}^k - \lambda^k$$

$$= -v^k + a^k$$

$$\begin{vmatrix} W a^k & W^3_{\rho_0 v_j} a^k + W_{\rho} h_j^k & 0^k & 0_j^k & 0^k \\ W^2 \gamma v_i a^k + h_i^k \chi & \tilde{A}^k_{ij} & (\rho_0+\chi) W^2 v_i a^k + h_i^k \chi & \tilde{B}^k_{ij} & 0^{ik} \\ (W^2\gamma - W - \chi) a^k + \chi v^k & \tilde{C}^k_j & [(\rho_0+\chi)W^2 - \chi] a^k + \chi r^k & \tilde{D}^k_j & 0^k \\ 0^{ik} & B^i h_j^k - B^k h_j^i & 0^{ik} & h^i_j a^k - h^k_j \bar{v}^i - h_j^k \psi^i & h^{ik} \\ 0^k & 0_j^k & 0^k & h_j^k & a^k - v^k \end{vmatrix}$$

: see Ryan's Thesis

$(a^k - v^k)\,|\text{Ryan's}| - h_j^k$

$$C_4 \stackrel{+}{=} -\left(\left((Bv)_{v_j} + B_j/W^2\right)/\chi\right) \cdot C_3$$

$$\begin{vmatrix} W a^k & W^3_{\rho_0 v_j} a^k + W_{\rho} h_j^k & 0^k & 0_j^k & 0^k \\ 0_i^k & \tilde{\alpha}^k_{ij} & \chi[h_i^k + v_i(a^k - v^k)] & B^k_{ij} & 0^{ik} \\ 0^k & \tilde{\xi}^k_j & [(\rho_0+\chi)W^2 - \chi] a^k + \chi r^k & \delta^k_j & 0^k \\ 0^{ik} & B^i h_j^k - B^k h_j^i & 0^{ik} & h^i_j a^k - h^k_j \bar{v}^i - h_j^k \psi^i & h^{ik} \\ 0^k & 0_j^k & 0^k & h_j^k & a^k - v^k \end{vmatrix}$$

Get rid of one of the off diagonal

$$\begin{array}{ccccc} 0^{1k} & B^1 h_j^k - B^k h_j^1 & 0^{1k} & h^1_j a^k - h^k_j \bar{v}^1 - h_j^k \psi^1 & 0 \\ 0^{2k} & B^2 h_j^k - B^k h_j^2 & 0^{2k} & h^2_j a^k - h^k_j \bar{v}^2 - h_j^k \psi^2 & 0 \\ 0^{kk} & B^k h_j^k - B^k h_j^k \to 0 & 0^{kk} & h^k_j a^k - h^k_j \bar{v}^k - h_j^k \psi^k & 1 \end{array}$$

$$\beta^k_{ij} = -\left(B_i v_j + (Bv) h_{ij}\right) a^k - \left(h_{ij} B^k + h_j^k B_i\right) \frac{1}{W^2}$$

$$\delta^k_j = 2B_j a^k - v_j B^k - (Bv) h_j^k - W^2 a^k \left((Bv) v_j + B_j \frac{1}{W^2}\right)\left(1 + \frac{\beta}{\kappa}\right)$$

New Territory from here on w/ row operations

$$R_4 - \frac{h^{ik}}{a^k - v^k} R_5$$

$$\begin{vmatrix}
W a^k & W^3_\beta v_j a^k + W_\beta h_j^k & 0^k & 0_j^k & 0^k \\
0_i^k & \tilde{\alpha}^k_{ij} & \kappa\left[h_i^k + v_i(a^k - v^k)\right] & B^k_{ij} & 0^{ik} \\
0^k & \tilde{\xi}^k_j & \left[(\beta+\kappa)W^2 - \kappa\right] a^k + \kappa v^k & \delta^k_j & 0^k \\
0^{ik} & B^i h_j^k - B^k h_j^i & 0^{ik} & h^i_j a^k - h^k_j v^i - h_j^k \psi^i - \frac{h_j^k h^{ik}}{a^k - v^k} & 0^{ik} \\
0^k & 0^k_j & 0^k & h_j^k & a^k - v^k
\end{vmatrix}$$

Since the final column is all zeros save the final value,

$$= \left(a^k - v^k\right) \begin{vmatrix} W a^k & W^3_{\beta_0 v_j} a^k + W_{\beta_0} h_j^k & 0^k & 0_j^k \\ 0_i^k & \tilde{\alpha}^k_{ij} & \chi\left[h_i^k + v_i(a^k - v^k)\right] & B^k_{ij} \\ 0^k & \tilde{\xi}^k_j & \left[(\beta_0 + \chi)W^2 - \chi\right]a^k + \chi v^k & \delta^k_j \\ 0^{ik} & B^i h_j^k - B^k h_j^i & 0^{ik} & h^i_j a^k - h^k_j \bar{v}^i - h_j^k \psi^i - \dfrac{h_j^k h^{ik}}{a^k - v^k} \end{vmatrix}$$

$$\xi^i = \bar{v}^i + \psi^i + \frac{h^{ik}}{a^k - v^k}$$

$$= \left(a^k - v^k\right) \begin{vmatrix} W a^k & W^3_{\beta_0 v_j} a^k + W_{\beta_0} h_j^k & 0^k & 0_j^k \\ 0_i^k & \tilde{\alpha}^k_{ij} & \chi\left[h_i^k + v_i(a^k - v^k)\right] & B^k_{ij} \\ 0^k & \tilde{\xi}^k_j & \left[(\beta_0 + \chi)W^2 - \chi\right]a^k + \chi v^k & \delta^k_j \\ 0^{ik} & B^i h_j^k - B^k h_j^i & 0^{ik} & h^i_j a^k - h^k_j \xi^i \end{vmatrix}$$

$$a^k C_2 \mathrel{+}= B^k \cdot C_4$$

$$\frac{1}{(a^k)^3} \left(a^k - v^k\right) \begin{vmatrix} W a^k & W^3_{\beta_0 v_j} a^k + W_{\beta_0} h_j^k & 0^k & 0_j^k \\ 0_i^k & \tilde{\alpha}^k_{ij} & \chi\left[h_i^k + v_i(a^k - v^k)\right] & B^k_{ij} \\ 0^k & \tilde{\xi}^k_j & \left[(\beta_0 + \chi)W^2 - \chi\right]a^k + \chi v^k & \delta^k_j \\ 0^{ik} & B^i h_j^k - B^k h_j^i & 0^{ik} & h^i_j a^k - h^k_j \xi^i \end{vmatrix}$$

$$a^k B^i h_j^k - \cancel{a^k B^k h_j^i} + \cancel{B^k h_j^i a^k} - B^k h_j^k f^i$$

$$C_2 R_4 \quad a^k B^i h_j^k - B^k h_j^k f^i$$

$B^j . C_4$

$$B^j h_j^i a^k = B^i a^k \qquad B^j h_j^k f^i = B^k f^i$$

$$C_4 R_4 \quad B^i a^k - B^k f^i$$

$$C_2 R_4 \quad a^k B^i h_j^k - B^k h_j^k f^i \qquad C_4 R_4 \quad B^i a^k - B^k f^i$$

○ ○ $a^k B^1 - B^k f^1$
○ ○ $a^k B^2 - B^k f^2$
○ ○ $a^k B^k - B^k f^k$

$B^1 a^k - B^k f^1$
$B^2 a^k - B^k f^2$
$B^k a^k - B^k f^k$

$$a^k = \bar{v}^k - \lambda^k$$

$$h^i_j a^k - h^k_j f^i$$

$$\begin{matrix} 0 & 0 & 0 & 0 & a^k & 0 & -f^1 \\ 0 & 0 & 0 & 0 & 0 & a^k & -f^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & a^k - f^k \end{matrix}$$

$$a^k - f^k = -\lambda^k + \psi^k + \frac{h^{kk}}{a^k - v^k}$$

New factors for Char. Eq.

$(a^k - v^k)(a^k - f^k)$ and remove λ^k

$$-\alpha^8 h^5 \rho_0 h_e W^3 (a^k - v^k)(a^k - f^k) a^k \Delta^{kk} \Big\{ h_e W^4 (a^k)^4 (1 - c_s^2) + \Big[(a^k)^2 \big(h_e W^2 c_s^2 + B^2 + W^2 (Bv)^2 \big) - c_s^2 \Big(a^k W (Bv) + \frac{B^k}{W} \Big)^2 \Big] \cdot \Big[(a^k - v^k)^2 - h^{kk} \Big] \Big\}$$

Right Eigenvectors

(4.1) $$e^0 W a^k + (ve) W^3_{\rho_0} a^k + e^k W_{\rho_0} = 0$$

(4.2)
$$
\begin{aligned}
0 = {} & e^0 \left[W^2 \gamma v_i a^k + \chi h_i^k \right] \\
& + \Big\{ e^i \left[Q a^k - (Bv) B^k \right] + (ve) \left[2 h_e W^4 v_i a^k - h_i^k B^2 + 2 B_i B^k \right] \\
& + (Be) \left[-B_i a^k + h_i^k (Bv) - v_i B^k \right] + e^k \left[Q v_i - (Bv) B_i \right] \Big\} \\
& + e^4 \left[(\rho_0 + \chi) W^2 v_i a^k + h_i^k \chi \right] \\
& + \Big\{ \hat{e}_i \left[-(Bv) a^k - \frac{1}{W^2} B^k \right] + (v\hat{e}) \left[-B_i a^k - v_i B^k + h_i^k (Bv) \right] \\
& + (B\hat{e}) \left[2 v_i a^k + \frac{1}{W^2} h_i^k \right] - \hat{e}^k \left[(Bv) v_i + \frac{1}{W^2} B_i \right] \Big\}
\end{aligned}
$$

$$(4.3)\quad 0 = e^{0}\left[(W^{2}\gamma - W - \chi)a^{k} + \chi v^{k}\right]$$

$$+ \left\{(ve)\left[(2h_e W^{4} + B^{2} - W^{3}_{p_0})a^{k} - B^{2}v^{k}\right] - (Be)\left[(Bv)(a^{k} - v^{k}) + B^{k}\right]\right.$$

$$\left. + e^{k}(Q - W_{p_0})\right\}$$

$$+ e^{4}\left[(p_0 + \chi)W^{2}a^{k} + \chi(v^{k} - a^{k})\right]$$

$$+ \left\{-(v\hat{e})\left[(Bv)(a^{k} - v^{k}) + B^{k}\right] + (B\hat{e})\left[\left(2 - \frac{1}{W^{2}}\right)a^{k} + \frac{1}{W^{2}}v^{k}\right]\right.$$

$$\left. - \hat{e}^{k}(Bv)\right\}$$

$$(4.4a)\quad B^{i}e^{k} - B^{k}e^{i} + \hat{e}^{i}a^{k} - \hat{e}^{k}\bar{v}^{i} - \hat{e}^{k}\psi^{i} + e^{8}h^{ik} = 0$$

$$(4.4b)\quad \hat{e}^{k} + e^{8}(a^{k} - v^{k}) = 0$$

Entropy Wave

(4.1) $e^k W_{\rho_0} = 0$

(4.2)

$$
\begin{aligned}
0 = {} & e^0 \left[\chi h_i^{\ k} \right] \\
& + \Big\{ e^i \left[-(Bv) B^k \right] + (ve) \left[-h_i^{\ k} B^2 + 2 B_i B^k \right] \\
& + (Be) \left[h_i^{\ k} (Bv) - v_i B^k \right] + e^k \left[Q v_i - (Bv) B_i \right] \Big\} \\
& + e^4 \left[h_i^{\ k} \mathcal{K} \right] \\
& + \Big\{ \hat{e}_i \left[-\tfrac{1}{W^2} B^k \right] + (v\hat{e}) \left[-v_i B^k + h_i^{\ k} (Bv) \right] \\
& + (B\hat{e}) \left[\tfrac{1}{W^2} h_i^{\ k} \right] - \hat{e}^k \left[(Bv) v_i + \tfrac{1}{W^2} B_i \right] \Big\}
\end{aligned}
$$

(4.3)

$$
\begin{aligned}
0 = {} & e^0 \left[\chi v^k \right] \\
& + \Big\{ (ve) \left[-B^2 v^k \right] - (Be) \left[(Bv)(-v^k) + B^k \right] \\
& + e^k (Q - W_{\rho_0}) \Big\} \\
& + e^4 \left[\mathcal{K} v^k \right] \\
& + \Big\{ -(v\hat{e}) \left[(Bv)(-v^k) + B^k \right] + (B\hat{e}) \left[\tfrac{1}{W^2} v^k \right] \\
& - \hat{e}^k (Bv) \Big\}
\end{aligned}
$$

(4.4a) $B^i e^k - B^k e^i - \hat{e}^k \bar{v}^i - \hat{e}^k \psi^i + e^8 h^{ik} = 0$

(4.4b) $\hat{e}^k - e^8 v^k = 0$

Eigen Vector:

$$
\begin{bmatrix} e^0 \\ e^i \\ e^4 \\ \hat{e}^i \\ e^8 \end{bmatrix}
$$

$e^k = 0$

$\hat{e}^k = 0$

$e^i = 0$

$e^8 = 0$

4.4b

$$\hat{e}^k = e^8 v^k$$

4.1

$$e^k W_{\rho_0} = 0$$

$$e^i = \hat{e}^k \left(\frac{h^{ik}}{v^k} - \bar{v}^i - \psi^i \right) \frac{1}{B^k}$$

4.2 v^i − 4.3

(4.2) $\cdot v^i$

$$-\left(1-\frac{1}{W^2}\right)B^k$$

$$0 = e^0[\chi \underbrace{h_i{}^k}_{v^k}] + \{\underbrace{e^i}_{(ve)}[-\cancel{(Bv)}B^k] + (ve)[-h_i{}^k B^2 + \cancel{2}\underbrace{B_i}B^k]_{(Bv)} + (Be)[\underbrace{h_i{}^k}_{v^k}(Bv) - \underbrace{v_i}_{-v^2}B^k]\}$$

$$-B^k + \frac{B^k}{W^2} \qquad Bv - \frac{Bv}{W^2}$$

$$+e^4[\underbrace{h_i{}^k}_{v^k} K] + \{\underbrace{\hat e_i}_{(v\hat e)}[-\frac{1}{W^2}B^k] + (v\hat e)[-\underbrace{v_i}_{-v^2}B^k + \underbrace{h_i{}^k}_{v^k}(Bv)] + (B\hat e)[\frac{1}{W^2}\underbrace{h_i{}^k}_{v^k}] - \hat e^k[(Bv)\underbrace{v_i}_{v^2} + \frac{1}{W^2}\underbrace{B_i}_{(Bv)}]\}$$

(4.3)

$$(4.3)\quad 0 = e^0[\chi v^k] + \{(ve)[-B^2 v^k] - (Be)[(Bv)(-v^k)+B^k]\} + e^4[K v^k]$$

$$+\{-(v\hat e)[(Bv)(-v^k)+B^k] + (B\hat e)[\frac{1}{W^2}v^k] - \hat e^k(Bv)\}$$

$$0 = (ve)[(Bv)B^k] + (Be)\left[2v^k(Bv) + \frac{B^k}{W^2}\right]$$

$$(ve) = \hat e^k\left(\cancel{\frac{v^k}{v^k}}^{\,1} - (\bar v v) - (v\psi)\right)\frac{1}{B^k} \to (ve)(Bv)B^k = \hat e^k(Bv)(1-(\bar v v)-(v\psi))$$

$$(Be) = \hat e^k\left(\frac{B^k}{v^k} - (\bar v B) - (B\psi)\right)\frac{1}{B^k} \to (Be)\left[2v^k(Bv)+\frac{B^k}{W^2}\right] = 2\hat e^k(Bv)\left(1 - \frac{v^k}{B^k}(\bar v B) - \frac{v^k}{B^k}(B\psi)\right)$$

$$(ve)\left[(Bv)B^k\right] + (Be)\left[2v^k(Bv) + \frac{B^k}{W^2}\right] = 0 = \hat{e}^k\left((Bv)\left[3 - (\bar{v}v) - (v\Psi) - \frac{2v^k}{B^k}\left((\bar{v}B) + (B\Psi)\right)\right] + \frac{1}{W^2}\left(\frac{B^k}{v^k} - (\bar{v}B) - (B\Psi)\right)\right)$$

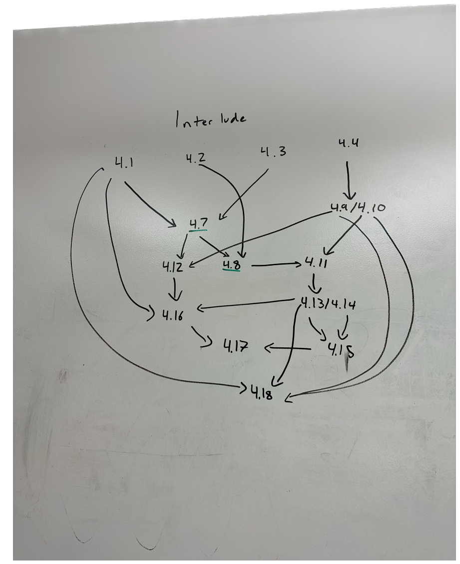
For this to be true, $\hat{e}^k = 0$

With this being the case, all of the other components of the entropy eigenvector are the same as in Ryan's thesis

Combine 4.2 v^i – 4.3,
get E_0 & E_4, then
rearrange 4.2 to get $\hat{e}^i$

Interlude

For 4.7 & 4.8, Ryan's equations don't depend on anything we've changed, so we keep these equations with the derivations located in the 'characteristic approach' document



$$
\begin{aligned}
(4.7)\quad 0 = {} & e^0\left[(W^2\gamma - \chi)a^k + \chi v^k\right] \\
& + \left\{(ve)\left[(2h_e W^4 + B^2)a^k - B^2 v^k\right] - (Be)\left[(Bv)(a^k - v^k) + B^k\right] + e^k Q\right\} \\
& + e^4\left[(p_0 + \mathcal{H})W^2 a^k + \mathcal{H}(v^k - a^k)\right] \\
& + \left\{-(v\hat{e})\left[(Bv)(a^k - v^k) + B^k\right] + (B\hat{e})\left[\left(2 - \frac{1}{W^2}\right)a^k + \frac{1}{W^2}v^k\right] - \hat{e}^k (Bv)\right\}
\end{aligned}
$$

$$(4.8)\quad 0 = e^0\left[-\chi\left((v^k - a^k)v_i - h_i{}^k\right)\right] + \Big\{ e^i\left[Q a^k - (B_v)B^k\right] + (v_e)\left[2B_i B^k + B^2\left((v^k - a^k)v_i - h_i{}^k\right)\right]$$

$$+ (B_e)\left[-B_i a^k + (B_v)\left((a^k - v^k)v_i + h_i{}^k\right)\right] - e^k (B_v) B_i \Big\} + e^4\left[-2\left((v^k - a^k)v_i - h_i{}^k\right)\right]$$

$$+ \Big\{ \hat{e}_i\left[-(B_v)a^k - \frac{1}{w^2}B^k\right] + (v\hat{e})\left[-(B_v)\left((v^k - a^k)v_i - h_i{}^k\right) - B_i a^k\right] + (B\hat{e})\left[-\frac{1}{w^2}\left((v^k - a^k)v_i - h_i{}^k\right)\right] - \hat{e}^k \frac{B_i}{w^2} \Big\}$$

From here on out, the equations will be different, so we will refer to equations that are different for us with a Roman numeral IV instead. For example, we will label this next equation as IV.9, bearing a similar role as Ryan's 4.9 but with modifications

Before continuing w/ Ryan's thesis, we will reduce 4.4a and 4.4b to one equation, IV.4

$$(4.4b)\quad \hat{e}^k + e^8(a^k - v^k) = 0$$

$$(4.4a)\quad B^i e^k - B^k e^i + \hat{e}^i a^k - \hat{e}^k \bar{v}^i - \hat{e}^k \psi^i + e^8 h^{ik} = 0$$

$$\frac{-\hat{e}^k}{a^k - v^k} = e^8$$

$$B^i e^k - B^k e^i + \hat{e}^i a^k - \hat{e}^k \bar{v}^i - \hat{e}^k \psi^i - \frac{\hat{e}^k h^{ik}}{a^k - v^k} = 0$$

$$\text{(IV.4)} \quad B^i e^k - B^k e^i + \hat{e}^i a^k - \hat{e}^k f^i = 0$$

where $f^i = \bar{v}^i + \psi^i + \frac{h^{ik}}{a^k - v^k}$

IV.4 $\cdot B_i$

$$\text{(IV.9)} \quad B^2 e^k - B^k (Be) + (B\hat{e}) a^k - \hat{e}^k (Bf) = 0$$

IV.4 $\cdot v_i$

$$\text{(IV.10)} \quad (Bv) e^k - B^k (ve) + (v\hat{e}) a^k - \hat{e}^k (vf) = 0$$

IV.10 to simplify 4.8

$$(4.8)\quad 0 = e^0\left[-\chi\left((v^k - a^k)_{v_i} - h_i{}^k\right)\right] + \Big\{e^i\left[Q a^k - (Bv)B^k\right] + (ve)\left[2B_i B^k + B^2\left((v^k - a^k)_{v_i} - h_i{}^k\right)\right]$$

$$+ (Be)\left[-B_i a^k + (Bv)\left((a^k - v^k)_{v_i} + h_i{}^k\right)\right] - e^k (Bv) B_i\Big\} + e^4\left[-\lambda\left((v^k - a^k)_{v_i} - h_i{}^k\right)\right]$$

$$+ \Big\{\hat{e}_i\left[-(Bv)a^k - \frac{1}{W^2}B^k\right] + (v\hat{e})\left[-(Bv)\left((v^k - a^k)_{v_i} - h_i{}^k\right) - B_i a^k\right] + (B\hat{e})\left[-\frac{1}{W^2}\left((v^k - a^k)_{v_i} - h_i{}^k\right)\right] - \hat{e}^k \frac{B_i}{W^2}\Big\}$$

$$-e^k(Bv)B_i - B_i a^k(v\hat{e}) + B_i B^k(ve) = -B_i \hat{e}^k (vf)$$

$$(IV.11)\quad 0 = (e^0\chi + e^4\kappa)\left((a^k - v^k)_{v_i} + h_i{}^k\right)$$

$$+ \Big\{e_i\left[(h_e W^2 + B^2)a^k - (Bv)B^k\right] + (ve)\left[B_i B^k - B^2\left((a^k - v^k)_{v_i} + h_i{}^k\right)\right]$$

$$+ (Be)\left[-B_i a^k + (Bv)\left((a^k - v^k)_{v_i} + h_i{}^k\right)\right]\Big\}$$

$$+ \Big\{\hat{e}_i\left[-(Bv)a^k - \frac{B^k}{W^2}\right] + (v\hat{e})\left[(Bv)\left((a^k - v^k)_{v_i} + h_i{}^k\right)\right] + (B\hat{e})\frac{1}{W^2}\left[(a^k - v^k)_{v_i} + h_i{}^k\right]\Big\}$$

$$- B_i \hat{e}^k \left(vf + \frac{1}{W^2}\right)$$