Modified terms for Characteristic Approach for ideal GRMHD

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Corrections for Ryan's Thesis

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Introduction

The purpose of this document is to provide clarity to the symbols used in the expanded derivation of the determinant in Ryan's thesis beginning on page 12. The following definitions correspond to various steps in the derivation where an intermediary result listed in the thesis differs from an expanded derivation done following the same actions.

Terms to be changed

The following coefficients take on two values each. One to correspond to the result found in the thesis and one to correspond to the result found in our extended derivation (i.e., if a term is present in Ryan's thesis and we don't find that term to actually be there, the value of the coefficient would be 1 for Ryan's thesis, and 0 for us). For the following list of terms, the first value is that for Ryan's thesis, and the second is for our extended derivation (see the previous mentioned example; in that case, the first value would be 1, and the second 0).

$$\epsilon = 1, 0$$
$$\mu = 0, 1$$
$$\zeta = 1, -1$$

These terms come up in several instances within the thesis, but seem to be automatically rectified in the thesis at various stages, suggesting that at least in two of the three cases, there was a typo of sorts. This being the case, two of the three instances are ultimately inconsequential, but will be included for the sake of clarity for anyone going through the notes and coming to similar conclusions.

The obsolete (as we will refer to the two that ultimately are inconsequential) terms are as follows:

$$\tilde{\xi}^k_j = (2h_eW^4 + \epsilon B^2)v_ja^k + Qh^k_j - B_jB^k - W^2(W^2v_ja^k + h^k_j)h_e(1 - \frac{\rho_0}{\kappa}c_s^2)$$

$$\beta^k_{ij} = -(B_iv_j + (Bv)h_{ij})a^k - (h_{ij}B^k + h^k_jB_i)\frac{1}{W^2} + \epsilon((Bv)B_j - B^2v_j)(h^k_i + v_i(a^k - v^k))$$

The following term is one that seems to (as of the most recent update of this document) not be automatically corrected, and potentially have an effect on the ultimate outcome of the characteristic equation

$$\begin{aligned} C^{kk}_j = {} & (a^k)^2[W^2(Wv_j\epsilon + (h_eW^2v_j + B^2v_j)\mu - (Bv)B_j) + \frac{\rho_0}{\kappa}W^2(h_eW^2c_s^2v_j + B^2v_j - (Bv)B_j)] \\ & + a^k[h^k_jW^2(Bv)^2 - W^2(Bv)v_jB^k + \frac{\rho_0}{\kappa}((B^2 + W^2(Bv)^2 + h_eW^2c_s^2)h^k_j - B^k(W^2(Bv)v_j + B_j))] \\ & - B^k[h^k_j(Bv) - v_jB^k]\zeta \end{aligned}$$