Package amsmath Error: llowed only in math modeSee the amsmath package documentation for explanation.You're in trouble here. Try typing <return> to proceed.If that doesn't work, type X <return> to quit.$^{*}293 EntropyWavesequation.2.29 \hat{e}^{kk} \hat{e}^{k} 303 EntropyWavesequation.2.30 \hat{e}^{kk} \hat{e}^{k} 313 EntropyWavesequation.2.31$

1. DETERMINENT

$$P^* = P + \frac{1}{2}B^2 \tag{1}$$

$$E = \frac{1}{2}\rho v^2 + \frac{P}{\gamma - 1} + \frac{1}{2}B^2 \tag{2}$$

By taking the individual derivatives we recieve this matrix whith $a^k = v^k - \lambda^k$:

$$\begin{pmatrix} a^k & \rho\delta^{jk} & 0 & 0^j \\ v_i a^k & \rho\delta_i^j a^k + \rho\delta^{jk} v_i & \delta_i^k & B^j\delta_i^k - B^k\delta_i^j - B_i\delta^{jk} \\ \frac{v^2}{2}a^k & \rho v^j a^k + (E+P^*)\delta^{jk} - B^k B^j & \frac{a^k}{\gamma-1} + v^k & B^j a^k + B^j v^k - \delta^{jk}(Bv) - B^k v^j \\ 0_i & B_i\delta^{jk} - B^k\delta_i^j & 0_i & \delta_i^j a^k - \delta^{jk} v_i \end{pmatrix}$$

Pull out a^k and let $R_2 = R_2 - v_i R_1$

$$a^k * \begin{pmatrix} 1 & \rho\delta^{jk} & 0 & 0^j \\ 0_i & \rho\delta_i^j a^k & \delta_i^k & B^j\delta_i^k - B^k\delta_i^j - B_i\delta^{jk} \\ \frac{v^2}{2} & \rho v^j a^k + (E+P^*)\delta^{jk} - B^k B^j & \frac{a^k}{\gamma-1} + v^k & B^j a^k + B^j v^k - \delta^{jk}(Bv) - B^k v^j \\ 0_i & B_i\delta^{jk} - B^k\delta_i^j & 0_i & \delta_i^j a^k - \delta^{jk} v_i \end{pmatrix} \tag{3}$$

Let $R_3 = R_3 - v^i R_2$

$$a^k * \begin{pmatrix} 1 & \rho\delta^{jk} & 0 & 0^j \\ 0_i & \rho\delta_i^j a^k & \delta_i^k & B^j\delta_i^k - B^k\delta_i^j - B_i\delta^{jk} \\ \frac{v^2}{2} & (E+P^*)\delta^{jk} - B^k B^j & \frac{a^k}{\gamma-1} & B^j a^k \\ 0_i & B_i\delta^{jk} - B^k\delta_i^j & 0_i & \delta_i^j a^k - \delta^{jk} v_i \end{pmatrix} \tag{4}$$

Let $R_3 = R_3 - \frac{v^2}{2} R_1$

$$a^k * \begin{pmatrix} 1 & \rho\delta^{jk} & 0 & 0^j \\ 0_i & \rho\delta_i^j a^k & \delta_i^k & B^j\delta_i^k - B^k\delta_i^j - B_i\delta^{jk} \\ 0 & (\frac{P}{\gamma-1} + B^2 + P)\delta^{jk} - B^k B^j & \frac{a^k}{\gamma-1} & B^j a^k \\ 0_i & B_i\delta^{jk} - B^k\delta_i^j & 0_i & \delta_i^j a^k - \delta^{jk} v_i \end{pmatrix} \tag{5}$$

Let $R_3 = R_3 - B^i R_4$

$$a^k * \begin{pmatrix} 1 & \rho\delta^{jk} & 0 & 0^j \\ 0_i & \rho\delta_i^j a^k & \delta_i^k & B^j\delta_i^k - B^k\delta_i^j - B_i\delta^{jk} \\ 0 & (\frac{P}{\gamma-1} + P)\delta^{jk} & \frac{a^k}{\gamma-1} & B^j a^k \\ 0_i & B_i\delta^{jk} - B^k\delta_i^j & 0_i & \delta_i^j a^k - \delta^{jk} v_i \end{pmatrix} \tag{6}$$

Now we expand on column one

$$a^k * \begin{pmatrix} \rho\delta_i^j a^k & \delta_i^k & B^j\delta_i^k - B^k\delta_i^j - B_i\delta^{jk} \\ (\frac{P}{\gamma-1}+P)\delta^{jk} & \frac{a^k}{\gamma-1} & \delta^{jk}(Bv) \\ B_i\delta^{jk} - B^k\delta_i^j & 0_i & \delta_i^j a^k - \delta^{jk}v_i \end{pmatrix} \tag{7}$$

Let $C_1 = a^k C_1 + B^k C_3$

$$a^k * \begin{pmatrix} (a^k)^2\rho\delta_i^j + B^k(B^j\delta_i^k - B^k\delta_i^j - B_i\delta^{jk}) & \delta_i^k & B^j\delta_i^k - B^k\delta_i^j - B_i\delta^{jk}a^k(\frac{P}{\gamma-1}+P) + (Bv)B^k > a^k(\frac{P}{\gamma-1}+P) + (Bv)B^k \\ \delta^{jk} & \frac{a^k}{\gamma-1} & \delta^{jk}(Bv) \\ B_i a^k\delta^{jk} - B^k\delta^{jk}v_i & 0_i & \delta_i^j a^k - \delta^{jk}v_i \end{pmatrix} \tag{8}$$

Let $C_1 = C_1 - \delta^{jk}(B_j C_3)$

$$a^k * \begin{pmatrix} (a^k)^2\rho\delta_i^j + B^k(B^j\delta_i^k - B^k\delta_i^j + B_i\delta^{jk}) - (B)^2\delta^{jk}\delta_i^k & \delta_i^k & B^j\delta_i^k - B^k\delta_i^j - B_i\delta^{jk} \\ a^k(\frac{P}{\gamma-1}+P)\delta^{jk} & \frac{a^k}{\gamma-1} & \delta^{jk}(Bv) \\ 0_i^j & 0_i & \delta_i^j a^k - \delta^{jk}v_i \end{pmatrix} \tag{9}$$

Expanding the indices to show the whole 7x7 matrix we get:

$$a^k * \begin{pmatrix} (a^k)^2\rho - (B^k)^2 & 0 & B^kB_1 & 0 & -B^k & 0 & -B_1 \\ 0 & (a^k)^2\rho - (B^k)^2 & B^kB_2 & 0 & 0 & -B^k & -B_2 \\ B^kB^1 & B^kB^2 & (a^k)^2\rho + (B^k)^2 - (B)^2 & 1 & B^1 & B^2 & -B^k \\ 0 & 0 & a^k(\frac{P}{\gamma-1}+P) & \frac{a^k}{\gamma-1} & 0 & 0 & (Bv) \\ 0 & 0 & 0 & 0 & a^k & 0 & -v_1 \\ 0 & 0 & 0 & 0 & 0 & a^k & -v_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & a^k - v^k \end{pmatrix} \tag{10}$$

Then using cofactor expansion upon the last three rows we recieve:

$$(a^k)^3(a^k - v^k) * \begin{pmatrix} (a^k)^2\rho - (B^k)^2 & 0 & B^kB_1 & 0 \\ 0 & (a^k)^2\rho - (B^k)^2 & B^kB_2 & 0 \\ B^kB^1 & B^kB^2 & (a^k)^2\rho + (B^k)^2 - (B)^2 & 1 \\ 0 & 0 & a^k(\frac{P}{\gamma-1}+P) & \frac{a^k}{\gamma-1} \end{pmatrix} \tag{11}$$

The characteristic equation is:

$$0 = a^k\lambda^k((a^k)^2\rho - (B^k)^2)[(a^k)^4\rho^2 + (a^k)^2\rho(P(\gamma-2) - B^2 - 2(B^k)^2) + (B^k)^2(B^2 - (B_1)^2 - (B_2)^2 + (B^k)^2 + P(2-\gamma))] \tag{12}$$

2. RIGHT EIGENVECTORS

Then we form the 4 equations of the Right Eigenvectors by dot producted each row by the vector $< e^0, e_j, e^4, \hat{e}_j >$. Then $II.1$ forms a relationship between e^0 and e^k which we use to cancel a few things in $II.2$ and $II.3$. Then doing $II.3 - v^i * II.2$ we were able to cancel more. Then we seperated the vector equations $II.2$ and $II.4$ into their singular components (multiplying by v^i forming the a equations and multiplying by B^i forming the b equations).

$$II.1)0 = a^k e^0 + \rho e^k \tag{13}$$

$$II.2)0 = v_i a^k e^0 + \rho a^k e_i + \rho e^k v_i + e^4\delta_i^k + (B\hat{e})\delta_i^k - B^k\hat{e}_i - B_i\hat{e}^k \tag{14}$$

$$II.3)0 = \frac{v^2}{2}a^k e^0 + \rho(ve)a^k + (E+P^*)e^k + (\frac{a^k}{\gamma-1} + v^k)e^4 + (B\hat{e})(a^k + v^k) - (Bv)\hat{e}^k - (v\hat{e})B^k \tag{15}$$

$$11.4)0 = B_i e^k - B^k e_i + a^k \hat{e}_i - v_i \hat{e}^k \tag{16}$$

After reducing these equations we are left with

$$II.1)0 = a^k e^0 + \rho e^k \tag{17}$$

$$II.2)0 = \rho a^k e_i + e^4 \delta_i^k + (B\hat{e})\delta_i^k - B^k \hat{e}_i - B_i \hat{e}^k \tag{18}$$

$$II.3)0 = (\frac{P}{\gamma - 1} + B^2 + P)e^k + \frac{a^k}{\gamma - 1} e^4 + (B\hat{e})a^k \tag{19}$$

$$II.4)0 = B_i e^k - B^k e_i + a^k \hat{e}_i - v_i \hat{e}^k \tag{20}$$

Seperating the vector equations into their components we recieve

$$II.2a)0 = \rho a^k (ve) + e^4 v^k + (B\hat{e})v^k - B^k (v\hat{e}) - (Bv)\hat{e}^k \tag{21}$$

$$II.2b)0 = \rho a^k (Be) + e^4 B^k + (B\hat{e})B^k - B^k (B\hat{e}) - B^2 \hat{e}^k \tag{22}$$

$$II.4a)0 = (Bv)e^k - B^k (ve) + a^k (v\hat{e}) - v^2 \hat{e}^k \tag{23}$$

$$II.4b)0 = B^2 e^k - B^k (Be) + a^k (B\hat{e}) - (Bv)\hat{e}^k \tag{24}$$

2.1. *Entropy Waves*

$a^k = 0$, $e^k = 0$, $II.1) = 0$, $II.3) = 0$ Because $e^k = 0$, when $i = k$, $\hat{e}^k = 0$. Therefore, $e^4 = 0$ and $e_i = 0$. Considerring the case where $i \neq k$, then $\delta_i^k = 0$ and therefore $\hat{e}_i = 0$.

$$II.2a)0 = e^4 v^k + (B\hat{e})v^k - B^k (v\hat{e}) - (Bv)\hat{e}^k \tag{25}$$

$$II.2b)0 = e^4 B^k - B^2 \hat{e}^k \tag{26}$$

$$II.4a)0 = -B^k (ve) - v^2 \hat{e}^k \tag{27}$$

$$II.4b)0 = -B^k (Be) - (Bv)\hat{e}^k \tag{28}$$

$$\hat{e}^k = \frac{-B^k (ve)}{v^2} \tag{29}$$

$$\hat{e}^k = \frac{-B^k (Be)}{Bv} \tag{30}$$

$$\hat{e}^k = e^4 \frac{B^k}{B^2} \tag{31}$$

Using equations $II.4$ we can see that $\hat{e}^k = 0$ and therefore using the last few equations $e^4 = 0$ and both (Be) and $(ve) = 0$

Seperating $\hat{e}^k$ into its components and solving we find

$$e^1 = (\hat{e}^k)[\frac{-(Bv)}{B^k B_1} - \frac{B_2((Bv)v_1 - B_1 v^2}{B^k[(B_1)^2 v_2 - v_1 B_1 B_2]}] = 0 \tag{32}$$

$$e^2 = (\hat{e}^k)\frac{(Bv)v_1 - v^2 B_1}{B_1 v_2 - v_1 B^k} = 0 \tag{33}$$

Then using equation $II.4$ and setting $i \neq k$, we find that $\hat{e}_i = 0$ therefore giving us the final eigenvector:

$$e =< 1, 0, 0, 0, 0, 0, 0, 0 > \tag{34}$$

2.2. *Alfvén Waves*

We start with the four base equations:

$$II.1)\ 0 = a^k e^0 + \rho e^k label II.1 \tag{35}$$

$$II.2)\ 0 = \rho e_i a^k + e^4 \delta_i^k + (B\hat{e})\delta_i^k - B^k \hat{e}_i - B_i \hat{e} k \tag{36}$$

$$II.3)\ 0 = \left(\frac{P}{\gamma - 1} + P + B^2\right) e^k + a^k e^4 \left(\frac{1}{\gamma - 1}\right) + (B\hat{e})a^k \tag{37}$$

$$II.4)\ 0 = B_i e^k - B^k e_i + \hat{e}_i a^k - \hat{e}^k v_i \tag{38}$$

Relationships:

$$II.2) \cdot B^i\ 0 = \rho(Be)a^k + e^4 B^k + (B\hat{e})B^k - B^k(B\hat{e}) \tag{39}$$

$\hat{e}^k = 0$ when $i = k$, found from equation II.4

$$\begin{aligned} II.2)^{i=k}\ 0 &= \rho e_i a^k + e^4 + B\hat{e} \\ &= -(a^k)^2 e^4 - \frac{\rho a^k}{B^k} Be + B\hat{e} \end{aligned} \tag{40}$$

$$\begin{aligned} II.4) \cdot B^i\ 0 &= B^2 (\frac{-a^k}{\rho}) e^0 - B^k(Be) + B\hat{e}a^k \\ &= -a^k e^0 \frac{(B^k)^2}{\rho} - \frac{\rho}{B^k} Be + B^2 \frac{a^k}{\rho} e^0 \\ &= \frac{a^k}{\rho} e^0 \left(-(B^k)^2 + B^2\right) \end{aligned} \tag{41}$$

$$e^0 = 0 \tag{42}$$

Using II.1 since $e^0 = 0$

$$e^k = 0 \tag{43}$$

By finding a relationship between e^k and e^4 we can prove $e^4 = 0$

$$II.3) - B^i \cdot II.4)\ 0 = \left(\frac{P}{\gamma - 1} + P\right) e^k + a^k e^4 \left(\frac{1}{\gamma - 1}\right) + B^k(Be) \tag{44}$$

$$II.3) - II.2 \cdot II.4)\ 0 = \left(\frac{P}{\gamma - 1} + P\right) e^k + e^4 \left(\frac{a^k}{\gamma - 1} - \frac{(B^k)^2}{\rho a^k}\right) \tag{45}$$

II.4) with e^k and $\hat{e}^k$ plugged in:

$$e_i = \hat{e}_i \frac{a^k}{B^k} \tag{46}$$

Plugging in a^k, we see:

$$II.4.1) e_i = \hat{e}_i \frac{1}{\sqrt{\rho}} \tag{47}$$

$$v_i \cdot II.4.1)\ 0 = (ve) - (v\hat{e}) \frac{1}{\sqrt{\rho}} \tag{48}$$

$$B_i \cdot II.4.1)\ 0 = (Be) - (B\hat{e}) \frac{1}{\sqrt{\rho}} \tag{49}$$

$$II.3)\ 0 = a^k \left(B\hat{e}\right) \rightarrow 0 = \frac{B^k}{\sqrt{\rho}} \left(B\hat{e}\right) \tag{50}$$

We have three linearly independent equations (48, 49, 50) in terms of e_1, e_2, $\hat{e}_1$, and $\hat{e}_2$. We can then solve for those components of the eigenvector. We are normalizing in terms of $\hat{e}_2$:

$$+a^k = \frac{B^k}{\sqrt{\rho}} \tag{51}$$

$$\begin{bmatrix} 0 \\ \frac{-B_2}{B_1\sqrt{\rho}} \\ \frac{1}{\sqrt{\rho}} \\ 0 \\ 0 \\ \frac{-B_2}{B_1} \\ 1 \\ 0 \end{bmatrix} \tag{52}$$

$$-a^k = \frac{B^k}{\sqrt{\rho}} \tag{53}$$

$$\begin{bmatrix} 0 \\ \frac{B_2}{B_1\sqrt{\rho}} \\ \frac{-1}{\sqrt{\rho}} \\ 0 \\ 0 \\ \frac{-B_2}{B_1} \\ 1 \\ 0 \end{bmatrix} \tag{54}$$

2.3. *Magnetosonic Waves*

We use the following equations

$$II.2 \cdot B^i)\ 0 = \rho(Be)a^k + e^4 B^k \tag{55}$$

$$II.2 \cdot v^i)\ 0 = \rho(ve)a^k + e^4 v^k + (B\hat{e})v^k - B^k(v\hat{e}) \tag{56}$$

$$II.4 \cdot B^i)\ 0 = (B^2)e^k - B^k(Be) + (B\hat{e})a^k \tag{57}$$

$$II.4 \cdot v^i)\ 0 = (Bv)e^k - B^k(ve) + (v\hat{e})a^k \tag{58}$$

Using those, we get e_1, e_2, $\hat{e}_1$, and $\hat{e}_2$.
Our normalization is e^4

e_0 comes from 17
e^k comes from 45
Then using Mathematica we are able to solve for the general eigenvector Magnetosonic waves and then substitute in the four values of a^k.

$$e^4 \cdot \begin{bmatrix} \frac{-(B^k)^2(\gamma-1)+(a^k)^2\rho}{(a^k)^2 P\gamma} \\ e^1 \\ e^2 \\ \frac{(B^k)^2(\gamma-1)+(a^k)^2\rho}{a^k P\gamma\rho} \\ 1 \\ \hat{e}^1 \\ \hat{e}^2 \\ 0 \end{bmatrix} \tag{59}$$

$$\alpha = a^k P(B_2 v_1 - B_1 v_2)\gamma\rho((a^k)^2\rho - (B^k)^2) \tag{60}$$

$$\hat{\alpha} = a^k \alpha \tag{61}$$

$$\begin{aligned} e_1 = \frac{e_4}{\alpha} \big[&(B^k)^2 \left(B_2 B^k (Bv) - B_2 B^2 v^k - B_1 B_2 B^k v_1 \gamma \right. \\ &\left. -\ B^k v_2 \gamma((B_2)^2 + P) + B_2 v^k \gamma(B^2 - (B^k)^2 + P) \right) \\ &+ \rho(a^k)^2 \left(B_1 B_2 B^k v_1 + (B_2)^2 B^k v_2 + B^k P v_2 \gamma - B_2 v^k \left(B^2 + (B^k)^2(\gamma-2) + P\gamma \right) \right) \\ &+ (a^k)^4 \rho^2 B_2 v^k \big] \end{aligned} \tag{62}$$

$$\begin{aligned} e_2 = \frac{e_4}{\alpha} \big[&(B^k)^2 \left((B_1)^2 B^k v_1(\gamma-1) + B_1 B_2 B^k v_2(\gamma-1) \right. \\ &\left. +\ B^k P v_1 \gamma - B_1 v^k((B^k)^2 - B^2 + \gamma(B^2 - (B^k)^2 + P)) \right) \\ &+ \rho(a^k)^2 \left(B_1 B^2 v^k - B_1 B^k(Bv + B^k v^k) - B^k P v_1 \gamma + B_1 v^k \gamma \left((B^k)^2 + P \right) \right) \\ &- (a^k)^4 \rho^2 B_1 v^k \big] \end{aligned} \tag{63}$$

$$\begin{aligned} \hat{e}_1 = \frac{e_4}{\hat{\alpha}} \big[&(B^k)^3 (B^k v_2 - B_2 v^k)(B^2 + (B^k)^2(\gamma-1) - B^2\gamma - P\gamma) \\ &-\ (a^k)^2 B^k \rho \left(B_1 B_2 v^k v_1(\gamma-1) + B^k v_2 \gamma \left((B^k)^2 - B^2 - P \right) + B_2 v^k \left(B^2 + (B^k)^2(\gamma-2) + P\gamma \right) \right) \\ &+\ (a^k)^4 \rho^2 (B_1 B_2 v_1 + B_2 B^k v^k + v_2((B_2)^2 - B^2 + (B^k)^2)) \big] \end{aligned} \tag{64}$$

$$\begin{aligned} \hat{e}_2 = \frac{e_4}{\hat{\alpha}} \big[&-(B^k)^3 (B^k v_1 - B_1 v^k)(B^2 + (B^k)^2(\gamma-1) - B^2\gamma - P\gamma) \\ &+\ (a^k)^2 B^k \rho \left(B_1 B_2 B^k v_2(\gamma-1) + B^k v_1 \gamma \left((B^k)^2 - B^2 - P \right) \right. \\ &\left. +\ B_1 v^k \left(B^2 + (B^k)^2(\gamma-2) + P\gamma \right) + (B_1)^2 B^k v_1(\gamma-1) \right) \\ &-\ (a^k)^4 \rho^2 (B_1 B_2 v_2 + B_1 B^k v^k + v_1((B_1)^2 - B^2 + (B^k)^2)) \big] \end{aligned} \tag{65}$$