

TABLE VIII Alias Relationships for 2^{k-p} Fractional Factorial Designs with $k \leq 15$ and $n \leq 64$

| Designs with 3 Factors | | |
|---|---|-----------------------|
| (a) 2^{3-1} ; 1/2 fraction of 3 factors in 4 runs | <u>Design Generators</u> $C = AB$ Defining relation: $I = ABC$ <u>Aliases</u> $A = BC$ $B = AC$ $C = AB$ | Resolution III |
| Designs with 4 Factors | | |
| (b) 2^{4-1} ; 1/2 fraction of 4 factors in 8 runs | <u>Design Generators</u> $D = ABC$ Defining relation: $I = ABCD$ <u>Aliases</u> $A = BCD$ $B = ACD$ $C = ABD$ $D = ABC$ $AB = CD$ $AC = BD$ $AD = BC$ | Resolution IV |
| Designs with 5 Factors | | |
| (c) 2^{5-2} ; 1/4 fraction of 5 factors in 8 runs | <u>Design Generators</u> $D = AB \quad E = AC$ Defining relation: $I = ABD = ACE = BCDE$ <u>Aliases</u> $A = BD = CE$ $B = AD = CDE$ $C = AE = BDE$ $D = AB = BCE$ $E = AC = BCD$ $BC = DE = ACD = ABE$ $CD = BE = ABC = ADE$ | Resolution III |
| (d) 2^{5-1} ; 1/2 fraction of 5 factors in 16 runs | <u>Design Generators</u> $E = ABCD$ Defining relation: $I = ABCDE$ <u>Aliases</u> Each main effect is aliased with a single 4-factor interaction. $AB = CDE \quad BD = ACE$ $AC = BDE \quad BE = ACD$ $AD = BCE \quad CD = ABE$ $AE = BCD \quad CE = ABD$ $BC = ADE \quad DE = ABC$ 2 blocks of 8: $AB = CDE$ | Resolution V |

TABLE VIII Alias Relationships for 2^{k-p} Fractional Factorial Designs with $k \leq 15$ and $n \leq 64$ (*Continued*)

| Designs with 6 Factors | |
|--|-----------------------|
| (e) 2^{6-3} ; 1/8 fraction of 6 factors in 8 runs | Resolution III |
| <u>Design Generators</u> $D = AB \quad E = AC \quad F = BC$ Defining relation: $I = ABD = ACE = BCDE = BCF = ACDF = ABEF = DEF$ | |
| <u>Aliases</u> $A = BD = CE = CDF = BEF \quad E = AC = DF = BCD = ABF$ $B = AD = CF = CDE = AEF \quad F = BC = DE = ACD = ABE$ $C = AE = BF = BDE = ADF \quad CD = BE = AF = ABC = ADE = BDF = CEF$ $D = AB = EF = BCE = ACF$ | |
| (f) 2^{6-2} ; 1/4 fraction of 6 factors in 16 runs | Resolution IV |
| <u>Design Generators</u> $E = ABC \quad F = BCD$ Defining relation: $I = ABCE = BCDF = ADEF$ | |
| <u>Aliases</u> $A = BCE = DEF \quad AB = CE$ $B = ACE = CDF \quad AC = BE$ $C = ABE = BDF \quad AD = EF$ $D = BCF = AEF \quad AE = BC = DF$ $E = ABC = ADF \quad AF = DE$ $F = BCD = ADE \quad BD = CF$ $ABD = CDE = ACF = BEF \quad BF = CD$ $ACD = BDE = ABF = CEF$ 2 blocks of 8: $ABD = CDE = ACF = BEF$ | |
| (g) 2^{6-1} ; 1/2 fraction of 6 factors in 32 runs | Resolution VI |
| <u>Design Generators</u> $F = ABCDE$ Defining relation: $I = ABCDEF$ | |
| <u>Aliases</u> Each main effect is aliased with a single 5-factor interaction. Each 2-factor interaction is aliased with a single 4-factor interaction. $ABC = DEF \quad ACE = BDF$ $ABD = CEF \quad ACF = BDE$ $ABE = CDF \quad ADE = BCF$ $ABF = CDE \quad ADF = BCE$ $ACD = BEF \quad AEF = BCD$ 2 blocks of 16: $ABC = DEF$ 4 blocks of 8: $AB = CDEF$ $ACD = BEF$ $AEF = BCD$ | |

TABLE VIII Alias Relationships for 2^{k-p} Fractional Factorial Designs with $k \leq 15$ and $n \leq 64$ (Continued)

| Designs with 7 Factors | |
|---|-----------------------|
| (h) 2^{7-4} ; 1/16 fraction of 7 factors in 8 runs | Resolution III |
| <p style="text-align: center;"><u>Design Generators</u></p> $D = AB \quad E = AC \quad F = BC \quad G = ABC$ | |
| <p>Defining relation: $I = ABD = ACE = BCDE = BCF = ACDF = ABEF = DEF = ABCG$ $= CDG = BEG = ADEG = AFG = BDFG = CEFG = ABCDEFG$</p> | |
| <p style="text-align: center;"><u>Aliases</u></p> $A = BD = CE = FG \quad E = AC = DF = BG$ $B = AD = CF = EG \quad F = BC = DE = AG$ $C = AE = BF = DG \quad G = CD = BE = AF$ $D = AB = EF = CG$ | |
| (i) 2^{7-3} ; 1/8 fraction of 7 factors in 16 runs | Resolution IV |
| <p style="text-align: center;"><u>Design Generators</u></p> $E = ABC \quad F = BCD \quad G = ACD$ | |
| <p>Defining relation: $I = ABCE = BCDF = ADEF = ACDG = BDEG = ABFG = CEFG$</p> | |
| <p style="text-align: center;"><u>Aliases</u></p> $A = BCE = DEF = CDG = BFG \quad AB = CE = FG \quad E = ABC = ADF = BDG = CFG \quad AF = DE = BG$ $B = ACE = CDF = DEG = AFG \quad AC = BE = DG \quad F = BCD = ADE = ABG = CEG \quad AG = CD = BF$ $C = ABE = BDF = ADG = EFG \quad AD = EF = CG \quad G = ACD = BDE = ABF = CEF \quad BD = CF = EG$ $D = BCF = AEF = ACG = BEG \quad AE = BC = DF$ $ABD = CDE = ACF = BEF = BCG = AEG = DFG$ 2 blocks of 8: $ABD = CDE = ACF = BEF = BCG = AEG = DFG$ | |
| (j) 2^{7-2} ; 1/4 fraction of 7 factors in 32 runs | Resolution IV |
| <p style="text-align: center;"><u>Design Generators</u></p> $F = ABCD \quad G = ABDE$ | |
| <p>Defining relation: $I = ABCDF = ABDEG = CEFG$</p> | |
| <p style="text-align: center;"><u>Aliases</u></p> $A = \quad AB = CDF = DEG \quad BC = ADF \quad CE = FG \quad ACE = AFG$ $B = \quad AC = BDF \quad BD = ACF = AEG \quad CF = ABD = EG \quad ACG = AEF$ $C = EFG \quad AD = BCF = BEG \quad BE = ADG \quad CG = EF \quad BCE = BFG$ $D = \quad AE = BDG \quad BF = ACD \quad DE = ABG \quad BCG = BEF$ $E = CFG \quad AF = BCD \quad BG = ADE \quad DF = ABC \quad CDE = DFG$ $F = CEG \quad AG = BDE \quad CD = ABF \quad DG = ABE \quad CDG = DEF$ $G = CEF$ 2 blocks of 16: $ACE = AFG$ 4 blocks of 8: $ACE = AFG$ $BCE = BFG$ $AB = CDF = DEG$ | |
| (k) 2^{7-1} ; 1/2 fraction of 7 factors in 64 runs | Resolution VII |
| <p style="text-align: center;"><u>Design Generators</u></p> $G = ABCDEF$ | |
| <p>Defining relation: $I = ABCDEFG$</p> | |
| <p style="text-align: center;"><u>Aliases</u></p> <p>Each main effect is aliased with a single 6-factor interaction. Each 2-factor interaction is aliased with a single 5-factor interaction. Each 3-factor interaction is aliased with a single 4-factor interaction.</p> | |
| <p>2 blocks of 32: ABC 4 blocks of 16: ABC CEF CDG</p> | |

TABLE VIII Alias Relationships for 2^{k-p} Fractional Factorial Designs with $k \leq 15$ and $n \leq 64$ (Continued)

| Designs with 8 Factors | |
|---|---------------|
| (l) 2^{8-4} ; 1/16 fraction of 8 factors in 16 runs | Resolution IV |
| <p style="text-align: center;"><u>Design Generators</u></p> $E = BCD \quad F = ACD \quad G = ABC \quad H = ABD$ <p>Defining relation: $I = BCDE = ACDF = ABEF = ABCG = ADEG = BDFG = CEFG = ABDH$ $= ACEH = BCFH = DEFH = CDGH = BEGH = AFGH = ABCDEFGH$</p> <p style="text-align: center;"><u>Aliases</u></p> $\begin{aligned} A &= CDF = BEF = BCG = DEG = BDH = CEH = FGH & AB &= EF = CG = DH \\ B &= CDE = AEF = ACG = DFG = ADH = CFH = EGH & AC &= DF = BG = EH \\ C &= BDE = ADF = ABG = EFG = AEH = BFH = DGH & AD &= CF = EG = BH \\ D &= BCE = ACF = AEG = BFG = ABH = EFH = CGH & AE &= BF = DG = CH \\ E &= BCD = ABF = ADG = CFG = ACH = DFH = BGH & AF &= CD = BE = GH \\ F &= ACD = ABE = BDG = CEG = BCH = DEH = AGH & AG &= BC = DE = FH \\ G &= ABC = ADE = BDF = CEF = CDH = BEH = AFH & AH &= BD = CE = FG \\ H &= ABD = ACE = BCF = DEF = CDG = BEG = AFG \end{aligned}$ <p style="text-align: center;">2 blocks of 8: $AB = EF = CG = DH$</p> | |
| (m) 2^{8-3} ; 1/8 fraction of 8 factors in 32 runs | Resolution IV |
| <p style="text-align: center;"><u>Design Generators</u></p> $F = ABC \quad G = ABD \quad H = BCDE$ <p>Defining relation: $I = ABCF = ABDG = CDFG = BCDEH = ADEFH = ACEGH = BEFGH$</p> <p style="text-align: center;"><u>Aliases</u></p> $\begin{aligned} A &= BCF = BDG & AE &= DFH = CGH & DE &= BCH = AFH \\ B &= ACF = ADG & AF &= BC = DEH & DH &= BCE = AEF \\ C &= ABF = DFG & AG &= BD = CEH & EF &= ADH = BGH \\ D &= ABG = CFG & AH &= DEF = CEG & EG &= ACH = BFH \\ E &= & BE &= CDH = FGH & EH &= BCD = ADF = ACG = BFG \\ F &= ABC = CDG & BH &= CDE = EFG & FH &= ADE = BEG \\ G &= ABD = CDF & CD &= FG = BEH & GH &= ACE = BEF \\ H &= & CE &= BDH = AGH & ABH &= CEF = DEG \\ AB &= CF = DG & CG &= DF = AEH & ABH &= CFH = DGH \\ AC &= BF = EGH & CH &= BDE = AEG & ACD &= BDF = BCG = AFG \\ AD &= BG = EFH & & & & \end{aligned}$ <p>2 blocks of 16: $ABE = CEF = DEG$ 4 blocks of 8: $ABE = CEF = DEG$ $ABH = CFH = DGH$ $EH = BCD = ADF = ACG = BFG$</p> | |

TABLE VIII Alias Relationships for 2^{k-p} Fractional Factorial Designs with $k \leq 15$ and $n \leq 64$ (Continued)

| Designs with 9 Factors (Continued) | | |
|---|------------------------------|------------------------------|
| (p) 2^{9-4} ; 1/16 fraction of 9 factors in 32 runs | | Resolution IV |
| <u>Design Generators</u> | | |
| $F = BCDE \quad G = ACDE \quad H = ABDE \quad J = ABCE$ | | |
| Defining relation: $I = BCDEF = ACDEG = ABFG = ABDEH = ACFH = BCGH = DEFGH = ABCEJ$ $= ADFJ = BDGJ = CEFJ = CDHJ = BEFHJ = AEGHJ = ABCDFGHJ$ | | |
| <u>Aliases</u> | | |
| $A = BFG = CFH = DFJ$ | $AD = CEG = BEH = FJ$ | $BJ = ACE = DG = EFH$ |
| $B = AFG = CGH = DGJ$ | $AE = CDG = BDH = BCJ = GHJ$ | $CD = BEF = AEG = HJ$ |
| $C = AFH = BGH = DHJ$ | $AF = BG = CH = DJ$ | $CE = BDF = ADG = ABJ = FGJ$ |
| $D = AFJ = BGJ = CHJ$ | $AG = CDE = BF = EHJ$ | $CJ = ABE = EFG = DH$ |
| $E =$ | $AH = BDE = CF = EGJ$ | $DE = BCF = ACG = ABH = FGH$ |
| $F = ABG = ACH = ADJ$ | $AJ = BCE = DF = EGH$ | $EF = BCD = DGH = CGJ = BHJ$ |
| $G = ABF = BCH = BDJ$ | $BC = DEF = GH = AEJ$ | $EG = ACD = DFH = CFJ = AHJ$ |
| $H = ACF = BCG = CDJ$ | $BD = CEF = AEH = GJ$ | $EH = ABD = DFG = BFJ = AGJ$ |
| $J = ADF = BDG = CDH$ | $BE = CDF = ADH = ACJ = FHJ$ | $EJ = ABC = CFG = BFH = AGH$ |
| $AB = FG = DEH = CEJ$ | $BH = ADE = CG = EFJ$ | $AEF = BEG = CEH = DEJ$ |
| $AC = DEG = FH = BEJ$ | | |
| 2 blocks of 16: $AEF = BEG = CEH = DEJ$ 4 blocks of 8: $AEF = BEG = CEH = DEJ$ | | |
| $AB = FG = DEH = CEJ$ | | |
| $CD = BEF = AEG = HJ$ | | |
| (q) 2^{9-3} ; 1/8 fraction of 9 factors in 64 runs | | Resolution IV |
| <u>Design Generators</u> | | |
| $G = ABCD \quad H = ACEF \quad J = CDEF$ | | |
| Defining relation: $I = ABCDG = ACEFH = BDEFGH = CDEFJ = ABEFGJ = ADHJ = BCGHJ$ | | |
| <u>Aliases</u> | | |
| $A = DHJ$ | $AC = BDG = EFH$ | $BF =$ |
| $B =$ | $AD = BCG = HJ$ | $BG = ACD = CHJ$ |
| $C =$ | $AE = CFH$ | $BH = CGJ$ |
| $D = AHJ$ | $AF = CEH$ | $BJ = CGH$ |
| $E =$ | $AG = BCD$ | $CD = ABG = EFJ$ |
| $F =$ | $AH = CEF = DJ$ | $CE = AFH = DFJ$ |
| $G =$ | $AJ = DH$ | $CF = AEH = DEJ$ |
| $H = ADJ$ | $BC = ADG = GHJ$ | $CG = ABD = BHJ$ |
| $J = ADH$ | $BD = ACG$ | $CH = AEF = BGJ$ |
| $AB = CDG$ | $BE =$ | $CJ = DEF = BGH$ |
| $DE = CFJ$ | $GJ = BCH$ | $AFJ = BEG = DFH$ |
| $DF = CEJ$ | $ABE = FGJ$ | $AGH = DGJ$ |
| $DG = ABC$ | $ABF = EGJ$ | $AGJ = BEF = DGH$ |
| $EF = ACH = CDJ$ | $ABH = BDJ$ | $BCE =$ |
| $EG =$ | $ABJ = EFG = BDH$ | $BCF =$ |
| $EH = ACF$ | $ACJ = CDH$ | $BDE = FGH$ |
| $EJ = CDF$ | $ADE = EHJ$ | $BDF = EGH$ |
| $FG =$ | $ADF = FHJ$ | $BEH = DFG$ |
| $FH = ACE$ | $AEG = BFJ$ | $BFH = DEG$ |
| $FJ = CDE$ | $AEJ = BFG = DEH$ | $CEG =$ |
| $GH = BCJ$ | $AFG = BEJ$ | $CFG =$ |
| 2 blocks of 32: CFG 4 blocks of 16: $CFG =$ | | |
| $AGJ = BEF = DGH$ | | |
| $ADE = EHJ$ | | |

TABLE VIII Alias Relationships for 2^{k-p} Fractional Factorial Designs with $k \leq 15$ and $n \leq 64$ (Continued)

| Designs with 10 Factors | |
|--|-----------------------|
| (r) 2^{10-6} ; 1/64 fraction of 10 factors in 16 runs | Resolution III |
| <u>Design Generators</u> | |
| $E = ABC \quad F = BCD \quad G = ACD \quad H = ABD \quad J = ABCD \quad K = AB$ Defining relation: $I = ABCE = BCDF = ADEF = ACDG = BDEG = ABFG = CEFH = ABDH$ $= CDEH = ACFH = BEFH = BCGH = AEGH = DFGH = ABCDEFGH = ABCDJ$ $= DEJ = AFJ = BCEFJ = BGJ = ACEGJ = CDFGI = ABDEFGJ = CHJ$ $= ABEHJ = BDFHJ = ACDEFHJ = ADGHJ = BCDEGHJ = ABCFGHJ = EFGHJ = ABK$ $= CEK = ACDFK = BDEFK = BCDGK = ADEGK = FGK = ABCEFGK = DHK$ $= ABCDEHK = BCFHK = AEFHK = ACGHK = BEGHK = ABDFGHK = CDEFGHK = CDJK$ $= ABDEJK = BFJK = ACEFJK = AGJK = BCEGJK = ABCDFGJK = DEFGJK = ABCHJK$ $= EHJK = ADFHJK = BCDEFHJK = BDGHJK = ACDEGHJK = CFGHJK = ABEFGHJK$ | |
| <u>Aliases</u> | |
| $A = FJ = BK \quad J = DE = AF = BG = CH$ $B = GJ = AK \quad K = AB = CE = FG = DH$ $C = HJ = EK \quad AC = BE = DG = FH$ $D = EJ = HK \quad AD = EF = CG = BH$ $E = DJ = CK \quad AE = BC = DF = GH$ $F = AJ = GK \quad AG = CD = BF = EH = JK$ $G = BJ = FK \quad AH = BD = CF = EG$ $H = CJ = DK$ 2 blocks of 8: $AG = CD = BF = EH = JK$ | |
| (s) 2^{10-5} ; 1/32 fraction of 10 factors in 32 runs | Resolution IV |
| <u>Design Generators</u> | |
| $F = ABCD \quad G = ABCE \quad H = ABDE \quad J = ACDE \quad K = BCDE$ Defining relation: $I = ABCDF = ABCEG = DEFG = ABDEH = CEFH = CDGH = ABFGH = ACDEJ$ $= BEFJ = BDGJ = ACFGJ = BCHJ = ADFHJ = AEGHJ = BCDEFGHJ = BCDEK$ $= AEFK = ADGK = BCFGK = ACHK = BDFHK = BEGHK = ACDEFGHK = ABJK$ $= CDFJK = CEGJK = ABDEFGJK = DEHJK = ABCEFHJK = ABCDGHJK = FGHJK$ | |
| <u>Aliases</u> | |
| $A = EFK = DGK = CHK = BJK \quad AH = BDE = BFG = DFJ = EGJ = CK$ $B = EFJ = DGJ = CHJ = AJK \quad AJ = CDE = CFG = DFH = EGH = BK$ $C = EFH = DGH = BHJ = AHK \quad AK = EF = DG = CH = BJ$ $D = EFG = CGH = BGJ = AGK \quad BC = ADF = AEG = HJ = DEK = FGK$ $E = DFG = CFH = BFJ = AFK \quad BD = ACF = AEH = GJ = CEK = FHK$ $F = DEG = CEH = BEJ = AEK \quad BE = ACG = ADH = FJ = CDK = GHK$ $G = DEF = CDH = BDJ = ADK \quad BF = ACD = AGH = EJ = CGK = DHK$ $H = CEF = CDG = BCJ = ACK \quad BG = ACE = AFH = DJ = CFK = EHK$ $J = BEF = BDG = BCH = ABK \quad BH = ADE = AFG = CJ = DFK = EGK$ $K = AEF = ADG = ACH = ABJ \quad CD = ABF = GH = AEJ = BEK = FJK$ $AB = CDF = CEG = DEH = FGH = JK \quad CE = ABG = FH = ADJ = BDK = GJK$ $AC = BDF = BEG = DEJ = FGJ = HK \quad CF = ABD = EH = AGJ = BGK = DJK$ $AD = BCF = BEH = CEJ = FHJ = GK \quad CG = ABE = DH = AFJ = BFK = EJK$ $AE = BCG = BDH = CDJ = GHJ = FK \quad DE = FG = ABH = ACJ = BCK = HJK$ $AF = BCD = BGH = CGJ = DHJ = EK \quad DF = ABC = EG = AHJ = BHK = CJK$ $AG = BCE = BFH = CFJ = EHJ = DK$ 2 blocks of 16: $AK = EF = DG = CH = BJ$ 4 blocks of 8: $AK = EF = DG = CH = BJ$ $AJ = CDE = CFG = DFH = EGH = BK$ $AB = CDF = CEG = DEH = FGH = JK$ | |

TABLE VIII Alias Relationships for 2^{k-p} Fractional Factorial Designs with $k \leq 15$ and $n \leq 64$ (Continued)

| Designs with 10 Factors (<i>Continued</i>) | | | Resolution IV |
|--|-----------------------|-------------------------|---------------|
| (t) 2 ¹⁰⁻⁴ ; 1/16 fraction of 10 factors in 64 runs | | | |
| <u>Design Generators</u> | | | |
| $G = BCDF \quad H = ACDF \quad J = ABDE \quad K = ABCE$ | | | |
| Defining relation: $I = BCDFG = ACDFH = ABGH = ABDEJ = ACEFGJ = BCEFHJ = DEGHJ = ABCEK$ $= ADEFGK = BDEFHK = CEGHK = CDJK = BFGJK = AFHJK = ABCDGHJK$ | | | |
| <u>Aliases</u> | | | |
| $A = BGH$ | $AD = CFH = BEJ$ | $BK = ACE = FGJ$ | |
| $B = AGH$ | $AE = BDJ = BCK$ | $CD = BFG = AFH = JK$ | |
| $C = DJK$ | $AF = CDH = HJK$ | $CE = ABK = GHK$ | |
| $D = CJK$ | $AG = BH$ | $CF = BDG = ADH$ | |
| $E =$ | $AH = CDF = BG = FJK$ | $CG = BDF = EHK$ | |
| $F =$ | $AJ = BDE = FHK$ | $CH = ADF = EGK$ | |
| $G = ABH$ | $AK = BCE = FHJ$ | $CJ = DK$ | |
| $H = ABG$ | $BC = DFG = AEK$ | $CK = ABE = EGH = DJ$ | |
| $J = CDK$ | $BD = CFG = AEJ$ | $DE = ABJ = GHJ$ | |
| $K = CDJ$ | $BE = ADJ = ACK$ | $DF = BCG = ACH$ | |
| $AB = GH = DEJ = CEK$ | $BF = CDG = GJK$ | $DG = BCF = EHJ$ | |
| $AC = DFH = BEK$ | $BJ = ADE = FGK$ | $DH = ACF = EGJ$ | |
| $EF =$ | $GJ = DEH = BFK$ | $AEG = BEH = CFJ = DFK$ | |
| $EG = DHJ = CHK$ | $GK = CEH = BFJ$ | $AEH = BEG$ | |
| $EH = DGJ = CGK$ | $HJ = DEG = AFK$ | $AFG = BFH = CEJ = DEK$ | |
| $EJ = ABD = DGH$ | $HK = CEG = AFJ$ | $AGJ = CEF = BHJ$ | |
| $EK = ABC = CGH$ | $ABF = FGH$ | $AGK = DEF = BHK$ | |
| $FG = BCD = BJK$ | $ACG = BCH = EFJ$ | $BCJ = EFH = BDK$ | |
| $FH = ACD = AJK$ | $ACJ = EFG = ADK$ | $BEF = CHJ = DHK$ | |
| $FJ = BGK = AHK$ | $ADG = BDH = EFK$ | $CDE = EJK$ | |
| $FK = BGJ = AHJ$ | $AEF = CGJ = DGK$ | $CFK = DFJ$ | |
| 2 blocks of 32: $AGJ = CEF = BHJ$ 4 blocks of 16: $AGJ = CEF = BHJ$ | | | |
| $AGK = DEF = BHK$ | | | |
| $CD = BFG = AFH = JK$ | | | |

TABLE VIII Alias Relationships for 2^{k-p} Fractional Factorial Designs with $k \leq 15$ and $n \leq 64$ (Continued)

| Designs with 11 Factors | |
|--|-----------------------|
| (u) 2^{11-7} ; 1/128 fraction of 11 factors in 16 runs | Resolution III |
| <u>Design Generators</u> | |
| $E = ABC \quad F = BCD \quad G = ACD \quad H = ABD \quad J = ABCD \quad K = AB \quad L = AC$ Defining relation: $I = ABCE = BCDF = ADEF = ACDG = BDEG = ABFG = CEFG = ABDH$ $= CDEH = ACFH = BEFH = BCGH = AEGH = DFGH = ABCDEFGH = ABCDJ$ $= DEJ = AFJ = BCEFJ = BGJ = ACEGJ = CDFGJ = ABDEFGJ = CHJ$ $= ABEHJ = BDFHJ = ACDEFHJ = ADGHJ = BCDEGHJ = ABCFGHJ = EFGHJ = ABK$ $= CEK = ACDFK = BDEFK = BCDGK = ADEGK = FGK = ABCEFGK = DHK$ $= ABCDEHK = BCFHK = AEFHK = ACGHK = BEGHK = ABDFGHK = CDEFGHK = CDJK$ $= ABDEJK = BFJK = ACEFJK = AGJK = BCEGJK = ABCDFGJK = DEFGJK = ABCHJK$ $= EHJK = ADFHJK = BCDEFHJK = BDGHJK = ACDEGHJK = CFGHJK = ABEFGHJK = ACL$ $= BEL = ABDFL = CDEFL = DGL = ABCDEGL = BCFGL = AEFGL = BCDHL$ $= ADEHL = FHL = ABCEFHL = ABGHL = CEGHL = ACDFGHL = BDEFGHL = BDJL$ $= ACDEJL = CFJL = ABEFJL = ABCGJL = EGJL = ADFGJL = BCDEFGJL = AHJL$ $= BCEHJL = ABCDFHJL = DEFHJL = CDGHJL = ABDEGHJL = BFGHJL = ACEFGHJL = BCKL$ $= AEKL = DFKL = ABCDEFKL = ABDGKL = CDEGKL = ACFGKL = BEFGKL = ACDHKL$ $= BDEHKL = ABFHKL = CEFHKL = GHKL = ABCEGHKL = BCDFGHKL = ADEFGHKL = ADJKL$ $= BCDEJKL = ABCFJKL = EFJKL = CGJKL = ABEGJKL = BDFGJKL = ACDEFGJKL = BHJKL$ $= ACEHJKL = CDFHJKL = ABDEFHJKL = ABCDGHJKL = DEGHJKL = AFGHJKL = BCEFGHJKL$ | |
| <u>Aliases</u> | |
| $A = FJ = BK = CL \quad J = DE = AF = BG = CH$ $B = GJ = AK = EL \quad K = AB = CE = FG = DH$ $C = HJ = EK = AL \quad L = AC = BE = DG = FH$ $D = EJ = HK = GL \quad AD = EF = CG = BH$ $E = DJ = CK = BL \quad AE = BC = DF = GH = KL$ $F = AJ = GK = HL \quad AG = CD = BF = EH = JK$ $G = BJ = FK = DL \quad AH = BD = CF = EG = JL$ $H = CJ = DK = FL$ 2 blocks of 8: $AE = BC = DF = GH = KL$ | |

TABLE VIII Alias Relationships for 2^{k-p} Fractional Factorial Designs with $k \leq 15$ and $n \leq 64$ (Continued)

| Designs with 11 Factors (Continued) | |
|---|---------------|
| (v) 2^{11-6} ; 1/64 fraction of 11 factors in 32 runs | Resolution IV |
| Design Generators | |
| $F = ABC \quad G = BCD \quad H = CDE \quad J = ACD \quad K = ADE \quad L = BDE$ | |
| Defining relation: | |
| $I = ABCF = BCDG = ADFG = CDEH = ABDEFH = BEGH = ACEFGH = ACDJ = BDFJ = ABGJ = CFGJ$ $= AEHJ = BCEFHJ = ABCDEGHJ = DEFGHJ = ADEK = BCDEFK = ABCEGK = EFGK = ACHK = BFHK$ $= ABDGHK = CDFGHK = CEJK = ABEFJK = BDEGJK = ACDEFGJK = DHJK = ABCDFHJK = BCGHJK$ $= AFGHJK = BDEL = ACDEFL = CEGJ = ABEFGL = BCHL = AFHL = DGHL = ABCDFGHL$ $= ABCEJL = EFJL = ADEGJL = BCDEFGJL = ABDHJL = CDFHJL = ACGHJL = BFGHJL = ABKL$ $= CFKL = ACDGKL = BDFGKL = ABCDEHKL = DEFHKL = AEGHKL = BCEFGHKL = BCDJKL$ $= ADFJKL = GJKL = ABCFGJKL = BEHJKL = ACEFHJKL = CDEGHJKL = ABDEFHJKL$ | |
| Aliases | |
| $A = BCF = DFG = CDJ = BGJ = EHJ = DEK = CHK = FHL = BKL$ $B = ACF = CDG = EGH = DFJ = AGJ = FHK = DEL = CHL = AKL$ $C = ABF = BDG = DEH = ADJ = FGJ = AHK = EJK = EGL = BHL = FKL$ $D = BCG = AFG = CEH = ACJ = BFJ = AEK = HJK = BEL = GHL$ $E = CDH = BGH = AHJ = ADK = FGK = CJK = BDL = CGL = FJL$ $F = ABC = ADG = BDJ = CGJ = EGK = BHK = AHL = EKL = CKL$ $G = BCD = ADF = BEH = ABJ = CFJ = EFK = CEL = DHL = JKL$ $H = CDE = BEG = AEJ = ACK = BFK = DJK = BCL = AFL = DGL$ $J = ACD = BDF = ABG = CFG = AEH = CEK = DHK = EFL = GKL$ $K = ADE = EFG = ACH = BFH = CEJ = DHJ = ABL = CFL = GJL$ $L = BDE = CEG = BCH = AFH = DGH = EFJ = ABK = CFK = GJK$ | |
| $AB = CF = GJ = KL \quad AE = HJ = DK \quad AH = EJ = CK = FL \quad AL = FH = BK \quad BH = EG = CL = FK$ $AC = BF = DJ = HK \quad AF = BC = DG = HL \quad AJ = CD = BG = EH \quad BD = CG = FJ = EL \quad CE = DH = JK = GL$ $AD = FG = CJ = EK \quad AG = DF = BJ \quad AK = DE = CH = BL \quad BE = GH = DL \quad EF = GK = JL$ | |
| $ABD = CDF = ACG = BFG = EFH = BCJ = AFJ = DGJ = BEK = GHK = AEL = HJL = DKL$ $ABE = CEF = DFH = AGH = EGJ = BHJ = BDK = CGK = FJK = ADL = FGL = CJL = EKL$ $ABH = DEF = AEG = CFH = BEJ = GHJ = BCK = AFK = DGK = ACL = BFL = DJL = HKL$ $ACE = BEF = ADH = FGH = DEJ = CHJ = CDK = BGK = EHK = AJK = DFL = AGL = BJL$ $AEF = BCE = DEG = BDH = CGH = FHJ = DFK = AGK = BJK = CDL = BGL = EHL = AJL$ | |
| 2 blocks of 16: $AB = CF = GJ = KL$ 4 blocks of 8: $AB = CF = GJ = KL$ | |
| $AD = FG = CJ = EK$ | |
| $BD = CG = FJ = EL$ | |

TABLE VIII Alias Relationships for 2^{k-p} Fractional Factorial Designs with $k \leq 15$ and $n \leq 64$ (*Continued*)

| Designs with 12 Factors | |
|---|-----------------------|
| (w) 2^{12-8} ; 1/256 fraction of 12 factors in 16 runs | Resolution III |
| <p style="text-align: center;"><u>Design Generators</u></p> $E = ABC \quad F = ABD \quad G = ACD \quad H = BCD$ $J = ABCD \quad K = AB \quad L = AC \quad M = AD$ <p style="text-align: center;"><u>Aliases</u></p> $A = HJ = BK = CL = DM$ $B = GJ = AK = EL = FM$ $C = FJ = EK = AL = GM$ $D = EJ = FK = GL = AM$ $E = DJ = CK = BL = HM$ $F = CJ = DK = HL = BM$ $G = BJ = HK = DL = CM$ $H = AJ = GK = FL = EM$ $J = DE = CF = BG = AH$ $K = AB = CE = DF = GH$ $L = AC = BE = DG = FH$ $M = AD = BF = CG = EH$ $AE = BC = FG = DH = KL = JM$ $AF = BD = EG = CH = JL = KM$ $AG = EF = CD = BH = JK = LM$ <p>2 blocks of 8: $AE = BC = FG = DH = KL = JM$</p> | |
| Designs with 13 Factors | |
| (x) 2^{13-9} ; 1/512 fraction of 13 factors in 16 runs | Resolution III |
| <p style="text-align: center;"><u>Design Generators</u></p> $E = ABC \quad F = ABD \quad G = ACD \quad H = BCD$ $J = ABCD \quad K = AB \quad L = AC \quad M = AD \quad N = BC$ <p style="text-align: center;"><u>Aliases</u></p> $A = HJ = BK = CL = DM = EN$ $B = GJ = AK = EL = FM = CN$ $C = FJ = EK = AL = GM = BN$ $D = EJ = FK = GL = AM = HN$ $E = DJ = CK = BL = HM = AN$ $F = CJ = DK = HL = BM = GN$ $G = BJ = HK = DL = CM = FN$ $H = AJ = GK = FL = EM = DN$ $J = DE = CF = BG = AH = MN$ $K = AB = CE = DF = GH = LN$ $L = AC = BE = DG = FH = KN$ $M = AD = BF = CG = EH = JN$ $N = BC = AE = FG = DH = KL = JM$ $AF = BD = EG = CH = JL = KM$ $AG = EF = CD = BH = JK = LM$ <p>2 blocks of 8: $AF = BD = EG = CH = JL = KM$</p> | |

TABLE VIII Alias Relationships for 2^{k-p} Fractional Factorial Designs with $k \leq 15$ and $n \leq 64$ (*Continued*)

| Designs with 14 Factors | |
|--|-----------------------|
| (y) 2^{14-10} ; 1/1024 fraction of 14 factors in 16 runs | Resolution III |
| <u>Design Generators</u> | |
| $E = ABC \quad F = ABD \quad G = ACD \quad H = BCD \quad J = ABCD$ $K = AB \quad L = AC \quad M = AD \quad N = BC \quad O = BD$ | |
| <u>Aliases</u> | |
| $A = HJ = BK = CL = DM = EN = FO$ $B = GJ = AK = EL = FM = CN = DO$ $C = FJ = EK = AL = GM = BN = HO$ $D = EJ = FK = GL = AM = HN = BO$ $E = DJ = CK = BL = HM = AN = GO$ $F = CJ = DK = HL = BM = GN = AO$ $G = BJ = HK = DL = CM = FN = EO$ $H = AJ = GK = FL = EM = DN = CO$ $J = DE = CF = BG = AH = MN = LO$ $K = AB = CE = DF = GH = LN = MO$ $L = AC = BE = DG = FH = KN = JO$ $M = AD = BF = CG = EH = JN = KO$ $N = BC = AE = FG = DH = KL = JM$ $O = BD = AF = EG = CH = JL = KM$ $AG = EF = CD = BH = JK = LM = NO$ 2 blocks of 8: $AG = EF = CD = BH = JK = LM = NO$ | |
| Designs with 15 Factors | |
| (z) 2^{15-11} ; 1/2048 fraction of 5 factors in 16 runs | Resolution III |
| <u>Design Generators</u> | |
| $E = ABC \quad F = ABD \quad G = ACD \quad H = BCD \quad J = ABCD$ $K = AB \quad L = AC \quad M = AD \quad N = BC \quad O = BD \quad P = CD$ | |
| <u>Aliases</u> | |
| $A = HJ = BK = CL = DM = EN = FO = GP$ $B = GJ = AK = EL = FM = CN = DO = HP$ $C = FJ = EK = AL = GM = BN = HO = DP$ $D = EJ = FK = GL = AM = HN = BO = CP$ $E = DJ = CK = BL = HM = AN = GO = FP$ $F = CJ = DK = HL = BM = GN = AO = EP$ $G = BJ = HK = DL = CM = FN = EO = AP$ $H = AJ = GK = FL = EM = DN = CO = BP$ $J = DE = CF = BG = AH = MN = LO = KP$ $K = AB = CE = DF = GH = LN = MO = JP$ $L = AC = BE = DG = FH = KN = JO = MP$ $M = AD = BF = CG = EH = JN = KO = LP$ $N = BC = AE = FG = DH = KL = JM = OP$ $O = BD = AF = EG = CH = JL = KM = NP$ $P = CD = EF = AG = BH = JK = LM = NO$ | |