

**NORTH CAROLINA STATE UNIVERSITY  
DEPARTMENT OF STATISTICS**

**Process Optimization for Manufacturing System**

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**Executive Summary:**

A precision plastic parts manufacturer requires our team's assistance to optimize a new design and eliminate variabilities in the product shapes. Our primary objective was to first understand the interactions and effects which each factor had on the geometrical integrity of the product. It is then that we built a model to forecast the variability in design and generate a list of optimal characteristics based on our model. Three different data requests were sent totaling 43 runs and thereby incurring \$43,000 out of a given budget of \$100,000. The first request was an initial screening design which helped our team discover the significant factors impacting the geometrical shape of the product. The factors discovered are: Cure Temp, Cure Time, Material Percent Resin. The second run analyzed the center, high and low points of the significant factors in order to try and find the minimum attenuation possible. The third run was a central composite design including axial points for Cure Time, Cure Temp and Material Percent Resin. In our final run the list of optimal settings to minimize attenuation are 28.84% for Material Percent Resin, 14 minutes for Cure Time and 40°C for Cure Temp. At the end we were able to execute a model with 27 data points to confirm the results and ensure that the lack of fit was insignificant.

**1. Introduction:**

Our team has been entrusted with the task of working for a manufacturer producing precision plastic parts and reducing the variability in their geometrical features. The manufacturing process starts with the heating of the plastic material. This plastic material was a combination of resin and glass filler. Once it has been heated, the plastic material is injected into a mold cavity and held for a specified dwell time under a given pressure and at a specific temperature. The plastic parts are then removed from the mold and put into a low temperature oven for several hours to stabilize their shape and geometry. These parts are then measured for geometrical integrity. In order to reduce the variation in geometrical features our team ran test experiments to determine the best combination of settings for the molding process that would produce the best quality products.

Some of the key features which we analyzed in our test experiments include: Material flow rate (ranges from 3-10 cc/sec), Mold temperature (ranges from 200-300 °C), Mold pressure (ranges from 50 to 150 Mpa), Dwell time (ranges from 5 to 20 sec), Cure temperature (ranges from 30 to 60 °C), Cure time (ranges from 5 to 15 hours), Material percent resin (ranges from 10% to 50%), Raw material supplier (Dexter or Polymax).

We approached this task by first identifying any significant effects and interactions which the above factors imposed on the product. Once this analysis was completed, we built a model to predict variability from the significant factors and generated a list of optimal characteristics based on the model. Our team has been constrained with a budget of \$100,00 for the experimental runs with each run costing \$1,000.

**2. Experimental Design**

**2.1. Data Request 1: Screening Design**

This initial screening design is a  $2^{8-4}$  factorial fractional design. In this case all the involved factors were varied for all the 16 runs using both their high and low values. This design included all the significant factors and some two factor interactions as well.

	Pattern	Flow Rate	Mold temp	Mold Press	Dwell Time	Cure Temp	Cure Time	Mat Resin %	Raw Mat Supplier	Y
1	-----	3	200	50	5	30	5	10	-1	502.5837528
2	-----+	3	200	50	20	60	15	50	-1	573.0142835
3	---+---+	3	200	150	5	60	15	10	1	520.1379054
4	---+---+	3	200	150	20	30	5	50	1	540.1103079
5	-+++--++	3	300	50	5	60	5	50	1	543.9133193
6	-+++--++	3	300	50	20	30	15	10	1	411.6581361
7	-+++--++	3	300	150	5	30	15	50	-1	481.4679112
8	-+++--++	3	300	150	20	60	5	10	-1	518.8075393
9	+-----+	10	200	50	5	30	15	50	1	459.3605413
10	+-----+	10	200	50	20	60	5	10	1	480.4903237
11	+-----+	10	200	150	5	60	5	50	-1	485.990983
12	+-----+	10	200	150	20	30	15	10	-1	368.0669686
13	++-----	10	300	50	5	60	15	10	-1	481.521957
14	++-----	10	300	50	20	30	5	50	-1	559.9916009
15	++++-----	10	300	150	5	30	5	10	1	474.512631
16	++++-----	10	300	150	20	60	15	50	1	541.1828653

Figure 1: First design screening all eight factors for significance using their low and high values

## 2.2. Data Request 2: Centerpoints

The second design was focused on investigating curvature. This is done by keeping all insignificant effects constant at the low level, and adding center points to screened main effects. Some replicates were used for centerpoints to increase power of the model.

	Pattern	Flow Rate	Mold temp	Mold Press	Dwell Time	Cure Temp	Cure Time	Mat Resin %	Raw Mat Supplier	Y
1	-----	3	200	50	5	30	5	10	-1	517.4201742
2	-----+	3	200	50	5	30	15	10	-1	392.5098541
3	-----+	3	200	50	5	60	5	10	-1	437.768426
4	-----+	3	200	50	5	60	15	10	-1	459.3673567
5	----00--	3	200	50	5	45	10	10	-1	414.3454422
6	----00--	3	200	50	5	45	10	10	-1	367.6383892
7	----00--	3	200	50	5	45	10	10	-1	396.5823763
8	----00--	3	200	50	5	45	10	10	-1	382.1844099
9	-----+-	3	200	50	5	30	5	50	-1	575.5397583
10	-----+-	3	200	50	5	30	15	50	-1	479.4119601
11	-----+-	3	200	50	5	60	5	50	-1	561.8724864
12	-----+-	3	200	50	5	60	15	50	-1	565.3551529
13	----00+-	3	200	50	5	45	10	50	-1	510.9778158
14	----00+-	3	200	50	5	45	10	50	-1	456.8276284
15	----00+-	3	200	50	5	45	10	50	-1	473.4834315
16	----00+-	3	200	50	5	45	10	50	-1	455.581038
17	-----0-	3	200	50	5	30	5	30	-1	147.4757005
18	-----0-	3	200	50	5	30	15	30	-1	33.74901671
19	-----0-	3	200	50	5	60	5	30	-1	99.92423385
20	-----0-	3	200	50	5	60	15	30	-1	74.81425063
21	----000-	3	200	50	5	45	10	30	-1	87.65581453

Figure 2: Second design using center points for three significant effects to measure curvature

## 2.3. Run 3: Central Composite Design

The third run was done for a central composite design with axial points taken for Cure Time, Cure Temperature, and Material Percent Resin. In this run the axial point for one factor was run with center points of

the other two factors and vice versa. Axial points for each continuous factor were calculated using the range of the factor and  $\pm\sqrt{3}$  [2]. For investigating the CCD, these 6 runs were combined with data from request 2.

	Flow Rate	Mold temp	Mold Press	Dwell Time	Cure Temp	Cure Time	Mat Resin %	Raw Mat Supplier	Y
1	3	200	50	5	45	10	0	-1	875.1638326
2	3	200	50	5	45	10	63.64	-1	1251.123959
3	3	200	50	5	45	1.59	30	-1	126.8457958
4	3	200	50	5	45	18.41	30	-1	60.74468832
5	3	200	50	5	70	10	30	-1	163.4456477
6	3	200	50	5	20	10	30	-1	101.6937426

Figure 3: Third request adding axial points for three significant effects to complete Central Composite Design

### 3. Results and Analysis

#### 3.1. Run 1: Screening Design

The 2k fractional factorial design showed four potentially significant effects via diagnostics with the half normal plot, including Flow Rate, Cure Time, Cure Temp, and Material Resin %. The potential aliases for the identified interaction include Flow Rate \* Mold Temp. , Material Resin % \* Dwell Time, Mold Press \* Material Resin Supplier. We choose to investigate the assumed interaction as Cure Temp and Cure Time are also classified as main effects, and therefore we assume a strong heredity of the interaction.

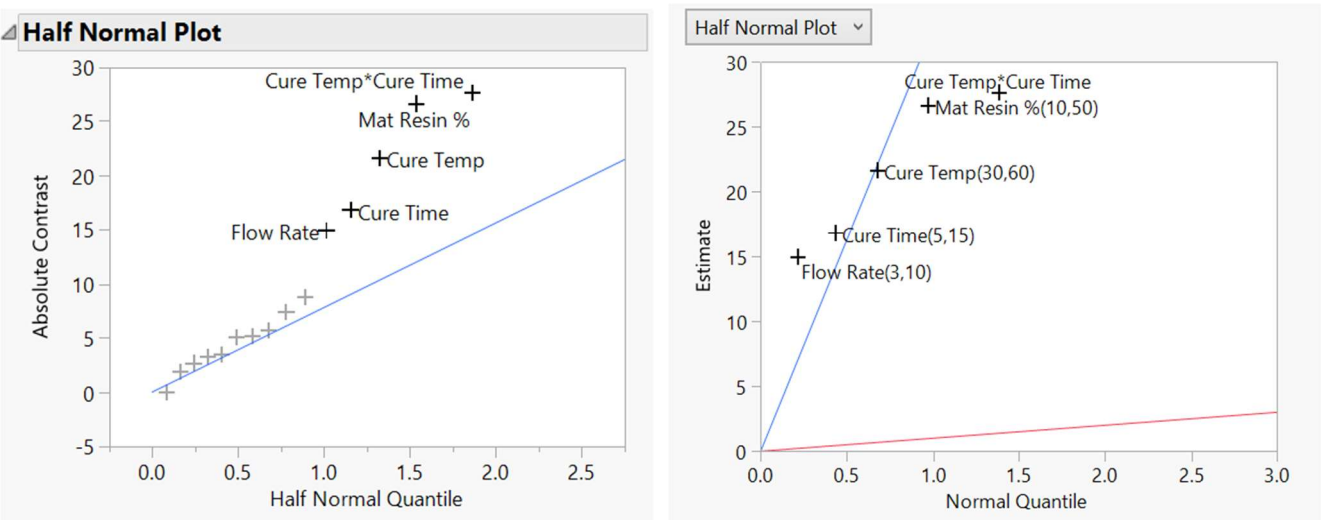


Figure 4: (a) Half normal plot of all screened factors. (b) Half normal plot of the main effects selected from a.

A model was run using the aforementioned effects. The effect summary shows the logworth significance of each of the identified effects in Figure 5. Material resin % and the interaction between Cure Temp and Cure Time are most significant. Although the flow rate is screened as significant, we choose to not include it in further data requests as we are limited in our experimentation runs. The other main effects are followed up with in Data Request 2 using center points and single replicates.

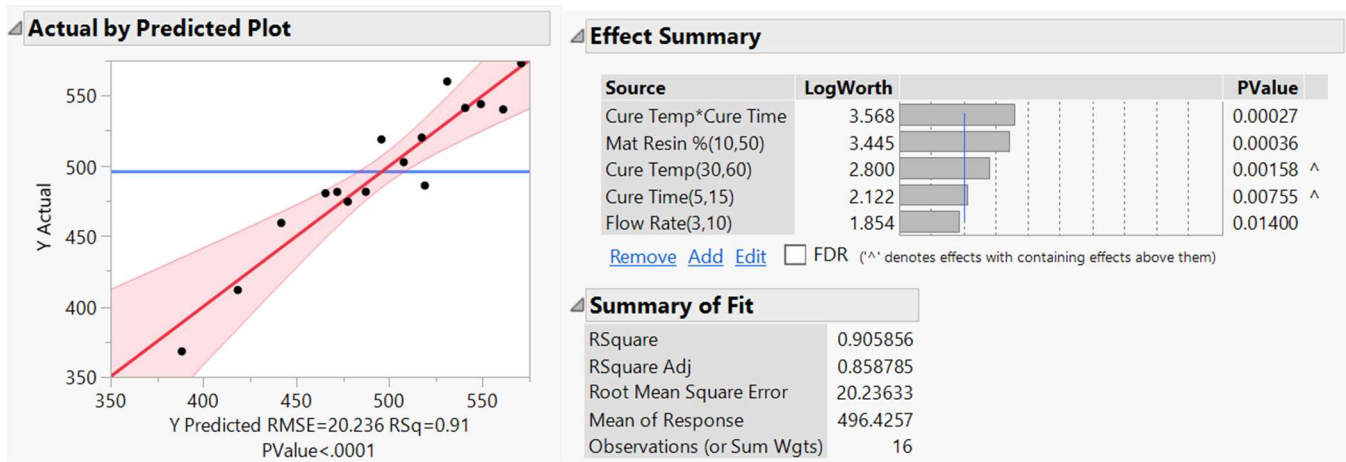


Figure 5: (a) Plot model effectiveness in predicting variation. (b) Summary of fit for first run model

### 3.2. Run 2: Centerpoints

For this design of centerpoint replicates for Cure Temp, Cure Time, and Material Resin %, we see an improved actual by predicted plot with lower variance. The significance of factors and their logworth effects are increased. We conclude that the model is accounting for most variance using the identified three main effects. Cure Time and Material Resin both show second order effects. This is a good sign as it means the model is operating in a region of curvature. Note third degree order effects were removed as assumed insignificant.

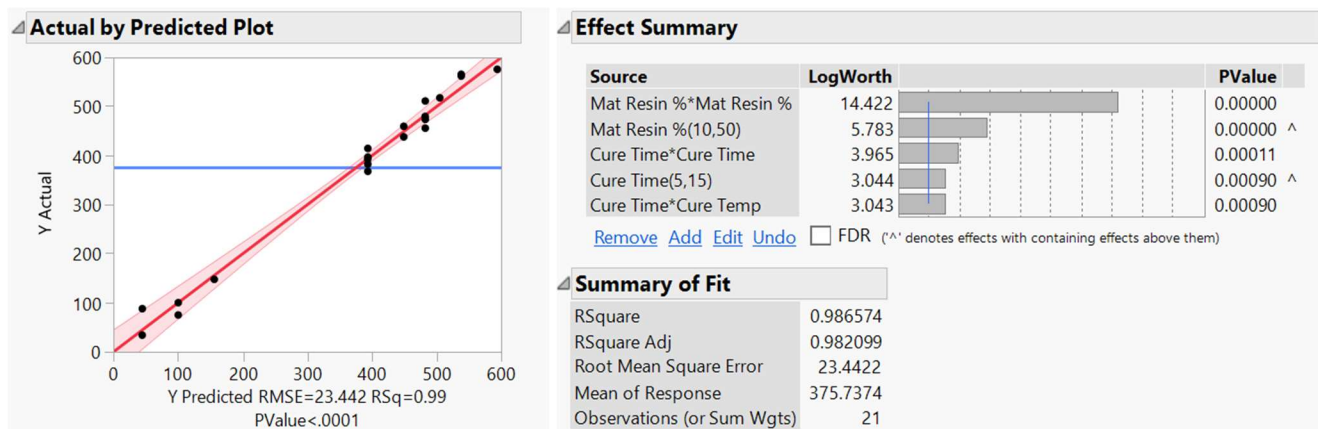


Figure 6: (a) Plot model effectiveness in predicting variation. (b) Summary of fit for second run model

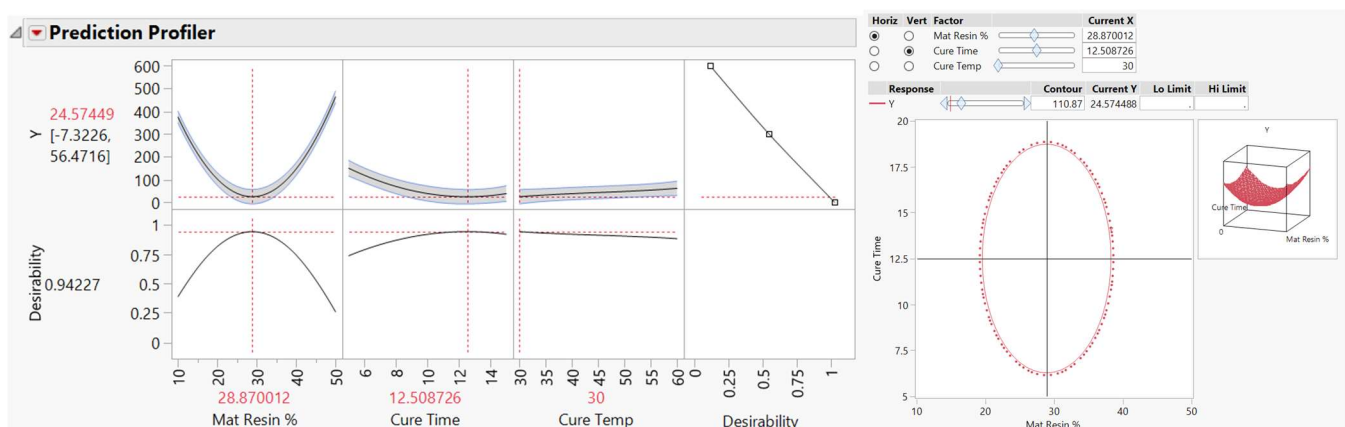


Figure 7: (a) Prediction profiler for the three main effects. (b) Contour profiler for

The purpose of the centerpoint analysis is to identify if we are operating in a region of curvature. The replicate centerpoints are used to increase power. Using the prediction profiler, we see that some curvature in the manufacturing process has been identified. We conclude that this design and its effects are sufficient for proceeding with our central composite design.

### 3.3. Run 3: Central Composite Design

The third run for the center points of the Cure Time, Cure Temperature, and Material Percent Resin, was used in combination with previous data points to complete a three factor Central Composite Design. The diagram shown in Figure 8, demonstrates our reasoning for selecting the 15 runs for this CCD. Included in the 15 runs are 6 axial points, 8 corner points, and one center point for all three factors [1].

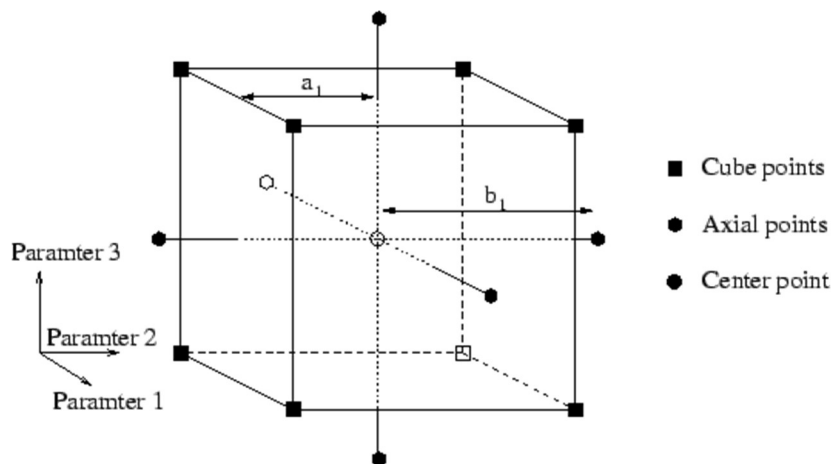


Figure 8: Visual representation of 15 points needed for three factor central composite design

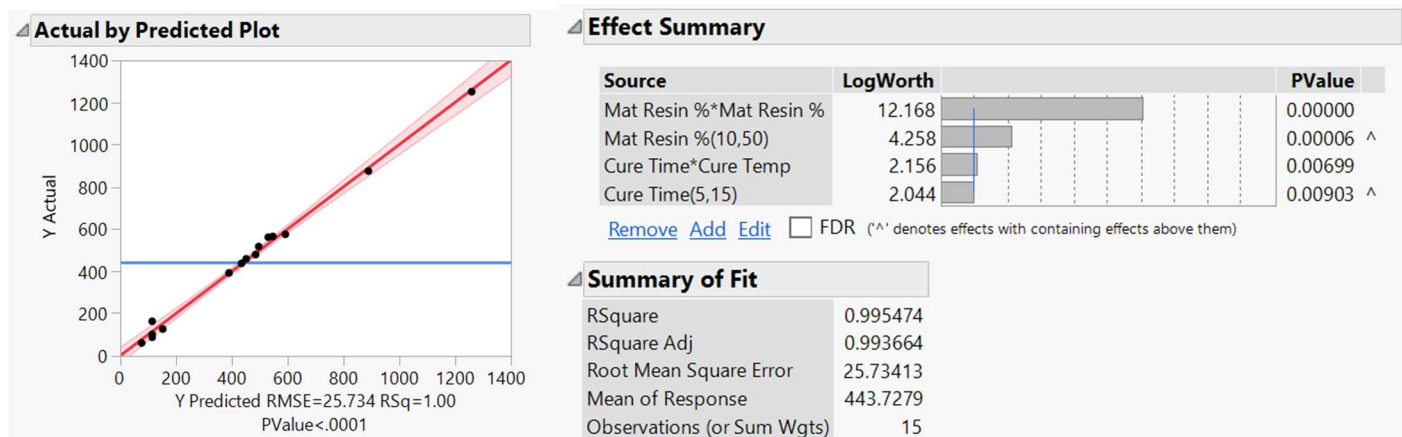


Figure 9: (a) Plot model effectiveness in predicting variation. (b) Summary of fit for third run model

Analysis of the CCD indicated that main effects, interactions, and Material Resin's quadratic term were significant in this run. With an R-Squared value of 0.9954, the model has improved in accounting for variance from the previous run. We note that Cure Time and Cure Temp no longer have significant second order effects using only the 15 runs for our CCD. This is concerning as we want to be operating in a region of curvature for the CCD. Note we address this problem of identifying curvature in the final model by adding additional data points from Request 1 and 2 to increase model power.



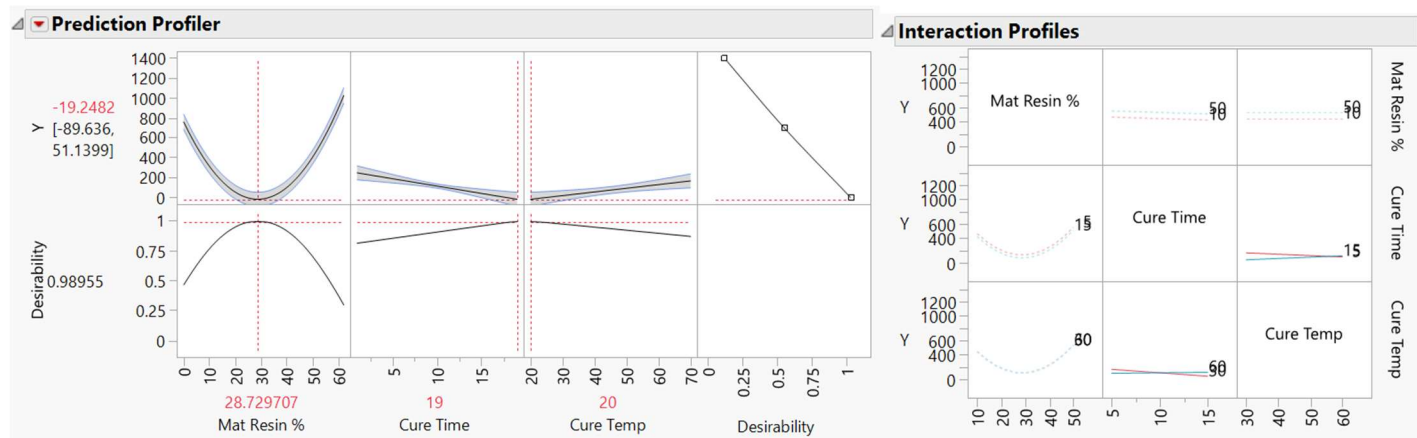


Figure 10: (a) Profiler of each main effect in model. (b) Cube plot showing response at extremes of the effects.

The prediction profiler after doing the Central Composite Design gave optimum value of 28.72% for Material Percent Resin, 19 Min for Cure Time, and 20°C for Cure Temperature. Integer values for Cure Time and Cure Temperature were obtained as a result of linearity in both of their profilers.

#### 4. Final Results

For the final design, we utilize all 27 runs from our second and third data request. The purpose here is to increase power of the model by doubling sample size from the above CCD, using only points relevant to the investigation of our three main effects. The RSquared value reduced marginally, along with error. The significance of our effects is improved, and we identify two new second order effects for cure time and temp.

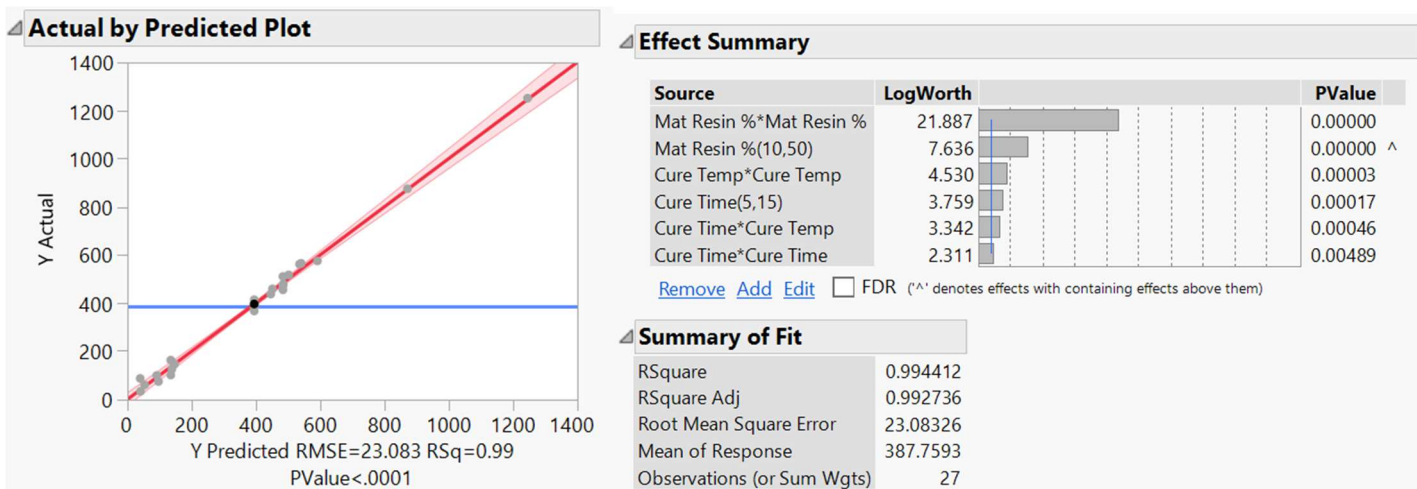


Figure 11: (a) Plot model effectiveness in predicting variation. (b) Summary of fit for final selected model

We now notice significant curvature for all three main effects, as indicated by the new Cure Time and Cure Temp second order sources. These are the results we intended to get by using all 27 data points for our model. The new model using replicate center points for all three of our factors shows we are operating in a region of curvature, which is accurately modeled in our expanded Central Composite Design. The ANOVA for our previous barebone CCD design and our final model are compared in Figure 13. The increased F ratio in our ANOVA is another diagnostic that identifies increased effectiveness of our final model. Our degrees of freedom demonstrate the increased power in our design, and the new effects serve to explain additional variance. We are satisfied with the final design based on our model summaries.

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	1456559.1	364140	549.8563
Error	10	6622.5	662	Prob > F
C. Total	14	1463181.6		<.0001*

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	115.04206	9.836438	11.70	<.0001*
Mat Resin %(10,50)	47.800307	7.158973	6.68	<.0001*
Cure Time(5,15)	-22.48741	6.963249	-3.23	0.0090*
Mat Resin %*Mat Resin %	376.2934	8.328469	45.18	<.0001*
Cure Time*Cure Temp	30.764964	9.098388	3.38	0.0070*

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	6	1896477.5	316080	593.2012
Error	20	10656.7	533	Prob > F
C. Total	26	1907134.3		<.0001*

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	40.328133	11.83303	3.41	0.0028*
Mat Resin %(10,50)	44.730052	5.046686	8.86	<.0001*
Cure Time(5,15)	-25.25592	5.493175	-4.60	0.0002*
Mat Resin %*Mat Resin %	399.44559	7.868899	50.76	<.0001*
Cure Time*Cure Time	19.898352	6.291008	3.16	0.0049*
Cure Time*Cure Temp	27.894701	6.663564	4.19	0.0005*
Cure Temp*Cure Temp	34.243786	6.376752	5.37	<.0001*

Figure 12: (a) ANOVA of previous model using barebone central composite design with 15 runs. (b) Final model ANOVA using all data points from request 1 and 2 to increase power of the central composite design

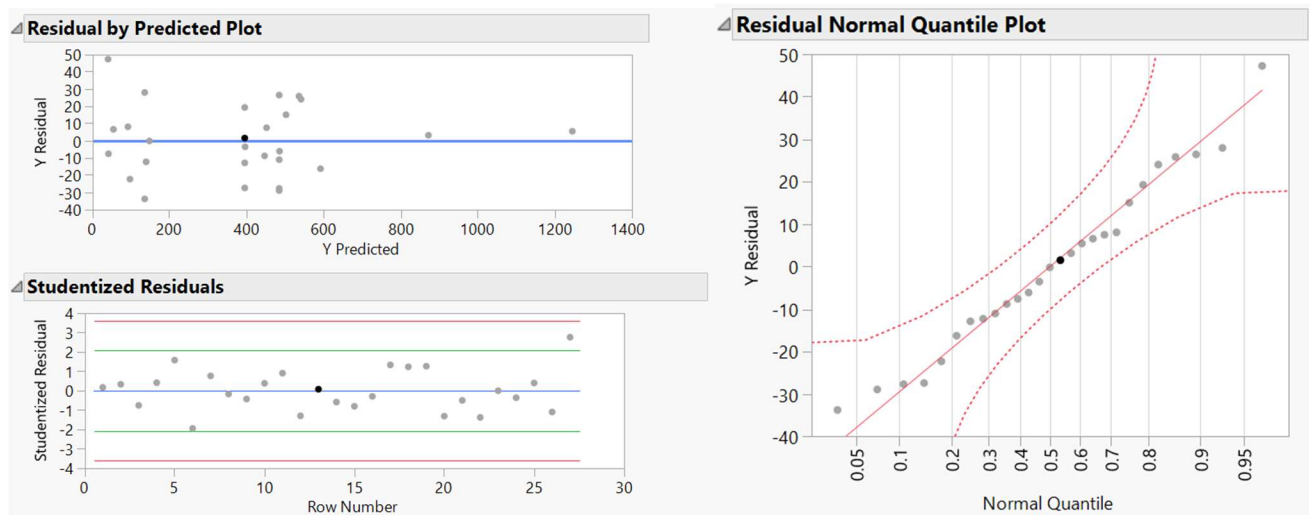


Figure 13: Residual plots for the final model

The residual plots are promising. They show approximately linear structure. We observe a zero mean across all residual plots. However, the Residual by Predicted Plot shows potential heteroscedasticity, as higher predicted values for response have lower variance. We decide to investigate using the Box-Cox plot.

This shows no transformation is needed, as lambda is about equal to 1. Therefore, we assume there is no heteroscedasticity, that we have an outlier at +50 on the low end of predicted values, and that our two points at the high end are close to zero, so results show generally constant variance.

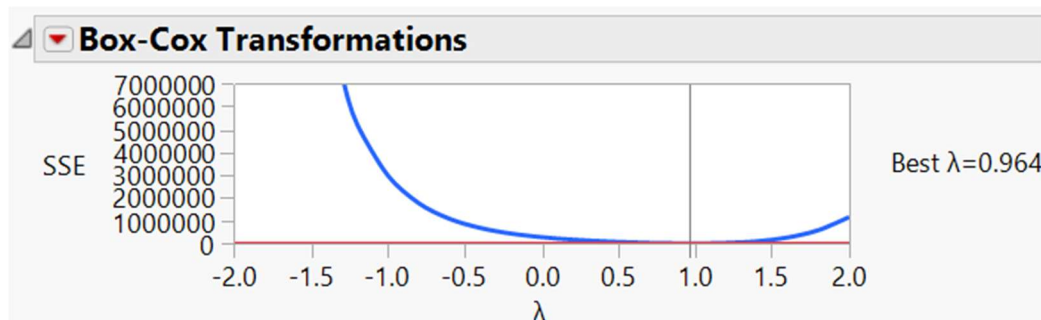


Figure 14: Box-Cox Plot for final model demonstrating no transforms on the response are needed

Our final model prediction profiler shows increased curvature. Desirability is maximized to find optimal operating points. These points are applied to the contour profiler to show optimization graphically. We notice that the Material Resin percentage dominates the contour of our response.

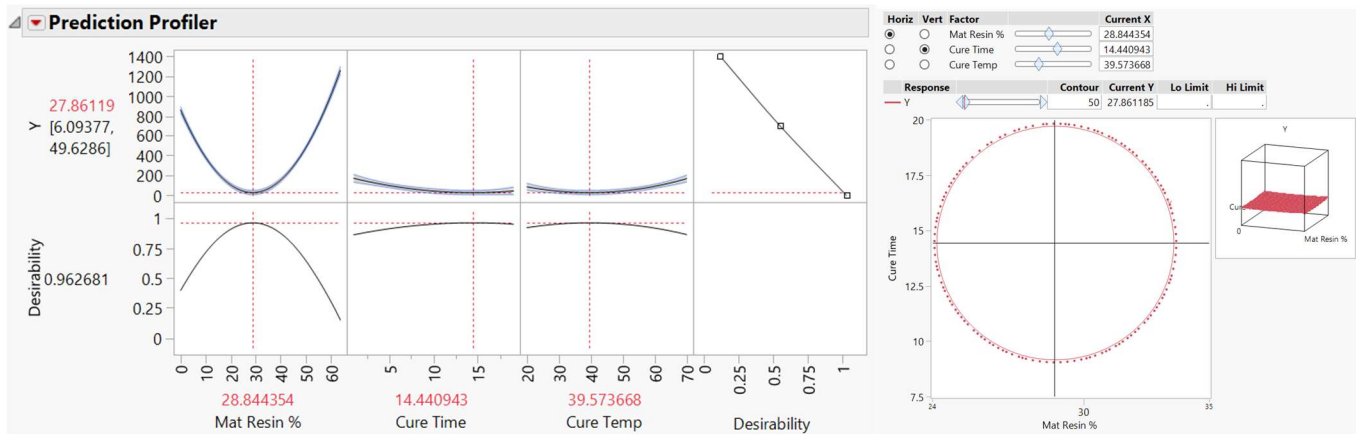


Figure 15: (a) Profiler of each main effect in model. (b) Contour showing predicted minimum at optimal points

The prediction profiler after doing the Central Composite Design gave optimum value of 28.84% for Material Percent Resin, 14.44 Min for Cure Time, and 39.57° C for Cure Temperature.

## 5. Recommendation and Conclusion

First objective of this project was to identify the significant main effects and interactions on the response, being variability of a certain geometric feature. From the initial screening we discovered four significant main effects, but decided to proceed with only Material Resin Percentage, Cure Temperature, and Cure Time for further designs, negating Flow Rate.

The Second objective was to build a model to predict variability from the significant factors and generate a list of optimal settings. Steepest descent technique was applied using a method of replicate centerpoints to find regions of curvature [3]. However after proceeding with standard central composite design utilizing 15 runs, we noticed only one effect with curvature. By adding together the CCD axial points with replicates and centerpoints to increase the power of our final model, we identified curvature for all three main effects, as well as an interaction for Cure Time and Cure Temp. This final design was used to predict variability with the following prediction expression:

$$\begin{aligned} \text{Variability} = & 1477.66 - 47.6804 \text{ Mat. Resin\%} - 37.7066 \text{ CureTime} - 17.4169 \text{ CureTemp} \\ & + 0.9986 \text{ Mat. Resin\%}^2 + .79592 \text{ CureTime}^2 + 0.15219 \text{ CureTemp}^2 \\ & + .3717933 \text{ CureTime} * \text{CureTemp} \end{aligned}$$

The optimal points for minimum variability in these three effects are 28.84% for Material Percent Resin, 14.44 Min for Cure Time, and 39.57° C for Cure Temperature. The expected response variability using these conditions is reduced to 27.86. Overall, the final model was created after 43 runs, significantly lower than the 100 run capacity. The design does have some limitations. We did not have time to proceed with dealiasing of the factors from the fractional factorial screening experiment, due to time constraints. We do notice after running a follow up screening design with all 43 runs that Mold Temperature and Dwell Time may also have some significant effect. Our recommendation to the customer is to proceed with further investigation via steepest descent for these now screened factors, utilizing replicate centerpoints to identify curvature.

## References



- [1] "Central Composite Designs." 3.1.5 *Central Composite Designs*, Institute for Microelectronics, [www.iue.tuwien.ac.at/phd/plasun/node32.html](http://www.iue.tuwien.ac.at/phd/plasun/node32.html).
- [2] "Central Composite Designs (CCD)." 5.3.3.6.1. *Central Composite Designs (CCD)*, National Institute of Standards and Technology, 2013, [www.itl.nist.gov/div898/handbook/pri/section3/pri3361.htm](http://www.itl.nist.gov/div898/handbook/pri/section3/pri3361.htm).
- [3] "Single Response: Path of Steepest Ascent." 5.5.3.1.1. *Single Response: Path of Steepest Ascent*, National Institute of Standards and Technology, 2013, [www.itl.nist.gov/div898/handbook/pri/section5/pri5311.htm](http://www.itl.nist.gov/div898/handbook/pri/section5/pri5311.htm).