EE2671 Transfer Function Estimation Using Steepest Descent

Summary In many signal processing applications, we are asked to find the transfer function of a system (e.g., the stock market, the human vocal tract) without having full control of the inputs to the system. We are often presented with the output of the system and if we are lucky the input which caused this output. In the problem below, we have the following information regarding a black box:

- the magnitude-squared of its frequency response at a few frequencies,
- the system well-modeled using an all-pole (i.e., denominator coefficients only) transfer function.

Details You are given $P(\omega_i), i = 1, \ldots, N$ and you are asked to minimize the cost function

$$E = \sum_{i=1}^{N} \frac{P(\omega_i)}{\hat{P}(\omega_i)} - \ln \frac{P(\omega_i)}{\hat{P}(\omega_i)} - 1$$

where

$$\hat{P}(\omega_i) = \frac{1}{\left|\sum_{k=0}^{p} a_k e^{-j\omega_i k}\right|^2} = \frac{1}{|a_0 + a_1 e^{-j\omega_i} + a_2 e^{-j2\omega_i}|^2} \text{ for } p = 2$$

is the magnitude-squared response of the proposed model.
For this week, minimize the distance using the method of steepest descent with your favorite line search algorithm. Here is the data

ω	$P(\omega)$
0.5236	0.6821
1.0472	4.3232
1.5708	3.7540
2.0944	0.4368
2.6180	0.1988

- Use $p = 2$ and start with $1, 0, 0$ as the initial estimate for the filter coefficients.
- The optimal values are $1, -0.5161, 0.9940$. Verify that these values produce $E = 0$ (or close enough).
- Plot the value of E vs. the iteration number.
- Plot the distance between your current estimate for the coefficients and the optimal values vs. the iteration number, use $\sum_{k=0}^{p}(a_k^{est} - a_k^{opt})^2$ to measure this distance.