

Yang-Mills Theory: A proof that the lowest excitations have a finite mass gap in regard to the vacuum state(Yungg-Millz)

J.I. Samuels

ABSTRACT

The Yang-Mills Theory hypothesis concerns itself with proving a finite mass gap relative to the vacuum state. The Yang-Mills theories are a special group of gauge theories with non-abelian symmetry, given by the Lagrangian. [1]

1. Proof of the existence of a finite mass gap

Proof.

The Lagrangian of the Yang-Mills Theories is

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}Tr(F^2) = -\frac{1}{4}F^{\alpha\mu\nu}F_{\mu\nu}^{\alpha} \\ \text{knowing } Tr(T^a)(T^b) &= \frac{1}{2}\delta^{ab}. \\ Tr(F^{\alpha\mu\nu}F_{\mu\nu}^{\alpha}) &= \frac{1}{2}\delta_{\mu\nu}^{\alpha^2} \\ &= \frac{1}{4}\delta_{\mu\nu}^{\alpha^2} = 0 \\ &\text{and} \\ \frac{1}{4}F^{\alpha\mu\nu}F_{\mu\nu}^{\alpha} &= 0\end{aligned}$$

When counting from the vacuum state, which is essentially 0, one must use a method that requires counting. The best methods for things like this is to take the square root or natural logarithm, as both these methods require counting to a total.

Taking the square root[2]

$$\begin{aligned}\frac{\sqrt{2}}{2}F^{\alpha\mu\nu\frac{1}{2}}F_{\mu\nu}^{\alpha\frac{1}{2}} &= \sqrt{0} \\ 0^{\frac{1}{2}} &= e^{i\ln(\cos(\frac{1}{2}\ln|0|)+isin(\frac{1}{2}\ln|0|))} \\ 0^{\frac{1}{2}} &= (\cos(\theta) - isin(\theta))e^{arg|w|} \\ \frac{\sqrt{2}}{2}F^{\alpha\mu\nu\frac{1}{2}}F_{\mu\nu}^{\alpha\frac{1}{2}} &= (\cos(\theta)isin(\theta))e^{arg|w|} \\ \frac{\sqrt{2}}{2}\delta_{\nu\mu}^{\alpha\frac{1}{2}\mu\nu} &= (\cos(\theta) - isin(\theta))e^{arg|w|} \\ \frac{\sqrt{2}}{2}\delta_{\mu\nu}^{\alpha\frac{1}{2}\mu\nu} &= (\cos(\theta) + isin(\theta))e^{arg|w|} \\ \delta_{\mu\nu}^{\alpha\frac{1}{2}\mu\nu} &= \sqrt{2}(\cos(\theta) + isin(\theta))e^{arg|w|}\end{aligned}$$

From here you are required to get the mass.

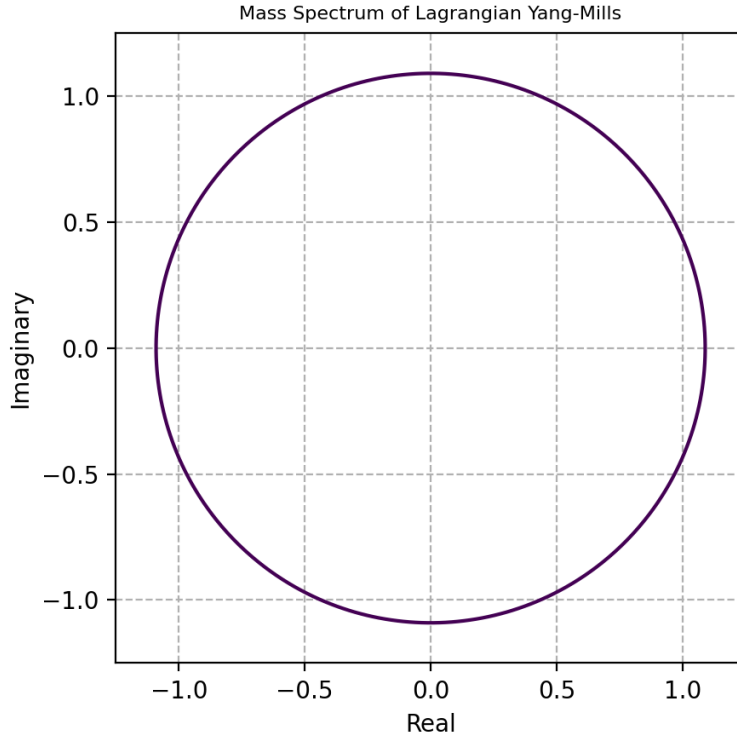
As δ is \equiv to the Force Tensor F , we can state. .. $F = ma$

$$\sqrt{F} = m$$

As at 0 $m=a$.

Continuing we can say...

$$\begin{aligned}\delta_{\mu\nu}^{\alpha\frac{1}{2}\mu\nu} &= F. \text{ As } \delta = \mathcal{O}(F). \text{ Therefore we now say that.} \\ \delta_{\mu\nu}^{\alpha\frac{1}{4}\mu\nu} &= \sqrt{2}^{\frac{1}{2}}(\cos(\frac{1}{2}\theta) + isin(\frac{1}{2}\theta))e^{arg|\frac{1}{2}w|}\end{aligned}$$

FIGURE 1. *Mass Spectrum*

$$\text{and then. } m = \sqrt{2}^{\frac{1}{2}} (\cos(\frac{1}{2}\theta) + i\sin(\frac{1}{2}\theta)) e^{arg|\frac{1}{2}w|}$$

This is the mass spectrum which we shall use to prove the finite mass gap in relation to the vacuum state.

□

The existence of a finite mass gap is proven by the 'precision' of the 'Ivan Orbit'. In short, any spectrum calculated produces a minimum number (finite mass gap). This proves the mass gap through fundamental mathematical logic, as any position along the axis can be chosen and any position will produce a spectrum with a finite mass gap between itself and 0.

A calculation of the mass gap can be done at any arbitrary point, but the points of $\frac{1}{2}$ and $\frac{1}{4}$ have been chosen.

The calculations performed are similar to the ones performed when calculating the Riemann Zeros[?] so I shall only provide a preliminary answer here. For $\frac{1}{2}$ the mass gap is ≈ 67 and for $\frac{1}{4}$ the mass gap is ≈ 1.148 .

This concludes the Yang Mills Hypothesis.

References

1. ARTHUR JAFFE AND EDWARD WITTEN, Quantum Yang-Mills Theory (Clay Mathematics, Year Not Said).
2. J I SAMUELS, 'A Proof of the Riemann Hypothesis', *ResearchGate* 2020