A PROOF FOR THE RIEMANN HYPOTHESIS

J.I.S

Abstract. The Riemann Hypothesis is a hypothesis first proposed by Bernhard Riemann in 1859 stating that the zeros of the Riemann Zeta Function exist as integers with values of $-2n$ and complex numbers where the real part is $\frac{1}{2}$. In this paper I demonstrate a new method of solving the problem through extending the complex analysis of zero using Jamell' Circles and Jamell's Distribution. The results are the complete proof of the Riemann Hypothesis and a new calculation of the non-trivial zeros which are congruent with the hypothesis and which are able to calculate the values of the primes.

Key words. Riemann Hypothesis

AMS subject classifications. 11M26

1. Proof for the trivial zeros.

THEOREM 1.1. *The Trivial Zeros of the Riemann zeta function are negative even integers whose formula can be expressed as $-2n$.*

Proof. Let the Riemann Zeta Functional equation be defined as $\zeta(1-s) = \frac{\Gamma(s)}{(2\pi)^s} 2cos(\frac{\pi s}{2})\zeta(s)$
[1]
We wish to prove that $\zeta(s) = \zeta(-2n)$
We first equate the function to 0.
$0 = \frac{\Gamma(s)}{(2\pi)^s} 2cos(\frac{\pi s}{2})\zeta(s)$
We then divide by the expressions attached to the cosine function.
$0 = cos(\frac{\pi s}{2})$
Next we substitute the cosine for the exponential equivalent.
$0 = \frac{e^{i\frac{\pi s}{2}} + e^{\frac{-i\pi s}{2}}}{2}$
And then we multiply by 2 to simplify.
$0 = e^{i\frac{\pi s}{2}} + e^{\frac{-i\pi s}{2}}$
We multiply by $e^{i\pi s}$. to simply again.
$0 = e^{i\pi s} + 1$
And solve the equation as a trigonometric function.
$-1 = cos(\pi s) + isin(\pi s)$
$s = 1 + 2n$
Finally we substitute the result back into the zeta function.
$\zeta(1-s) = \zeta(1-(1-2n))$
$\zeta(-2n)$ □

2. The Jamell Circle.

THEOREM 2.1. *The square root of 0 can be transformed into a trigonometric function known as a Jamell Circle. All Jamell Circles have the properties of center$(0,0)$ and a radius of e^w.*

Proof. Let the expression for the square root of 0 be $x = 0^{\frac{1}{2}}$.
And let us manipulate the expression using logarithms and exponentials
so that$e^{i\frac{1}{2}ln|0|} = cos(\frac{1}{2}ln|0|) + isin(\frac{1}{2}ln|0|)$.
$i\frac{1}{2}ln|0| = ln|cos(\frac{1}{2}ln|0|) + isin(\frac{1}{2}ln|0|)$.
$\frac{1}{2}ln|0| = -iln|cos(\frac{1}{2}ln|0|) + isin(\frac{1}{2}ln|0|)$.

$0^{\frac{1}{2}} = e^{-iln|cos(\frac{1}{2}ln|0|)+isin(\frac{1}{2}ln|0|)}$.
$= e^{iln|u+iv|}$.
$= e^{iln|\omega|}e^{u+iv}$.
$= e^{arg|\omega|}(cos(\theta) - isin(\theta))$.
Where ω = u+iv.
$-\pi \geq \pi < \pi$.
If $arg|\omega|$ is taken as 0 which is the most reasonable assumption given the nature of what we are calculating and the fact that $\zeta(s) = -2n$ and $\zeta(s) = \frac{1}{2} + iy$ both lay on the line. Then the square root of zero can be presented as a standard normal distribution.

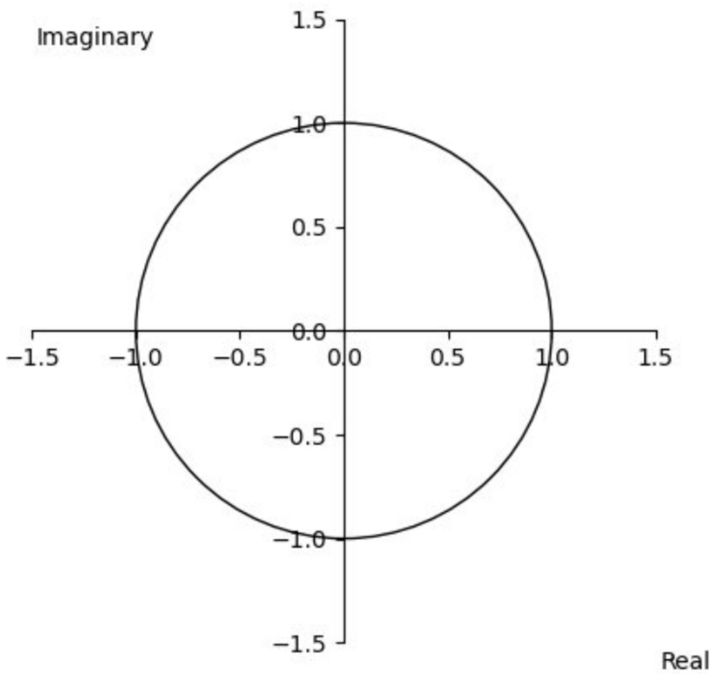

Fig. 1. *Jamell Circle.*

3. The Riemann Circle Theorem.

Theorem 3.1. *The Riemann Zeta Function $\zeta(s) = \sum_{n=1} n^{-s}$ can be shown to be a circle.*

Lemma 3.2. *The integrand of the Riemann Zeta Function can be shown to be a circle.*

Proof. Let the Riemann Zeta Function be defined as $\zeta(s) = \sum_{n=1} n^{-s}$.
As a continuous integral the equation the function is defined as. $\zeta(s) = \int_{n=1} n^{-s}ds$.
Assigning the integral term as x as in a substitution. $x = n^{-s}$.
This can then be manipulated to $ln|x| = -sln|n|$.
And it follows that $x = e^{-sln|n|}$.
$= cos(sln|n|) - isin(sln|n|)$.

$= cos(A\theta) - isin(A\theta)$.

We can now say here that the subject of the Riemann Zeta Equation has been shown to be equivalent to a circle. Now that we have manipulated the Riemann Zeta integrand into an exponential form, it should be easier to integrate.

Proof. Let the Riemann Zeta Function as a continuous sum be $\int_{n=1} e^{-sln|n|}$.
$\int_{n=1} e^{-sln|n|} = \frac{-s}{n} e^{-sln|n|}$.
or equivalently $\int \frac{1}{n^s} ds = \frac{-s}{n} cos(A\theta) - isin(A\theta)$.
$\substack{n=\infty \\ n=1} \left[\frac{-s}{n} cos(A\theta) - isin(A\theta)\right]$.
The result of which is $0 + s(cos(A\theta) - isin(A\theta)$.
And which can be written as $s(cos(A\theta) - isin(A\theta)$.
This form is equivalent to $e^w(cos(\theta) - sin(\theta))$.

This circle when drawn is the same as a Jamell Circle.

4. Proof for the non trivial zeros.

THEOREM 4.1. *The non-trivial zeros of the Riemann Zeta Function are all complex numbers with a real part of* $\frac{1}{2}$.

Proof. Let the Riemann Zeta Functional equation be $\zeta(1-s) = \frac{\Gamma(s)}{(2\pi)^s} 2cos(\frac{\pi s}{2})\zeta(s)$.
Equating it to the square root of zero $\sqrt{0} = \frac{(s)}{(2\pi)^s} 2cos(\frac{\pi s}{2})\zeta(s)$.
Using the value of the normal standard deviation which is also the value of the standard deviation in a Jamell Circle.
$cos(\frac{\pi s}{2}) = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$
Then re-arranging the equation.
$\frac{e^{i\frac{\pi s}{2}} + e^{-i\frac{\pi s}{2}}}{2} = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$
$e^{i\frac{\pi s}{2}} + e^{-i\frac{\pi s}{2}} = \sqrt{2} + i\sqrt{2}$
$e^{i\pi s} - (\sqrt{2} + i\sqrt{2})e^{\frac{\pi s}{2}} + 1 = 0$
Using the quadratic formula to solve for $e^{\frac{i\pi s}{2}}$.
$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = e^{\frac{i\pi s}{2}}$
$e^{\frac{i\pi s}{2}} = \frac{\sqrt{2}+i\sqrt{2} \pm \sqrt{(\sqrt{2}(1+i))^2 - 4}}{2}$
$e^{\frac{i\pi s}{2}} = \frac{\sqrt{2}+i\sqrt{2}}{2} \pm \frac{(4i-4)^{\frac{1}{2}}}{2}$
$e^{\frac{i\pi s}{2}} = \frac{\sqrt{2}+i\sqrt{2}}{2} \pm (i-1)^{\frac{1}{2}}$
We now have two terms. Substituting the exponential expression for it's trigonometric function and focusing on the first term gives us
$cos(\frac{\pi s}{2}) + isin(\frac{\pi s}{2}) = \frac{\sqrt{2}+i\sqrt{2}}{2}$.
And then solving this expression gives gives us a result of
$s = \frac{1}{2}$.
We now have calculated a value of $\frac{1}{2}$ for the first set of the zeros. We shall now use DeMoivre's Theorem to calculate the second set of zeros.
Focusing on the second term$cos(\frac{\pi s}{2}) + isin(\frac{\pi s}{2}) = (i-1)^{\frac{1}{2}}$.
And applying DeMoivre's Theorem to give us $cos(\frac{\pi s}{4}) + isin(\frac{\pi s}{4}) = (i-1)$.
And solving this sinusoidal expression gives us a result of $s = 2n + 1$.

As the second set of zeros $(2n+1)$ are the trivial zeros that were calculated before and they were calculated alongside the first set of zeros which were a $\frac{1}{2}$ we can say that the second set of zeros (the trivial zeros) have verified the first set of zeros (the non-trivial zeros) and can conclude that the Riemann Hypothesis has been satisfied.

5. The method for calculating the non trivial zeros. In order to calculate the imaginary parts of every zero of the Riemann Zeta Function it is necessary to take a look back at the Riemann Circle. The non trivial zeros all have a real part of a $\frac{1}{2}$ and when drawn on Riemann Circle with radius 1 this gives an imaginary part of $\frac{\sqrt{3}}{2}$. When transforming a circle into an infinite line the arc length for points at a $\frac{1}{2}$ are calculated and the formula for this is $s = \frac{1}{2} + iS$, where S is the arc length. There are two formulas for counting the non trivial zeros as the zeros above and below the x axis must be counted. Another way of looking at it is, as it is exponential two formulas are required to calculate it by polynomially. The equations for calculating the non-trivial zeros are presented below:

$$s_1 = \frac{\pi}{3}(1 + 6n) \tag{5.1}$$

$$s_2 = \frac{5\pi}{3}(1 + \frac{6n}{5}) \tag{5.2}$$

$$y = \sqrt{\frac{y^2}{\theta^2} - \frac{1}{4}} \tag{5.3}$$

s_1, s_2 are the arc lengths above and below the x axis respectively and therefore their position on the y axis n is the integer value. The results of the two formulas disprove the current calculation of the non-trivial zeros. The first formula verifies the twin prime formula $(1 + 6n$. An example of the code is presented below and the first 50 non trivial zeros is included in the appendix.

Algorithm 5.1 Algorithm for calculating the non trivial zeros

DEFINE FUNCTION RIEMANN(number=20);
INITIALISE EMPTY ARRAY Y
FOR ELEM IN X:
$\mathrm{Y}_1 = S_1$
Y.APPEND(Y_1)
$\mathrm{Y}_2 = S_2$
Y.APPEND(Y_2)
SORT(Y)
PLOT $\frac{1}{2}$ vs Y

5.1. A summary of the non-trivial zeros.

- All of the non trivial zeros are ± 1 away from a prime number, similar to the Twin Prime formula and as suggested by the result $2n + 1$.
- $-1, 0$ and 1 are considered prime numbers by the Riemann Zeta Function, the next prime predicted is 3. The only prime not predicted is 2.
- Because of the nature of primes -1 is the only negative prime number.
- The primes follow a cycle with a period of approximately 30.
- All non trivial zeros are even when rounded down (floored) as implied by the trivial zeros.
- All non trivial zeros are irrational which is due to them all being multiples of π.

6. Conclusions. The Riemann Hypothesis was examined and proven and the formulae for calculating the non trivial zeros was worked out and the results compared against the current/mainstream results. The new zeros satisfied the Riemann Hypothesis better than the current zeros and in conclusion the Riemann Hypothesis has been satisfied.

REFERENCES

[1] Easy Proofs of Riemann's Functional Equation for $\eta(s)$ and Lipschitz Summation, S. Robins and M. Knopp, Proceedings of the American Mathematical Society, 2001