The Birch and Swinnerton - Dyer Conjecture

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Abstract

The Birch and Swinnerton Conjecture describes the set of rational solutions that describe an elliptic curve,[2] it asserts that L(C, 1) = 0 C(Q) is infinite. [1] In this paper I shall prove this conjecture.

1. *Proof of Birch Swinnerton Conjecture*

Proof.

[1]The equation for an elliptic curve is: $C : y^2 = x^3 + ax + b$

The number of solutions modulus p is: N_p solutions of C mod p

The co-efficient a_p is $p - N_p$

The L function $L(C, s) = \Pi_{p|2\Delta}(1 - a_p p^{-s} + p^{1-2s})^{-s}$

and the case we are discussing $L(C, 1) = \Pi_{p|2\Delta}(1 - a_p p^{-1} + p^{-1})^{-1}$

We shall begin by solving C

$$y^2 = x^3 + ax + b = 0$$

$$x^3 + ax = -b$$

$$x(x^2 + a) = -b$$

$$x(x^2 + a) + b = 0$$

solutions for b = 0 a$\neq$ 0 and a = 0

$$x = 0, x = \pm\sqrt{a}$$

3 solutions.

solutions for b $\neq$ 0, a = 0

$$x^3 + b = 0, x = \pm b^{\frac{1}{3}}$$

MAX 2 solutions.

So far we have 2 outcomes.

$$x(x^2 + a) = -b$$

$$(x^2 + a) = \frac{-b}{a}$$

$$\frac{1}{x^2+a} = \frac{x}{-b}$$

as $x \to \sqrt{a}$ $b \to -\infty$

Notice this function is similar to ln(0)

$$\frac{-b}{x^2+a} = x$$

$$\frac{x^2+a}{-b} = \frac{1}{x}$$

as $x \to 0$ $\frac{a}{-b} \to \infty$ Limit Testing

1)$\frac{1}{x^2+a} = \frac{-x}{b}$ as $x \to \sqrt{a}$ $b \to -\infty$

2)$\frac{x^2+a}{-b} = \frac{1}{x}$ as $x \to 0 \frac{a}{-b} \to -\infty$ and $a \to -\infty$ (if considering the limits of 1) as well)

$$\frac{f(x)}{f(x)} = 1$$

2020 *Mathematics Subject Classification* 11G40 (primary).

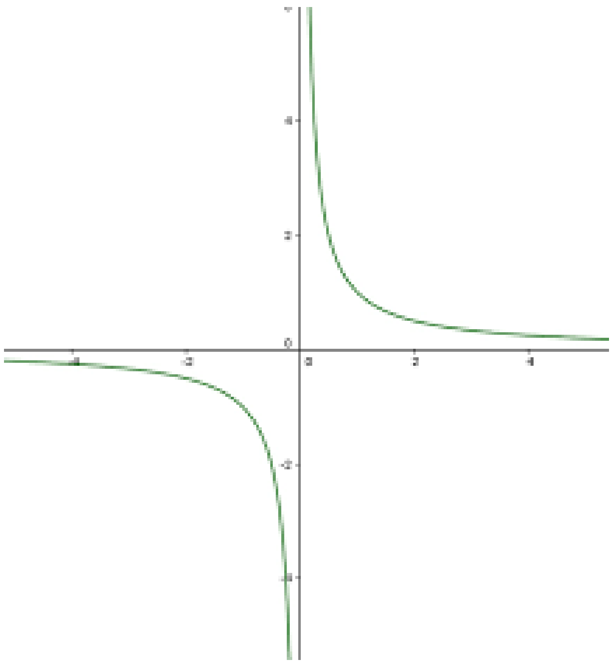

FIGURE 1. *1/x suggested distribution of solutions to an elliptic curve*

$$\frac{\frac{1}{x^2+a}}{\frac{(x^2+a}{-b}} = \frac{\frac{-x}{b}}{\frac{1}{x}}$$

$$\frac{-b}{(x^2+a)^2} = \frac{-x^2}{b} \; -b^2 = -x^2(x^2+a)^2$$

$$x^2(x^2+a^2) - b^2 = 0$$

this is $f^2(x)$. Here a = 0 which reinforces the likelihood of there only being 2 solutions.

$$\frac{x^2+a}{1} = \frac{-b}{x} \text{ as } x \to 0 \; a \to \infty$$

$$\frac{\frac{x^2+a}{1} = \frac{-b}{x}}{\frac{x^2+a}{-b} = \frac{1}{x}}$$

$$-b = -b \text{ good}$$

We have now ascertained that there is a maximum of 2 solutions. as $x \to 0$ $a \to \infty$ $b \to -\infty$

So far:

The set of rational solutions behaves as a $\frac{1}{x}$ graph or an $ln(0)$ graph. As $a >> b$.

$N_p = 2$ mod p and in special cases 3 mod p.

$$L(C,1) = \Pi_{p|2\Delta}(1 - a_p p^{-1} + p^{1-2*1})^{-1}$$

$$L(C,1) = \Pi_{p|2\Delta}(1 - p.p^{-1} + p^{-1})^{-}1$$

Here we have to notice that the answer is a prime distribution.

Simply $a_p.p^{-}1$ as $a_p = p - N_p$ where $N_p = 1$

$$1 - 2 = -1$$

$$2 - 2 = -0$$

$$3 - 2 = 1$$

$$5 - 2 = 3$$

$$7 - 2 = -5$$

$$L(C,1) = \Pi_{p|2\Delta}(1 - p.p^{-1} + p^{-1})^{-1}$$

$$L(C,1) = \Pi_{p|2\Delta}(1 - 1 + p^{-1})^{-1}$$

$L(C,1) = \Pi_{p|2\Delta}(p^{-}1)^{-1}$
$L(C,1) = \Pi_{p|2\Delta}(p)$
The sum of all primes tends to infinity
$L(C,1) = \Pi_{p|2\Delta}(p) \to \infty$
This proof also holds for $(p(p-N_p))$
As $p^2 - 2$ results in a distribution of the integers of the Riemann Zeros [3].
Where the result is the Riemann Non-Trivial Zeros. Which leads to the same result.
$L(C,1) = \Pi_{p|2\Delta}(Re(Zeros))$ Which the sum of would equal infinity.

$\square$

References

1. Andrew WIles, Clay Mathematics, https://www.claymath.org/sites/default/files/birchswin.pdf, 2021
2. Wikipedia, $en.wikipedia.org/wiki/Birch_and_Swinnerton - Dyer_conjecture, 2021 Jamell Ivan Samuels, A Proof of the Riemann H$