AMS 102.05 – Final Exam **NAME:**________________________
SCORE:______________ **STUDENT ID #:**________________

1. Enrique is a volunteer worker at the local blood-donor center. According to the American Red Cross, 42% of blood donors have type O blood. Enrique is also a medical student and is doing a project as part of his studies. He will record the blood type for 150 randomly selected blood donors at the center. What is the probability that the sample proportion of donors with type O blood is between 35% and 50%?

2. For adult females, the red blood cell count obtained in a medical blood test has a mean of 4,500,000 per mm^3 and a standard deviation of 350,000 per mm^3. Suppose that the cell counts are approximately normally distributed.

    a. What is the probability that a randomly chosen adult female has a red blood cell count of more than 5,000,000 per mm^3?
    b. A simple random sample of 40 adult females will be chosen. What is the probability that the mean red blood cell count for these 40 sampled women will be 4,400,000 per mm^3 or lower?
    c. Suppose 20 adult females are selected at random. What would be the distribution of the total red blood cell count across the 20 females sampled? (**Hint:** Think about how the sample mean and the total are related.)

3. A small locally owned pizza company is trying a new pizza crust. An experiment is conducted to find the optimal baking time (20, 25, or 30 minutes), baking temperature (400˚F, 425˚F, or 450˚F), and amount of cheese (2 cups or 2.5 cups). Five batches of pizza-crust dough will be assigned to each treatment.

    a. How many treatments does this experiment have?
    b. How many experimental units are used in this experiment?

4. A company employs 800 people, 75% of whom are males and 25% are females. A management task force officer polls a stratified random sample of 90 males and 20 females.
    a. What is the chance that a particular female employee will be polled?
    b. Using the following random numbers list the first five females to be included in the sample. 52102 51916 46369 18586 21216 14068 83149 98736 23495
    c. Suppose that the average response for the sample females was 5, while the average response for the sampled males was 8. Give the overall estimate of the average response.

5. Social workers claim that 15% of the beds in homeless shelters in San Francisco are assigned to families. In a simple random sample of 1395 shelter beds, 196 were assigned to families.
   a. Give the estimate for the population proportion, p, of beds assigned to families among all homeless-shelter beds in San Francisco.
   b. Give a 99% confidence interval for the population proportion p of beds assigned to families among all homeless-shelter beds in San Francisco. Show all steps involved in getting your answer.
   c. What is the width of your confidence interval in part (b)?
   d. What is the margin of error for your confidence interval in part (b)?
   e. If you constructed a 90% confidence interval using the same data, the margin of error would be (select one) smaller, larger, the same, can't tell.

6. The 10 measurements that follow are the furnace temperatures recorded on successive batches in a semiconductor-manufacturing process. The temperature unit is degrees Fahrenheit (˚F). (Recall $˚\mathrm{C} = \frac{5}{9}(˚\mathrm{F} - 32) = \frac{5}{9}˚\mathrm{F} - \frac{160}{9}$)

Observation #	1	2	3	4	5	6	7	8	9	10
Temperature (˚F)	949	949	951	950	954	953	955	955	959	957

a) On average, the temperatures were about ________˚F from their mean of __________˚F.
b) Give the five-number summary of this data. (Clearly label each of the five numbers.)
c) On average the temperatures were about _________ ˚C from their mean of _______˚C.
d) Give the five-number summary of these data in ˚C. (Clearly label each of the five numbers.)

7. One-fifth of a particular breed of rabbits has long hair. Suppose that 10 of these rabbits are born.
   a. What is the expected number of long-haired rabbits?
   b. What is the probability that the number of long-haired rabbits will be more than or equal to 3?

8. Suppose that there are exactly two boxes, Box A and Box B. Each box contains 25 same sized tokens. Each token is inscribed with a dollar value. The frequency table (distribution) for each box is as follows:

Box A

Value ($)	Frequency
0	1
5	1
10	2
15	3
20	4
25	6
30	8

Box B

Value ($)	Frequency
0	8
5	6
10	4
15	3
20	2
25	1
30	1

Suppose that you are shown just one of the boxes. A claim is made that the box is Box A. You must decide (judge) the probable truth of this claim *statistically* by selecting at random one token from the shown box.

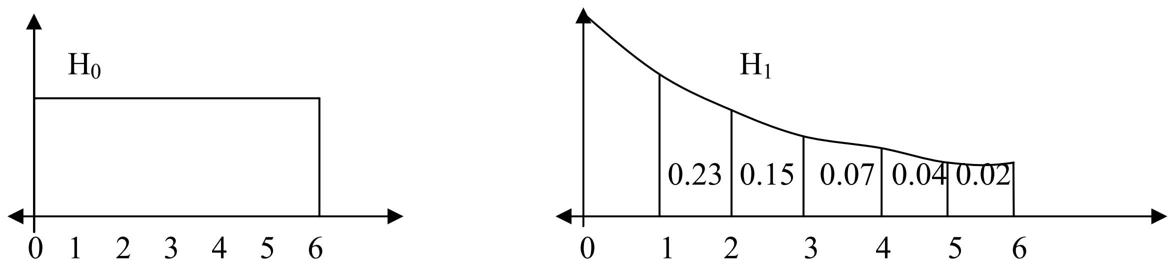
a. Write the two hypotheses for the statistical test.
b. What is the direction of extreme?
c. Write a decision rule so that the probability of a Type I error is as close to, but no more than, 10%.
d. What is the numerical value for α for your decision rule in part c?
e. What is the chance of committing a Type II error using your decision rule?
f. Suppose that you select a token inscribed with $5. Using the decision rule from part c, what is your decision?

9. A long-term study of the environmental conditions in a bay gave the following annual average salinity readings for 12 consecutive years (given in order from left to right):

9, 12, 14, 15, 13, 15, 15, 15, 13, 17, 17, 16

a. Obtain a time plot for the average salinity readings. Be sure that you label the axes and provide the minimum and maximum values on each axis.
b. Which of the following is (are) an appropriate description of the plot in (a):
    i. Increasing trend.
    ii. Decreasing variability.

10. Consider two competing models to describe a population of values for the lifetime of a computer chip. You will be allowed to select one chip at random from the population at random and must decide which of the two models to support.



a. What is the height of the density specified under the null hypothesis?
b. What proportion of chips last less than one year under the alternative hypothesis?
c. You select a chip at random from the population, and the resulting lifetime is 0.5 years. Calculate the p-value for this test.
d. Give your decision at a 10% significance level and state your conclusion using a well-written sentence.

$0 \leq P(A) \leq 1$ $\qquad$ $P(S) = 1$ $\qquad$ $P(A) = 1 - P(\text{not } A)$

$P(A \text{ and } B) = P(B)P(A|B) = P(A)P(B|A)$ $\qquad$ $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}, P(B) > 0$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$E(X) = \mu = x_1p_1 + x_2p_2 + \cdots x_kp_k = \sum x_ip_i$, $\quad \bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$, $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}$

$Var(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - E(X)^2 = \sum x_i^2 p_i - \mu^2$, $\qquad s^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}$

Linear Transformations If Y = aX + b, then $\bar{y} = a\bar{x} + b$ and $s_y = |a|s_x$

$Z = \frac{x-\mu}{\sigma}$ Standardization Transformation, Changes $X \sim N(\mu,\sigma)$ to $Z \sim N(0,1)$

Binomial Random Variables

$P(X = x) = nCx\, p^x (1-p)^{n-x}$ $\qquad$ $n! = n(n-1)(n-2) \ldots (2)(1)$

$E(X) = np$ $\qquad$ 0! = 1, 1!=1, 2!=2, 3!=6, 4!=24, 5! = 120

$Var(X) = np(1-p)$

$nCx = \frac{n!}{x!\,(n-x)!}$

Geometric Random Variables

$P(X = x) = p(1-p)^{x-1}$ $\qquad$ $E(X) = \frac{1}{p}$ $\qquad$ $Var(X) = \frac{1-p}{p^2}$

Uniform Distributions U(a,b)

Areas correspond to probabilities, $\qquad \bar{u} = \frac{b+a}{2}$, $\qquad s_u = \frac{(b-a)^2}{12}$

Empirical Rule for Normal Distributions

68% of observations will fall within σ of the mean, 95% within 2σ and 99.7% within 3σ

Sampling Distributions of Statistics

Sample Proportions

$\hat{p} \sim N(p, \sqrt{\frac{p(1-p)}{n}})$ Sample proportion is approximately normal for large n.

Standard Error is $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, Margin of Error is $z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, Test statistic is $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$

Confidence interval is $\hat{p} \pm z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ where $P(Z < z^*) = 1 - \frac{\alpha}{2}$ $\qquad Z \sim N(0,1)$

Sample Means

$\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$ For normal populations, sample mean is normal for any n.

For non-normal populations sample mean is approximately normal for large n. ($n \geq 30$)

Standard Normal Table Transformations

$P(Z > A) = 1 - P(Z < A)$

$P(A < Z < B) = P(Z < B) - P(Z < A)$

Table entry for z is the area under the standard normal curve to the left of z.

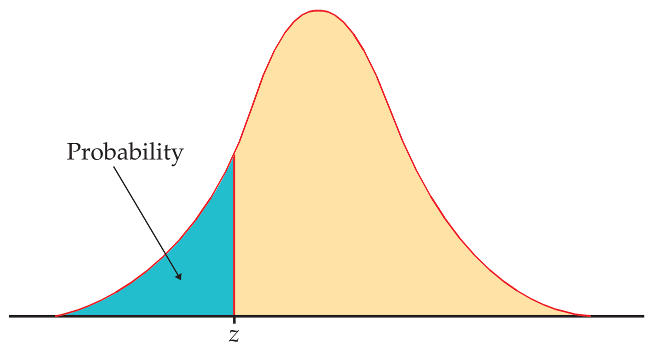


TABLE A

Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the area under the standard normal curve to the left of z.

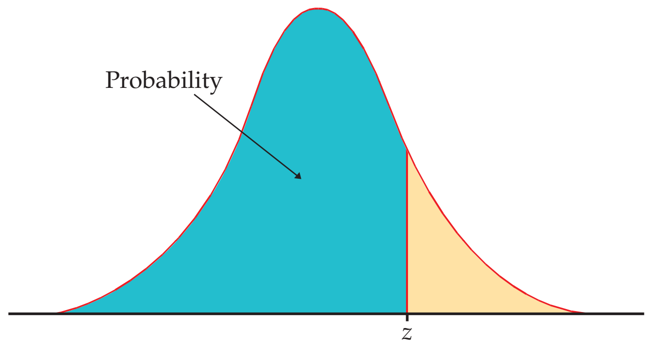


TABLE A

Standard normal probabilities (continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998