1) Our goal is to estimate the average lifespan of a certain bacteria under exposure to extreme heat. We take a sample of 500 bacteria and record an average lifespan of 1.3 minutes. The believed standard deviation of the lifespan of bacteria is 24 seconds.

   a. Develop a 99% Confidence Interval for the true average lifespan.

$\overline{\boldsymbol{x}} = \mathbf{1.3}\ \boldsymbol{minutes} = \mathbf{78}\ \boldsymbol{seconds} \qquad \boldsymbol{n} = \mathbf{500} \qquad \boldsymbol{Z}^*(99\%) = \boldsymbol{P}(\boldsymbol{Z} < .\mathbf{995}) = \mathbf{2.576}\ (\boldsymbol{or}\ \mathbf{2.575})$

$$SD_{\overline{x}} = \frac{24}{\sqrt{500}} = 1.07$$

Confidence Interval is given by

$\overline{\boldsymbol{x}} \pm \boldsymbol{Z}^*\boldsymbol{SD}_{\overline{x}} \Rightarrow \mathbf{78} \pm \mathbf{2.576}(\mathbf{1.07}) \Rightarrow \quad (\mathbf{75.24}\ \boldsymbol{seconds}, \mathbf{80.76}\ \boldsymbol{seconds})$

   b. What assumptions are required for this test to be valid? Are they satisfied here?

We assume that $\overline{x} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$ which is only a good approximation when n is sufficiently large. Since $n \geq 30$, the assumptions are met.

   c. Explain what it means to be 99% Confident.

We are 99% confident in the method. Therefore 99% of the random samples we could draw will yield an interval that contains the true mean.

2) You work in the admissions office at a state university. You are in charge of estimating the proportion of in-state students that will be entering the university in the following year. You want to ensure that you are within 5 percent of the true proportion, so that an accurate value can be placed on the budget.

   a. If you wish to construct a 95% Confidence Interval, what sample size would be required assuming you have no prior information about the true proportion of in-state students.

$\boldsymbol{Error} = \mathbf{0.05} \qquad \boldsymbol{Z}^*(95\%) = \boldsymbol{P}(\boldsymbol{Z} < .\mathbf{975}) = \mathbf{1.96} \qquad \boldsymbol{p} = \boldsymbol{unknown}, \boldsymbol{so\ set\ to}\ \mathbf{0.5}$

$$n = \frac{Z^{*2}p(1-p)}{Error^2} = \frac{1.96^2(0.5)(0.5)}{0.05^2} = 384.16, \text{round up to } 385$$

   b. If you look up the current year's admissions, you find that 72% of students paid in-state tuition. Calculate the modified sample size with this new piece of information.

$\boldsymbol{Error} = \mathbf{0.05} \qquad \boldsymbol{Z}^*(95\%) = \boldsymbol{P}(\boldsymbol{Z} < .\mathbf{975}) = \mathbf{1.96} \qquad \boldsymbol{p} = \boldsymbol{prior\ info}, \boldsymbol{set\ to}\ \mathbf{0.72}$

$$n = \frac{Z^{*2}p(1-p)}{Error^2} = \frac{1.96^2(0.72)(0.28)}{0.05^2} = 309.7, \text{round up to } 310$$

3) **REVIEW QUESTION (FROM CHAPTER 7)**
You are trying to throw a party for a group of 250 individuals. You want to buy soda for the party, and you get a discount if you buy in bulk and only 1 brand (ie, either Coke or Pepsi). You decide to buy Coke only, since you came across a study that said that 54% of people prefer Coke to Pepsi. Assuming this proportion is correct, what is the probability of having ***more than*** 125 people in your group (ie, a majority) that actually prefer Pepsi to Coke. Use the ***Normal Approximation to the Binomial*** to solve this. Don't forget to check your assumptions!!

$$\boldsymbol{n = 250 \qquad p = 0.46 = the\ probability\ of\ a\ person\ preferring\ Pepsi}$$
$$\boldsymbol{X \sim Bin(250, 0.46)}$$

$$\boldsymbol{For\ Binomial, we\ want\ P(X > 125)}$$

Normal Approximation

$$\boldsymbol{Y \sim N\left(np, \sqrt{np(1-p)}\right) = N(115, 7.88)}$$

$$\boldsymbol{For\ Normal, we\ want\ P(Y > 125.5), due\ to\ continuity\ correction}$$

$$\boldsymbol{P(Y > 125.5) = P\left(\frac{Y - \mu}{\sigma} > \frac{125.5 - 115}{7.88}\right) = P(Z > 1.332) = 1 - 0.9086 = 0.0914}$$