AMS 102.4 Midterm II **NAME:____________________**
11/16/2011 **ID Number:____________________**

SHOW ALL WORK! The more work you show, the more chances you have to earn partial credit.

(12 points) True or False. Answer **"TRUE"** or **"FALSE,"** not "T" or "F" and provide an explanation for your choice. Good explanations consist of properties, definitions or mathematics that backs up your claim.

a) The normal approximation to the binomial is $\boldsymbol{Y \sim N(np, \sqrt{\frac{p(1-p)}{n}})}$ where n = number of independent trials and p = the probability of success on each trail, in the binomial experiment.

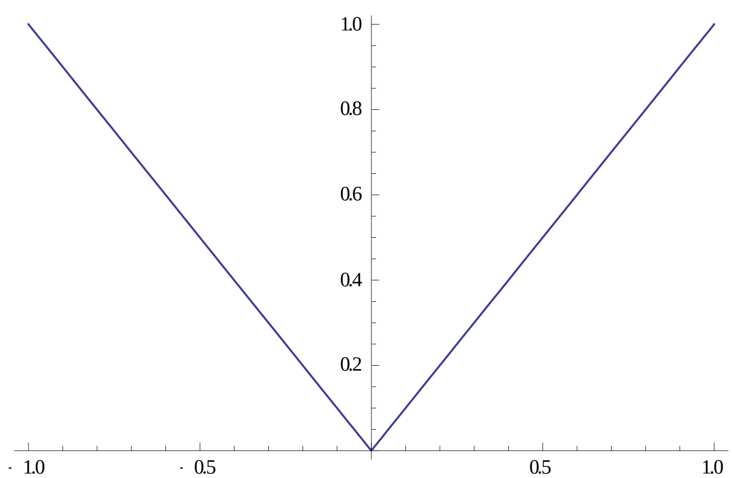
b) I have a sample space with only two possible events, A and B. Therefore, $P(B) = P(A^c)$.

c) If I want to get higher precision in my sample mean estimator, I should increase the sample size.

d) Let $f(x) = |x|, -1 \leq x \leq 1$, where $|x|$ = absolute value of x.

So, $f(x) = \begin{cases} -x & for -1 \leq x < 0 \\ x & for \quad 0 \leq x \leq 1 \end{cases}$, (see graph below).

This is a probability density function (pdf).



1) **(17 points)** Consider a large bag of coins, consisting of only **quarters** ($0.25), **dimes** ($0.10) and **nickels** ($0.05). 40% of the coins are nickels, 35% are dimes.

   a. **(7 points)** Randomly select **5** coins from the bag. What is the **probability** that your selection is worth **at least $1**. (Note that the only way to make $1 with 5 coins is Q,Q,Q,Q,x, ie 4 quarters and 1 coin of any type)

   b. **(10 points)** Find the **average value** and **standard deviation** of a coin in the bag. (Hint: **Use Expected Values**)

2) **(12 points)** Consider the following experiment, you **flip a fair coin and roll a fair die**. If the coin is **Heads**, you **add 1** to the value of the roll, if the coin is **Tails** you **subtract 1** from the value of the roll. Let X = the value determined by this experiment.

   a. **(7 points)** Provide a table of the possible values of X and the associated probabilities.

   b. **(5 points)** Are **X = 4** and **Heads** independent events, provide justification for your claim?

3) **(19 points)** Let Y be distributed as a Continuous **Uniform** RV between 0 and 2. So, $\boldsymbol{Y \sim U(0, 2)}$.

a. **(6 points)** Find $P(1.25 < Y \leq 1.75)$.

b. **(8 points)** Now you take a **sample** from this distribution of size **300**. Let $\boldsymbol{\bar{Y}}$ be the mean of this sample. Find $P(\boldsymbol{\bar{Y}} \geq 1.05)$. (Note that the $Var(Y) = \frac{1}{3}$).

c. **(5 points)** Justify the reasoning behind your answer in part b, i.e. what **assumptions** did you make **about** $\boldsymbol{\bar{Y}}$ and **why were they valid**?

EXTRA CREDIT

1) **(3 points)** The Geometric Distribution is also related to Bernoulli trials (Repeated trials of the same experiment that only has two possible outcomes, a "failure" or a "success")

   The probability mass function is defined as follows: (note that p is your probability of success)

$$P(X = k) = (1 - p)^{k-1}p \quad k = 1,2,3,\ldots$$

   Interpret what a particular value k represents.

2) **(3 points)** Consider a situation where I have 5 shirts I can wear during the 5-day work week, 2 Red, 2 Blue and 1 White. I want to know the number of different ways I can order the colors I wear through the week, assuming that the Reds are indistinguishable from each other and the Blues are indistinguishable from each other.

So, 2 ways I could wear them are R,R,B,B,W or R,R,B,W,B. How many overall different ways can I order them?

Hint: This is related to factorials. First, assume each shirt is unique, what would be the number of ways you can order them in this situation? That's an upper bound for the particular situation asked in the question. Now you need to figure out how to decrease that so that you only represent the value for the case specified above. A good place to start may be an easier example where you have 4 of 1 color and 1 of the other, write down all the different ways and then figure out how to write a formula for that, then extend it.