AMS 102.7 Homework #5

DUE: 3/22/2012

1) Consider a large bag of coins, consisting of only quarters ($0.25), dimes ($0.10) and nickels ($0.05). 40% of the coins are nickels, 35% are dimes.

   a. Randomly select 5 coins from the bag with replacement (ie, the probabilities don't change after a coin is selected). What is the probability that your selection is worth at least $1. (Hints: Use Binomial Distribution, think about the different ways you can make a dollar and how you can translate that into a success/failure problem)

   b. Find the average value and standard deviation of a coin in the bag. (Hint: The coins in the bag form a PMF).

a) Two ways to solve this. First we note that 1 - .40 - .35 = .25 so there are 25% quarters.

<u>Method 1 (Binomial Method)</u>

Observe that we make at least $1 only if we select 4 or 5 quarters, thus we can formulate

$\boldsymbol{Y \sim Binomial\left(5, \frac{1}{4}\right)} \qquad \boldsymbol{n = 5\ trials,\ p = \frac{1}{4}}$ **(Probability of selecting a quarter)**

$$\boldsymbol{Now, P(X \geq 4) = P(X = 4) + P(X = 5) = \binom{5}{4}.25^4.75^1 + \binom{5}{5}.25^5.75^0 = \frac{1}{64} \approx 0.016}$$

<u>Method 2 (Probabilities)</u>

Observe that we make $1 only if we select 4 quarters and any other type of coin.

$$\boldsymbol{P(Q,Q,Q,Q,D) = \frac{1}{4} * \frac{1}{4} * \frac{1}{4} * \frac{1}{4} * \frac{7}{20} \approx 0.00137}$$
$$\boldsymbol{P(Q,Q,Q,Q,N) = \frac{1}{4} * \frac{1}{4} * \frac{1}{4} * \frac{1}{4} * \frac{2}{5} \approx 0.00156}$$
$$\boldsymbol{P(Q,Q,Q,Q,Q) = \frac{1}{4} * \frac{1}{4} * \frac{1}{4} * \frac{1}{4} * \frac{1}{4} \approx 0.00098}$$

$$\boldsymbol{Now, we\ note\ that\ there\ are\ 5\ ways\ to\ observe\ Q,Q,Q,Q,D\ or\ Q,Q,Q,Q,N,}$$
$$\boldsymbol{ie, Q,Q,Q,Q,D\ or\ Q,Q,Q,D,Q, etc \ldots.}$$

$$\boldsymbol{So, P(More\ than\ \$1) = 5 * (.00137) + 5 * (.00156) + .00098 \approx 0.016}$$

b) Average value of coin in the bag is

$$\boldsymbol{E[X] = (25¢ * .25) + (10¢ * .35) + (5¢ * .40) = 11.75¢}$$

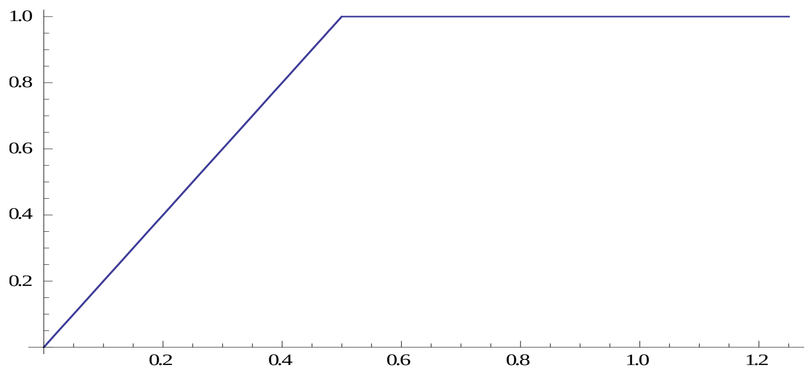
Variance of coin in the bag is

$$\boldsymbol{E[X]^2 - E[X] = (25^2 * .25) + (10^2 * .35) + (5^2 * .40) - 11.75^2 = 63.1875}$$

So Standard Deviation of coin in the bag is

$$\boldsymbol{\sqrt{E[X]^2 - E[X]} = \sqrt{63.1875} \approx 7.949¢}$$

2) Consider the following function: $f(x) = \begin{cases} 2x & 0 \le x \le \frac{1}{2} \\ 1 & \frac{1}{2} \le x \le \frac{5}{4} \\ 0 & otherwise \end{cases}$



a. Verify this function satisfies the properties of a PDF
   i) Always positive
   ii) Area under curve =1

i) This is always positive as the entire graph lies above the x-axis
ii) Area under the curve is the sum of the triangle and rectangle the graph can be split into

Triangle: **Area =** $\left(\frac{1}{2}\right) \boldsymbol{bh} = \frac{1}{2} * \left(\frac{1}{2} - \mathbf{0}\right) * (\mathbf{1}) = \frac{1}{2} * \frac{1}{2} * \mathbf{1} = \frac{1}{4}$

Rectangle: **Area =** $\boldsymbol{bh} = \left(\frac{5}{4} - \frac{1}{2}\right) * \mathbf{1} = \frac{3}{4}$

Total Area: **Area =** $\frac{3}{4} + \frac{1}{4} = \mathbf{1}$

Therefore this function is a PDF

b. What is the probability of a RV described by the function above taking on a value between 0 and 1?

$$\boldsymbol{P(0 \le X \le 1) = \ Area\ between\ 0\ and\ 1 = \frac{1}{4} + \left(\frac{1}{2}\right) * 1 = 0.75}$$

0 to 1 encompasses the whole triangles and a rectangle with a base of ½ .

c. What is the median of this distribution? (Hint: Think about what the median represents in terms of a percentile!)

The median = 50th percentile
To find a percentile we set the Area = 0.5 and solve for a particular value.
The area of the triangle is 0.25 so we need the area of the rectangle to equal 0.25 as well. Thus we want the base of the rectangle to be 0.25, so

$$\boldsymbol{X - 0.5 = 0.25 \rightarrow X = 0.75}$$

So the median of the distribution is 0.75