“NEWTON’S SECOND LAW PROBLEMS: MULTIPLE OBJECTS”

full solutions

Step-by-step discussions for each of these solutions are available in the videos. You can find links to these resources at my website: www.freelance-teacher.com

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Solutions begin on next page.

Video (1)

Here is a summary of some of the main steps in the solution.
We used the Addition Method to solve the system of equations.

$$W_A = m_A g = 4.5(9.8) = 44.1\text{ N}$$

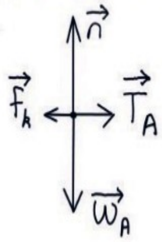
$$f_k = \mu_k n = 0.3n$$

$$W_B = m_B g = 2.5(9.8) = 24.5\text{ N}$$

$$a_{Ax} = +a$$

$$a_{By} = +a$$

Free-body diagram showing all the forces exerted on block A



Free-body diagram showing all the forces exerted on block B

Force Table for block A ($\rightarrow x$, $\downarrow y$)

$W_A = 44.1\text{N}$	n	$f_k = 0.3n$	$T_A = T$
$W_{Ax} = 0$	$n_x = 0$	$f_{kx} = -0.3n$	$T_{Ax} = +T$
$W_{Ay} = +44.1\text{N}$	$n_y = -n$	$f_{ky} = 0$	$T_{Ay} = 0$

Force Table for block B ($\rightarrow x$, $\downarrow y$)

$W_B = 24.5\text{N}$	$T_B = T$	← magnitudes of the overall force vectors
$W_{Bx} = 0$	$T_{Bx} = 0$	} components of the forces
$W_{By} = +24.5\text{N}$	$T_{By} = -T$	

$$\Sigma F_{Ax} = m_A a_{Ax}$$
$$0 + 0 + (-.3n) + T = 4.5a$$
$$-.3n + T = 4.5a$$
$$-.3(44.1) + T = 4.5a$$
$$-13.23 + T = 4.5a$$

$$\Sigma F_{Ay} = m_A a_{Ay}$$
$$44.1 + (-n) + 0 + 0 = 4.5(0)$$
$$44.1 - n = 0$$
$$+n \qquad +n$$
$$44.1\text{ N} = n$$

$$\Sigma F_{By} = m_B a_{By}$$
$$24.5 + (-T) = 2.5a$$
$$\left.\begin{aligned} 24.5 - T &= 2.5a \\ -13.23 + T &= 4.5a \end{aligned}\right\}\text{add}$$
$$11.27 = 7a$$
$$\frac{11.27}{7} = \frac{\cancel{7}a}{\cancel{7}}$$
$$a = 1.61\ \frac{\text{m}}{\text{s}^2}$$

$$-13.23 + T = 4.5a$$
$$-13.23 + T = 4.5(1.61)$$
$$-13.23 + T = 7.245$$
$$+13.23 \qquad +13.23$$
$$T = 20.475\text{ N}$$

Block B

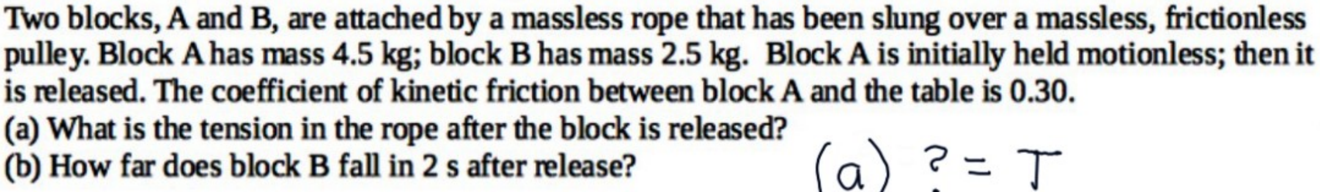
Δt	Δy	v_{iy}	v_{fy}	a_y
2s	Δy?	0	v_{iy}	$+1.61\ \frac{\text{m}}{\text{s}^2}$

} knowns

$$\Delta y = v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
$$\Delta y = 0(2) + \tfrac{1}{2}(1.61)(2)^2$$
$$\Delta y = \tfrac{1}{2}(1.61)4$$
$$\Delta y = +3.22\text{ m}$$

A fully explained solution begins on the next page.

Here is a fully explained solution to the problem. **Part (a):**

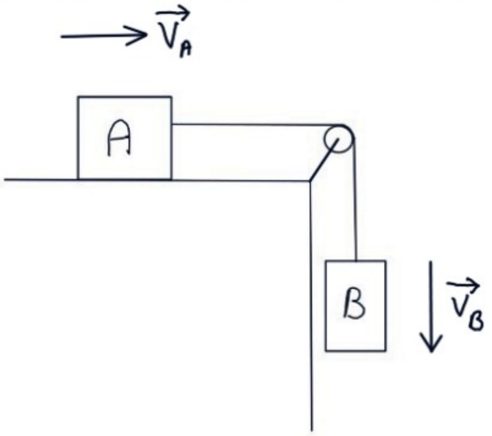


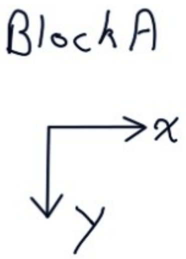
Two blocks, A and B, are attached by a massless rope that has been slung over a massless, frictionless pulley. Block A has mass 4.5 kg; block B has mass 2.5 kg. Block A is initially held motionless; then it is released. The coefficient of kinetic friction between block A and the table is 0.30.

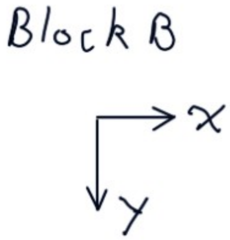
(a) What is the tension in the rope after the block is released?

(b) How far does block B fall in 2 s after release?

(a) ? = T







The problem refers to the concepts of mass and friction force, and part (a) refers to tension force, all of which fit into a Newton's Second Law framework, so we plan to use the Newton's Second problem-solving framework to solve part (a).

The problem asks for "the tension in the rope". This wording is probably intended to refer to the *magnitude* of the tension force exerted by the rope. We identify what part (a) is asking us for by writing: ? = T

The symbol T, written without an arrow on top, stands for the magnitude of the tension force. Remember that the definition of a "magnitude" is: a number that can be positive or zero, but that can never be negative.

Block A is initially held motionless, then released; the problem tells us that, when it is released, it begins sliding. Common sense tells us that, after block A is released, block B will begin falling downward, which will drag block A to the right.

The direction of an object's velocity vector indicates the object's direction of motion. Therefore, we have drawn velocity vectors pointing to the right and down in the sketch above, to indicate block A's and block B's directions of motion after they are released.

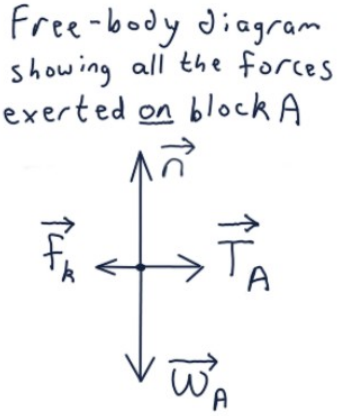
Choose positive axes pointing in the objects' directions of motion. Block A is moving right, so we choose a positive x-axis pointing to the right. Block B is moving down, so we choose a positive y-axis pointing down. *It would be permissible to choose different axes for block B than for block A*—but for this particular problem there is no need to do that, so we have chosen the same axes for block B as for block A. *Write down* your axes for each block, as shown above.

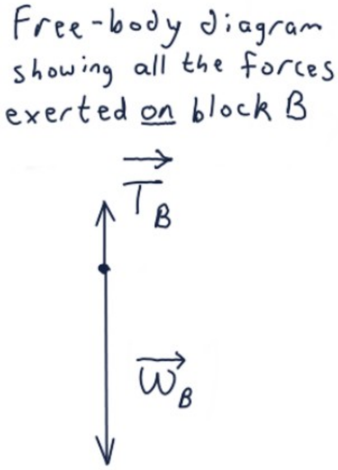
(Some professors or textbooks might prefer to choose "up" as the positive y-direction for this problem. But for a beginning student it is probably best to choose "down" as positive, since that is block B's direction of motion.)

Draw two separate Free-body Diagrams, one diagram showing all the forces being exerted on block A, and a *separate* diagram showing all the forces being exerted on block B.

Use subscripts to distinguish the forces being exerted on block A from the forces being exerted on block B:

$\vec{w}_A$ vs. $\vec{w}_B$, $\vec{T}_A$ vs. $\vec{T}_B$





General two-step process for identifying the forces for your Free-body Diagram for a particular object:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.
(This method works for most *first-semester* problems.)

In this case, block A is being touched by the surface of the table, which exerts both a normal force and a frictional force; and by the rope, which exerts a "tension force". We know that *kinetic* friction applies for this problem because block A is *sliding*.

Block B is being touched only by the rope, which exerts a "tension force".

The rule for determining the direction of the weight force is: The weight force always points down.

The rule for determining the direction of the normal force is: The normal force points *perpendicular* to, and away from, the surface that is touching the object. (In math, "normal" means "perpendicular".)

In this problem, the surface touching block A is the table. So the normal force points perpendicular to, and away from, the surface of the table. Therefore, the normal force on this problem points "up".

The rule for determining the direction of kinetic friction is: Kinetic friction points parallel to the surface, and opposite to the direction that the object is sliding. **Friction opposes sliding.**

Block A is sliding to the right, so for this problem the kinetic friction points to the left.

The rule for determining the direction of the tension force is: The tension force points parallel to the rope, and away from the object.

This rule is based on the common sense idea that a rope can only "pull" an object, not "push" it. The rope exerts a rightward pulling force on block A, and an upward pulling force on block B.

Do not draw the forces on your "main sketch" (drawn on the previous page). Instead, draw the forces in two free-body diagrams that are separate from the main sketch, as shown above.

Velocity is not a force, so do *not* include the velocity vectors in your Free-body diagrams. Instead, include the velocity vectors in your "main sketch", as illustrated on the previous page.

NEWTON'S SECOND LAW PROBLEMS: MULTIPLE OBJECTS Solution for Video (1)

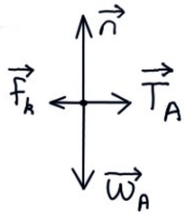
Complete a Force Table for block A, and a Force Table for block B. The purpose of the Force Tables is to organize the data that we will need to plug into our Newton's Second Law equations.

$$\begin{aligned} w_A &= m_A g \\ &= 4.5(9.8) \\ &= 44.1\text{ N} \end{aligned}$$

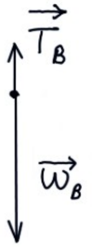
$$\begin{aligned} f_k &= \mu_k n \\ &= 0.3n \end{aligned}$$

$$\begin{aligned} w_B &= m_B g \\ &= 2.5(9.8) \\ &= 24.5\text{ N} \end{aligned}$$

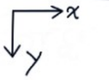
Free-body diagram showing all the forces exerted on block A



Free-body diagram showing all the forces exerted on block B

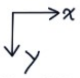


Force Table for block A



$w_A = 44.1\text{N}$	n	$f_k = 0.3n$	$T_A = T$
$w_{Ax} = 0$	$n_x = 0$	$f_{kx} = -0.3n$	$T_{Ax} = +T$
$w_{Ay} = +44.1\text{N}$	$n_y = -n$	$f_{ky} = 0$	$T_{Ay} = 0$

Force Table for block B



$w_B = 24.5\text{N}$	$T_B = T$	← magnitudes of the overall force vectors
$w_{Bx} = 0$	$T_{Bx} = 0$	} components of the forces
$w_{By} = +24.5\text{N}$	$T_{By} = -T$	

A vector symbol with an arrow on top (e.g., $\vec{n}$) stands for the complete vector, including both magnitude and direction. A vector symbol written without an arrow on top (e.g., n), stands for just the *magnitude* of the overall vector.

In our Free-body diagrams, we write the vector symbols with arrows on top, because the purpose of the Free-body diagram is to represent the *directions* of the overall vectors. In the first row of our Force Tables, we write the vector symbols without arrows on top, because the first row of the Force Table indicates the *magnitudes* of the overall vectors.

For a massless rope stretched over a massless, frictionless pulley, **the magnitude of the tension force is the same at both ends of the rope**, although the direction of the tension force may be different at the two ends of the rope. In our Force Tables above, the symbols T_A and T_B, written without arrows on top, stand for the *magnitudes* of the tension forces on block A and on block B. Because these magnitudes are equal, we can write $T_A = T$, and $T_B = T$, using the same symbol, T, to represent both magnitudes, as shown in the tables above.

It is crucial to include the negative signs on n_y and T_{By}, because $\vec{n}$ and $\vec{T}_B$ point up, which we have chosen to represent the *negative* y-direction. **It is crucial to include the negative sign on f_{kx}**, because $\vec{f}_k$ points left, which we have chosen to represent the *negative* x-direction.

Include a "+" sign in front of all positive components. This will help you to remember to include a "-" sign in front of negative components.

The signs of the components depend on the axes you choose. If we had chosen different axes (for example, if we had decided to choose "up" as our positive y-direction), then we would have obtained different signs for the components.

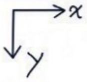
Next, write the Newton's Second Law equations, as shown below. Block A experiences forces in both the x- and y-components, so we write Newton's Second Law equations for block A for both the x- and y-components. Block B experiences no forces in the x-component, so for block B we write the Newton's Second Law equation for the y-component only.

On the left side of each equation, add the individual force components, which we determined in our Force Tables, as shown below.

Now we need to determine what to plug in for a_{Ax}, a_{Ay}, and a_{By} in our Newton's Second Law equations.

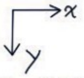
Because block A is vertically motionless, $a_{Ay} = 0$. Substitute this value for a_{Ay} into the Newton's Second Law y-equation for block A, as shown below.

Force Table for block A



$W_A = 44.1N$	n	$f_k = 0.3n$	$T_A = T$
$W_{Ax} = 0$	$n_x = 0$	$f_{kx} = -0.3n$	$T_{Ax} = +T$
$W_{Ay} = +44.1N$	$n_y = -n$	$f_{ky} = 0$	$T_{Ay} = 0$

Force Table for block B



$W_B = 24.5N$	$T_B = T$	← magnitudes of the overall force vectors
$W_{Bx} = 0$	$T_{Bx} = 0$	} components of the forces
$W_{By} = +24.5N$	$T_{By} = -T$	

$$\sum_i F_{AX} = m_A a_{AX}$$

$$0 + 0 + (-.3n) + T = 4.5$$

$$\sum_i F_{Ay} = m_A a_{Ay}$$

$$44.1 + (-n) + 0 + 0 = 4.5(0)$$

$$\sum_i F_{By} = m_B a_{By}$$

$$24.5 + (-T) = 2.5$$

Notice that, in the equations above, we haven't substituted yet for a_{Ax} or for a_{By}. On the next page, we will determine what to substitute for a_{Ax} and for a_{By}.

Now we need to determine what to substitute for a_{Ax} and a_{By}.

Since a_{Ax} and a_{By} are components, we should begin by determining whether they are positive or negative. To figure that out, we need to determine the directions of $\vec{a}_A$ and $\vec{a}_B$.

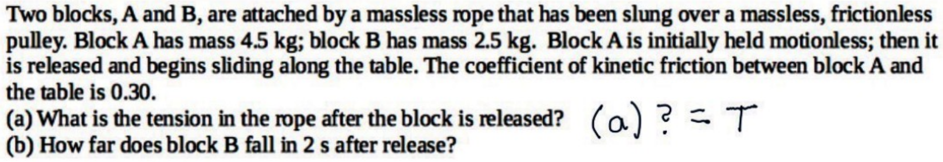
In general, the direction of the acceleration vector does *not* necessarily point in the object's direction of movement. But, if an object starts from rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving.

Block A begins at rest and then starts moving to the right. In order to *begin* moving to the right, block A must have an acceleration vector that points to the right, which is our positive x-direction. Therefore, a_{Ax} is positive.

$a_{Ax} = +$

Block B begins at rest and then starts moving down. In order to *begin* moving down, block B must have an acceleration vector that points down, which is our positive y-direction. Therefore, a_{By} is positive.

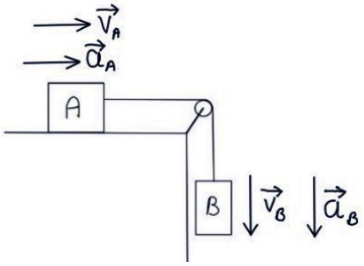
$a_{By} = +$



Two blocks, A and B, are attached by a massless rope that has been slung over a massless, frictionless pulley. Block A has mass 4.5 kg; block B has mass 2.5 kg. Block A is initially held motionless; then it is released and begins sliding along the table. The coefficient of kinetic friction between block A and the table is 0.30.

(a) What is the tension in the rope after the block is released? (a) ? = T

(b) How far does block B fall in 2 s after release?



***The magnitude of the acceleration will be the same* for two objects connected by an unstretchable rope,**[1] although the directions of the accelerations may be different for the two objects. For an introductory course, we assume that the rope is unstretchable, unless the problem indicates otherwise.

Because block A and block B are connected by the rope, they will have the same magnitude of acceleration. Let's represent the magnitude of the acceleration for block A and for block B with the symbol a. Then $a_{Ax} = +a$, and $a_{By} = +a$. **Substitute these values into the Newton's Second Law equations, as shown below.**

$a_{Ax} = +a$

$a_{By} = +a$

$$\begin{array}{c|c|c} \sum_1 F_{Ax} = m_A a_{Ax} & \sum_1 F_{Ay} = m_A a_{Ay} & \sum_1 F_{By} = m_B a_{By} \\ 0 + 0 + (-.3n) + T = 4.5\,a & 44.1 + (-n) + 0 + 0 = 4.5(0) & 24.5 + (-T) = 2.5\,a \end{array}$$

When we write positive components by themselves, we include the "+" sign, to emphasize that they are positive. When we substitute positive components into an equation, we leave out the plus sign, to avoid cluttering the equation.

(Now we can see why it was helpful to choose object B's direction of motion, "down", as our positive y-direction. If we had chosen "up" as our positive y-direction, then we would get that $a_{By} = -a$. By choosing "down" as our positive direction for object B, we avoid having to deal with a negative acceleration component for this problem.)

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[1] There are some exceptions to this rule, but the rule will apply to all the problems we will see in this video series.

The Newton's Second Law y-equation for object A has only one unknown, so we begin by solving that equation for *n*.

Force Table for block A $\downarrow y \rightarrow x$

$w_A = 44.1\text{N}$	n	$f_k = 0.3n$	$T_A = T$
$w_{Ax} = 0$	$n_x = 0$	$f_{kx} = -0.3n$	$T_{Ax} = +T$
$w_{Ay} = +44.1\text{N}$	$n_y = -n$	$f_{ky} = 0$	$T_{Ay} = 0$

Force Table for block B $\downarrow y \rightarrow x$

$w_B = 24.5\text{N}$	$T_B = T$	← magnitudes of the overall force vectors
$w_{Bx} = 0$	$T_{Bx} = 0$	components of the forces
$w_{By} = +24.5\text{N}$	$T_{By} = -T$	

$$\sum F_{Ax} = m_A a_{Ax}$$

$$0 + 0 + (-.3n) + T = 4.5a$$

$$-.3n + T = 4.5a$$

3 unknowns

$$\sum F_{Ay} = m_A a_{Ay}$$

$$44.1 + (-n) + 0 + 0 = 4.5(0)$$

$$44.1 - n = 0$$

$$+n \qquad +n$$

$$44.1\text{ N} = n$$

$$\sum F_{By} = m_B a_{By}$$

$$24.5 + (-T) = 2.5a$$

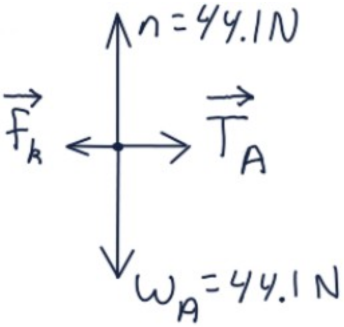
$$24.5 - T = 2.5a$$

2 unknowns

Check: Does the sign of our result for *n* make sense? The symbol *n* stands for the *magnitude* of the normal force, and a magnitude can never be negative, so, yes, it makes sense that our result for *n* came out positive. (The symbol $\vec{n}$ stands for the "complete" normal force vector including both magnitude and direction. The symbol *n,* written without an arrow on top, stands just for the *magnitude* of the normal force.)

Check: Does the size of our result for *n* make sense? To prevent block A from beginning to move down into the surface of the table, $\vec{n}$ must cancel $\vec{w}_A$. So, yes, it makes sense that:

$n = 44.1\text{ N} = w_A$



Therefore, in the Free-body diagram on the right, I have drawn the length of the $\vec{n}$ arrow equal to the length of the $\vec{w}_A$ arrow.

Do not say, "The weight of A equals the normal force on A". The two forces point in different directions, so the forces are *not* equal. Instead, say "The *magnitude* of the weight force on object A equals the *magnitude* of the normal force on object A."

Don't assume that the magnitude of the normal force will always equal the magnitude of the weight. On this problem, the magnitude of the normal force *does* equal the magnitude of the weight. But you will see other problems in which the magnitude of the normal force does *not* equal the magnitude of the weight. For each problem, use the Newton's Second Law equations to figure out the correct value for the magnitude of the normal force.

Next, we substitute our value for *n* into the Newton's Second Law x-equation for object A.

$\sum_i F_{Ax} = m_A a_{Ax}$	$\sum_i F_{Ay} = m_A a_{Ay}$	$\sum_i F_{By} = m_B a_{By}$
$0 + 0 + (-.3n) + T = 4.5a$	$44.1 + (-n) + 0 + 0 = 4.5(0)$	$24.5 + (-T) = 2.5a$
$-.3n + T = 4.5a$	$44.1 - n = 0$	$24.5 - T = 2.5a$
$-.3(44.1) + T = 4.5a$	$+n \qquad +n$	
	$44.1\,N = n$	

2 equations
2 unknowns

The Newton's Second Law equation for object A now has two unknowns, and the Newton's Second Law equation for object B still has two unknowns. Taken together, those two equations form a system of two equations in two unknowns (*a* and *T*), which we can solve simultaneously by using the "Addition method", as illustrated on the next page.

We solve the system of equations using the Addition Method, as illustrated below.

First, recopy the Newton's Second Law x-equation for block A underneath the Newton's Second Law y-equation for block B.

Add the two equations. Notice that, when we add the equations, the *T* terms cancel out, leaving an equation that involves only *a*. This is the reason that the Addition Method works well for these particular equations.

Solve the new equation for *a*.

$\sum F_{Ax} = m_A a_{Ax}$	$\sum F_{Ay} = m_A a_{Ay}$	$\sum F_{By} = m_B a_{By}$
$0 + 0 + (-.3n) + T = 4.5a$	$44.1 + (-n) + 0 + 0 = 4.5(0)$	$24.5 + (-T) = 2.5a$
$-.3n + T = 4.5a$	$44.1 - n = 0$	$24.5 - T = 2.5a$ } add these
$-.3(44.1) + T = 4.5a$	$+n \qquad +n$	$-13.23 + T = 4.5a$ } two equations
$-13.23 + T = 4.5a$	$44.1\text{ N} = n$	$11.27 = 7a$
		$\frac{11.27}{7} = \frac{7a}{7}$
		$a = 1.61 \frac{m}{s^2}$

Now take your result for *a*, and substitute it into the Newton's Second Law x-equation for block A. Then solve that equation for *T*.

$\sum F_{Ax} = m_A a_{Ax}$	$\sum F_{Ay} = m_A a_{Ay}$	$\sum F_{By} = m_B a_{By}$
$0 + 0 + (-.3n) + T = 4.5a$	$44.1 + (-n) + 0 + 0 = 4.5(0)$	$24.5 + (-T) = 2.5a$
$-.3n + T = 4.5a$	$44.1 - n = 0$	$24.5 - T = 2.5a$ } add these
$-.3(44.1) + T = 4.5a$	$+n \qquad +n$	$-13.23 + T = 4.5a$ } two equations
$-13.23 + T = 4.5a$	$44.1\text{ N} = n$	$11.27 = 7a$
$-13.23 + T = 4.5a$		$\frac{11.27}{7} = \frac{7a}{7}$
$-13.23 + T = 4.5(1.61)$		$a = 1.61 \frac{m}{s^2}$
$-13.23 + T = 7.245$		
$+13.23 \qquad +13.23$		
$T = 20.475\text{ N}$		

Notice how we continue to organize our math in three adjacent columns. You should imitate this "adjacent column approach" in your own scratchwork.

The Addition Method works well for these equations because the *T* variables cancel out when we add the two equations. If the two equations were not already in a form such that the *T* variables would cancel when the equations are added, then the Addition Method would work less well and it might be preferable to use the Substitution Method to solve the equations.

For completeness, the Substitution Method for solving these two equations is illustrated on the next page.

On this particular problem, the simplest method for solving the equations is the Addition Method. But for completeness, the Substitution Method for solving the equations is illustrated below.

The "Substitution method" for solving a system of two equations in two unknowns simultaneously:

1. Begin by solving one of the equations for one of the unknowns. Solve for the variable that is the easiest to solve for in the two equations.
2. Substitute the algebraic expression obtained in step 1 into the other equation.
3. Solve the equation obtained in step 2 for the second unknown.
4. If you care about the remaining unknown, then substitute the value or expression obtained in step 3 into the equation obtained in step 1 to determine the remaining unknown.

$$\sum_{1}^{\rightarrow} F_{Ax} = m_A a_{Ax}$$

$$0 + 0 + (-.3n) + T = 4.5a$$
$$-.3n + T = 4.5a$$
$$-.3(44.1) + T = 4.5a$$
$$-13.23 + T = 4.5a$$
$$\underline{+13.23 \qquad +13.23}$$
$$T = 4.5a + 13.23$$
$$T = 4.5(1.61) + 13.23$$
$$T = 20.475\text{ N}$$

$$\sum_{1}^{\uparrow} F_{Ay} = m_A a_{Ay}$$

$$44.1 + (-n) + 0 + 0 = 4.5(0)$$
$$44.1 - n = 0$$
$$\underline{+n \qquad +n}$$
$$44.1\text{ N} = n$$

$$\sum_{1}^{\uparrow} F_{By} = m_B a_{By}$$

$$24.5 + (-T) = 2.5a$$
$$24.5 - T = 2.5a$$
$$24.5 - (4.5a + 13.23) = 2.5a$$
$$24.5 + (-4.5a) + (-13.23) = 2.5a$$
$$24.5 + (-13.23) + (-4.5a) = 2.5a$$
$$11.27 - 4.5a = 2.5a$$
$$\underline{+4.5a \qquad +4.5a}$$
$$11.27 = 7a$$
$$\frac{11.27}{7} = \frac{\cancel{7}a}{\cancel{7}}$$
$$a = 1.61\ \frac{m}{s^2}$$

For the benefit of students who find this algebra to be challenging, I have illustrated every little step in the algebra above. If you thought the algebra for this problem was easy, naturally it would be OK to skip or combine some of the steps illustrated above.

On this problem, the Addition Method (illustrated on the previous page) is simpler than the Substitution Method (illustrated on this page). There are many other problems, however, in which in which the Substitution Method might be preferable to the Addition Method, or in which the Addition Method might not work at all.

Force Table for block A $\begin{array}{l}\to x\\ \downarrow y\end{array}$

$w_A = 44.1\text{N}$	n	$f_k = 0.3n$	$T_A = T$
$w_{Ax} = 0$	$n_x = 0$	$f_{kx} = -0.3n$	$T_{Ax} = +T$
$w_{Ay} = +44.1\text{N}$	$n_y = -n$	$f_{ky} = 0$	$T_{Ay} = 0$

Force Table for block B $\begin{array}{l}\to x\\ \downarrow y\end{array}$

$w_B = 24.5\text{N}$	$T_B = T$
$w_{Bx} = 0$	$T_{Bx} = 0$
$w_{By} = +24.5\text{N}$	$T_{By} = -T$

$T_B = T$ ← magnitudes of the overall force vectors

T_{Bx}, T_{By}: components of the forces

$$\sum F_{Ax} = m_A a_{Ax}$$

$$0 + 0 + (-.3n) + T = 4.5a$$
$$-.3n + T = 4.5a$$
$$-.3(44.1) + T = 4.5a$$
$$-13.23 + T = 4.5a$$

$$\sum F_{Ay} = m_A a_{Ay}$$

$$44.1 + (-n) + 0 + 0 = 4.5(0)$$
$$44.1 - n = 0$$
$$+n \qquad +n$$
$$44.1\text{ N} = n$$

$$\sum F_{By} = m_B a_{By}$$

$$24.5 + (-T) = 2.5a$$
$$24.5 - T = 2.5a$$
$$-13.23 + T = 4.5a$$

add these two equations

$$11.27 = 7a$$
$$\frac{11.27}{7} = \frac{\cancel{7}a}{\cancel{7}}$$
$$a = 1.61\,\frac{\text{m}}{\text{s}^2}$$

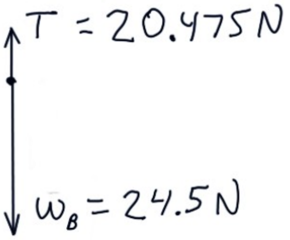
$$-13.23 + T = 4.5a$$
$$-13.23 + T = 4.5(1.61)$$
$$-13.23 + T = 7.245$$
$$+13.23 \qquad +13.23$$
$$T = 20.475\text{ N}$$

Check: Does the sign of *a* make sense? We are using the symbol *a* to stand for the *magnitude* of the acceleration. A magnitude can never be negative, so, yes, it makes sense that our result for *a* came out to be positive.

Check: Does the size of *a* make sense? Because block B is being held back by the rope, rather than falling freely, we would expect that block B will fall with an acceleration that is smaller in magnitude than free-fall acceleration. Our result for *a* (1.61 m/s^2) is indeed less than free-fall acceleration (9.8 m/s^2), so, yes, our result for the size of *a* does make sense.

Check: Does the sign of *T* make sense? We are using the symbol *T* to stand for the *magnitude* of the tension force. A magnitude can never be negative, so, yes, it makes sense that our result for *T* came out to be positive.

Check: Does the size of *T* make sense?

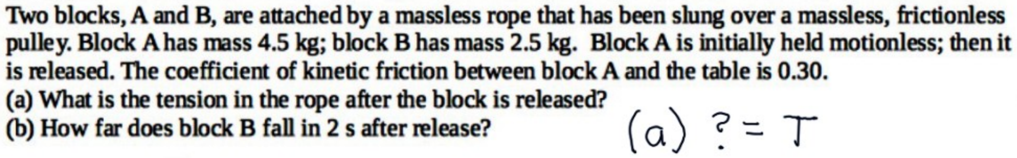


In general, the direction of the net force on an object does not necessarily indicate the object’s direction of motion. But if an object begins at rest, then the direction of the net force on the object *does* indicate the direction that the object will *begin* moving.

Block B starts at rest, and then begins falling downward. In order for block B to *begin* falling downward, $\vec{w}_B$ must be greater than the tension force in magnitude. So, yes, it does make sense that the magnitude of $\vec{w}_B$ (24.5 N) is greater than the magnitude of the tension force (20.475 N).

Therefore, in the Free-body diagram at the right, I have drawn the arrow for $\vec{w}_B$ longer than the arrow for the tension force.

Now we can answer part (a).



Two blocks, A and B, are attached by a massless rope that has been slung over a massless, frictionless pulley. Block A has mass 4.5 kg; block B has mass 2.5 kg. Block A is initially held motionless; then it is released. The coefficient of kinetic friction between block A and the table is 0.30.
(a) What is the tension in the rope after the block is released?
(b) How far does block B fall in 2 s after release?

(a) ? = T

Answer for (a):

After the block is released, the tension has magnitude 20N.

I have rounded the answer to two digits.

<u>Recap</u> for part (a):

For part (a), we learned how to deal with a problem that involves two objects connected by a rope. For such a problem, **draw two *separate* Free-body diagrams.**

It will simplify your solution if you **choose positive axes pointing in the objects' directions of motion.**

It is OK to choose different axes for different objects. Remember that, although we chose the same axes for both objects for this problem, you will see other problems in which it will be helpful to choose different axes for the different objects.

For a massless rope stretched over a massless, frictionless pulley, **the magnitude of the tension force is the same at both ends of the rope** (but the direction of the tension force may be different at the two ends of the rope). In our Force Tables, we used this rule to write $T_A = T$, and $T_B = T$, using the same symbol, T, to represent both magnitudes.

The magnitude of the acceleration will be the same **for two objects connected by an unstretchable rope** (but the directions of the accelerations may be different for the two objects)**.** We used this rule to write $a_{Ax} = +a$, and $a_{By} = +a$, using the same symbol, a, to represent both magnitudes. Then we substituted a in for both a_{Ax} and a_{By} in our Newton's Second Law equations.

$a_{Ax} = +a$

$a_{By} = +a$

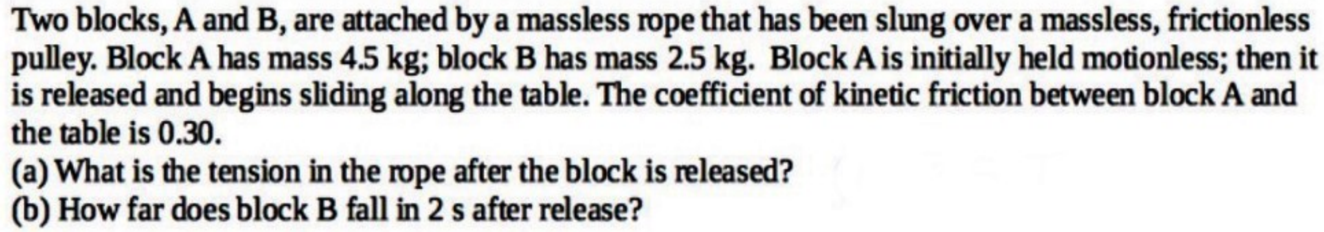
We can use the **Addition Method** to solve our system of simultaneous equations.

Always try to use the exact right symbol, including the exact right subscripts.

Notice how we used subscripts to distinguish between variables that referred to block A, and variables that referred to block B. (For example, a_{Ay} versus a_{By}).

Notice how we used subscripts to distinguish between variables that referred to the x-component, and variables that referred to the y-component. (For example, a_{Ax} versus, a_{Ay}).

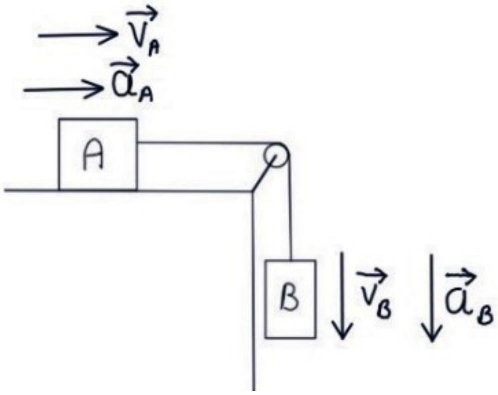
Part (b):

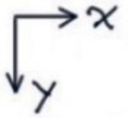


Two blocks, A and B, are attached by a massless rope that has been slung over a massless, frictionless pulley. Block A has mass 4.5 kg; block B has mass 2.5 kg. Block A is initially held motionless; then it is released and begins sliding along the table. The coefficient of kinetic friction between block A and the table is 0.30.

(a) What is the tension in the rope after the block is released?

(b) How far does block B fall in 2 s after release?





Part (b) introduces the concepts of time (2 s) and distance (“how far does block B fall”). These concepts do not fit into the Newton’s Second Law equations, but they do fit into the kinematics equations. This indicates that, for part (b) of the problem, we need to shift from the Newton’s Second Law problem-solving framework to a Kinematics problem-solving framework.

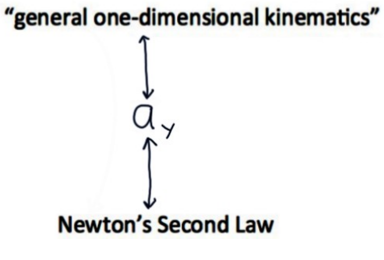
Part (b) is asking about block B, so we will apply kinematics to block B.

We will use “general” kinematics, as opposed to “projectile motion” kinematics.

(“Projectile motion” applies when the only force on the object is the force of the Earth’s gravity; i.e., “projectile motion” applies when the only force on the object is the force of the object’s weight. Projectile motion does not apply to this problem because there are other forces on block B besides the weight force.)

We will use “one-dimensional” kinematics, because block B is moving in a straight line.

The connecting link between Newton’s Second Law and kinematics is the concept of acceleration. Block B is moving in the *y*-component, so the connecting link for this problem will be a_y. We already determined Block B’s acceleration in our work on part (a); now, we can take this result for acceleration and substitute it for a_y in our kinematics framework.



There are two types of kinematics in an introductory course: (1) "constant velocity", and (2) "constant acceleration with changing velocity".

In this problem, the blocks begin at rest, and then start moving. This means that blocks' velocity changes. So the velocity is changing, not constant.

Is the acceleration constant? The acceleration is determined by the net force. The forces we have identified in our force table are all constant. Since the forces are all constant, the net force on the object is constant. According to Newton's Second Law, the net force determines the acceleration, so when the net force is constant, we know that the acceleration is constant. In fact, in part (a) we have already determined that the acceleration has a constant magnitude of 1.61 m/s^2.

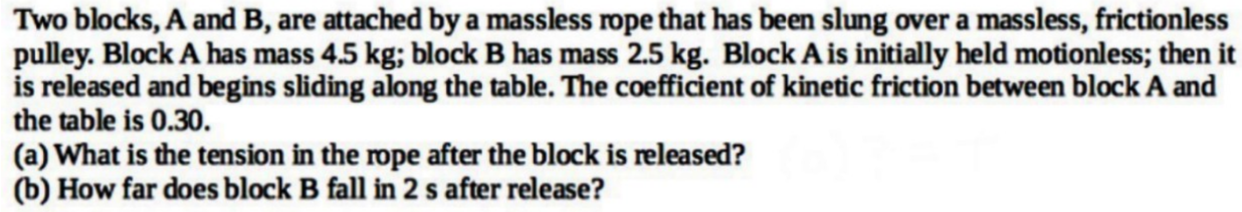
So for this problem we apply "constant acceleration with changing velocity" kinematics. Block B is moving only in the y-component, so we apply kinematics only to the y-component.

We draw block B's path of motion in our sketch, as shown below.

Part (b) tells us that a time of 2 seconds is elapsing. We indicate the key points in time in our sketch: $t_0 = 0$, when the blocks are released, and $t_1 = 2$ s; we choose these as the "initial" (*i*) and "final" (*f*) points that we will substitute into our kinematics equations.

The question is asking us for how far block B will fall, which we can measure by block B's displacement, Δy. (We know Δy will be positive, because we have chosen "down" as our positive direction. This is another example of why it was a good idea to choose "down" as the positive direction.)

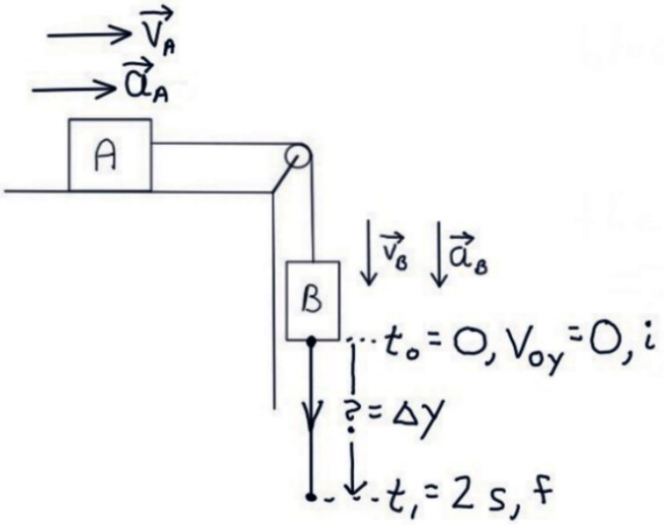
Because the problem tells us that block B begins from rest, we know that $v_{oy} = 0$. Add this information to your sketch, as shown below.

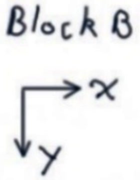


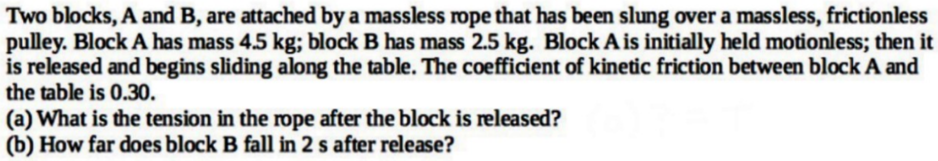
Two blocks, A and B, are attached by a massless rope that has been slung over a massless, frictionless pulley. Block A has mass 4.5 kg; block B has mass 2.5 kg. Block A is initially held motionless; then it is released and begins sliding along the table. The coefficient of kinetic friction between block A and the table is 0.30.

(a) What is the tension in the rope after the block is released?

(b) How far does block B fall in 2 s after release?



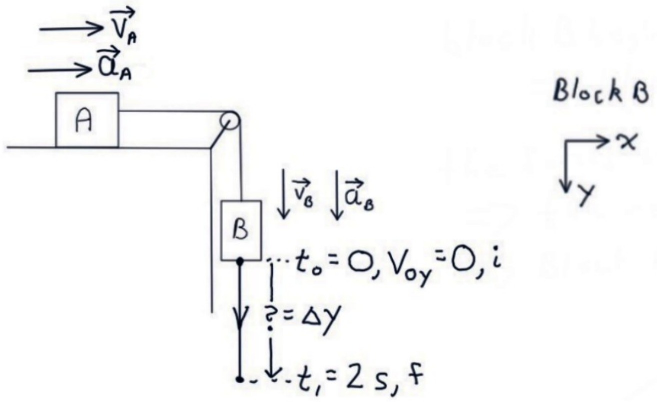




Two blocks, A and B, are attached by a massless rope that has been slung over a massless, frictionless pulley. Block A has mass 4.5 kg; block B has mass 2.5 kg. Block A is initially held motionless; then it is released and begins sliding along the table. The coefficient of kinetic friction between block A and the table is 0.30.

(a) What is the tension in the rope after the block is released?

(b) How far does block B fall in 2 s after release?



Write down your kinematics "setup" by listing the five kinematics variables:

$\Delta t,\ \Delta y,\ v_{iy},\ v_{fy},\ a_y$

We have determined that object B is accelerating downward, which is the positive y-direction we chose for Block B, so a_y is positive for Block B. In part (a), we found that the magnitude of the acceleration for both block A and block B is 1.61 m/s^2. So, for block B, $a_y = +1.61$ m/s^2.

(Again, we see here that is was useful to choose "down" as the positive y-direction for Block B. If we had chosen "up" as positive, then a_y for block B would be negative.)

Remember that $v_{iy} = 0$ because the blocks begin at rest.

Part (b) tells us that a time of 2 seconds is elapsing, so Δt = 2 s.

$\sum F_{Ax} = m_A a_{Ax}$

$0 + 0 + (-.3n) + T = 4.5a$

$-.3n + T = 4.5a$

$-.3(44.1) + T = 4.5a$

$\boxed{-13.23 + T = 4.5a}$

$\sum F_{Ay} = m_A a_{Ay}$

$44.1 + (-n) + 0 + 0 = 4.5(0)$

$44.1 - n = 0$

$+n \qquad +n$

$\boxed{44.1\text{ N} = n}$

$\sum F_{By} = m_B a_{By}$

$24.5 + (-T) = 2.5a$

$24.5 - T = 2.5a$

$-13.23 + T = 4.5a$

$11.27 = 7a$

$\frac{11.27}{7} = \frac{\cancel{7}a}{\cancel{7}}$

$a = 1.61\ \frac{m}{s^2}$

$-13.23 + T = 4.5a$

$-13.23 + T = 4.5(1.61)$

$-13.23 + T = 7.245$

$+13.23 \qquad +13.23$

$T = 20.475$ N

Block B

?

$\Delta t,\ \Delta y,\ V_{iy},\ V_{fy},\ a_y$

$2\text{ s},\ \Delta y,\ 0,\ V_{fy},\ +1.61\ \frac{m}{s^2}$

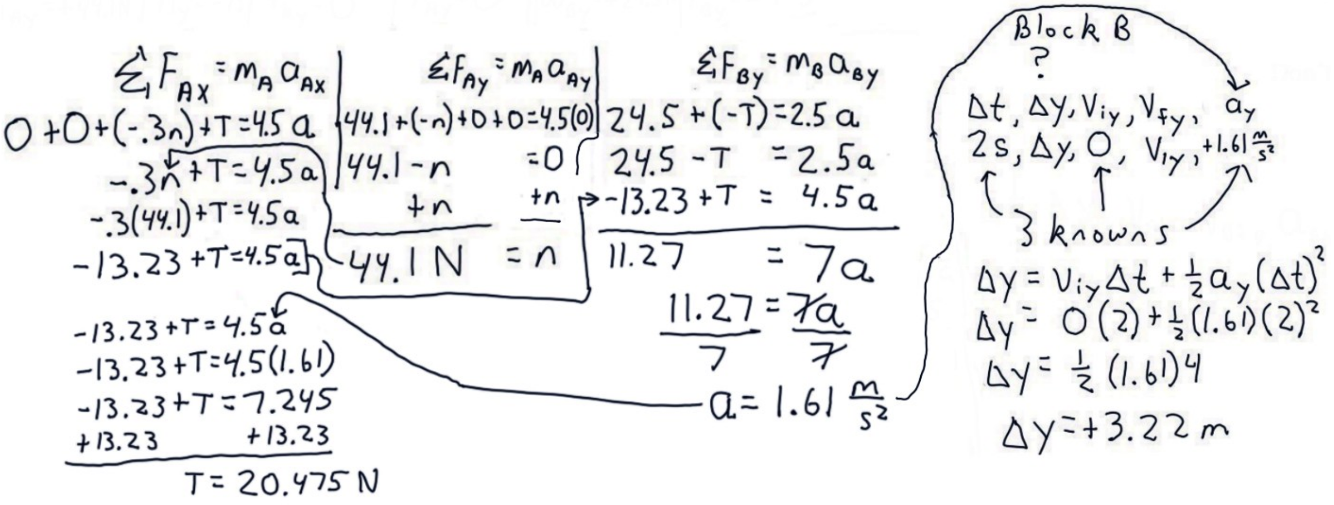
3 knowns

When we know values for three kinematics variables, we are ready to choose a kinematics equation. We do know three kinematics values (Δ*t*, v_{iy}, and a_y), so we are ready to choose a kinematics equation.

Kinematics Equations for constant a_y with changing v_y

***y* equations**	**missing variables**
$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$	v_{fy}
$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y$	Δt
$v_{fy} = v_{iy} + a_y\Delta t$	Δy

On this problem, the kinematics variable that we do *not* care about is v_{fy}, so we choose the kinematics equation that is missing v_{fy}: $\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$



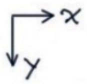
$\sum_1 F_{AX} = m_A a_{AX}$
$0 + 0 + (-.3n) + T = 4.5a$
$-.3n + T = 4.5a$
$-.3(44.1) + T = 4.5a$
$-13.23 + T = 4.5a$

$-13.23 + T = 4.5a$
$-13.23 + T = 4.5(1.61)$
$-13.23 + T = 7.245$
$+13.23 \qquad +13.23$
$T = 20.475\text{ N}$

$\sum_1 F_{Ay} = m_A a_{Ay}$
$44.1 + (-n) + 0 + 0 = 4.5(0)$
$44.1 - n = 0$
$+n \qquad +n$
$44.1\text{ N} = n$

$\sum_1 F_{By} = m_B a_{By}$
$24.5 + (-T) = 2.5a$
$24.5 - T = 2.5a$
$-13.23 + T = 4.5a$
$11.27 = 7a$
$\frac{11.27}{7} = \frac{\cancel{7}a}{\cancel{7}}$
$a = 1.61\ \frac{m}{s^2}$

Block B
?
$\Delta t,\ \Delta y,\ V_{iy},\ V_{fy},\ a_y$
$2s,\ \Delta y,\ 0,\ V_{iy},\ +1.61\frac{m}{s^2}$
3 knowns
$\Delta y = V_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$
$\Delta y = 0(2) + \frac{1}{2}(1.61)(2)^2$
$\Delta y = \frac{1}{2}(1.61)4$
$\Delta y = +3.22\text{ m}$

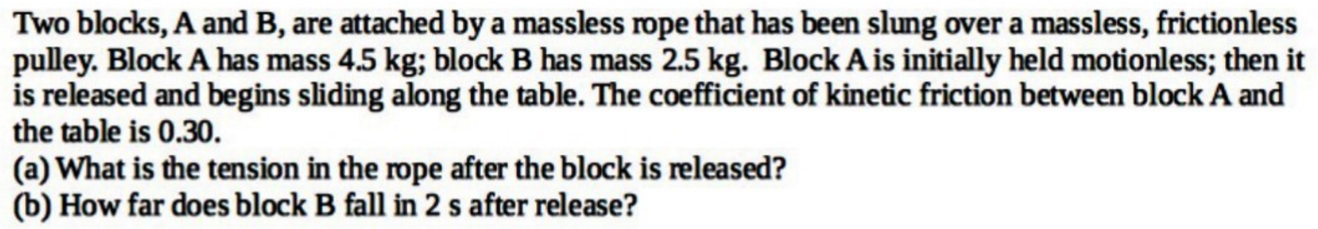


Check: Does the sign of Δ*y* make sense? Δ*y* came out to be positive. We expected that the object will be displaced downward, and downward is our positive direction, so, yes, it makes sense that our result for Δ*y* is positive.

Check: Does the magnitude of Δ*y* make sense? 1 meter is roughly 1 yard, and 1 yard equals 3 feet, so 3.22 m is roughly 3 yards, which is 9 feet. 9 feet would actually be an unrealistic height for a table, but on this problem we haven't made any mistakes. The problem simply was written with an unrealistically tall table. (We could imagine that the table is positioned next to a ledge, so that block B has room to fall 9 feet even if the table is not 9 feet tall.)

It is good to check whether your results seem realistic, but you should also keep in mind that professors don't always take the time to make sure that a problem will generate a realistic answer!

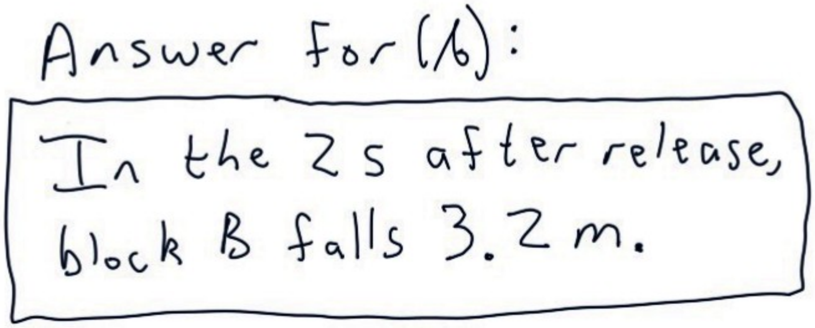
Now we can answer the question for part (b).



Two blocks, A and B, are attached by a massless rope that has been slung over a massless, frictionless pulley. Block A has mass 4.5 kg; block B has mass 2.5 kg. Block A is initially held motionless; then it is released and begins sliding along the table. The coefficient of kinetic friction between block A and the table is 0.30.

(a) What is the tension in the rope after the block is released?

(b) How far does block B fall in 2 s after release?



Answer for (b):

In the 2 s after release, block B falls 3.2 m.

Problem Recap on next page.

Recap:

For this problem, we learned how to deal with a problem that involves two objects connected by a rope. For such a problem, **draw two *separate* Free-body diagrams**.

It will simplify your solution if you **choose positive axes pointing in the objects' directions of motion.** It is OK to choose different axes for different objects. Remember that, although we chose the same axes for both objects for this problem, you will see other problems in which it will be helpful to choose different axes for the different objects.

For a massless rope stretched over a massless, frictionless pulley, **the magnitude of the tension force is the same at both ends of the** rope, although the direction of the tension force may be different at the two ends of the rope. In our Force Tables, we used this rule to write $T_A = T$, and $T_B = T$, using the same symbol, T, to represent both magnitudes.

***The magnitude of the acceleration will be the same* for two objects connected by an unstretchable rope**, although the directions of the accelerations may be different for the two objects**.** We used this rule to write $a_{Ax} = +a$, and $a_{By} = +a$, using the same symbol, a, to represent both magnitudes. Then we substituted $+a$ in for both a_{Ax} and a_{By} in our Newton's Second Law equations.

$a_{Ax} = +a$

$a_{By} = +a$

We obtained a system of two equations in two unknowns. For the particular equations that we obtained, the most convenient way to solve the system of equations is the **Addition Method**.

Always try to use the exact right symbol, including the exact right subscripts. Study the symbols we used in our solution for this problem, and make sure you understand why each symbol and subscript was appropriate.

Notice how we used subscripts to distinguish between variables that referred to block A, and variables that referred to block B. (For example, a_{Ay} versus a_{By}). Notice how we used subscripts to distinguish between variables that referred to the x-component, and variables that referred to the y-component. (For example, a_{Ax} versus, a_{Ay}). Notice how we used subscripts to distinguish between between the object's initial and final velocities (v_{iy} versus v_{fy}). (For this problem, only the initial velocity turned out to be important.)

Think in terms of components. Use careful subscripts to identify which component you are focusing on in each part of your solution.

For this problem, notice that each Newton's Second Law equation refers specifically either to the x-component or to the y-component. And notice that we applied kinematics specifically to the y-component for object B. Notice that, in this problem, a_{Ax} (which was zero) is quite different from a_{Ay} (which we represented as $+a$).

For kinematics problems, organize the kinematics data with a list of the five kinematics variables, as shown at right.

Use acceleration as the connecting link between Newton's Second Law and kinematics.

Block B

?

Δt	Δy	V_{iy}	V_{fy}	a_y
$2s$	Δy	0	V_{iy}	$+1.61\frac{m}{s^2}$

Video (2)

Here is a summary of some of the main steps in the solution.

Notice that we use different axes for mass 1 and for mass 2. (The axes are drawn in the Force Tables.)

$W_1 = m_1 g$
$= 30(9.8)$
$= 294\text{ N}$

$W_2 = m_2 g$
$= 50(9.8)$
$= 490\text{ N}$

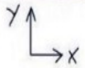
$a_{1y} = +a$

$a_{2y} = +a$

Free-body diagram showing all the forces exerted on mass 1

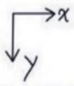
Free-body diagram showing all the forces exerted on mass 2

Force Table for mass 1



$W_1 = 294\text{ N}$	$T_1 = T$
$W_{1x} = 0$	$T_{1x} = 0$
$W_{1y} = -294\text{N}$	$T_{1y} = +T$

Force Table for mass 2



$W_2 = 490\text{ N}$	$T_2 = T$
$W_{2x} = 0$	$T_{2x} = 0$
$W_{2y} = +490\text{N}$	$T_{2y} = -T$

← magnitudes of the overall force vectors

} components of the forces

$\Sigma F_{1y} = m_1 a_{1y}$

$-294 + T = 30\,a$

$\Sigma F_{2y} = m_2 a_{2y}$

$490 + (-T) = 50\,a$

$$\left.\begin{aligned} 490 - T &= 50a \\ -294 + T &= 30a \end{aligned}\right\}\text{add}$$

$196 = 80a$

$\frac{196}{80} = \frac{\cancel{80}a}{\cancel{80}}$

$a = 2.45\ \frac{m}{s^2}$

mass 1

need

Δt,	Δy,	v_{iy},	v_{fy},	a_y
1.5 s,	Δy,	0,	v_{iy},	$+2.45\frac{m}{s^2}$

3 knowns

$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$

$\Delta y = 0(1.5) + \frac{1}{2}(2.45)(1.5)^2$

$\Delta y = \frac{1}{2}(2.45)(1.5)^2$

$\Delta y = +2.76\text{ m}$

mass 2

need

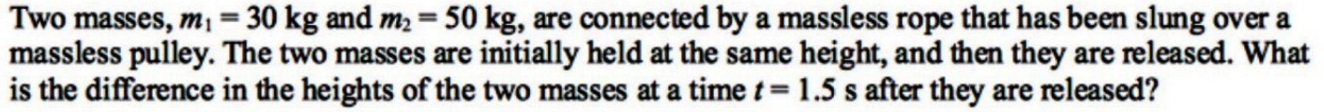
Δt,	Δy,	v_{iy},	v_{fy},	a_y
1.5 s,	Δy,	0,	v_{iy},	$+2.45\frac{m}{s^2}$

$\Delta y = +2.76\text{ m}$

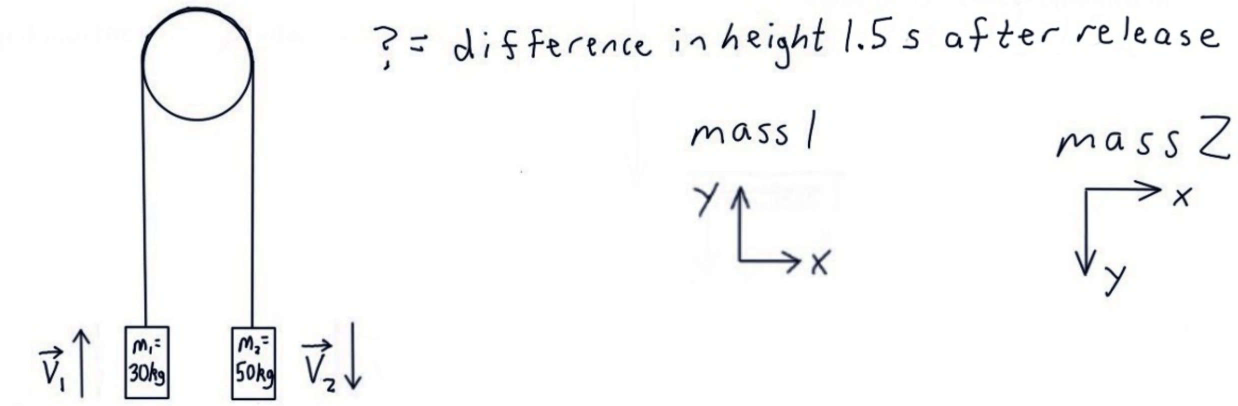
difference in height $= 2.76\text{ m} + 2.76\text{ m}$
$= 5.52\text{ m}$

A fully explained solution begins on the next page.

Here is a fully explained solution for the problem.



Two masses, m_1 = 30 kg and m_2 = 50 kg, are connected by a massless rope that has been slung over a massless pulley. The two masses are initially held at the same height, and then they are released. What is the difference in the heights of the two masses at a time t = 1.5 s after they are released?



?= difference in height 1.5 s after release

Make a note of what the question is asking for, as shown above.

Because object 2 is more massive than object 1, common sense tells us that, when they are released, object 2 will fall downwards, dragging object 1 upwards.

The direction of an object's velocity vector indicates the object's direction of motion. Therefore, we have drawn $\vec{v}_1$ pointing up and $\vec{v}_2$ pointing down in the sketch above, to indicate the objects' directions of motion after they are released.

Choose positive axes pointing in the objects' directions of motion. Mass 1 is moving up, so for mass 1 we choose a positive y-axis pointing up. But mass 2 is moving down, so **for mass 2 we choose a positive y-axis pointing down**. Notice that **it is OK to choose different axes for the different objects**. *Write down* your axes for each mass, as shown above.

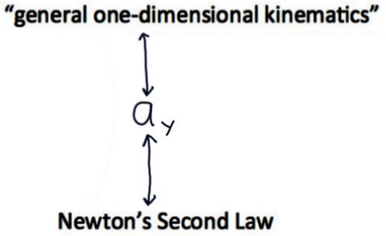
(Some professors or textbooks might prefer to choose "up" as the positive y-direction for both objects. But for a beginning student it is probably best to choose "down" as positive for mass 2, since that is mass 2's direction of motion.)

The problem refers to the concept of mass, which fits into a Newton's Second Law problem-solving framework. The problem also refers to the concepts of time and distance ("difference in height"), which fit in a kinematics framework. Therefore, we expect to use both the Newton's Second Law problem-solving framework, and the kinematics framework, for solving the problem.

We will use "general" kinematics, as opposed to "projectile motion" kinematics. "Projectile motion" applies when the only force on the object is the force of the Earth's gravity; i.e., "projectile motion" applies when the only force on the object is the force of the object's weight. Projectile motion does not apply to this problem because there are other forces on the masses besides the weight forces.

We will use "one-dimensional" kinematics, because each mass is moving in a straight line

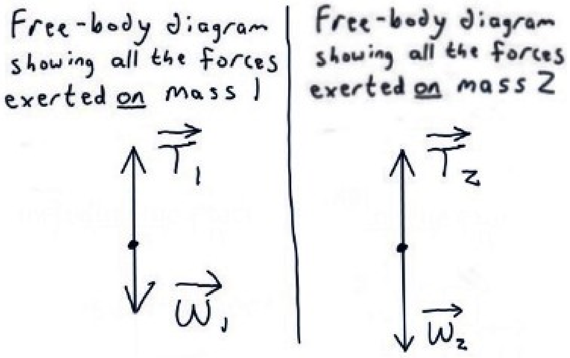
The connecting link between Newton's Second Law and kinematics is the concept of acceleration. The masses are moving in the y-component, so the connecting link for this problem will be a_y.



Draw two separate Free-body Diagrams, one diagram showing all the forces being exerted on mass 1, and a *separate* diagram showing all the forces being exerted on mass 2.

Use subscripts to distinguish the forces being exerted on mass 1 from the forces being exerted on mass 2.

$\vec{w}_1$ vs. $\vec{w}_2$, $\vec{T}_1$ vs. $\vec{T}_2$



General two-step process for identifying the forces for your Free-body Diagram for a particular object:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

Mass 1 is being touched only by the rope, which exerts a "tension force".
Mass 2 is also being touched only by the rope, which again exerts a tension force.

The rule for determining the direction of the weight force is: The weight force always points down.

The rule for determining the direction of the tension force is: The tension force points parallel to the rope, and away from the object

This rule is based on the common sense idea that a rope can only "pull" an object, not "push" it. The rope exerts an upward pulling force on mass 1, and an upward pulling force on mass 2.

Notice that neither object is in contact with a "surface", so neither object experiences a "normal force". Don't assume that every problem will involve a normal force!

Complete a Force Table for block A, and a Force Table for block B.

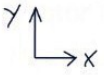
$$w_1 = m_1 g$$
$$= 30(9.8)$$
$$= 294\text{ N}$$

$$w_2 = m_2 g$$
$$= 50(9.8)$$
$$= 490\text{ N}$$

Free-body diagram showing all the forces exerted on mass 1

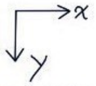
Free-body diagram showing all the forces exerted on mass 2

Force Table for mass 1



$w_1 = 294\text{ N}$	$T_1 = T$
$w_{1x} = 0$	$T_{1x} = 0$
$w_{1y} = -294\text{N}$	$T_{1y} = +T$

Force Table for mass 2



$w_2 = 490\text{ N}$	$T_2 = T$	← magnitudes of the overall force vectors
$w_{2x} = 0$	$T_{2x} = 0$	} components of the forces
$w_{2y} = +490\text{N}$	$T_{2y} = -T$	

For a massless rope stretched over a massless, frictionless pulley, **the magnitude of the tension force is the same at both ends of the rope**, although the direction of the tension force may be different at the two ends of the rope. The problem does not explicitly state that the pulley is massless, but for an introductory course, we assume that the pulley is frictionless unless the problem indicates otherwise.[1]

In our Force Tables above, the symbols T_1 and T_2, written without arrows on top, stand for the *magnitudes* of the tension forces on mass 1 and on mass 2. Because these magnitudes are equal, we can write $T_1 = T$, and $T_2 = T$, using the same symbol, T, to represent both magnitudes, as shown in the tables above.

We use this rule to determine the components: If a vector is parallel or anti-parallel to one of the axes, then the component for that axis has the same magnitude and direction as the overall vector; and the component for the *other* axis is zero.

Notice that w_{1y} is negative, while T_{1y} is positive, because “up” is our positive direction for mass 1. **In contrast, w_{2y} is positive, while T_{2y} is negative, because “down” is our positive direction for mass 2.**

Include a “+” sign in front of all positive components. This will help you to remember to include a “-” sign in front of negative components.

Remember that *the signs of the components depend on the axes you choose*. If we had chosen different axes (for example, if we had decided to choose “up” as our positive y-direction for *both* objects), then we would have obtained different signs for the components.

---

[1] The assumption that the pulley is frictionless also justifies our earlier conclusion that the masses will definitely begin moving after they are released.

Now we need to determine what to plug in for a_{1y} and a_{2y} in our Newton's Second Law equations.

Since a_{1y} and a_{2y} are components, we should begin by determining whether they are positive or negative. To figure that out, we need to determine the directions of $\vec{a}_1$ and $\vec{a}_2$.

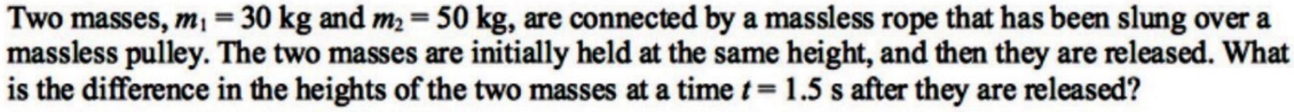
In general, the direction of the acceleration vector does *not* necessarily indicate the object's direction of movement. But, if an object starts from rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving.

Mass 1 begins at rest and then starts moving up. In order to *begin* moving up, mass 1 must have an acceleration vector that points up, which is the positive y-direction for mass 1. Therefore, a_{1y} is positive.

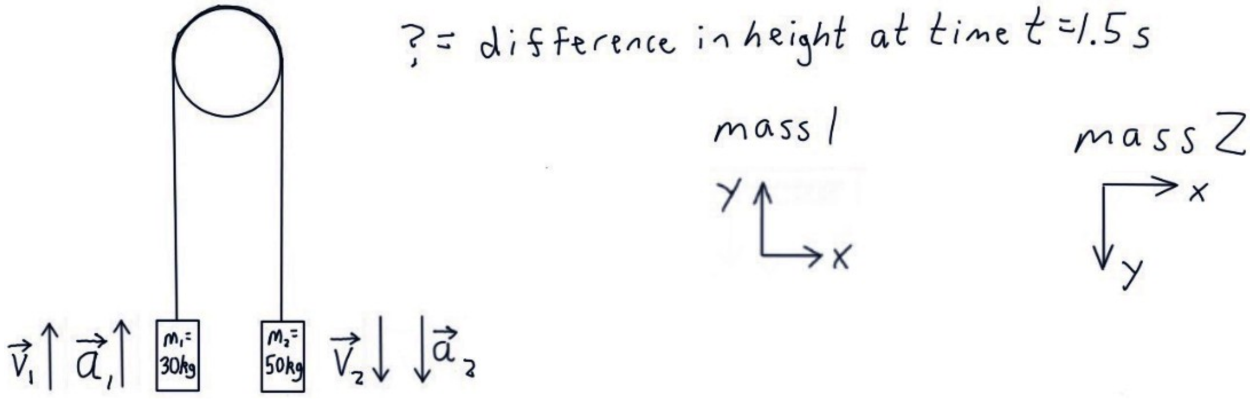
$a_{1y} = +$

Mass 2 begins at rest and then starts moving down. In order to *begin* moving down, mass 2 must have an acceleration vector that points down, which is the positive y-direction for mass 2. Therefore, a_{2y} is positive.

$a_{2y} = +$



Two masses, m_1 = 30 kg and m_2 = 50 kg, are connected by a massless rope that has been slung over a massless pulley. The two masses are initially held at the same height, and then they are released. What is the difference in the heights of the two masses at a time t = 1.5 s after they are released?



The magnitude of the acceleration will be the same **for two objects connected by an unstretchable rope**[1], although the directions of the accelerations may be different for the two objects.

Because mass 1 and mass 2 are connected by the rope, they will have the same magnitude of acceleration. Let's represent the magnitude of the acceleration for mass 1 and mass 2 with the symbol a. Then $a_{1y} = +a$, and $a_{2y} = +a$. Substitute these values into the Newton's Second Law equations, as shown on the next page.

$a_{1y} = +a$

$a_{2y} = +a$

Now we can see why it was helpful to choose mass 2's direction of motion, "down", as the positive y-direction for mass 2. If we had chosen "up" as our positive y-direction for mass 2, then we would get that $a_{2y} = -a$. By choosing "down" as our positive direction for mass 2, we avoid having to deal with a negative acceleration component for this problem.

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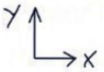
[1] There are some exceptions to this rule, but the rule will apply to all the problems we will see in this video series.

Next, write the Newton's Second Law equations, as shown below. Neither block experiences any forces in the x-component, so we write the Newton's Second Law equations only for the y-components.

On the left side of each equation, add the individual force components, which we determined in our Force Tables, as shown below.

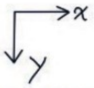
As discussed on the previous page, we substitute $a_{1y} = +a$, and $a_{2y} = +a$, as shown below.
When we write positive components by themselves, we include the "+" sign, to emphasize that they are positive. When we substitute positive components into an equation, we leave out the plus sign, to avoid cluttering the equation.

Force Table for mass 1



$W_1 = 294\text{ N}$	$T_1 = T$
$W_{1x} = 0$	$T_{1x} = 0$
$W_{1y} = -294\text{N}$	$T_{1y} = +T$

Force Table for mass 2



$W_2 = 490\text{ N}$	$T_2 = T$	← magnitudes of the overall force vectors
$W_{2x} = 0$	$T_{2x} = 0$	components of the forces
$W_{2y} = +490\text{N}$	$T_{2y} = -T$	

$$\Sigma F_{1y} = m_1 a_{1y}$$

$$-294 + T = 30\,a$$

$$\Sigma F_{2y} = m_2 a_{2y}$$

$$490 + (-T) = 50\,a$$

$$490 - T = 50\,a$$

$$\sum F_{1y} = m_1 a_{1y} \qquad \sum F_{2y} = m_2 a_{2y}$$

$$-294 + T = 30\,a \qquad 490 + (-T) = 50\,a$$

$$\underbrace{-294 + T = 30\,a \qquad 490 - T = 50a}_{\text{2 equations, 2 unknowns}}$$

The Newton’s Second Law equations form a system of two equations in two unknowns (*a* and *T*). We can solve this system of simultaneous equations using the Addition method, as illustrated below.

$$\sum F_{1y} = m_1 a_{1y} \qquad \sum F_{2y} = m_2 a_{2y}$$

$$-294 + T = 30\,a \qquad 490 + (-T) = 50\,a$$

$$\left.\begin{aligned} 490 - T &= 50a \\ -294 + T &= 30a \end{aligned}\right\}\text{add}$$

$$196 = 80a$$

$$\frac{196}{80} = \frac{\cancel{80}a}{\cancel{80}}$$

$$a = 2.45\ \frac{m}{s^2}$$

The Addition Method works well for these equations because the *T* variables cancel out when we add the two equations. If the two equations were not already in a form such that the *T* variables would cancel when the equations are added, then the Addition Method would work less well and it might be preferable to use the Substitution Method to solve the equations.

For completeness, the Substitution Method for solving these two equations is illustrated on the next page.

On this particular problem, the simplest method for solving the equations is the Addition Method. But for completeness, the Substitution Method for solving the equations is illustrated below.

The “Substitution method” for solving a system of two equations in two unknowns simultaneously:

1. Begin by solving one of the equations for one of the unknowns. Solve for the variable that is the easiest to solve for in the two equations.
2. Substitute the algebraic expression obtained in step 1 into the other equation.
3. Solve the equation obtained in step 2 for the second unknown.
4. If you care about the remaining unknown, then substitute the value or expression obtained in step 3 into the equation obtained in step 1 to determine the remaining unknown. In this problem, we do not care about the value of *T*, so we don’t bother to carry out step 4 of the method.

$$\sum F_{1y} = m_1 a_{1y}$$
$$-294 + T = 30a$$
$$\underline{+294 \qquad +294}$$
$$T = 30a + 294$$

$$\sum F_{2y} = m_2 a_{2y}$$
$$490 + (-T) = 50a$$
$$490 - T = 50a$$
$$490 - (30a + 294) = 50a$$
$$490 + (-30a) + (-294) = 50a$$
$$490 + (-294) + (-30a) = 50a$$
$$196 - 30a = 50a$$
$$\underline{+30a \quad +30a}$$
$$196 = 80a$$
$$\frac{196}{80} = \frac{\cancel{80}a}{\cancel{80}}$$
$$a = 2.45\ \frac{m}{s^2}$$

For the benefit of students who find this algebra to be challenging, I have illustrated every little step in the algebra above. If you thought the algebra for this problem was easy, naturally it would be OK to skip or combine some of the steps illustrated above.

On this problem, the Addition Method (illustrated on the previous page) is simpler than the Substitution Method (illustrated on this page). There are many other problems, however, in which in which the Substitution Method might be preferable to the Addition Method, or in which the Addition Method might not work at all.

$$a = 2.45 \frac{m}{s^2}$$

Check: Does the sign of *a* make sense? We are using the symbol *a* to stand for the *magnitude* of the acceleration. A magnitude can never be negative, so, yes, it makes sense that our result for *a* came out to be positive.

Check: Does the size of *a* make sense? Because mass 2 is being held back by the rope, rather than falling freely, we would expect that mass 2 will fall with an acceleration that is smaller in magnitude than free-fall acceleration. Our result for *a* (2.45 m/s^2) is indeed less than free-fall acceleration (9.8 m/s^2), so, yes, our result for the size of *a* does make sense.

We haven’t yet answered the question, which is asking for the difference in height after 1.5 s. To answer this question, we must now shift to a general one-dimensional kinematics framework.

There are two types of kinematics in an introductory course: (1) “constant velocity”, and (2) “constant acceleration with changing velocity”.

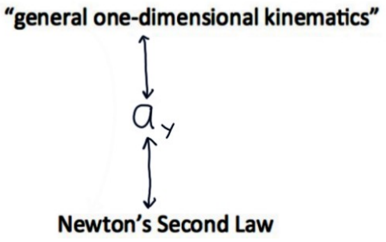
In this problem, the masses begin at rest, and then start moving. This means that masses’ velocity changes. So the velocity is changing, not constant.

Is the acceleration constant? The acceleration is determined by the net force. The forces we have identified in our force table are all constant. Since the forces are all constant, the net force on the object is constant. According to Newton’s Second Law, the net force determines the acceleration, so when the net force is constant, we know that the acceleration is constant. In fact, in our solution so far we have already determined that the acceleration has a constant magnitude of 2.45 m/s^2.

So for this problem we apply “constant acceleration with changing velocity” kinematics. The masses are moving only in the y-component, so we apply kinematics only to the y-component.

The connecting link between Newton’s Second Law and constant-acceleration kinematics is the concept of acceleration. The masses are moving in the *y*-component, so the connecting link for this problem will be a_y.

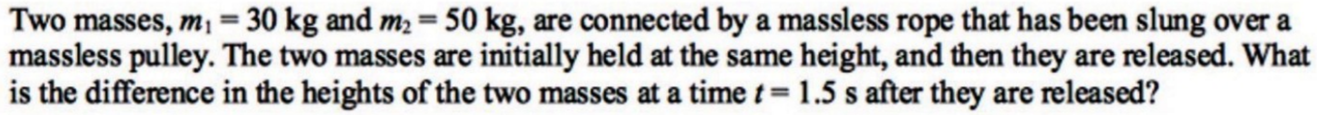
We already determined the block’s acceleration in our solution so far, so we can take this result for acceleration and use it to get a value for a_y in our kinematics framework.



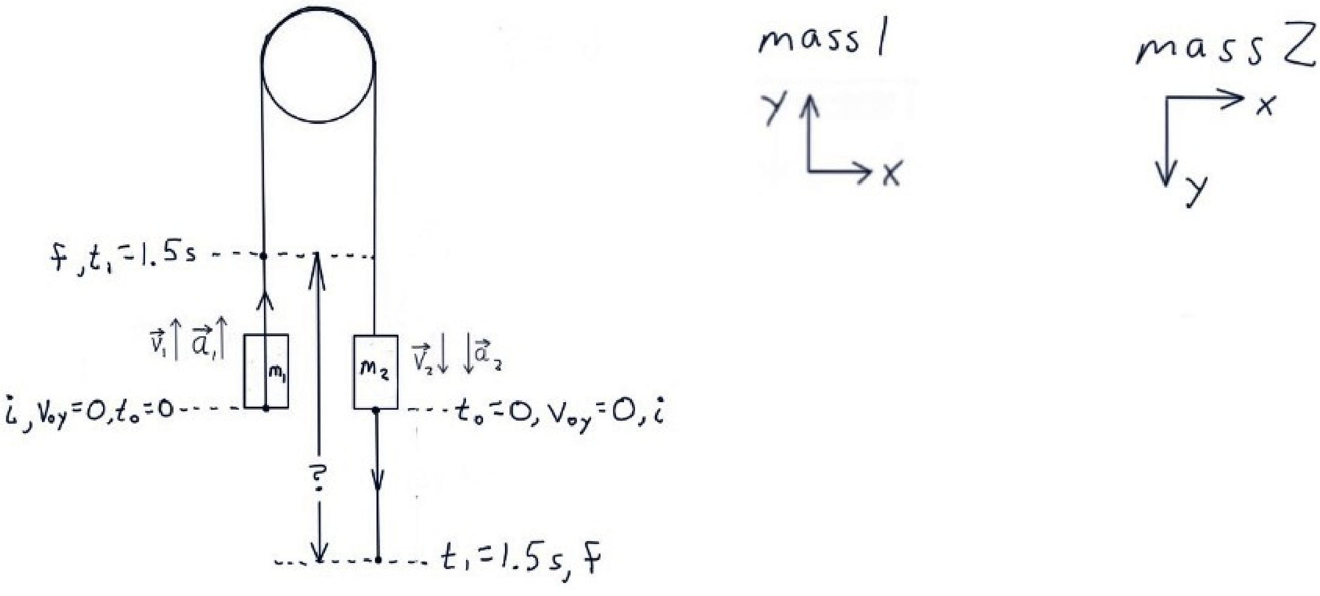
The problem asks us to compare the final heights of both masses, so we will apply kinematics to both of the masses.

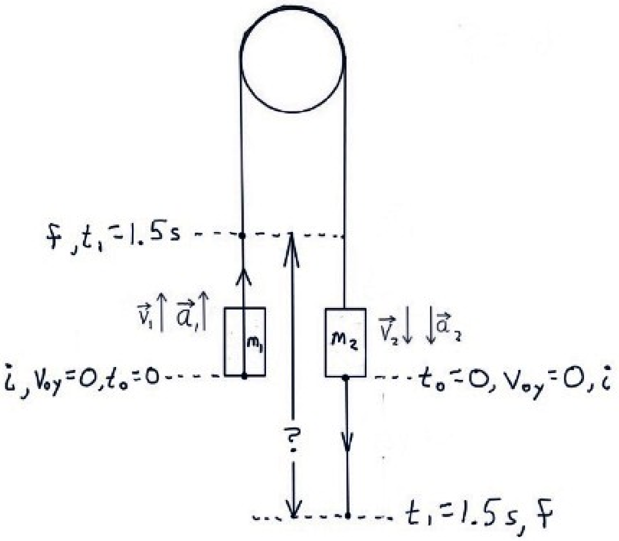
We draw the masses’ paths of motion in our sketch. We indicate the key points in time in our sketch: $t_0 = 0$, when the masses are released, and $t_1 = 1.5$ s; we choose these as the “initial” (*i*) and “final” (*f*) points that we will substitute into our kinematics equations. The question is asking us for the difference in height between the two blocks after 1.5 s; we can build this question in to the sketch, as shown below.

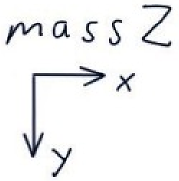
The problem says that the masses are “held” and then “released”. This wording implies that the masses begin moving *from rest*. **Because the problem implies that the masses begin from rest, we know that $v_{oy} = 0$ for both masses.** Add this information to your sketch, as shown below.



Two masses, $m_1 = 30$ kg and $m_2 = 50$ kg, are connected by a massless rope that has been slung over a massless pulley. The two masses are initially held at the same height, and then they are released. What is the difference in the heights of the two masses at a time $t = 1.5$ s after they are released?







Write down your kinematics “setups” for the two masses, as shown in the two rightmost columns below.[1]

In order to determine the final difference in heights, we need to find Δ*y* for both masses. **Indicate that Δ*y* is the kinematics variable you need to determine**, as shown in the setups below.

The problem implies the objects begin at rest, so v_{iy} = 0 for both objects.

We have determined mass 1 is accelerating upward, which is the positive y-direction for mass 1, so a_y is positive for mass 1. We have determined mass 2 is accelerating downward, which is the positive y-direction for mass 2, so a_y is positive for mass 2. We have already determined that the magnitude of the acceleration is 2.45 m/s^2 for both masses. Therefore, **a_y = +2.45 m/s^2 for both masses.**

(Again, we see here that is was useful to choose “down” as the positive y-direction for mass 2. If we had chosen “up” as positive for mass 2, then a_y for mass 2 would be negative.)

$$\sum F_{1y} = m_1 a_{1y}$$
$$-294 + T = 30a$$
$$+294 \qquad +294$$
$$T = 30a + 294$$

$$\sum F_{2y} = m_2 a_{2y}$$
$$490 + (-T) = 50a$$
$$490 - T = 50a$$
$$490 - (30a + 294) = 50a$$
$$490 + (-30a) + (-294) = 50a$$
$$490 + (-294) + (-30a) = 50a$$
$$196 - 30a = 50a$$
$$+30a \qquad +30a$$
$$196 = 80a$$
$$\frac{196}{80} = \frac{\cancel{80}a}{\cancel{80}}$$
$$a = 2.45 \frac{m}{s^2}$$

mass 1
need ↓
$\Delta t, \Delta y, V_{iy}, V_{fy}, a_y$
$1.5\text{s}, \Delta y, 0, V_{1y}, +2.45 \frac{m}{s^2}$
3 knowns

mass 2
need ↓
$\Delta t, \Delta y, V_{iy}, V_{fy}, a_y$
$1.5\text{s}, \Delta y, 0, V_{1y}, +2.45 \frac{m}{s^2}$

---

[1] If we try to use subscripts to distinguish the kinematics variables for mass 1 from the kinematics variables for mass 2, we will have to use three subscripts for v_{1iy} and v_{2iy}. Writing three subscripts is awkward, so, instead, it is preferable to distinguish the kinematics variables by simply writing “mass 1” and “mass 2” above the kinematics variables for each object, as shown above.

When we know values for three kinematics variables, we are ready to choose a kinematics equation. We do know three kinematics values (Δt, v_{iy}, and a_y), so we are ready to choose a kinematics equation.

Kinematics Equations for constant a_y with changing v_y

y equations	missing variables
$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$	v_{fy}
$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y$	Δt
$v_{fy} = v_{iy} + a_y\Delta t$	Δy

On this problem, the kinematics variable that we do *not* care about is v_{fy}, so we choose the kinematics equation that is missing v_{fy}: $\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$

$\Sigma F_{1y} = m_1 a_{1y}$

$-294 + T = 30a$

$+294 \qquad +294$

$T = 30a + 294$

$\Sigma F_{2y} = m_2 a_{2y}$

$490 + (-T) = 50a$

$490 - T = 50a$

$490 - (30a + 294) = 50a$

$490 + (-30a) + (-294) = 50a$

$490 + (-294) + (-30a) = 50a$

$196 - 30a = 50a$

$+30a \qquad +30a$

$196 = 80a$

$\frac{196}{80} = \frac{\cancel{80}a}{\cancel{80}}$

$a = 2.45\ \frac{m}{s^2}$

mass 1

need

$\Delta t, \Delta y, v_{iy}, v_{fy}, a_y$

$1.5\ s, \Delta y, 0, v_{iy}, +2.45\ \frac{m}{s^2}$

3 knowns

$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$

$\Delta y = 0(1.5) + \frac{1}{2}(2.45)(1.5)^2$

$\Delta y = \frac{1}{2}(2.45)(1.5)^2$

$\Delta y \approx +2.76\ m$

mass 2

need

$\Delta t, \Delta y, v_{iy}, v_{fy}, a_y$

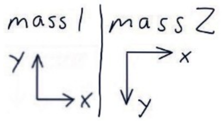
$1.5\ s, \Delta y, 0, v_{iy}, +2.45\ \frac{m}{s^2}$

$\Delta y \approx +2.76\ m$

Since the values of the kinematics variables for mass 2 are identical to the values of the kinematics variables for mass 1, we can see that Δy for mass 2 must be the same as Δy for mass 1.

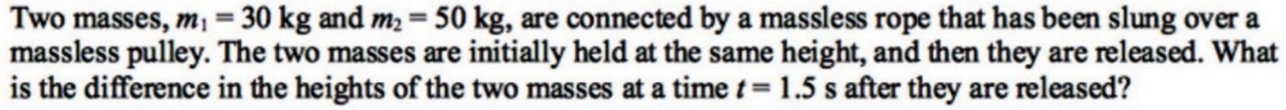
(If it was obvious to you that that was going to be the result, it would be OK to write only the kinematics setup for mass 1, without taking the time to explicitly write out the kinematics setup for mass 2.)

Check: Does the sign of Δy make sense? Δy is positive for both mass 1 and for mass 2. We expected that mass 1 will be displaced upward, which is the positive direction for mass 1; and we expected that mass 2 will be displaced downward, which is the positive direction for mass 2; so, yes, it makes sense that our results for Δy are positive for both masses.



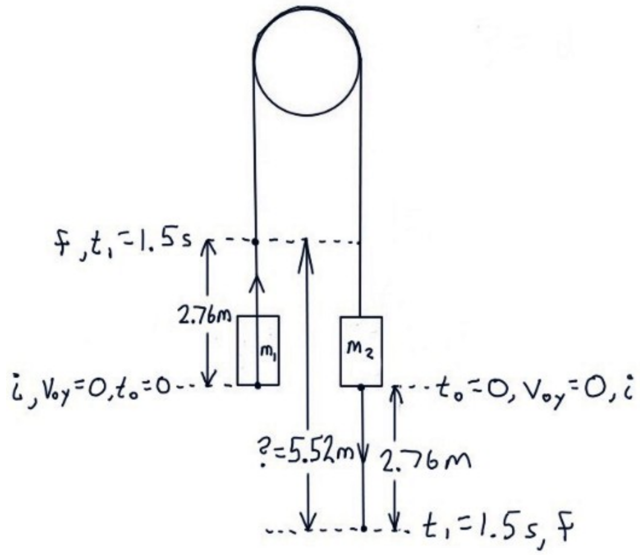
Again we see the value of choosing "down" as our positive direction for mass 2. If we had chosen "up" as the positive direction for mass 2, then Δy for mass 2 would be negative.

If Δy for both masses is +2.76 m, then the total difference in height after 1.5 s is 2.76 m + 2.76 m, which is 5.52 m. Build this conclusion into your sketch, as shown below.



Two masses, m_1 = 30 kg and m_2 = 50 kg, are connected by a massless rope that has been slung over a massless pulley. The two masses are initially held at the same height, and then they are released. What is the difference in the heights of the two masses at a time t = 1.5 s after they are released?

? = difference in height after 1.5 s



Answer:

The difference in height at time $t = 1.5$ s will be 5.5m.

Check: Does the size of our answer make sense? 1 meter is roughly 1 yard, and 1 yard equals 3 feet, so 5.5 m is roughly 5 yards, which is 15 feet. Therefore the pulley must be mounted at least 15 feet above the floor in order for there to be enough room for the masses to move. 15 feet is fairly high, but not so high as to be particularly implausible.

Problem Recap on next page.

Recap:

It simplified our solution to **choose positive axes pointing in each object's direction of motion.** For this problem, that meant choosing *different* y-axes for mass 1 and for mass 2.

For a massless rope stretched over a massless, frictionless pulley, **the magnitude of the tension force is the same at both ends of the rope**, although the direction of the tension force may be different at the two ends of the rope. In our Force Tables, we used this rule to write $T_1 = T$, and $T_2 = T$, using the same symbol, T, to represent both magnitudes.

Since we chose different axes for the two masses, **we had to be extra careful to get the correct "+" and "-" signs in our Force Table**. We had to be careful to determine the signs for the components for each mass based on the positive y-direction we had chosen for that mass.

The magnitude of the acceleration will be the same **for two objects connected by an unstretchable rope,** although the directions of the accelerations may be different for the two objects**.** We used this rule to write $a_{1y} = +a$, and $a_{2y} = +a$, using the same symbol, a, to represent both magnitudes. Then we substituted $+a$ in for both a_{1y} and a_{2y} in our Newton's Second Law equations.

$$a_{1y} = +a$$

$$a_{2y} = +a$$

We obtained a system of two equations in two unknowns. For the particular equations that we obtained, the most convenient way to solve the system of equations is the **Addition Method**.

For kinematics problems, organize the kinematics data with a list of the five kinematics variables, as shown at right.

Use acceleration as the connecting link between Newton's Second Law and kinematics.

mass 1

need

↓

Δt	Δy	v_{iy}	v_{fy}	a_y
1.5 s	Δy	0	v_{1y}	$+2.45 \frac{m}{s^2}$

As a beginning student, you will make fewer mistakes, and have better understanding, if you follow this advice: **Write the *general* equation before you plug in specific numbers or symbols.**

For example, write the general kinematics equation, $\Delta y = v_{iy} \Delta t + \frac{1}{2} a_y (\Delta t)^2$, before you plug in specifics.

For another example, write the general Newton's Second Law equations, $\Sigma F_{1y} = m_1 a_{1y}$ and $\Sigma F_{2y} = m_2 a_{2y}$, before you plug in specifics.

It is even a good idea to write the general equations for the magnitude of the weight, $w_1 = m_1 g$ and $w_2 = m_2 g$, before you plug in specifics.

This advice is particularly important when you are first learning how to solve a particular type of problem. Once you are comfortable with a problem type, it becomes less important to write the general equations for those problems.

Video (3)

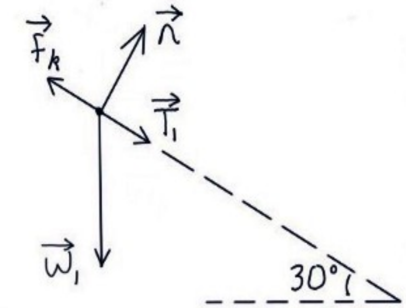
Here is a summary of some of the key steps in the solution:

$$W_1 = m_1 g \quad \approx 3(9.8) \quad \approx 29.4\text{N}$$

$$W_2 = m_2 g \quad \approx 2(9.8) \quad \approx 19.6\text{N}$$

$$f_k = \mu_k n \quad = 0.4n$$

Free-body diagram showing all the forces exerted on mass 1

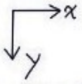


Free-body diagram showing all the forces exerted on mass 2

Force Table for mass 1

$W_1 = 29.4\text{N}$	n	$f_k = .4n$	$T_1 = T$
$W_{1x} = +14.7\text{N}$	$n_x = 0$	$f_{kx} = -.4n$	$T_{1x} = +T$
$W_{1y} = -25.5\text{N}$	$n_y = +n$	$f_{ky} = 0$	$T_{1y} = 0$

Force Table for mass 2



$W_2 = 19.6\text{N}$	$T_2 = T$	← magnitudes of the overall force vectors
$W_{2x} = 0$	$T_{2x} = 0$	components of the forces
$W_{2y} = +19.6\text{N}$	$T_{2y} = -T$	

$$\Sigma F_{1x} = m_1 a_{1x}$$
$$14.7 + (-.4n) + T = 3a_{1x}$$
$$14.7 - .4n + T = 3a_{1x}$$
$$14.7 - .4(25.5) + T = 3a_{1x}$$
$$4.5 + T = 3a_{1x}$$

$$\Sigma F_{1y} = m_1 a_{1y}$$
$$-25.5 + n = 3(0)$$
$$-25.5 + n = 0$$
$$+25.5 \qquad +25.5$$
$$n = 25.5\text{N}$$

$$\Sigma F_{2y} = m_2 a_{2y}$$
$$19.6 + (-T) = 2a_{1x}$$
$$\left.\begin{array}{l} 19.6 - T = 2a_{1x} \\ 4.5 + T = 3a_{1x} \end{array}\right\} \text{add}$$
$$24.1 = 5a_{1x}$$
$$\frac{24.1}{5} = \frac{\cancel{5}a_{1x}}{\cancel{5}}$$
$$a_{1x} = +4.82 \frac{m}{s^2}$$

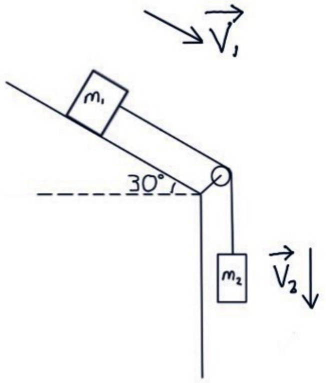
$$a_{2y} = a_{1x}$$
$$a_{2y} = +4.82 \frac{m}{s^2}$$

$$4.5 + T = 3a_{1x}$$
$$4.5 + T = 3(4.82)$$
$$4.5 + T = 14.46$$
$$-4.5 \qquad -4.5$$
$$T = 9.96\text{N}$$

Full solution begins on the next page.

Here is a full solution to the problem:

In the diagram, $m_1 = 3.0$ kg and $m_2 = 2.0$ kg. The pulley is massless and frictionless. Mass 1 is sliding down the incline; mass 2 is falling. There is a coefficient of kinetic friction $\mu_k = 0.40$ between mass 1 and the incline. Find the acceleration of the masses, and the tension in the rope.



$? = T$

$? = a_1, \; ? = a_2$

$? = \underbrace{\text{direction of } \vec{a}_1, \; ? = \text{direction of } \vec{a}_2}$

mass1 mass2

The problem refers to the concepts of mass, friction force, acceleration, and tension force, all of which fit into a Newton’s Second Law framework, so we plan to use the Newton’s Second problem-solving framework to solve the problem. Although the concept of acceleration also fits into a kinematics framework, there are no other kinematics concepts mentioned in the problem, so it appears that we will not need to apply a kinematics framework to this problem.

The problem asks for the tension, by which the professor probably means the *magnitude* of the tension force. The problem also asks for the masses’ acceleration. I will interpret this question to be asking for both the magnitude and the direction of the overall acceleration vectors for both mass 1 and mass 2. Remember that writing a vector symbol without the arrow on top specifically refers to the *magnitude* of the vector, so we can identify what the question is asking for with the following notation:
$? = T$
$? = a_1, \; ? = a_2$
$? =$ direction of $\vec{a}_1$, $? =$ direction of $\vec{a}_2$

The problem tells us that mass 1 is sliding down the incline and that mass 2 is falling. The direction of an object’s velocity vector indicates the object’s direction of motion. Therefore, we have drawn velocity vectors pointing down the incline, and straight down, in the sketch above, to indicate mass 1’s and mass 2’s directions of motion, respectively, after they are released.

Choose positive axes pointing in the objects’ directions of motion. Mass 1 is moving parallel to, and down, the incline, so we choose a positive x-axis pointing parallel to, and down, the incline for mass 1. We choose a positive y-axis pointing perpendicular to, and away, from the incline for mass 1.

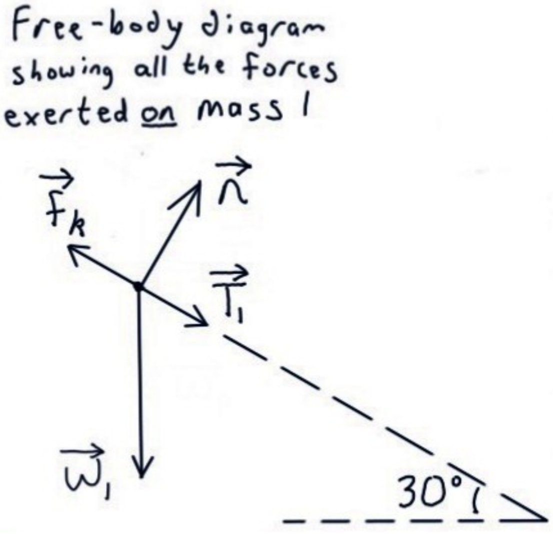
Mass 2 is moving straight down, so we choose a positive y-axis pointing straight down for mass 2. We choose a positive x-axis pointing to the right for mass 2.

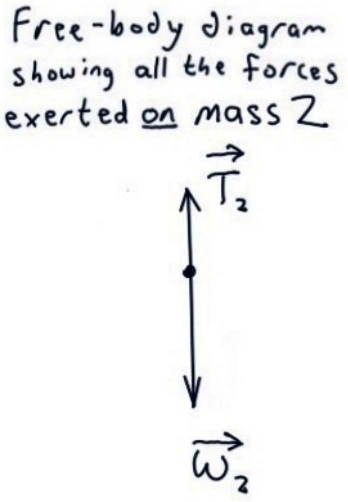
Notice that **it is OK to choose different axes for the two masses**. *Write down* your axes for each mass, as shown above.

(Some professors or textbooks might prefer to choose “up” as the positive y-direction for mass 2. But for a beginning student it is probably best to choose “down” as positive for mass 2, since that is mass 2’s direction of motion.)

Draw two separate Free-body Diagrams, one diagram showing all the forces being exerted on mass 1, and a *separate* diagram showing all the forces being exerted on mass 2.

Use subscripts to distinguish the forces being exerted on mass 1 from the forces being exerted on mass 2: $\vec{w}_1$ vs. $\vec{w}_2$, $\vec{T}_1$ vs. $\vec{T}_2$





General two-step process for identifying the forces for your Free-body Diagram for a particular object:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

In this case, mass 1 is being touched by the surface of the incline, which exerts both a normal force and a frictional force; and by the rope, which exerts a "tension force". We know that *kinetic* friction applies for this problem because mass 1 is *sliding*.

Mass 2 is being touched only by the rope, which exerts a "tension force".

The rule for determining the direction of the weight force is: The weight force always points down.

The rule for determining the direction of the normal force is: The normal force points *perpendicular* to, and away from, the surface that is touching the object. (In math, "normal" means "perpendicular".)

In this problem, the surface touching mass 1 is the incline. So the normal force points perpendicular to, and away from, the surface of the incline.

The rule for determining the direction of kinetic friction is: Kinetic friction points parallel to the surface, and opposite to the direction that the object is sliding. **Friction opposes sliding.**

Mass 1 is sliding parallel to, and down, the incline, so for this problem the kinetic friction points parallel to, and *up*, the incline.

The rule for determining the direction of the tension force is: The tension force points parallel to the rope, and away from the object.

This rule is based on the common sense idea that a rope can only "pull" an object, not "push" it. The rope exerts a pulling force down the incline on mass 1, and an upward pulling force on mass 2.

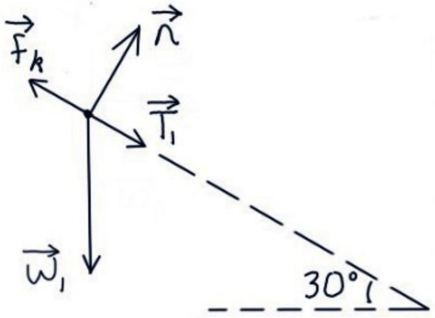
Begin a Force Table for mass 1, and a Force Table for mass 2.

$w_1 = m_1 g$
$= 3(9.8)$
$= 29.4\text{N}$

$w_2 = m_2 g$
$= 2(9.8)$
$= 19.6\text{N}$

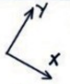
$f_k = \mu_k n$
$= 0.4n$

Free-body diagram showing all the forces exerted on mass 1



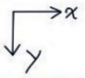
Free-body diagram showing all the forces exerted on mass 2

Force Table for mass 1



w_1	n	f_k	T_1
$w_1 = 29.4\text{ N}$	n	$f_k = .4n$	$T_1 = T$
$w_{1x} =$	$n_x = 0$	$f_{kx} = -.4n$	$T_{1x} = +T$
$w_{1y} =$	$n_y = +n$	$f_{ky} = 0$	$T_{1y} = 0$

Force Table for mass 2



w_2	T_2	
$w_2 = 19.6\text{ N}$	$T_2 = T$	← magnitudes of the overall force vectors
$w_{2x} = 0$	$T_{2x} = 0$	} components of the forces
$w_{2y} = +19.6\text{N}$	$T_{2y} = -T$	

For a massless rope stretched over a massless, frictionless pulley, **the magnitude of the tension force is the same at both ends of the rope**, although the direction of the tension force may be different at the two ends of the rope. In our Force Tables above, the symbols T_1 and T_2, written without arrows on top, stand for the *magnitudes* of the tension forces on mass 1 and on mass 2. Because these magnitudes are equal, we can write $T_1 = T$, and $T_2 = T$, using the same symbol, T, to represent both magnitudes, as shown in the tables above. (The problem does not explicitly state that the rope is massless. But, for an introductory physics course, we typically assume that ropes are massless unless the problem indicates otherwise.)

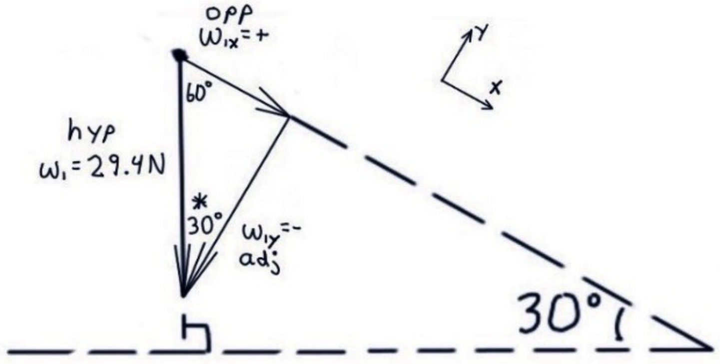
We used this rule to determine the components: If a vector is parallel or anti-parallel to one of the axes, then the component for that axis has the same magnitude and direction as the overall vector; and the component for the *other* axis is zero.

It is crucial to include the negative sign on f_{kx}, because the friction force is pointing up the incline, which we have chosen to represent the *negative* x-direction for mass 1. **It is crucial to include the negative sign on T_{2y}**, because the tension force exerted on mass 2 points up, which we have chosen to represent the *negative* y-direction for mass 2.

Include a "+" sign in front of all positive components. This will help you to remember to include a "-" sign in front of negative components.

Remember that *the signs of the components depend on the axes you choose*. If we had chosen different axes (for example, if we had decided to choose "up" as our positive y-direction for mass 2), then we would have obtained different signs for the components.

$\vec{w}_1$ is neither parallel nor anti-parallel to the x- and y-axes for mass 1. Therefore, to break $\vec{w}_1$ into components, we must draw a right triangle and use SOH CAH TOA, as illustrated below.



SOH CAH TOA

$$\sin 30° = \frac{opp}{hyp} \qquad \cos 30° = \frac{adj}{hyp}$$

$$\sin 30° = \frac{|w_{1x}|}{29.4} \qquad \cos 30° = \frac{|w_{1y}|}{29.4}$$

$$29.4 \cdot \sin 30° = \frac{|w_{1x}|}{\cancel{29.4}} \cdot \cancel{29.4} \qquad 29.4 \cdot \cos 30° = \frac{|w_{1y}|}{\cancel{29.4}} \cdot \cancel{29.4}$$

$$|w_{1x}| = 14.7 \text{ N} \qquad |w_{1y}| = 25.5 \text{ N}$$

$$w_{1x} = +14.7 \text{ N} \qquad w_{1y} = -25.5 \text{ N}$$

We use absolute value signs in our SOH CAH TOA equations to remind ourselves that the SOH CAH TOA equations do not determine the "+" or "-" signs on the components. Therefore, we need to determine the "+" or "-" signs ourselves in a separate step.

It is crucial to include the negative sign on w_{1y}, because w_{1y} points *into* the incline, which we have chosen to represent the *negative* y-direction for mass 1.

The video explains in more detail how to draw the correct right triangle, how to determine the directions of the components, and how to use geometry to determine the angles inside the right triangle.

Now we can substitute these components into our Force Table for mass 1.

Force Table for mass 1

$w_1 = 29.4$ N	n	$f_k = .4n$	$T_1 = T$
$w_{1x} = +14.7$ N	$n_x = 0$	$f_{kx} = -.4n$	$T_{1x} = +T$
$w_{1y} = -25.5$ N	$n_y = +n$	$f_{ky} = 0$	$T_{1y} = 0$

Force Table for mass 2

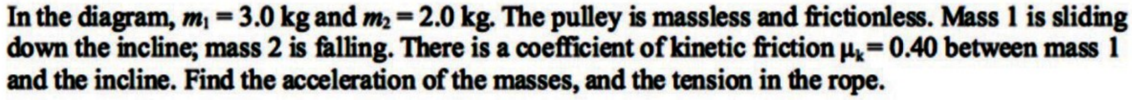
$w_2 = 19.6$ N	$T_2 = T$	← magnitudes of the overall force vectors
$w_{2x} = 0$	$T_{2x} = 0$	components of the forces
$w_{2y} = +19.6$ N	$T_{2y} = -T$	

Now we need to determine what to plug in for a_{1x}, a_{1y}, and a_{2y} in our Newton's Second Law equations.

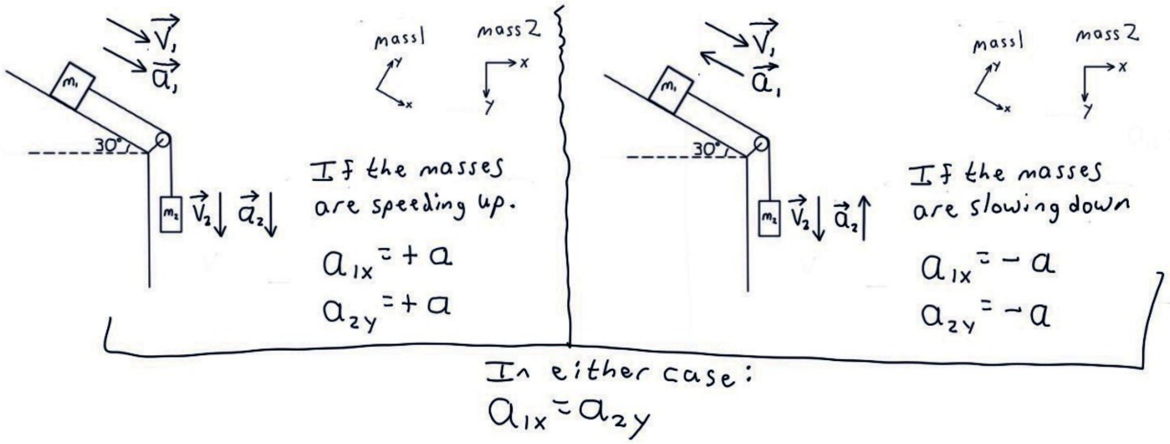
Mass 1 is moving parallel to the incline, in the x-component. Mass 1 has no motion perpendicular to the incline, in the y-component. Because mass 1 is motionless in the y-component, $\boldsymbol{a_{1y} = 0}$.

Substitute this value for a_{1y} into the Newton's Second Law y-equation for mass 1, as shown on the next page.

***The magnitude of the acceleration will be the same* for two objects connected by an unstretchable rope**, although the directions of the accelerations may be different for the two objects. Because mass 1 and mass 2 are connected by the rope, they will have the same magnitude of acceleration. Let's represent the magnitude of the acceleration for mass 1 and for mass 2 with the symbol a.



In the diagram, $m_1 = 3.0$ kg and $m_2 = 2.0$ kg. The pulley is massless and frictionless. Mass 1 is sliding down the incline; mass 2 is falling. There is a coefficient of kinetic friction $\mu_k = 0.40$ between mass 1 and the incline. Find the acceleration of the masses, and the tension in the rope.



If the acceleration vector is parallel to the velocity vector, then the object is moving with increasing speed. If the acceleration vector is anti-parallel to the velocity vector, then the object is moving with decreasing speed.

It is possible that both objects are speeding up. In that case, $\vec{a}_1$ will be parallel to $\vec{v}_1$, and $\vec{a}_2$ will be parallel to $\vec{v}_2$. Then, $a_{1x} = +a$, and $a_{2y} = +a$.

Based on the wording of the problem, it is also possible that both objects are slowing down. In that case, $\vec{a}_1$ will be anti-parallel to $\vec{v}_1$, and $\vec{a}_2$ will be anti-parallel to $\vec{v}_2$. Then, $a_{1x} = -a$, and $a_{2y} = -a$.

In either case, we can say that $a_{1x} = a_{2y}$. Use this equation to substitute into the Newton's Second Law y-equation for mass 2, as shown on the next page.

Now we can see why it was helpful to choose mass 2's direction of motion, "down", as our positive y-direction for mass 2. If we had chosen "up" as our positive y-direction for mass 2, then we would get that $a_{1x} = -a_{2y}$. By choosing "down" as our positive y-direction for mass 2, we avoid having to deal with this negative sign.

On the previous problems in this series, the objects started moving from rest, so we knew for sure that the objects must be speeding up. On this problem, we do not know that the objects began motion from rest, so it is possible that the objects might be either speeding up or slowing down as they move.

Next, write the Newton’s Second Law equations, as shown below. Mass 1 experiences forces in both the x- and y-components, so we write Newton’s Second Law equations for mass 1 for both the x- and y-components. Mass 2 experiences no forces in the x-component, so for mass 2 we write the Newton’s Second Law equation only for the y-component.

On the left side of each equation, add the individual force components, which we determined in our Force Tables, as shown below.

*F*or this problem, mass 1 is motionless in the x-component, so **a_{1y} = 0**. Substitute this value into the Newton’s Second Law y-equation for mass 1, as shown below.

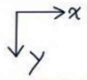
As discussed on the previous page, for this problem, $a_{1x} = a_{2y}$. **We can use this equation to substitute a_{1x} in for a_{2y} in the Newton’s Second Law y-equation for object 2, as shown below.** This helps us by reducing the total number of unknowns remaining in our Newton’s Second Law equations.

On the previous problems in this series, the objects started moving from rest, so we knew for sure that the objects must be speeding up. On this problem, we do not know that the objects began motion from rest, so it is possible that the objects might be either speeding up or slowing down as they move. This is the reason that we are dealing with the acceleration components slightly differently in this problem than we did in the two previous problems in this series.

Force Table for mass 1

$W_1 = 29.4\text{N}$	n	$f_k = .4n$	$T_1 = T$
$W_{1x} = +14.7\text{N}$	$n_x = 0$	$f_{kx} = -.4n$	$T_{1x} = +T$
$W_{1y} = -25.5\text{N}$	$n_y = +n$	$f_{ky} = 0$	$T_{1y} = 0$

ForceTable for mass 2



$W_2 = 19.6\text{N}$	$T_2 = T$	← magnitudes of the overall force vectors
$W_{2x} = 0$	$T_{2x} = 0$	} components of the forces
$W_{2y} = +19.6\text{N}$	$T_{2y} = -T$	

$\Sigma F_{1x} = m_1 a_{1x}$	$\Sigma F_{1y} = m_1 a_{1y}$	$\Sigma F_{2y} = m_2 a_{2y}$	$a_{1x} = a_{2y}$
$14.7 + (-.4n) + T = 3a_{1x}$	$-25.5 + n = 3(0)$	$19.6 + (-T) = 2a_{1x}$	
$14.7 - .4n + T = 3a_{1x}$	$-25.5 + n = 0$	$19.6 - T = 2a_{1x}$	

The Newton’s Second Law y-equation for mass 1 has only one unknown, so we begin by solving that equation for *n*.

Force Table for mass 1 (y, x)

$w_1 = 29.4\,N$	n	$f_k = .4n$	$T_1 = T$
$w_{1x} = +14.7\,N$	$n_x = 0$	$f_{kx} = -.4n$	$T_{1x} = +T$
$w_{1y} = -25.5\,N$	$n_y = +n$	$f_{ky} = 0$	$T_{1y} = 0$

Force Table for mass 2 (x, y)

$w_2 = 19.6\,N$	$T_2 = T$	← magnitudes of the overall force vectors
$w_{2x} = 0$	$T_{2x} = 0$	components of the forces
$w_{2y} = +19.6\,N$	$T_{2y} = -T$	

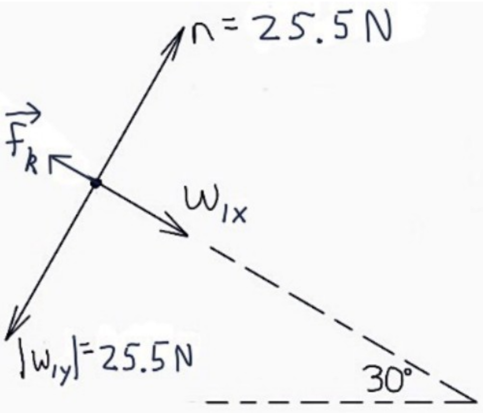
$\Sigma F_{1x} = m_1 a_{1x}$	$\Sigma F_{1y} = m_1 a_{1y}$	$\Sigma F_{2y} = m_2 a_{2y}$	$a_{1x} = a_{2y}$
$14.7 + (-.4n) + T = 3\,a_{1x}$	$-25.5 + n = 3(0)$	$19.6 + (-T) = 2\,a_{1x}$	
$14.7 - .4n + T = 3a_{1x}$	$-25.5 + n = 0$	$19.6 - T = 2\,a_{1x}$	
2 unknowns	$+25.5 \quad +25.5$	2 unknowns	
	$n = 25.5\,N$		

Check: Does the sign of our result for *n* make sense? The symbol *n* stands for the *magnitude* of the normal force, and a magnitude can never be negative, so, yes, it makes sense that our result for *n* came out positive.

Check: Does the size of our result for *n* make sense? To prevent mass 1 from beginning to move into the surface of the incline, $\vec{n}$ must cancel w_{1y}. So, yes, it makes sense that:

$n = 25.5\text{ N} = |w_{1y}|$

Therefore, in the Free-body diagram on the right, I have drawn the length of the $\vec{n}$ arrow equal to the length of the w_{1y} arrow.



Notice that this problem demonstrates that you should not assume that the magnitude of the normal force will always equal the magnitude of the weight force. There are many problems in which the magnitude of the normal force does equal the magnitude of the weight force. But there are also many problems, such as this one, in which the magnitude of the normal force (25.5 N) does *not* equal the magnitude of the overall weight force (29.4 N). For each problem, use the Newton’s Second Law equations to figure out the correct value for the magnitude of the normal force.

Next, we substitute our value for *n* into the Newton's Second Law x-equation for mass 1.

$$\begin{aligned} \sum F_{1x} &= m_1 a_{1x} \\ 14.7+(-.4n)+T &= 3a_{1x} \\ 14.7-.4n+T &= 3a_{1x} \\ 14.7-.4(25.5)+T &= 3a_{1x} \\ 4.5+T &= 3a_{1x} \end{aligned}$$

$$\begin{aligned} \sum F_{1y} &= m_1 a_{1y} \\ -25.5+n &= 3(0) \\ -25.5+n &= 0 \\ +25.5 \quad & \quad +25.5 \\ \hline n &= 25.5\text{ N} \end{aligned}$$

$$\begin{aligned} \sum F_{2y} &= m_2 a_{2y} \\ 19.6+(-T) &= 2a_{1x} \\ 19.6-T &= 2a_{1x} \end{aligned}$$

$$a_{1x} = a_{2y}$$

$$\underbrace{\qquad\qquad}_{\text{2 equations, 2 unknowns}}$$

2 equations
2 unknowns

As they are now written, the Newton's Second Law x-equation for mass 1 and the Newton's Second Law y-equation for mass 2 form a system of two equations in two unknowns (a_{1x} and *T*). We can solve this system of simultaneous equations using the Addition method, as illustrated below.

$$\begin{aligned} \sum F_{1x} &= m_1 a_{1x} \\ 14.7+(-.4n)+T &= 3a_{1x} \\ 14.7-.4n+T &= 3a_{1x} \\ 14.7-.4(25.5)+T &= 3a_{1x} \\ 4.5+T &= 3a_{1x} \end{aligned}$$

$$\begin{aligned} \sum F_{1y} &= m_1 a_{1y} \\ -25.5+n &= 3(0) \\ -25.5+n &= 0 \\ +25.5 \quad & \quad +25.5 \\ \hline n &= 25.5\text{ N} \end{aligned}$$

$$\begin{aligned} \sum F_{2y} &= m_2 a_{2y} \\ 19.6+(-T) &= 2a_{1x} \\ 19.6-T &= 2a_{1x} \\ 4.5+T &= 3a_{1x} \quad \text{add} \\ \hline 24.1 &= 5a_{1x} \\ \frac{24.1}{5} &= \frac{\cancel{5}a_{1x}}{\cancel{5}} \\ a_{1x} &= 4.82\,\frac{\text{m}}{\text{s}^2} \end{aligned}$$

$$\begin{aligned} a_{2y} &= a_{1x} \\ a_{2y} &= 4.82\,\frac{\text{m}}{\text{s}^2} \end{aligned}$$

$$\begin{aligned} 4.5+T &= 3a_{1x} \\ 4.5+T &= 3(4.82) \\ 4.5+T &= 14.46 \\ -4.5 \quad & \quad -4.5 \\ \hline T &= 9.96\text{ N} \end{aligned}$$

The Addition Method works well for these equations because the *T* variables cancel out when we add the two equations. If the two equations were not already in a form such that the *T* variables would cancel when the equations are added, then the Addition Method would work less well and it might be preferable to use the Substitution Method to solve the equations.

For completeness, the Substitution Method for solving these two equations is illustrated on the next page.

On this particular problem, the simplest method for solving the equations is the Addition Method. But for completeness, the Substitution Method for solving the equations is illustrated below.

The "Substitution method" for solving a system of two equations in two unknowns simultaneously:

1. Begin by solving one of the equations for one of the unknowns. Solve for the variable that is the easiest to solve for in the two equations.
2. Substitute the algebraic expression obtained in step 1 into the other equation.
3. Solve the equation obtained in step 2 for the second unknown.
4. If you care about the remaining unknown, then substitute the value or expression obtained in step 3 into the equation obtained in step 1 to determine the remaining unknown. In this problem, we do not care about the value of *T*, so we don't bother to carry out step 4 of the method.

$$\sum_i F_{1x} = m_1 a_{1x}$$
$$14.7 + (-.4n) + T = 3a_{1x}$$
$$14.7 - .4n + T = 3a_{1x}$$
$$14.7 - .4(25.5) + T = 3a_{1x}$$
$$4.5 + T = 3a_{1x}$$
$$T = 3a_{1x} - 4.5$$
$$T = 3(4.82) - 4.5$$
$$T = 9.96\text{N}$$

$$\sum_i F_{1y} = m_1 a_{1y}$$
$$-25.5 + n = 3(0)$$
$$-25.5 + n = 0$$
$$+25.5 \qquad +25.5$$
$$n = 25.5\text{ N}$$

$$\sum_i F_{2y} = m_2 a_{2y}$$
$$19.6 + (-T) = 2a_{1x}$$
$$19.6 - T = 2a_{1x}$$
$$19.6 - (3a_{1x} - 4.5) = 2a_{1x}$$
$$19.6 + (-3a_{1x}) + 4.5 = 2a_{1x}$$
$$19.6 + 4.5 + (-3a_{1x}) = 2a_{1x}$$
$$24.1 - 3a_{1x} = 2a_{1x}$$
$$+3a_{1x} \qquad +3a_{1x}$$
$$24.1 = 5a_{1x}$$
$$\frac{24.1}{5} = \frac{5a_{1x}}{5}$$
$$a_{1x} = +4.82\,\frac{\text{m}}{\text{s}^2}$$

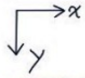
$$a_{1x} = a_{2y}$$
$$+4.82\,\frac{\text{m}}{\text{s}^2} = a_{2y}$$

On this problem, the Addition Method (illustrated on the previous page) is simpler than the Substitution Method (illustrated on this page). There are many other problems, however, in which in which the Substitution Method might be preferable to the Addition Method, or in which the Addition Method might not work at all.

Force Table for mass 1

$w_1 = 29.4\text{ N}$	n	$f_k = .4n$	$T_1 = T$
$w_{1x} = +14.7\text{ N}$	$n_x = 0$	$f_{kx} = -.4n$	$T_{1x} = +T$
$w_{1y} = -25.5\text{ N}$	$n_y = +n$	$f_{ky} = 0$	$T_{1y} = 0$

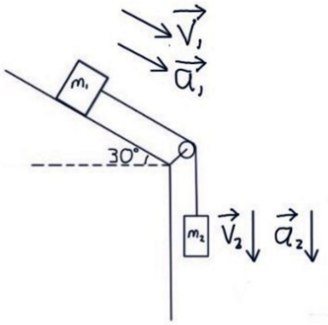
Force Table for mass 2

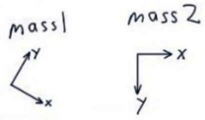


$w_2 = 19.6\text{ N}$	$T_2 = T$	← magnitudes of the overall force vectors
$w_{2x} = 0$	$T_{2x} = 0$	components of the forces
$w_{2y} = +19.6\text{N}$	$T_{2y} = -T$	

$T = 9.96\text{ N}$, $a_{1x} = +4.82\ \frac{\text{m}}{\text{s}^2}$, $a_{2y} = +4.82\ \frac{\text{m}}{\text{s}^2}$

In the diagram, $m_1 = 3.0$ kg and $m_2 = 2.0$ kg. The pulley is massless and frictionless. Mass 1 is sliding down the incline; mass 2 is falling. There is a coefficient of kinetic friction $\mu_k = 0.40$ between mass 1 and the incline. Find the acceleration of the masses, and the tension in the rope.



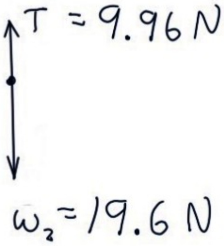


If the masses are speeding up.

$a_{1x} = +a$

$a_{2y} = +a$

Free-body diagram showing all the forces exerted on mass 2



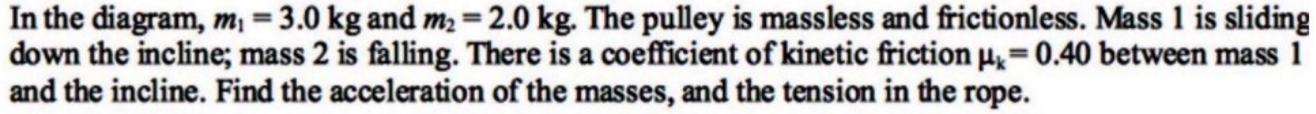
Check: Does the magnitude of a_{1x} and a_{2y} make sense? Because mass 2 is being held back by the rope, rather than falling freely, we would expect that mass 2 will fall with an acceleration that is smaller in magnitude than free-fall acceleration. Our result for the magnitude of a_{2y} (4.82 m/s^2) is indeed less than free-fall acceleration (9.8 m/s^2), so, yes, our result for the magnitude of a_{2y} does make sense.

Check: Does the sign of T make sense? We are using the symbol T to stand for the *magnitude* of the tension force. A magnitude can never be negative, so, yes, it makes sense that our result for T came out to be positive.

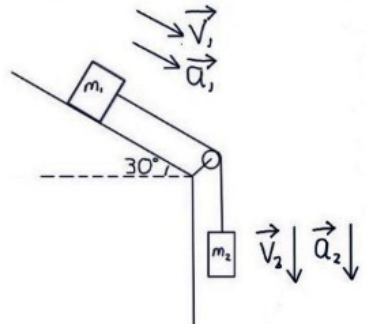
Check: Does the sign of a_{1x} and a_{2y} make sense? Does the size of T make sense? a_{1x} and a_{2y} both came out to be positive, which means that $\vec{a}_1$ is parallel to $\vec{v}_1$, and $\vec{a}_2$ is parallel to $\vec{v}_2$, which means that the masses are speeding up. This is consistent with our result for T. The magnitude of the downward weight force on mass 2 (19.6 N) is greater than the magnitude of the upward tension force on mass 2 (9.96 N). This confirms that mass 2 will experience a downward net force, and hence a downward acceleration, and hence will be speeding up in this problem. So yes, our result for the sign of a_{2y} is consistent with our result for the size of T.

Notice that, in the Free-body diagram above, I have drawn the arrow for $\vec{w}_2$ longer than the arrow for $\vec{T}_2$, to match these results.

We have determined the *components* of the acceleration. But we are interpreting the question to be asking about the *overall* acceleration vectors. So, next, we will use the acceleration components to determine the magnitude and direction of the overall acceleration vectors.



In the diagram, $m_1 = 3.0$ kg and $m_2 = 2.0$ kg. The pulley is massless and frictionless. Mass 1 is sliding down the incline; mass 2 is falling. There is a coefficient of kinetic friction $\mu_k = 0.40$ between mass 1 and the incline. Find the acceleration of the masses, and the tension in the rope.



$? = T$

$? = a_1,\ ? = a_2$

$\underbrace{? = \text{direction of } \vec{a}_1}_{\text{mass1}},\ \underbrace{? = \text{direction of } \vec{a}_2}_{\text{mass 2}}$

Components → Overall vectors

$$\begin{array}{l} a_{1x} = +4.82\,\frac{m}{s^2} \\ a_{1y} = 0 \end{array} \rightarrow \left\{ \begin{array}{l} a_1 = 4.82\,\frac{m}{s^2} \\ \text{direction of } \vec{a}_1 = \text{down the incline} \end{array} \right.$$

$$\begin{array}{l} a_{2x} = 0 \\ a_{2y} = +4.82\,\frac{m}{s^2} \end{array} \rightarrow \left\{ \begin{array}{l} a_2 = 4.82\,\frac{m}{s^2} \\ \text{direction of } \vec{a}_2 = \text{straight down} \end{array} \right.$$

a_{1y} and a_{2x} are zero because mass 1 is motionless in the y-component and mass 2 is motionless in the x-component.

We used this rule: If one component of a vector is zero, then the magnitude and direction of the overall vector is the same as the magnitude and direction of the nonzero component.

Notice that I have drawn vector arrows showing the directions of $\vec{a}_1$ and $\vec{a}_2$ in the sketch above.

Now we can answer the question.

The magnitude of the tension is 10N.
Mass 1 has acceleration of $4.8\,\frac{m}{s^2}$, down the incline.
Mass 2 has acceleration of $4.8\,\frac{m}{s^2}$, straight down.

Problem Recap on next page.

Recap:

It simplified our solution to **choose positive axes pointing in each object's direction of motion.** For this problem, that meant choosing *different* axes for mass 1 and for mass 2.

For a massless rope stretched over a massless, frictionless pulley, **the magnitude of the tension force is the same at both ends of the rope**, although the direction of the tension force may be different at the two ends of the rope. In our Force Tables, we used this rule to write $T_1 = T$, and $T_2 = T$, using the same symbol, T, to represent both magnitudes.

Since we chose different axes for the two masses, we had to be extra careful to get the correct "+" and "-" signs in our Force Table. We had to be careful to determine the signs for the components for each mass based on the positive directions we had chosen for that mass.

The magnitude of the acceleration will be the same **for two objects connected by an unstretchable rope**, although the directions of the accelerations may be different for the two objects**.** We used this rule to determine that, based on the positive directions we chose for each object, $a_{1x} = a_{2y}$. Then we substituted a_{1x} in for a_{2y} in our Newton's Second Law y-equation for mass 2.

In the previous problems in this series, we knew that the objects began motion from rest, so we knew that the objects were speeding up. In this problem, we did not know that the motion began from rest. Therefore, for this problem, it was possible that the objects might be speeding up, or that the objects might be slowing down. This caused us to use a slightly different approach to dealing with the acceleration on this problem than the approach we used for the previous problems.

Think in terms of components. We drew a right triangle and used SOH CAH TOA in order to break $\vec{w}_1$ into components.

We obtained a system of two equations in two unknowns. For the particular equations that we obtained, the most convenient way to solve the system of equations is the **Addition Method**.

See next page for an additional piece of advice about how to avoid the most common mistake that students make in physics.

The most common mistake that students make in physics is “confusing the concepts”.

To avoid confusing the concepts, **don’t use the word “it”.**
Instead, when thinking about a concept, make an effort to always *label* exactly which concept you are thinking about the with the correct name or the exact right symbol.

For example, don’t confuse the weight of mass 1 (which has to broken into components using SOH CAH TOA) and the weight of mass 2 (for which we do not need SOH CAH TOA). Don’t say “it is 29.4 N” or “it is 19.6 N”. Instead, say “w_1 is 29.4 N” and “w_2 is 19.6 N”.

Don’t confuse the tension force on mass 1 (which has x-component +T and y-component zero) with the tension force on mass 2 (which has x-component zero and y-component -T). Don’t say “it is +T” or “it is zero”. Instead, say “T_{1x} is +T” and “T_{2x} is zero”.

Don’t confuse a_{1y} (which is zero), with a_{2y} (which is equal to a_{1x}). Don’t say “it is zero” or “it is equal to a_{1x}”. Instead, say “a_{1y} is zero” and “a_{2y} is equal to a_{1x}”.

Don’t confuse a_{1y} (which is zero) with a_{1x} (which is equal to a_{2y}). (Think in terms of components!)

Don’t confuse mass (a scalar, with units kg) with weight (a vector with a direction and a magnitude, units N). Don’t say “it is 3 kg” or “it is 29.4 N”. Instead, say “the mass of object 1 is 3 kg” or “the magnitude of the weight of object 1 is 29.4 N”.

Don’t confuse the *coefficient* of kinetic friction (a scalar, symbol μ_k, no units) with the *force* of kinetic friction (a vector, symbol $\vec{f}_k$, with a direction and a magnitude, units N). Don’t say “it is 0.4” or “it is 0.4n”. Instead, say “μ_k is 0.4” and “the magnitude of the kinetic friction is 0.4n”.
(Remember that the symbol μ is pronounced “mu”.)

I try to model this technique (avoid using the word “it”) in the solutions document and in the videos.

Here are two other techniques to help you avoid confusing the concepts:

Always try to use the exact right symbol, including the exact right subscripts.

Think in terms of components.

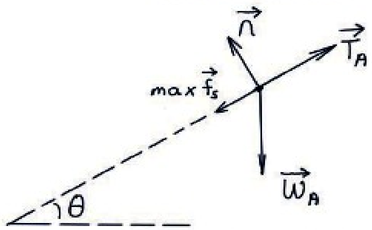
Video (4)

Here is a summary of some of the key steps in the solution:

$$w_A = m_A g$$
$$= Mg$$

$$w_B = m_B g$$
$$= Mg$$

Free-body diagram showing all the forces exerted on block A



Free-body diagram showing all the forces exerted on block B

Force Table for block A (y, x)

Force Table for block B (x, y)

$w_A = Mg$	n	max $f_s = \mu_s n$	$T_A = T$	$w_B = Mg$	$T_B = T$	← magnitudes of the overall force vectors
$w_{Ax} = -Mg\sin\theta$	$n_x = 0$	max $f_{sx} = -\mu_s n$	$T_{Ax} = +T$	$w_{Bx} = 0$	$T_{Bx} = 0$	} components of the forces
$w_{Ay} = -Mg\cos\theta$	$n_y = +n$	max $f_{sy} = 0$	$T_{Ay} = 0$	$w_{By} = +Mg$	$T_{By} = -T$	

Givens: M, θ, g

$$\Sigma F_{Ax} = m_A a_{Ax}$$
$$-Mg\sin\theta + (-\mu_s n) + T = M(0)$$
$$-Mg\sin\theta - \mu_s n + T = 0$$
$$-Mg\sin\theta - \mu_s(Mg\cos\theta) + (Mg) = 0$$
$$-Mg\sin\theta - \mu_s(Mg\cos\theta) + Mg = 0$$
$$+\mu_s(Mg\cos\theta) \qquad +\mu_s(Mg\cos\theta)$$
$$-Mg\sin\theta + Mg = \mu_s(Mg\cos\theta)$$
$$\frac{-Mg\sin\theta + Mg}{Mg\cos\theta} = \frac{\mu_s\cancel{(Mg\cos\theta)}}{\cancel{Mg\cos\theta}}$$

$$\Sigma F_{Ay} = m_A a_{Ay}$$
$$-Mg\cos\theta + n = M(0)$$
$$-Mg\cos\theta + n = 0$$
$$+Mg\cos\theta \qquad +Mg\cos\theta$$
$$n = Mg\cos\theta$$

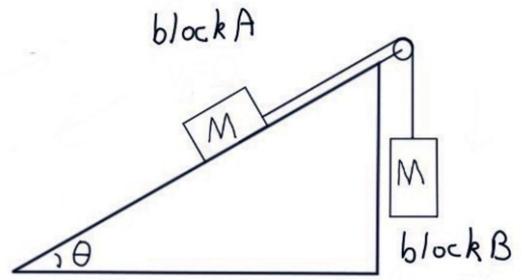
$$\Sigma F_{By} = m_B a_{By}$$
$$Mg + (-T) = M(0)$$
$$Mg - T = 0$$
$$+T \qquad +T$$
$$Mg = T$$

$$\mu_s = \frac{-Mg\sin\theta + Mg}{Mg\cos\theta}$$
$$\mu_s = \frac{\cancel{Mg}(-\sin\theta + 1)}{\cancel{Mg}\cos\theta}$$
$$\mu_s = \frac{-\sin\theta + 1}{\cos\theta}$$
$$\mu_s = \frac{1 + (-\sin\theta)}{\cos\theta}$$
$$\mu_s = \frac{1 - \sin\theta}{\cos\theta}$$

} Most professors would accept any of these expressions as the answer to the problem.

Here is a full solution to the problem:

Two blocks, both with the same mass *M*, are attached by a rope that has been slung over a massless, frictionless pulley. What is the smallest coefficient of static friction μ_s that will prevent the blocks from moving?



? = smallest μ_s
that will prevent the blocks from moving
= borderline μ_s
such that block A is on the borderline
between sliding and not sliding

Assume μ_s equals the borderline value.

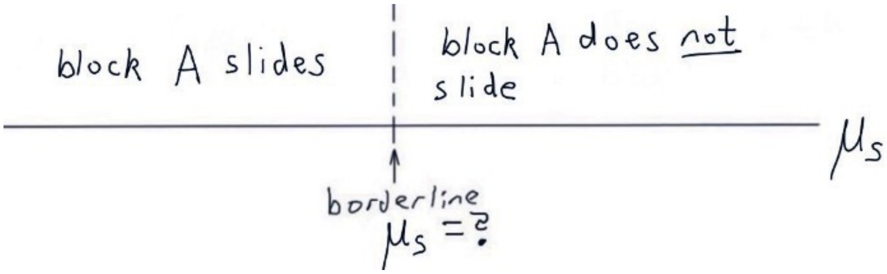
The problem mentions the concepts of mass and friction force, which fit into a Newton’s Second Law framework, so we plan to use the Newton’s Second Law framework to solve the problem.

To distinguish between the two blocks, we have labeled them “block A” and “block B”, as shown in the diagram above.

We identify what part (a) is asking us for by writing down a “?” and a symbol for the what the problem is asking, as shown above:
? = smallest μ_s that will prevent the blocks from moving

Although the problem refers to the “smallest” μ_s, what the problem is “really” asking for is the “borderline” μ_s—the value of μ_s for which the blocks are just on the borderline between starting to move and not starting to move. So we can rewrite the question as shown above:
? = borderline μ_s, such that block A is on the borderline between sliding and not sliding

Therefore, in order to solve the problem, we will **assume** that μ_s is at the borderline value, at which block A is on the borderline between sliding and not sliding. We have written down this assumption, as shown above.



As shown in the diagram above, when μ_s is less than the borderline value, block A will begin to slide. And when μ_s is greater than the borderline value, block A will *not* begin to slide.

What happens if μ_s is *equal* to the borderline value, as in part (a)? Surprisingly, at the "borderline" μ_s, we can assume *either* that block A will slide, *or* that block A will *not* slide, whichever is *convenient* for solving the problem.

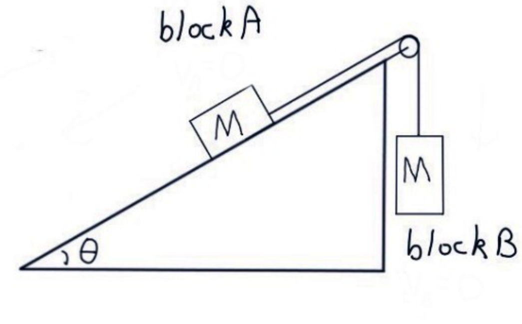
It turns out that, for a "minimum or maximum problem involving whether an object will slide", it is usually convenient to assume that the object will **not** slide. Therefore, for this problem, **we will assume that block A will *not* slide at the borderline μ_s**.

Since we will assume that block A does *not* slide, our plan for this problem is to use *static* friction, rather than kinetic friction.

Since block A will be on the *borderline* of sliding, for this problem we should apply the *maximum* static friction. The reason that block A is on the verge of sliding is because static friction is "maxed out".

Write down all the assumptions we are making for the problem, as shown below:

Two blocks, both with the same mass *M*, are attached by a rope that has been slung over a massless, frictionless pulley. What is the smallest coefficient of static friction μ_s that will prevent the blocks from moving?

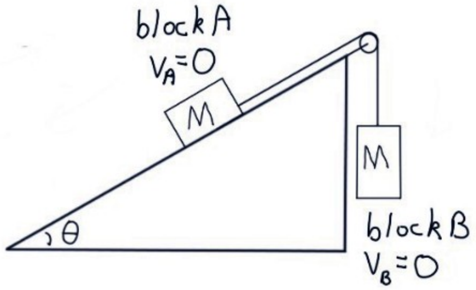


? = smallest μ_s
that will prevent the blocks from moving
= borderline μ_s
such that block A is on the borderline
between sliding and not sliding

Assume μ_s equals the borderline value.
Assume that at this borderline μ_s,
block A will not slide.

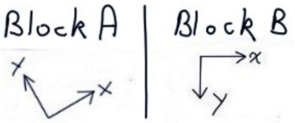
(Why is it convenient to assume that block A will *not* slide? Well, the question is asking us for μ_s. To solve for μ_s, we will need to write down an equation involving μ_s. Since we are assuming that the object is on the borderline of sliding, but is *not* sliding, we can apply maximum static friction in our solution. This allow us to use the special formula for the magnitude of static friction, "max $f_s = \mu_s n$", which is the equation involving μ_s that we need to solve the problem. If we had assumed that block A *does* slide, then we would not be able to apply any equations involving μ_s, so we wouldn't be able to answer the question.)

Two blocks, both with the same mass *M*, are attached by a rope that has been slung over a massless, frictionless pulley. What is the smallest coefficient of static friction μ_s that will prevent the blocks from moving?



? = smallest μ_s that will prevent the blocks from moving
= borderline μ_s such that block A is on the borderline between sliding and not sliding

Assume μ_s equals the borderline value.
Assume that at this borderline μ_s, block A will not slide.

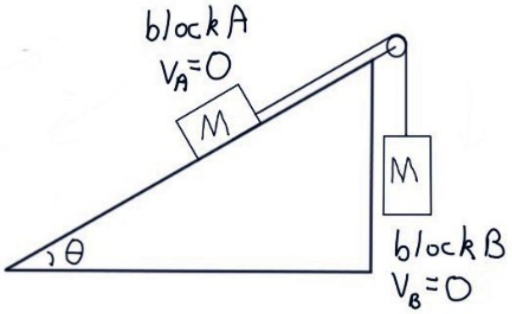


We are assuming for this problem that block A does not slide. In this problem, if block A is not sliding, then it is motionless. Since block B is connected by block A by the rope, if block A is motionless, then block B will be motionless as well. Therefore $v_A = 0$, and $v_B = 0$, as noted in the sketch above.

Because block A is on the borderline of moving up the ramp, we choose the positive x-axis for block A pointing up the ramp. Because Block B is on the borderline of moving downward, we will choose a positive y-direction for block B pointing downward. *Write down* your axes for each block, as shown above.

(How do we know that block A is on the verge of sliding up the ramp, rather than down the ramp, and that block B is on the verge of moving downward, rather than moving upward? We discuss this issue later in this solution, on the page dealing with the Free-body diagrams.)

Two blocks, both with the same mass *M*, are attached by a rope that has been slung over a massless, frictionless pulley. What is the smallest coefficient of static friction μ_s that will prevent the blocks from moving?



? = smallest μ_s that will prevent the blocks from moving

= borderline μ_s such that block A is on the borderline between sliding and not sliding

---

Assume μ_s equals the borderline value.
Assume that at this borderline μ_s, block A will not slide.

Givens: M, θ, g

This problem is a “symbolic” problem, rather than a “numeric” problem, because the problem gives us symbols to work with (*M* and θ) rather than giving us numbers. Therefore, our solution to this problem will give us a chance to illustrate the techniques appropriate for solving symbolic problems. Symbolic problems are common on physics exams!

For a *symbolic* problem, such as this one, we should **make a list of the given symbols**, as shown above.

Rules for which symbols to treat as givens:

A symbol that is explicitly mentioned in the problem is treated as a given.

A symbol that is not explicitly mentioned in the problem is *not* treated as a given.

Exception: A symbol that represents what the question is asking you to determine is *not* treated as a given, even when the symbol is explicitly mentioned in the problem.

Another exception: Physical constants, such as *g*, are treated as givens even when they are not explicitly mentioned in the problem.

For this problem, *M* is treated as a “given”, because it is mentioned in the problem. θ is treated as a given, because it is used in the sketch provided by the problem.

g is treated as a given because it is a known physical constant (9.8 m/s^2).

μ_s is obviously *not* treated as a given, even though the symbol μ_s is mentioned in the problem, since μ_s is what the problem is asking you to determine.

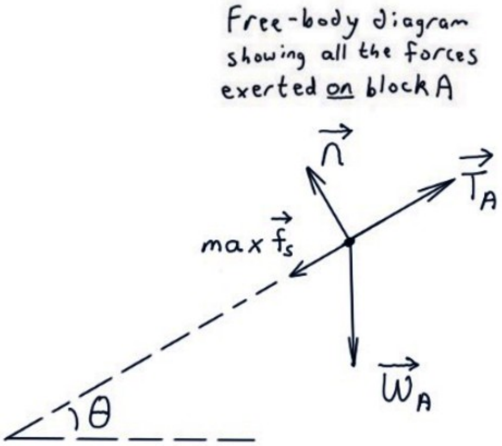
Write down your list of given symbols, as shown above.

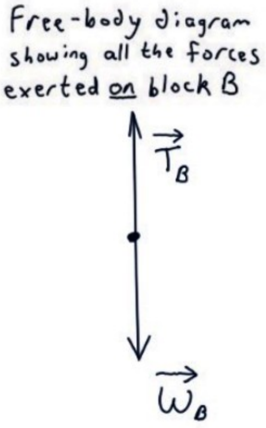
Notice that a symbol that is used in the provided sketch, such as θ, is treated as a given!

We can treat the “given” symbols as “knowns”. Therefore, listing the givens for a symbolic problem will help you to identify which symbols in your equations stand for “knowns” and which symbols stand for “unknowns”.

Furthermore, only symbols that are treated as givens should be included in your final answer.

Draw two separate Free-body Diagrams, one diagram showing all the forces being exerted on block A, and a *separate* diagram showing all the forces being exerted on mass block B.





General two-step process for identifying the forces for your Free-body Diagram for a particular object:
(1) Draw a downward vector for the object’s weight.
(2) Draw a force vector for each thing that is *touching* the object.

In this case, block A is being touched by the surface of the incline, which exerts both a normal force and a frictional force; and by the rope, which exerts a “tension force”. We know that *static* friction applies for this problem because we are assuming that block A does not slide at the borderline μ_s. We apply *maximum* static friction because we are assuming that block A is on the *verge* of sliding.

Block B is being touched only by the rope, which exerts a “tension force”.

The rule for determining the direction of the weight force is: The weight force always points down.

The rule for determining the direction of the normal force is: The normal force points *perpendicular* to, and away from, the surface that is touching the object.

In this problem, the surface touching block A is the incline. So the normal force points perpendicular to, and away from, the surface of the incline.

The rule for determining the direction of static friction is: Static friction points parallel to the surface, in the direction that is necessary to prevent the object from sliding.

Ask yourself, in what direction would block A slide if there were *no* friction? With no friction, would block A slide down the incline, dragging block B behind it? Or would block B fall downwards, dragging block A behind it?

Since both blocks have equal mass, neither is heavier. In that case, your common sense should tell you that, without friction, the freely hanging object, block B, would fall downwards, dragging block A behind it. (Technically, this is because block B is being pulled straight down by the *full* weight force, while block A is being pulled down the incline only by the x-component of the weight force.)

So, without friction, block A would be dragged *up* the incline. Therefore, to prevent block A from sliding, the static friction force must point parallel to, and *down*, the incline.

The rule for determining the direction of the tension force is: The tension force points parallel to the rope, and away from the object. (Ropes can only “pull”; they cannot “push”).

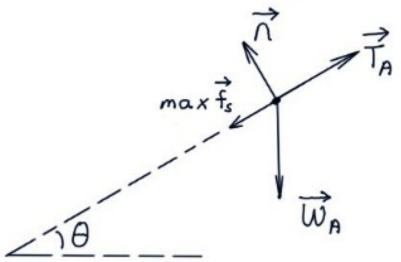
The rope exerts a pulling force up the incline on block A, and an upward pulling force on block B.

Begin a Force Table for block A, and a Force Table for block B.

$$W_A = M_A g$$
$$= Mg$$

$$W_B = M_B g$$
$$= Mg$$

Free-body diagram showing all the forces exerted on block A

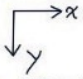


Free-body diagram showing all the forces exerted on block B

Force Table for block A

$W_A = Mg$	n	max $f_s = \mu_s n$	$T_A = T$
$W_{Ax} =$	$n_x = 0$	max $f_{sx} = -\mu_s n$	$T_{Ax} = +T$
$W_{Ay} =$	$n_y = +n$	max $f_{sy} = 0$	$T_{Ay} = 0$

Force Table for block B



$W_B = Mg$	$T_B = T$	← magnitudes of the overall force vectors
$W_{Bx} = 0$	$T_{Bx} = 0$	} components of the forces
$W_{By} = +Mg$	$T_{By} = -T$	

We are applying static friction to this problem, because we are assuming that at the borderline μ_s, block A does not slide. We are assuming that static friction is at its *maximum* value, because we are assuming that μ_s is at its borderline value, at which the object is just on the *verge* of sliding because static friction is "maxed out".

Because we are assuming that static friction is at its maximum value, we can apply the special formula for finding the magnitude of maximum static friction: "max $f_s = \mu_s n$", as shown in the table above.

For a massless rope stretched over a massless, frictionless pulley, **the magnitude of the tension force is the same at both ends of the rope** (but the direction of the tension force may be different at the two ends of the rope). In our Force Tables above, the symbols T_A and T_B, written without arrows on top, stand for the magnitudes of the tension forces on block A and on block B. Because these magnitudes are equal, we can write $T_A = T$, and $T_B = T$, using the same symbol, T, to represent both magnitudes, as shown in the tables above.

We use this rule to determine the components: If a vector is parallel or anti-parallel to one of the axes, then the component for that axis has the same magnitude and direction as the overall vector; and the component for the *other* axis is zero.

It is crucial to include the negative sign on f_{kx}, because the friction force is pointing down the incline, which we have chosen to represent the *negative* x-direction for block A. **It is crucial to include the negative sign on T_{By}**, because the tension force exerted on block B points up, which we have chosen to represent the *negative* y-direction for block B.

Include a "+" sign in front of all positive components. This will help you to remember to include a "-" sign in front of negative components.

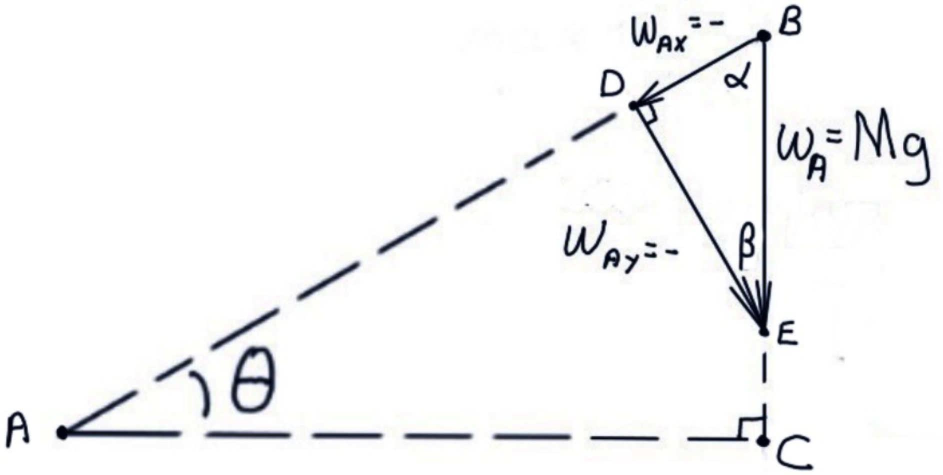
$\vec{w}_A$ is neither parallel nor anti-parallel to the x- and y-axes for block A. Therefore, to break $\vec{w}_A$ into components, we must draw a right triangle and use SOH CAH TOA, as illustrated below.

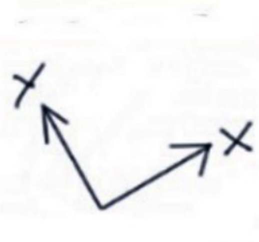
First, draw a right triangle to represent the components.

Use this rule to draw the legs of the right triangle: Draw the legs of the right triangle parallel (or anti-parallel) to the axes. The overall vector forms the hypotenuse of the right triangle.

Use this rule to determine the directions of the components: The components are supposed to represent the overall vector. Therefore, the head of a component arrow should be at the head of the overall vector, or the tail of a component arrow should be at the tail of the overall vector.

Use geometry to determine how to represent the angles inside the right triangle, as described below.





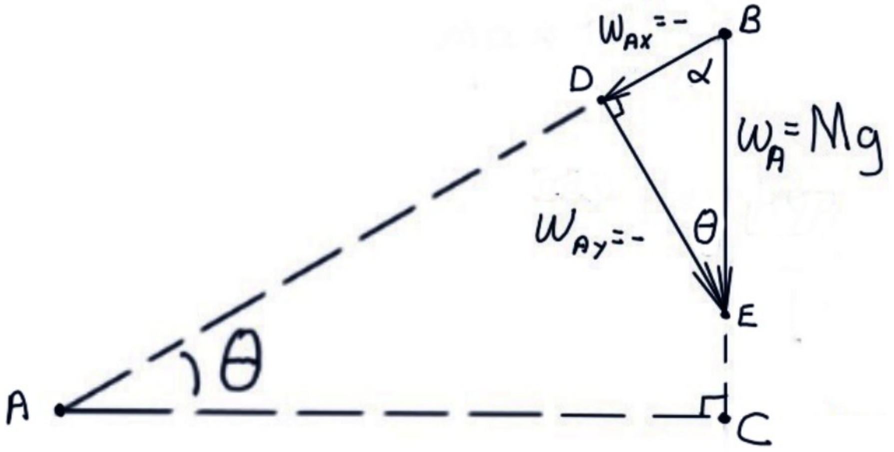
The acute angles of a right triangle add up to 90°.

Angles θ and α are the acute angles in right triangle ΔABC, so: $\theta + \alpha = 90°$

Angles α and β are the acute angles in right triangle ΔBDE, so: $\beta + \alpha = 90°$

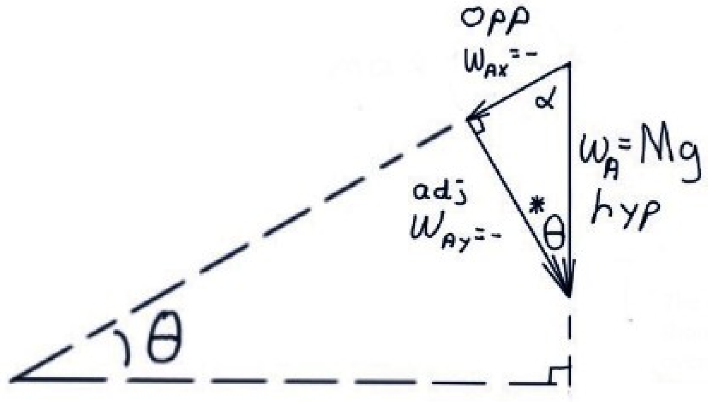
From these equations we see that θ is the angle which adds to α to give 90°, and that β is also the angle which adds to α to give 90°. This tells us that β and θ represent the same size angle.

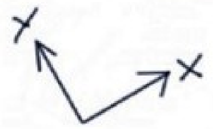
Therefore, β = θ. This allows us to relabel angle β with the symbol θ, as shown below.



Now we can apply the SOH CAH TOA equations, using the angle we have labeled as θ in ΔBDE, as illustrated on the next page.

To indicate that we choose to focus on the angle labeled as θ, we label that angle with an asterisk (*).





SOH CAH TOA

$$\sin\theta = \frac{opp}{hyp}$$
$$\sin\theta = \frac{|w_{Ax}|}{Mg}$$
$$Mg\cdot\sin\theta = \frac{|w_{Ax}|}{\cancel{Mg}}\cdot\cancel{Mg}$$
$$|w_{Ax}| = Mg\sin\theta$$
$$w_{Ax} = -Mg\sin\theta$$

$$\cos\theta = \frac{adj}{hyp}$$
$$\cos\theta = \frac{|w_{Ay}|}{Mg}$$
$$Mg\cos\theta = \frac{|w_{Ay}|}{\cancel{Mg}}\cdot\cancel{Mg}$$
$$|w_{Ay}| = Mg\cos\theta$$
$$w_{Ay} = -Mg\cos\theta$$

We use absolute value signs in our SOH CAH TOA equations to remind ourselves that the SOH CAH TOA equations do not determine the "+" or "-" signs on the components. Therefore, we need to determine the "+" or "-" signs ourselves in a separate step.

It is crucial to include the negative signs on w_{Ax} and w_{Ay}, because w_{Ay} points *into* the incline, which we have chosen to represent the *negative* y-direction for block A, and w_{Ax} points *down* the incline, which we have chosen to represent the negative x-direction for block A.

Now we can substitute these components into our Force Table for block A, as shown below.

Force Table for block A

$w_A = Mg$	n	$\max f_s = \mu_s n$	$T_A = T$
$w_{Ax} = -Mg\sin\theta$	$n_x = 0$	$\max f_{sx} = -\mu_s n$	$T_{Ax} = +T$
$w_{Ay} = -Mg\cos\theta$	$n_y = +n$	$\max f_{sy} = 0$	$T_{Ay} = 0$

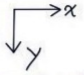
Force Table for block B

$w_B = Mg$	$T_B = T$	← magnitudes of the overall force vectors
$w_{Bx} = 0$	$T_{Bx} = 0$	} components of the forces
$w_{By} = +Mg$	$T_{By} = -T$	

Force Table for block A

$w_A = Mg$	n	$\max f_s = \mu_s n$	$T_A = T$
$w_{Ax} = -Mg\sin\theta$	$n_x = 0$	$\max f_{sx} = -\mu_s n$	$T_{Ax} = +T$
$w_{Ay} = -Mg\cos\theta$	$n_y = +n$	$\max f_{sy} = 0$	$T_{Ay} = 0$

Force Table for block B



$w_B = Mg$	$T_B = T$	← magnitudes of the overall force vectors
$w_{Bx} = 0$	$T_{Bx} = 0$	components of the forces
$w_{By} = +Mg$	$T_{By} = -T$	

Givens: M, θ, g

Now we need to determine what to plug in for a_{Ax}, a_{Ay}, and a_{By} in our Newton’s Second Law equations.

We are assuming that the blocks are not moving.

So, both blocks will be motionless in both the x- and the y-components.

So, we can substitute $a_{Ax} = 0$, $a_{Ay} = 0$, and $a_{By} = 0$ into our Newton’s Second Law equations.

$$\sum_i F_{Ax} = m_A a_{Ax}$$
$$-Mg\sin\theta + (-\mu_s n) + T = M(0)$$
$$-Mg\sin\theta - \underset{\uparrow}{\mu_s} \underset{\uparrow}{n} + \underset{\uparrow}{T} = 0$$

3 unknowns

$$\sum_i F_{Ay} = m_A a_{Ay}$$
$$-Mg\cos\theta + n = M(0)$$
$$-Mg\cos\theta + \underset{\uparrow}{n} = 0$$

1 unknown

$$\sum_i F_{By} = m_B a_{By}$$
$$Mg + (-T) = M(0)$$
$$Mg - \underset{\uparrow}{T} = 0$$

1 unknown

Remember that for this problem the givens, which we treat as knowns, are the symbols M, θ, and g.

(M is a given because it is mentioned in the problem. θ is a given because it is used the sketch that was provided for the problem. g is a given because it is a known physical constant.)

Therefore the symbols in our Newton’s Second Law equations that represent unknowns are μ_s, n, and T.

(Remember that μ_s is not treated a given, even though it was explicitly mentioned in the problem, because μ_s represents what the problem is asking us for.)

Therefore, the Newton’s Second Law x-equation for block A has three unknowns (μ_s, n, and T).

The Newton’s Second Law y-equation for block A has only one unknown (n).

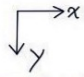
And the Newton’s Second Law y-equation for block B has only one unknown (T).

Therefore, we are not ready yet to solve the Newton’s Second Law x-equation for block A. Instead, now is a good time to solve the Newton’s Second Law y-equation for block A for n, and the Newton’s Second Law y-equation for block B for T, as shown on the next page.

Force Table for block A

$w_A = Mg$	n	$\max f_s = \mu_s n$	$T_A = T$
$w_{Ax} = -Mg\sin\theta$	$n_x = 0$	$\max f_{sx} = -\mu_s n$	$T_{Ax} = +T$
$w_{Ay} = -Mg\cos\theta$	$n_y = +n$	$\max f_{sy} = 0$	$T_{Ay} = 0$

Force Table for block B



$w_B = Mg$	$T_B = T$	← magnitudes of the overall force vectors
$w_{Bx} = 0$	$T_{Bx} = 0$	components of the forces
$w_{By} = +Mg$	$T_{By} = -T$	

Givens: M, θ, g

$$\sum F_{Ax} = m_A a_{Ax}$$

$$-Mg\sin\theta + (-\mu_s n) + T = M(0)$$

$$-Mg\sin\theta - \mu_s n + T = 0$$

↑ ↑ ↑

3 unknowns

$$\sum F_{Ay} = m_A a_{Ay}$$

$$-Mg\cos\theta + n = M(0)$$

$$-Mg\cos\theta + n = 0$$

$$+Mg\cos\theta \quad +Mg\cos\theta$$

$$n = Mg\cos\theta$$

$$\sum F_{By} = m_B a_{By}$$

$$Mg + (-T) = M(0)$$

$$Mg - T = 0$$

$$+T \quad +T$$

$$Mg = T$$

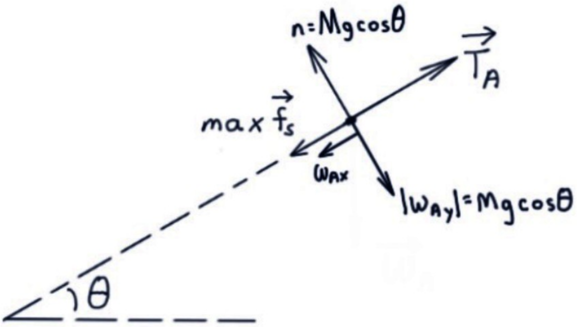
Check: Do the signs of the these results for *n* and *T* make sense? *M* and *g* both represent positive numbers. θ is an acute angle, so cos θ is also positive. (You should know that, for a positive acute angle, both the cosine and the sine of the angle will be positive.) Therefore, our results for *n* and *T* both came out to be positive. Does that make sense?

n and *T* both stand for magnitudes. A magnitude can never be negative, so, yes, it makes sense that our results for *n* and *T* both came out to be positive.

Check: Does our result for the size of *n* make sense? In order to prevent block A from beginning to move into the surface of the incline, $\vec{n}$ must cancel w_{Ay}. So, yes, it makes sense that:

$n = Mg\cos\theta = |w_{Ay}|$

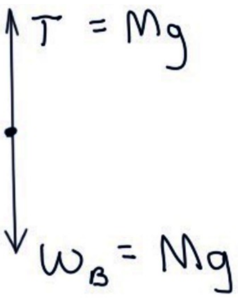
In the version of the Free-body Diagram on the right, I have drawn the arrow for $\vec{n}$ the same length as the arrow for w_{Ay} to reflect this relationship.



Check: Does our result for the size of *T* make sense? We are assuming that block B is motionless. In order to prevent block B from beginning to move downward, $\vec{T}_B$ must cancel w_{By}. So, yes, it makes sense that:

$T_B = T = Mg = w_B$

In the version of the Free-body Diagram on the right, I have drawn the arrow for $\vec{T}_B$ the same length as the arrow for w_B to reflect this relationship.



Next, we substitute our results for *n* and *T* into the Newton’s Second Law x-equation for block A.

After these substitutions, the Newton’s Second Law x-equation for block A has only one unknown remaining, μ_s. (Remember, *M*, θ, and *g* are treated as givens in this problem.) So we are now ready to solve the Newton’s Second Law x-equation for block A for μ_s.

$$\sum F_{Ax} = m_A a_{Ax}$$

$$-Mg\sin\theta + (-\mu_s n) + T = M(0)$$

$$-Mg\sin\theta - \mu_s n + T = 0$$

$$-Mg\sin\theta - \mu_s(Mg\cos\theta) + (Mg) = 0$$

$$\begin{aligned} -Mg\sin\theta - \mu_s(Mg\cos\theta) + Mg &= 0 \\ +\mu_s(Mg\cos\theta) \qquad\quad &\;\; +\mu_s(Mg\cos\theta) \\ \hline -Mg\sin\theta + Mg &= \mu_s(Mg\cos\theta) \end{aligned}$$

$$\frac{-Mg\sin\theta + Mg}{Mg\cos\theta} = \frac{\mu_s\cancel{(Mg\cos\theta)}}{\cancel{Mg\cos\theta}}$$

$$\mu_s = \frac{-Mg\sin\theta + Mg}{Mg\cos\theta}$$

$$\mu_s = \frac{\cancel{Mg}(-\sin\theta + 1)}{\cancel{Mg}\cos\theta}$$

$$\mu_s = \frac{-\sin\theta + 1}{\cos\theta}$$

$$\mu_s = \frac{1 + (-\sin\theta)}{\cos\theta}$$

$$\mu_s = \frac{1 - \sin\theta}{\cos\theta}$$

Most professors would accept any of these expressions as the answer to the problem.

$$\sum F_{Ay} = m_A a_{Ay}$$

$$-Mg\cos\theta + n = M(0)$$

$$\begin{aligned} -Mg\cos\theta + n &= 0 \\ +Mg\cos\theta \qquad &\;\; +Mg\cos\theta \\ \hline n &= Mg\cos\theta \end{aligned}$$

$$\sum F_{By} = m_B a_{By}$$

$$Mg + (-T) = M(0)$$

$$\begin{aligned} Mg - T &= 0 \\ +T \quad &\;\; +T \\ \hline Mg &= T \end{aligned}$$

We have simplified and rearranged our result for μ_s to obtain the most “elegant” way to express the answer. But most professors would probably accept any of the expressions for μ_s listed above as the answer to the problem.

Our simplification of the result is based on “cancelling” the *Mg* term from the numerator and denominator of the fraction. The video contains further discussion of how you can tell when cancelling is or is not a mathematically “legal” technique.

$$\mu_s = \frac{1-\sin\theta}{\cos\theta}$$

Check: Does the sign of our result for μ_s make sense?

For any positive acute angle, the cosine and sine of the angle are both positive numbers between 0 and 1.

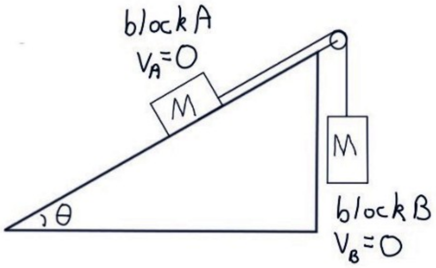
On this problem θ is an acute angle, so we know that $0 < \cos\theta < 1$ and $0 < \sin\theta < 1$. Since sin θ is less than 1, we know that $1 - \sin\theta$ is positive. Therefore, both the numerator and the denominator of our result for μ_s represent positive numbers, so we know that our result for μ_s is positive.

This does makes sense, because the coefficient of friction can never be a negative number.

It is interesting to note that neither *M* nor *g* appears in our final expression for μ_s. This means that, according to our answer, neither *M* nor *g* has any effect on the minimum μ_s that will prevent block A from sliding. This means that changing the mass of the blocks (changing *M*) does not affect whether block A will slide. Also, redoing the experiment on a different planet or on the moon (i.e., changing *g*), would not affect whether block A will slide!

For the sake of simplicity, we do not try to explain these interesting results in the video.

Two blocks, both with the same mass *M*, are attached by a rope that has been slung over a massless, frictionless pulley. What is the smallest coefficient of static friction μ_s that will prevent the blocks from moving?



? = smallest μ_s that will prevent the blocks from moving

= borderline μ_s such that block A is on the borderline between sliding and not sliding

Answer:

$$\boxed{\text{The smallest coefficient of static friction that will prevent the blocks from moving is } \mu_s = \frac{1-\sin\theta}{\cos\theta}}$$

Givens: M, θ, g

Most professors would probably accept any of the expressions for μ_s listed on the previous page as a correct answer to the problem.

Make sure your final answer includes only the given symbols. In this problem, our final answer should include only the symbols *M*, θ, and *g*. In fact, as we have seen, we were able to simplify our answer so that it included only the symbol θ, which is indeed on our list of givens.

If your final answer included the symbol *n* or the symbol *T*, then your answer would be considered incorrect, because *n* and *T* are not treated as “givens” in this problem.

Problem Recap on next page.

Recap:

Notice that **we needed to choose different axes for block A and for block B.**

For a massless rope stretched over a massless, frictionless pulley, **the magnitude of the tension force is the same at both ends of the rope** (but the direction of the tension force may be different at the two ends of the rope). In our Force Tables, we used this rule to write $T_A = T$, and $T_B = T$, using the same symbol, T, to represent both magnitudes.

The problem asks for the minimum μ_s that will prevent block A from sliding. We can interpret this question as asking for the “borderline” μ_s, at which block A is on the borderline between sliding and not sliding.

To answer that question, we assumed that μ_s was equal to the borderline value.

And we assumed that, at the borderline value for μ_s, block A will *not* slide.

The assumption that block A is not sliding allowed us to apply static friction in our solution.

The assumption that block A is on the *verge* of sliding allowed us to apply *maximum* static friction, which meant that we could use **the special formula “max $f_s = \mu_s n$”**.

The assumption that block A is not sliding meant that neither block is moving, which allowed us to **substitute $a_{Ax} = 0$, $a_{Ay} = 0$, and $a_{By} = 0$** in our Newton’s Second Law equations.

In summary, **to solve a maximum or minimum problem involving whether an object will slide:**

Assume that the object is on the *borderline* between sliding and not sliding.

Assume that, at the borderline, the object will *not* slide.

Apply maximum static friction, and use the special formula “max $f_s = \mu_s n$”.

Use the assumption that the object is not sliding to substitute for the acceleration components in your Newton’s Second Law equations.

This problem was a “symbolic”, rather than a numeric, problem. **For a symbolic problem, make a list of the symbols that you will be treating as “givens”.** The givens are treated as “knowns”; symbols that are not on the list of givens are treated as unknowns. This allows you to count the number of unknowns in your Newton’s Second Law equations, which helps you to determine which equations are ready to be solved.

Also remember that your final answer to the problem should include only symbols that are on the list of givens.

Think in terms of components. Make sure you understand **how to break the weight vector into components for a symbolic problem involving an inclined plane**. The key step was using geometry to determine which angle to label as θ in the right triangle that we drew to represent the components.

For a symbolic problem, students often forget to include the negative signs on negative components. Remember, **include a positive sign in front of all your positive components**, so that you will be more likely to notice when you need a negative sign in front of a negative component.

Write the general equation before you plug in specific numbers or symbols. We illustrated this technique in our solution to this problem.