Epistemic Numbers

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1 Some basic facts

- ***Epistemic Numbers (ENs)***
  Syncatically complex numbers formed by some combination of number and (non-numerical) indefinite expressions that convey uncertainty, vagueness or ignorance as to the exact value in question.

(1) a. Twenty-some people came to the party.

b. I have won over thirty-some thousand dollars at the local bingo.

- Like other numbers, ENs denote lower (and can denote) upper bounds. Unlike other numbers, ENs seem to denote a range of values determined by the numbers that fit in the same numeral structure.[1]

(2) a. S knows that only 19 people came (or e.g. $[15, 29]$, etc.) ✘(1a)

b. S knows that 32 people came (or e.g. $[21, 32]$. etc.) ✘(1a)

c. S knows that $[21, 29]$ people came ✓(1a)

2 Mendia (2018)

- ***ENs as minimal domain wideners***
  The indefinite *some* denotes an anti-singleton subset selection function over numbers, f. It takes a set of numbers and returns some non-singleton subset (based on Alonso-Ovalle and Menéndez-Benito 2010).

(3) $[\![f]\!] = \lambda D_{\langle dt\rangle}.\lambda d_d : |f(D)| > 1\,.\,f(D)(d)$

- ***Implementation***
  Additive numbers, such as *twenty-two* and *twenty-some*, are syntactically complex (Hurford 1987) and semantically interpreted by an operation of ADD(ITION) (Rothstein 2017); e.g. [20 [ADD 2]].

- ENs come come in [Spec, NumP] (e.g. Solt 2015 a.o.) and combine with NP through the mediation of a silent NUM head (e.g. Kayne 2005, Zweig 2005, Schwarzschild 2006, a.o.).

(4) a. $[_{DP}\ [_{NumP}\ [_{DP}\ \text{numeral}][_{Num'}\ \textsc{Num}\ \text{NP}]]]$

b. $[\![\textsc{Num}]\!] = \lambda P_{\langle et\rangle}.\lambda d_d.\lambda x_e.P(x) \wedge |x| = d$

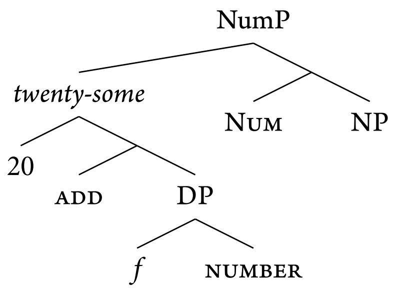
- Due to its condition as anti-singleton, f(NUMBER) is itself a set. Accordingly, the addition operation ADD operates on sets, rather than entities.

1 In this sense they are not approximate numbers like *around/about twenty* (Sauerland and Stateva 2007, Krifka 2009).

(5) a. $[\![\textsc{add}]\!] = \lambda D'.\lambda D''.\ \lambda d.\exists d'd''[\ d = d' + d'' \wedge D'(d') \wedge D''(d'')]$

b. $[\![23]\!]\ =\ \lambda d.d = 23$

(6)



(7) a. $[\![DP]\!] = [\![f]\!]([\![\textsc{number}]\!])\ =\ \lambda d\ .f(0 < d < 10)$

b. $[\![\textit{twenty-some}]\!]$
$= [\![\textsc{add}]\!]([\![\mathrm{DP}]\!])([\![20]\!])$
$= \lambda d.\exists d'd''[d = d' + d'' \wedge [\![20]\!](d')\ \wedge f(0 < d'' < 10)]$

(8) $[_{CP}\ A(\text{twenty-some})\ [_{CP}\ \lambda d\ [_{TP}\ \exists\ t_d$ students came $]]]$

(9) $\exists d[$I paid $20 +$ *d-many* dollars $\wedge d \in f(\{d : 0 < d < 10\})]$

- ***Epistemic effect***
(9) triggers a competition with all the singleton number values in the range of the epistemic number, as expressed in (10) (Alonso-Ovalle and Menéndez-Benito 2010, a.o.).

(10) a. $\exists d[$I paid $20 +$ *d-many* dollars $\wedge d \in \{1\}]$

…

b. $\exists d[$I paid $20 +$ *d-many* dollars $\wedge d \in \{9\}]$

- Assuming the speaker was truthful (11), the singleton (more informative) alternatives to (9) must have been disregarded by S for some reason—presumably because S wasn't in a position to commit to the truth of any one of them—allowing the addressee to infer a set of Primary Implicatures:

(11) $\Box_{epi}(\exists d[$I paid $20 +$ *d-many* dollars $\wedge d \in f(\{d : 0 < d < 10\})])$

(12) a. $\neg \Box_{epi}\ (\exists d[$I paid $20 +$ *d-many* dollars $\wedge d \in \{1\}])$

…

b. $\neg \Box_{epi}\ (\exists d[$I paid $20 +$ *d-many* dollars $\wedge d \in \{9\}])$

➠ Together (11) and (12) correspond to modal variation: $\neg\exists d \in D\ \Box_{epi}\ [\varphi(d)]$ (not Free Choice).

- The speaker's epistemic state is incompatible with full knowledge about the number in question.
- The speaker need not be ignorant about *all* the values in the relevant range (cf. disjunction).

➠ The epistemic effect arises as a conversational **quantity** implicature; the range of values considered depends on (language-specific) properties of cardinal number systems.

3 Assessment

❶ ***Scope***
In Mendia (2018) the DP *twenty-some* in [Spec, NumP] QRs to CP. However, ENs seem to be interpretable in positions where QR cannot take them:

(13) We have organized a big party at our place. We don't want to get a fine so I asked the building manager what our maximum capacity is. She told me the exact number, but all I can remember is that it was somewhere between 21 and 29.

a. If twenty-some people come to the party, we'll get a fine.

b. ✓There is a number n of people st. if n-many people come, we will get a fine.
$\rightsquigarrow$ we get a fine for some specific unknown number between 21 and 29.

c. #If there is a number n of people st. n-many people come to the party, we will get a fine.
$\rightsquigarrow$ If 21-29 people come, we get a fine.

- In (13b) we have a claim about a *specific unkown* **number** of people, certainly not a claim about specific (known or unknown) people. This is only possible if the EN is interpreted outside *if*. (Note also that there is no inverse distribution of the EN over the indefinite *a fine*.)

(14) I should know about the warm-up routine in PE class, but I can't recall it exactly.

a. Most students do ten pull-ups and twenty-some push-ups in their warm-up routine.

b. ✓There is a number n st. most students do 10 pull ups and n-many push-ups.
$\rightsquigarrow$ most students do the same amount n of push-ups.

c. ✓Most students are st. they do 10 pull ups and 21-29 push-ups.
$\rightsquigarrow$ most students do anywhere between 21 and 29 push-ups.

- In fact we can anchor the epistemic effect at different levels, the widest of which cannot be reached by QR.

(15) Sam said that there were twenty-some people at the party... (Anderson 2020)

a. ...but I don't know exactly how many she said there were. *speaker ignorance*

b. ...but she didn't know exactly how many (actually, there were 25). *subject ignorance*

➠ ENs pattern with other indefinites in that they can take exceptional scope.

❷ ***The persistence of the epistemic effect***

With a variety of ignorance-inducing expressions, the epistemic effect vanishes in certain contexts. With ENs obviation seems to be harder, if possible at all.

(16) a. Liz must read <u>Kafka or Poe</u>, it doesn't matter which one.

b. Liz must read <u>some poem</u>, it doesn't matter which one.

c. Liz must read <u>at least three papers</u>, as long as she reads three she'll pass.

d. Liz must do <u>twenty-some push-ups</u> to pass PE, #as long as she does 21 she'll pass.

(17) a. Every student read <u>Kafka or Poe</u>, Liz read A, Sam read B, etc.

b. Every student read <u>some poem</u>, Liz read A, Sam read B, etc.

c. Every student read <u>at least three papers</u>, Liz read 3, Sam read 4, etc.

d. Every student did <u>twenty-some push-ups</u>, #Liz did 22, Sam did 27, etc.

➠ The epistemic effect of ENs does not align squarely with that of other ignorance-inducing expressions.

4 Rethinking ENs

- ***Rationale***
  Intuitively, indefinite *some* in *twenty-some* is picking some value from a set of allowable options. The value picked by *some* in ENs is (*i*) specific but (*ii*) unknown to the speaker; i.e. it can be "scopally specific" wrt. its number value—to use Farkas's (2002) term—but it must be "epistemically non-specific."

- In this sense it resembles other "specific unknown" indefinites such as *wh-oo* series in Kannada (Haspelmath 1997), Russian *-to* (Yanovich 2005), *khi* series in Tiwa (Dawson 2020), a.o. We can also find uses of (stressed) *some* outside ENs (Warfel 1972).[2]

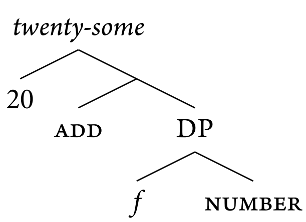
➠ It seems quite natural then to attribute the task of "picking a value from a set" to a Choice Function (CF). Two things to do: fix the semantics, account for the epistemic effect.

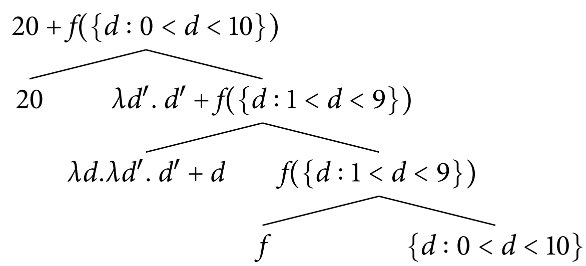
➠ ***Choice Functions for ENs***
As above, indefinites in ENs range over a set of numbers in the language. A CF applies to a set of numbers and delivers a single *d*-type value. We keep the syntactic configuration as before, but the semantics are simplified substantially:

(18) A function f is a Choice Function (CF) if for any non-empty set A, $f(A) \in A$.

(19) a. $[\![23]\!] = 23$ (numbers are now simplex degrees, type d)

b. $[\![\textsc{add}]\!] = \lambda d.\lambda d'.\, d' + d$

(20)


(21)


- Existential Closure binds f (Reinhart 1997, Winter 1997, Matthewson 1999).

(22) a. Twenty-some students came.

b. $\exists f\,[\exists x\,[came(x) \wedge students(x) \wedge |x| = 20 + f(\{d: 0 < d < 10\})]]$
$\rightsquigarrow$ There is a function f picking number n s.t. n-many students came.

- We saw above that ENs in embedded positions may be interpreted with both wide and narrow scope.

(23) Every student wrote twenty-some rhymes.

a. $\exists f$ [every student] $\lambda 1$ [t_1 wrote twenty-f(*some*) rhymes]]]] *wide scope*
$\rightsquigarrow$ There is a function f picking number n s.t. every student wrote n-many rhymes

2 E.g. *Ralph is worried because he lost* ***some*** *letters he was supposed to mail (#but I have them right here).*

- There are various options to capture the narrow scope of (23). The simplest one is to allow quantification over f to be scopally mobile.[3]

(24) Every student wrote twenty-some rhymes.

a. [every student] $\lambda 1\ \exists f$ [t_1 wrote twenty-f(*some*) rhymes]]] *narrow scope*
$\rightsquigarrow$ For every student, there is a function f picking number n s.t. every student wrote n-many rhymes

- The account also predicts intermediate scope interpretations with ENs, but these are difficult to detect.

(25) a. Every professor rewarded every student who solved twenty-some questions.

b. For every professor x there is a specific number n of questions between 21 and 29 s.t. every student who solved n-many questions was rewarded by x.

➠ ***Epistemic effect***
We want to know how ENs give rise to an epistemic effect under a CF analysis.[4]

- ***Not informativity***
With CFs, ENs are *specific unknown* indefinites. But as we saw above and unlike run-of-the-mill indefinites, the epistemic effect of ENs is quite pervasive.

- In the earlier account, *some* in ENs was constrained to provide anti-singleton sets, and the epistemic effect arose pragmatically by competition with more informative (singleton) alternatives.

- It is not so clear whether a similar reasoning could apply here as well. With CFs ENs no longer deliver a set of values. Competition happens by comparing propositions that pick individual values; in the case of ENs, there is no way to discern what the value is (at least not without having access to a full model). This presents an obstacle to competition in terms of informational strength.

- ***A secondary route: manner***
Rather than in strength, ENs and its cardinal number alternatives compete by virtue of their different *forms*.
  - Suppose some forms are preferred over others. In the case of numerals, we may conjecture that cardinal numbers are *generally* preferred over ENs, approximates, etc.[5]
  - If so, all else equal, a speaker should chose a form *cn* over a form *en*.
  - Thus, whenever a speaker chooses the dispreferred form *en*, it must be that **not all else is equal**: The preference for *cn* must have been defeated by more important concerns, and the addressee must infer the existence of such higher-pressing matters.

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3 We could also parametrize the CF (Kratzer 1998, Chierchia 2001). Parametrized CFs are functions from individuals to choice functions, where the individual argument can be provided by a covert variable bound by a higher quantifier. E.g.:

(i) $\exists f$ [every student$_x$] $\lambda 1$ [t_1 wrote twenty-$f_1(x, \textit{some})$ rhymes]]]].

4 Appealing to CFs does not commit us to epistemic specificity.

> [...] there is a crucial difference between saying that the speaker has a particular choice function in mind, and saying that the speaker has a particular individual in mind. [...] (one) might have had a particular choice function in mind, yet might not necessarily have known what individual was picked out by that function. (Kratzer 2003)

Still, we should not allow ENs to be epistemically specific.

5 Additional preferences may include shorter/less complex structures *à la* Katzir (2007), Fox and Katzir (2011), Katzir (2014).

- ***Pressing matters***
  What could those pressing matters be? What drives lack of commitment to a preferred form? Lack of knowledge, avoidance to make a false claim, irrelevance, indifference, etc.

(26) For some (exact) number value n:

a. *Cooperativity*: The speaker is willing to commit n to the common ground.

b. *Authority*: The speaker is informed about n.

c. *Relevance*: n is relevant to the conversational purposes of the interlocutors.

- If these conditions are met, a rational speaker would **not** chose an EN. Thus, the use of an EN signals that at least one of (26) does not hold.
  - A failure to abide by *authority* would result in an ignorance implicature.
  - A failure in *relevance* would result in an indifference implicature.
  - Failing to be *cooperative* may signal specific scenarios, such as clue-providing/taunting and "TV game show" scenarios.
- General schema for when this type of pervasive epistemic effects arises:
  - *S* is faced with a choice between two forms *en* and *cn*.[6]
  - There is a **preference** for *cn* over *en* **that applies across contexts**.
  - Uttering *en* will make the preferred form *cn* **salient**.
  - The asserted content in *en* does **not asymmetrically entail** *cn* (i.e. *cn* is not weaker).

5 Final thoughts

- ***What we achieved***
  Resorting to CFs provides a straightforward semantic analysis.
- The epistemic effect is a manner rather than a quantity implicature.
- ***Moving forward***
  The price to pay for the simplified semantics is a potential proliferation of interpretations, some of which may not be available.
- If we can argue that generally CF indefinites compete with other forms (rigid designators? definite descriptions? referential expressions?) we might expect similar behavior to ENs in the right contexts. (E.g. restrictions in utilizing a wide-scope specific indefinite when the witness is known to all.)
- We must think about embedding (e.g. (15)). This seems to be a general property of manner implicatures:

(27) The judge believes that Jane caused the sheriff to die.

a. *Local*: the judge does not believe that she killed him.

b. *Global*: *S* does not believe that she killed him.

6 Assuming all ordinary cardinal numerals share the same form.

- More generally, weak implicatures are non-uniform not only in their logical signatures—e.g. disjunction *vs.* modal variation—but also in their driving forces—quantity *vs.* manner considerations.
- We now have two "black boxes": a notion of *preference* that drives competition between forms and a notion of *saliency* that brings attention to certain forms.

A Cross-linguistic variation in ENs

A.1 Base systems

- Basque, unlike English or Spanish, has a predominantly base 20 number system.

(28) *hogei - ta - hamaika*
twenty and eleven
'thirty-one'

(29) *ber - hogei - ta - hamaika*
two twenty and eleven
'fifty-one'

- Western Basque can also forms ENs, but the possible values vary accordingly with the base.

(30) *hogei - ta - sa - k*
twenty and INDF PL
✓ [21,39] ✘ ≤ 20 ✘ ≥ 40

A.2 Types of indefinites

- Unlike English, Spanish allows indefinites other than *some* in ENs.

(31) *sesenta y mucho*
sixty and many
✓ [66,69] ✘ ≤ 65 ✘ ≥ 70

(32) *sesenta y poco*
hundred and few
✓ [61,64] ✘ ≤ 60 ✘ ≥ 65

A.3 Additive *vs.* multiplicative environments

- In Spanish/English, indefinites cannot usually precede the number.

(33) *veinte - i - algo*
twenty and some
twenty-some

(34) **algo - i - seis*
some and six
Int.: some-six

- Languages such as Japanese seem to generally allow such ENs.

(35) a. *nan - juu - nin - ka - ga*
what ten CL-HUMAN PART NOM
⇝ 10, 20, 30, 40, 50, 60, 70, 80, 90

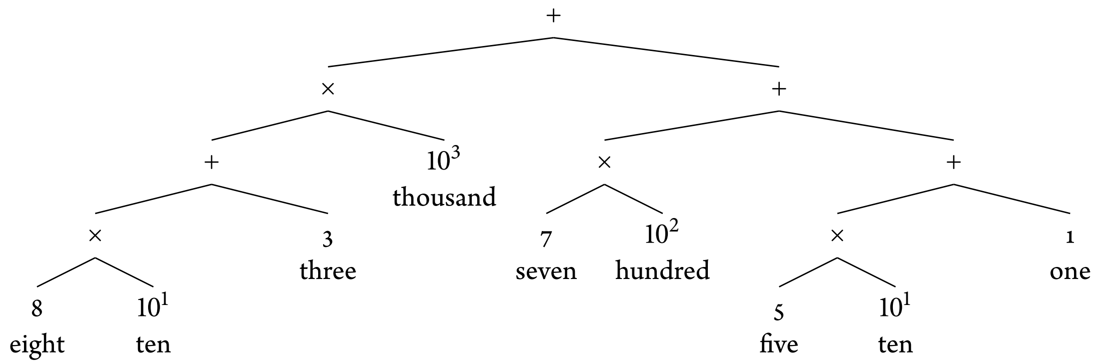
b. *hyaku - nan - juu - nin - ka - ga*
hundred what ten CL-HUMAN PART NOM
⇝ 110, 120, 130, 140, 150, 160, 170, 180, 190

B Some more details on Mendia (2018)

- ***Hurford's insight***
  Hurford (1975, 1987) provides evidence for two fundamental aspects of complex numerals:

❶ Syntactically, complex numerals have a complex syntactic structure.

(36) Structure of the number 83751



❷ Semantically, multiplicative and additive environments are interpreted as multiplication and addition operations on numbers.

(37) $[\![83751]\!] = \Bigg(\bigg(\Big(\big((8 \times 10) + 3\big) \times 10^3\Big) + \Big(\big(7 \times 10^2\big) + \big((5 \times 10) + 1\big)\Big)\bigg)\Bigg)$

- ***Implementation***
  Numbers come in [Spec, NumP] (e.g. Solt 2015 a.o.). They are interpreted as singleton sets of degrees.

(38) a. $[_{DP}\ \mathrm{D}\ [_{NumP}\ [_{DP}\ \text{numeral}][_{Num'}\ \textsc{Num}\ \mathrm{NP}]]]$

b. $[\![23]\!] = \lambda d.d = 23$

c. $[\![\textsc{Num}]\!] = \lambda P_{\langle et\rangle}.\lambda d_d.\lambda x_e.P(x) \wedge |x| = d$

- Complex cardinals have a coordinate structure (Ionin and Matushansky 2006), headed by an additive head ADD (cf. Anderson 2015). Multiplicative cardinals are similarly syntactized (Rothstein 2017, Mendia 2018).

(39) a. $[\![\textsc{add}]\!] = \lambda D'.\lambda D''.\ \lambda d.\exists d'd''[\ d = d' + d'' \wedge D'(d') \wedge D''(d'')]$

b. $[\![\textsc{mul}]\!] = \lambda D'.\lambda D''.\ \lambda d.\exists d'd''[\ d = d' \times d'' \wedge D'(d') \wedge D''(d'')]$

- There is more structure to numbers than meets the eye. Cardinal numbers are the product of some integer $n \in \{1, 2, \ldots, i\}$ and a numerical *base* B^n, where $B^n = 10^n$ for some numbers B and n. In the decimal system (Spanish, English), B is a power of 10: $[\![B^n]\!] = 10^n$. Different number systems may require different bases.

(40) a. **English/Spanish**
$[\![\ 30\ ]\!] = [\ 3\ [\textsc{mul}\ B^1]]\ = [\ 3\ [\textsc{mul}\ 10]]$ *three tens*

b. **Basque/Georgian**
$[\![\ 30\ ]\!] = [\ 20\ [\textsc{add}\ 10]]$ *twenty and ten*

c. **Kanum (Tondan)**
$[\![\ 30\ ]\!] = [\ 5\ [\textsc{mul}\ B^1]]\ = [\ 5\ [\textsc{mul}\ 6]]$ *five sixes*

- In decimal, for 83751 we have:

(41) $[\![83751]\!] = \Big[\Big[\big[(8\times 10)+3\big]\times 10^3\Big] + \Big[\big[7\times 10^2\big] + \big[(5\times 10)+1\big]\Big]\Big]$

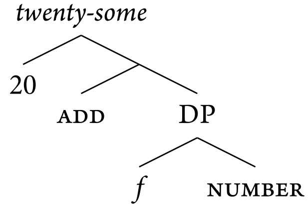
(42) $[\![83751]\!] = \Big[\Big[\big[[8 \text{ MUL } B^1] \text{ ADD } 3\big] \text{ MUL } B^3\Big] \text{ ADD } \Big[\big[[7 \text{ MUL } B^2] \text{ ADD } \big[[5 \text{ MUL } B^1] \text{ ADD } 1\big]\big]\Big]\Big]$

- ***Extension to ENs***
How does an indefinite ever get to appear in the place of a number?

(43) a. twenty-some books = [[twenty ADD [some NUMBER]] NUM books]

b. $[\![\text{NUMBER}]\!] = \{d : B^i > d > B^j\}$, where i and j depend on the base positions syntactically available in each case. (E.g. for *twenty-some*, $[\![\text{NUMBER}]\!] = \{d : B^0 > d > B^1\}$, for *a hundred-some* $[\![\text{NUMBER}]\!] = \{d : B^1 < d < B^2\}, \ldots$)

(44) *twenty-some*



(45) a. $[\![\text{DP}]\!] = [\![f]\!]([\![\text{NUMBER}]\!]) = \lambda d\,.\,f(0 < d < 10)$

b. $[\![\textit{twenty-some}]\!]$
$= [\![\text{ADD}]\!]([\![\text{DP}]\!])([\![20]\!])$
$= \lambda d.\exists d'd''[d = d' + d'' \wedge [\![20]\!](d') \wedge f(0 < d'' < 10)]$

- ***Final touches***
There is a type mismatch between the EN ($\langle dt\rangle$) in [Spec, NumP] and it's complement $[_{Num'}$ NUM NP$]$ ($\langle d, et\rangle$). Lowering operations like ι/MAX cannot apply since they both return unique values. So we raise:[7,8]

(46) a. Generalized A: Shift $P_{\langle\sigma t\rangle}$ to $\lambda Q_{\langle\sigma t\rangle}.\exists\sigma[P(\sigma)\wedge Q(\sigma)]$. [cf. Partee 1987]

b. $[_{CP}\ A(\text{twenty-some})\ [_{CP}\ \lambda d\ [_{TP}\ \exists\ t_d$ students came $]]]$

7 A third solution: allow NUM to provide existential quantification over degrees: $\lambda P_{\langle et\rangle}.\lambda D_{\langle dt\rangle}.\lambda x\,.\,P(x)\wedge\exists d[|x| = D(d)]$.

8 Note that, independent of the scope facts related to above, QR-ing ENs is not problem free either: the operation involves left-branch extraction of a numeral, which is usually not allowed (*[How many]$_i$ did you eat t$_i$ bananas?). Moreover, QR-ing the EN runs the risk of violating the Heim-Kennedy Constraint (Bhatt and Pancheva 2004, 15): if the scope of a quantificational DP contains the trace of a DegP, it also contains that DegP itself.

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