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Epistemic numbers*

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Abstract I provide an analysis of *epistemic numbers*, such as *twenty-some NP*. Epistemic numbers are syntactically complex numbers formed by some combination of number and (non-numerical) indefinite expressions that convey uncertainty, vagueness or ignorance. I examine these constructions in a number of languages and provide an analysis that capitalizes on independently motivated properties of their two sub-components: epistemic indefinites and complex numerals. I argue that variability in these two components explain the cross-linguistics variability in the nature and interpretation of epistemic numbers across languages.

Keywords: Epistemic indefinites, numerals, implicatures, ignorance, uncertainty

1 Introduction

Natural languages allow a variety of ways of expressing indeterminacy, uncertainty and ignorance. Epistemic indefinites, also referred to occasionally as anti-specific indefinites, figure prominently among these. Epistemic indefinites are indefinite determiners or pronouns that convey information about the epistemic state of the speaker, by signaling that she is ignorant or uncertain about some claim. These type of indefinites are moreover, well attested in a multitude of typologically unrelated languages (e.g. see the survey in Haspelmath 1997). For instance, in ordinary out of the blue contexts, English *some*, German *irgendein* and Spanish *algún* all signal that the speaker does not know what the witness of the existential claim is. In the case of (1), that the speaker does not know what doctor María married, as signaled by the (in)appropriateness of the follow-ups in (2).

(1) a. María married some doctor.
b. Maria hat irgendeinen Arzt geheiratet.
Maria has IRGENDEINEN doctor married
c. María se ha casado con algún médico.
María REFL has married with ALGÚN doctor

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(2) a. But I don't know who.
b. # She married Dr. Smith.

Epistemic indefinites however need not convey ignorance about particular individuals. As Weir (2012) discusses, English *some* may also combine with subkind denoting NPs to convey epistemic ignorance of the speaker about the relevant subkind, but not about the witness of the existential claim. In the two examples below (from Weir 2012: 180), there is no doubt that the speaker knows for a fact what is the particular individual she is referring to; instead, *some* conveys ignorance about the subkind those individuals belong to.

(3) a. I saw some contraption in the copy room this morning.
b. Doctor, some growth appeared on my arm. Should I be worried?

This papers contributes to the growing body of literature on epistemic indefinites by discussing an understudied type of epistemic indefinite: epistemic numbers. A variety of languages allow constructions where non-numerical expressions combine with numbers in order to form complex cardinal numerals. In such constructions, instead of denoting a single value, the resulting number denotes a range of possible values. In English, for instance, the indefinite determiner *some* can combine not only with individual denoting and subkind denoting nouns, but also with numbers, as illustrated in (4).

(4) a. Twenty-some people came to the party.
b. I have won over thirty-some thousand dollars at the local bingo.

The two constructions in (4) have in common that they convey ignorance about the exact quantity in question, either about the exact number of people that came to the party (4a), or about the exact amount of dollars that she won (4b). For instance, if the speaker knew that between twenty-one and twenty-six people came to the party, she could truthfully and felicitously use either (4a). And in a situation where the speaker knew the exact number of people that came to the party, it would be infelicitous to use (4a) as an answer to a question like *how many people came to the party*? For convenience, then, we can offer the following working definition of epistemic numbers:

(5) **Epistemic Numbers**
Syntactically complex numbers formed by some combination of number and (non-numerical) indefinite expressions that convey uncertainty or ignorance.

In this respect, what all the indefinites in (1) through (4) have in common is that they are incompatible with full knowledge, and so they require of the speaker to be in a certain belief/knowledge state. My goal in this paper is to provide an analysis of epistemic numbers that captures these similarities by making use of independently attested properties of epistemic indefinites and the syntax/semantics of complex numerals.

Before I turn to the main section of the paper, I discuss first some key properties that epistemic numbers display across languages. Then I provide a specific proposal for the syntax/semantics of ordinary cardinal numbers to form the foundation for the later analysis of epistemic numbers. In the final portion, I given an account of the epistemic effect and discuss outstanding questions and issues.

2 Epistemic numbers across languages

Take a brief Q&A dialog like the following:

(6) a. How many people came to the party?

b. Twenty-some people came.

Given that the response in (6b) did not mention an exact number, how much information did the speaker convey? Like with other numbers, epistemic numbers convey a lower bound. In the case of (6b)'s response, the speaker knows that no less than twenty-one people came. It also seems that if the speaker knew that more than twenty-nine people came, she would not have answered with (6b); thus, we may conclude that epistemic numbers may also convey upper bounds.[1] The resulting state of affairs is one where the speaker who answered with (6b) knew that between twenty-one and twenty-nine people came to the party, no less, no more.

(7) The answer in (6b) is true iff...

a. ✓ $[21,29]$ people came

b. ✘ $\leq [20]$ people came

c. ✘ $\geq [30]$ people came

The fact that epistemic numbers denote lower and upper bounds suggests that they are not to be confused with approximate numbers (cf. Sauerland & Stateva 2007, Krifka 2009, a.o.). For instance, the expressions in the right hand side of (8) may all be true of nineteen people:

1 I will make the simplifying assumption that this upper bound emerges semantically and postpone discussion of whether this is the best characterization for a future occasion. For discussion on ordinary numerals, see Breheny (2008), Spector (2013), a.o.

(8) twenty-some people $\neq \left\{ \begin{array}{l} \text{some twenty people[2]} \\ \text{around twenty people} \\ \text{about twenty people} \\ \text{more or less twenty people} \\ \text{twenty-odd people} \end{array} \right.$

That epistemic numbers in English are constrained to the range of values between twenty-one and twenty-nine indicates a connection between the range of values that they denote and the numbers that *could* have taken the place of the indefinite. In English, no grammatical combination of the number *twenty* with any other number is able to form a complex numeral denoting values either below twenty-one or above twenty-nine. In other words, it seems that we can impute the upper bound of epistemic number to the lack of a grammatical variant of *twenty-eleven* and such.

There are two cross-linguistic axes of variation with respect to the upper/lower bounds conveyed by epistemic numbers. The first one concerns the base of the language at hand. English is a base-ten language, as witnessed by the limitation noted above to form numbers such as *twenty-eleven*. But there are languages with bases other than ten that also possess epistemic numbers. What we observe in such cases is that the range of available values varies accordingly to the base of the language. For instance, Basque, unlike English, is a predominantly base-twenty language, where forms such as *twenty-eleven* are the only way to express the number *thirty-one*.

(9) a. hogei - ta - hamaika
twenty and eleven
'thirty-one'

b. ber - hogei - ta - hamaika
two twenty and eleven
'fifty-one'

The western variety of Basque (not others) possesses an epistemic number formed by combining a number, a conjunctive particle and plural marker (Azkue 1905/1969). As indicated below, the range of values that an epistemic number of the form *twenty-indefinite* is compatible with in Western Basque is limited to the next multiple of twenty, not the next multiple of ten.

(10) a. hogei - ta - sa - k
twenty and and PL
'twenty-some or thirty-some' ✓ $[21,39]$ ✘ $\leq [20]$; ✘ $\geq [40]$

2 But see Stevens & Solt (2018) for reasons why *some-n* is not always an approximator.

b. ber - hogei - ta - sa - k
two twenty and and PL
'forty-some or fifty-some' ✓ $[41, 59]$ ✘ $\leq [40]$; ✘ $\geq [60]$

This contrasts between English and Basque makes good our earlier observation the range of possible values denoted by an epistemic number is determined by the base of the number system in the language.

The second axis of cross-linguistic variation corresponds to the indefinite itself. Some language, such as Spanish, permit epistemic numbers to be constructed from a variety of indefinite expressions. These are the epistemic indefinite *algún* ("some"), and the nouns *pico* and *tantos*, both of which denote indeterminate quantities.

(11) a. sesenta y { pico / tanto / algo }
sixty and some
'sixty-some'

b. ciento y { pico / tanto / algo }
hundred and some
'hundred and some'

But unlike English, Spanish also allows epistemic indefinites with quantifier-determiners other than *some*, in particular with the vague quantifiers *muchos* and *pocos*, the counterparts of English *many* and *few* respectively. In these cases the upper and lower bounds must be adjusted, as they are further limited by the type of determiner that forms the epistemic numbers. Below I represent *muchos* as limiting the available values of (12a) to the upper half of those in (11a), whereas *pocos* is represented as selecting their lower half.

(12) a. sesenta y mucho ✓ $[66, 69]$ ✘ $\leq [65]$; ✘ $\geq [70]$
sixty and many

b. sesenta y poco ✓ $[61, 64]$ ✘ $\leq [60]$; ✘ $\geq [65]$
sixty and few

It should be noted, however, that intuitions about what the bounds of epistemic numbers formed with *muchos* and *pocos* are, unfortunately, not as sharp. While all speakers admit that a sentence containing the number (12a) is false for a value like sixty-two, it is less clear where the exact lower bound is. The same holds of the upper bound of epistemic indefinites formed with quantifier *pocos*. Moreover, given the vague nature of *muchos* and *pocos*, one could think of cases where values in the lower half, e.g. *sixty-four*, are judged to be significantly more in the relevant respect than just *sixty*. In that case, then, (12a) should be felicitous in such contexts. For these reasons, the bounds expressed in (12) must be taken to be an idealization rather than set on stone.[3]

Summing up, what we find cross-linguistically is that (*i*) epistemic numbers denote lower and upper bounds and convey ignorance on the speaker part as to the exact value; (*ii*) the range of available numbers is determined by the base of the number system in the language; and (*iii*) more than one type of indefinite may be allowed.

In order to account for the properties in (*i*) through (*iii*) above, I begin by providing a full syntactic/semantic account of complex cardinal numbers. I then show how we can integrate epistemic numbers within this general account, and what exactly the connection to epistemic indefinites is. Finally, relying on this close resemblance to epistemic indefinites, I show that their pragmatic behavior, i.e. the epistemic effect, can be captured as a conversational implicature conveying modal variation.

3 A syntax/semantics for numbers

3.1 Background assumptions

In spelling out my particular take on the syntax/semantics of complex numeral expressions I rely on two main background assumptions. The first one is a general design of number systems. As Hurford (1975, 1987) showed, number systems in most languages share two fundamental aspects when it comes to constructing complex numerals. First, syntactically, complex numerals have a complex syntactic structure. In languages like English, for instance, there are two different syntactic environments, additive and multiplicative,[4] as illustrated in Figure 1.

Second, from a semantic standpoint, these additive and multiplicative environments are interpreted as addition and multiplication operations on numbers.[5]

(13) $$[\![83751]\!] = \Big(\Big(\big((8\times 10)+3\big)\times 10^3\Big) + \Big(\big(7\times 10^2\big) + \big((5\times 10)+1\big)\Big)\Big)$$

3 The complications increase the with epistemic numbers ranging over higher values. Take a numeral equivalent to "a hundred and few euros". Would the sentence be true in case the real value was 145?

4 I will gloss over the third environment, exponentiation. The reason is that, unlike additive and multiplicative environments, exponentiation is rather opaque, as there is a strong cross-linguistic tendency to use portmanteau forms for the lower powers (such as *hundred*, *thousand* and *million* in English).

5 A minority of languages use other environments as well. Latin shows subtractive environments (*un-de-viginti*, one-from-twenty, i.e. 19), Welsh shows division (*hanner cant*, "half hundred", i.e. 50) and a strategy often referred to as overcounting, as old Danish (*halv-tred-sinds-tyve*; "half-third-times-twenty", i.e. 50, or "half of the third twenty"). Examples from Comrie (1999).

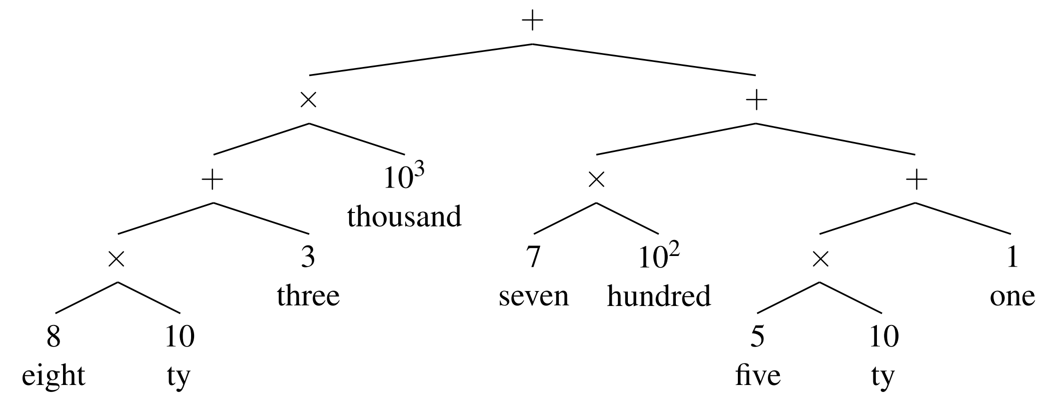


Figure 1 Structure of number 83751 (Hurford 1975, 1987)

My second assumption concerns the general geometry of the DPs modified by numerals. Following ideas in Ionin & Matushansky (2006) and Solt (2015), I represent numerals as specifiers of a Number projection headed by NUM, which then combines directly with an NP.

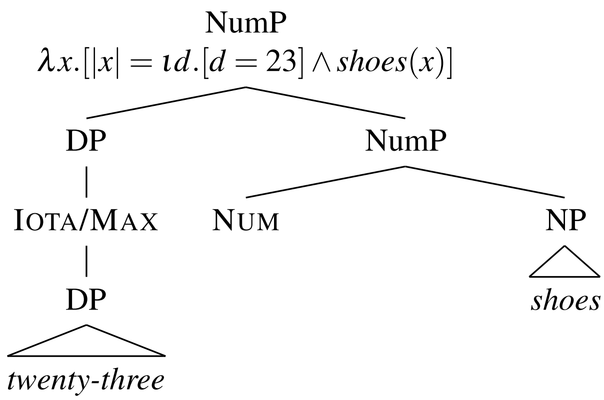
(14) $[_{DP}$ D $[_{NumP}$ $[_{DP}$ numeral$][_{NumP}$ NUM NP$]]]$

The head NUM is similar to a gradable predicate in that it expresses a relation between a property of individuals, the NP, and a degree along some scale; in this case, the cardinality scale.

(15) $[\![\text{NUM}]\!] = \lambda P_{\langle et \rangle}.\lambda d_d.\lambda x_e.[P(x) \wedge |x| = d]$

Numbers themselves are born as predicates of degrees, type $\langle d,t \rangle$, which then are lowered by a generalized version of the iota type-shift originally proposed by Partee (1987) so as to serve as arguments to NUM (i.e. shift $P_{\langle dt \rangle}$ to $\iota d[P(d)]$). With these assumptions, the representation of a simple Number Phrase like *twenty-three shoes* is given below.

(16)



3.2 Complex numbers

We turn now to the internal structure of complex numbers. In this respect I make two contributions that depart from earlier proposals in the literature. First, I represent additive and multiplicative environments as syntactic heads with selecting for number denotations and that add and multiple them, respectively. In a sense, this is a fully-faithful instantiation of Hurford's suggestion couched in current syntactic/semantic assumptions. This was already suggested by Anderson (2015) for the additive environment, I simply extend it here to the multiplicative environment as well; the definitions of the respective heads are provided below.

(17) a. $[\![\text{ADD}]\!] = \lambda D'.\lambda D''.\lambda d.\exists d'd''[d = d' + d'' \wedge D'(d') \wedge D''(d'')]$
b. $[\![\text{MUL}]\!] = \lambda D'.\lambda D''.\lambda d.\exists d'd''[d = d' \times d'' \wedge D'(d') \wedge D''(d'')]$

Semantically, the task of ADD and MUL is no other than taking two singleton properties of degrees and to add/multiple them, resulting in the property that is true of the degree that is equal to the sum/multiplication.

Second, I propose that there is more structure to numbers than meets the eye, also in the case of simplex numbers. Specifically, all cardinal numbers are the product of some integer $n \in \{1, 2, ..., 9\}$ and a numerical *base* B^n. This amounts to saying that natural languages construct numbers like we represent them in the positional notation. For instance, in the decimal system, as in Spanish and English, B is a power of 10, and so $[\![B^n]\!] = \lambda d.[d = 10^n]$. Different number systems, however, may make use of different bases, and thus identical underlying structures for a number such as 40 may be expressed in different ways.

(18) a. **English/Spanish**
$[\![40]\!] = [\ 4\ [\text{MUL}\ B^1]]\ = [\ 4\ [\text{MUL}\ 10]]$ *four tens*
b. **Basque**
$[\![40]\!] = [\ 2\ [\text{MUL}\ B^1]]\ = [\ 2\ [\text{MUL}\ 20]]$ *two twentys*

As an illustration, consider the following semantic representation of the number 83751 below (cf. Figure 1).[6]

(19) **Full structure of 83751**

$$\Big[[8\ \textsc{mul}\ B^4]\ \textsc{add}\ \Big[[3\ \textsc{mul}\ B^3]\ \textsc{add}\ \Big[[7\ \textsc{mul}\ B^2]\ \textsc{add}\ \big[[5\ \textsc{mul}\ B^1]\ \textsc{add}\ [1\ \textsc{mul}\ B^0]\big]\Big]\Big]\Big]$$

Arriving at the resulting (singleton) property of degrees is now straightforward. All numbers start off in a multiplicative environment combining with a base that then get added. So for the number 51 we have:

(20) a. $[\![5]\!] = \lambda d.[d = 5]$
b. $[\![B^1]\!] = [\![10]\!] = \lambda d.[d = 10]$
c. $[\![50]\!] = [5\ [\textsc{mul}\ B^1]] = [5\ [\textsc{mul}\ 10]] = [\![\textsc{mul}]\!](]\!]10]\!])([\![5]\!]) \leftrightarrow\ \lambda d\ .[d = 50]$

(21) $[\![51]\!] = [5\ [\textsc{add}\ 1]] = [\![\textsc{add}]\!](]\!]1]\!])([\![5]\!]) \leftrightarrow\ \lambda d\ .[d = 50+1]$

It is clear how to proceed for the rest of the complex number by recursively applying the same operations.

Directly referencing the base in the structure of the number makes it straightforward to extend the analysis to languages with uncommon bases. For instance, Ngkolmpu, a Kanum language spoken in southern New Guinea, has a base-6 number system, where the portmanteau forms correspond to the powers of 6: *traowo* is 6 (6^1), *ptae* is 36 (6^2), *tarumpao* is 216 (6^3), etc. Correspondingly, the number 13 is expressed as *yempoka traowo naempr* ("two six one"), whose structure looks as below:

(22) $\big[[2\ \textsc{mul}\ B^1]\ \textsc{add}\ [1\ \textsc{mul}\ B^0]\big] = \big[[2\ \textsc{mul}\ 6]\ \textsc{add}\ [1\ \textsc{mul}\ 1]\big]$
$\leftrightarrow \lambda d\ .[d = 12+1]$

Summing up, the two characterizing features of the syntax/semantics of complex numerals that I proposed above are (i) that if follows to the end Hurford's suggestion of having two distinct environments (additive and multiplicative) and (ii) that numbers are constructed off of a combination with a (power of) the base.

4 Epistemic Numbers

We are now ready to extend the analysis of numbers to epistemic numbers. The obvious question is: how does an indefinite ever get to appear in the place of a number? In order to answer this question, it is worth thinking about what the indefinite

6 This is not the only possible representation, for instance the following would also work:
$\Big[\big[[8\ \textsc{mul}\ B^1]\ \textsc{add}\ [3\ \textsc{mul}\ B^0]\big]\ \textsc{mul}\ [1\ \textsc{mul}\ B^3]\ \Big]\textsc{add}\ \Big[[7\ \textsc{mul}\ B^2]\ \textsc{add}\ \big[[5\ \textsc{mul}\ B^1]\ \textsc{add}\ [1\ \textsc{mul}\ B^0]\big]\Big]$

does when it combines with a number. I suggest that the indefinites in epistemic numbers are in fact identical to epistemic indefinites elsewhere. For concreteness, I follow the analysis of the epistemic indefinite *algún* in Spanish provided by Alonso-Ovalle & Menéndez-Benito (2010, 2013) (see also Anderson 2015). That is, I treat the indefinite as denoting an anti-singleton subset selection function over numbers. (I will use INDEF as a blanket term for *some* and *algo/tantos*, etc.) The semantic task of INDEF is to take a set of numbers and return some non-singleton subset.

(23) $[\![\text{INDF}]\!] = \lambda f_{\langle dt,dt\rangle}.\lambda D_{\langle dt\rangle}.\lambda d_d : |f(D)| > 1 \,.\, [f(D)(d)]$

Thus, indefinites in epistemic numbers are identical to epistemic indefinites at large in coming with the same anti-singleton constraint.

But then, if f selects a value from a set of numbers where do those numbers come from? Here I follow work by Kayne (2005), Zweig (2005) and Schwarzschild (2006) a.o., who show that indefinites and vague quantifier-determiners such as *many* and *few* never combine directly with a property, but they do so through the mediation of a silent noun NUMBER. Thus, like other quantity expressions, indefinites in epistemic indefinites can be taken to combine with nominals via a null element.

(24) a. few books = [[few NUMBER] books]

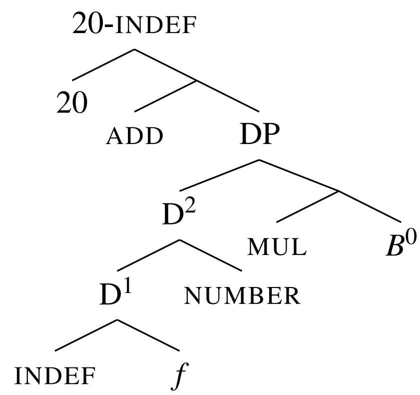
b. twenty-some books = [[two [some NUMBER]] books]

It is this NUMBER null head that provides the relevant set of numbers from which the indefinite must pick a non-singleton subset. Specifically, I assume that the denotation of NUMBER is simply the set of “basic” numbers (those **not** requiring ADD, i.e. those combining with the “zero” base, B^0), which varies depending on the base of the system.

(25) $[\![\text{NUMBER}]\!] = \{1,2,3,4,5,6,7,8,9\}$ [$\leftrightarrow \{d \,:\, B^0 > d > B^1\}$ for base-10]

We now have all the pieces in place to provide a semantics for epistemic numbers. Consider the numeral *twenty-some*.

(26)



(27) a. $[\![\mathrm{D}^2]\!]$
$= [\![\textsc{indf}]\!]([\![f\,]\!])([\![\textsc{number}]\!])$
$= \lambda d.[f\ (0 < d < 10)]$

b. $[\![\mathrm{DP}]\!] = [\![\textsc{mul}]\!]([\![B^0]\!])([\![\mathrm{D}^2]\!])$
$\leftrightarrow\ \lambda d.[f\ (0 < d < 10)]$

c. $[\![\text{20-}\textsc{indef}[\![$
$= [\![\textsc{add}[\![([\![\mathrm{DP}]\!])([\![20]\!])$
$\leftrightarrow\ \lambda d.[20\ +\ f\ (0 < d < 10)]$

In prose: the indefinite (with the function *f* syntactically represented) combines with NUMBER to return some non-singleton subset, which then continues its life throughout the derivation as any ordinary number does. This results in adding 20 to some number in the range $[1,10]$.

Notice that by confining INDEF to a position that only another number could take we capture the limitation of epistemic numbers to appear only with complex cardinals. That is, epistemic numbers only combine with numerals that can independently combine by addition with some other number. If a number like *six* can combine with *twenty* to form *twenty-six*, then an epistemic number may be formed. However, numbers where the addition is not performed in the syntax, as with simplex or grammaticalized forms, epistemic numbers are ruled out. The number *quince* ("fifteen") in Spanish is one such case: *quince y dos* ("fifteen and two") is ungrammatical because *quince* is a simplex form, unlike e.g. *diecieseis* ("ten-and-six"; 16); accordingly, *quince-algo* and other variants are ungrammatical.

Accounting for the different values available in languages with other bases is straightforward. A number expression like *hogeitazak* ("twenty-some" in Western Basque) has the same structure as that of (26), with the only difference that NUMBER now contains integers in the $[1,19]$ range corresponding to a base-20 number system.

(28) a. $[\![\mathrm{D}^2]\!] = [\![\textsc{indf}]\!]([\![f\,]\!])([\![\textsc{number}]\!]) = \lambda d.[f\ (0 < d < 20)]$

b. $[\![\mathrm{DP}]\!] = [\![\textsc{mul}]\!]([\![B^0]\!])([\![\mathrm{D}^2]\!]) \quad \leftrightarrow\ \lambda d.[f\ (0 < d < 20)]$

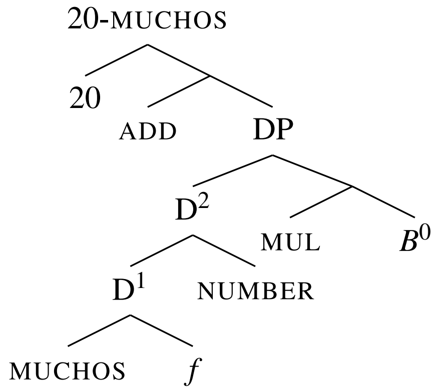
c. $[\![\text{20-}\textsc{indef}[\![= [\![\textsc{add}[\![([\![\mathrm{DP}]\!])([\![20]\!]) \quad \leftrightarrow\ \lambda d.[20\ +\ f\ (0 < d < 20)]$

In the case of variation in the indefinite itself in languages like Spanish, the derivation proceeds exactly as before, with the difference that the range of values made available by the epistemic number is further restricted. To account for this, I define *muchos* and *pocos* by determining the property of numbers that remains above/below certain threshold. For concreteness–but see discussion in §2–assume that the threshold is established by the *median* M of NUMBER, which corresponds to the n^{th} ordered value of some set of numbers A: $n = \frac{1}{2}(|A|+1)$. In the decimal system, the median of NUMBER is always 5.

(29) a. $[\![\text{MUCHOS}]\!] = \lambda f_{\langle dt,dt\rangle}\lambda D_{\langle dt\rangle}.\lambda d_d : |f(D)| > 1 \ . \ [f(D)(d) \wedge d \ > \ M(D)]$
b. $[\![\text{POCOS}]\!] = \lambda f_{\langle dt,dt\rangle}\lambda D_{\langle dt\rangle}.\lambda d_d : |f(D)| > 1 \ . \ [f(D)(d) \wedge d \ < \ M(D)]$

The derivation of Spanish *twenty-many* is provided below.

(30)



(31) a. $[\![\text{D}^2]\!]$
$= [\![\text{INDF}]\!]([\![f\,]\!])([\![\text{NUMBER}]\!])$
$= \ \lambda d.[f\ (5 < d < 10)]$
b. $[\![\text{DP}]\!] = [\![\text{MUL}]\!]([\![B^0]\!])([\![\text{D}^2]\!])$
$\leftrightarrow \ \lambda d.[f\ (5 < d < 10)]$
c. $[\![\text{-INDF}]\!]$
$= [\![\text{ADD}]\!]([\![\text{DP}]\!])([\![20]\!])$
$\leftrightarrow \ \lambda d.[20 \ + \ f\ (5 < d < 10)]$

Summing up, this section has shown that we can account for the semantic behavior of epistemic numbers by relying on (*i*) a specific syntax/semantics of cardinal numbers where they are built from syntactic heads comprising their bases, and (*ii*) a specific semantics of epistemic indefinites, where they denote subset-selection functions that may apply to numbers as well as individuals/kinds. Points of variation in (*i*) and (*ii*) across languages also account for the observed variation in the nature of epistemic numbers. By factoring in base values, the analysis is flexible enough to account for number systems other than the decimal. The indefiniteness of epistemic numbers is then due to the fact that f, just like ordinary epistemic indefinites, is an anti-singleton subset selection function, thus impeding the specific interpretation of the indefinite.

5 The epistemic effect

So far we have seen that epistemic numbers are "numbers" just like cardinal numbers are. This section shows that, in addition, their epistemic effect is also closely related to the epistemic effects of epistemic indefinites. Above the epistemic effect was informally referred to as conveying ignorance or uncertainty of the speaker about some particular number. In this section I show that the epistemic effect is a case of modal variation, and is best captured as a conversational implicature, triggered by the fact that the speaker chose to use anti-singleton domain, in parallel with epistemic indefinites and other determiners (e.g. *algún que otro*, "some or other", in Spanish; Alonso-Ovalle & Menéndez-Benito 2013).

5.1 The epistemic effect as a conversational implicature

The epistemic effect of epistemic numbers behaves like other quantity conversational implicatures in a number of respects. It bears their three main signature properties: unlike entailments and presuppositions the effect can be reinforced without fear of redundancy (32a), it can be canceled (32b), and it disappears in downward-entailing contexts (32c).

(32) a. I paid twenty-some dollars for this shirt, I don't know how much exactly.

b. I did pay twenty-some dollars for this shirt, twenty-seven to be precise.

c. I didn't pay twenty-some dollars for this shirt. $\rightsquigarrow$ *no epistemic effect*

This conversational implicature is better understood as a case of modal variation (von Fintel 2000, Alonso-Ovalle & Menéndez-Benito 2010):

(33) **Modal Variation**:
$$\exists w', w'' \in Epi_{s,w}\left[\ \{d : P(w')(x) \wedge |x| = d\} \neq \{d : P(w'')(x) \wedge |x| = d\}\ \right]$$

As expressed by (33) modal variation requires that there be at least two epistemic worlds with distinct values accessible. That this is the case is shown by the fact that epistemic numbers do not require ignorance about any one particular epistemic alternative within the range of possible values. This is in contrast with other epistemic determiners such *at least n*, which does require ignorance about whether *n* is the case.

(34) a. Sam paid at least 61 \$, #but I'm certain she didn't pay exactly 61\$.
$\rightsquigarrow$ *the speaker* ***must*** *consider 61\$ as a possibility*.

b. Sam paid 60-some \$, but I'm certain she didn't pay {61\$/62\$...}
$\rightsquigarrow$ *the speaker* ***must not*** *consider {61\$/62§...} as a possibility*.

5.2 Derivation

Following the analysis of epistemic indefinites in Alonso-Ovalle & Menéndez-Benito (2010), I suggested that epistemic numbers select an non-singleton subset of values from a set of numbers. This is an important step towards deriving the modal variation of epistemic numbers. Building on the analysis of German *irgendein* by Kratzer & Shimoyama (2002), Alonso-Ovalle & Menéndez-Benito (2010) suggest the epistemic effect of Spanish *algún* arises because this indefinite signals that its domain of quantification cannot be a singleton set. Due to the limitation that this anti-singleton constraint imposes on the domain of quantification, the epistemic indefinite triggers a competition between the assertion and alternative propositions that would have resulted from restricting the domain to a singleton.

5.2.1 Epistemic indefinites: the case of *algún*

As illustration, consider sentence (35a) in a context where there are three rooms under consideration: the living room, the bedroom and the kitchen (Alonso-Ovalle & Menéndez-Benito 2013). This sentence is interpreted as (35b), where f selects a non-singleton subset of the relevant rooms.

(35) a. Juan está en alguna habitación.
Juan is in ALGÚN room
'Juan is in some room or other.'
b. Juan is in $f(\{\text{living room, bedroom, kitchen}\})$

The anti-singleton constraint triggers a competition between (35a) and the stronger (and relevant) alternatives in (36).

(36) a. Juan is in $\{\text{living room}\}$
b. Juan is in $\{\text{bedroom}\}$
c. Juan is in $\{\text{kitchen}\}$

Given that any one of the alternatives in (36) is more informative than the assertion in (35a), why did the speaker not chose one of them? The intuition, first pursued by Kratzer & Shimoyama (2002), is that by choosing (35a) the speaker is avoiding making a false claim: the speaker does not have enough evidence for any one of the stronger alternatives in (36), while still being certain that one of them must be true. Assume that upon hearing (35b), hearers assume that speakers believe that their utterance is true–e.g. by following the mandate of the Maxim of Quality. Moreover, in accordance to basic principles of cooperation and efficiency, hearers may conclude that the speaker does not believe the truth of any of the more informative propositions in (36), since otherwise she would have chosen to utter one such sentence.

(37) $\Box_{epi}(\text{Juan is in } f(\{\text{living room, bedroom, kitchen}\}))$

(38) a. $\neg\Box_{epi}(\text{Juan is in } \{\text{living room}\})$
b. $\neg\Box_{epi}(\text{Juan is in } \{\text{bedroom}\})$
c. $\neg\Box_{epi}(\text{Juan is in } \{\text{kitchen}\})$

Together with (37), the Primary Implicatures from (38) entail that the speaker does not know in which room Juan is.

5.2.2 Modal variation with epistemic numbers

A similar reasoning can apply to epistemic numbers as well, given the earlier proposal of treating them as imposing an anti-singleton on the domain of quantification. A sentence like *I paid twenty-some dollars*, interpreted as in (39), triggers a competition with all the singleton number values in the range of epistemic number, as expressed in (40).

(39) $\exists d[\text{I paid } 20+d\$ \wedge d \in f(\{0<d<10\})]$

(40) a. $\exists d[\text{I paid } 20+d\$ \wedge d \in \{1\}]$
b. $\exists d[\text{I paid } 20+d\$ \wedge d \in \{2\}]$
...
c. $\exists d[\text{I paid } 20+d\$ \wedge d \in \{9\}]$

As before, we assume the speaker believes in the truth of her assertion, and conclude that all the singleton alternatives to (39) were disregarded precisely because the speaker was not in a position to commit to the truth of any one of them. This results in the set of Primary Implicatures in (42).

(41) $\Box_{epi}(\exists d[\text{I paid } 20+d\$ \wedge d \in f(\{0<d<10\})])$

(42) a. $\neg\Box_{epi}(\exists d[\text{I paid } 20+d\$ \wedge d \in \{1\}])$
b. $\neg\Box_{epi}(\exists d[\text{I paid } 20+d\$ \wedge d \in \{2\}])$
...
c. $\neg\Box_{epi}(\exists d[\text{I paid } 20+d\$ \wedge d \in \{9\}])$

Together (41) and (42) correspond to modal variation: $\forall d \in D\ \neg\Box_{epi}[\phi(d)]$ or alternatively $\neg\exists d \in D\ \Box_{epi}[\phi(d)]$.

A number of consequences follow from this strengthened meaning. First, the speaker's epistemic state is predicted to be incompatible with full knowledge about the number in question, as desired. Second, the epistemic effect is predicted to be weaker than that of total ignorance; that is, the epistemic state of the speaker need not be such that she is ignorant about *all* the values in the relevant range. This type of absolute ignorance, familiar from multiple disjuncts, is compatible with epistemic numbers, but they do not require it. In fact, epistemic numbers are compatible with a fair amount of knowledge in the right context. Consider, for instance, a sentence like (43).

(43) That night Michael Jordan scored twenty-some points in triples.

In basketball, triples are three-point scores, as opposed to other field scores which are worth only two points. If (43) was uttered by a basketball enthusiast, the sentence should be odd if it brought attention to a putative epistemic state where all the values in $[21,29]$ were taken to be epistemic possibilities for the speaker. This is not the case, however, and (43) is felicitous in such context. In this case, we need to add the premise that the speaker knows that, for all numbers $d \in [21,29]$ and $d \notin \{21,24,27\}$, $\Box_{epi}\neg[\phi(d)]$.

(44) a. Premise:
$\Box_{epi}\neg(\exists d[\text{Michael Jordan scored } 20+d \text{ points} \wedge d \in \{2,3,5,6,8,9\}])$

b. Assertion:
$\Box_{epi}(\exists d[\text{Michael Jordan scored } 20+d \text{ points} \wedge d \in f(\{0<d<10\})])$

c. Primary Implicatures:
$\neg\Box_{epi}(\exists d[\text{Michael Jordan scored } 20+d \text{ points} \wedge d \in \{1\})])$
$\neg\Box_{epi}(\exists d[\text{Michael Jordan scored } 20+d \text{ points} \wedge d \in \{2\})])$
$\ldots$
$\neg\Box_{epi}(\exists d[\text{Michael Jordan scored } 20+d \text{ points} \wedge d \in \{9\})])$

In this case, the strengthened meaning of (43) conveys an epistemic inference about a much reduced domain of quantification.[7] As a consequence, the epistemic possibilities accessible to the speaker are accordingly reduced.[8]

(45) Epistemic possibilities accessible from w_0

a. w_1: Michael Jordan scored 21 points.

b. w_2: Michael Jordan scored 24 points.

c. w_3: Michael Jordan scored 27 points.

Summing up, the analysis captures the epistemic effect of epistemic numbers as a conversational implicature conveying modal variation: a conversational implicature triggered by the anti-singleton constraint on f, which results in the listener reasoning that the speaker wanted to avoid a false claim.

6 Further work

The main take-away from this paper is that epistemic numbers are “epistemic” just like epistemic indefinites are, and they are “numbers” just like cardinal numbers

7 Since $\Box_{epi}\neg[\phi]$ entails $\neg\Box_{epi}[\phi]$ (Hintikka 1962), the Primary Implicatures corresponding to values not in $\{21,24,27\}$ are rendered superfluous.

8 Of course, if the hearer ignores the rules of basketball she will lack the relevant premise in (44) and so the epistemic inference will be about the whole range of values.

are. The main focus of the paper has been to provide an analysis of epistemic numbers flexible enough to account for their cross-linguistic variation. While this much has been accomplished, there are further aspects of epistemic numbers that future work should address. For instance, given the syntax/semantics of numbers defended here, many kinds of structures may be generated. One may thus wonder, why do indefinites need to be complements to ADD? Why can't we say *some-three*? This seems to be a matter of cross-linguistic variation as well. Japanese is language that allows such combinations, by forming epistemic numbers with the indeterminate pronoun *nan* ("what") may also precede a numeral in a multiplicative environment. In such cases, available values vary accordingly, as expected by the present analysis.

(46) a. nan - juu - nin - ka - ga
what ten CL-HUMAN PART NOM

$[[some \times B^1] + 4] \rightsquigarrow$ 10, 20, 30, 40, 50, 60, 70, 80, 90

b. hyaku - nan - juu - nin - ka - ga
hundred what ten CL-HUMAN PART NOM

$[[[1 \times B^2] + [some \times B^1]] + 4]$
$\rightsquigarrow$ 110, 120, 130, 140, 150, 160, 170, 180, 190

Such strategies are of course not allowed in other languages, just like the flexibility of Spanish to use a number of different indefinite expressions is not attested elsewhere (to the best of my knowledge). I will these matters as open questions.

Many other questions remain. One may wonder, for instance, why epistemic numbers of the form INDEF-*number*, with the indefinite in a multiplicative environment, are typologically rarer than *number*-INDEF, with the indefinite in an additive environment. Similarly, this paper does not have much to add about what exactly regiments the availability epistemic numbers across languages, or what regiments the availability of various types of indefinites in epistemic numbers across languages.

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