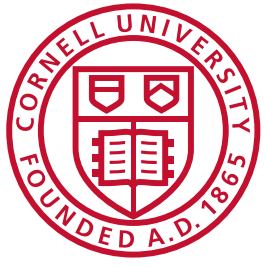
STRUCTURING IGNORANCE

Rutgers Linguistics Colloquium

06 March 2020



Jon Ander Mendia

THE PLOT

Pragmatics as the “science of the **un**said”.

Conversational Implicatures

A form of inference that arises from expectations about how speakers will behave, based on general assumptions about cooperative behavior.

Grice (1975)

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(1) A. I am out of gas.
B. There is a gas station around the corner.
$\rightsquigarrow$*the gas station is open and serviceable*

Grice (1975)

Informativity Implicatures

(2) **Strong Implicatures**

A. How many children do you have?

B. I have three children.

↝*B doesn't have four children*

a.k.a. Scalar Implicatures

Informativity Implicatures

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A. How many children do you have?

B. I have three children.

↝*B doesn't have four children*

(3) **Weak Implicatures** a.k.a. Uncertainty/Ignorance Inferences

A. Where is Sue? I need to track her down.

B. She is in Europe.

↝*B doesn't know where exactly Sue is*

Dedicated expressions of ignorance

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⇝ *Broccoli? Banana? Both?*

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d. Al arrived at least to the base camp
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- These are all **Ignorance Inferences** (IIs).

General Questions

Empirical

What propositions **can** and **must** we be ignorant about to felicitously use "ignorance inducing" expressions?

Theoretical

What is the right (formal) characterization of IIs? How do we explain convergence among speakers?

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- Why AL?
  - Flexibility: modifies *all* sorts of constituents.
  - Robust: `II`s of AL are “obligatory”.

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  - Closing the descriptive gap between AL modifying numerals and other cases.

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- First: Description
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  - Closing the descriptive gap between AL modifying numerals and other cases.

- Second: Theory
  - A Gricean pragmatic calculus that accounts for all the data.
  - Two sources of alternatives; emphasis on the role of **order** over **entailment**.

Not today!

Not all constructions involving AL convey ignorance.

① Concessive AL

(5) Sue's flight got canceled but at least she was reimbursed.

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② Embedded AL

(6) a. Every student wrote at least two papers.
b. You must write at least two QPs to be ABD.

THE BASICS OF AT LEAST

At least (AL)

Three undisputed facts about AL and AL-statements:

① AL may combine with a variety of expressions.

② AL denotes a lower bound.

③ AL-statements come with `II`s.

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⇒ The speaker knows that Sue does not have less than four dogs
⇝ *The speaker is ignorant about the exact number of dogs*

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⇝ *The speaker is ignorant about the exact number of dogs*

(8) At least Liz and Sue read the paper
⇒ The speaker knows that both Liz and Sue read the paper
⇝ *The speaker is ignorant about exactly who read the paper*

Agreements and Disagreements

Somewhat disputed is what the exact division of labor between semantics and pragmatics is. A commonly held view:

- The lower bound is part of the *semantic* content of AL.
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Much disputed is *how* to derive the inferences:

- AL as disjunction [Büring 2007]
- AL as a (double) modal [Geurts and Nouwen 2007]
- AL as a meta-speech act operator [Cohen and Krifka 2014]
- AL as an epistemic indefinite [Nouwen 2015]
- AL as "inquisitive" [Coppock and Brochhagen 2013b,a]
- AL as (more or less) "classical" quantifier:
  - (Neo-)Gricean [Mendia 2015, Kennedy 2015, Schwarz 2016]
  - Grammatical [Mayr and Meyer 2014, Buccola and Haida 2019]

The descriptive gap

Most attention has been devoted to AL modifying numerals. AL may combine with a number of different types of complements or associates, consistently leading to IIs in all these environments.

(9) a. At least *some* students came to the party. [Horn Scales]
   ⇝ *the speaker S is ignorant about whether all students came*
   b. At least *Bill and Sue* came to the party. [Cardinality Scales]
   ⇝ *S is ignorant about whether someone else came to the party*
   c. Sue won at least *the silver medal*. [Lexical Scales]
   ⇝ *S is ignorant about whether Sue won the gold medal*
   d. Bill ate at least *broccoli*. [Evaluative Scales]
   ⇝ *S is ignorant about whether Bill ate candy*

Krifka (1999), Coppock and Brochhagen (2013b).

Two questions

Predictability

Is there any proposition in particular about which the speaker **must** be ignorant so that they can successfully use an AL-statement? If so, what are they?

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Uniformity

Are the inferences that come with AL the same across the board, regardless of its type of associate?

CHARACTERIZING IGNORANCE

Total *vs* Partial Ignorance

Disjunction demands "equal rights for each disjunct": A speaker uttering (10) is committed to be ignorant about each individual disjunct.

(10) Liz read Tintin, Asterix, or Gaston.
- ⇝ *S doesn't know whether Liz read Tintin*
- ⇝ *S doesn't know whether Liz read Asterix*
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Zimmermann (2000), Alonso-Ovalle (2006), a.o.

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➔ This is **Total Ignorance**.

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Total *vs* Partial Ignorance

Indefinites do not impose such stringent restrictions:

(11) Liz paid twenty-some dollars for the book, but less than 27.
⇝ *21? 22? 23? ... 26?*

(12) Some friend of yours called; it wasn't Louise.
⇝ *Cynthia? Liz? Zoe?*

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↝ *21? 22? 23? ... 26?*

(12) Some friend of yours called; it wasn't Louise.
↝ *Cynthia? Liz? Zoe?*

The speaker is unable to identify the witness of an existential claim, but ignorance is not required about every individual in the domain.

- This is **Partial Ignorance**.

Total *vs* Partial Ignorance

(13) Last night LeBron James scored at least 30 points in three-pointers.

Coppock and Brochhagen (2013b), Mendia (2015), Nouwen (2015), Kennedy (2015), Schwarz (2016).

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The password is at least six characters long.

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b. ⌜ and _____⌝ ⊨ ⌜*only P*⌝ ②

c. ⌜ and _____⌝ ⊭ ⌜*more than P*⌝, ⌜*only P*⌝ ③

Assertibility conditions of AL

(16) Sue ate at least two apples...

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b. # and she did not eat more than two.
   ⇒ *S* knows ⌜*exactly 2*⌝ ✘

c. but she didn't eat {four/three or four/six/...}. ✓

Assertibility conditions of AL

Thus, for an AL-statement *at least P*, the speaker **must be** ignorant as to whether:

- *only P* is the case.
- *more than P* is the case.

Büring (2007), Geurts and Nouwen (2007), Kennedy (2015), Nouwen (2015).

Beyond numerals

Some non-numeral cases pattern with numerals:

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(17) Sue ate at least some of the apples...

a. # but I know that she didn't eat just some.
   ⇒ *S* knows that ⌜*more than just some*⌝ ✘

b. # but she didn't eat all.
   ⇒ *S* knows ⌜*just some*⌝ ✘

Beyond numerals

Numerals and quantifiers like *some* share two important logical properties:

① Asymmetric logical entailment.

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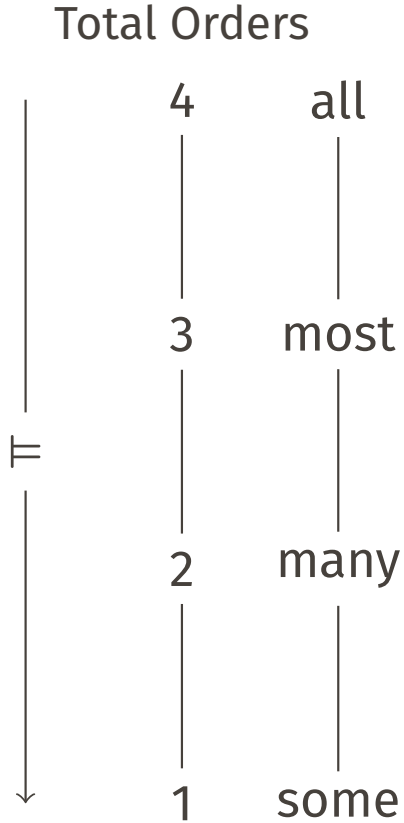

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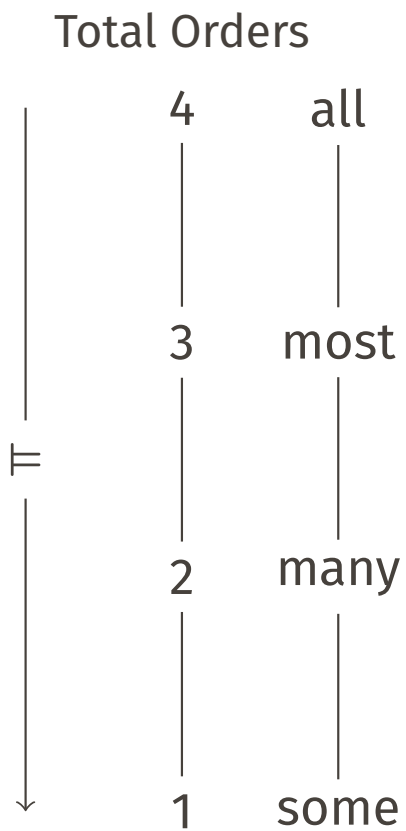

❓ Which property is responsible for the exact form of AL's `II`s?

❓ Which of the two notions **matters** for implicature calculation?

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Contextual entailment = Common Ground information + Logical entailment

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$$\not\therefore \ \frac{\text{Sue has a college degree}}{\text{Sue has a high school degree}}$$

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(19)
$$\frac{\text{Sue has a college degree}}{\not\therefore\ \text{Sue has a high school degree}}$$

(20)
$$\frac{\begin{array}{l}\text{Sue has a college degree}\\ \text{All college graduates have high school degrees}\end{array}}{\therefore\ \text{Sue has a high school degree}}$$

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c. but I know that she does not have a PhD. ✓

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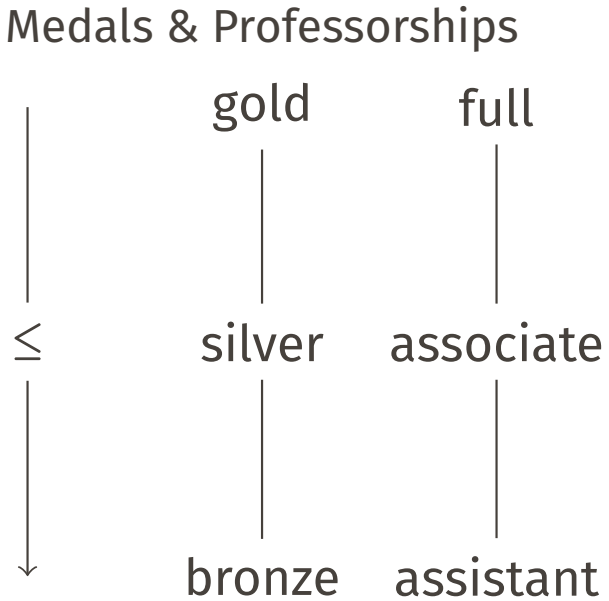
- Like with numerals/quantifiers, the speaker is ignorant as to whether:
  - *only P* is the case.
  - *more than P* is the case.

No entailment

Logically and contextually independent alternatives can also be ordered:



No entailment

(22) Sue won the gold medal ⊭ Sue won the silver medal

No entailment

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(23)

	[CG???]
	Sue won the gold medal
∴	Sue won the silver medal

- No amount of Common Ground information may be supplied so as to make *gold* entail *silver*, these are *logically* **and** *contextually* independent.

No entailment

(24) Sue won at least the bronze medal...

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a. #she placed first or second.

⇒ *S* knows that ⌜*more than bronze*⌝ ✘

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b. #she didn't place first or second.
   ⇒ *S* knows that ⌜*only bronze*⌝ ✘

c. but I know that she did not place second. ✓

- Once more, the speaker is ignorant as to whether:
  - *only P* is the case.
  - *more than P* is the case.

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- Once more, the speaker is ignorant as to whether:
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Conclusion about entailment

AL is blind to entailment (contextual or logical), all it needs is an ordered structure.

Ordering: Partial Orders

If AL is blind about **entailment**, is it also blind about different types of **order**?

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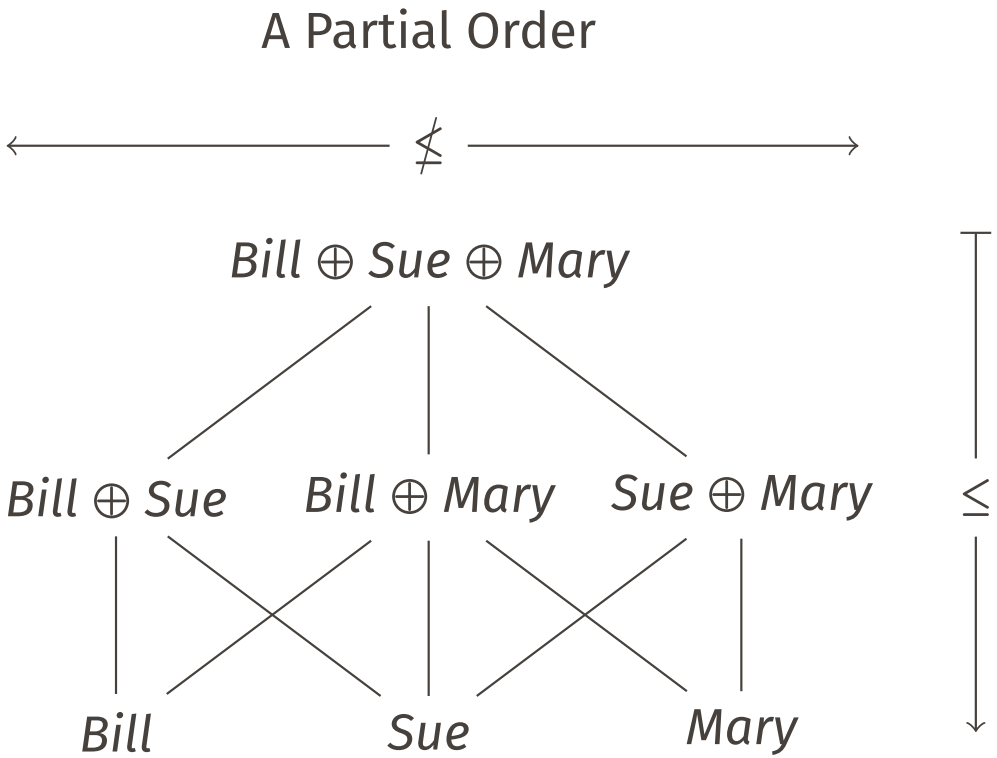

Ordering: Partial Orders

Context: Sherlock Holmes went on vacation for a couple of days and let some of his friends celebrate dinner at 221B Baker Street: Dr. Watson, Mrs. Hudson, Mycroft, Irene Adler and some of the Baker Street Irregulars. After vacation, he returns to his room only to discover that somebody has been messing with his chemistry set. Inspector Lestrade from Scotland Yard is with him, and asks: *Who do you think touched the chemistry set?*

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a. but certainly not only them.

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   ⇒ *S* knows that ⌜*more than M&H*⌝ ✓

b. # and nobody else.
   ⇒ *S* knows that ⌜*only M&H*⌝ ✘

c. but I know that it wasn't Irene Adler. ✓

Ordering: Total *vs.* Partial

(26) a. Who touched the chemistry set?

b. At least Mycroft and Mrs. Hudson, but certainly not only them.
SH knows that ⌜*only M&H*⌝ is **false** ✓

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SH knows that ⌜*only M&H*⌝ is **false** ✓

(27) a. How many people touched the chemistry set?

b. # At least two people, but certainly not just two.

SH knows that ⌜*only 2*⌝ is **false** ✘

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(29) *Assertibility conditions of* ⌜at least P⌝ *with* ***Partial Orders***:
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(28) *Assertibility conditions of* ⌜at least P⌝ *with* ***Total Orders***:
   a. *S* **must** be ignorant about ⌜*more than P*⌝.
   b. *S* **must** be ignorant about ⌜*only P*⌝.

(29) *Assertibility conditions of* ⌜at least P⌝ *with* ***Partial Orders***:
   a. *S* **must** be ignorant about ⌜*more than P*⌝.
   b. *S* **may but need not** be ignorant about ⌜*only P*⌝.

- The use of AL when modifying Totally Ordered domains is more restricted wrt. the **exhaustive interpretation of the prejacent**.

Interim Conclusion

① Alternatives of AL-statements need not be ordered by (logical/contextual) entailment.

② Alternatives must be ordered by some (possibly *ad hoc*) relation $\leq$.

③ Structural differences between orderings of alternatives correlate with different `II`s.

Answer to Predictability

Predictability

Is there any proposition in particular about which the speaker **must** be ignorant so that they can successfully use an AL-statement?

- Yes.

`II`s indicate the minimal epistemic conditions that speakers must meet to successfully use AL; in turn, these minimal conditions constitute what is *minimally* predictable about the speaker's epistemic state.

Answer to Uniformity

Uniformity

Are the inferences that come with AL the same across the board, regardless of its type of associate?

- No.

New Generalization

The exact form of AL's `II`s depends on the **order** of AL's associate.

CALCULATING INFERENCES

An observation

AL is Conventionally Associated with Focus: `II`s vary accordingly.

(30) a. The chair at least invited the postdoc$_F$ to lunch.

⇝ *ignorance about whether someone else was invited*

b. The chair at least invited the postdoc to lunch$_F$.

⇝ *ignorance about whether she got invited to something else*

Beaver and Clark (2008), Coppock and Brochhagen (2013b), Mendia (2017)

Semantics: focus

A two tier semantic system delivering an ordinary semantic value $[\![\cdot]\!]^o$ and a focus semantic value $[\![\cdot]\!]^f$.

(31) Sue ate broccoli$_F$

a. $[\![[\textit{Sue ate } [\textsc{broccoli}]_F]\!]^o = \textit{ate}(\textit{Sue}, \textit{broccoli})$

b. $[\![[\textit{Sue ate } [\textsc{broccoli}]_F]\!]^f = \{x \in D_e \mid \textit{ate}(\textit{Sue}, x)\}$

Rooth (1985, 1992) et seq.

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A contextually determined set of relevant alternatives C:

(32) A proposition with some focused constituent φ_F is defined only if C is a subset of $[\![\varphi]\!]^f$ containing $[\![\varphi]\!]^o$ and at least one other element.

Rooth (1985, 1992) et seq.

Semantics: AL

(33) $[_{S^1}$ *at least C* $[_{S^2}$ $[_{S^3}$ Sue is [an assistant professor]$_F$] ~ C]]

(34) $[\![\textit{at least}]\!] = \lambda C_{\langle st,t\rangle}.\lambda p_{\langle st\rangle}.\lambda w.\exists q[q \in C \wedge q(w) \wedge p \leq q]$

(35) a. $[\![33]\!]^f = \{P(Sue) \mid P \in D_{\langle e,t\rangle}\}$

b. $[\![33]\!]^o =$ *Sue is an assistant professor*

c. $[\![33]\!]^{c_f} = \left\{ \begin{array}{l} \ldots \\ \textit{Sue is a visiting professor,} \\ \textit{Sue is an assistant professor,} \\ \textit{Sue is an associate professor,} \\ \ldots \end{array} \right\}$

$[\![(33)]\!] = \lambda w.\exists q[q \in [\![(33)]\!]^{c_f} \wedge q(w) \wedge [\![(33)]\!]^o \leq q]$

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$[\![(33)]\!] = \lambda w.\exists q[q \in [\![(33)]\!]^{c_f} \wedge q(w) \wedge [\![(33)]\!]^o \leq q]$

order

Pragmatics: Preliminaries

- I enclose propositions in square brackets, such that $[\varphi]$ stands for some proposition containing an expression φ. For instance:
  - $[\mathtt{4}]$ *4 students came.*
  - $[\mathtt{Al} \oplus \mathtt{Mary}]$, sometimes $[\mathtt{A} \oplus \mathtt{M}]$ *Al and Mary came.*

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- With modifiers, I use:
  - $[\geq \varphi]$ $[\textit{at least } \varphi]$
  - $[\textit{EX } \varphi]$ [*only/exactly* φ; the *exhaustive* interpretation]

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- With modifiers, I use:
  - $[\geq \varphi]$ — $[$*at least* $\varphi]$
  - $[EX\ \varphi]$ — [*only/exactly* φ; the *exhaustive* interpretation]
- I also use the familiar epistemic operators K_S and P_S:
  - $K_S[\varphi]$ — *S knows that* φ
  - $P_S[\varphi]$ — φ *is possible according to S*

Pragmatics: Ignorance

Ignorance is a stronger notion that mere *uncertainty*:

- Uncertainty: $\neg \mathsf{K}_S[\varphi]$
- Ignorance: $\neg \mathsf{K}_S[\varphi] \wedge \neg \mathsf{K}_S \neg [\varphi] \qquad \equiv \mathsf{P}_S[\varphi] \wedge \mathsf{P}_S \neg [\varphi]$

Pragmatics: A basic Gricean-style calculus

(36) Sue read Tintin or Asterix. $\rightsquigarrow$*S doesn't know which*

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a. ASSERTION: $\left[\mathrm{T} \vee \mathrm{A}\right]$

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> A proposition ψ constitutes a Stronger Alternative to φ if it conveys more information (i.e. both being true, ψ is compatible with less situations). If ψ is relevant and true, S should choose ψ over φ.

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> The inference that $\neg K_S[\varphi]$, for some Stronger Alternative φ of ψ.

e. Implicature Base: $K_S[T \vee A] \wedge \neg K_S[T] \wedge \neg K_S[A]$

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> The inference that $\neg K_S[\varphi]$, for some Stronger Alternative φ of ψ.

e. Implicature Base: $K_S[T \vee A] \wedge \neg K_S[T] \wedge \neg K_S[A]$

⚠ Ignorance requires **both** $\neg K_S[\varphi]$ **and** $\neg K_S \neg [\varphi]$! $(\equiv P_S \neg [\varphi] \wedge P_S[\varphi])$

Pragmatics: A basic Gricean-style calculus

Symmetric alternatives

$$\overbrace{[T \vee A]}^{\text{assertion}} \Rightarrow \overbrace{[T] \quad [A]}^{\text{exhaust the space of possibilities}}$$

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- Suppose $K_S[A]$: then $K_S[A] \wedge \neg K_S[A] \vDash \bot$
- Suppose $K_S\neg[A]$: then $K_S[T]$, and thus $K_S[T] \wedge \neg K_S[T] \vDash \bot$

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- Suppose $K_S\neg[A]$: then $K_S[T]$, and thus $K_S[T] \wedge \neg K_S[T] \vDash \bot$
- Therefore, neither $K_S[A]$ nor $K_S\neg[A]$ can be true: $\neg K_S[A] \wedge \neg K_S\neg[A]$

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$$\overbrace{[T \vee A]}^{\text{assertion}} \Rightarrow \overbrace{[T] \quad [A]}^{\text{exhaust the space of possibilities}}$$

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- Suppose $K_S\neg[A]$: then $K_S[T]$, and thus $K_S[T] \wedge \neg K_S[T] \vDash \bot$
- Therefore, neither $K_S[A]$ nor $K_S\neg[A]$ can be true: $\neg K_S[A] \wedge \neg K_S\neg[A]$
- The missing inferences from the IMPLICATURE BASE are **entailed**:

$$\underbrace{K_S[T \vee A]}_{\text{EI}} \wedge \underbrace{\neg K_S[T]}_{\text{PI}} \wedge \underbrace{\neg K_S[A]}_{\text{PI}} \wedge \underbrace{\neg K_S\neg[T]}_{\text{Entailment}} \wedge \underbrace{\neg K_S\neg[A]}_{\text{Entailment}}$$

Ignorance through symmetry!

Pragmatics: Two steps for AL's alternatives

Alternatives pertinent to the pragmatic calculus are obtained from two independent sources:

- Focus alternatives.
- Substitution of AL-propositions by their corresponding *EX*-propositions.

Pragmatics: Step I –Use focus alternatives

For a proposition of the form $[AL\ p]$, replace the prejacent p by every proposition in $[\![p]\!]^{c_f}$.

(37) $Alt_{FOC}([\![33]\!]) = \{[\![\text{at least } p]\!] \mid p \in [\![33]\!]^{c_f}\} =$

$$\left\{\begin{array}{l} \ldots \\ \textit{Sue is at least a visiting professor,} \\ \textit{Sue is at least an assistant professor,} \\ \textit{Sue is at least an associate professor,} \\ \ldots \end{array}\right\}$$

Pragmatics: Step II –Trade-in with EX

AL-statements stand in an asymmetric entailment relation with EX-statements: trade AL for EX.

(38) $Alt_{EX}(\llbracket 33 \rrbracket) = \{\llbracket \text{only } p \rrbracket \mid p \in \llbracket 33 \rrbracket^{c_f}\} =$

$$\left\{ \begin{array}{l} \ldots \\ \textit{Sue is only a visiting professor}, \\ \textit{Sue is only an assistant professor}, \\ \textit{Sue is only an associate professor}, \\ \ldots \end{array} \right\}$$

Pragmatics: Step III –Informativity

We now have lots of alternatives to p: $Alt_{EX} \cup Alt_{FOC}$:

(39) $Alt([\![33]\!]) = \left\{ \begin{array}{l} \textit{Sue is at least an assisstant professor,} \\ \textit{Sue is at least an associate professor,} \\ \textit{Sue is at least a full professor,} \\ \textit{Sue is at least a distiguished professor} \\ \textit{Sue is only a visiting professor,} \\ \textit{Sue is only an assistant professor,} \\ \textit{Sue is only an associate professor,} \\ \textit{Sue is only a full professor,} \\ \textit{Sue is only a distinguished professor} \end{array} \right\}$

Pragmatics: Step III –Informativity

- **Informativity**
  A proposition φ is more informative than an alternative ψ iff, both are true and φ is compatible with less worlds/situations than ψ.

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  A proposition φ is more informative than an alternative ψ iff, both are true and φ is compatible with less worlds/situations than ψ.

- $[\texttt{Assoc}]$ ranks higher than $[\texttt{Assist}]$
- $[\texttt{Assoc}] \not\models [\texttt{Assist}]$
- But $[\geq \texttt{Assoc}]$ is more informative than $[\geq \texttt{Assis}]$: it excludes $[\mathit{EX}\,\texttt{Assist}]$.

Pragmatics: Step III –Informativity

The final set of Stronger Alternatives of *p*:

(40) $SA([\![33]\!]) = \left\{ \begin{array}{l} \textit{Sue is at least an associate professor,} \\ \textit{Sue is at least a full professor,} \\ \textit{Sue is at least a distiguished professor} \\ \textit{Sue is only an assistant professor,} \\ \textit{Sue is only an associate professor,} \\ \textit{Sue is only a full professor,} \\ \textit{Sue is only a distinguished professor} \end{array} \right\}$

- This set of stronger alternatives is sufficient to derive the right kind of IIs simply by following our previous pragmatic calculus.

Pragmatics: Calculating Implicatures

(41) Sue is at least as assistant professor.

Pragmatics: Calculating Implicatures

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a. ASSERTION: $[\geq \texttt{Assis}]$

Pragmatics: Calculating Implicatures

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c. $SA([\geq \texttt{Assis}]) = (40)$

Pragmatics: Calculating Implicatures

(41) Sue is at least as assistant professor.

a. Assertion: $[\geq \texttt{Assis}]$

b. Epistemic Implication: $K_S[\geq \texttt{Assis}]$

c. $SA([\geq \texttt{Assis}]) =$ (40)

d. Primary Implicatures:

$\neg K_S[\geq \texttt{Assoc}] \wedge \neg K_S[\geq \texttt{Assis}] \wedge \neg K_S[\geq \texttt{Full}] \wedge \neg K_S[\geq \texttt{Dist}]$

$\neg K_S[\textit{Ex}\ \texttt{Assis}] \wedge \neg K_S[\textit{Ex}\ \texttt{Assoc}] \wedge \neg K_S[\textit{Ex}\ \texttt{Full}] \wedge \neg K_S[\textit{Ex}\ \texttt{Dist}]$

Pragmatics: Calculating Implicatures

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$\neg K_S[\geq \texttt{Assoc}] \wedge \neg K_S[\geq \texttt{Assis}] \wedge \neg K_S[\geq \texttt{Full}] \wedge \neg K_S[\geq \texttt{Dist}]$
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Together with the Epistemic Implication, these implicatures constitute the Implicature Base.

(42) Implicature Base:
$K_S[\geq \texttt{Assis}] \wedge$
$\neg K_S[\geq \texttt{Assoc}] \wedge \neg K_S[\geq \texttt{Assist}] \wedge \neg K_S[\geq \texttt{Full}] \wedge \neg K_S[\geq \texttt{Dist}] \wedge$
$\neg K_S[\textit{Ex}\ \texttt{Assis}] \wedge \neg K_S[\textit{Ex}\ \texttt{Assoc}] \wedge \neg K_S[\textit{Ex}\ \texttt{Full}] \wedge \neg K_S[\textit{Ex}\ \texttt{Dist}]$

Pragmatics: Entailments

Two of the stronger alternatives in the Implicature Base are **symmetric**: $[\textit{Ex}\ \texttt{Assis}]$ and $[\geq\ \texttt{Assoc}]$.

$$\underbrace{[\geq\ \texttt{Assis}]}_{\text{assertion}} \Rightarrow \overbrace{\underbrace{[\textit{Ex}\ \texttt{Assis}]}_{\text{SA}}\quad\underbrace{[\geq\ \texttt{Assoc}]}_{\text{SA}}}^{\text{exhaust the space of possibilities}}$$

- The speaker **must** contend that:
  - It's possible that Sue is just an assistant professor
  - It's possible that Sue is not just an assistant professor
  - It's possible that Sue is more than just an assistant professor
  - It's possible that Sue is not more than just an assistant professor

Pragmatics: Conclusion I

When AL modifies a totally ordered associate:

① Two alternatives **must** constitute epistemic possibilities for the speaker:

- The exhaustive interpretation of the prejacent ($[\![\textit{Ex}\ \texttt{Assis}]\!]$)
- The immediately stronger AL-alternative ($[\![\geq\ \texttt{Assoc}]\!]$)

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② Other alternatives **may but need not** constitute epistemic possibilities.

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② Other alternatives **may but need not** constitute epistemic possibilities.

➔ **Partial Ignorance**

Pragmatics: AL with Partial Orders

Assume a context with domain $\{Bill, Sue, Ed\}$.

(43) Liz saw at least $[\text{Bill}]_F$

Pragmatics: AL with Partial Orders

Assume a context with domain $\{Bill, Sue, Ed\}$.

(43) Liz saw at least [Bill]$_F$

The derivation of the ordinary and focus semantic values proceeds as usual.

(44) a. $[\![43]\!]^f = \{saw(Liz, x) \mid x \in D_e\}$

b. $[\![43]\!]^o = Liz\ saw\ Bill$

c. $[\![43]\!]^{c_f} = \left\{ \begin{array}{l} L\ saw\ B,\ L\ saw\ S,\ L\ saw\ E, \\ L\ saw\ B\ and\ S,\ L\ saw\ B\ and\ E,\ L\ saw\ S\ and\ E, \\ L\ saw\ B\ and\ S\ and\ E \end{array} \right\}$

AL with Partial Orders

(45) Liz saw at least Bill

a. ASSERTION: $[\geq \text{B}]$

AL with Partial Orders

(45) Liz saw at least Bill

a. ASSERTION: $[\geq \text{B}]$

b. EPISTEMIC IMPLICATION: $\text{K}_S[\geq \text{B}]$

AL with Partial Orders

(45) Liz saw at least Bill

a. ASSERTION: $[\geq \text{B}]$

b. EPISTEMIC IMPLICATION: $\text{K}_S[\geq \text{B}]$

c. $SA([\geq \text{B}]) =$

$$\left\{ \begin{array}{l} [Ex\ \text{B}], \\ [Ex\ \text{B} \oplus \text{S}], [Ex\ \text{B} \oplus \text{E}], [Ex\ \text{B} \oplus \text{S} \oplus \text{E}], \\ [\geq\ \text{B} \oplus \text{S}], [\geq\ \text{B} \oplus \text{E}], [\geq\ \text{B} \oplus \text{S} \oplus \text{E}] \end{array} \right\}$$

AL with Partial Orders

(45) Liz saw at least Bill

a. Assertion: $[\geq \text{B}]$

b. Epistemic Implication: $\text{K}_\text{S}[\geq \text{B}]$

c. $SA([\geq \text{B}]) =$

$$\left\{ \begin{array}{l} [\textit{Ex}\ \text{B}], \\ [\textit{Ex}\ \text{B} \oplus \text{S}], [\textit{Ex}\ \text{B} \oplus \text{E}], [\textit{Ex}\ \text{B} \oplus \text{S} \oplus \text{E}], \\ [\geq \text{B} \oplus \text{S}], [\geq \text{B} \oplus \text{E}], [\geq \text{B} \oplus \text{S} \oplus \text{E}] \end{array} \right\}$$

d. Primary Implicatures:

$$\left\{ \begin{array}{l} \neg \text{K}_\text{S}[\textit{Ex}\ \text{B}] \wedge \\ \neg \text{K}_\text{S}[\textit{Ex}\ \text{B} \oplus \text{S}] \wedge \neg \text{K}_\text{S}[\textit{Ex}\ \text{B} \oplus \text{E}] \wedge \neg \text{K}_\text{S}[\textit{Ex}\ \text{B} \oplus \text{S} \oplus \text{E}] \wedge \\ \neg \text{K}_\text{S}[\geq \text{B} \oplus \text{S}] \wedge \neg \text{K}_\text{S}[\geq \text{B} \oplus \text{E}] \wedge \neg \text{K}_\text{S}[\geq \text{B} \oplus \text{S} \oplus \text{E}] \end{array} \right\}$$

Exhausting possibilities

The crucial difference between TOs and POs: there's no pair of symmetric alternatives for PO associates!

(46) $\underbrace{[\geq \mathrm{B}]}_{\text{assertion}} \Rightarrow \overbrace{\underbrace{[\mathit{Ex}\ \mathrm{B}]}_{\text{SA}} \wedge \underbrace{[\geq\ \mathrm{B} \oplus \mathrm{S}]}_{\text{SA}} \wedge \underbrace{[\geq\ \mathrm{B} \oplus \mathrm{E}]}_{\text{SA}}}^{\text{exhaust the space of possibilities}}$

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- No entailments of the form $\neg \mathrm{K}_S \neg [\varphi]$ ($\equiv \mathrm{P}_S[\varphi]$)! E.g.: *S* **could** know that:
  - it's not the the case that Liz saw *only Bill*.
    $\rightsquigarrow$ *S* doesn't know who else Liz saw

Exhausting possibilities

The crucial difference between TOs and POs: there's no pair of symmetric alternatives for PO associates!

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- No entailments of the form $\neg \mathrm{K}_S \neg [\varphi]$ ($\equiv \mathrm{P}_S[\varphi]$)! E.g.: *S* **could** know that:
  - it's not the the case that Liz saw *only Bill*.
    $\rightsquigarrow$ *S* doesn't know who else Liz saw
  - it's not the case that Liz saw *at least Bill and Sue*.
    $\rightsquigarrow$ *S* doesn't know whether Liz saw *only Bill* or *Bill and Sue*.

Exhausting possibilities

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    $\rightsquigarrow$ *S* doesn't know who else Liz saw
  - it's not the case that Liz saw *at least Bill and Sue*.
    $\rightsquigarrow$ *S* doesn't know whether Liz saw *only Bill* or *Bill and Sue*.
- However, the speaker **must not** know that e.g. Liz saw *only Bill*.

Pragmatics: Conclusion II

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① No ignorance about any one particular alternative is predicted: they all **may but need not** constitute epistemic possibilities.

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② The resulting IIs are weaker than those of AL with totally ordered associates.

- AL with TO: **Strong Partial Ignorance** ⇒ IIs about **two** alternatives.
- AL with PO: **Weak Partial Ignorance** ⇒ Uncertainty inferences.

Pragmatics: Conclusion II

① **New Empirical Generalization**

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AL involves a pragmatic reasoning procedure that relies on **both** focus and exhaustive alternatives:

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- EX-alternatives are required to create *symmetric* alternatives.
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④ With the two-alternative system we get:

- Flexibility: we may draw IIs about any focused constituent.
- Strength of Ignorance: we account for the Partial Ignorance of AL (*viz.* Total Ignorance of disjunction).
- Non-uniformity: the status of the IIs differs across different types of associates.

CONCLUSION & RAMIFICATIONS

Pragmatics as the “science of the **un**said”.

Conclusions

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- *Contextualism* [e.g. Hirschberg 1991, 93]
  "...the orderings [that support scalar implicatures] are partially [contextually] ordered sets [...] and any poset can support scalar implicatures."

- *Logicism* [e.g. Magri 2017, 10]
  "...the algorithm for the computation of scalar implicatures must be purely logical, namely blind to common knowledge."

Conclusions

Contextualism

orderings

Logicism

algorithm

Conclusions: the lesson from AL

The question is ill-posed, it's not either one or the other!

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reconciliation

We do not rely on logic to establish order,
we do rely on logic to reason about order.

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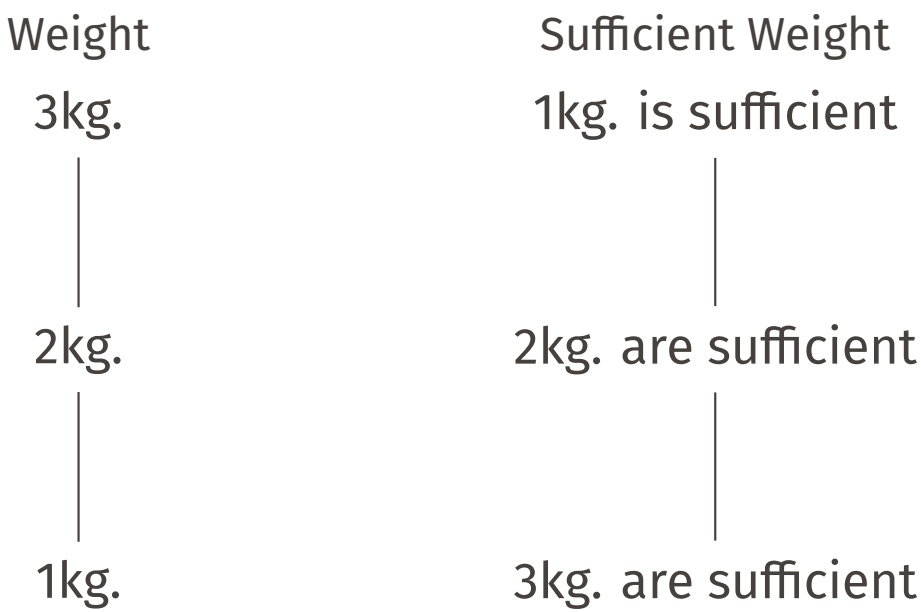

Looking forward: the *locus* of order

(47) a. The apples weigh at least 2kg
$\rightsquigarrow$ *S* doesn't know whether exactly 2kgs. or more.

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b. ? At least 2kg. of apples are sufficient.

$\not\rightsquigarrow$ *S doesn't know whether exactly 2kg. or less are sufficient.*

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   ⇝ *S* doesn't know whether exactly 2kgs. or more.
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(48) a. ?? At least 20mph is fast for this road.
   b. ?? At least 20 people are too many.

(49) a. 20mph or more is fast for this road.
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- The ordering is induced solely on the basis of its focus associate, not on the whole proposition.

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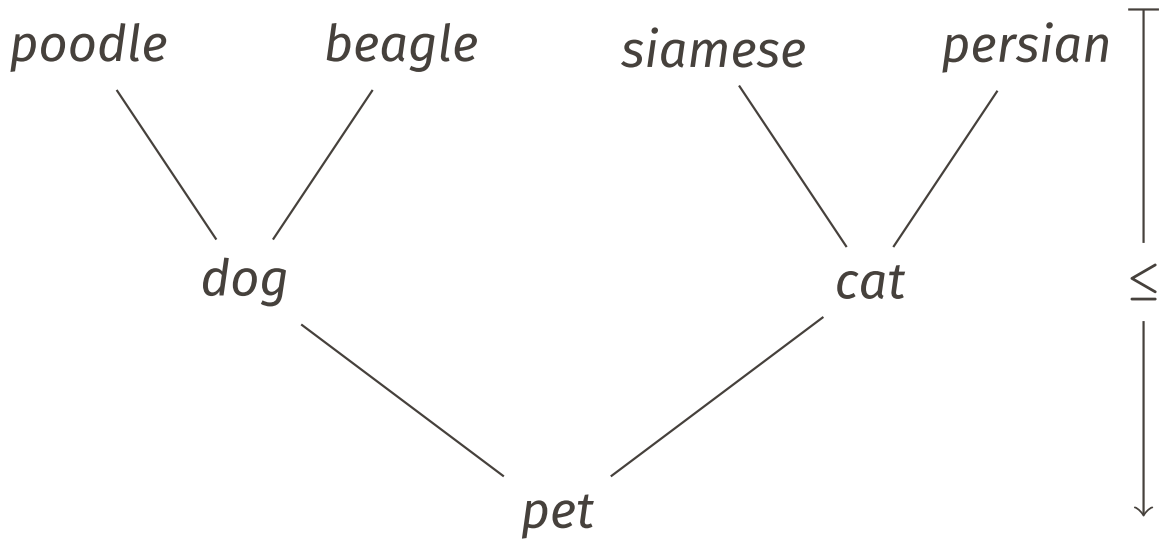

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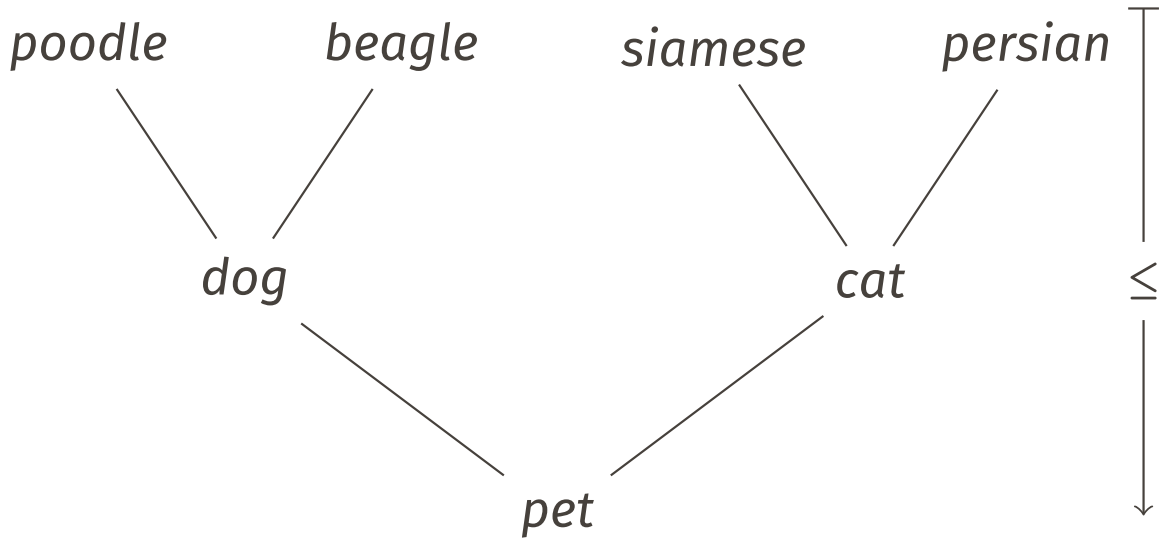
❓ Can we identify a natural class of structures that support `II`s with AL?

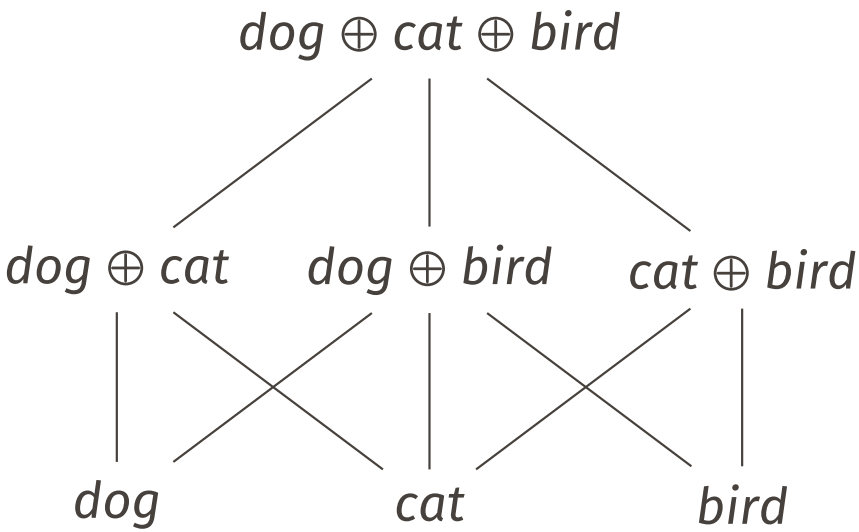


(50) a. What pet did Sue get?

b. # She got at least a dog.

⇝*S doesn't know whether Sue got a poodle or a beagle*

Looking forward: *impossible* orderings?



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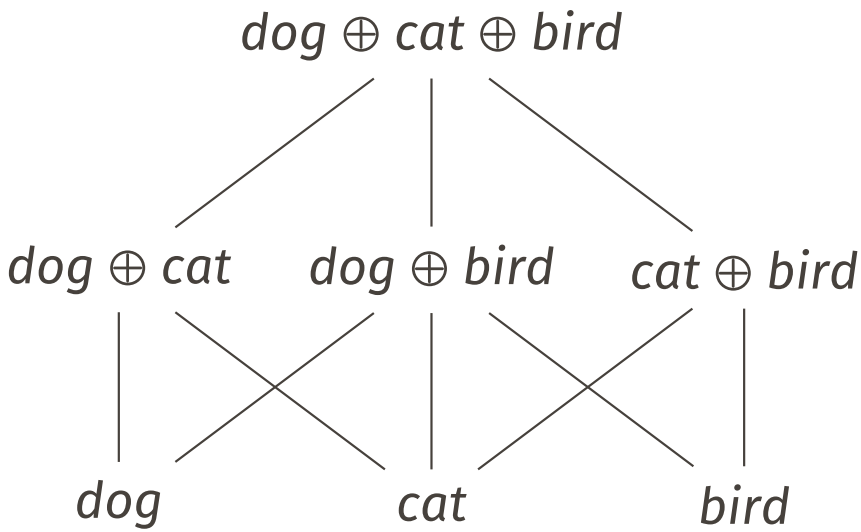

(51) a. What pet did Sue get?

b. She got at least a dog.

⇝*S doesn't know whether Sue got a cat and a bird as well*

Looking forward: *impossible* orderings?

- Not any PO set would do. IIs of AL seem to prefer structures with a **unique maximal element** (i.e. a join-semilattice).

The science of the unsaid is partly
the science of finding and reasoning about order.

Thank you!

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Embedding

(52) Every student read at least two papers.

- IIs are derived if there is a pair of **symmetric** Stronger Alternatives.
- But $\forall\left[Ex\ 2\right]$ and $\forall\left[\geq 3\right]$ are not symmetric, since $\forall\left[\geq 2\right]$ could be true by virtue of some students reading exactly two papers while other students read more than two.

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- The interpretation conveying ignorance may be achieved by interpreting the universal quantifier under the scope of AL: $[\geq 2\forall]$.

Scalar Implicature: A problem

(54) At least two people came to the party.
↛ *It is not the case that at least three people came to the party*

- Prediction: `II`s about two SAs, $[Ex\ 2]$ and $[\geq 3]$:
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- In principle, any other additional alternative φ could be strengthened, $K_S \neg [\varphi]$. There are **many** such alternatives. E.g. $[Ex\ 3]$ and $[\geq\ 4]$.
- What if we strengthen **both**? Contradiction:
  $K_S \neg [Ex\ 3] \wedge K_S \neg [\geq\ 4] \vDash K_S \neg [\geq\ 3]$

Scalar Implicature: Towards a solution

① Schwarz's (2016) solution: import a consistency preservation mechanism from grammatical approaches to implicature (cf. Innocent Exclusion).

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- ES_1 the speaker is **maximally knowledgeable** about the question that the proposition she is uttering is making a contribution to.
  - What if the task of *at least* is precisely to preempt these assumption?
- ES_2 The listener is free to consider the speaker an authority about **any** SA, unless it is in conflict with IIs.
  - Then for $K_S[\geq 2]$ the listener will not infer $K_S\neg[Ex\ 3] \wedge K_S\neg[\geq\ 4]$ because this conjunction is in conflict with the $II\neg K_S[\geq 3] \wedge \neg K_S\neg[\geq 3]$.

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  - *Answer:*
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    - ...maybe one. — Condition BAD
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    - ...but not only two. — Condition TARGET

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- **Type** Partial Order (Conjunction):
  - *Question*:
    Who completed the quiz?
  - *Answer:*
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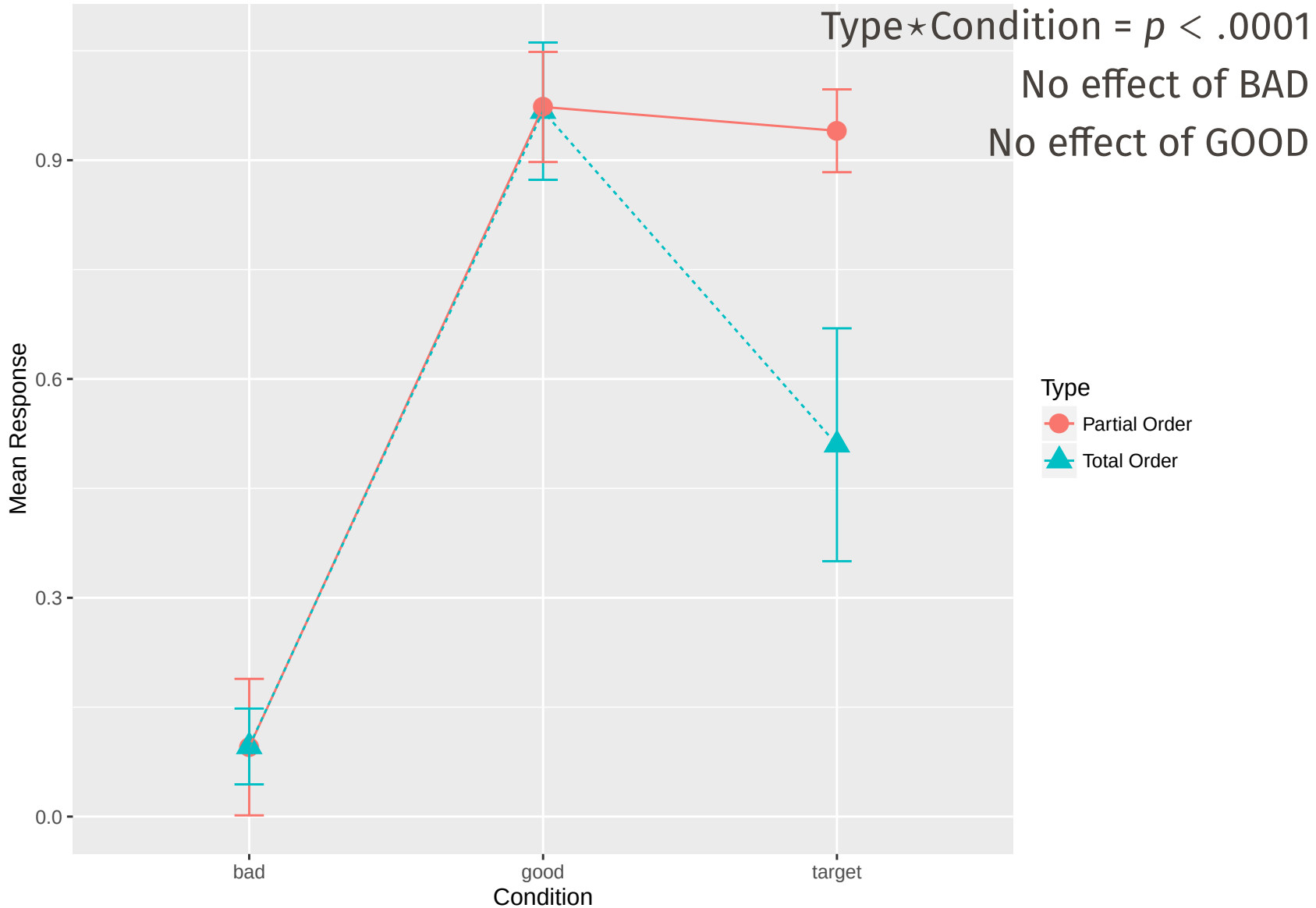
    - ...maybe only Liz. — Condition BAD
    - ...maybe somebody else too. — Condition GOOD
    - ...but not only them. — Condition TARGET

Results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No effect of GOOD
Mean Response
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target
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