Inspire Create Transform

A note on "Scheduling the South American Qualifiers to the 2018 FIFA World Cup by integer programming"

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1. PROBLEM DESCRIPTION

- In South America, every 4 years, 10 national soccer teams compete for one of the 4.5 spots in the World Cup.
- This process is made via a tournament “all against all”, where all teams play twice against each other.
- The configuration of the matches, have to keep some restrictions and also favorably, a symmetry scheme.

1. PROBLEM DESCRIPTION

- In 2017, Durán et al. (2017) used integer lineal programming to propose schedules that meet all the requirements.
- They introduced some symmetry schemes to finally adopt the French One.
- In this research, we want to analyze the symmetry schemes proposed in order to compare time convergence and features.
- Also, additional features are proposed based on the observed results.

Durán et al. (2017) proposed the followings symmetry schemes

Scheme	Schedule										
Mirrored scheme	1	2	3	...	9	1	2	3	...	8	9
French scheme	1	2	3	...	9	2	3	4	...	9	1
English scheme	1	2	3	...	9	9	1	2	...	7	8
Inverted scheme	1	2	3	...	9	9	8	7	...	2	1
Back to back scheme	1	1	2	...	5	5	6	6	...	9	9

Table 1. Symmetry schemes

2. STATE OF THE ART

Some of the restrictions incompatible with some schemes, for example

- The back-to-back scheme is not compatible with the first double robin constraint: every team faces every other once in the first half and once in the second half.
- The mirrored scheme is incompatible with the balance constraints.

3. MATHEMATICAL MODEL

We got a set of teams $I = \{1, \ldots, n\}$ (in our case $n = 10$), a set of dates (rounds) $K = \{1, \ldots, 2(n-1)\}$ which have a subset K_{odd} which would be the odd rounds. The main decision variable X_{ijk} is a binary variable that is equal to 1 if the team i plays against team j at home in round k and 0 otherwise.

3. MATHEMATICAL MODEL

Double round robin constraints:

$$\sum_{k \in K : k \leq n-1} (X_{i,j,k} + X_{j,i,k}) = 1 \qquad \forall\, i \in I, j \in I : i \neq j \tag{1}$$

$$\sum_{k \in K : k > n-1} (X_{i,j,k} + X_{j,i,k}) = 1 \qquad \forall\, i \in I, j \in I : i \neq j \tag{2}$$

$$\sum_{k \in K} X_{i,j,k} = 1 \qquad \forall j \in I, k \in K \tag{3}$$

3. MATHEMATICAL MODEL

Compactness. The schedule must also be compact, that is, all teams must play one match in each round.

$$\sum_{i \in I : i \neq j} (X_{i,j,k} + X_{j,i,k}) = 1 \qquad \forall j \in I, k \in K \tag{4}$$

Top team constraints. The set of best teams I_s is conformed by Argentina and Brazil.

$$\sum_{j \in I_s} (X_{i,j,k} + X_{j,i,k} + X_{i,j,k+1} + X_{j,i,k+1}) \leq 1 \qquad \forall i \in I \setminus Is, k \in K : k \leq |K| \tag{5}$$

3. MATHEMATICAL MODEL

Balance constraints. These are included to maintain a balanced distribution among the teams of H-A sequences in double rounds.

$$n/2 - 1 \leq \sum_{k \in K_{odd}} y_{i,k} \leq n/2 \qquad \forall i \in I \tag{6}$$

$$\sum_{j \in I : i \neq j} (X_{i,j,k} + X_{j,i,k+1}) \leq 1 + y_{i,k} \qquad \forall i \in I, k \in K_{odd} \tag{7}$$

$$y_{i,k} \leq \sum_{j \in I : i \neq j} x_{j,i,k+1} \qquad \forall i \in I, k \in K_{odd} \tag{8}$$

3. MATHEMATICAL MODEL

Objective function. They aim at minimizing the total number of away breaks within double across all teams.

$$\sum_{j \in I : i \neq j} (X_{i,j,k} + X_{j,i,k+1}) \leq 1 + w_{i,k} \qquad \forall i \in I, k \in K_{odd} \tag{9}$$

$$w_{i,k} \leq \sum_{j \in I : i \neq j} X_{j,i,k} \qquad \forall i \in I, k \in K_{odd} \tag{10}$$

$$w_{i,k} \leq \sum_{j \in I : i \neq j} X_{j,i,k+1} \qquad \forall i \in I, k \in K_{odd} \tag{11}$$

The minimization of the total number of breaks in double rounds is captured by the following objective function:

$$\min \sum_{i \in I} \sum_{k \in K_{odd}} w_{i,k} \tag{12}$$

3. MATHEMATICAL MODEL

Mirrored scheme.

$$X_{i,j,k} = X_{j,i,k+n-1} \qquad \forall i,j \in I : i \neq j, k \in K : 1 \leq k \tag{13}$$

French scheme.

$$X_{i,j,1} = X_{j,i,2n-2}, \quad X_{i,j,k} = X_{j,i,k+n-2}, \qquad \forall i,j \in I : i \neq j, 2 \leq k \leq n-1 \tag{14}$$

English scheme.

$$X_{i,j,n-1} = X_{j,i,n}, \quad X_{i,j,k} = X_{j,i,k+n}, \qquad \forall i,j \in I : i \neq j, 2 \leq k \leq n-2 \tag{15}$$

Inverted scheme.

$$X_{i,j,k} = X_{j,i,k+1} \qquad \forall i,j \in I : i \neq j, k \in K_{odd} \tag{16}$$

4. COMPUTATIONAL EXPERIMENTS

We did an implementation of the model and run it for different values of n: 10 (original), 20, 30, 50 and 100.
The results are presented in the table **??**, UK stands for unknown (we don't get a solution in an hour) and TLE is for Time Limit Exceed (the model will take more than an hour to get a solution).

n	**10**		**20**		**30**		**50**	
SCHEME	**OF**	**Time (s)**	**OF**	**Time (s)**	**OF**	**Time (s)**	**OF**	**Time (s)**
No symmetry	0	183.08	4	TLE	0	TLE	UK	TLE
Mirrored	8	TLE	37	TLE	91	TLE	UK	TLE
French	0	2.96	0	39.99	0	213.24	UK	TLE
English	0	0.78	0	35.4	0	446.96	UK	TLE
Inverted	0	114.04	0	241.79	0	1383.02	UK	TLE
Back to back	0	0.15	0	7.73	0	82.71	0	1585.44

With $n = 100$, all the schemes get the Time Limit Exceed and no solution founded.

5. ADDITIONAL FEATURES

We proposed two additional features to add to the model

- In half of the matches, every team must play against the best teams once as local once as visitant.

$$\sum_{j \in I_s} \sum_{k \in K} X_{i,j,kp} = 1 \qquad \forall i \in I : i \neq j \quad k \leq n-1 \tag{17}$$

- No more than 2 H-A or A-H consecutive sequences.

$$\sum_{k \leq kp \leq k+2} y_{i,kp} \leq 2 \qquad \forall i \in I \, \forall j \in I : i \neq j \, \forall k \in K_{odd} \; k < |K| - 4 \tag{18}$$

$$\sum_{k \leq kp \leq k+2} y_{i,kp} \geq 1 \qquad \forall i \in I \, \forall j \in I : i \neq j \, \forall k \in K_{odd} \; k < |K| - 4 \tag{19}$$

This features keep the feasibility of the model and don't increase the computational time.

REFERENCES

Durán, G., Guajardo, M., & Sauré, D. (2017). Scheduling the south american qualifiers to the 2018 fifa world cup by integer programming. *Elsevier*.

Muchas gracias.
¿Preguntas?

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