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A note on “Scheduling the South American Qualifiers to the 2018 FIFA World Cup by integer programming”

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1. PROBLEM DESCRIPTION

- In South America, every 4 years, 10 national soccer teams compete for one of the 4.5 spots in the World Cup.
- This process is made via a tournament “all against all”, where all teams play twice against each other.
- The configuration of the matches, have to keep some restrictions and also favorably, a symmetry scheme.

1. PROBLEM DESCRIPTION

- In 2017, Durán et al. (2017) used integer lineal programming to propose schedules that meet all the requirements.
- They introduced some symmetry schemes to finally adopt the French One.
- In this research, we want to analyze the symmetry schemes proposed in order to compare time convergence and features.
- Also, additional features are proposed based on the observed results.

2. STATE OF THE ART

Durán et al. (2017) proposed the followings symmetry schemes

Scheme	Schedule										
Mirrored scheme	1	2	3	...	9	1	2	3	...	8	9
French scheme	1	2	3	...	9	2	3	4	...	9	1
English scheme	1	2	3	...	9	9	1	2	...	7	8
Inverted scheme	1	2	3	...	9	9	8	7	...	2	1
Back to back scheme	1	1	2	...	5	5	6	6	...	9	9

Table 1. Symmetry schemes

2. STATE OF THE ART

Some of the restrictions incompatible with some schemes, for example

- The back-to-back scheme is not compatible with the first double robin constraint: every team faces every other once in the first half and once in the second half.
- The mirrored scheme is incompatible with the balance constraints.

3. MATHEMATICAL MODEL

We got a set of teams $I = \{1, \ldots, n\}$ (in our case $n = 10$), a set of dates (rounds) $K = \{1, \ldots, 2(n-1)\}$ which have a subset K_{odd} which would be the odd rounds. The main decision variable X_{ijk} is a binary variable that is equal to 1 if the team i plays against team j at home in round k and 0 otherwise.

3. MATHEMATICAL MODEL

Double round robin constraints:

$$\sum_{k \in K : k \leq n-1} (X_{i,j,k} + X_{j,i,k}) = 1 \qquad \forall\, i \in I, j \in I : i \neq j \tag{1}$$

$$\sum_{k \in K : k > n-1} (X_{i,j,k} + X_{j,i,k}) = 1 \qquad \forall\, i \in I, j \in I : i \neq j \tag{2}$$

$$\sum_{k \in K} X_{i,j,k} = 1 \qquad \forall j \in I, k \in K \tag{3}$$

Compactness. The schedule must also be compact, that is, all teams must play one match in each round.

$$\sum_{i \in I : i \neq j} (X_{i,j,k} + X_{j,i,k}) = 1 \qquad \forall j \in I, k \in K \tag{4}$$

Top team constraints. The set of best teams I_s is conformed by Argentina and Brazil.

$$\sum_{j \in I_s} (X_{i,j,k} + X_{j,i,k} + X_{i,j,k+1} + X_{j,i,k+1}) \leq 1 \qquad \forall i \in I \setminus Is, k \in K : k \leq |K| \tag{5}$$

3. MATHEMATICAL MODEL

Balance constraints. These are included to maintain a balanced distribution among the teams of H-A sequences in double rounds.

$$n/2 - 1 \leq \sum_{k \in K_{odd}} y_{i,k} \leq n/2 \qquad \forall i \in I \tag{6}$$

$$\sum_{j \in I : i \neq j} (X_{i,j,k} + X_{j,i,k+1}) \leq 1 + y_{i,k} \qquad \forall i \in I, k \in K_{odd} \tag{7}$$

$$y_{i,k} \leq \sum_{j \in I : i \neq j} x_{j,i,k+1} \qquad \forall i \in I, k \in K_{odd} \tag{8}$$

3. MATHEMATICAL MODEL

Objective function. They aim at minimizing the total number of away breaks within double across all teams.

$$\sum_{j \in I : i \neq j} (X_{i,j,k} + X_{j,i,k+1}) \leq 1 + w_{i,k} \qquad \forall i \in I, k \in K_{odd} \tag{9}$$

$$w_{i,k} \leq \sum_{j \in I : i \neq j} X_{j,i,k} \qquad \forall i \in I, k \in K_{odd} \tag{10}$$

$$w_{i,k} \leq \sum_{j \in I : i \neq j} X_{j,i,k+1} \qquad \forall i \in I, k \in K_{odd} \tag{11}$$

The minimization of the total number of breaks in double rounds is captured by the following objective function:

$$\min \sum_{i \in I} \sum_{k \in K_{odd}} w_{i,k} \tag{12}$$

3. MATHEMATICAL MODEL

Mirrored scheme.

$$X_{i,j,k} = X_{j,i,k+n-1} \qquad \forall i,j \in I : i \neq j, k \in K : 1 \leq k \tag{13}$$

French scheme.

$$X_{i,j,1} = X_{j,i,2n-2}, \quad X_{i,j,k} = X_{j,i,k+n-2}, \qquad \forall i,j \in I : i \neq j, 2 \leq k \leq n-1 \tag{14}$$

English scheme.

$$X_{i,j,n-1} = X_{j,i,n}, \quad X_{i,j,k} = X_{j,i,k+n}, \qquad \forall i,j \in I : i \neq j, 2 \leq k \leq n-2 \tag{15}$$

Inverted scheme.

$$X_{i,j,k} = X_{j,i,k+1} \qquad \forall i,j \in I : i \neq j, k \in K_{odd} \tag{16}$$

4. MAIN ADVANCES

We run several experiments to measure the impact of certain factors in the model run time, these factors are:

- Size (number of teams)
- Number of top teams
- Impact of new constraints

Where it was determined that an increase in the size costs much more than an increase in the number of top teams. Also, depending on the used scheme it could be noticed that some of them help in the convergence when the solver are exploring new solutions. Also, the new constraints added by us doesn't appear to increase the execution time.

5. ADDITIONAL FEATURES

We proposed two additional features to add to the model

- In half of the matches, every team must play against the best teams once as local once as visitant.

$$\sum_{j \in I_s} \sum_{k \in K} X_{i,j,kp} = 1 \qquad \forall i \in I : i \neq j \quad k \leq n-1 \tag{17}$$

- No more than 2 H-A or A-H consecutive sequences.

$$\sum_{k \leq kp \leq k+2} y_{i,kp} \leq 2 \qquad \forall i \in I \, \forall j \in I : i \neq j \, \forall k \in K_{odd} \; k < |K| - 4 \tag{18}$$

$$\sum_{k \leq kp \leq k+2} y_{i,kp} \geq 1 \qquad \forall i \in I \, \forall j \in I : i \neq j \, \forall k \in K_{odd} \; k < |K| - 4 \tag{19}$$

This features keep the feasibility of the model and don't increase the computational time.

REFERENCES

Durán, G., Guajardo, M., & Sauré, D. (2017). Scheduling the south american qualifiers to the 2018 fifa world cup by integer programming. *Elsevier*.

Thanks.
Any questions?

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