Innovative Applications of O.R.

Scheduling the South American Qualifiers to the 2018 FIFA World Cup by integer programming

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A B S T R A C T

Every four years, the 10 national teams members of the South American Football Confederation (CONMEBOL) compete for one of the South American slots in the final phase of the FIFA World Cup. The qualifying competition consists of a double round robin tournament. The matches are scheduled in 9 closely spaced pairs known as double rounds. Every team plays twice in each double round. The tournament is spread over 2 years, so the double rounds are months apart. After using the same mirrored schedule for about twenty years, and persistent complaints from its members, CONMEBOL decided to change the schedule for the 2018 World Cup. Supported by one of CONMEBOL's members, we used integer programmming to construct schedules that overcome the main drawbacks of the previous approach. After exploring many design criteria, we proposed a candidate schedule based on a *French scheme*. The main feature of the proposed schedule is that every team plays once at home and once away on each double round, a departure from traditional symmetric (mirrored) schemes. This proposal was unanimously approved by CONMEBOL members and is currently being used in the qualifier tournament for the 2018 FIFA World Cup in Russia.

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1. Introduction

Organized every 4 years by the Fédération Internationale de Football Association (FIFA), the FIFA World Cup (or simply, *World Cup*) is the most important football competition in the world. With dozens of sponsors and intense media coverage, the final phase of the World Cup receives more international attention than any other single-sport event. Some 3.4 million spectators filled the stadiums for the 2014 World Cup finals matches in Brazil and the television coverage reached 3.2 billion people around the world (FIFA, 2014a; 2014b).

The World Cup begins with a qualifying stage in which each of FIFA's six continental federations selects its representatives for the World Cup's final stage. Out of FIFA's 209 member associations, only 32 teams qualify for this final stage. In the South American qualifying stage, the teams that will represent the South American Football Confederation (CONMEBOL) are determined by the results of a double round-robin tournament organized by the Confederation. The tournament, which is followed by the great majority of the nearly 420 million inhabitants of South America, consists of 10 national teams and is played over nine FIFA *double rounds*, each one consisting of two games. Although these double rounds are usually played once every several months, the two games within a double round are scheduled only 4 or 5 days apart.

In all of the World Cups from 1998 through 2014, CONMEBOL used a *mirrored* double round-robin schedule for the qualifying tournament. Under this format, a team would face every rival once over the first 9 rounds in some defined order, and then again over the last 9 rounds in the same order but with the home-away status reversed. Moreover, in the tournaments of 2002 through 2010 the schedules themselves were identical in the sense that the teams played their rivals in precisely the same order each time. This setup elicited many complaints from CONMEBOL members over the years as it was perceived to unfairly benefit certain teams at the expense of others. Not surprisingly, most of the complaints came from teams that repeatedly failed to qualify for the World Cup finals.

In light of this situation, CONMEBOL decided to change the tournament's fixture for the 2018 World Cup. Whatever format

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might eventually be adopted, team assignments would be made by random draw. To ensure the new process was fair, CONMEBOL members were invited to submit candidate fixture templates (that is, a tournament fixture with the team assignments to the template positions left blank), which all of the members would then vote on.

In partnership with Chile's National Professional Football Association (ANFP), we used integer programming (IP) to develop a series of generic schedules among which Chile's proposal was selected. There is a relatively long tradition of using IP to schedule sports tournaments. Its use has been particularly popular in football; some examples are tournaments in Holland (Schreuder, 1992), Austria and Germany (Bartsch, Drexl, & Kröger, 2006), Chile (Alarcón et al., 2017; Durán et al., 2007; Durán, Guajardo, & Wolf-Yadlin, 2012), Denmark (Rasmussen, 2008), Belgium (Goossens & Spieksma, 2009), Norway (Flatberg, Nilssen, & Stølevik, 2009), Honduras (Fiallos, Pérez, Sabillón, & Licona, 2010), Brazil (Ribeiro & Urrutia, 2012) and Ecuador (Recalde, Torres, & Vaca, 2013). We refer the reader to Rasmussen and Trick (2008), Wright (2009), Kendall, Knust, Ribeiro, and Urrutia (2010), and Ribeiro (2012) for reviews of the literature on the use of IP in sport scheduling.

In our work, candidate proposals were designed so as to balance several sporting fairness considerations. One of our primary concerns was to eliminate the occurrence of *double round breaks* in which a team plays the two games in a double round either both at home or both away. Unfortunately, such a feature cannot be achieved in a mirrored fixture, as will be shown in this paper. We therefore devised our schedule proposals to eliminate double round breaks while maintaining a certain symmetry that would *resemble* a mirrored scheme. Some of these designs were inspired by the symmetric schemes used in certain European Leagues (Goossens & Spieksma, 2012).

The proposal finally chosen for submission by the ANFP, which followed a *French* symmetric scheme (explained below), was voted unanimously by CONMEBOL members and adopted for scheduling the 2018 World Cup qualifying tournament, which at the time of writing is currently being played.

The remainder of this paper is organized as follows. Section 2 provides background on the World Cup qualification tournament. Section 3 describes the basic principles and symmetric schemes for scheduling, and sets out the IP formulations. Section 4 summarizes our results and analyzes the schedule adopted by CONMEBOL. Section 5 presents our conclusions.

2. Background

2.1. South American Qualifier Tournament

CONMEBOL, the South American Football Confederation, is the organization overseeing the 2018 World Cup qualifying phase for its ten member nations: Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Paraguay, Peru, Uruguay and Venezuela.

It is arguably one of the strongest confederations in FIFA, its members having won 9 out of the 20 World Cup championships held between 1939 and 2014 (note that FIFA has 209 nation members, more than the United Nations). Thus, despite being the confederation with the fewest members, four of its teams qualify directly for the World Cup final phase while a fifth competes in an intercontinental playoff for an additional place in the finals.

CONMEBOL selects the teams that advance to the final phase of the World Cup based on the results of a double round-robin tournament in which each team plays all the other teams twice, once at home and once away. The tournament is played over 18 *rounds*, each consisting of 5 games and in which each team plays exactly once. Thus, the entire tournament consists of 90 matches. A match victory is worth 3 points, a draw 1 point, and a loss no points at all. At the end of the tournament, the 4 teams with the most points advance to the final phase of the World Cup (various tie-breaking rules apply) and the fifth team plays a two-legged tie against a team from the Asian or Oceanian confederation for an additional place.

Many players on CONMEBOL members' national teams play for the best football clubs in the world. Since these clubs are always reluctant to grant their players permission to participate in international games, FIFA publishes in advance a calendar of *double rounds* listing one-week periods in which club players must be allowed to represent their national teams. These dates have to be carefully chosen so as to not interfere with international club-level competitions and national club leagues, and are thus rather few in number. As a result, consecutive double rounds are usually months apart.

The South American qualifying tournament games takes place in 9 of FIFA's double rounds, which are spread over a period of 2–3 years. Within each double round, two rounds of the qualifying tournament are separated by only 4 or 5 days. We refer to the first (second) round in a double round as the *even* (*odd*) round, in reference to the parity of their index numbers in the series of 18 single game rounds making up the tournament. The practice of playing two games during a double round has been promoted by FIFA not only for CONMEBOL but for other confederations as well.

2.2. Qualifier Tournament Fixture 1998–2014

For all five World Cup tournaments held between 1998 and 2014, CONMEBOL used a mirrored scheme. That is, in the first 9 rounds each team played every other team once, while in the second 9 rounds the same matchups were repeated in the same order but with the home-away status reversed. Moreover, the same schedule was used for all four World Cups between 2002 and 2014 (the only variation being that since Brazil was exempted from the process in 2014 as the host team, the side that in each round would otherwise have played the Brazilians had a bye). Table 1 shows this fixture. The columns are the rounds while the rows are the teams, the cells thus giving the corresponding team's

Table 1
The 2002–2014 schedule of the South American Qualifier Tournament. The columns represent the rounds while the rows show the teams and their opponents in each round. "@" indicates an away game.

Team	1	2	3	4	5	6	7	8	9	10	...	18
ARG	CHI	@VEN	BOL	@COL	ECU	@BRA	PAR	@PER	URU	@CHI		@URU
BOL	@URU	COL	@ARG	@VEN	CHI	PAR	@ECU	@BRA	PER	URU		@PER
BRA	@COL	ECU	@PER	URU	@PAR	ARG	@CHI	BOL	@VEN	COL		VEN
CHI	@ARG	PER	@URU	PAR	@BOL	@VEN	BRA	COL	@ECU	ARG		ECU
COL	BRA	@BOL	VEN	ARG	@PER	@ECU	URU	@CHI	PAR	@BRA		@PAR
ECU	VEN	@BRA	@PAR	PER	@ARG	COL	BOL	@URU	CHI	@VEN		@CHI
PAR	@PER	URU	ECU	@CHI	BRA	@BOL	@ARG	VEN	@COL	PER		COL
PER	PAR	@CHI	BRA	@ECU	COL	@URU	VEN	ARG	@BOL	@PAR		BOL
URU	BOL	@PAR	CHI	@BRA	VEN	PER	@COL	ECU	@ARG	@BOL		ARG
VEN	@ECU	ARG	@COL	BOL	@URU	CHI	@PER	@PAR	BRA	ECU		@BRA

Table 2
Double-round breaks in the 2002–2014 South American Qualifier Tournament.

Team	B_h	B_a	B	H-A	A-H
ARG	0	0	0	9	0
BOL	2	2	4	2	3
BRA	0	0	0	0	9
CHI	1	1	2	1	6
COL	1	1	2	6	1
ECU	1	1	2	4	3
PAR	1	1	2	3	4
PER	1	1	2	6	1
URU	1	1	2	4	3
VEN	1	1	2	1	6

opponent in each round with away games indicated by an "@" sign. As an example, Argentina played a home game against Chile in the first round and an away game against Venezuela in the second round.

To the best of our knowledge, there is no official documentation detailing the criteria behind the construction of this fixture. This lack of transparency has given rise to much speculation. For example, Gilardi (2013) suggests that the fixture was designed so as to favor a particular team that did not qualify to the 1998 World Cup. (The author lists a series of factors that arguably favors a particular country, which would be behind the construction of the fixture and that, paradoxically, subsequently failed to qualify to World Cups to date.)

The 2002–2014 fixture has some positive features. For example, no team plays more than two consecutive home or away games; also, each team plays once at home and once away in the first and last double rounds. This latter feature is perceived as contributing to the fairness of the tournament. Also, no team plays consecutively against Argentina and Brazil, which are considered the two strongest South American teams.

Unfortunately, the 2002–2014 fixture also has many undesirable features. Traditionally, the literature in sports scheduling has paid special attention to the number of *breaks* in a fixture. A break occurs when a team plays two games in a row either at home or away (see Section 4 in Rasmussen & Trick (2008)). Given the long separation between successive double rounds, in the context of the qualifying tournament, it stands to reason that the focus ought to be on double-round breaks; that is, double rounds in which both games are played either at home or away. In the 2002–2014 fixture, there are 18 double-round breaks. This is largely a feature of the mirrored structure of the fixture: it can be shown that the minimum number of such breaks in a mirrored tournament is 16.

Compounding such a problem, double-round breaks in the 2002–2014 fixture are unevenly distributed among the teams. Table 2 provides summary statistics on the number of home (B_h), away (B_a) and total (B) double-round breaks per team. We see, for example, that Bolivia has the most (4) double-round breaks while Argentina and Brazil have none.

In terms of sporting fairness, away double-round breaks are undesirable as they mean not having a home-field advantage for two consecutive games (the significance of home-field advantage in football has been widely documented in the literature; see Pollard, 2006). Moreover, in the context of the South American qualifying tournament, they imply that teams might not play at home for up to a year. For example, in the qualifiers for the 2010 World Cup, Uruguay did not play at home for more than 6 months (from September 10, 2008 to March 28, 2009) due to an away break in double round 5. Given all the local excitement and the economic benefits that matches at home entail (ticket sales, advertising, etc.), away double-round breaks are also unpopular with fans, team officials and the media.

Away double-round breaks might also involve long travel sequences to foreign countries within just a few days, exacerbating player fatigue. For example, in double round 2, Bolivia has an away break against Argentina and Venezuela. This means a trip totalling 7340 kilometers (slightly more than the distance from London to Mumbai), and the fifth longest of the side's 72 possible travel sequences. By contrast, Peru's only double-round away break is against Bolivia and Paraguay in double round 5, the second shortest of the Peruvian side's 72 possible trips.

Table 2 also reveals an unbalanced distribution of home-away (H-A) and away-home (A-H) sequences in double rounds (for example, Argentina had nine H-A sequences while Brazil did not have any). For logistical reasons, national teams generally prefer the H-A sequence. Since players typically return to their home countries before the first game of a double round, playing first at home means more time to prepare and only one additional flight (to the away game) before heading back to their respective club teams.

2.3. Controversy and the CONMEBOL decision

Aside from the structural deficiencies of the 2002–2014 fixture, its repeated use gave rise over the years to other problems that might have been avoided had there been a random draw before each qualifying tournament. For example, the fixture had Uruguay playing at home against Argentina in the final round. As one of the strongest teams, Argentina usually came to the final round already qualified while Uruguay typically was still seeking qualification, either directly or in the intercontinental play-off. In the 2002 qualifiers, Argentina had already won a place in the finals while Uruguay needed a draw to play in the intercontinental play-off, and the two teams drew 1–1. In the 2006 edition, Argentina again had already qualified but Uruguay, this time needing a win to make the intercontinental play-off, defeated Argentina by a score of 1–0. Then, in the 2010 qualification, Argentina had not yet qualified and took the match 1–0 over the Uruguayans, who later secured a place in the finals by winning the intercontinental play-off. Finally, in the 2014 qualification, whereas Argentina's berth in the finals was already assured by the time of the match, Uruguay needed 3 points to keep their hopes of qualifying directly alive. The Argentineans fielded a team replete with reserve players that lost 3–2, and only because of results in other teams' matches did Uruguay have to win in the intercontinental play-off in order to qualify. This type of situation is not uncommon in decisive football tournament games (e.g., Kendall and Lenten, 2017 recount the cases of *tacit collusion* between Austria and West Germany in the 1982 World Cup, and Sweden and Denmark in the 2004 UEFA Euro championship).

The foregoing is just one of many examples of controversies arising from the repeated use of an unbalanced (in terms of sporting fairness) fixture. Whether or not the suspicions of football fans and the media are well-founded, CONMEBOL took notice of the many complaints received over the years. In January 2015, CONMEBOL members agreed that a new methodology would be used for scheduling the 2018 qualifier tournament. Under this new approach, the fixture would be drawn up initially as a template in which the teams are represented by numbers and the actual team assignments would be made by random draw. It was also agreed that the definitive template would be selected from template proposals made by the ten CONMEBOL members. By the time CONMEBOL made the decision to change its methodology, our research team had been working with the ANFP in the application of IP techniques to the scheduling of Chile's professional football leagues for more than ten years (Durán et al., 2007; Durán et al., 2012). The impact of such application and general features on its portability to other sports competitions such as the World Cup qualifiers and the Argentinean leagues of volleyball and basketball have been

recounted in Alarcón et al. (2017). Collaborating with the ANFP we developed a series of fixture templates for the 2018 qualifiers, and the one finally submitted by the Association was chosen by CONMEBOL as the winning proposal. The technical approach to the formulation of this proposal is discussed in the next section.

3. Integer programming approach

Encouraged by our results in scheduling the Chilean professional leagues, we approach the scheduling of the World Cup qualifiers using IP. As mentioned in Section 1, the use of IP in sports scheduling has been widely documented. The double-round feature of the setting, however, has received little attention. Wright (2006) introduces the concept to represent two matches played by a single team over two consecutive days in New Zealand's National Basketball League. In this league, the teams prefer to play both matches in a double round away, in contrast to our case where it is desirable to avoid breaks in double rounds. Goossens and Spieksma (2011) generalize the concept of a break by defining it as any arbitrarily chosen pair of rounds with the same home-away status instead of just a consecutive pair with such status. In our problem, then, double rounds are arbitrary pairs formed by an odd round and its following even round. Their approach differs from ours, however, in that they focus on minimizing generalized breaks in a single round robin tournament with no additional constraints. Kendall (2008) and Kendall and Westphal (2013)), attempting to minimize the travel distances in English football, analyze fixtures where there is exactly one home and one away game in two consecutive matches. These fixtures, however, are limited to only one or two pairs of rounds around the Christmas period rather than an entire tournament.

3.1. Base IP formulation

Let $I = \{1, \ldots, n\}$ be the set of teams ($n = 10$). Also, let $K = \{1, \ldots, 18\}$ be the set of rounds and K_{odd} the set of first rounds in a double round. For $i, j \in I$ and $k \in K$, define the binary variable $x_{i,j,k}$ as equal to 1 if team i plays at home against team j in round k, and zero otherwise.

Double round robin constraints. In a double-round robin tournament, all teams must play twice against any other team. Here, we maintain traditional features of the tournament: every team faces every other team once in the first half and once on the second half, and exactly one of the two games is played at home while the other one is played away. These constraints are formulated below.

$$\sum_{k \in K: k \leq n-1} (x_{i,j,k} + x_{j,i,k}) = 1 \quad \forall i \in I, j \in I : i \neq j \tag{1}$$

$$\sum_{k \in K: k > n-1} (x_{i,j,k} + x_{j,i,k}) = 1 \quad \forall i \in I, j \in I : i \neq j \tag{2}$$

$$\sum_{k \in K} x_{i,j,k} = 1 \quad \forall i \in I, j \in I : i \neq j \tag{3}$$

Compactness. The schedule must also be compact, that is, all teams must play one match in each round.

$$\sum_{i \in I: i \neq j} (x_{i,j,k} + x_{j,i,k}) = 1 \quad \forall j \in I, k \in K \tag{4}$$

Top team constraints. The set of top teams, denoted I_S, contains Argentina and Brazil, the acknowledged powerhouses of South American football. These two teams have by far the best historical records in the qualifying tournaments, one or other of them

winning first place in every tournament since 1998. Thus, it is desirable that no other team be required to play against them in consecutive matches.

$$\sum_{j \in I_S} (x_{i,j,k} + x_{j,i,k} + x_{i,j,k+1} + x_{j,i,k+1}) \leq 1 \quad \forall i \in I \setminus I_S, k \in K : k < |K| \tag{5}$$

Balance constraints. These are included to maintain a balanced distribution among the teams of H-A sequences in double rounds. Given that there are $n - 1$ double rounds, we impose that each team has between $n/2 - 1$ and $n/2$ such sequences. These constraints are encoded with the help of auxiliary variables $y_{i,k}$, which take a value of 1 if team i has an H-A sequence in the double round that start at round k and zero otherwise. The auxiliary variables must satisfy the following constraints:

$$n/2 - 1 \leq \sum_{k \in K_{odd}} y_{i,k} \leq n/2, \qquad \forall i \in I, \tag{6}$$

$$\sum_{j \in I: i \neq j} (x_{i,j,k} + x_{j,i,k+1}) \leq 1 + y_{i,k}, \quad \forall i \in I, k \in K_{odd}, \tag{7}$$

$$y_{i,k} \leq \sum_{j \in I: i \neq j} x_{i,j,k}, \qquad \forall i \in I, k \in K_{odd}, \tag{8}$$

$$y_{i,k} \leq \sum_{j \in I: i \neq j} x_{j,i,k+1}, \qquad \forall i \in I, k \in K_{odd}. \tag{9}$$

Objective function. We aim at minimizing the total number of away breaks within double rounds across all teams. To accomplish this, we first define auxiliary variables $w_{i,k}$, which take a value of 1 if team i has an away break in the double round starting in round k and zero otherwise. These variables must then satisfy the following constraints:

$$\sum_{j \in I: i \neq j} (x_{j,i,k} + x_{j,i,k+1}) \leq 1 + w_{i,k}, \quad \forall i \in I, k \in K_{odd}, \tag{10}$$

$$w_{i,k} \leq \sum_{j \in I: i \neq j} x_{j,i,k}, \qquad \forall i \in I, k \in K_{odd}, \tag{11}$$

$$w_{i,k} \leq \sum_{j \in I: i \neq j} x_{j,i,k+1}, \qquad \forall i \in I, k \in K_{odd}. \tag{12}$$

Note also that since the existence of an away break for a given team in a double round necessarily implies a home break for some another team in the same double round (de Werra, 1988), the minimization of the total number of breaks in double rounds is captured by the following objective function:

$$\min \sum_{i \in I} \sum_{k \in K_{odd}} w_{i,k}. \tag{13}$$

3.2. Symmetric schemes

Unlike the mirrored structure of previous World Cup qualifier tournaments, the base formulation above does not impose any type of symmetry between the first 9 rounds and the last 9. Some form of *symmetry* is present in most tournament formats, however. In a survey of the schedules of 25 European football leagues for the 2008–2009 season, Goossens and Spieksma (2012) found that this was the case for 20 of them, the mirrored scheme being the one most frequently used (in 15 leagues).

Anticipating reluctance on the part of CONMEBOL to adopt a scheme that did not incorporate some element of symmetry, we studied several possibilities and designed our proposals accordingly.

Table 3
Representation of the symmetric schemes.

Scheme	Schedule									
Mirrored scheme	1	2	3	…	9	1	2	3	…	8	9
French scheme	1	2	3	…	9	2	3	4	…	9	1
English scheme	1	2	3	…	9	9	1	2	…	7	8
Inverted scheme	1	2	3	…	9	9	8	7	…	2	1
Back-to-back scheme	1	1	2	…	5	5	6	6	…	9	9

Mirrored scheme. The mirroring condition can be incorporated into the base IP formulation by adding the following constraints:

$$x_{i,j,k} = x_{j,i,k+n-1}, \quad \forall i,j \in I : i \neq j,\ k \in K : 1 \le k \le n-1. \quad (14)$$

Unfortunately, imposing only constraints (1) to (5) in addition to the above condition results in at least 16 double-round breaks, the very feature we want to minimize. Also, adding the H-A balance constraints leads to infeasibility. We therefore explored several other forms of symmetry, which are set out below.

French scheme. In this scheme, the first and last rounds are identical but their home-away status is reversed; the same holds for rounds k and $k+n-2$, $k = 2, \ldots, n-1$. That is,

$$x_{i,j,1} = x_{j,i,2n-2},\ x_{i,j,k} = x_{j,i,k+n-2},\ \forall i,j \in I : i \neq j,\ 2 \le k \le n-1. \quad (15)$$

English scheme. In this scheme, rounds $n/2 - 1$ and $n/2$ are identical but their home-away status is reversed; the same holds for rounds k and $k+n$, $k = 2, \ldots, n-2$. That is,

$$x_{i,j,n-1} = x_{j,i,n},\ x_{i,j,k} = x_{j,i,k+n},\ \forall i,j \in I : i \neq j,\ 2 \le k \le n-2. \quad (16)$$

Inverted scheme. In this scheme, round k and $2n - k - 1$ are identical but their home-away status is reversed, for $k = 1, \ldots, n-1$. That is,

$$x_{i,j,k} = x_{j,i,2n-1-k}, \quad \forall i,j \in I : i \neq j,\ 1 \le k \le n-1. \quad (17)$$

Back-to-back scheme. In this scheme, round k and $k+1$ are identical but their home-away status is reversed, for $k \in K_{odd}$. That is,

$$x_{i,j,k} = x_{j,i,k+1}, \quad \forall i,j \in I : i \neq j,\ k \in K_{odd}. \quad (18)$$

Min–Max separation scheme. In this scheme, the two games between any two teams are separated by at least c rounds and at most d consecutive rounds. That is,

$$\sum_{\bar{k} \in K : k \le \bar{k} \le k+c} (x_{i,j,\bar{k}} + x_{j,i,\bar{k}}) \le 1, \quad \forall i,j \in I : i \neq j,\ k \le |K| - c, \quad (19)$$

$$\sum_{\bar{k} \in K : (k-d) \wedge 1 \le \bar{k} \le (k+d) \vee 2(n-1) : \bar{k} \neq k} x_{i,j,\bar{k}} \ge x_{j,i,k},$$

$$\forall i,j \in I : i \neq j,\ k \in K. \quad (20)$$

A summary of the conditions imposed by each symmetric scheme is presented in Table 3. Here, a schedule for a generic team is illustrated by an ordered array of its opponents (represented by the digits $1, \ldots, 9$), where the position in the array refers to the round an opponent is faced and the second appearance of a digit indicates a match with the same opponent but with the home-away status reversed. (We have excluded the Min–Max scheme as it does not necessarily repeats rounds generically throughout the fixture.)

Although not as popular as the mirrored scheme, some of the schemes above have been used in the professional football leagues of various countries. For example, the French scheme has been used in Czech Republic, France, Luxembourg and Russia, while the English and the inverted schemes have been used in Austria and

Switzerland, respectively (Goossens & Spieksma, 2012). In South America, the inverted scheme has been used for the prestigious *Copa Libertadores* (international clubs) tournament.

We solved the base formulation in combination with the various symmetry-related constraints and obtained a number of template schedules with an objective function value of zero. As some constraints are incompatible with some symmetric schemes, we relaxed the former when necessary. For example, for the back-to-back scheme, we relaxed the constraint that every team faces every other team once in the first half of the tournament and once in the second half, which is clearly incompatible with the scheme. Note also that for every symmetry scheme above, there might exist multiple schedules. In each setting, we obtained schedules that eliminated double-round breaks. For the Min–Max separation scheme we use $c = 6$ and $d = 12$, as these were the closest possible values to the infeasible mirrored case ($c = 9$ and $d = 9$) that eliminated double-round breaks. Among all schemes, the French one allowed us to satisfy all constraints and, more important, was well accepted by ANFP officials, as we detail next.

4. Results and implementation in practice

Among the templates presented in the previous section, ANFP officials selected the one following the French scheme. The ANFP proposed this schedule at a meeting of CONMEBOL's members in April 2015. Although other countries also presented proposals, ours was unanimously selected by all members for the 2018 World Cup qualifiers.

In July 2015, the random draw for the assignment of teams to the template fixture took place in Saint Petersburg at an event involving the 209 FIFA-affiliated national associations, which was broadcast worldwide. Former *Best World Cup Award* players Ronaldo and Forlán drew balls from two pots to define the South American Qualifiers schedule. One pot contained the names of the participating teams while the other contained the numbers indicating the position each team would take in the schedule we constructed. The outcome of the draw was the schedule presented in Table 4.

By construction, the definitive schedule departs from a mirrored scheme only in that the last round of matches in the second half is the reverse of the first round in the first half. All teams play once at home and once away in each double round. The schedule also has a series of features that significantly improve upon the 2002–2014 setup, as can be seen in the comparisons in Table 5. First, it achieves a much better balance of H-A and A-H sequences by assigning every team at least 4 and at most 5 of each of these sequences. Second, the number of breaks in double rounds was reduced from 18 to 0. This feature in particular significantly enhances sporting fairness. Our schedule also performs well on g-ranking-balancedness, a fairness concept introduced by Goossens and Spieksma (2012) that compares the number of home games played by each team upon completion of each round. A schedule is considered to be *g-ranking-balanced* if, immediately after each round, the difference between the number of home games played by any two teams up till then is at most g. Of the 25 leagues the authors studied, only the English league had the minimum possible g value of 1. Our schedule also achieves a g value of 1. Moreover, at the end of every even round, all of the teams have played exactly the same number of home and away games. This is perhaps positive when people look at the standing and compare the number of points of every team. In addition, our schedule balances the H-A sequences within double rounds, especially for Brazil, Chile and Venezuela, which instead of having either 0 or 1 such sequence, as in the 2002–2014 fixture, now have 4 or 5 of them. As with the 2002–2014 schedule, in the new schedule no team plays consecutively against Argentina and Brazil. In the draw, a special colour

Table 4
Schedule of the South American Qualifiers to the 2018 FIFA World Cup Russia.

Team	1	2	3	4	5	6	7	8	9	10	...	18
ARG	ECU	@PAR	BRA	@COL	@CHI	BOL	URU	@VEN	@PER	PAR		@ECU
BOL	URU	@ECU	VEN	@PAR	COL	@ARG	PER	@CHI	@BRA	ECU		@URU
BRA	@CHI	VEN	@ARG	PER	URU	@PAR	@ECU	COL	BOL	@VEN		CHI
CHI	BRA	@PER	COL	@URU	ARG	@VEN	@PAR	BOL	@ECU	PER		@BRA
COL	PER	@URU	@CHI	ARG	@BOL	ECU	VEN	@BRA	@PAR	URU		@PER
ECU	@ARG	BOL	URU	@VEN	PAR	@COL	BRA	@PER	CHI	@BOL		ARG
PAR	@VEN	ARG	@PER	BOL	@ECU	BRA	CHI	@URU	COL	@ARG		VEN
PER	@COL	CHI	PAR	@BRA	VEN	@URU	@BOL	ECU	ARG	@CHI		COL
URU	@BOL	COL	@ECU	CHI	@BRA	PER	@ARG	PAR	VEN	@COL		BOL
VEN	PAR	@BRA	@BOL	ECU	@PER	CHI	@COL	ARG	@URU	BRA		@PAR

Table 5
Comparison of the 2018 and 2002–2014 South American qualifying tournament schedules.

Team	Schedule 2002–2014					Schedule 2018				
	B_h	B_a	B	H-A	A-H	B_h	B_a	B	H-A	A-H
ARG	0	0	0	9	0	0	0	0	5	4
BOL	2	2	4	2	3	0	0	0	5	4
BRA	0	0	0	0	9	0	0	0	4	5
CHI	1	1	2	1	6	0	0	0	5	4
COL	1	1	2	6	1	0	0	0	5	4
ECU	1	1	2	4	3	0	0	0	4	5
PAR	1	1	2	3	4	0	0	0	4	5
PER	1	1	2	6	1	0	0	0	4	5
URU	1	1	2	4	3	0	0	0	4	5
VEN	1	1	2	1	6	0	0	0	5	4
Total	9	9	18	36	36	0	0	0	45	45

was used to identify the balls associated with these two teams (which constituted set I_s in our formulation) and their two predefined numbers in the template.

Finally, although our work focused on the particular case of the South American Qualifiers, which involve 10 teams, we have also explored the possibilities of the French system for generating schedules using the same criteria for other tournament sizes. Through numerical experiments, and also through some simple logical reasoning, it can easily be shown that for $n = 4$ and $n = 6$, the problem becomes infeasible. However, numerical experiments allows us to find feasible schedules under the French system without double-round breaks for all other even n between 8 and 38. Any general result about this and the other formats remain worthy of investigation.

5. Conclusions

In this paper, we have shown our use of IP to the design of a new schedule for the South American qualification phase of the FIFA World Cup football championship. Our work contributes to the theory and practice of sports scheduling by presenting what is, to the best of our knowledge, the first real-world application of OR to football scheduling at the *international* level. The proposed schedule replaced a fixture used by CONMEBOL for the last two decades. Resistance to change, a barrier to the adoption of technology, is specially strong in the culture of an international organization such as CONMEBOL, which must align the interests of its ten members, each with their own cultural identity and point of view.

Our proposal introduced a series of modifications that, from a rather objective perspective, improved upon the traditional schedule, and was selected unanimously by CONMEBOL members. The proposed schedule, with teams assigned as in Table 4, is currently being used in the qualification tournament for the 2018 World Cup. The tournament started in October 2015 and will conclude in October 2017. At the time of writing, with 4 rounds remaining, there

are 8 teams with plausible possibilities to qualify, in what is being arguably one of the tightest qualification tournaments in the recent history.

While having fewer teams than regular football league tournaments, the scheduling problem for the South American Qualifiers has features that pose interesting technical challenges. One of these is the temporal spacing of the games and the structure derived from the double rounds. Our focus on minimizing double-round breaks resulted in a fixture that has all teams playing at least once at home in every double round. This is in sharp contrast to mirrored schedules, where teams can go without playing at home for several months. With the double round feature also adopted by other confederations, such as UEFA in Europe and AFC in Asia, solutions addressing this issue are all the more necessary.

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