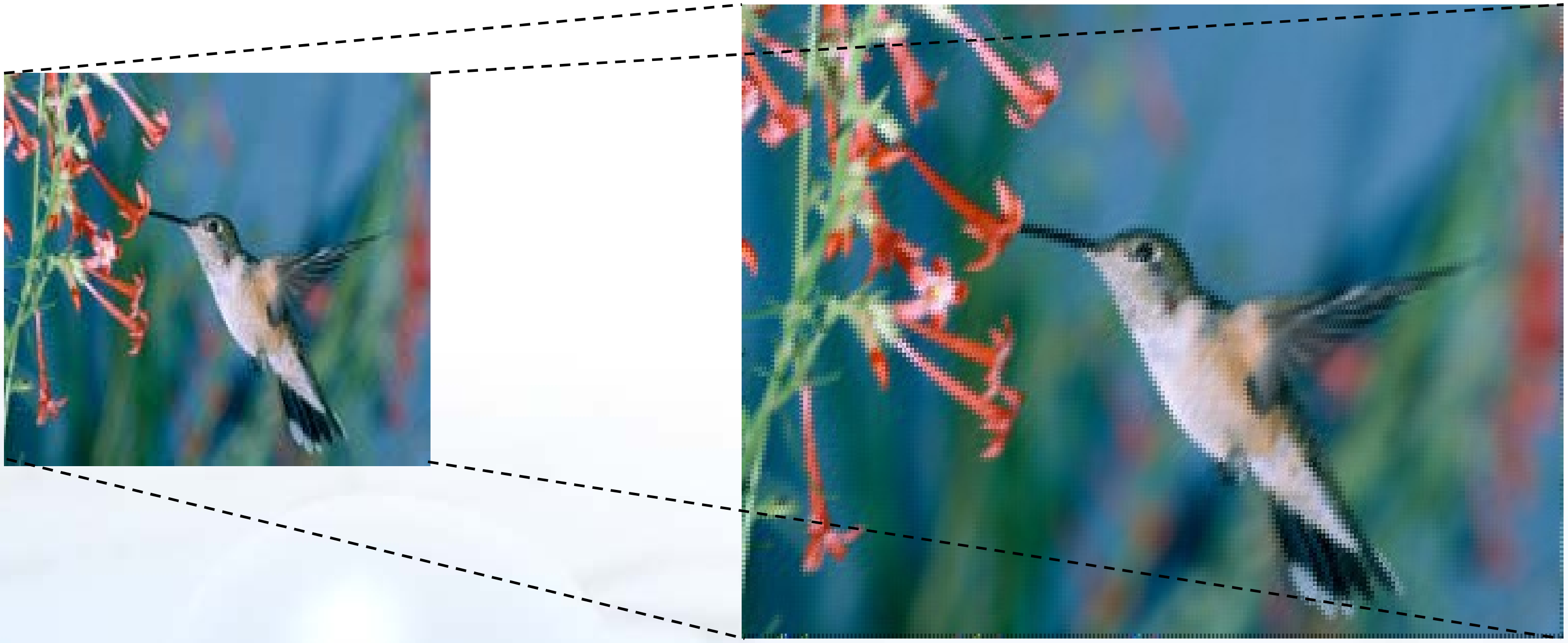

Wavelet and PDE based approach for Image and Video Scaling and De-noising

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Introduction:

- Video and Image scaling are widely used in numerous applications
- Various methods are used for these purposes eg: Bilinear, Bicubic etc
- Scaling decreases the quality of images and videos and inserts unwanted artifacts called noise
- In this project, we aim to improve the quality of image further by using de-noising techniques like PDE, Wavelets.
- We compare different results.

Videos and Images

- A image of mxn consists of mxn number of pixels.
- Each pixel defines the value different components that uniquely determine the color at that pixel like RGB, CMYK
- For example of 24-bit RGB image has three components R, G, B each of 8-bit determining intensity of each.
- A video file consists of set of consecutive frames.
- Each frame determines the image displayed at a particular time.

Noise

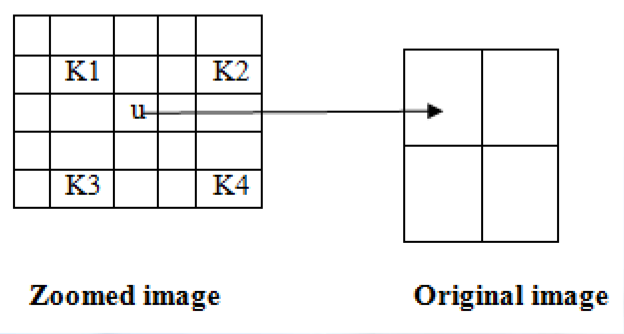
- Unwanted artifacts in the image are called noise.
- Usually introduced by using different transformations of the image
- Some examples of noise are:
    - Aliasing
    - Posterization
    - Ringing Artifacts
- Removal of such artifacts is called de-noising

Scaling

- Resizing a particular image or video to a new resolution.
- Images are resized by interpolating the pixel intensities using various algorithms
- Video are scaled by scaling each of its frame to the desired size
- Quality of scaled output decreases with increasing scaling factor
- Scaling inserts noise in the image
- Some scaling algorithms: Bilinear, Bicubic, Nearest Neighbor, Lanczos (sinc 3)

Scaling: Nearest Neighbor

- Value of pixel in resultant image is determined using 4 neighboring pixels to the corresponding point in original



- Value of pixel is selected which is nearest to the point in original image

Scaling: Bilinear

- Interpolation is first done in one direction and then in other direction using 4 neighboring points

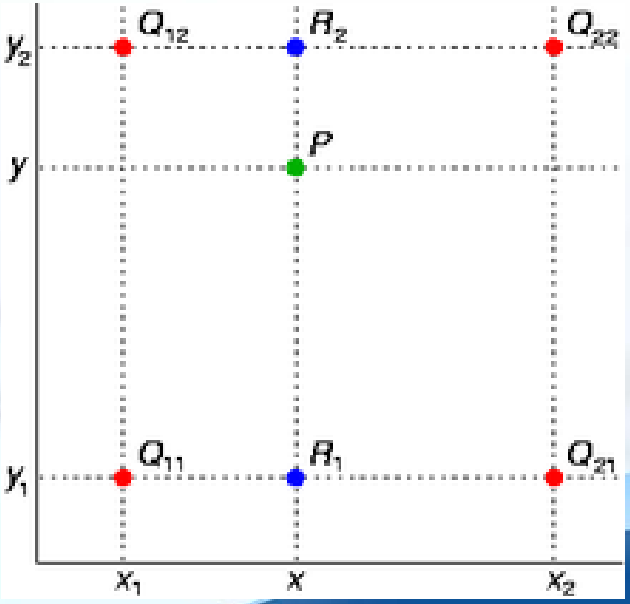
Horizontal Interpolation

$$f(R_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21})$$

$$f(R_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22})$$

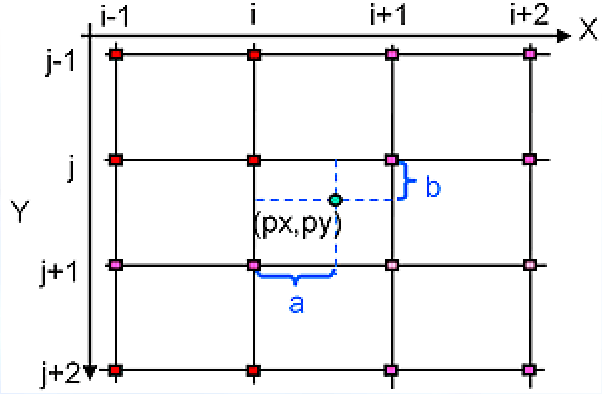
Vertical Interpolation

$$f(P) \approx \frac{y_2 - y}{y_2 - y_1} f(R_1) + \frac{y - y_1}{y_2 - y_1} f(R_2).$$



Scaling: Bicubic

- Uses 16 neighboring points to calculate the interplated value.



- 16 coefficients aij is calculated using derivatives and cross-derivatives and the 16 equations.
- Interpolated value is then formulated as:

$$p(x,y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} x^i y^j.$$

Scaling: Lanczos (sinc 3)

- Uses Lanczos filter which is windowed form of sinc filter[7
- Lanczos window is the *central* lobe of a horizontally-stretched sinc, sinc(*x*/*a*) for $-a \le x \le a$.**[7]**
- Lanczos filter is given by:

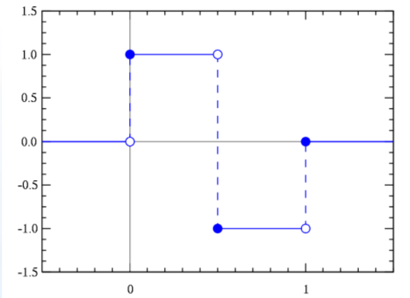
$$L(x) = \begin{cases} \operatorname{sinc}(x)\operatorname{sinc}(x/a) & -a < x < a, x \neq 0 \\ 1 & x = 0 \\ 0 & \text{otherwise} \end{cases}$$

- Where $\operatorname{sinc}(x)\operatorname{sinc}(x/a) = \frac{a\sin(\pi x)\sin(\pi x/a)}{\pi^2 x^2}.$
- Interpolation Equation:

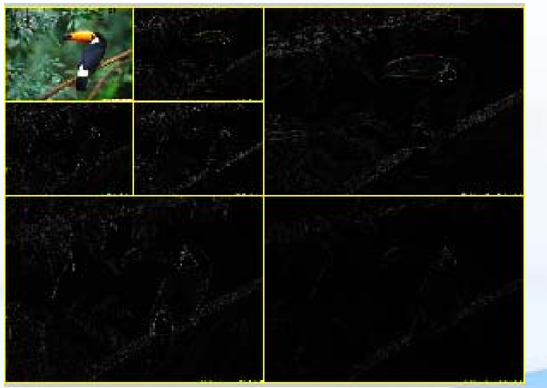
$$\hat{I}(x_0, y_0) = \sum_{i=\lfloor x_0 \rfloor - a + 1}^{\lfloor x_0 \rfloor + a} \sum_{j=\lfloor y_0 \rfloor - a + 1}^{\lfloor y_0 \rfloor + a} I(i, j) L(x_0 - i) L(y_0 - j).$$

Haar Wavelet Transform

- The Haar Wavelet and 2x2 Haar matrix are:**[6]**



$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$



- Haar wavelet transform is calculated using the basis filters (1/sqrt(2), 1/sqrt(2)) and (1/sqrt(2), -1/sqrt(2)) for different levels
- Image is reconstructed using the inverse of this process

Denoising using Haar (Level 3)

- Denoising is done for transformed image using either Soft or Hard thresholding and then de-noised image is constructed**[9]**
    - Hard Thresholding:

$$f_h(\mathbf{x}) = \begin{cases} \mathbf{x}, & \text{if } |\mathbf{x}| \geq \lambda \\ 0, & \text{otherwise.} \end{cases}$$

    - Soft Thresholding:

$$f_s(\mathbf{x}) = \begin{cases} \mathbf{x} - \lambda, & \text{if } \mathbf{x} \geq \lambda \\ 0, & |\mathbf{x}| < \lambda \\ \mathbf{x} + \lambda, & \text{if } \mathbf{x} \leq -\lambda. \end{cases}$$

- Threshold if not known for the image can be calculated as:**[9][8]**

λ = б * √2*log(n)/n

where б = Median of coefficients/(0.6745*n)

PDE Based De-noising

- Implements de-noising through diffusion of intensity in images.
- Intra-region smoothing in Anisotropic diffusion**[2][4]**
- Partial Differential Equation is given by:

$$du/dt = c * Du - P^1 u_0 + u_0$$

$$=> \; u_{n+1} = u_n +$$

$$\Delta t *((c^N * Du_n^N + c^S * Du_n^S + c^E * Du_n^E + c^W * Du_n^W) - P^1 u_0 + u_0)$$

$$c^i = e^{-(x/K)^2}$$

$$x = Du_n^i$$

Performance Metrics

- Performance measurement is done using the following metrics:
  - Mean Square Error(MSE)**[3]**

$$MSE = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} [I'(i,j) - I(i,j)]^2$$

  - Peak Signal Noise Ratio**[3]**

$$PSNR = 20 \log_{10} \left[\frac{255}{RMSE} \right]$$

$$RMSE = \sqrt{MSE}$$

  - Correlation Parameter**[3]**

$$CP = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} (\Delta I - \Delta \bar{I}) \times (\Delta \hat{I} - \Delta \bar{\hat{I}})}{\sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} (\Delta I - \Delta \bar{I})^2 \times \sum_{i=1}^{m} \sum_{j=1}^{n} (\Delta \hat{I} - \Delta \bar{\hat{I}})^2}}$$

  - Structure Similarity Index Map**[3]**

$$SSIM(X,Y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)}$$

Result Analysis

- Analysis of the algorithm done on both still image and video frames
- Performance Metrics generated for zoom Levels : 2x, 4x, 8x, 10x
- Still Image :

Performance Metrics	2x	4x	8x	10x
MSE	BL	NN	LZ	LZ
PSNR	BL	NN	LZ	LZ
CP	PDE+NN	HAAR+NN	HAAR+BC	HAAR+NN
SSIM	BL	PDE+LZ	PDE+ LZ	PDE+ LZ

Result Analysis

- Video Frames :
- **Frame1**

Performance Metrics	2x	4x	8x	10x
MSE	HAAR+LZ	HAAR+LZ	HAAR+LZ	HAAR+LZ
PSNR	HAAR+LZ	HAAR+LZ	HAAR+LZ	HAAR+LZ
CP	PDE+LZ	NN	PDE+NN	PDE+NN
SSIM	LZ	PDE+LZ	PDE+ LZ	PDE+ LZ

- **Frame2**

Performance Metrics	2x	4x	8x	10x
MSE	HAAR+LZ	HAAR+LZ	HAAR+LZ	HAAR+LZ
PSNR	HAAR+LZ	HAAR+LZ	HAAR+LZ	HAAR+LZ
CP	PDE+BC	PDE+NN	PDE+NN	PDE+NN
SSIM	LZ	PDE+LZ	PDE+ LZ	PDE+ LZ

Conclusion

- De-noising improved the quality of image and video-image after scaling
- Partial differentiation based approach used here performs better than de-noising done by haar wavelets for static images
- For still images, it observed that PDE + Lanczos and Lanczos give better results as the zoom factor increases, though it is also observed that the performance of Haar increases consistently as Zoom Level increases.
- For the chosen video sequence of two frames, Lanczos followed by Haar wavelet denoising consistently out performs the other approaches

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Thanks!