REVIEW: SCALAR DARK MATTER AND NEUTRINO MASSES

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Abstract

Two of the key issues facing Standard Model physics that need to be resolved are (i) the appearance of a small but non-zero neutrino mass, and, (ii) the missing mass problem in the Universe. If both of these challenges require physics beyond the Standard Model, it is certainly aesthetically pleasing if the two could be solved within the same extension to our current frontier. This review focuses on some of the model building strategies to realise this goal as presented in the literature. The main focus will be a low energy effective theory that couples a dark scalar to Standard Model neutrinos. In this context we will present the most important observational bounds on the model to provide a realistic parameter spaces to explore.

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I   NEUTRINO MASSES

1.1   Starting Point: The Standard Model

We will start with a very brief review of the minimal set up [1], where we simply consider the Standard Model. In particular, we will consider the structure of the lepton and scalar sectors. These sectors are charged[1.1] under the gauge group $\mathscr{G}_{\text{EW}} = \text{SU}(2)_L \times \text{U}(1)_Y$. The particle content is as follows;

- There are 3 *left-handed* $\text{SU}(2)$ doublets[1.2];

$$\psi_i^{\text{T}} = (\nu^i, \ell^i)_L \tag{1.1}$$

  where i runs over the neutrino species, $\{\nu_e, \nu_\mu, \mu_\tau\}$, and $l_i = (e, \mu, \tau)$. There are also 3 *right-handed* lepton fields, ℓ_R^i, which will form the chiral partner to the left handed ones above.

- In the Higgs (*scalar*) sector, there is a scalar doublet;

$$\Phi^{\text{T}} = (\phi^+, \phi^0) \tag{1.2}$$

  where the superscripts refer to the respective charges of the complex scalar fields $\phi^{+,0}$. The additional physics in this sector is the electroweak phase transition where the neutral component obtains a VEV. We will make use of both the Higgs mechanism, as well as general ideas regarding spontaneous symmetry breaking, but discussion is not in the scope of this review, see [2] for more.

Now, in reference to the title of the section, within this guage realisation of the standard model, with this particle content, neutrinos are massless. The issue however, is that observations of neutrino oscillations [3], contradict this fact. The short reasoning behind this is that neutrino oscillations imply that the flavour eigenstates which govern the interactions, and the mass eigenstates, which are eigenstates of the mass matrix, are *not simultaneous*.

1.2   Majorana Neutrino Masses

In the neutrino sector, there are two possible types of mass term in the Lagrangian;

- *Dirac Neutrino Masses* arise after the introduction of a right-handed neutrino field ν_R^i with $i = e, \mu, \tau$. The mass then arises under the Higgs mechanism due to a coupling;

$$\mathcal{L}_{\text{lept},\phi} \propto \lambda^{ij} \bar{\psi}^i \Phi \ell^j + \lambda_\nu^{ij} \bar{\psi}^i \Phi^c \nu_R^i \tag{1.3}$$

  After electroweak symmetry breaking (EWSB), the neutrinos get a mass term;

$$-\sum_i m_\nu^i \left(\bar{\nu}_R^i \nu_L^i + \bar{\nu}_L^i \nu_R^i \right) \tag{1.4}$$

- *Majorana Masses* are a qualitatively different scenario that is possible if the fermion is neutral, as in the case of the neutrino. In this case, the right-handed neutrino field is not independent of the left-handed one.[1.3] The mass term becomes;

$$-\frac{1}{2} \sum_i m_\nu^i \left(\bar{\nu}_L^{i,c} \nu_L^i + \bar{\nu}_L^i \nu_L^{i,c} \right) \tag{1.5}$$

[1.1]In other words, the relevant fields are in some representation (e.g. fundamental, trivial, adjoint etc.) of the gauge group

[1.2]In the fundamental representation of $\text{SU}(2)$, we have suppresed the discussion of the hypercharge ($\text{U}(1)_Y$ representation) for now

[1.3]To be precise, $\nu_R(x) = \nu_L^c(x)$ where c represents the charge conjugated field.

Figure 1.1 *The tree level diagram which generates a Majorana mass term for the neutrinos after EWSB. This arises due to the introduction of a new heavy fermion N, which interacts with the neutrinos and the neutral Higgs scalar.*
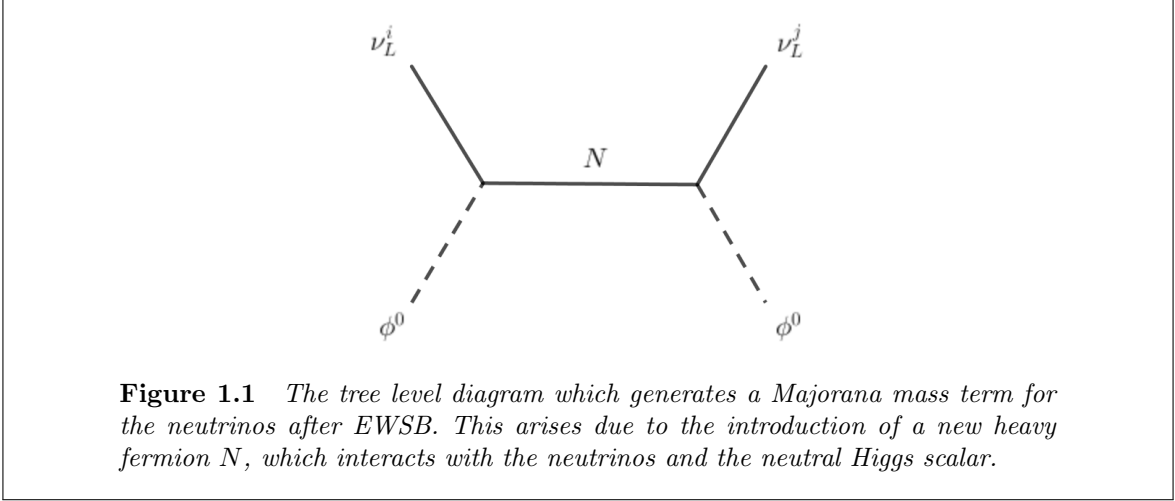
We will focus on the second scenario, with the added motivation that we would like it to arise from some manifestation of the Higgs mechanism. As discussed above, this can't arise at tree level in the Standard Model. The lowest order for which this sort of term can be generated by a Higgs VEV is at dimension 5 [4] of the form;

$$\Lambda^{-1}\phi^0\phi^0\nu_L^i\nu_L^j \tag{1.6}$$

Being dimension 5, the operator is not renormalisable, and as such this can only be an effective coupling, valid up to some large mass scale Λ. Amongst other things, we will be interested as to exactly what happens above this mass scale, when we consider possible UV completions that lead to this regime.

1.3   Tree Level Realisations of Majorana Masses

If we are to generate the Majorana mass terms in 1.2 from a UV theory, then the initial couplings in the high energy Lagrangian should themselves be renormalisable. To see what sort of interactions (1.5) might come from, it is instructive to consider the set of fully gauge invariant terms, of which it is one [5];

$$\Lambda^{-1}(\nu_L^i\phi^0 - \ell_L^i\phi^+)(\nu_L^j\phi^0 - \ell_L^j\phi^0) \tag{1.7}$$

In this section we will illustrate the simplest realisation of this structure at tree level, although in fact there are three possibilities [1]. In our case, we introduce a new heavy fermion singlet N, with some mass m_N. If we suppose it has some coupling;

$$f_i\bar{N}\nu_L^i\phi^0 \tag{1.8}$$

Then the operator in (1.7) can be generated via the diagram shown in Figure 1.1. This method of generating neutrino masses via a Higgs VEV is known generically as the *canonical seesaw mechanism*, and as we alluded to above, there are a number of tree level possibilities, as well as more options at one-loop. In this section we have introduced the problem of neutrino mass generation within the Standard Model, as well as considered the form of an effective operator which would realise such a Majorana mass for the neutrino. We now need to understand where Dark Matter comes into the story, and why, for example the canonical seesaw mechanism does not have a suitable Dark Matter candidate.

II Linking Dark Matter and Majorana Neutrinos

2.1 Introduction to Dark Matter

The fact that approximattely 23% of the universe at the current time is formed of non-baryonic matter generates a number of questions about the nature of this hidden sector;

1) Is there a particle content to the sector, or is Dark Matter composite as in e.g. theories of Primordial Black Holes? [6]

2) Do we need physics beyond the Standard Model?

3) If it does have a new particle content, how does this sector interact with the Standard Model?

4) What evidence can be used to distinguish between and constriain different models?

We are primarily interested in the last two questions posed, and will be considering extensions to the Standard Model. In particular we are concerned with phenomenological scenarios where the following two conditions hold;

- The model contains a suitable dark matter candidate, in the sense that the particle is stable and whose interactions are not ruled out by the various astrophysical, thermal, collider, and cosmological bounds.

- The model *also* contains a mechanism that generates the small Majorana neutrino mass. Whilst this is not necessary for the dark matter candidate to therefore be coupled directly to the Standard Model neutrinos, we will consider scenarios in which this is the case.

Of course, the last point is by no means a necessary requirement in a theory of dark matter, however the focus of this review is understanding to what extent we can build models that consistently, and naturally solve both issues.

Furthermore, we will spend a decent proportion of time later discussing the phenomenological consequences of both the low energy, and UV complete theories. Especially in the context of the effective low energy theory we present in Section III, any conclusions are greatly strengthened if they probe a parameter space not categorically ruled out by other bounds.

2.2 Understanding the Role of $\mathbb{Z}_2$

One of the key properties mentioned above is that particle Dark Matter should be stable. In the majority of the rest of the review, we will claim that our Dark Matter candidate is stable "as a result of a $\mathbb{Z}_2$ symmetry". The aim of this section is to illustrate how this manifests itself within a UV extension to the Standard Model [5, 7–9].

The big picture is as follows; we will introduce a new scalar into the Standard Model which will be odd under a new global $\mathbb{Z}_2$ symmetry. Through interactions with Standard Model neutrinos and new heavy fermions, it will generate a very similar dimension 5 operator in a completely analogous way to the neutral Higgs field, ϕ^0. Unlike the Higgs however, we shall see that the effect of the $\mathbb{Z}_2$ symmetry on the Higgs potential for the model ensures that the new scalar cannot develop a VEV. It also means that the lightest degree of freedom is rendered stable in a way to be explained further below. These two facts have the combined effect that (i) The neutrino mass is still generated in this process, but at one loop instead of at tree level, (ii) The lightest degree of freedom in the scalar sector is stable and can therefore act as our Dark Matter candidate.

2.3 The Toy Model: $SU(2)_L \times U(1)_Y \times \mathbb{Z}_2$

In what follows, one should bear in mind the case illustrated in 1.3 to compare which features are retained, and which features are constrained by the new $\mathbb{Z}_2$ symmetry. In particular, we will find that (i) an analogous dimension 5 operator to $\phi^0\phi^0\nu_L^i\nu_L^j$ is generated, but, (ii) unlike the neutral component of the Higgs doublet, the new scalar will not acquire a VEV due to the $\mathbb{Z}_2$ symmetry. These two considerations along with an additional assumption regarding the lepton number of the new particle content will result in the neutrino masses only being generated at one loop.

2.3.1 Particle Content

Describing the particle content of the theory is equivalent to noting the different representations of the fields with respect to the group $\mathscr{H} = SU(2)_L \times U(1)_Y \times \mathbb{Z}_2$.[2.1] In what follows, we shall use the following notation; consider some arbitrary field χ, then we say that $\chi \sim (R_{SU(2)}, R_{U(1)}, R_{\mathbb{Z}_2})$ if;

- χ is in the $R_{SU(2)}$ representation of $SU(2)$. For example, if χ were an $SU(2)$ doublet, then we would take $R_{SU(2)} = 2$ etc.

- χ has hypercharge $R_{U(1)}$ with respect to $U(1)_Y$, i.e. $\chi \to e^{iR_{U(1)}\alpha}\chi$ under a $U(1)_Y$ gauge transformation.

- χ has parity $R_{\mathbb{Z}_2}$ under $\mathbb{Z}_2$. For example, if χ were odd under $\mathbb{Z}_2$, then $R_{\mathbb{Z}_2} = -1$.

As an aside that we will use implicitly, a term in the Lagrangian is gauge invariant under transformations by elements of $\mathscr{H}$ if it lies in the trivial representation $(1, 1, +1)$. To check whether this is the case, by our definition, it is simply a matter of adding the respective R values together for each field in the coupling. The particle content in the lepton and and scalar sectors is then as follows;

- The three electroweak doublets, $\psi_i = (\nu_L^i, \ell_L^i)^T \sim (2, -\frac{1}{2}, +1)$, and the associated right-handed lepton fields, $\ell_R^i \sim (1, 1, +1)$;

- The Higgs electroweak doublet, $\Phi = (\phi^+, \phi^0)^T \sim (2, \frac{1}{2}, +1)$;

- The first new particle is a right-handed lepton, $N_R^i \sim (1, 0, -1)$, *with lepton number* $l = 0$ [9];

- A scalar electroweak doublet, $\eta = (\eta^+, \eta^0)^T \sim (2, \frac{1}{2}, -)$ with lepton number $l = -1$.

Note that the new particles are odd under the $\mathbb{Z}_2$ symmetry, whilst the Standard Model particles are even. Now, at this point, it is important to note why no term $\phi^0\phi^0\nu_l^i\nu_L^j$ can be generated in this model. As we saw in 1.3, to generate such a term at tree level, we need a coupling of the form $\bar{N}_R^i(\nu_L^j\phi^0 - \ell_L\phi^+)$.[2.2] However, lepton number conservation forbids such a term, since $l(\bar{N}\nu\phi) = 0 + 1 + 0 = 1 \neq 0$. The absence of this term means that at tree level, we do not generate Majorana neutrino masses after EWSB.

On the other hand, the fact that $l(\eta) = -1$ means that an identical term with ϕ interchanged η is not forbidden and will therefore generate an effective dimension 5 operator $\Lambda^{-1}\eta^0\eta^0\nu_L^i\nu_L^j$ via a diagram like Figure 1.1. If η^0 were to acquire a VEV $\langle\eta^0\rangle \neq 0$, this would generate neutrino mass at tree level. In the next few sections, we will see why this is not the case in this model.

[2.1] We are supressing the $SU(3)_C$ part of the gauge group simply because in the sectors under consideration, all fields are in the trivial representation of $SU(3)_C$.

[2.2] We will see where such a term comes from shortly

2.3.2   The Higgs Potential

To begin we shall consider the most general $SU(2) \times U(1)$ invariant scalar Lagrangian. This will therefore necessarily include terms that break the $\mathbb{Z}_2$ symmetry explicitly. We will then derive the expected VEVs that arise from this general framework to understand why $\mathbb{Z}_2$ symmetry sets the η^0 VEV to zero. This will be equivalent to setting the couplings of the $\mathbb{Z}_2$-breaking terms to zero. The most general Higgs potential is of the form;

$$V_s = m_1^2 \Phi^\dagger \Phi + m_2^2 \eta^\dagger \eta + \frac{1}{2}\lambda_1(\Phi^\dagger\Phi)^2 + \frac{1}{2}\lambda_2(\eta^\dagger\eta)^2 + \lambda_3(\Phi^\dagger\Phi)(\eta^\dagger\eta) + \lambda_4(\Phi^\dagger\eta)(\eta^\dagger\Phi) + \mu_{12}^2(\Phi^\dagger\eta + \eta^\dagger\Phi)$$
(2.1)

Note that only the last term explicity breaks the $\mathbb{Z}_2$ symmetry, so moving to the $\mathbb{Z}_2$-symmetric case is equivalent to setting $\mu_{12}^2 = 0$ in the calculations that follow. To find show that the VEV of η^0 does indeed vanish for $\mu_{12}^2 = 0$, we consider the constraint equations $\partial_\Phi V_s = \partial_\eta V_s = 0$. Letting v and u be the VEV of ϕ^0 and η^0 respectively, we find [9];

$$m_1^2 v + \lambda_1 v^3 + (\lambda_3 + \lambda_4)vu^2 + \mu_{12}^2 u = 0 \tag{2.2}$$
$$m_2^2 u + \lambda_2 u^3 + (\lambda_3 + \lambda_4)uv^2 + \mu_{12}^2 v = 0 \tag{2.3}$$

In the case that the Higgs field acquires a VEV, $m_1^2 < 0$, which of course certainly occurs, and we have $m_2^2 > 0$ with $\mu_{12}^2 \ll m_2^2$, we find that;

$$v^2 \simeq -\frac{m_1^2}{\lambda_1}, \quad u \simeq -\frac{\mu_{12}^2 v}{m_2^2 + (\lambda_3 + \lambda_4)v^2} \tag{2.4}$$

From this we immediately see that indeed if $\mu_{12}^2 = 0$, the VEV of η^0, $\langle\eta^0\rangle = 0$. This has a couple of immediate consequences which will be discussed more in the next section;

1) Although the coupling $\bar{N}_R^i \nu_L^j \eta^0$ arises in a gauge invariant manner, the fact that the VEV of η^0 vanishes implies that there is no Dirac mass linking ν_L^i and N_R^i.

2) It also leads to the effect discussed above where the neutrinos do not acquire a mass at tree level, only at one loop.

2.3.3   Yukawa Sector

Along with a Majorana mass term for the right-handed N_R^i of the form;

$$\frac{1}{2}M_{ij}N_R^i N_R^j + \text{h.c.} \tag{2.5}$$

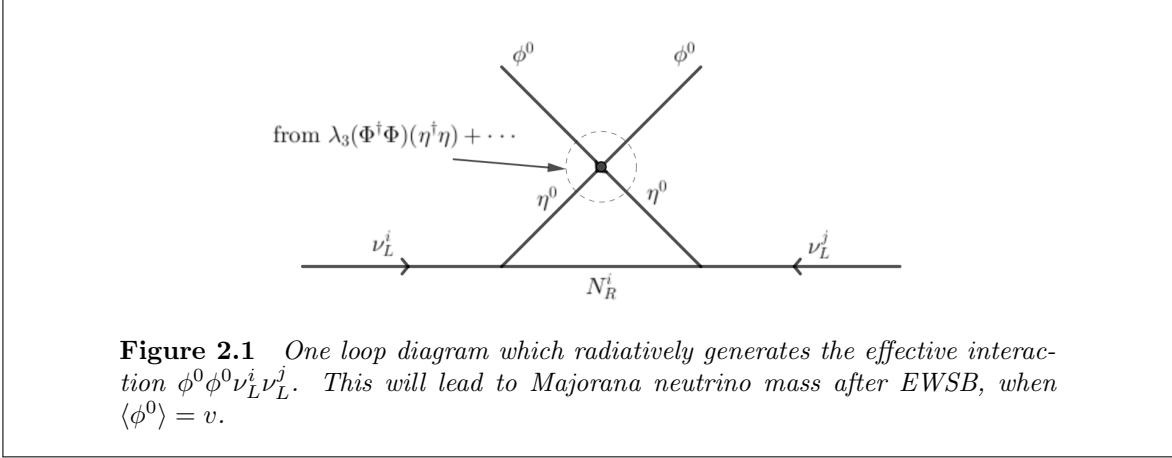We also have a Yukawa sector of the Lagrangian that takes the form;

$$\mathcal{L}_Y = f_{ij}\Phi^\dagger\psi^i\ell_R^j + h_{ij}\bar{\Psi}^i(i\sigma_2)\eta N^j + \text{h.c.} \tag{2.6}$$

Where σ_2 is one of the Pauli matrices.[2.3] There are a couple of things to note about this Lagrangian;

- Each of the terms is indeed gauge invariant under $\mathscr{H}$. For example the first term has a combined representation $(-2+2+1, -\frac{1}{2}+1-\frac{1}{2}, +1\cdot+1\cdot+1) = (1, 1, +1)$, and similarly for the second term.

- By expanding the definitions of Φ and ψ^i, as well as using the definitions of σ_2, we see exactly where the term of the form $h_{ij}(\nu_L^i\eta^0 - \ell_L^j\eta^+)N_R^j$ comes from. This term will then generate the effective vertex $\Lambda^{-1}\eta^0\eta^0\nu_L^i\nu_L^j$ discussed 1.3.

[2.3]

$$\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Figure 2.1 *One loop diagram which radiatively generates the effective interaction $\phi^0\phi^0\nu_L^i\nu_L^j$. This will lead to Majorana neutrino mass after EWSB, when $\langle\phi^0\rangle = v$.*

2.3.4 Generating Neutrino Mass

To understand where the neutrino mass comes from in this model, we first make it clear that the Majorana mass does indeed still come from the effective operator $\phi^0\phi^0\nu_L^i\nu_L^j$. This ensures that after EWSB, the neutrino does acquire a mass. The point is however that this only happens at one loop. To see this, note;

- There are 4 point interactions in V_s of the form $\lambda_3\Phi^\dagger\Phi\eta^\dagger\eta + \lambda_4(\Phi^\dagger\eta)(\eta^\dagger\Phi)$ which are not prohibited by the $\mathbb{Z}_2$ symmetry.

- We have an interaction vertex in the Yukawa sector of the form $\bar{N}_R^i\eta^0\nu_L^j$, which again is not prohibited by gauge invariance of violation of lepton number.

The combination of these two observations lead to a diagram of the form shown in Figure 2.1. One can then use the associated Feynman rules to compute the value of the diagram and sum over the different neutrino species to obtain a mass matrix for the neutrinos below the electroweak scale. We will postpone this calculation until we consider a completely anaologous one in the next section.

2.3.5 Scalar Degrees of Freedom

The only thing left is to investigate the Dark Matter candidate in the model [5]. To do so we should study the low energy scalar degrees of freedom. The correct regime to work in for the low energy spectrum is after EWSB. By gauge invariance, we can consider the Higgs doublet to take the form $\Phi = (0, v + \Re\phi^0)^{\mathrm{T}}$ and expand the scalar Lagrangian in (2.1) using this ansatz. Firstly note that we have removed $\Im\phi^0$ using the residual gauge freedom. The remaining physical scalar degrees of freedom are then;

$$\Re\phi^0, \quad \eta^\pm, \quad \Re\eta^0, \quad \Im\eta^0 \tag{2.7}$$

By expanding the Lagrangian and collecting together the quadratic terms, we find;

$$
\begin{aligned}
V_s = {} & m_1^2(\Re\phi^0)^2 + \frac{1}{2}m_2^2\left((\Re\eta^0)^2 + (\Im\eta^0)^2 + (\eta^+)^2 + (\eta^-)^2\right) \\
& + \lambda_1(\Re\phi^0)^2 + \lambda_2 v^2\left((\Re\eta^0)^2 + (\Im\eta^0)^2 + (\eta^+)^2 + (\eta^-)^2\right) \\
& + \lambda_3 v^2\left((\Re\eta^0)^2 + (\Im\eta^0)^2\right) + \lambda_4 v^2\left((\Re\eta^0)^2 - (\Im\eta^0)^2\right) + \text{non mass terms}
\end{aligned}
\tag{2.8}
$$

From this we can read off the masses of the physical degrees of freedom;

- $m^2(\sqrt{2}\Re\phi^0) = 2\lambda_1 v^2$, using the relationship $v^2 = -m_1^2/\lambda_1$

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- $m^2(\eta^\pm) = m_2^2 + \lambda_3 v^2$

- $m^2(\sqrt{2}\Re\eta^0) = m_2^2 + (\lambda_3 + \lambda_4 + \lambda_5)v^2$

- $m^2(\sqrt{2}\Im\eta^0) = m_2^2 + (\lambda_3 + \lambda_4 - \lambda_5)v^2$

We are now in a position to make the conclusion that, depending on the sign of λ_5 and suitable choices for the other parameters, either $\Re\eta^0$ or $\Im\eta^0$ is the lightest stable particle. Again, we emphasise that this is stabilised by the $\mathbb{Z}_2$ symmetry. To see this imagine a diagram with one external $\Re/\Im\eta^0$ leg. The Ward identity for the $\mathbb{Z}_2$ symmetry prohibits any decay into a final state that has none of the Dark Matter candidate particle. Hence it is stable to decay in this sense.

III   The Low Energy Effective Field Theory

The discussion of UV complete theories presents a concrete scenario to implement the ideas of radiatively generating neutrino mass, as well as providing a dark matter candidate. On the other hand, at a given low energy scale, our experiments will not be able to probe the UV structure of the theory, only the effective degrees of freedom remaining at the scale in question (e.g. the energy scale at the LHC, or in cosmological scenarios). This is ultimately a statement about renormalisation flow in that we can construct a number of UV theories whose low energy degrees of freedom nonetheless appear to be the same.

In this section, we will present a low energy effective theory that contains a scalar Dark Matter candidate coupled to neutrinos [10–14] that generates a one-loop neutrino mass. This will include a discussion of both real and complex scalars, since the two have qualitatively different constraints. We shall explain why it must be an effective theory, as well as how the neutrino mass is generated. More importantly, we will focus on the phenomenological consequences of the model and be as clear as possible as to exactly what constraints these place on the couplings and masses within the theory. These include astrophsyical, particle physics, and cosmological bounds [10–14]. This will provide the basis for further numerical investigation.

This is not to say that providing a clear embedding of the low energy theory is not useful at all. Indeed in Section IV, we will provide a review of one way to embed the dark matter candidate in a UV theory to illustrate where the relevant degrees of freedom may come from [14, 15]. Furthermore, we will also discuss the possible phenomenological implications of these *UV theories*, although necessarily the bounds they imply must be weaker than in the effective field theory case.

3.1   The Lagrangian and Particle Content

The overarching motivation for this work is that whatever model we consider should link neutrino mass generation and scalar dark matter particle content. In this regard, our focus is on a low energy particle spectrum that consists of the Standard Model along with;

- A scalar field, δ, which may be real or complex;

- Two or more massive right-handed fermions N_R^i, note that this is not the same fermion as in Section 2.3.1, but it is consistent across the literature.

In addition to this, we assume that there is a residual $\mathbb{Z}_2$ symmetry under which the new particles are odd, and the Standard Model is even. This has a number of effects, in a fashion completely analagous to the discussion in Section 2.2;

1) It ensures that the lightest particle in the spectrum is stable, providing a Dark Matter candidate

2) It prohibits a term of the form $\phi^0 \bar{N}_R^i \nu_L^\alpha$, so that after EWSB, there is no Dirac mass term linking N_R^i and ν_L^α. This means that the neutrino mass is not generated at tree level.

3) The new scalar, δ, cannot acquire a VEV, due to the structure of the Higgs sector

The key feature we are interested in however is the interaction part of the Lagrangian which couples this new dark sector to the neutrino sector in the Standard Model. In our effective theory, we take the couplings to be of the form;

$$g_{i\alpha}\delta\bar{N}_R^i\nu_L^\alpha + \text{h.c.} \tag{3.1}$$

Where there is an implicit sum over the right-handed fermion species, i, and the Standard Model neutrino species, α. Later we will be interested in constraining the values of the coupling constants.

3.1.1 Why must it be an effective theory?

To understand why the Lagrangian in (3.1) must be effective, i.e. only valid up to some high energy scale Λ, we make an assumption regarding the representations of the new fields. We assume that there are no electroweak interactions between δ, N_R^i and the electroweak gauge bosons. In other words, they lie in the trivial representation of $\text{SU}(2)_L \times \text{U}(1)_Y$. This is important because the Standard Model neutrinos are charged under the electroweak symmetry. Therefore the coupling in (3.1) cannot be gauge invariant. It must therefore be an effective theory that is part of some $\text{SU}(2)_L \times \text{U}(1)_Y$ invariant UV theory. The implication of this is that when we calculate for example loop diagrams, we should bear in mind that the theory does not hold up to arbitrarily high energies. Instead there is some cutoff, Λ, below which our Lagrangian is a valid description.

3.2 Generation of Neutrino Masses

In this section we will explicitly calculate the contribution to the neutrino mass matrix that arises due to interaction term in (3.1). The Majorana mass term for the neutrino is of the form;

$$\frac{1}{2}(m_\nu)_{\alpha\beta}\left(\bar{\nu}_L^\alpha\nu_L^\beta + \text{h.c.}\right) \tag{3.2}$$

We are interested in calculating the mass matrix $(m_\nu)_{\alpha\beta}$. To do so, we will need to distinguish between the two cases where either (i) δ is a real scalar field, or, (ii) δ is a complex scalar field.

3.2.1 Case 1: Real Scalar Field

In this case, there is only one diagram that contributes to the neutrino mass, as shown in Figure 3.1. We note a couple of points regarding the the diagram;

- Any of the species of right-handed fermion N_R^i can run around the loop, so we should sum over these possibilities

- As indicated, the diagram should be evaluated at zero external momenta. This is in order to extract only the mass contribution. For non-zero external momenta, there would also generically be higher order[3.1] derivative terms generated by the loop integral.

With this in mind, the contribution to the neutrino mass is given by;

$$\frac{1}{2}(m_\nu)_{\alpha\beta} = \sum_i g_{i\alpha}g_{i\beta}\int^\Lambda \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\slashed{p} - \slashed{k} + m_{N^i}}{\left((p-k)^2 - m_{N^i}^2\right)(p^2 - m_\delta^2)}\Bigg|_{k=0} \tag{3.3}$$

[3.1]Higher order in the sense of derivates, the terms will still be quadratic in the neutrino fields.
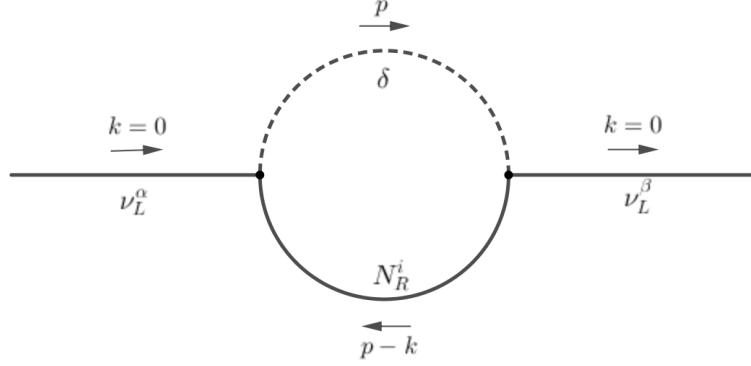
Figure 3.1 *The one loop diagram contributing to the neutrino mass in the case that δ is a real scalar. The external neutrinos are evaluated at zero incoming momenta so as to extract only the mass contribution as opposed to the quadratic derivative interactions.*

Note that the factor of $\frac{1}{2}$ in this formula comes from the same factor in (3.2). To evaluate this first note that the term involving $\not{p}$ is odd in p so when it is integrated over the full space, it vanishes. This leaves only terms of the schematic form;

$$g_{i\alpha}g_{i\beta} \int^{\Lambda} \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{m_{N^i}}{\left(p^2 - m_{N^i}^2\right)\left(p^2 - m_\delta^2\right)} \tag{3.4}$$

We can compute the integral as follows, noting in particular the explicit dependence on the cutoff of the effective theory, Λ;

$$\int^{\Lambda} \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{\left(p^2 - m_{N^i}^2\right)\left(p^2 - m_\delta^2\right)} = \frac{\mathrm{Vol}(S^3)}{32\pi^4} \int_0^{\Lambda} p^3 \mathrm{d}p \, \frac{1}{\left(p^2 - m_{N^i}^2\right)\left(p^2 - m_\delta^2\right)}$$

$$= \frac{1}{16\pi^2} \int_0^{\Lambda} p^3 \mathrm{d}p \, \frac{1}{\left(p^2 - m_{N^i}^2\right)\left(p^2 - m_\delta^2\right)}$$

Now;

$$\frac{1}{16\pi^2} \int_0^{\Lambda} p^3 \mathrm{d}p \, \frac{1}{\left(p^2 - m_{N^i}^2\right)\left(p^2 - m_\delta^2\right)} = \frac{1}{32\pi^2(m_{N^i}^2 - m_\delta^2)} \left(m_{N^i}^2 \log(\Lambda^2 - m_{N^i}^2) - m_{N^i}^2 \log m_{N^i}^2 \right.$$

$$\left. - m_\delta^2 \log(\Lambda^2 - m_\delta^2) + m_\delta^2 \log m_\delta^2 \right) \tag{3.5}$$

We now assume that our effective scale, which is likely above the electroweak scale, satisfies $Lambda \gg m_{N^i}, m_\delta$. In this case, we can simplify this to;

$$\frac{1}{32\pi^2(m_{N^i}^2 - m_\delta^2)} \left(m_{N^i}^2 \log \Lambda^2 - m_{N^i}^2 \log m_{N^i}^2 - m_\delta^2 \log \Lambda^2 + m_\delta^2 \log m_\delta^2 \right)$$

$$= \frac{1}{32\pi^2(m_{N^i}^2 - m_\delta^2)} \left((m_{N^i}^2 - m_\delta^2) \log \frac{\Lambda^2}{m_{N^i}^2} - m_\phi^2 \log \frac{m_{N^i}^2}{m_\phi^2} \right) \tag{3.6}$$

We can now put this together with the expression in (3.3) to find the result quoted in [11–13];

$$(m_\nu)_{\alpha\beta} = \sum_i \frac{g_{i\alpha}g_{i\beta}}{16\pi^2} m_{N^i} \left(\log \frac{\Lambda^2}{m_{N^i}^2} - \frac{m_\delta^2}{m_{N^i}^2 - m_\delta^2} \log \frac{m_{N^i}^2}{m_\delta^2} \right) \tag{3.7}$$

3.2.2 Case 2: Complex Scalar Field

To investigate the complex scalar field, we should first think about the scalar degrees of freedom. Since the field is in the trivial representation of the electroweak gauge group, the most general hermitian mass term for δ can be written as [12];

$$V_m = M^2 \delta^\dagger \delta - \frac{1}{2}(m^2 \delta\delta + \text{h.c.}) \tag{3.8}$$

Consider expanding the scalar field as $\delta = \frac{1}{\sqrt{2}}(\delta_1 + i\delta_2)$ where $\delta_{1,2}$ are both real fields. Expanding the terms above in terms of these real degrees of freedom;

$$V_m = \frac{1}{2}M^2(\delta_1 + i\delta_2)(\delta_1 - i\delta_2) - \frac{1}{4}m^2\left((\delta_1 + i\delta_2)(\delta_1 + i\delta_2) + (\delta_1 - i\delta_2)(\delta_1 - i\delta_2)\right) \tag{3.9}$$

Collecting the terms together for each field we find;

$$V_m = \frac{1}{2}(M^2 - m^2)\delta_1^2 + \frac{1}{2}(M^2 + m^2)\delta_2^2 \tag{3.10}$$

We can then immediately see that the mass eigenstates are simply δ_1 and δ_2 themselves, with masses $m_{\delta_1}^2 = M^2 - m^2$, $m_{\delta_2}^2 = M^2 + m^2$. The lighter of these will be our Dark Matter candidate. Before we do eventually calculate the neutrino mass in this case, we note that interaction term (3.1) is diagonal in this mass basis;

$$\mathcal{L}_{\text{int}} = g_{i\alpha}\bar{N}_R^i \nu_L^\alpha (\delta_1 + i\delta_2) \tag{3.11}$$

The contributions to the neutrino mass are then two diagrams of the form in Figure 3.1. One will have δ_1 running round the loop, whilst the other will have δ_2. We note that the only difference between them is that the second diagram will have two couplings with an extra factor of i, $(ig_{i\alpha})(ig_{i\beta}) = -g_{i\alpha}g_{i\beta}$. As such the second will come with a negative sign. The total contribution is then a sum of two contributions of the form (??);

$$\begin{aligned}
(m_\nu)_{\alpha\beta} = &\sum_i \frac{g_{i\alpha}g_{i\beta}}{16\pi^2} m_{N^i} \left(\log \frac{\Lambda^2}{m_{N^i}^2} - \frac{m_{\delta_1}^2}{m_{N^i}^2 - m_{\delta_1}^2} \log \frac{m_{N^i}^2}{m_{\delta_1}^2} \right) \\
&- \sum_i \frac{g_{i\alpha}g_{i\beta}}{16\pi^2} m_{N^i} \left(\log \frac{\Lambda^2}{m_{N^i}^2} - \frac{m_{\delta_2}^2}{m_{N^i}^2 - m_{\delta_2}^2} \log \frac{m_{N^i}^2}{m_{\delta_2}^2} \right)
\end{aligned} \tag{3.12}$$

Importantly, we see that the dependence on the cutoff drops out in the complex case and we find;

$$(m_\nu)_{\alpha\beta} = \sum_i \frac{g_{i\alpha}g_{i\beta}}{16\pi^2} m_{N^i} \left(\frac{m_{\delta_2}^2}{m_{N^i}^2 - m_{\delta_2}^2} \log \frac{m_{N^i}^2}{m_{\delta_2}^2} - \frac{m_{\delta_1}^2}{m_{N^i}^2 - m_{\delta_1}^2} \log \frac{m_{N^i}^2}{m_{\delta_1}^2} \right) \tag{3.13}$$

3.3 Astrophysical, Cosmological, and Particle Physics Constraints

So far we have illustrated (i) how neutrino mass is generated within the effective framework, and, (ii) discussed which are the relevant degrees of freedom when it comes to suitable dark matter candidates. We now discuss the vital question as to what constraints have already been placed on the model across a range of scenarios. We will investigate the constraints that arise from the two key scenarios [10–14, 16];

1) The cosmological bound on the Dark Matter annihilation cross section;

2) The bounds that arise due to light meson and tau decay.

We will also discuss some of the other astrophysical bounds in slightly more general terms.

3.3.1   Dark Matter Annihilation Cross Section

Within this effective model, there are three annihilation channels in the case that N_R^i is a Majorana fermion, given by $\delta\delta \to \{\nu\nu, \nu\bar{\nu}, \bar{\nu}\bar{\nu}\}$. According to [**13**], the cross section for the process $\delta\delta \to \nu\bar{\nu}$ is suppresed relative to the other two, so we consider just those. Suppressing somewhat the distinction between the different flavours of neutrinos, as well as the different types of new fermion, the thermally averaged cross section is given by [**13**];

$$\langle\sigma v\rangle \simeq 2\langle\sigma(\delta\delta \to \nu\nu)\rangle \simeq \frac{g^4}{4\pi}\frac{m_N^2}{(m_\delta^2 + m_N^2)^2} \tag{3.14}$$

Note that this implies that;

$$m_N^2 \simeq \frac{2\pi}{g^4}(m_\delta^2 + m_N^2)^2\langle\sigma v\rangle$$
$$\Rightarrow m_N \simeq \frac{\sqrt{2\pi}}{g^2}(m_\delta^2 + m_N^2)\sqrt{\langle\sigma v\rangle} \tag{3.15}$$

Real Scalar Dark Matter

In the real case, we make a simplification that the first term in (3.7) dominates, then we can use (3.15) to approximate;

$$m_\nu \simeq \frac{g^2 m_N}{16\pi^2}\log\frac{\Lambda^2}{m_N^2} \simeq \frac{g^2}{16\pi^2}\sqrt{2\pi\langle\sigma v\rangle}\frac{1}{g^2}(m_\delta^2 + m_N^2)\log\frac{\Lambda^2}{m_N^2} \tag{3.16}$$

The rationale behind this is that after simplifying the expression above, we have removed the dependence on g. We see that;

$$m_\nu \simeq \sqrt{\frac{\langle\sigma v\rangle}{128\pi^3}}m_N^2\left(1 + \frac{m_\delta^2}{m_N^2}\right)\log\frac{\Lambda^2}{m_N^2} \tag{3.17}$$

We can take a number of key insights from this relation;

- The neutrino mass is very closely related to the Dark Matter abundance, since this is determined at freeze out in terms of the annihilation cross section;

- We have some bound on m_ν from neutrino experiments and Cosmology;

- $\langle\sigma v\rangle$ is set by the requirement that the relic density of Dark Matter agrees with the observed value;

- The free parameters in the model are m_N, m_δ/m_N, and Λ. However note that (i) $m_\delta/m_N \in [0,1]$ in order that δ remains the lightest particle and hence the Dark Matter candidate, (ii) the dependence on Λ is logarithmic. We deduce that m_N is therfore the most important parameter.

Finally then, we take [**13**] $\Lambda \sim 200\,\text{GeV}$, $0.01\,\text{eV} < m_\nu < 1\,\text{eV}$ and the fact that sub-GeV Dark Matter requires $\langle\sigma v\rangle \simeq 10^{-26}\,\text{cm}^3\text{s}^{-1}$. This allows us to read off bounds on the mass of;

$$\mathcal{O}(1\,\text{MeV}) \lesssim m_\phi < m_N \lesssim 10\,\text{MeV} \qquad (3.18)$$

Notice that this bound can be reduced by increasing the precision of the neutrino mass. Rearanging for the coupling, we find;

$$g \simeq 10^{-3}\sqrt{\frac{m_N}{10\,\text{MeV}}}\left(\frac{\langle\sigma v\rangle}{10^{-26}\,\text{cm}^3\text{s}^{-1}}\right)^{\frac{1}{4}}\left(1+\frac{m_\delta^2}{m_N^2}\right)^{\frac{1}{2}} \qquad (3.19)$$

With the range of neutrino masses chosen, we find the bound;

$$3\times 10^{-4} \lesssim g \lesssim 10^{-3} \qquad (3.20)$$

Complex Scalar Dark Matter

In the complex case, we can apply a similar idea, except that the correction to the neutrino mass is now given by;

$$(m_\nu)_{\alpha\beta} \simeq \frac{g^2}{16\pi^2}m_N\left(\frac{m_{\delta_2}^2}{m_N^2-m_{\delta_2}^2}\log\frac{m_N^2}{m_{\delta_2}^2}-\frac{m_{\delta_1}^2}{m_N^2-m_{\delta_1}^2}\log\frac{m_N^2}{m_{\delta_1}^2}\right) \qquad (3.21)$$

Making the same substitution for the m_N prefactor in terms of the annihilation cross-section, we find;

$$m_\nu \simeq \sqrt{\frac{\langle\sigma v\rangle}{128\pi^3}}\left(m_{\delta_2}^2\log\frac{m_N^2}{m_{\delta_2}^2}-m_{\delta_1}^2\log\frac{m_N^2}{m_{\delta_1}^2}\right) \qquad (3.22)$$

Now we can make a similar deduction by varying the neutrino mass to find in this scenario;

$$(1\,\text{MeV})^2 \lesssim \left|m_{\delta_2}^2-m_{\delta_1}^2\right| \lesssim (20\,\text{MeV})^2 \qquad (3.23)$$

We note that now m_N is a free parameter, so is far less constrained than in the real case. In turn, this implies that g is less constrained to be so weak, although we shall see that there are different bounds due to light meson decay. As a final comment on this scenario, we do note that although the masses of the scalars are not necessarily constrained, the mass difference is $\mathcal{O}(\text{MeV})$, suggesting on naturalness grounds, that the masses themselves are probably of a similar order.

3.3.2   *Light Meson and Tau Decay*

Consider the decay $A \to B + \nu$ [10–12], then generically if the coupling in (3.1) is present, eventually $\nu \to N_R^i\delta$. This means we should see $A \to B + \text{missing energy}$. To see how this provides bounds on the couplings, we shall provide the specific example of kaon decay. We will then quote the generic bounds found in the literature due to this process in the context of other decay channels. Consider the ratio;

$$\frac{\Gamma(K^+ \to e^+ + \text{missing energy})}{\Gamma(K^+ \to \mu^+ + \text{missing energy})} = \frac{\Gamma_{\text{SM}}(K^+ \to e^+\nu_e) + \sum_i \Gamma(K^+ \to e^+ N_R^i\delta)}{\Gamma_{\text{SM}}(K^+ \to \mu^+\nu_e) + \sum_i \Gamma(K^+ \to \mu^+ N_R^i\delta)} \qquad (3.24)$$

Note that we can rearrange this as the ratio;

$$\frac{\frac{\Gamma_{\mathrm{SM}}(K^+ \to e^+ \nu_e)}{\Gamma_{\mathrm{SM}}(K^+ \to \mu^+ \nu_e)} + \sum_i \frac{\Gamma(K^+ \to e^+ N_R^i \delta)}{\Gamma_{\mathrm{SM}}(K^+ \to \mu^+ \nu_e)}}{1 + \sum_i \frac{\Gamma(K^+ \to \mu^+ N_R^i \delta)}{\Gamma_{\mathrm{SM}}(K^+ \to \mu^+ \nu_e)}} \tag{3.25}$$

Now the restricted phase space for the decay $K^+ \to \mu^+ N_R^i \delta$ compared to $K^+ \to \mu^+ \nu_e$ ensures that;

$$\frac{\Gamma(K^+ \to \mu^+ N_R^i \delta)}{\Gamma_{\mathrm{SM}}(K^+ \to \mu^+ \nu_e)} \ll 1 \tag{3.26}$$

So we can approximate;

$$\frac{\Gamma(K^+ \to e^+ + \text{missing energy})}{\Gamma(K^+ \to \mu^+ + \text{missing energy})} \simeq \frac{\Gamma_{\mathrm{SM}}(K^+ \to e^+ \nu_e) + \sum_i \Gamma(K^+ \to e^+ N_R^i \delta)}{\Gamma_{\mathrm{SM}}(K^+ \to \mu^+ \nu_e)} \tag{3.27}$$

This is an important simplification in the sense that the new ratio depends on only one of the couplings, in this case g_{ie}. Experiments such as KLOE [17] can constrain this ratio. The most recent constraints give;

$$\sum_i |g_{ie}|^2 < 10^{-5} \tag{3.28}$$

We can use the same methodology for $K^+ \to \mu^+ +$ missing energy to constrain the $g_{i\mu}$ couplings. In this case, the results come from [18] the LBL Bevatron;

$$\sum_i |g_{i\mu}|^2 \lesssim 10^{-4} \tag{3.29}$$

Finally, by considering decays such as $\tau \to \mu \nu N_R^i \delta$ [12], we can put constraints on the tau coupling, although these are much weaker;

$$\sum_i |g_{i\tau}|^2 < 10^{-1} \tag{3.30}$$

There are a couple of important observations that need to be made with respect to these bounds;

- Provided $\max(m_\delta^2, m_{N^i}^2) \ll m_{K,\pi} \simeq \mathcal{O}(500\,\mathrm{MeV})$, the bounds are similar in both the complex and the real case;

- With the bounds as above, we note that the couplings themselves can have magnitudes;

$$|g_{ie}| \lesssim 3 \times 10^{-3}, \quad |g_{i\mu}| \lesssim 10^{-2}, \quad |g_{i\tau}| \lesssim 3 \times 10^{-1} \tag{3.31}$$

- In the heavy case [10], $m_K < m_\delta + m_{N^i} < m_D$, where m_D is the mass of the D meson, the bounds above do not apply. Instead the strongest bounds have $|g_{ie}| \lesssim 0.4$ and $g_{i\mu} \lesssim \mathcal{O}(1)$.

3.3.3 Additional Constraints

The two methods of constraining the effective theory given above provide the most stringent bounds on the couplings and the masses. Furthermore, they also provide two very distinct scenarios for comparison. The first investigates the phenomenology in the setting of thermalizing early universe cosmology, whilst the second is a pure particle physics test. Within the literature [12, 13], there are a couple of other suggestions for additional, or future methods of constraint. These include;

1) *Supernova Core Collapse:* It is suggested [12, 16] that we could search for a dip in the neutrino energy spectrum coming from neutrinos produced during supernova core collapse.

2) *Large Scale Structure:* This places constraints on the mass [19] due to the damping length of $m \lesssim \mathcal{O}(\text{keV})$.

3) *Big Bang Nucleosynthesis:* This can restrict the masses of the particles [13] due to the fact that baryon-to-photon ratio and light element abundance measurements place an upper bound on the number of additional degrees of freedom below the MeV scale.[3.2]

4) *Detection at Neutrino Experiments:* This is more of future work, but there is a suggestion [12] that experiments such as IceCube may be able generate constraints.

IV   UV Completions of the Effective Model

In the last section, we studied the effective field theory and the observational constraints on it. We will spend the remainder of the review providing a concrete realisation of two UV completions of the effective Lagrangian; one for the real case, and the other for the complex scalar field. The two models will be qualitatively different, with a different particle content. We will also consider breifly some of the possible phenomenology, although we emphasise that the key constraints lie within the effective theory that will ultimately be the focus of any future work.

4.1   Real Scalar Dark Matter

As we explained in Section 3.1.1, the effective theory is not $\text{SU}(2) \times \text{U}(1)$ invariant. We must therefore embed this in an $\text{SU}(2) \times \text{U}(1)$ invariant UV completion. As we have done across the different settings, we also include a $\mathbb{Z}_2$ symmetry under which the new particles introduced below are odd, and Standard Model particles are even. The new particle content is as follows [11, 14];

• An electroweak singlet χ;

• Two Majorana fermions, E^i;

• An electroweak doublet $\Psi^{\text{T}} = (\psi^0, \psi^-)$ where $\psi^0 = \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2)$

In the scalar sector, we also have the Higgs doublet Φ, then the most general $\text{SU}(2) \times \text{U}(1)$ invariant, renormalisable Lagrangian is given by;

$$
\begin{aligned}
\mathcal{L}_s = &-m_\Psi^2 \Psi^\dagger \Psi - \frac{m_s^2}{2}\chi^2 - (m_{\chi\Psi}\chi\Phi^{\text{T}}(i\sigma_2)\Psi + \text{h.c.}) - \lambda_1(\Phi^{\text{T}}(i\sigma_2)\Psi)^2 - \mathscr{R}[\lambda_2(\Phi^{\text{T}}(i\sigma_2)\Psi)^2] \\
&- \lambda_3\chi^2\Phi^\dagger\Phi - \lambda_4\Psi^\dagger\Psi\Phi^\dagger\Phi - \frac{\lambda_1'}{2}(\Psi^\dagger\Psi)^2 - \frac{\lambda_2'}{2}\chi^4 - \lambda_3'\chi^2\Psi^\dagger\Psi - m_H^2\Phi^\dagger\Phi - \frac{\lambda}{2}(\Phi^\dagger\Phi)^2
\end{aligned}
\tag{4.1}
$$

[3.2]To be more precise, combinations of CMB, baryon-to-photon ratio, and light element abundance data place an upper bound of an additional 1.5 degrees of freedom at the 95% confidence level.

There are conditions for stability at large field values that arise from setting e.g. all fields except Φ to zero. These conditions are given by [14];

$$\lambda_1', \lambda_2' > 0, \quad \lambda_3' > -\sqrt{\lambda_1' \lambda_2'}, \quad \lambda_3 > -\sqrt{\lambda \lambda_2'}, \quad \lambda_1 - |\lambda_2| + \lambda_4 > -\sqrt{\lambda \lambda_1'} \tag{4.2}$$

WE also have a Yukawa sector which couples the Standard Model neutrinos to the new scalars and the new fermions. If we let $L^\alpha = (\nu^\alpha, \ell^\alpha)_L$ be the Standard Model lepton doublet, then the most general gauge invariant Yukawa Lagrangian is;

$$\mathcal{L}_Y = -h_{i\alpha} \bar{E}^i \Psi^\dagger L^\alpha - \frac{M_{ij}}{2} \bar{E}^i E^j \tag{4.3}$$

where the second term encodes the Majorana mass of the new fermions.

4.1.1 The Low Energy Spectrum

To relate this to the effective field theory, we need to derive the low energy spectrum i.e. the masses of the relevant degrees of freedom. To do this, we recall that due to the $\mathbb{Z}_2$ symmetry, the new scalar cannot acquire a VEV, as such the relevant process to consider is EWSB where the Higgs doublet can be written as $\Phi = (0, \frac{v_H}{\sqrt{2}})$. The relevant mass terms in the scalar Lagrangian after spontaneous symmetry breaking are then;

$$\mathcal{L}_m = -m_{\psi^-}^2 \left|\psi^-\right|^2 - \frac{m_{\psi_2}^2}{2}\psi_2^2 - \frac{m_\chi^2}{2}\chi^2 - \frac{m_{\psi_1}^2}{2}\psi_1^2 - m_{\chi\Psi}v_H\psi_1\chi + \text{non mass terms} \tag{4.4}$$

To derive this, we simply extract the terms quadratic in the fields in the scalar Lagrangian, which will include contributions from quartic terms that contain two Higgs doublets for example. The parameters above can be related to the ones in the high energy Lagrangian via;[4.1]

$$m_{\psi^-}^2 = m_\Psi^2 + \lambda_4 \frac{v_H^2}{2} \tag{4.5}$$

$$m_\chi^2 = m_\Psi^2 + \lambda_3 \frac{v_H^2}{2} \tag{4.6}$$

$$m_{\psi_1}^2 = m_\Psi^2 + \lambda_1 \frac{v_H^2}{2} + \lambda_2 \frac{v_H^2}{2} \tag{4.7}$$

$$m_{\psi_2}^2 = m_\Psi^2 + \lambda_1 \frac{v_H^2}{2} - \lambda_2 \frac{v_H^2}{2} \tag{4.8}$$

The final step to extract the spectrum is to note that there is some mixing between χ and ψ_1. The mass eigenstates are then the original ψ^-, ψ_2 as well as the mixed real scalar fields δ_1 and δ_2, where;

$$\begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \chi \\ \psi_1 \end{pmatrix} \tag{4.9}$$

Where the angle α is determined by;

$$\tan 2\alpha = \frac{2v_H m_{\chi\Psi}}{(m_{\psi_1}^2 - m_\chi^2)} \tag{4.10}$$

The masses of the mass eigenstates $\delta_{1,2}$ are then;

$$m_{\delta_1}^2 \simeq m_\chi^2 - \frac{(m_{\chi\Psi}v_H)^2}{(m_{\psi_1}^2 - m_\chi^2)}, \quad m_{\delta_2}^2 \simeq m_\chi^2 + \frac{(m_{\chi\Psi}v_H)^2}{(m_{\psi_1}^2 - m_\chi^2)} \tag{4.11}$$

[4.1]A techinicality that is considered in [14] is that we should tune the parameters to avoid also spontaneously breaking the $\mathbb{Z}_2$ symmetry.

Finally then we can conclude that due to the $\mathbb{Z}_2$ symmetry (i) the lightest particle out of δ_1, δ_2 will be the Dark Matter candidate, and, (ii) this particle will be stable. We are also now able to see where the coupling in (3.1) comes from. Expanding the Yukawa Lagrangian, we find a term;

$$-h_{i\alpha}\bar{E}^i\Psi^\dagger L^\alpha = -h_{i\alpha}\bar{E}^i\frac{1}{\sqrt{2}}\psi_1\nu_L^\alpha + \cdots = -\kappa_{i\alpha}\delta_1\bar{E}^i\nu_L^\alpha + \cdots \quad (4.12)$$

Where we have assumed for simplicity that δ_1 is the lightest in this case. We see this reproduces the coupling between the new fermions, the Dark Matter scalar, and the Standard Model neutrinos, in a gauge invariant way.

4.1.2   Phenomenology

We will not go into great details about the phenomenology of the particular model, since the aim was more to illustrate how the low energy degrees of freedom arise. Nonetheless, it is interesting, and informing to discuss some of the implications. A more detailed account can be found in [**14**]. It should be noted that in the context of this review, none of the following are necessarily self-evident, however it is helpful to have a record of them;

- *Dark Matter Self Interaction:* The self interaction cross section of the model $\sigma(\delta_1\delta_1 \to \delta_1\delta_1)$ is not in conflict with present bounds from galaxy mass profiles.

- *Electron-Positron Annihilation:* There is a loop suppression when it comes to $\delta_1\delta_1 \to e^+e^-$ annihilation which prohibits the model from explaining the $511\,\text{keV}$ line.

- *Interaction with the Z-boson:* There is an interaction term between $\psi_{1,2}$ and the Z-boson since Ψ is charged under the electroweak gauge group. However, ψ_2 is heavy in this context, so there is no new decay mode for the Z-boson.

- *Lepton Flavour Violating Rare Decays:* A charged scalar, δ_1, interacts with a charged lepton, so we can have lepton violating decays. Branching ratios for e.g. $\mu \to e\gamma$ can constrain the couplings $h_{i\alpha}$.

- *Muon Magnetic Moment:* If we consider the other term in $\bar{E}^i\Psi^\dagger L^\alpha$, we have a charged scalar ψ^- interacting with a muon field, μ, so there can be a correction to the muon magnetic moment proportional to the couplings $h_{i\alpha}$.

4.2   Complex Scalar Dark Matter

In this case, the main references are [**11**, **15**], which provides a scenario to embed the complex scalar Dark Matter candidate within a UV theory. As before, there will be a $\mathbb{Z}_2$ symmetry under which the new particles are odd. In this scenario, this is actually a residual symmetry arising from a larger symmetry group. We will discuss this point more later. The new particle content is;

- Two fermionic doubets R, R', with;

$$R = (\nu_R, E_R^-), \quad R' = (E_R^+, \nu_R') \quad (4.13)$$

  Where $\nu_R, E_R^-, \nu_R', E_R^+$ are new right-handed fermion fields.

- An electroweak triplet, Δ;

$$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix} \quad (4.14)$$

- A complex singlet, $\zeta \sim (1, 0, -1)$

4.2.1 $\mathscr{G}$-Symmetry Breaking

There is an additional important feature to the model. We introduce a global symmetry;

$$\mathscr{G} = \mathrm{U}(1)_R \times \mathrm{U}(1)_\zeta \times \mathrm{U}(1)_\Delta \times \mathrm{U}(1)_\ell \tag{4.15}$$

Each of the new fields is charged under this symmetry, so lies in some representation relative to each subgroup in the product. We can understand this as follows;

- Under the $\mathrm{U}(1)_R$, only R and R' are charged and they have equal and opposite quantum numbers;

- ζ is the only field charged under $\mathrm{U}(1)_\zeta$;

- Δ is only charged under $\mathrm{U}(1)_\Delta$

- $\mathrm{U}(1)_\ell$ is the usual lepton number symmetry group

With these definitions in mind, we can write down the most general scalar $\mathscr{G}$-invariant potential;

$$V = -\mu_\Phi^2 \Phi^\dagger \Phi + \mu_\Delta^2 \mathrm{Tr}(\Delta^\dagger \Delta) + \mu_\zeta^2 \zeta^\dagger \zeta + \frac{\lambda}{4}\Phi^\dagger \Phi + \frac{\lambda_\zeta}{4}(\zeta^\dagger \zeta)^2 + \frac{\lambda_{\Delta_1}}{2}(\mathrm{Tr}\Delta^\dagger \Delta)^2 + \frac{\lambda_{\Delta_2}}{2}\mathrm{Tr}(\Delta^\dagger[\Delta^\dagger, \Delta]\Delta)$$
$$+ \lambda_{\Phi\Delta_1}\Phi^\dagger \Phi \mathrm{Tr}(\Delta^\dagger \Delta) + \lambda_{\Phi\Delta_2}\Phi^\dagger[\Delta^\dagger, \Delta]\Phi + \lambda_{\zeta\Delta}\zeta^\dagger \zeta \mathrm{Tr}(\Delta^\dagger \Delta) + \lambda_{\Phi\zeta}\zeta^\dagger \zeta \Phi^\dagger \Phi \tag{4.16}$$

As a quick aside, one should convince oneself that all the $\mathrm{SU}(2)$ structure works correctly. We also have a fermionic part to the Lagrangian;

$$\mathcal{L}_R = -m_{RR'}(R')^\dagger R + \mathrm{h.c.} \tag{4.17}$$

Now, the dynamics of the model are as follows;

1) At some high energy scale, the symmetry group $\mathscr{G}$ is broken to a subgroup $\mathrm{U}(1)_X$ under which $\{R, \Delta\}$ have the same charge and $\{R', \zeta\}$ have the equal and opposite charge. The Standard Model is neutral under this symmetry. The exact details of this are not specified here or in [11, 15]. This generates terms of the form;

$$V_{\Phi\Delta\zeta} = \lambda_{\Phi\Delta\zeta}\Phi^\mathrm{T} i\sigma_2 \Delta^\dagger \Phi \zeta^\dagger + \mathrm{h.c.} \tag{4.18}$$
$$V_{L\zeta} = g_\alpha \zeta^\dagger R^\dagger L^\alpha + \mathrm{h.c.} \tag{4.19}$$

Where L^α is the usual electroweak lepton doublet.

2) At an even lower energy, $\mathrm{U}(1)_X$ breaks to $\mathbb{Z}_2$, under which only the new particles are odd under the residual symmetry. The scalar potential now contains the terms;

$$V_s = \tilde{\lambda}_{\Phi\Delta\zeta}\Phi^\mathrm{T} i\sigma_2 \Delta^\dagger \Phi \zeta + \tilde{\mu}_\zeta^2 \zeta^2 + \tilde{\lambda}_{\zeta_1}\zeta^4 + \tilde{\lambda}_{\zeta_2}\zeta^3 \zeta^\dagger + \tilde{\lambda}_{\Phi\zeta}\Phi^\dagger \Phi \zeta^2 + \tilde{\lambda}_{\Delta\zeta}\Delta^\dagger \Delta \zeta^2 + \mathrm{h.c.} \tag{4.20}$$

And the Yukawa part of the Lagrangian now contains;

$$\mathcal{L}_{L\zeta} = \tilde{g}_\alpha \zeta R^\dagger L^\alpha + \mathrm{h.c.} \tag{4.21}$$
$$\mathcal{L}_{L\Delta} = \tilde{g}_{\Delta\alpha} R'^\dagger \Delta L^\alpha + \mathrm{h.c.} \tag{4.22}$$

4.2.2   The Scalar Spectrum

As in the real case, to deduce the spectrum, we investigate the mass eigenstates in the scalar mass Lagrangian after EWSB. In other words we should set $\Phi = (0, \frac{v_H}{\sqrt{2}})$. The calculation is more involved than in the real case [11, 15] since there is now mixing between ζ_1, ζ_2, Δ_1, and Δ_2 where $\zeta = \frac{1}{\sqrt{2}}(\zeta_1 + i\zeta_2)$ and $\Delta^0 = \frac{1}{\sqrt{2}}(\Delta_1 + i\Delta_2)$. This introduces 4 new scalar fields $\delta_{1,2,3,4}$ which are linear combinations of these degrees of freedom. These fields, along with Δ^+ and Δ^{++} are the scalar mass eigenstates. After EWSB, we can write the masses of these fields in terms of the Lagrangian parameters defined above, along with the Higgs VEV [15]. The lightest of $\delta_{1,2,3,4}$ will be the Dark Matter candidate. We will take this to be δ_1 without loss of generality. Finally then, expanding the definitions in (4.21) and (4.22), we find the scalar, fermino, Standard Model neutrino interaction;

$$\mathcal{L}_{L\zeta} + \mathcal{L}_{L\Delta} = \tilde{h}_{1\alpha}\delta_1\bar{\nu}_R\nu^\alpha + \tilde{h}_{2\alpha}\delta_1\bar{\nu}'_R\nu^\alpha + \cdots \tag{4.23}$$

4.2.3   Phenomenology

Again, we postpone a detailed analysis of the phenomenology of this embedding to [11], however we give a qualitative explanation of the sort of detection features one might expect. In this case we have;

- *Neutrino Massses and Lepton Flavour Violation:* As always within these models, there is a one loop contribution to the neutrino mass matrix. This will depend on all the parameters of the model such as the masses of the scalars and the heavy fermions. This can place constraints on the couplings and the masses.

- *Electroweak Precision Tests:* The fields are charged under the electroweak gauge group, so there must be couplings to the electroweak gauge bosons. These can affect the precision electroweak parameters, whose measurement can place constraints on for example the triplet mass splitting.

- *Signatures at Colliders:* The model provides a relatively developed collider phenomenology due to decays such as $h \to \delta_1\delta_2$. This can constrain the coupling structure.

V   Concluding Remarks

The aim of this review was to provide a coherent summary of models which link scalar Dark Matter to small neutrino masses. The big picture here revolves around two key unanswered questions in theoretical physics currently;

1) What is the nature of Dark Matter?

2) In light of observations such as neutrino oscillations, what mechanism generates small masses for the Standard Model neutrinos?

Taking an economical view that a scenario that both generates neutrino mass and presents a candidate for Dark Matter is a pleasing solution. It also seems more likely that it is easier to bring a single model in line with observations, than multiple, constrained models solving each problem separately.

With this in mind, we illustrated how the introduction of a $\mathbb{Z}_2$ symmetry with a simple toy model could radiatively generate Majorana neutrino masses. We also discussed how this symmetry stabilised the lowest mass particles in the spectrum of the theory, and how therefore the model naturally prsented a Dark Matter candidate.

Up to this point, our discussion had revovled around fully UV complete theories. The nature of the renormalisation flow ultimately hides much of the UV structure however. It is therefore important to focus on, and constrain, an effective field theory at some lower energy scale that nonetheless generates the phenomenology of neutrino masses and Dark Matter. Now we are working with effective field theories, we no longer require the theory to be gauge invariant under the electroweak gauge group, since it is only valid up to some (high) energy scale Λ.

As in the discussion above, it is important to constrain the effective field theory using observations across different physical regimes. We focussed on cosmological, astrophysical, and particle physics experiments that could place bounds on the coupling structure in (3.1). These observations included the thermal evolution of the early universe, as well as branching ratios at particle colliders. With these explicit bounds in both the real, and the complex case, one can present a more concrete numerical investigation. This can be used for example to examine the model testing possibilities at neutrino experiments such as IceCube.

We did finally present a concrete realisation of these models that were gauge invariant and UV complete in both the real and complex cases. Indeed, we also discussed to some extent whether these models were falisfiable in a phenomenological sense. We would emphasise however that the key points to take away from this review are at the level of effective field theory, viewing the UV completion as merely an illustration of plausibiity. It remains to consider ways to further contrain and test such effective models, as well as extend the possible couplings to, for example, gauge bosons or fermion Dark Matter.

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