

BIG-BANG NUCLEOSYNTHESIS

4.1 Nuclear Statistical Equilibrium

As a first step to understanding primordial nucleosynthesis we will consider the consequences of nuclear statistical equilibrium (NSE) among the light nuclear species. In *kinetic* equilibrium, the number density of a very nonrelativistic nuclear species $A(Z)$ with mass number A and charge Z is given by

$$n_A = g_A \left(\frac{m_A T}{2\pi} \right)^{3/2} \exp \left(\frac{\mu_A - m_A}{T} \right), \quad (4.1)$$

where μ_A is the chemical potential of the species. If the nuclear reactions that produce nucleus A out of Z protons and $A - Z$ neutrons occur rapidly compared to the expansion rate, *chemical* equilibrium also obtains. In chemical equilibrium, the chemical potential of the species $A(Z)$ is related to the neutron and proton chemical potentials by

$$\mu_A = Z\mu_p + (A - Z)\mu_n. \quad (4.2)$$

Equation (4.1) also applies to the neutron and proton. Using this fact, when chemical equilibrium pertains we can express $\exp(\mu_A/T)$ in terms of the neutron and proton number densities:¹

$$\begin{aligned} \exp(\mu_A/T) &= \exp[(Z\mu_p + (A - Z)\mu_n)/T] \\ &= n_p^Z n_n^{A-Z} \left(\frac{2\pi}{m_N T} \right)^{3A/2} 2^{-A} \exp[(Zm_p + (A - Z)m_n)/T]. \end{aligned} \quad (4.3)$$

¹In all pre-exponential factors, the difference between m_n , m_p , and m_A/A is not important, and all will be taken to be equal to a common mass, the nucleon mass, m_N .

AZ	B_A	g_A
${}^2\text{H}$	2.22 MeV	3
${}^3\text{H}$	6.92 MeV	2
${}^3\text{He}$	7.72 MeV	2
${}^4\text{He}$	28.3 MeV	1
${}^{12}\text{C}$	92.2 MeV	1

Table 4.1: The binding energies of some light nuclei.

Recalling the definition of the binding energy of the nuclear species $A(Z)$,

$$B_A \equiv Zm_p + (A - Z)m_n - m_A, \quad (4.4)$$

and substituting (4.3) into (4.1), the abundance of species $A(Z)$ is

$$n_A = g_A A^{3/2} 2^{-A} \left(\frac{2\pi}{m_N T} \right)^{3(A-1)/2} n_p^Z n_n^{A-Z} \exp(B_A/T). \quad (4.5)$$

A list of binding energies of some light nuclei is given in Table 4.1.

Since particle number densities in the expanding Universe decrease as R^{-3} (for constant number per comoving volume), it is useful to use the total nucleon density, $n_N = n_n + n_p + \sum_i (A n_A)_i$, as a fiducial quantity and to consider the mass fraction contributed by nuclear species $A(Z)$,

$$X_A \equiv \frac{n_A A}{n_N}$$

$$\sum_i X_i = 1. \quad (4.6)$$

Using this definition we find that in NSE the mass fraction of species $A(Z)$ is given by

$$X_A = g_A [\zeta(3)^{A-1} \pi^{(1-A)/2} 2^{(3A-5)/2}] A^{5/2} (T/m_N)^{3(A-1)/2} \\ \times \eta^{A-1} X_p^Z X_n^{A-Z} \exp(B_A/T), \quad (4.7)$$

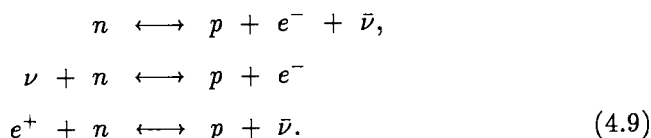
where as usual,

$$\eta \equiv \frac{n_N}{n_\gamma} = 2.68 \times 10^{-8} (\Omega_B h^2) \quad (4.8)$$

is the present baryon-to-photon ratio. The fact that the Universe is “hot” ($\eta \ll 1$, i.e., very high entropy per baryon) is of the utmost significance to primordial nucleosynthesis.² After considering the “initial conditions” for primordial nucleosynthesis we will describe nucleosynthesis in three simple steps.

4.2 Initial Conditions ($T \gg 1$ MeV, $t \ll 1$ sec)

The ratio of neutrons to protons is of particular importance to the outcome of primordial nucleosynthesis, as essentially all the neutrons in the Universe become incorporated into ${}^4\text{He}$. The balance between neutrons and protons is maintained by the weak interactions (here $\nu \equiv \nu_e$):



When the rates for these interactions are rapid compared to the expansion rate H , chemical equilibrium obtains,

$$\mu_n + \mu_\nu = \mu_p + \mu_e, \quad (4.10)$$

from which it follows that in chemical equilibrium

$$n/p \equiv n_n/n_p = X_n/X_p = \exp[-Q/T + (\mu_e - \mu_\nu)/T] \quad (4.11)$$

where $Q \equiv m_n - m_p = 1.293$ MeV. Based upon the charge neutrality of the Universe we can infer that $\mu_e/T \sim (n_e/n_\gamma) = (n_p/n_\gamma) \sim \eta$, from which it follows that $\mu_e/T \sim 10^{-10}$, cf. (3.55). The electron neutrino number of the Universe is similarly related to μ_ν/T ; however, since the relic neutrino background has not been detected, none of the neutrino lepton numbers (e , μ , or τ) is known. For the moment we will assume that the lepton numbers, like the baryon number, are small ($\ll 1$), so that $|\mu_\nu|/T \ll 1$. In our discussion of non-standard scenarios of nucleosynthesis we will return to the possibility that they might be of order unity (or larger). Having

²For purposes of comparison, in a star like our sun, $n_\gamma/n_N \sim 10^{-2}$; even in the post-collapse core of a supernova n_N/n_γ is only a few. Indeed, the entropy of the Universe is enormous.

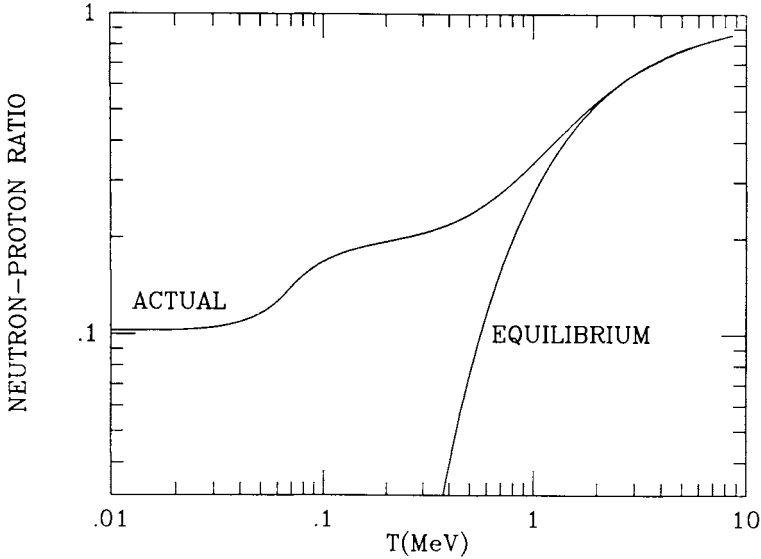


Fig. 4.1: The equilibrium and actual values of the neutron to proton ratio.

made this assumption, the equilibrium value of the neutron-to-proton ratio is

$$\left(\frac{n}{p}\right)_{EQ} = \exp(-Q/T) \quad (4.12)$$

which is shown in Fig. 4.1.

Now consider the rates for the weak interactions that interconvert neutrons and protons; the rates (per nucleon per time) for these reactions are found by integrating the square of the matrix element for a given process, weighted by the available phase-space densities of particles (other than the initial nucleon), while enforcing four-momentum conservation. As an example, the rate for $pe \rightarrow \nu n$ is given by

$$\begin{aligned} \Gamma_{pe \rightarrow \nu n} &= \int f_e(E_e)[1 - f_\nu(E_\nu)] |\mathcal{M}|_{pe \rightarrow \nu n}^2 (2\pi)^{-5} \delta^4(p + e - \nu - n) \\ &\quad \times \frac{d^3 p_e}{2E_e} \frac{d^3 p_\nu}{2E_\nu} \frac{d^3 p_n}{2E_n}. \end{aligned} \quad (4.13)$$

All of these processes have in common a factor from the nuclear matrix

element for the β -decay of the neutron,

$$|\mathcal{M}|^2 \propto G_F^2(1 + 3g_A^2), \quad (4.14)$$

where $g_A \simeq 1.26$ is the axial-vector coupling of the nucleon. This factor can be expressed in terms of the mean neutron lifetime τ_n (or the neutron half life $\tau_{1/2}(n) = \ln(2)\tau_n$) as

$$\tau_n^{-1} = \Gamma_{n \rightarrow pe\nu} = \frac{G_F^2}{2\pi^3}(1 + 3g_A^2)m_e^5\lambda_0 \quad (4.15)$$

where

$$\lambda_0 \equiv \int_1^q d\epsilon \epsilon(\epsilon - q)^2(\epsilon^2 - 1)^{1/2} \simeq 1.636 \quad (4.16)$$

simply represents a numerical factor from the phase space integral for neutron decay [9].

The neutron-proton mass difference and the electron mass determine the limits of integration for these rates. In terms of the dimensionless quantities $q = Q/m_e$, $\epsilon = E_e/m_e$, $z = m_e/T$, and $z_\nu = m_e/T_\nu$,

$$\Gamma_{pe \rightarrow \nu n} = (\tau_n \lambda_0)^{-1} \int_q^\infty d\epsilon \frac{\epsilon(\epsilon - q)^2(\epsilon^2 - 1)^{1/2}}{[1 + \exp(\epsilon z)][1 + \exp((q - \epsilon)z_\nu)]}, \quad (4.17)$$

In the high-temperature and low-temperature limits

$$\Gamma_{pe \rightarrow \nu n} \longrightarrow \begin{cases} \tau_n^{-1}(T/m_e)^3 \exp(-Q/T) & T \ll Q, m_e \\ \frac{7}{60}\pi(1 + 3g_A^2)G_F^2 T^5 \simeq G_F^2 T^5 & T \gg Q, m_e. \end{cases} \quad (4.18)$$

By comparing Γ to the expansion rate of the Universe, $H \simeq 1.66g_*^{1/2}T^2/m_{Pl} \simeq 5.5T^2/m_{Pl}$, we find that

$$\Gamma/H \sim (T/0.8 \text{ MeV})^3 \quad (4.19)$$

for $T \gtrsim m_e$. Thus at temperatures greater than about 0.8 MeV one expects the neutron-to-proton ratio to be equal to its equilibrium value, which at temperatures much greater than an MeV implies $X_n \simeq X_p$.

At temperatures greater than about an MeV, not only are the rates for the weak interactions more rapid than the expansion rate, but so are the rates for the nuclear reactions that build up the light elements, and so NSE should obtain. For purposes of illustration, consider the following system

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of light elements: neutrons, protons, deuterons, ^3He nuclei, ^4He nuclei, and ^{12}C nuclei. We choose to represent mass 3 with the more tightly-bound ^3He nucleus, and “metals” with ^{12}C . In NSE the mass fractions of the various nuclear species are:

$$X_n/X_p = \exp(-Q/T) \quad (4.20)$$

$$X_2 = 16.3(T/m_N)^{3/2}\eta \exp(B_2/T)X_nX_p \quad (4.21)$$

$$X_3 = 57.4(T/m_N)^3\eta^2 \exp(B_3/T)X_nX_p^2 \quad (4.22)$$

$$X_4 = 113(T/m_N)^{9/2}\eta^3 \exp(B_4/T)X_n^2X_p^2 \quad (4.23)$$

$$X_{12} = 3.22 \times 10^5 (T/m_N)^{33/2}\eta^{11} \exp(B_{12}/T)X_n^6X_p^6 \quad (4.24)$$

$$1 = X_n + X_p + X_2 + X_3 + X_4 + X_{12}. \quad (4.25)$$

In Fig. 4.2 the NSE abundances for this system are displayed. It is significant to note that although the binding energies per nucleon are of the order of 1 to 8 MeV, the equilibrium abundance of nuclear species does not become of order unity until a temperature of order 0.3 MeV. This is due to the high entropy of the Universe, that is, the very small value of η . Although for temperatures less than a few MeV nuclei are favored on energetic grounds, entropy considerations favor free nucleons, and the entropy of the Universe is very high. As a rough estimate of when a nuclear species A becomes thermodynamically favored, let us solve for the temperature T_{NUC} when $X_A \sim 1$ (assuming $X_n \sim X_p \sim 1$):

$$T_{NUC} \simeq \frac{B_A/(A-1)}{\ln(\eta^{-1}) + 1.5 \ln(m_N/T)}. \quad (4.26)$$

For deuterium, $T_{NUC} \simeq 0.07$ MeV, for ^3He $T_{NUC} \simeq 0.11$ MeV, for ^4He $T_{NUC} \simeq 0.28$ MeV, and for ^{12}C $T_{NUC} \simeq 0.25$ MeV. The fact that the abundances of the light elements did not begin to build up until temperatures of much less than an MeV is often blamed on the very small binding energy of the deuteron—the so called “deuterium bottleneck.” In fact, the NSE abundances of ^4He and ^{12}C (nuclei with large binding energies) are very small until temperatures that are less than about 0.3 MeV—a fact that traces to the high entropy of the Universe and not the small binding energy of the deuteron. At a somewhat lower temperature ($T \sim 0.1$ MeV), the low abundances of D and ^3He delay nucleosynthesis briefly; the myth of the “deuterium bottleneck” is just that.

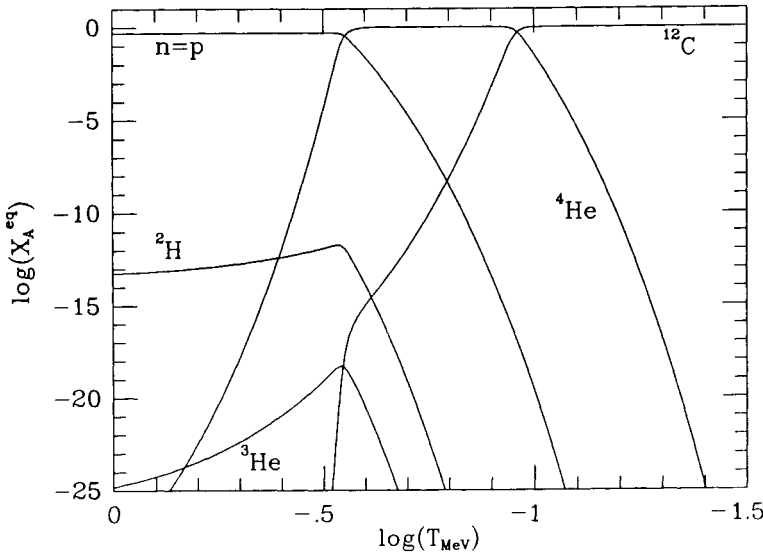


Fig. 4.2: The NSE mass fractions for the system of n , p , D , ${}^3\text{He}$, ${}^4\text{He}$, and ${}^{12}\text{C}$ as a function of temperature. For simplicity we have taken $X_n = X_p$.

4.3 Production of the Light Elements: 1–2–3

• *Step 1* ($t = 10^{-2}\text{sec}$, $T = 10\text{ MeV}$): At this epoch, the energy density of the Universe is dominated by radiation, and the relativistic degrees of freedom are: e^\pm , γ , and 3 neutrino species, so that $g_* = 10.75$.³ All the weak rates are much larger than the expansion rate H , so $(n/p) = (n/p)_{EQ} \simeq 1$ and $T_\nu = T$. The light elements are in NSE, but they have very small abundances due to the fact that η is so small. For example, with $\eta = 10^{-9}$

$$X_n, X_p = 0.5$$

$$X_2 = 4.1(T/m_N)^{3/2}\eta \exp(2.22/T_{\text{MeV}}) \simeq 6 \times 10^{-12}$$

³These are the light ($\lesssim \text{MeV}$) particle species known to exist. The upper limits to the masses of the 3 neutrino species are about: 13 eV (electron neutrino); 0.25 MeV (muon neutrino); and 35 MeV (tau neutrino). The electron and muon neutrinos are clearly “light;” we will also take the tau neutrino to be light. We will use $g_* = 10.75$ as the value for the standard scenario; later, we consider the possibility of additional, hypothetical light species. If the tau neutrino is heavy ($\gg \text{MeV}$), then $g_* = 9$.

$$\begin{aligned}
X_3 &= 7.2(T/m_N)^3 \eta^2 \exp(7.72/T_{\text{MeV}}) \simeq 2 \times 10^{-23} \\
X_4 &= 7.1(T/m_N)^{9/2} \eta^3 \exp(28.3/T_{\text{MeV}}) \simeq 2 \times 10^{-34} \\
X_{12} &= 79(T/m_N)^{33/2} \eta^{11} \exp(92.2/T_{\text{MeV}}) \simeq 2 \times 10^{-126} \quad (4.27)
\end{aligned}$$

where $T_{\text{MeV}} = T/\text{MeV}$.

• *Step 2* ($t \simeq 1$ sec, $T = T_F \simeq 1$ MeV): Shortly before this epoch, the 3 neutrino species decouple from the plasma, and as we have discussed in the previous Chapter, a little later ($T \simeq m_e/3$) the e^\pm pairs annihilate, transferring their entropy to the photons alone and thereby raising the photon temperature relative to that of the neutrinos by a factor of $(11/4)^{1/3}$. At about this time the weak interactions that interconvert neutrons and protons freeze out (Γ becomes smaller than H). When this occurs, the neutron-to-proton ratio is given approximately by its equilibrium value,

$$\left(\frac{n}{p}\right)_{\text{freeze-out}} = \exp(-Q/T_F) \simeq \frac{1}{6}. \quad (4.28)$$

After freeze-out the neutron-to-proton ratio does not remain truly constant, but actually slowly decreases due to occasional weak interactions (eventually dominated by free neutron decays). The deviation of n/p from its equilibrium value becomes significant by the time nucleosynthesis begins (see Fig. 4.1). At this time the light nuclear species are still in NSE, with very small abundances:

$$\begin{aligned}
X_n &\simeq 1/7 \\
X_p &\simeq 6/7 \\
X_2 &\simeq 10^{-12} \\
X_3 &\simeq 10^{-23} \\
X_4 &\simeq 10^{-28} \\
X_{12} &\simeq 10^{-108} \quad (4.29)
\end{aligned}$$

• *Step 3* ($t = 1$ to 3 minutes, $T = 0.3$ to 0.1 MeV): At about this time g_* has decreased to its value today, 3.36, because the e^\pm pairs have disappeared and transferred their entropy to the photons. The neutron-to-proton ratio has decreased from $\sim 1/6$ to $\sim 1/7$ due to occasional weak interactions; compare this to the equilibrium value for $T = 0.3$

MeV, $(n/p)_{EQ} = 1/74$. At a temperature of 0.3 MeV the NSE value of the mass fraction of ${}^4\text{He}$ rapidly approaches unity. Shortly before this ($T \sim 0.5$ MeV), the actual amount of ${}^4\text{He}$ present first falls below its NSE value. This occurs because the rates for the processes that synthesize ${}^4\text{He}$ [$D(D, n){}^3\text{He}(D, p){}^4\text{He}$, $D(D, p){}^3\text{H}(D, n){}^4\text{He}$, and $D(D, \gamma){}^4\text{He}$] are not fast enough to keep up with the rapidly increasing “NSE demand” for ${}^4\text{He}$. The reaction rates, Γ , which are proportional to $n_A \langle \sigma |v| \rangle$, are not fast enough for two reasons: (1) While the abundances of D, ${}^3\text{He}$, and ${}^3\text{H}$ are actually beginning to exceed their NSE values, the NSE abundances are still very small, $X_i = 10^{-12}$, 2×10^{-19} , 5×10^{-19} , respectively. For this reason the number densities of these fuels, $n_A = (X_A/A)\eta n_\gamma$, are small. (2) Coulomb-barrier suppression is beginning to become significant: The thermal average of the barrier-penetration factor is given by

$$\langle \sigma |v| \rangle \propto \exp[-2\bar{A}^{1/3}(Z_1 Z_2)^{2/3} T_{\text{MeV}}^{-1/3}], \quad (4.30)$$

where $\bar{A} = A_1 A_2 / (A_1 + A_2)$, and $\langle \sigma |v| \rangle$ indicates the thermally-averaged cross section times relative velocity. Until the abundances of D, ${}^3\text{He}$, and ${}^3\text{H}$ become of order unity at $T \simeq T_{\text{NUC}} \sim 0.1$ MeV, these reactions cannot produce sufficient ${}^4\text{He}$ to establish its NSE abundance. Once these abundances build up, essentially all the available neutrons are quickly bound into ${}^4\text{He}$, the most tightly bound light nuclear species. Assuming that all the neutrons wind up in ${}^4\text{He}$, the resulting mass fraction of ${}^4\text{He}$ is easy to estimate:

$$X_4 \simeq \frac{4n_n}{n_N} = \frac{4(n_n/2)}{n_n + n_p} = \frac{2(n/p)_{\text{NUC}}}{1 + (n/p)_{\text{NUC}}} \quad (4.31)$$

where $(n/p)_{\text{NUC}} \simeq 1/7$ is the ratio of neutrons to protons at the time ${}^4\text{He}$ synthesis finally takes place ($T \simeq 0.1$ MeV).

While the binding energies per nucleon of ${}^{12}\text{C}$, ${}^{16}\text{O}$, etc. are larger than that of ${}^4\text{He}$, by the time “the light-element bottleneck” is finally broken and ${}^4\text{He}$ is produced, Coulomb-barrier suppression is very significant. This fact, together with the absence of tightly-bound isotopes with mass 5 and 8, prevents significant nucleosynthesis beyond ${}^4\text{He}$. Moreover, the low nucleon density suppresses the triple-alpha reaction by which stellar nuclear burning bridges these mass gaps.

Some ${}^7\text{Li}$ is synthesized, ${}^7\text{Li}/\text{H} \sim 10^{-10}$ to 10^{-9} , produced for $\eta \lesssim 3 \times 10^{-10}$ by the process ${}^4\text{He}({}^3\text{H}, \gamma){}^7\text{Li}$, and for $\eta \gtrsim 3 \times 10^{-10}$ by ${}^4\text{He}({}^3\text{He}, \gamma){}^7\text{Be}$ (followed by the eventual β -decay of ${}^7\text{Be}$ to ${}^7\text{Li}$ by electron capture). As we shall soon see, even this trace amount of ${}^7\text{Li}$ proves to be a very valuable

probe of primordial nucleosynthesis. In addition, substantial amounts of both D and ^3He are left unburnt, $\text{D}, ^3\text{He}/\text{H} \sim 10^{-5}$ to 10^{-4} , as the rates for the reactions that burn them to ^4He , $\Gamma \propto X_{2,3}(\eta n_\gamma)\langle\sigma|v|\rangle$, become small as X_2, X_3 become small and the reactions freeze out. Since these rates are proportional to η , it is clear that the amounts of D, ^3He left unburnt should decrease with increasing η .

4.4 Primordial Abundances: Predictions

The idea of primordial nucleosynthesis dates back to Gamow [2] in 1946.⁴ In the 1950's Alpher, Follin, and Herman [3] all but wrote a code to calculate the synthesis of ^4He . Shortly before the discovery of the CMBR, Hoyle and Tayler estimated the amount of ^4He that would be synthesized in the early stages of a hot big bang [4]. Almost immediately after the discovery of the CMBR Peebles [5] wrote a very simple code to follow ^4He synthesis, and in 1967 Wagoner, Fowler, and Hoyle [6] wrote a very detailed reaction network to follow primordial nucleosynthesis all the way up the periodic table. Other independent nucleosynthesis codes have been written since [7]. Wagoner's 1973 version of the code [8] has become the "standard code" for primordial nucleosynthesis. The nuclear reaction rates have been updated periodically and the weak rates corrected for finite temperature and radiative/Coulomb corrections [9]. The code has recently been "modernized" (faster integration procedures, more user-friendly operation, better documentation, etc.) by Kawano [10], and FORTRAN versions of the code and documentation are available. All of the results presented here were calculated using the most up to date version of the code.

How accurate are the predicted abundances? The numerical accuracy of the code is better than 1%. In principle the abundances are sensitive to the input nuclear physics data. In practice, the relevant cross sections are known to sufficient accuracy that, with a few important exceptions, the theoretical uncertainties are irrelevant. The predicted ^4He abundance essentially only depends upon the weak interaction rates (which determine the neutron-to-proton ratio at freeze out). In turn, all of these rates depend upon the same matrix element, which also sets the neutron half life, $\tau_{1/2}(n)$.

⁴Gamow originally proposed that all of the elements in the periodic table be built up during big bang nucleosynthesis. Of course, it is now generally believed that the heavy elements ($A \geq 12$) are produced by massive stars, and that the light elements B, Be, and ^6Li are produced by cosmic ray spallation processes.

At present, the uncertainty in the neutron half life is surprisingly large:

$$\tau_{1/2}(n) = 10.5 \pm 0.2 \text{ min.} \quad (4.32)$$

In addition, some recent determinations suggest an even lower half life, $\tau_{1/2}(n) = 10.1$ to 10.3 min [11]. Shortly, we will discuss the sensitivity of ${}^4\text{He}$ production to the neutron half life. The only other significant sensitivity of the predicted abundances to input parameters involves ${}^7\text{Li}$: due to uncertainties in the cross sections for several reactions that both destroy and produce ${}^7\text{Li}$, there is about a 50% uncertainty in the predicted abundance of ${}^7\text{Li}$.

In Fig. 4.3 we display the time evolution of the light-element abundances for 3 neutrino species, $\eta = 3 \times 10^{-10}$, and $\tau_{1/2}(n) = 10.6$ min, and in Fig. 4.4 their final abundances as a function of the present baryon-to-photon ratio, η . We note that the predicted abundances are a function of η alone; this is simple to understand. During primordial nucleosynthesis the Universe is radiation dominated, so that the expansion rate $H = H(T)$. The various reaction rates are proportional to thermally-averaged cross sections, $\langle \sigma |v| \rangle = f(T)$, times the number densities of various nuclear species, $n_A = (X_A/A)\eta n_\gamma = n_A(\eta, T)$. Thus, all reaction rates are only a function of the density parameter η and the temperature T . Of course, η can be expressed in terms of Ω_B (at the expense of the unknown factor h), or in terms of ρ_B (by specifying the present photon temperature, which is now rather well determined).

Before comparing the predicted abundances with the observed abundances, it is useful to consider the sensitivity of the abundances to the one free cosmological parameter, η , and the two physical parameters $g_*(T \sim \text{MeV})$ and $\tau_{1/2}(n)$.

- $\tau_{1/2}(n)$: As discussed previously, all the weak rates are proportional to $G_F^2(1 + 3g_A^2)$, or in terms of the neutron half life, $\Gamma \propto T^5/\tau_{1/2}(n)$. An increase in the input value of $\tau_{1/2}(n)$ decreases all of the weak rates that interconvert neutrons and protons, and thereby leads to a freeze out of the neutron-to-proton ratio at a higher temperature, $T_F \propto \tau_{1/2}(n)^{1/3}$, and larger value of (n/p) . Since the final ${}^4\text{He}$ abundance depends upon the value of $(n/p)_{\text{freeze-out}}$, cf. (4.28), this leads to an increase in the predicted ${}^4\text{He}$ abundance. The abundances of the other light elements change also, but since their present abundances are known far less precisely than that of ${}^4\text{He}$, the changes are not yet of great interest.

- g_* : Since $H \propto g_*^{1/2}T^2$, an increase in the input value of g_* leads to a faster expansion rate (for the same temperature); this too leads to an

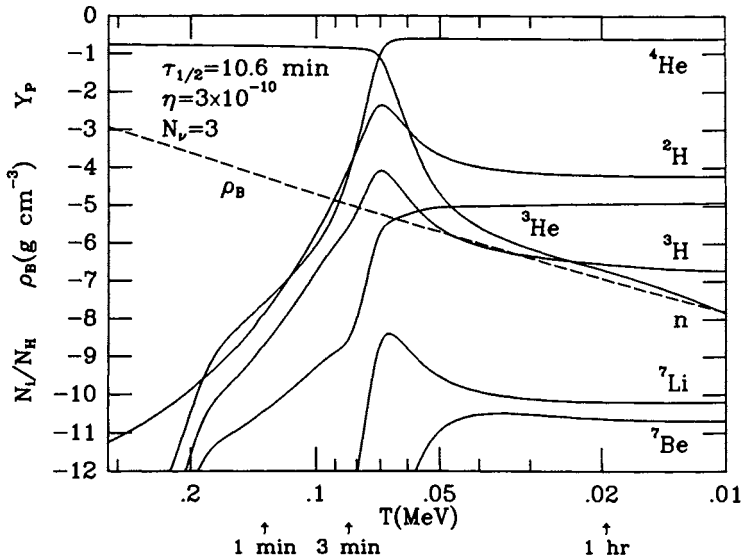


Fig. 4.3: The development of primordial nucleosynthesis. The dashed line is the baryon density, and the solid lines are the mass fraction of ${}^4\text{He}$, and the number abundance (relative to H) for the other light elements.

earlier freeze out of the neutron-to-proton ratio: $T_F \propto g_*^{1/6}$, at a higher value, and hence more ${}^4\text{He}$. Later we will use the dependence of T_F upon g_* to study the effects of the possible existence of additional light particle species (e.g., additional light neutrino species).

• η : In NSE the abundances of species $A(Z)$, $X_A \propto \eta^{A-1}$. For a larger value of η , the abundances of D, ${}^3\text{He}$, and ${}^3\text{H}$ build up slightly earlier, cf. (4.26), and thus ${}^4\text{He}$ synthesis commences earlier, when the neutron-to-proton ratio is larger, resulting in more ${}^4\text{He}$. Around the time ${}^4\text{He}$ synthesis begins in earnest ($T \simeq 0.1$ MeV), the neutron-to-proton ratio is only slowly decreasing (due to neutron decays), and the sensitivity of ${}^4\text{He}$ production to η is only slight. As mentioned earlier, the amount of D and ${}^3\text{He}$ left unburnt depends upon η (decreasing with increasing η). This sensitivity to η is much more significant, with the yields of D and ${}^3\text{He}$ decreasing as η^{-n} ($n \sim 1$ to 2). The ${}^7\text{Li}$ “trough” at a value of $\eta \simeq 3 \times 10^{-10}$ results because of the two different production processes, one which dominates at small η and one which dominates at large η .

An accurate analytic fit to the primordial mass fraction of ${}^4\text{He}$ is $Y_P = 0.230 + 0.025 \log(\eta/10^{-10}) + 0.0075(g_* - 10.75) + 0.014[\tau_{1/2}(n) - 10.6 \text{ min}]$.

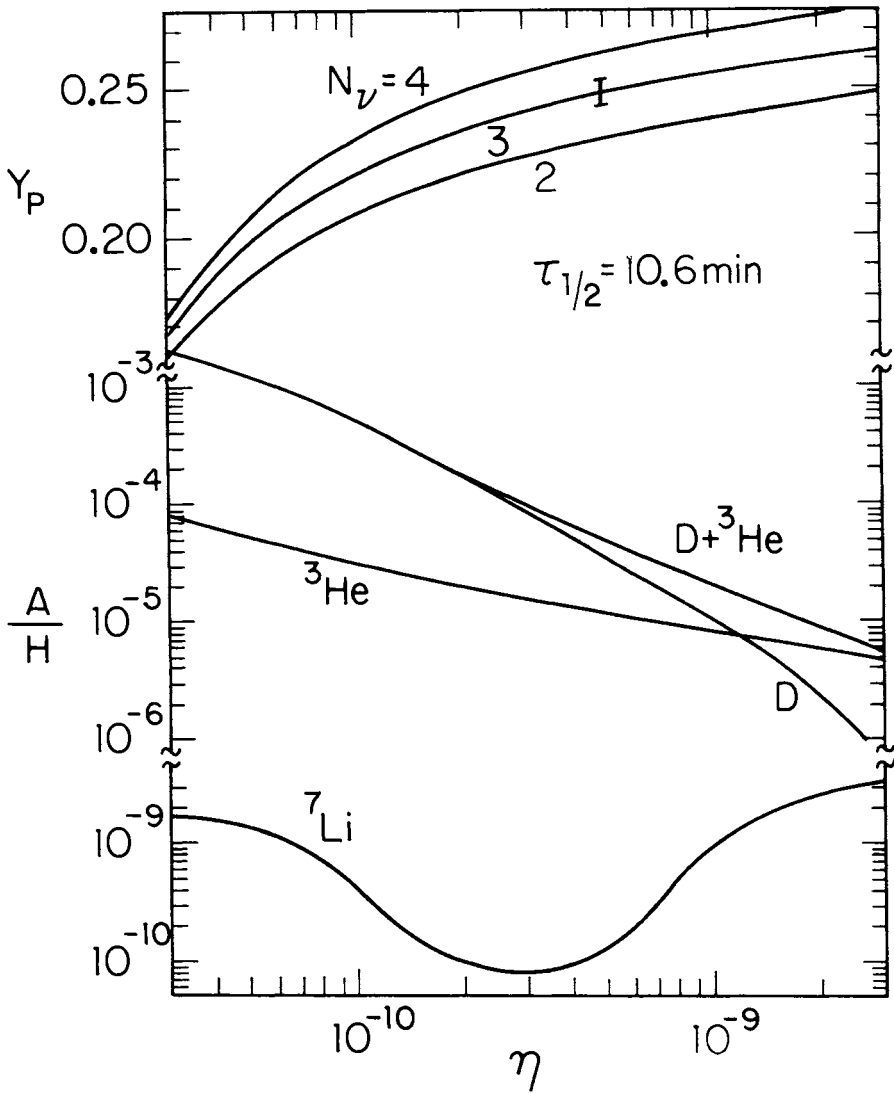


Fig. 4.4: The predicted primordial abundances of the light elements as a function of η . The error bar indicates the change in Y_P for $\Delta\tau_{1/2} = \pm 0.2$ min.

4.5 Primordial Abundances: Observations

Unlike the predicted primordial abundances, where within the context of the standard cosmology the theoretical uncertainties are well defined—and small, the uncertainties in the “observed” primordial abundances are far less certain—and large. As the reader by now must appreciate, to a first approximation, all observables in cosmology are impossible to measure! The problems with the primordial abundances of the light elements are manifold. What we would like to measure are the primordial, cosmic abundances, i.e., abundances at an epoch before other astrophysical processes, e.g., stellar production and destruction, became important. What we can measure are present-day abundances in selected astrophysical sites. From these we must try to infer primordial abundances.

We will not review the important work of numerous astrophysicists which led to the modern view that the present abundances of the light elements D, ^3He , ^4He , and ^7Li are predominantly cosmological in origin (for such a review see, e.g., [12] and [13]). However, it is interesting to note that even in the mid-1960's it was not generally accepted that ^4He had a primordial component, and it wasn't until the early 1970's that it was realized that no contemporary astrophysical process could account for the observed D abundance. And, as we shall mention below, the primordial component of ^7Li was not determined until the early 1980's.

- *Deuterium:* The abundance of D has been measured in solar system studies, in UV absorption studies of the local interstellar medium (ISM), and in studies of deuterated molecules (DCO, DHO) in the ISM. The solar system determinations are based upon measuring the abundances of deuterated molecules in the atmosphere of Jupiter: $\text{D}/\text{H} \simeq (1 \text{ to } 4) \times 10^{-5}$, and inferring the pre-solar (i.e., at the time of the formation of the solar system) D/H ratio from meteoritic and solar data on the abundance of ^3He : $\text{D}/\text{H} \simeq (1.5 \text{ to } 2.9) \times 10^{-5}$. These determinations are consistent with a pre-solar value of $(\text{D}/\text{H}) \simeq (2 \pm 1) \times 10^{-5}$. An average ISM value for $(\text{D}/\text{H}) \simeq 2 \times 10^{-5}$ has been derived from UV absorption studies of the local ISM (distances less than a few hundred pc), with individual measurements spanning the range $(1 \text{ to } 4) \times 10^{-5}$. Note that these measurements are consistent with the solar system determinations of D/H. The studies of deuterated molecules in the ISM that $\text{D}/\text{H} \sim 1 \times 10^{-5}$ with about a factor of 2 uncertainty. While isotopic chemical effects have been taken into account, it should be noted that they can be significant, cf. the abundance of deuterium in the ocean, $\text{D}/\text{H} \simeq 1.5 \times 10^{-4}$, is about a factor of 10 above

cosmic,⁵ and the abundance of D in the atmosphere of Venus is 10^{-2} , a factor of 10^3 above cosmic. Very recently, studies of QSO absorption line systems, clouds of gas backlit by QSO's, have been used to try to determine the extragalactic deuterium abundance. An absorption system with red shift $z_{abs} = 0.03$ associated with Mrk 509 yielded an upper limit to the D abundance of: $D/H \lesssim 10^{-4}$, and a tentative detection in an absorption system with red shift $z_{abs} = 3.09$ associated with Q0420-388 gave $D/H \sim 4 \times 10^{-5}$ [14].

Since the deuteron is very weakly-bound, it is easily destroyed (burned at temperatures greater than about 0.5×10^6 K) and hard to produce. Thus, it has been difficult to find a contemporary astrophysical site to produce even the small cosmic abundance of D.⁶ Therefore, it seems highly plausible that the presently-observed deuterium abundance provides a *lower* bound to the primordial abundance. Using $(D/H)_P \gtrsim 1 \times 10^{-5}$ it follows that η must be less than about 10^{-9} in order for primordial nucleosynthesis to account for the observed abundance of D.⁷ [Note: because of the rapid variation of $(D/H)_P$ with η , this upper bound to η is rather insensitive to the precise lower bound to $(D/H)_P$ assumed.] Since $\eta = 2.68 \times 10^{-8} \Omega_B h^2$, an upper bound to Ω_B also follows: $\Omega_B \lesssim 0.037 h^{-2} \leq 0.20$, indicating that baryons alone cannot close the Universe. One is also tempted to exploit the sensitive dependence of $(D/H)_P$ upon η to derive a *lower* bound to η ; this is not possible because D is so easily destroyed, and the present abundance could be significantly lower than the primordial abundance. As we will see this end can be accomplished by instead using the abundances of both D and ^3He .

- **Helium-3:** The abundance of ^3He has been measured in solar system studies and by observations of the $^3\text{He}^+$ hyperfine line in galactic HII regions⁸ (the analog of the 21 cm line of H). The abundance of ^3He in the oldest meteorites, carbonaceous chondrites, has been determined to be

⁵The D abundance in some ocean-class lakes (e.g., Lake Michigan!) is about a factor of 2 higher than the "ocean" value.

⁶One might wonder why D can be produced in the big bang if it is so easily destroyed elsewhere. As is apparent from Fig. 4.2, the primordial NSE value of D/H should never exceed about 10^{-13} . However, because reaction rates cannot keep pace with the changing temperature, large departures from NSE occur. In stars, densities and time scales are much larger, and NSE is more closely tracked.

⁷Here and throughout the subscript *P* refers to the *primordial* abundance of an element.

⁸An HII region is a cloud of ionized hydrogen gas; typical dimensions are 10's of pc and typical temperatures are order 10,000 to 30,000 K. The energy input is usually provided by several young O or B stars at the center.

${}^3\text{He}/\text{H} = 1.4 \pm 0.4 \times 10^{-5}$. Since these objects are believed to have formed at about the same time as the solar system, they provide a sample of pre-solar material. The abundance of ${}^3\text{He}$ in the solar wind has been determined by analyzing gas-rich meteorites, lunar soil, and the foil placed upon the surface of the moon by the Apollo astronauts. Since D is burned to ${}^3\text{He}$ during the sun's approach to the main sequence, these measurements represent the pre-solar sum of D and ${}^3\text{He}$. These determinations of $\text{D} + {}^3\text{He}$ are all consistent with a pre-solar $[(\text{D} + {}^3\text{He})/\text{H}] \simeq (3.6 \pm 0.6) \times 10^{-5}$. The 3.46 cm hyperfine transition of ${}^3\text{He}^+$ is difficult to detect. Bania, Rood, and Wilson [15] have searched 17 galactic HII regions for this line, and have 9 detections with ${}^3\text{He}^+/\text{H} \simeq (1.2 \text{ to } 15) \times 10^{-5}$, and 8 upper limits (i.e., no detection) with ${}^3\text{He}^+/\text{H} \lesssim (0.4 \text{ to } 6.2) \times 10^{-5}$.

${}^3\text{He}$ is much more difficult to destroy than D. It is very hard to efficiently dispose of ${}^3\text{He}$ without also producing heavy elements or large amounts of ${}^4\text{He}$ (environments hot enough to burn ${}^3\text{He}$ are usually hot enough to burn protons to ${}^4\text{He}$). When the kind of stars we see today process material they return more than 50% of their original ${}^3\text{He}$ to the ISM. In the absence of a generation of very exotic Pop III stars that process essentially all the material in the Universe and in so doing destroy most of the ${}^3\text{He}$ without overproducing ${}^4\text{He}$ or heavy elements, ${}^3\text{He}$ can have been astrated (i.e. reduced by stellar burning) by a factor of no more than $f_a \simeq 2$.⁹ Using this argument, the inequality

$$\begin{aligned} \left[\frac{(\text{D} + {}^3\text{He})}{\text{H}} \right]_P &\leq \left(\frac{\text{D}}{\text{H}} \right)_{pre-\odot} + f_a \left(\frac{{}^3\text{He}}{\text{H}} \right)_{pre-\odot} \\ &\leq (1 - f_a) \left(\frac{\text{D}}{\text{H}} \right)_{pre-\odot} + f_a \left[\frac{(\text{D} + {}^3\text{He})}{\text{H}} \right]_{pre-\odot} \end{aligned} \quad (4.33)$$

and the presolar abundances of D and $\text{D} + {}^3\text{He}$, we can derive an upper bound to the primordial abundance of $\text{D} + {}^3\text{He}$:

$$\left[\frac{(\text{D} + {}^3\text{He})}{\text{H}} \right]_P \lesssim 8 \times 10^{-5}. \quad (4.34)$$

For a very conservative astration factor, $f_a \simeq 4$, the upper limit becomes 13×10^{-5} . Using 8×10^{-5} as an upper bound to the primordial $\text{D} + {}^3\text{He}$

⁹The youngest stars, e.g., our sun, are called Pop I; the oldest observed stars are called Pop II. Pop III refers to a yet to be discovered, hypothetical first generation of stars.

production implies that for concordance, η must be greater than 4×10^{-10} (for the very conservative upper bound of 13×10^{-5} , η must be greater than 3×10^{-10}). To summarize, consistency between the predicted big bang abundances of D and ${}^3\text{He}$ and their abundances observed today requires η to lie in the range $\simeq (4 \text{ to } 10) \times 10^{-10}$.

• *Lithium-7*: Until very recently, our knowledge of the ${}^7\text{Li}$ abundance was limited to observations of meteorites, the local ISM, and Pop I stars, with a derived present abundance of ${}^7\text{Li}/\text{H} \simeq 10^{-9}$ (to within a factor of 2), which is about a factor of 10 greater than that predicted by primordial nucleosynthesis. Given that ${}^7\text{Li}$ is produced by cosmic ray spallation and some stellar processes (e.g., in novae outbursts), and is easily destroyed in environments where $T \gtrsim 2 \times 10^6$ K (e.g., ${}^7\text{Li}/\text{H} \simeq 10^{-11}$ in the atmosphere of our sun), there was not the slightest reason to suspect (or even hope!) that this value accurately reflects the primordial abundance. In 1982 Spite and Spite [16] attempted to observe ${}^7\text{Li}$ lines in the atmospheres of 13 unevolved halo and old disk stars with very low metal abundances ($Z = Z_\odot/12$ to $Z_\odot/250$), and masses in the range $(0.6 \text{ to } 1.1)M_\odot$. To the surprise of many they were successful. For these objects they saw an interesting correlation in the ${}^7\text{Li}$ abundance with stellar mass (actually surface temperature): a plateau in the ${}^7\text{Li}$ abundance for the highest mass stars. While there was clear evidence that the lower mass stars had astrated their ${}^7\text{Li}$ (rapidly decreasing lithium abundance with decreasing mass), the plateau provided very good evidence that the more massive stars had not. Further observations of other very metal-poor, old stars have confirmed their initial results. While a fundamental quantitative understanding of the plateau and the mass where the ${}^7\text{Li}$ abundance starts to decrease is lacking, a qualitative explanation does exist: lower mass stars have deeper surface convective zones and convect their ${}^7\text{Li}$ deep enough to burn it. The observational evidence for this qualitative explanation is very convincing. From the plateau a primordial ${}^7\text{Li}$ abundance follows: ${}^7\text{Li}/\text{H} \simeq (1.1 \pm 0.4) \times 10^{-10}$. This measured abundance represents the pre-pop II ${}^7\text{Li}$ abundance, which, barring the existence of a generation of Pop III stars that very efficiently destroyed (or produced) ${}^7\text{Li}$, is then the primordial abundance.¹⁰

Remarkably, this is the predicted big bang production of ${}^7\text{Li}$ for η in the range $(2 \text{ to } 5) \times 10^{-10}$. If we take this to be the primordial ${}^7\text{Li}$ abundance,

¹⁰Recently, SN1987A provided the opportunity to determine the ${}^7\text{Li}$ abundance in the metal-poor ISM of the Large Magellanic Cloud. Absorption studies of the light from SN1987A indicate $({}^7\text{Li}/\text{H})_{\text{LMC}} \lesssim 1 \times 10^{-10}$, consistent with the pop II abundance in our own galaxy [17].

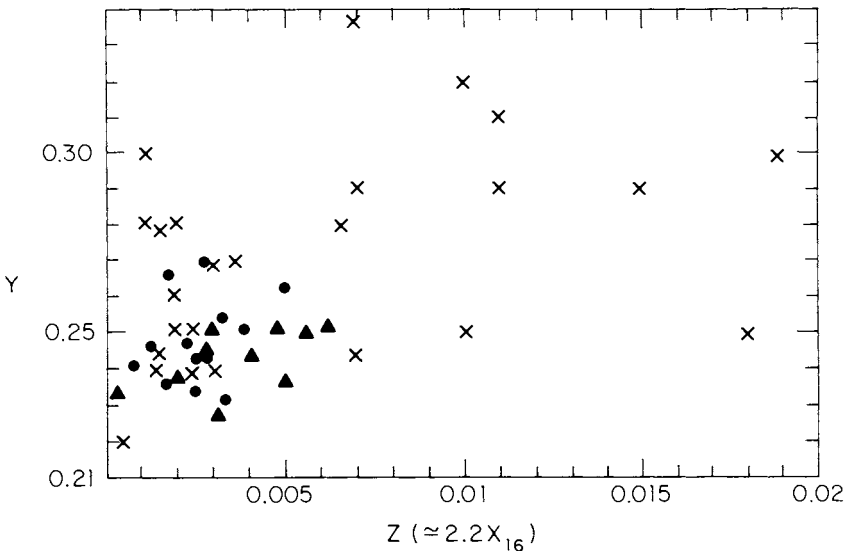


Fig. 4.5: Summary of ${}^4\text{He}$ determinations for galactic and extragalactic HII regions. The filled circles and triangles are two carefully studied sets of metal-poor extragalactic HII regions.

and allow for a possible 50% uncertainty in the predicted abundance of ${}^7\text{Li}$ (due to estimated uncertainties in some of the reaction rates that affect ${}^7\text{Li}$), then consistency for ${}^7\text{Li}$ restricts η to the range $(1 \text{ to } 7) \times 10^{-10}$. It is interesting to note that this range of values for η lies at the bottom of the ${}^7\text{Li}$ trough, making ${}^7\text{Li}$ a very powerful probe of the nucleon density: Any value of η away from the trough results in the overproduction of ${}^7\text{Li}$.

To summarize thus far, concordance of the big bang nucleosynthesis predictions with the derived abundances of D and ${}^3\text{He}$ requires $\eta \simeq (4 \text{ to } 10) \times 10^{-10}$ (or being more conservative, $(3 \text{ to } 10) \times 10^{-10}$); moreover, concordance for D, ${}^3\text{He}$, and ${}^7\text{Li}$ together further restricts η : $\eta \simeq (4 \text{ to } 7) \times 10^{-10}$, with the more conservative range being $(3 \text{ to } 10) \times 10^{-10}$.

• *Helium-4:* In the past ten years the quality and quantity of ${}^4\text{He}$ observations has increased markedly. In Fig. 4.5 all the ${}^4\text{He}$ abundance determinations derived from observations of recombination lines in HII regions (galactic and extragalactic) are shown as a function of metallicity Z (more precisely, 2.2 times the mass fraction of ${}^{16}\text{O}$).

Since ${}^4\text{He}$ is also synthesized in stars, some of the observed ${}^4\text{He}$ is most certainly *not* primordial. Since stars also produce metals, one would expect

some correlation between Y and Z , or at least a trend: lower Y where Z is lower. Such a trend is apparent in Fig. 4.5. From Fig. 4.5 it is also clear that there is a large primordial component to ${}^4\text{He}$: $Y_P \simeq 0.22$ to 0.26 . Is it possible to pin down the value of Y_P more precisely?

There are many steps in going from the line strengths (what the observer actually measures), to a mass fraction of ${}^4\text{He}$ (e.g., corrections for neutral ${}^4\text{He}$, ${}^4\text{He}^{++}$, reddening, etc.). In galactic HII regions where abundances can be determined for various positions within a region, variations are seen within a given region. Observations of extragalactic HII regions are actually observations of a superposition of *several* HII regions. Although observers have quoted statistical uncertainties of $\Delta Y \simeq \pm 0.01$ (or smaller), from the scatter in Fig. 4.5 it is clear that the systematic uncertainties must be larger. For example, different observers have derived ${}^4\text{He}$ abundances of between 0.22 and 0.25 for I Zw18, an extremely metal-poor dwarf emission line galaxy.¹¹

Perhaps the safest way to estimate Y_P is to concentrate on the ${}^4\text{He}$ determinations for metal-poor objects. From Fig. 4.5 $Y_P \simeq 0.23$ to 0.25 appears to be consistent with all the data (although Y_P as low as 0.22 or high as 0.26 could not be ruled out). Kunth and Sargent have studied 13 metal-poor ($Z \lesssim Z_\odot/5$) Blue Compact galaxies (the solid circles in Fig. 4.5). From a weighted average for their sample they derive a *primordial abundance* $Y_P \simeq 0.245 \pm 0.003$; allowing for a 3σ variation this suggests $0.236 \leq Y_P \leq 0.254$.

For the range deduced from D, ${}^3\text{He}$, and ${}^7\text{Li}$: $\eta \geq 4 \times 10^{-10}$ (and $\tau_{1/2}(n) \geq 10.3$ min), the predicted ${}^4\text{He}$ abundance is (see Fig. 4.6)

$$Y_P \geq \begin{cases} 0.227 & \text{for } N_\nu = 2 \\ 0.242 & \text{for } N_\nu = 3 \\ 0.254 & \text{for } N_\nu = 4. \end{cases} \quad (4.35)$$

[Using the more conservative lower bound, $\eta \geq 3 \times 10^{-10}$, the above numbers become 0.224, 0.238, and 0.251, respectively.] Since $Y_P \simeq 0.23$ to 0.25 there are values of η , N_ν , and $\tau_{1/2}(n)$ for which there is agreement between the abundances predicted by big bang nucleosynthesis and the primordial abundances of D, ${}^3\text{He}$, ${}^4\text{He}$, and ${}^7\text{Li}$ derived from observational data.

• *Summary of the Confrontation Between Theory and Observation:* The only isotopes that are predicted to be produced in significant amounts

¹¹An excellent discussion of the difficulties and uncertainties associated with ${}^4\text{He}$ determinations in HII regions (using I Zw18 as a case study) is given in [18].

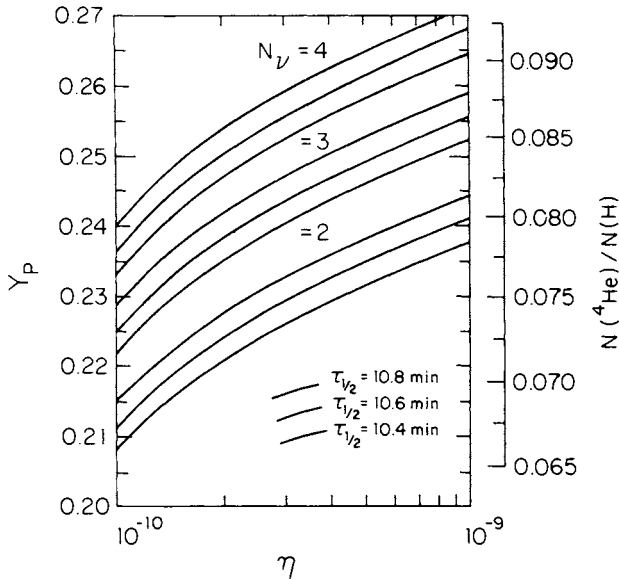


Fig. 4.6: Predicted ${}^4\text{He}$ abundance.

($A/H > 10^{-12}$) during the epoch of primordial nucleosynthesis are: D, ${}^3\text{He}$, ${}^4\text{He}$, and ${}^7\text{Li}$, and their predicted abundances span some 9 orders of magnitude. At present there is good agreement between the predicted primordial abundances of all 4 of these elements and their observed abundances for values of N_ν , $\tau_{1/2}(n)$, and η in the following intervals: $2 \leq N_\nu \leq 4$; $10.3 \text{ min} \leq \tau_{1/2}(n) \leq 10.7 \text{ min}$; and $4 \times 10^{-10} \leq \eta \leq 7 \times 10^{-10}$ (or, with the more conservative range, $(3 \text{ to } 10) \times 10^{-10}$). This is a truly remarkable achievement, and strong evidence that the standard model is valid at times as early as 10^{-2} sec after the bang.

The existence of a concordant range for η is a spectacular success for the standard cosmology. The narrowness of the range is both reassuring—in the end there can be but one set of concordant values—and a cause for continued close scrutiny. For example, focussing on the ${}^4\text{He}$ abundance; should the primordial abundance be unambiguously determined to be 0.22 or less, then the present agreement would disappear. What recourses exist if $Y_P \leq 0.22$? If a generation of Pop III stars that efficiently destroyed ${}^3\text{He}$ and ${}^7\text{Li}$ existed, then the lower bound to η based upon D, ${}^3\text{He}$ (and ${}^7\text{Li}$) no longer exists. The only solid lower bound to η would then be that based upon the amount of luminous matter in galaxies: $\eta \geq 0.3 \times 10^{-10}$.

In this case the predicted Y_P could be as low as 0.15 or 0.16. Although small amounts of anisotropy increase the primordial production of ${}^4\text{He}$, recent work suggests that larger amounts could *decrease* the primordial production of ${}^4\text{He}$ [19]. Another possibility is neutrino degeneracy; as mentioned earlier, a large electron lepton number, $|\mu_\nu| \gtrsim T$, modifies the equilibrium value of the neutron-to-proton ratio,

$$\left[\left(\frac{n}{p} \right)_{\mu_\nu \neq 0} \right]_{EQ} = \exp(-\mu_\nu/T) \left[\left(\frac{n}{p} \right)_{\mu_\nu = 0} \right]_{EQ} \quad (4.36)$$

and therefore the value of (n/p) at freeze out. Since the yield of ${}^4\text{He}$ is determined by this value, it is possible to dial in the desired ${}^4\text{He}$ abundance.¹² Finally, one might have to discard the standard cosmology altogether.

4.6 Primordial Nucleosynthesis as a Probe

If, based upon its apparent success, we accept the validity of the standard model, we can use primordial nucleosynthesis as a probe of conditions in the early Universe, and thereby of cosmology and particle physics. For example, concordance requires: $4 \times 10^{-10} \lesssim \eta \lesssim 7 \times 10^{-10}$ (or being more conservative, $(3 \text{ to } 10) \times 10^{-10}$). This is the most precise determination we have of the present baryon density

$$\begin{aligned} 4(3) \times 10^{-10} &\leq \eta \leq 7(10) \times 10^{-10} \\ 0.015(0.011) &\leq \Omega_B h^2 \leq 0.026(0.037) \\ 0.015(0.011) &\leq \Omega_B \leq 0.16(0.21) \\ 6(4) \times 10^{-11} \leq n_B/s &\simeq \eta/7 \lesssim 1(1.4) \times 10^{-10}, \end{aligned} \quad (4.37)$$

where the more conservative bounds are given in parentheses.

Recall that luminous matter contributes less than about 0.01 of critical density, and that the dynamical determinations of the cosmic density suggest that, $\Omega_0 = 0.2 \pm 0.1$. First, since primordial nucleosynthesis indicates that $\Omega_B \gtrsim 0.015 h^{-2} \gtrsim 0.015$, there must be baryonic matter that is not luminous—not much of a surprise. Second, if, as some dynamical studies suggest, $\Omega_0 > 0.15$ to 0.20, then some other, non-baryonic, form of matter

¹²Large chemical potentials for any, or several, of the 3 neutrino species also affects the energy density in the neutrino species, and thereby the expansion rate.

must account for the difference between Ω_0 and Ω_B . Numerous candidates have been proposed for the dark matter, including primordial black holes, axions, quark nuggets, photinos, gravitinos, higgsinos, relativistic debris, massive neutrinos, sneutrinos, monopoles, pyrgons, maximons, etc. We will return to this intriguing possibility in the next Chapter.

Let us turn now to probing particle physics; the best known constraint from primordial nucleosynthesis is the limit to the number of light neutrino flavors: $N_\nu \leq 4$, which we will now discuss. In their 1964 paper, Hoyle and Tayler realized from their estimates of primordial ${}^4\text{He}$ production that an additional neutrino species would lead to greater ${}^4\text{He}$ production [4]. Waggoner, Fowler, and Hoyle were more quantitative: They calculated primordial nucleosynthesis with an arbitrary “speed-up” factor, which is equivalent to changing g_* [6]. Schvartsman [21] and Peebles [22] also emphasized the dependence of the yield of ${}^4\text{He}$ on the expansion rate of the Universe during nucleosynthesis: ${}^4\text{He}$ production increases with increasing expansion rate, or g_* . Steigman, Schramm, and Gunn used this fact to place a limit to the number of light neutrino species (originally $N_\nu \leq 7$) [20]. As discussed earlier, the reason for increased ${}^4\text{He}$ involves the freeze out of the neutron-to-proton ratio, which occurs at a temperature of ~ 0.8 MeV. At this epoch the known relativistic degrees of freedom are: γ , 3 species of $\nu\bar{\nu}$'s, and e^\pm pairs, and of course there may be additional, as of yet unknown, light (mass less than about a MeV) particle species. The possibilities include, additional neutrino species, axions, majorons, right-handed neutrinos, etc.

Now let us be more quantitative. Recall that Y_P increases with increasing values of η , $\tau_{1/2}(n)$, and $g_*(T)$, shown in Fig. 4.6. An upper limit to $g_*(T \sim \text{MeV})$ can be obtained from the following: (i) a lower limit to η —based upon the present $\text{D} + {}^3\text{He}$ abundance $\eta \geq (3 \text{ to } 4) \times 10^{-10}$; (ii) a lower limit to $\tau_{1/2}(n)$ —present results suggest $\tau_{1/2}(n) \geq 10.3 \text{ min}$ (1σ); (iii) an upper bound to Y_P —present observations suggest that $Y_P \leq 0.25$. The resulting limit, stated in terms of g_* , or the equivalent number of neutrino species, is

$$N_\nu \leq 4 \quad \text{or} \quad g_*(T \sim \text{MeV}) \leq 12.5. \quad (4.38)$$

Recall the definition of g_* ,

$$g_* = 10.75 \text{ (std model; } N_\nu = 3) \\ + \sum_{\text{new bosons}} g_i(T_i/T)^4 + \frac{7}{8} \sum_{\text{new fermions}} g_i(T_i/T)^4. \quad (4.39)$$

Then $N_\nu \leq 4$, $g_* \leq 12.5$, implies that

$$1.75 \geq 1.75(N_\nu - 3) + \sum_{\text{new bosons}} g_i(T_i/T)^4 + \frac{7}{8} \sum_{\text{new fermions}} g_i(T_i/T)^4. \quad (4.40)$$

While at most 1 additional light ($m \lesssim 1$ MeV) neutrino species can be tolerated, many more additional species can be tolerated if their temperatures T_i are less than the photon temperature T . For example, consider a particle that decoupled at a temperature $T_D \gtrsim 300$ GeV, when $g_* \gtrsim 106.75$. Mimicking the calculation we carried out for neutrino decoupling in Chapter 3, we find that $(T_i/T)^4 \leq (10.75/106.75)^{4/3} \simeq 0.047$, so that such a species makes a very small contribution to $g_*(T \sim \text{MeV})$.

The number of neutrino species can also be determined by measuring the width of the Z^0 boson: Each neutrino flavor less massive than $m_Z/2$ contributes about 190 MeV to the width of the Z^0 . Present UA(1) and CDF collider data on the production of W^\pm and Z^0 bosons and e^+e^- experiments that look for the process $e^+ + e^- \rightarrow \text{nothing} + \gamma$ (which includes $e^+ + e^- \rightarrow \nu\bar{\nu} + \gamma$) have placed limits to N_ν which are of the order of 4 to 7 [23]. When the width of the Z^0 boson is measured to high precision at SLC and LEP, N_ν should be determined very accurately, $\Delta N_\nu \lesssim 0.2$, and the big bang constraint put to the test.¹³

4.7 Concluding Remarks

Primordial nucleosynthesis is both the earliest and most stringent test of the standard cosmology, and an important probe of cosmology and particle physics. The present state of agreement between theory and observation indicates that the standard cosmology is a valid description of the Universe at least back to times as early as 10^{-2} sec after the bang and temperatures as high as 10 MeV. As a probe it provides us with the best determination of Ω_B , a very stringent limit to N_ν , and numerous other constraints on particle physics and cosmology. Given the important role occupied by

¹³While big bang nucleosynthesis and the width of the Z^0 both provide information about the number of neutrino flavors, they “measure” slightly different quantities. Big bang nucleosynthesis is sensitive to the number of light ($m \lesssim 1$ MeV) neutrino species and all other light degrees of freedom, whereas the width of the Z^0 is determined by the number of particles less massive than about 46 GeV that couple to the Z^0 (neutrinos among them). As we go to press, SLC reports $N_\nu = 2.79 \pm 0.63$, and the four groups at LEP report $N_\nu = 3.27 \pm 0.30$, 3.42 ± 0.48 , 3.1 ± 0.4 , and $2.4 \pm 0.4 \pm 0.5$ [24].

big bang nucleosynthesis, it is clear that continued scrutiny is in order. The importance of new observational data cannot be overemphasized: extragalactic D abundance determinations (Is the D abundance universal? What is its value?); more measurements of the ^3He abundance (What is its primordial value?); continued improvement in the accuracy of ^4He abundances in very metal poor HII regions (recall, a difference between $Y_P = 0.22$ and $Y_P = 0.23$ is crucial); and further study of the ^7Li abundance in very old stellar populations (Has the primordial abundance of ^7Li actually been measured?). Data from particle physics will prove useful too: A high precision determination of $\tau_{1/2}(n)$ (i.e., $\Delta\tau_{1/2}(n) \leq \pm 0.05$ min) will all but eliminate the uncertainty in the predicted ^4He primordial abundance; an accurate measurement of the width of the Z^0 will determine the total number of neutrino species (less massive than about 46 GeV) and thereby bound the total number of light neutrino species. All these data will not only make primordial nucleosynthesis a more stringent test of the standard cosmology, but they will also make primordial nucleosynthesis a more powerful probe of the early Universe.

On the theoretical side, one should not let the success of the standard scenario keep us from at least exploring other possibilities. For example, standard nucleosynthesis precludes the possibility of $\Omega_B = 1$. If our theoretical prejudice is correct and $\Omega_0 = 1$, primordial nucleosynthesis then makes a very bold prediction: Most of the material in the cosmos today is non-baryonic. Rather than blindly embracing this profound prediction, one should keep an open mind to variants of the standard scenario of nucleosynthesis. Recently, two non-standard scenarios have been suggested as a means to allow $\Omega_B = 1$. (1) A relic, decaying particle species with mass \gtrsim few GeV and lifetime $\tau \sim 10^4$ to 10^6 sec. The hadronic decays of this relic, which occur well after the standard epoch of nucleosynthesis, initiate a second epoch of nucleosynthesis, resetting the light-element abundances—to acceptable values the authors hope [25]. (2) Large, local inhomogeneities in the baryon-to-photon ratio produced by a strongly, first order quark/hadron phase transition. In this scenario, η is highly inhomogeneous (varying by a factor of greater than 30 or so); in addition, due to the ability of neutrons to diffuse through the cosmic plasma more easily than protons, the high density regions become proton rich and the low density regions neutron rich [26]. Because of the inhomogeneities, the cosmic light-element abundances represent the average over the two very different zones of nucleosynthesis, and the authors hope that the resulting abundances are compatible with the observations and $\Omega_0 = \Omega_B = 1$. At present both scenarios have “serious lithium problems;” in the first scenario ^7Li

tends to be underproduced (by a factor of 2 to 3) and ${}^6\text{Li}$ overproduced¹⁴ (by a factor of about 10). In the second scenario, the ${}^7\text{Li}$ yield is (10 to 1000) $\times 10^{-10}$, compared to the observed abundance in old pop II stars of about 10^{-10} . (It also appears that in addition to the lithium problem, ${}^4\text{He}$ may be overproduced). An interesting aside is that both scenarios probably would have been viable 6 years ago prior to determination of the primordial ${}^7\text{Li}$ abundance. The lithium abundance is proving to be a very powerful probe of the early Universe. The success of the very simple, standard scenario of nucleosynthesis is all that more impressive in the light of the apparent inability of scenarios with adjustable parameters to achieve the same success.

4.8 References

References to some historical papers on primordial nucleosynthesis are given in *Early Universe: Reprints*.

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