Degrees of Freedom

A Gentle Introduction to Effective Field Theory

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Outline

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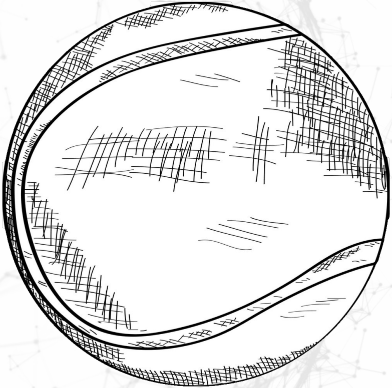
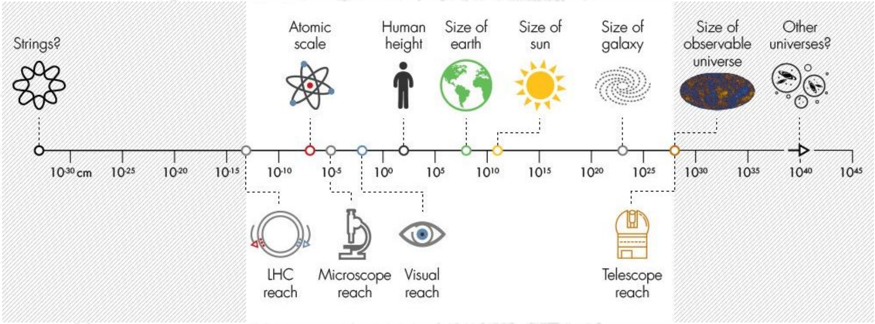
What do tennis balls have to do with anything?



How long is a piece of string?



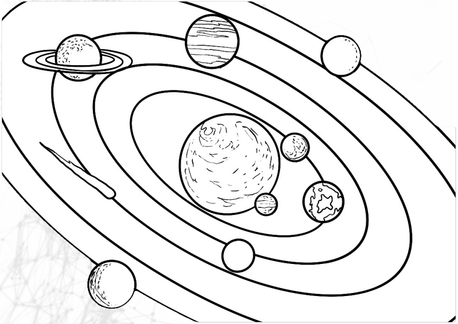
Not very heavy, not very small, not very fast

Classical Mechanics has many successes, for example;

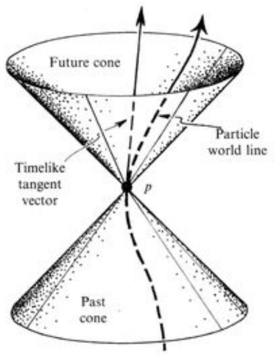
- Describing planetary orbits
- Classical Electromagnetism

But experiments show it is not perfect, something is missing;

- Atomic Clocks
- Precession of Mercury



The very fast



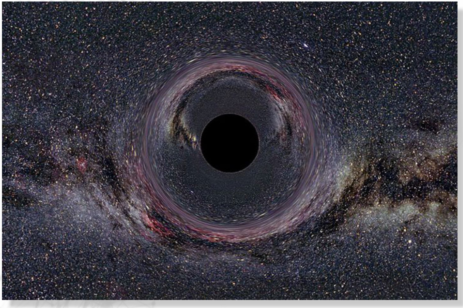
Imagine taking your nice, easy to understand classical system and boosting it to somewhere close to the speed of light;

$$c = 300,000,000\,\mathrm{ms}^{-1}$$

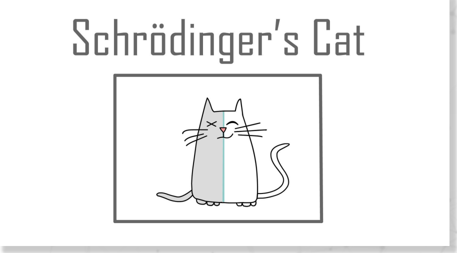
You will find some very weird effects such as **length contraction** and **time dilation**. This is the regime of **Special Relatvity**.

The very heavy, but not very small

The extension of Einstein's special relatvity happens when the gravitational effects get very strong. This is **General Relatvity**. Special Relativity can be thought of an effective theory for weak gravitational fields.



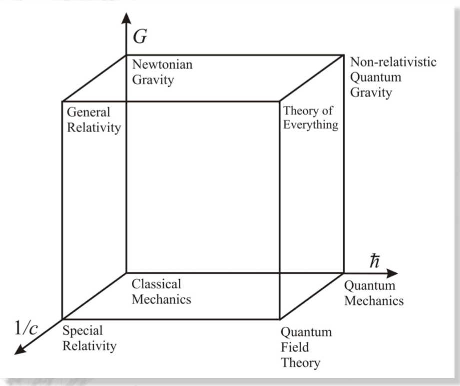
The very small



When things get very small, e.g. the size of an atom (10^{-9} m), the world becomes a bit fuzzy. We lose our ability to predict with certainty and work in the framework of **Quantum Mechanics**.

The very everything

We have been incredibly successful in describing almost every corner of this cube using e.g. **Quantum Field Theory** to unify Special Relativity and Quantum Mechanics. But the top corner where we find the ultimate extremes of gravity, speed, and quantum effects, still eludes us.



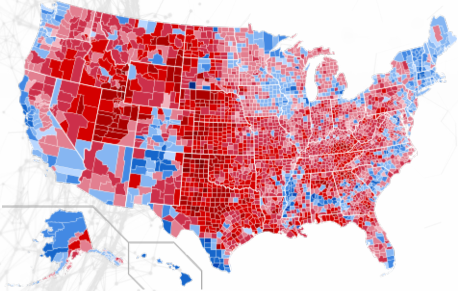
How do we describe a problem?

Each framework has relevant **degrees of freedom**, or parameters if you will, to describe the problem. This is best understood with a few examples;

- *A tennis ball thrown in the air:* best described using classical mechanics by giving its position $\vec{x}$ and its momentum $\vec{p}$
- *Two neutron stars orbiting each other:* these are very compact objects and require general relativity for a good description. The relevant degrees of freedom are in the so called metric, h_{ij}
- *Particle Physics:* Now the degrees of freedom are even more esoteric quantum fields $\phi(x^\mu)$, $W^\pm(x^\mu)$, ...

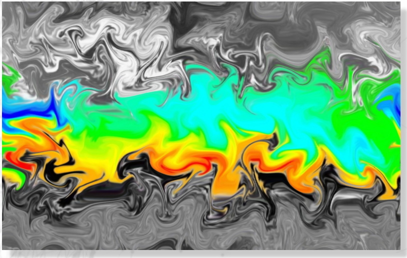
The Key Idea: Coarse Graining

We motivate the idea by considering a very different scenario: **Politics**

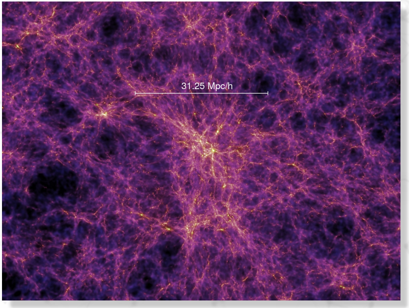


Fluid Dynamics

We don't try and describe fluids in terms of the motion of the individual particles $\{x_i, p_i\}$. Instead we use averaged, or **coarse grained** variables such as **temperature**, **pressure**, **fluid velocity**, and **temperature**.



Cosmology and Dark Matter



Another great example is dark matter distribution in the universe. Even the details of where whole **galaxies** are is completely unimportant to the overall description, just their average abundance.

Note that $1\ Mpc \simeq 3,000,000,000,000,000,000,000,000\ cm!$

The Best Idea in Physics

So what is the real story here? The key is the following expression;

$$e^{-S_{\Lambda^-}[\{\phi_i^-\}]} = \int \mathcal{D}\psi_j^+ \; e^{-S_{\Lambda^+}[\{\psi_j^+,\phi_i^-\}]}$$

The formalism is due to *Kenneth Wilson*, and encodes the idea we've been getting at all along. This is the **Renormalisation Group Flow**, and is probably the most important idea in 20$^{\text{th}}$ century physics.

Quantum Gravity, The Multiverse, and some reading