# *Multitask Pointer Network for Discontinuous Constituent Parsing*

*An application to the German language*

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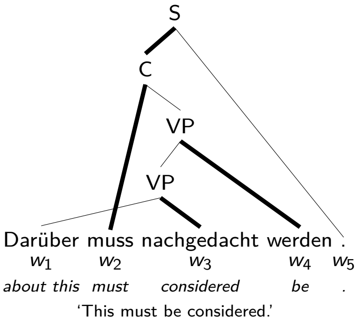
- My model is based on Fernández-González and Gómez-Rodríguez (2022), who propose an architecture based on pointer neural networks in a multi-task setting.
- I achieve state-of-the-art performance across several metrics.

# *Linguistic preliminaries: constituent trees*



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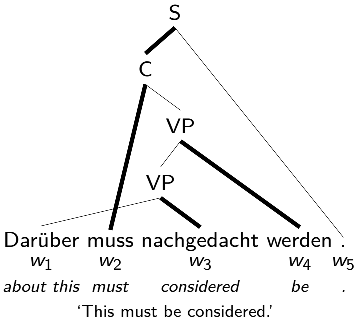


Let $w = (w_1, \ldots, w_L)$ be a sentence.

### *Definition*

- A *constituent tree* is a rooted tree whose leaves are the words $(w_i)_{i=1}^{L}$ and internal nodes are constituents satisfying some constraints.
- A *constituent* is a triple $(Z, \mathcal{Y}, h)$ containing, respectively, its label, yield, and lexical head.
- A constituent is *discontinuous* if its yield is not contiguous.

# Reduction to dependency parsing

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A *dependency tree* is a rooted tree spanning the words in the sentence $(w_i)_{i=1}^{L}$. Each edge is labelled and connects a *head word* (parent) to a *dependency* (child).

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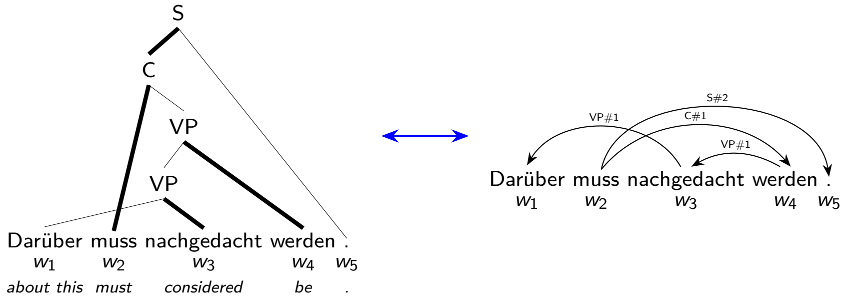
Fernández-González and Martins (2015) show that constituent trees are isomorphic to dependency trees in which the edges contain information about constituent labels and attachment order.



## *Mathematical formalisation*

- Regressor: sentence $(w_i)_{i=1}^{L}$.

[1] For notational simplicity we have written $p_w(\cdot) \equiv p(\cdot \mid w)$ and $p_w(\cdot \mid \cdot) \equiv p(\cdot \mid \cdot, w)$.

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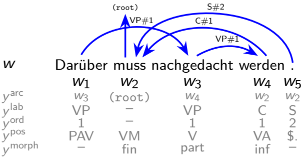


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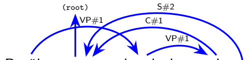


| | | | | | |
|---|---|---|---|---|---|
| $w$ | Darüber | muss | nachgedacht | werden | . |
| | $w_1$ | $w_2$ | $w_3$ | $w_4$ | $w_5$ |
| $y^{\text{arc}}$ | $w_3$ | (root) | $w_4$ | $w_2$ | $w_2$ |
| $y^{\text{lab}}$ | VP | – | VP | C | S |
| $y^{\text{ord}}$ | 1 | – | 1 | 1 | 2 |
| $y^{\text{pos}}$ | PAV | VM | V | VA | $. |
| $y^{\text{morph}}$ | – | fin | part | inf | – |

Denote the regressand by $y = (y_i)_{i=1}^{L}$ where $y_i = (y_i^{\text{arc}}, y_i^{\text{lab}}, y_i^{\text{ord}}, y_i^{\text{pos}}, y_i^{\text{morph}})$ and with mild abuse of notation let $y_{<i} = (y_1, \ldots, y_{i-1})$ for each $i = 2, \ldots, L$ and $y_{<1} = 0$.

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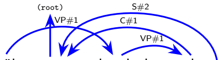


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### *Assumption*

(conditional independence) For each $i = 1, \dots, L$, the random variables $y_i^{\text{lab}}, y_i^{\text{ord}}, y_i^{\text{pos}}$ and $y_i^{\text{morph}}$ are mutually independent conditional on $y_i^{\text{arc}}$, $y_{<i}$ and $w$.

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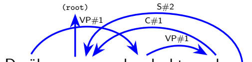


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We can decompose the conditional probability of $y$ given $w$:[1]

$$p_w(y) = \prod_{i=1}^{L} p_w(y_i \mid y_{<i})$$

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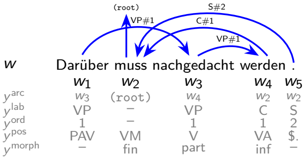


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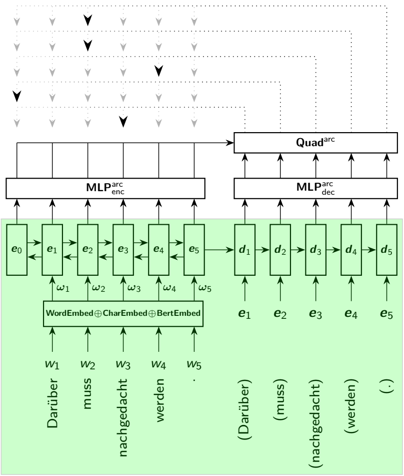
*Building on Dozat and Manning (2016)'s bi-affine mechanism and drawing from Vinyals et al. (2015).*



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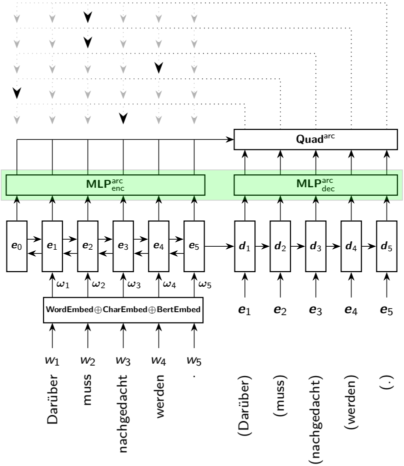

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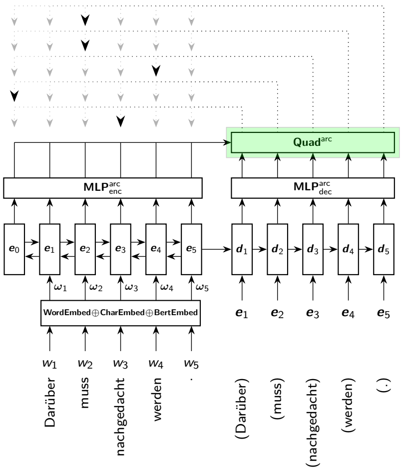

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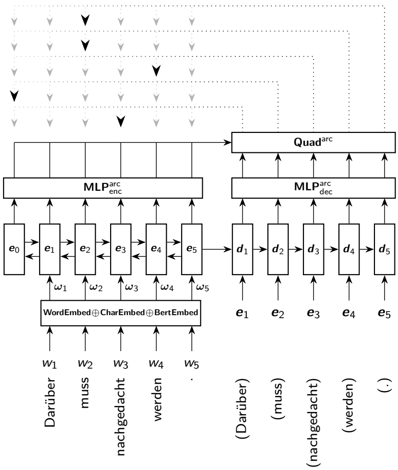

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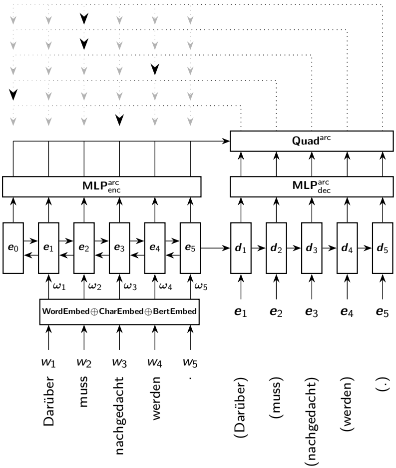


- Fixing child $w_j$, the vector $\mathbf{softmax}(\boldsymbol{s}^{\text{arc}}_{:,j})$ can be interpreted as a probability distribution over potential parents:

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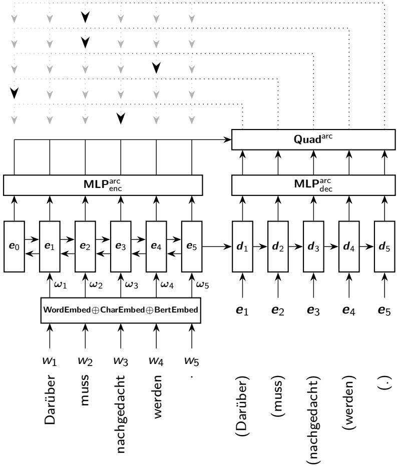


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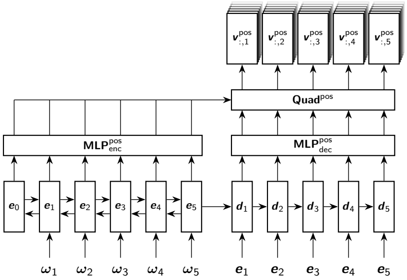
# *Classification tasks: quadratic classifier*

*Building on Dozat and Manning (2016).*

- Encoder and decoder are shared across tasks. The remaining network is separately trained.



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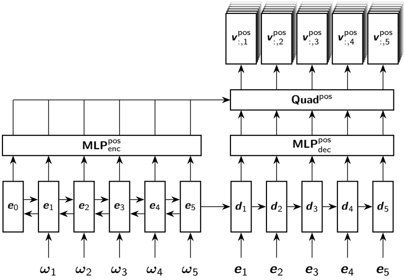

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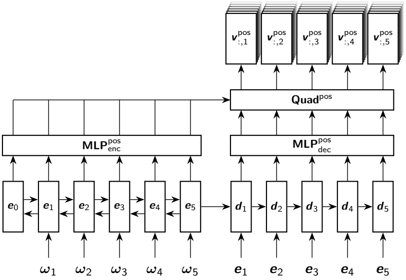

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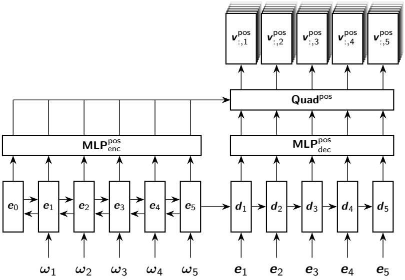


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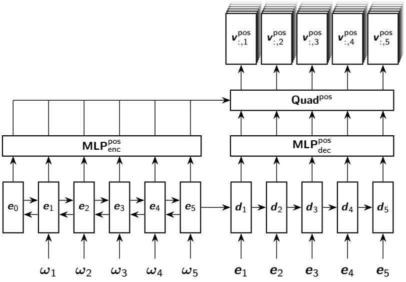


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- Fixing child $w_j$, the vector $\mathbf{softmax}(\boldsymbol{v}^{\text{pos}}_{i,j})$ can be interpreted as a probability distribution over its parts of speech conditional on having an arc to $w_i$:

$$p^{\text{pos}}(c \mid w_i, y_{<j}, w) = \mathbf{softmax}(\boldsymbol{v}^{\text{pos}}_{i,j})_c.$$

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## *Training*

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The feasible region is very large, so the maximisation is approximated via *beam search*.

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| **Model** | **F1** | **Disc. F1** |
|---|---|---|
| Coavoux et al. (2019) | 82.7 | 55.9 |
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| Schnabel & Schütze (2014) | 97.50 | 97.76 |
| Kondratyuk et al. (2018) | 98.58 | 98.97 |
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*Table:* Comparison of part of speech (`pos`) and morphology accuracies (%). Morphology accuracies are the average of accuracies for `case`, `degree`, `gender`, `mood`, `number`, `person` and `tense`.

# *Appendix: Mathematical justification for quadratic classifier*

Notation has been simplified for presentational clarity. Take a single class $c$ and suppose

$$\begin{bmatrix} \boldsymbol{e}_i \\ \boldsymbol{d}_j \end{bmatrix} \Bigg| \, c \sim \mathcal{N}\left( \begin{bmatrix} \boldsymbol{\mu}_c \\ \phi_c \end{bmatrix}, \begin{bmatrix} A_c & Q_c^{\mathsf{T}} \\ Q_c & B_c \end{bmatrix}^{-1} \right).$$

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**Memory-efficient implementation:** under conditional normality, the affine quadratic transformation can be computed as

$$v_{i,j}^c = k^c + \left\| W_h^c \boldsymbol{e}_i + W_d^c \boldsymbol{d}_j + \boldsymbol{w}^c \right\|_2^2.$$

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