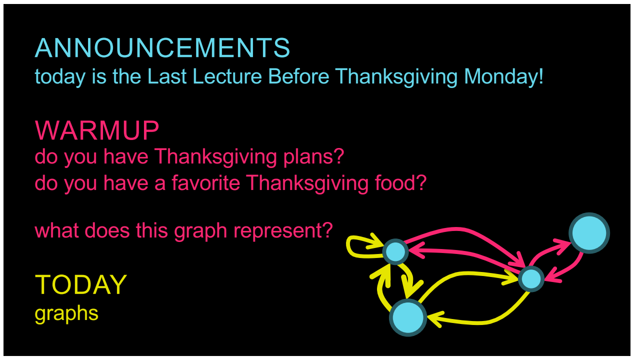


ANNOUNCEMENTS

today is the Last Lecture Before Thanksgiving Monday!

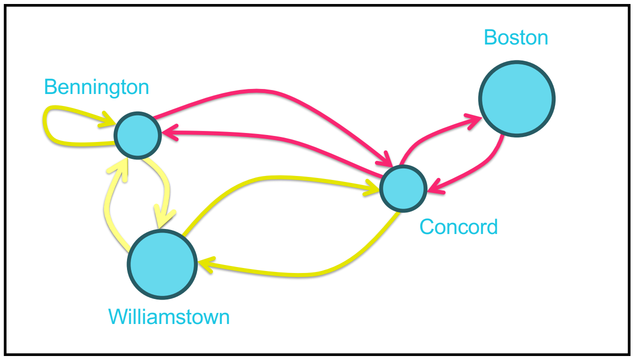
WARMUP

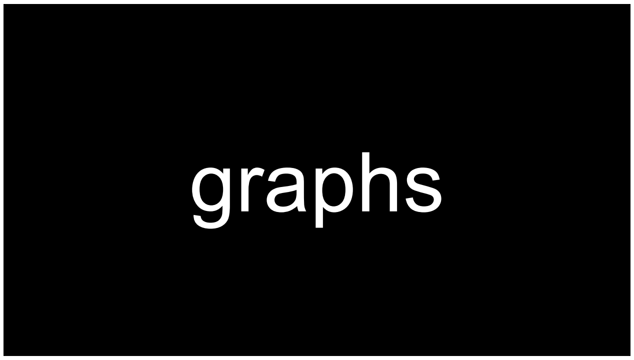
do you have Thanksgiving plans?
do you have a favorite Thanksgiving food?

what does this graph represent?

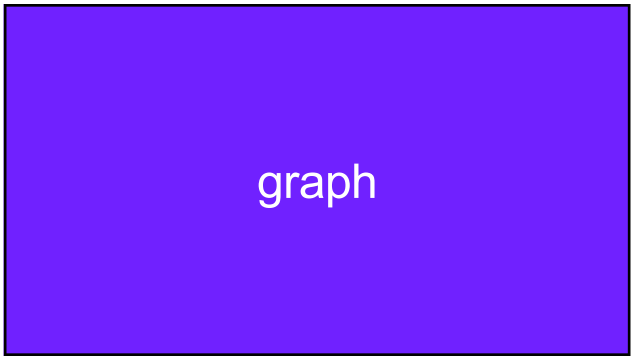
TODAY

graphs

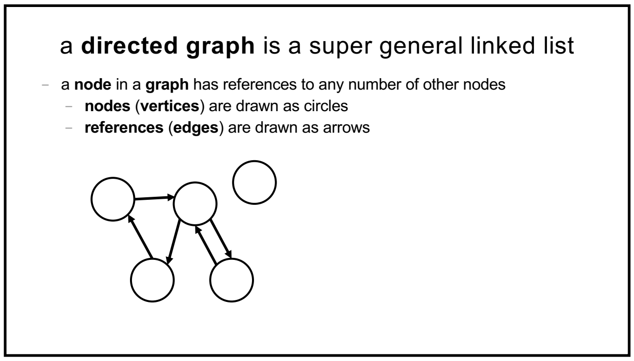




graphs

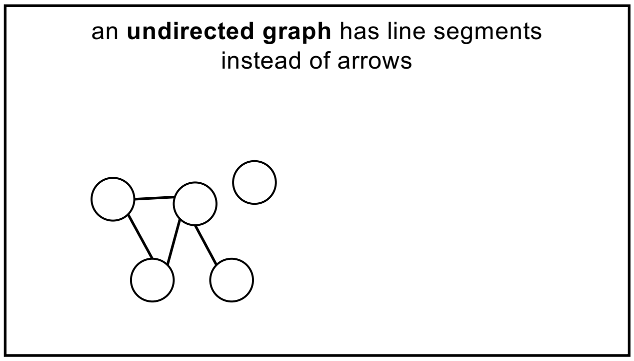


graph



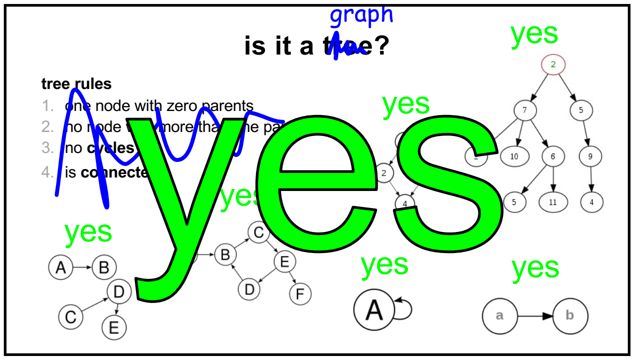
a **directed graph** is a super general linked list

- a **node** in a **graph** has references to any number of other nodes
  - **nodes** (**vertices**) are drawn as circles
  - **references** (**edges**) are drawn as arrows



an **undirected graph** has line segments instead of arrows is it a graph?

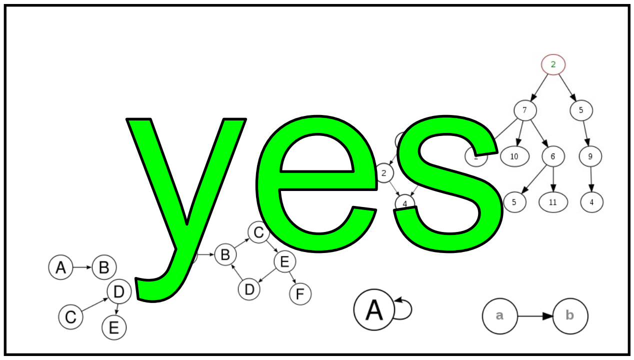
time for everyone's favorite home game...



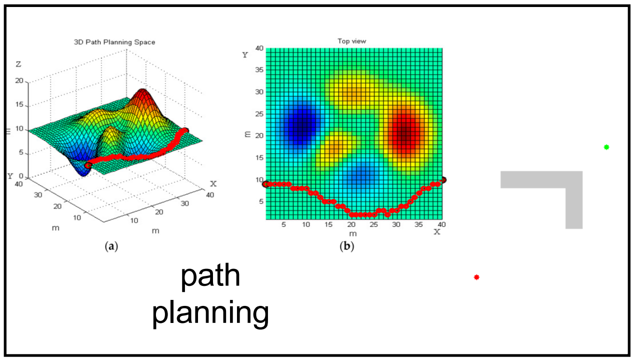
is it a ~~tree~~ graph?

tree rules

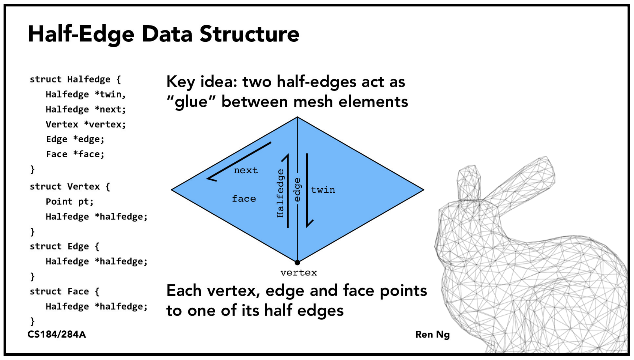
1. ~~one node with zero parents~~
2. ~~no node with more than one parent~~
3. ~~no **cycles**~~
4. ~~is **connected**~~



examples



path planning



Half-Edge Data Structure

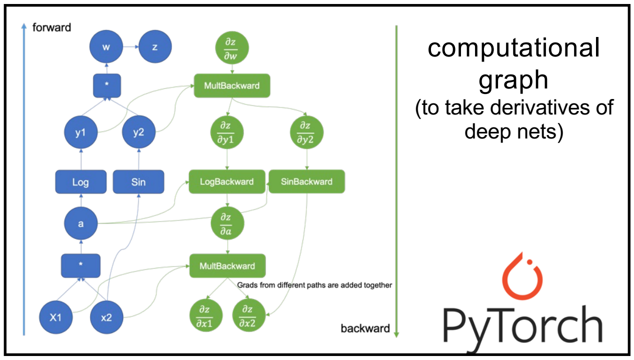
```
struct Halfedge {
   Halfedge *twin,
   Halfedge *next;
   Vertex *vertex;
   Edge *edge;
   Face *face;
}
struct Vertex {
   Point pt;
   Halfedge *halfedge;
}
struct Edge {
   Halfedge *halfedge;
}
struct Face {
   Halfedge *halfedge;
}
```

Key idea: two half-edges act as "glue" between mesh elements

Each vertex, edge and face points to one of its half edges

CS184/284A

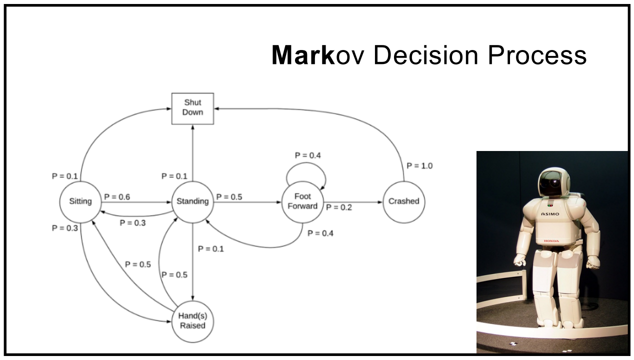
Ren Ng



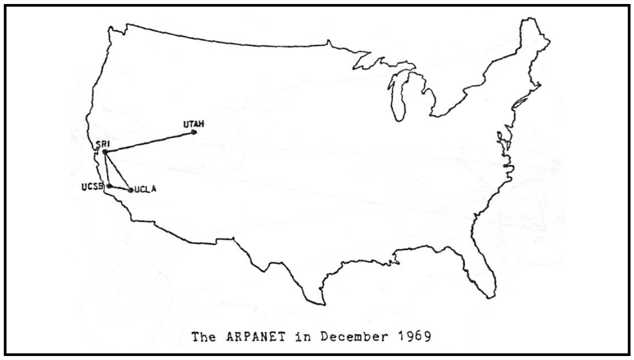
forward
w
z
*
y1
y2
Log
Sin
a
*
X1
x2
$\frac{\partial z}{\partial w}$
MultBackward
$\frac{\partial z}{\partial y1}$
$\frac{\partial z}{\partial y2}$
LogBackward
SinBackward
$\frac{\partial z}{\partial a}$
MultBackward
Grads from different paths are added together
$\frac{\partial z}{\partial x1}$
$\frac{\partial z}{\partial x2}$
backward

computational graph
(to take derivatives of deep nets)

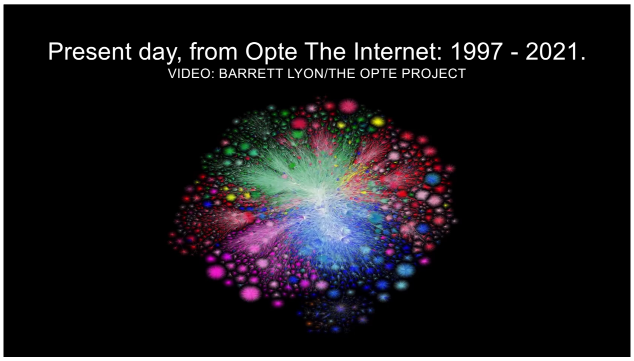
PyTorch



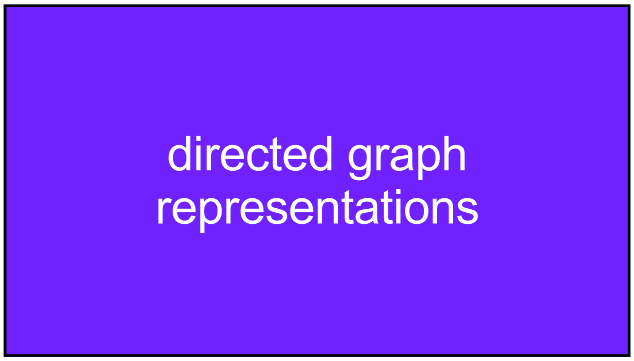
Markov Decision Process



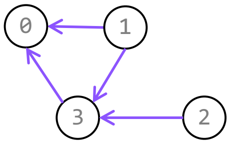
The ARPANET in December 1969



Present day, from Opte The Internet: 1997 - 2021.
VIDEO: BARRETT LYON/THE OPTE PROJECT



directed graph representations

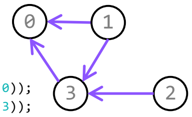


Object-Oriented list of nodes

```
class Graph {
    ArrayList<Node> nodes;
    Graph() { ... }
}
```

```
class Node {
    ArrayList<Node> neighbors;
    Node() { ... }
}
```

```
Graph graph = new Graph();
graph.nodes.add(new Node());
graph.nodes.add(new Node());
graph.nodes.add(new Node());
graph.nodes.add(new Node());
graph.nodes.get(1).neighbors.add(graph.nodes.get(0));
graph.nodes.get(1).neighbors.add(graph.nodes.get(3));
graph.nodes.get(3).neighbors.add(graph.nodes.get(0));
graph.nodes.get(2).neighbors.add(graph.nodes.get(3));
```

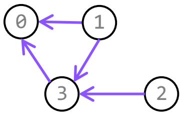


Object-Oriented list of nodes

```
class Graph {
    ArrayList<Node> nodes;
    Graph() { ... }
}
```

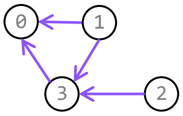
```
class Node {
    ArrayList<Node> neighbors;
    Node() { ... }
}
```

```
Graph graph = new Graph();
graph.addNode();
graph.addNode();
graph.addNode();
graph.addNode();
graph.addEdge(1, 0);
graph.addEdge(1, 3);
graph.addEdge(3, 0);
graph.addEdge(2, 3);
```



list of lists

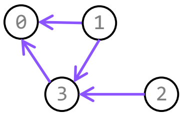
```
ArrayList<ArrayList<Integer>> graph = new ArrayList<>();
graph.add(new ArrayList<>());
graph.add(new ArrayList<>());
graph.add(new ArrayList<>());
graph.add(new ArrayList<>());
graph.get(1).add(0);
graph.get(1).add(3);
graph.get(3).add(0);
graph.get(2).add(3);
```



list of edges

```
class Edge {
    int i;
    int j;
    Edge(int i, int j) { ... }
}
```

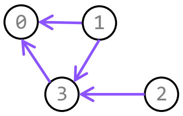
```
ArrayList<Edge> graph = new ArrayList<>();
graph.add(new Edge(1, 0));
graph.add(new Edge(1, 3));
graph.add(new Edge(3, 0));
graph.add(new Edge(2, 3));
```



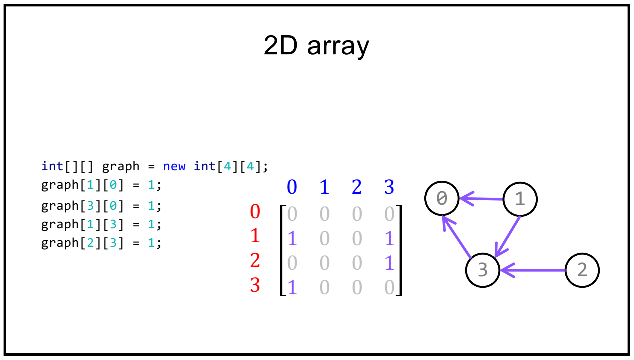
math: adjacency matrix

- an adjacency matrix is a square matrix used to represent a graph
  - a graph with n nodes has corresponding $n{\times}n$ adjacency matrix $\boldsymbol{G}$
  - $G_{i,j} = \begin{cases} 1 & \text{if there is an edge from node i} \rightarrow \text{node j} \\ 0 & \text{otherwise} \end{cases}$

$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 & 0 \end{array}$$



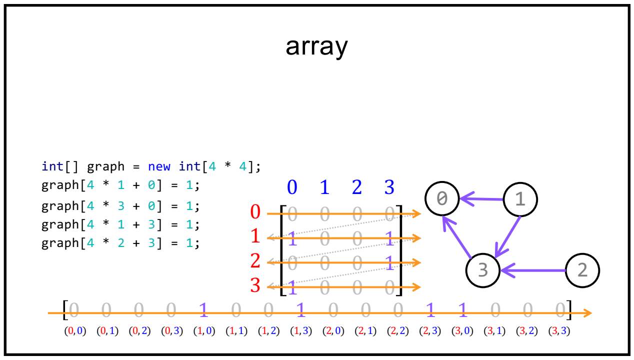
  - **note:** an undirected graph's adjacency matrix is **symmetric**



2D array

```
int[][] graph = new int[4][4];
graph[1][0] = 1;
graph[3][0] = 1;
graph[1][3] = 1;
graph[2][3] = 1;
```

$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 & 0 \end{array}$$



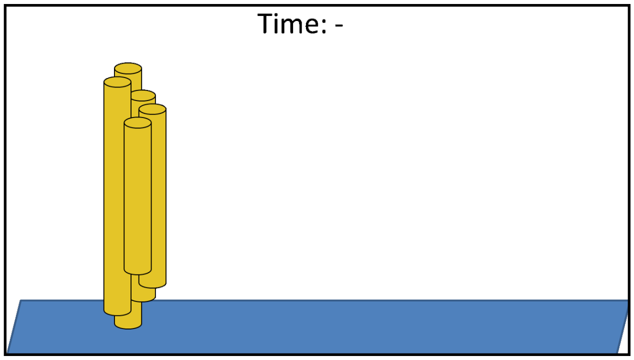
array

```
int[] graph = new int[4 * 4];
graph[4 * 1 + 0] = 1;
graph[4 * 3 + 0] = 1;
graph[4 * 1 + 3] = 1;
graph[4 * 2 + 3] = 1;
```

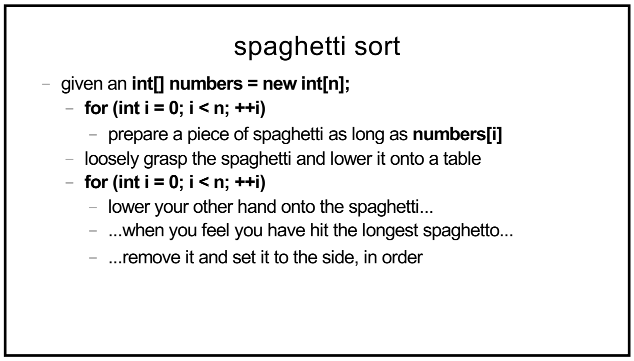
$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 & 0 \end{array}$$

0	0	0	0	1	0	0	1	0	0	0	1	1	0	0	0
(0,0)	(0,1)	(0,2)	(0,3)	(1,0)	(1,1)	(1,2)	(1,3)	(2,0)	(2,1)	(2,2)	(2,3)	(3,0)	(3,1)	(3,2)	(3,3)

spaghetti sort

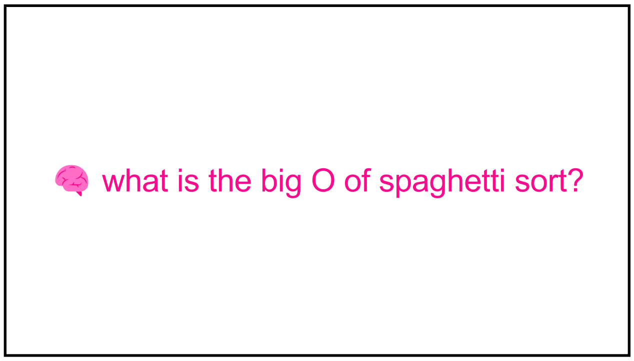


Time: -



spaghetti sort

- given an **int[] numbers = new int[n];**
  - **for (int i = 0; i < n; ++i)**
    - prepare a piece of spaghetti as long as **numbers[i]**
  - loosely grasp the spaghetti and lower it onto a table
  - **for (int i = 0; i < n; ++i)**
    - lower your other hand onto the spaghetti...
    - ...when you feel you have hit the longest spaghetto...
    - ...remove it and set it to the side, in order



🧠 what is the big O of spaghetti sort?

at least O(n * L)

where L is the length of the longest spaghetto

🍝 what?

hint: how do you *know* that your hand has hit the spaghetto?

gamedev update