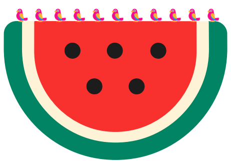
Week11a

- the Fibonacci sequence
- recursion
- dynamic programming



WARMUP

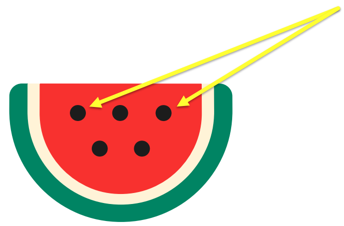
What does this print?

```
PRINT(0.1 + 0.2);
PRINT(SQRT(2.0) * SQRT(2.0));
```

WARMUP

What is F_7?

$F_0 = 0$
$F_1 = 1$
$F_k = F_{k-1} + F_{k-2}$



what are these?

---

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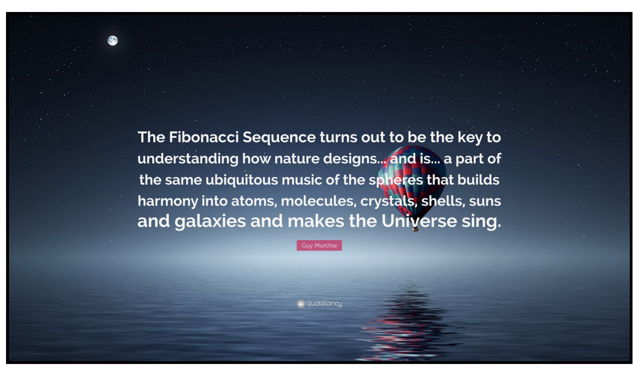
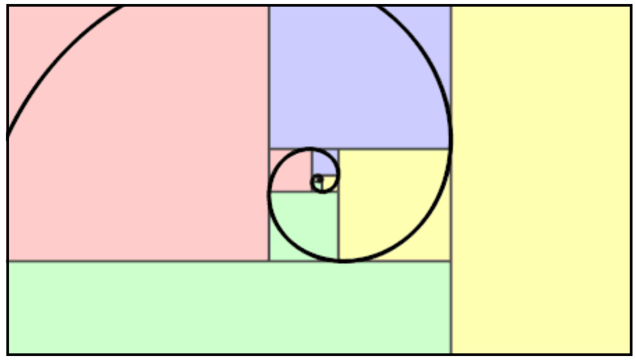
---

the Fibonacci sequence





```
// F_0                       =  0
// F_1                       =  1
// F_2 = F_1 + F_0 = 1 + 0 =  1
// F_3 = F_2 + F_1 = 1 + 1 =  2
// F_4 = F_3 + F_2 = 2 + 1 =  3
// F_5 = F_4 + F_3 = 3 + 2 =  5
// F_6 = F_5 + F_4 = 5 + 3 =  8
// F_7 = F_6 + F_5 = 8 + 5 = 13
// ...
```

recursion

review: recursion basics

recursion

- a **recursive function** is a function that calls itself
  - each call must make progress towards a **base case** (when the function finally returns without calling itself)
    - ✨ when in doubt, try something like zero for your base case

```
class Main {
    static int digitSum(int n) {
        if (n == 0) {
            return 0;
        }
        return digitSum(n / 10) + (n % 10);
    }

    public static void main(String[] arguments) {
        PRINT(digitSum(256)); // 13
    }
}
```

```
static int digitSum(int n) {
    if (n == 0) {
        return 0;
    }
    return digitSum(n / 10) + (n % 10);
}
```

return 0;

return digitSum(0) + 2;

return digitSum(2) + 5;

return digitSum(25) + 6;

int a = digitSum(256);

```
static int digitSum(int n) {
    if (n == 0) {
        return 0;
    }
    return digitSum(n / 10) + (n % 10);
}
```

return 0;

return digitSum(0) + 2;

return digitSum(2) + 5;

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int a = digitSum(256);

```
static int digitSum(int n) {
    if (n == 0) {
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return 0 + 2;

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int a = digitSum(256);

```
static int digitSum(int n) {
    if (n == 0) {
        return 0;
    }
    return digitSum(n / 10) + (n % 10);
}
```

return 2;

return digitSum(2) + 5;

return digitSum(25) + 6;

int a = digitSum(256);

```
static int digitSum(int n) {
    if (n == 0) {
        return 0;
    }
    return digitSum(n / 10) + (n % 10);
}
```

return 2;

return digitSum(2) + 5;

return digitSum(25) + 6;

int a = digitSum(256);

```
static int digitSum(int n) {
    if (n == 0) {
        return 0;
    }
    return digitSum(n / 10) + (n % 10);
}
```

```
return 2 + 5;
return digitSum(25) + 6;
int a = digitSum(256);
```

```
static int digitSum(int n) {
    if (n == 0) {
        return 0;
    }
    return digitSum(n / 10) + (n % 10);
}
```

```
return 7;
return digitSum(25) + 6;
int a = digitSum(256);
```

```
static int digitSum(int n) {
    if (n == 0) {
        return 0;
    }
    return digitSum(n / 10) + (n % 10);
}
```

```
return 7;
return digitSum(25) + 6;
int a = digitSum(256);
```

```
static int digitSum(int n) {
    if (n == 0) {
        return 0;
    }
    return digitSum(n / 10) + (n % 10);
}
```

```
return 7 + 6;
int a = digitSum(256);
```

```
static int digitSum(int n) {
    if (n == 0) {
        return 0;
    }
    return digitSum(n / 10) + (n % 10);
}
```

```
return 13;
int a = digitSum(256);
```

```
static int digitSum(int n) {
    if (n == 0) {
        return 0;
    }
    return digitSum(n / 10) + (n % 10);
}
```

```
return 13;
int a = digitSum(256);
```

```
static int digitSum(int n) {
    if (n == 0) {
        return 0;
    }
    return digitSum(n / 10) + (n % 10);
}
```

```
int a = 13;
```

fin.

⚠️ recursion hazard 1

repeated computation

🐌 **example:** slow very slow fibonnaci

(couldn't we just use a for loop...?)

```
// F_0 = 0
// F_1 = 1
// F_2 = F_1 + F_0 = 1 + 0 = 1
// F_3 = F_2 + F_1 = 1 + 1 = 2
// F_4 = F_3 + F_2 = 2 + 1 = 3
// F_5 = F_4 + F_3 = 3 + 2 = 5
// ...
static long fib(long k) {
    if (k == 0) { return 0; }
    if (k == 1) { return 1; }
    return fib(k - 1) + fib(k - 2);
}
```

```
fib(4)
```

```
(fib(3) + fib(2))
```

```
((fib(2) + fib(1)) + (fib(1) + fib(0)))
```

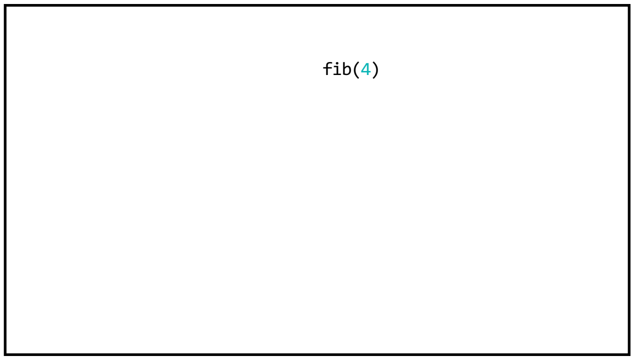
```
(((fib(1) + fib(0)) + fib(1)) + (fib(1) + fib(0)))
```

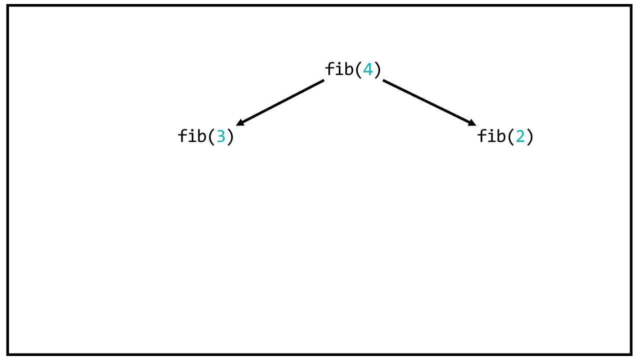
```
(((1 + 0) + 1) + (1 + 0))
```

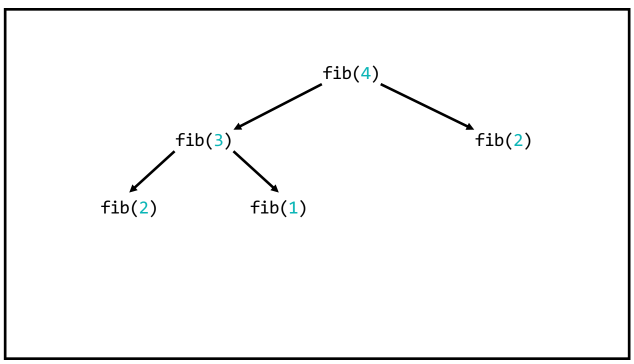
```
((1 + 1) + 1)
```

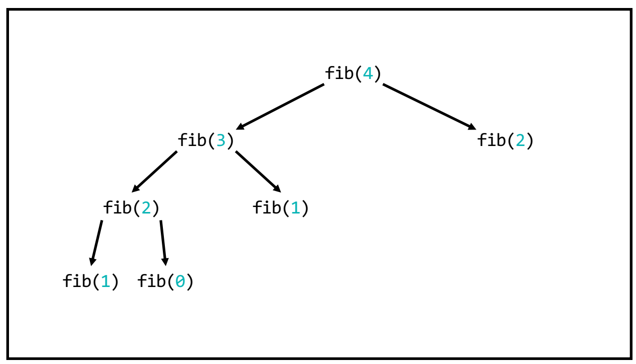
```
(2 + 1)
```

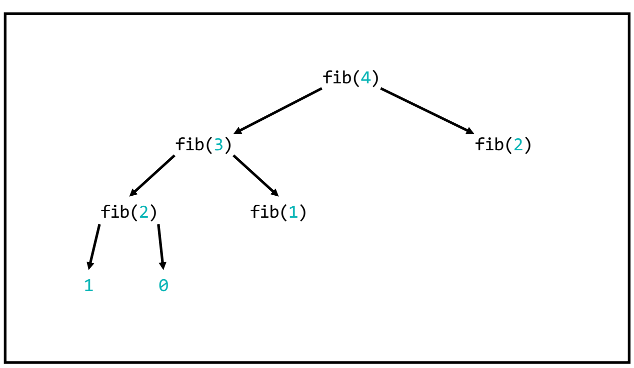
```
3
```

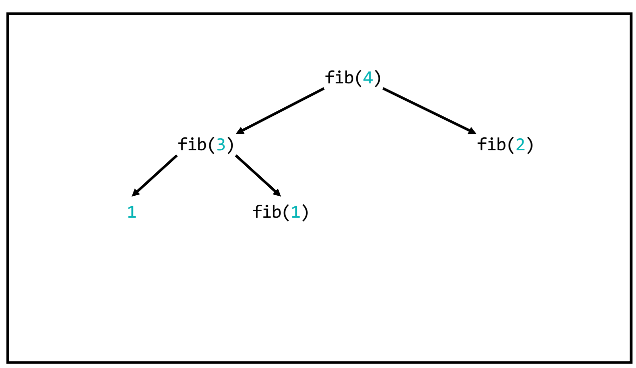

fib(4)

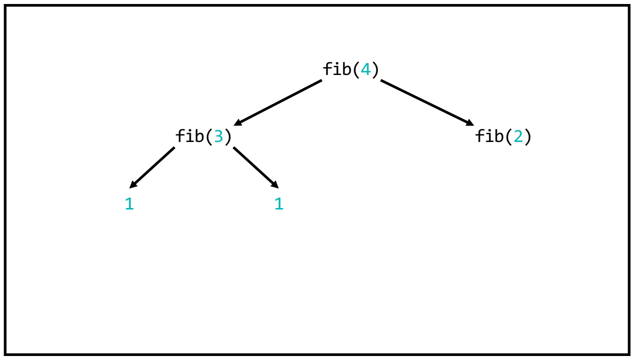

fib(4)
fib(3)
fib(2)

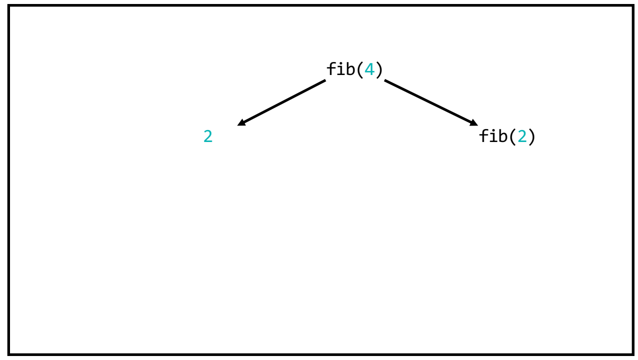

fib(4)
fib(3)
fib(2)
fib(2)
fib(1)

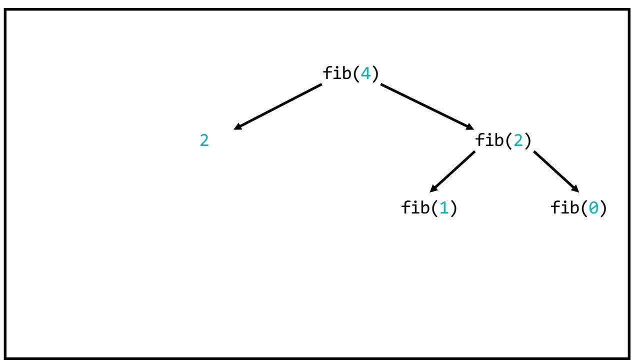

fib(4)
fib(3)
fib(2)
fib(2)
fib(1)
fib(1)
fib(0)

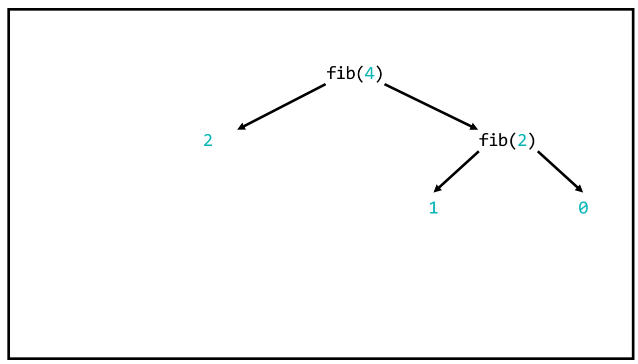

fib(4)
fib(3)
fib(2)
fib(2)
fib(1)
1
0

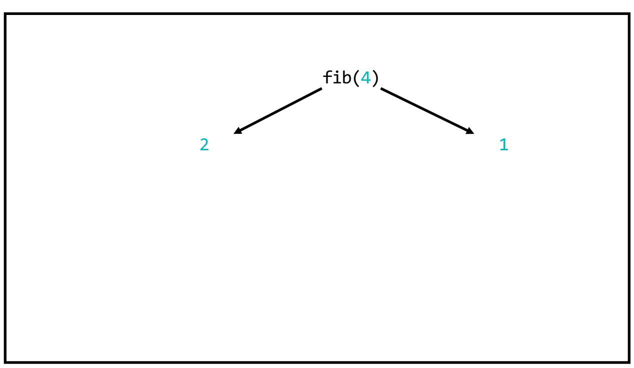

fib(4)
fib(3)
fib(2)
1
fib(1)

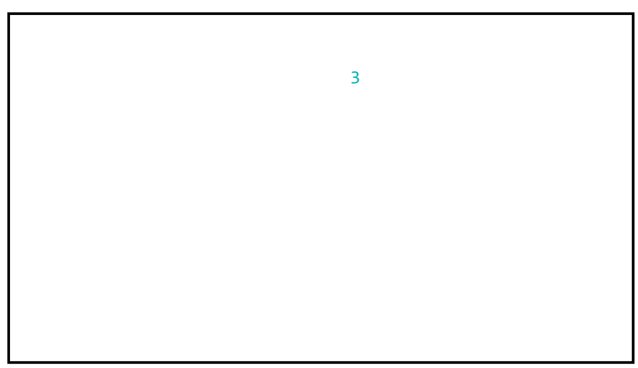

fib(4)
fib(3)
fib(2)
1
1

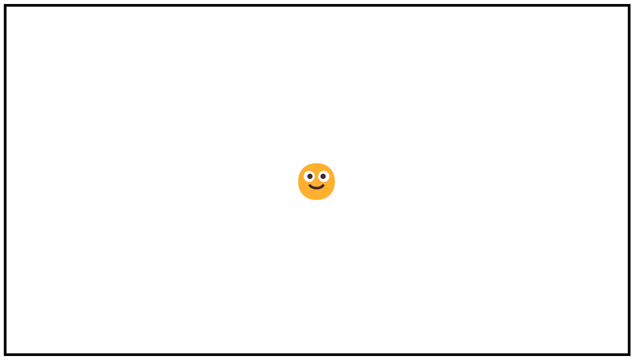

fib(4)
2
fib(2)

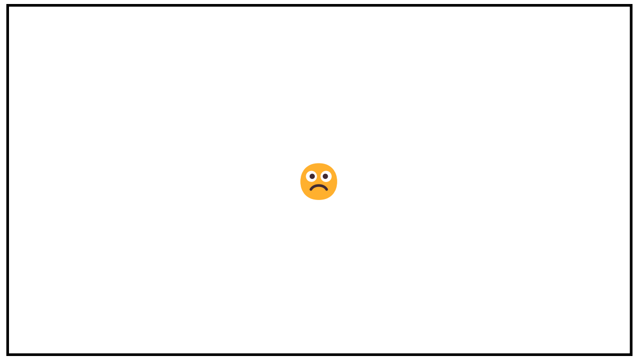

fib(4)
2
fib(2)
fib(1)
fib(0)

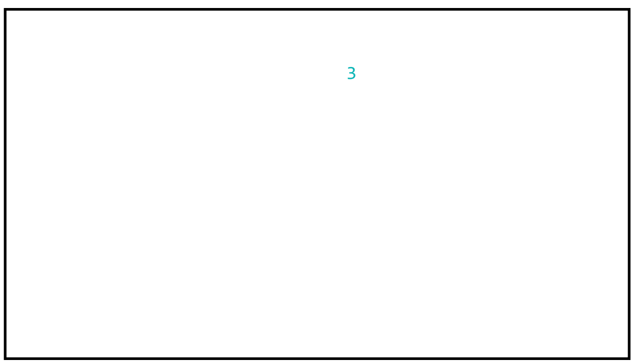

fib(4)
2
fib(2)
1
0

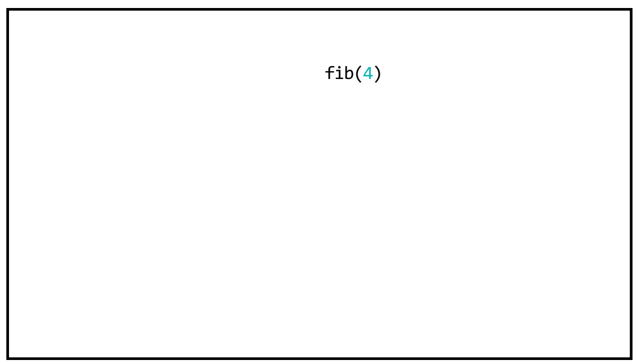

fib(4)
2
1

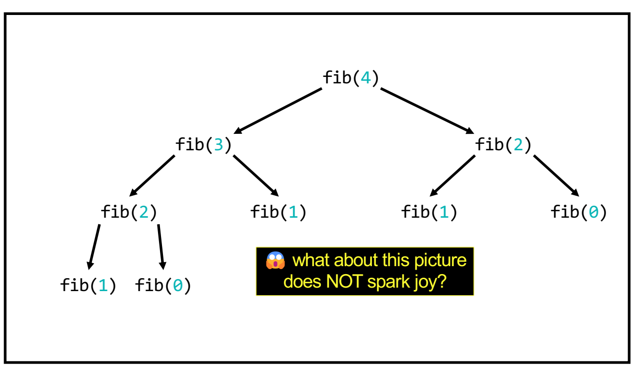

3

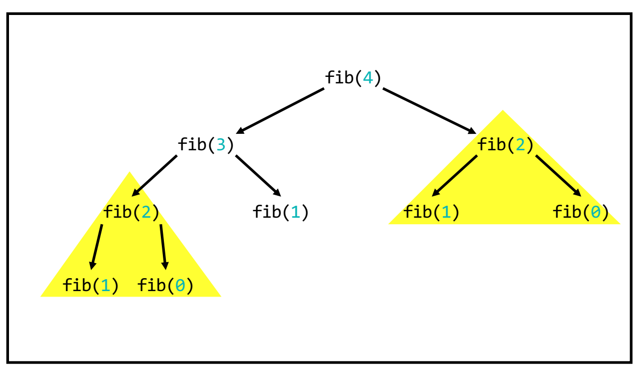





3


fib(4)


fib(4)
fib(3)
fib(2)
fib(2)
fib(1)
fib(1)
fib(0)
fib(1)
fib(0)
what about this picture does NOT spark joy?

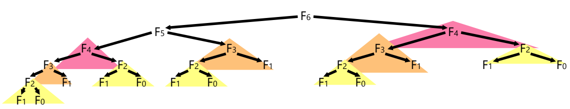

fib(4)
fib(3)
fib(2)
fib(2)
fib(1)
fib(1)
fib(0)
fib(1)
fib(0)

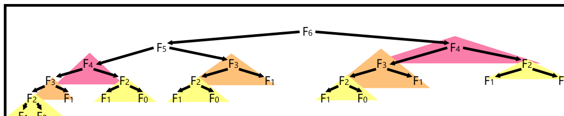
we computed `fib(2)` from scratch two seperate times

😱 we are **repeating computation**!

(what a waste 😞)

and for bigger n...
there is (much) more repetition





🧠 who's that
big O runtime?

exponential 🙀

`[fib(5), fib(36), fib(77) demo]`

⚠️ recursion hazard 2

stack overflow

☠️🐌 dangerous slow $\sum_{i=1}^{n} i = 1 + \cdots + n$

```
// 1 + ... + n
static int sum(int n) {
    if (n == 0) return 0;
    return n + sum(n - 1);
}
```

sum(100000)

sum(99999)
↑
sum(100000)

sum(0)
↑
⋮
↑
sum(99999)
↑
sum(100000)

😱 what about this picture does NOT spark joy?

the height of the callstack is O(n)

😱 big n → **stack overflow**!

😞

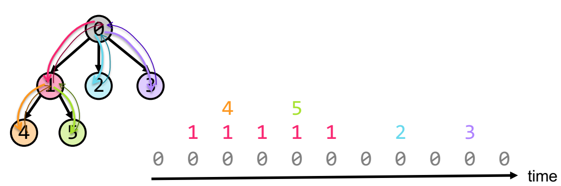
[sum(100000) demo]

🧠 our depth-first binary (search) tree traversals were recursive...

...should we be worried about them overflowing the callstack?

no. (assuming your tree is ~balanced)

callstack height for depth-first **balanced** binary tree traversal is log(n)



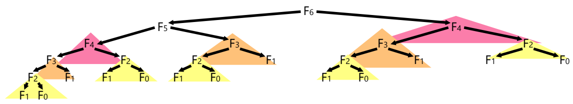
*additionally, some languages/compilers** have "tail-call optimization," which would prevent a stack overflow for `sum(100000)`

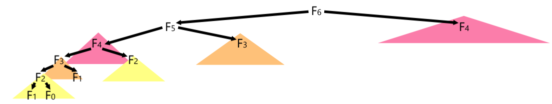
**Java is not one of these languages (as our demo showed)

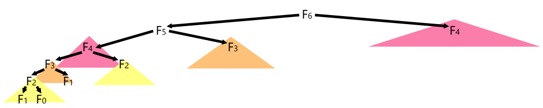
dynamic programming

dynamic programming is when you use the result of previous computation

(this is a squishy definition)







🧠 who's that
big O runtime?

linear 🙄 👍

memoization

memoization means storing the results of previous functions calls, so we don't have to repeat work when the function is called again

memoized fibonacci

```
static HashMap<Long, Long> table = new HashMap<>();

static long memoizedFib(long k) { // NOTE: long is an integer type
                                  //       that can store larger numbers than int
    if (k == 0) { return 0; }
    if (k == 1) { return 1; }

    long Fkm1;
    if (table.containsKey(k - 1)) {
        Fkm1 = table.get(k - 1);
    } else {
        Fkm1 = memoizedFib(k - 1);
        table.put(k - 1, Fkm1);
    }

    long Fkm2;
    if (table.containsKey(k - 2)) {
        Fkm2 = table.get(k - 2);
    } else {
        Fkm2 = memoizedFib(k - 2);
        table.put(k - 2, Fkm2);
    }

    return Fkm1 + Fkm2;
}
```

what's the runtime of each call to memoizedFib(...)?

```
public static void main(...) {
    int n = ...;
    long a = memoizedFib(n);
    long b = memoizedFib(n - 1);
    long c = memoizedFib(n + 1);
}
```

```
int n = ...;
long a = memoizedFib(n);      // O(n)
long b = memoizedFib(n - 1);  // O(1)
long c = memoizedFib(n + 1);  // O(1)
```

closed form fibonacci

Computation by rounding [edit]

Since

$$\frac{|\psi|^n}{\sqrt{5}} < \frac{1}{2}$$

for all $n \geq 0$, the number F_n is the closest integer to

$$\frac{\varphi^n}{\sqrt{5}}.$$

Therefore it can be found by rounding, or in terms of the floor function:

$$F_n = \left\lfloor \frac{\varphi^n}{\sqrt{5}} + \frac{1}{2} \right\rfloor,\ n \geq 0.$$

Or the nearest integer function:

$$F_n = \left[\frac{\varphi^n}{\sqrt{5}} \right],\ n \geq 0.$$

Similarly, if we already know that the number $F > 1$ is a Fibonacci number, we can determine its index within the sequence by

$$n(F) = \left\lfloor \log_\varphi \left(F \cdot \sqrt{5} + \frac{1}{2} \right) \right\rfloor$$

approximate closed-form fibonnaci

```
// NOTE: Because of floating point error, this does not work for big n.
//       (On my computer, returns wrong result for n > 70.)
static long closedFormFib(long n) {
    final double goldenRatio = (1.0 + Math.sqrt(5.0)) / 2.0;
    return Math.round(Math.pow(goldenRatio, n) / Math.sqrt(5.0));
}
```

exponentiation by squaring

note: there is actually a log(n) algorithm using matrices and "exponentiation by squaring"

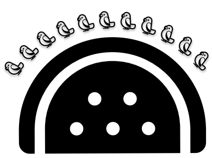
this algorithm does NOT have floating point problems (all numbers are integers)

$$
\begin{array}{cccc}
N=1 & N=2 & N=4 & N=8 \\
\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} & \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} & \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix} & \begin{pmatrix} 34 & 21 \\ 21 & 13 \end{pmatrix} \\
1 & 2 & 4 & 8
\end{array}
$$

Times matrix is multiplied with itself

Week11b

- Fibonacci wrapup
- recursion example



WARMUP

Say n = 64

Your goal is to compute 3^n ("3 to the n-th power") using only the multiplication operator (*)

Give an O(n) algorithm

Give an O(log n) algorithm

recursive fibonacci wrapup

review:
bad bad very bad recursive O(2^n) fibonacci

recursive O(2^n) fibonacci

```
static long fib(long k) {
    if (k == 0) { return 0; }
    if (k == 1) { return 1; }
    return fib(k - 1) + fib(k - 2);
}
```

review:
recursive memoized
O(n) fibonacci

memoized recursive O(n) fibonacci

```
// NOTE: Could also have used an array, with, for example, value 0 meaning "not yet computed."
static HashMap<Integer, Long> table = new HashMap<>();
static long fib(int k) {
  if (k == 0) { return 0; }
  if (k == 1) { return 1; }

  long Fkm1;
  if (table.containsKey(k - 1)) {
    Fkm1 = table.get(k - 1);
  } else {
    Fkm1 = fib(k - 1);
    table.put(k - 1, Fkm1);
  }

  long Fkm2;
  if (table.containsKey(k - 2)) {
    Fkm2 = table.get(k - 2);
  } else {
    Fkm2 = fib(k - 2);
    table.put(k - 2, Fkm2);
  }

  return Fkm1 + Fkm2;
}
```

can we get the best of both worlds?

(easy to read, fast)

[add a helper function]

recursive O(n) fibonnaci

```
static HashMap<Integer, Long> table = new HashMap<>();

static long fibHelper(int k) {
  long result;
  if (table.containsKey(k)) {
    result = table.get(k);
  } else {
    result = fib(k);
    table.put(k, result);
  }
  return result;
}


static long fib(int k) {
  if (k == 0) { return 0; }
  if (k == 1) { return 1; }
  return fibHelper(k - 1) + fibHelper(k - 2);
}
```

alternate fibonacci approaches

iterative (not-recursive) O(n) fibonacci

iterative O(n) fibonnaci

```java
static long fib(int n) {
  long F_i   = 1;
  long F_im1 = 0;
  for (int i = 2; i <= n + 1; ++i) {
    // (F_i, F_im1) <- (F_i + F_im1, F_i)
    long tmp = F_i;
    F_i += F_im1;
    F_im1 = tmp;
  }
  return F_im1;
}
```

TODO make better

```python
def fib(n):
    F_i = 1
    F_im1 = 0
    for i in range(2, n + 2):
        F_i, F_im1 = (F_i + F_im1), F_i
    return F_im1
```

$(F_1, F_0) = (1, 0)$

$(F_2, F_1) \leftarrow (1 + 0, 1) = (1, 1)$

$(F_3, F_2) \leftarrow (1 + 1, 1) = (2, 1)$

$(F_4, F_3) \leftarrow (2 + 1, 2) = (3, 2)$

$(F_5, F_4) \leftarrow (3 + 2, 3) = (5, 3)$

closed form fibonacci

Computation by rounding [edit]

Since

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approximate closed-form fibonacci

```java
// NOTE: Because of floating point error, this does not work for big n.
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static long closedFormFib(long n) {
    final double goldenRatio = (1.0 + Math.sqrt(5.0)) / 2.0;
    return Math.round(Math.pow(goldenRatio, n) / Math.sqrt(5.0));
}
```

O(log n) matrix exponentiation by squaring (oh no)

problem: calculate fib(64)

observation: 64 = 2 * 2 * 2 * 2 * 2 * 2

[matrix multiplication review]

[update rule as a matrix multiplication]

[matrix exponentiation by squaring]

recursion example

subset sum

problem overview

given a finite set of numbers $\{a, b, c, \ldots\}$,
is there **any** subset that sums to target T?

no. --Mark

examples

- is there any subset of { 1, 3, 5 } that sums to 4?
  - yes; { 1, 3 }
- is there any subset of { 1, 3, 5 } that sums to 9?
  - yes; { 1, 3, 5 }
- is there any subset of { 1, 3, 5 } that sums to 0?
  - yes; {}
- is there any subset of { 1, 3, 5 } that sums to 7?
  - no

solution method

given a finite set of numbers $\{a, b, c, \ldots\}$,
is there **any** subset that sums to target T?

any

any

- **for each subset...**
  - if it sums to target...
    - `return true;` 😀
- `return false;`

- **for each subset...**
  - if it sums to target...
    - `return true;` 😀
- `return false;`

🧠 why is this maybe not so easy?

"hard" means "hard for a computer to solve quickly"

---

what are the subsets of $\{ 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$?

∅ | {1} | {2} | {3} | {4} | {5} | {6} | {7} | {8} | {9} | {1, 2} | {1, 3} | {1, 4} | {1, 5} | {1, 6} | {1, 7} | {1, 8} | {1, 9} | {2, 3} | {2, 4} | {2, 5} | {2, 6} | {2, 7} | {2, 8} | {2, 9} | {3, 4} | {3, 5} | {3, 6} | {3, 7} | {3, 8} | {3, 9} | {4, 5} | {4, 6} | {4, 7} | {4, 8} | {4, 9} | {5, 6} | {5, 7} | {5, 8} | {5, 9} | {6, 7} | {6, 8} | {6, 9} | {7, 8} | {7, 9} | {8, 9} | {1, 2, 3} | {1, 2, 4} | {1, 2, 5} | {1, 2, 6} | {1, 2, 7} | {1, 2, 8} | {1, 2, 9} | {1, 3, 4} | {1, 3, 5} | {1, 3, 6} | {1, 3, 7} | {1, 3, 8} | {1, 3, 9} | {1, 4, 5} | {1, 4, 6} | {1, 4, 7} | {1, 4, 8} | {1, 4, 9} | {1, 5, 6} | {1, 5, 7} | {1, 5, 8} | {1, 5, 9} | {1, 6, 7} | {1, 6, 8} | {1, 6, 9} | {1, 7, 8} | {1, 7, 9} | {1, 8, 9} | {2, 3, 4} | {2, 3, 5} | {2, 3, 6} | {2, 3, 7} | {2, 3, 8} | {2, 3, 9} | {2, 4, 5} | {2, 4, 6} | {2, 4, 7} | {2, 4, 8} | {2, 4, 9} | {2, 5, 6} | {2, 5, 7} | {2, 5, 8} | {2, 5, 9} | {2, 6, 7} | {2, 6, 8} | {2, 6, 9} | {2, 7, 8} | {2, 7, 9} | {2, 8, 9} | {3, 4, 5} | {3, 4, 6} | {3, 4, 7} | {3, 4, 8} | {3, 4, 9} | {3, 5, 6} | {3, 5, 7} | {3, 5, 8} | {3, 5, 9} | {3, 6, 7} | {3, 6, 8} | {3, 6, 9} | {3, 7, 8} | {3, 7, 9} | {3, 8, 9} | {4, 5, 6} | {4, 5, 7} | {4, 5, 8} | {4, 5, 9} | {4, 6, 7} | {4, 6, 8} | {4, 6, 9} | {4, 7, 8} | {4, 7, 9} | {4, 8, 9} | {5, 6, 7} | {5, 6, 8} | {5, 6, 9} | {5, 7, 8} | {5, 7, 9} | {5, 8, 9} | {6, 7, 8} | {6, 7, 9} | {6, 8, 9} | {7, 8, 9} | {1, 2, 3, 4} | {1, 2, 3, 5} | {1, 2, 3, 6} | {1, 2, 3, 7} | {1, 2, 3, 8} | {1, 2, 3, 9} | {1, 2, 4, 5} | {1, 2, 4, 6} | {1, 2, 4, 7} | {1, 2, 4, 8} | {1, 2, 4, 9} | {1, 2, 5, 6} | {1, 2, 5, 7} | {1, 2, 5, 8} | {1, 2, 5, 9} | {1, 2, 6, 7} | {1, 2, 6, 8} | {1, 2, 6, 9} | {1, 2, 7, 8} | {1, 2, 7, 9} | {1, 2, 8, 9} | {1, 3, 4, 5} | {1, 3, 4, 6} | {1, 3, 4, 7} | {1, 3, 4, 8} | {1, 3, 4, 9} | {1, 3, 5, 6} | {1, 3, 5, 7} | {1, 3, 5, 8} | {1, 3, 5, 9} | {1, 3, 6, 7} | {1, 3, 6, 8} | {1, 3, 6, 9} | {1, 3, 7, 8} | {1, 3, 7, 9} | {1, 3, 8, 9} | {1, 4, 5, 6} | {1, 4, 5, 7} | {1, 4, 5, 8} | {1, 4, 5, 9} | {1, 4, 6, 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{1, 3, 4, 6, 8} | {1, 3, 4, 6, 9} | {1, 3, 4, 7, 8} | {1, 3, 4, 7, 9} | {1, 3, 4, 8, 9} | {1, 3, 5, 6, 7} | {1, 3, 5, 6, 8} | {1, 3, 5, 6, 9} | {1, 3, 5, 7, 8} | {1, 3, 5, 7, 9} | {1, 3, 5, 8, 9} | {1, 3, 6, 7, 8} | {1, 3, 6, 7, 9} | {1, 3, 6, 8, 9} | {1, 3, 7, 8, 9} | {1, 4, 5, 6, 7} | {1, 4, 5, 6, 8} | {1, 4, 5, 6, 9} | {1, 4, 5, 7, 8} | {1, 4, 5, 7, 9} | {1, 4, 5, 8, 9} | {1, 4, 6, 7, 8} | {1, 4, 6, 7, 9} | {1, 4, 6, 8, 9} | {1, 4, 7, 8, 9} | {1, 5, 6, 7, 8} | {1, 5, 6, 7, 9} | {1, 5, 6, 8, 9} | {1, 5, 7, 8, 9} | {1, 6, 7, 8, 9} | {2, 3, 4, 5, 6} | {2, 3, 4, 5, 7} | {2, 3, 4, 5, 8} | {2, 3, 4, 5, 9} | {2, 3, 4, 6, 7} | {2, 3, 4, 6, 8} | {2, 3, 4, 6, 9} | {2, 3, 4, 7, 8} | {2, 3, 4, 7, 9} | {2, 3, 4, 8, 9} | {2, 3, 5, 6, 7} | {2, 3, 5, 6, 8} | {2, 3, 5, 6, 9} | {2, 3, 5, 7, 8} | {2, 3, 5, 7, 9} | {2, 3, 5, 8, 9} | {2, 3, 6, 7, 8} | {2, 3, 6, 7, 9} | {2, 3, 6, 8, 9} | {2, 3, 7, 8, 9} | {2, 4, 5, 6, 7} | {2, 4, 5, 6, 8} | {2, 4, 5, 6, 9} | {2, 4, 5, 7, 8} | {2, 4, 5, 7, 9} | {2, 4, 5, 8, 9} | {2, 4, 6, 7, 8} | {2, 4, 6, 7, 9} | {2, 4, 6, 8, 9} | {2, 4, 7, 8, 9} | {2, 5, 6, 7, 8} | {2, 5, 6, 7, 9} | {2, 5, 6, 8, 9} | {2, 5, 7, 8, 9} | {2, 6, 7, 8, 9} | {3, 4, 5, 6, 7} | {3, 4, 5, 6, 8} | {3, 4, 5, 6, 9} | {3, 4, 5, 7, 8} | {3, 4, 5, 7, 9} | {3, 4, 5, 8, 9} | {3, 4, 6, 7, 8} | {3, 4, 6, 7, 9} | {3, 4, 6, 8, 9} | {3, 4, 7, 8, 9} | {3, 5, 6, 7, 8} | {3, 5, 6, 7, 9} | {3, 5, 6, 8, 9} | {3, 5, 7, 8, 9} | {3, 6, 7, 8, 9} | {4, 5, 6, 7, 8} | {4, 5, 6, 7, 9} | {4, 5, 6, 8, 9} | {4, 5, 7, 8, 9} | {4, 6, 7, 8, 9} | {5, 6, 7, 8, 9} | {1, 2, 3, 4, 5, 6} | {1, 2, 3, 4, 5, 7} | {1, 2, 3, 4, 5, 8} | {1, 2, 3, 4, 5, 9} | {1, 2, 3, 4, 6, 7} | {1, 2, 3, 4, 6, 8} | {1, 2, 3, 4, 6, 9} | {1, 2, 3, 4, 7, 8} | {1, 2, 3, 4, 7, 9} | {1, 2, 3, 4, 8, 9} | {1, 2, 3, 5, 6, 7} | {1, 2, 3, 5, 6, 8} | {1, 2, 3, 5, 6, 9} | {1, 2, 3, 5, 7, 8} | {1, 2, 3, 5, 7, 9} | {1, 2, 3, 5, 8, 9} | {1, 2, 3, 6, 7, 8} | {1, 2, 3, 6, 7, 9} | {1, 2, 3, 6, 8, 9} | {1, 2, 3, 7, 8, 9} | {1, 2, 4, 5, 6, 7} | {1, 2, 4, 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---

there are 2^9 of them

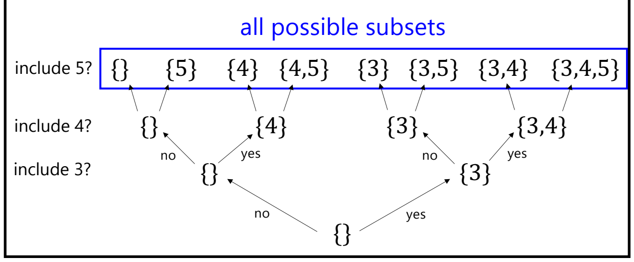
🧠 why?

each of the 9 elements is either included or not included (excluded) in the subset

9 include/exclude decisions => 2^9

what are the subsets of $\{3, 4, 5\}$?

what are the subsets of $\{3, 4, 5\}$?



hint

Question: "can any subset of $\{a, b, c, \ldots\}$ sum to T?"

key insight

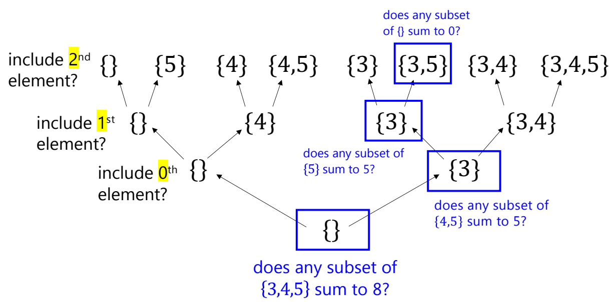
Considering just a, we have **two cases:**

1) exclude a

in this case, **Equivalent Question:** "can any subset of $\{b, c, \ldots\}$ sum to T?"

2) include a

in this case, **Equivalent Question:** "can any subset of $\{b, c, \ldots\}$ sum to $T - a$?"



okay cool good luck

final project

final project

You may **do your final project on whatever you like,** provided you can **answer the following questions**.

1. What is the **title** of my project?
2. What **data structures** will I use?
   Note: Arrays count.
3. **What** is the game/app that I am proposing?
   What does it do?
   How does it *feel*?
4. Will the viewer/player **interact** with my project?
   How so?
5. Does Jim think my project is **doable**?
   What is my fallback plan if my project ends up being harder than I expect?
   What extensions can I do if my project ends up being easier than I expect?
6. What is the very **first thing I will implement**?
   (Drawing "the data" is usually a good first step.)

do your final project on whatever you like
answer the following questions

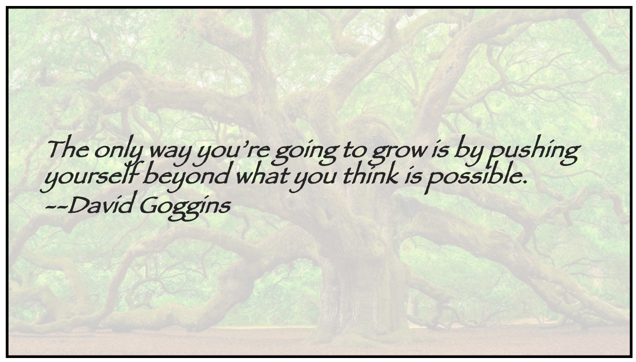
1. **title**
2. **data structures**
3. **What**
4. **interact**
5. **doable?**
6. **first thing I will implement**

example

- Woo!-doku
- 2D array to represent the board.
- A colorful sudoku board, that does a happy dance when you solve it.
- Click to select cells. Type numbers on the keyboard to fill in numbers.
- Yes! And you can write a sudoku solver or automatic board generation if you have extra time!
- Store a board I found on the internet as a 2D array (-1's for empty cells) and draw it to the Terminal using System.out.println.

how are we feeling?

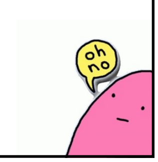
why am i making you make a thing



also you were warned 😳

this course will give you the power to make things!

what do you want to make?



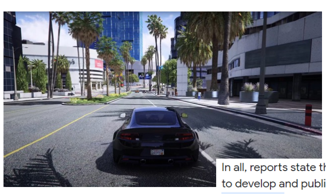
how to
make a thing

how not to
make a thing

note: this advice is like...just advice
feel free to ignore (at your own peril) 😛

project selection

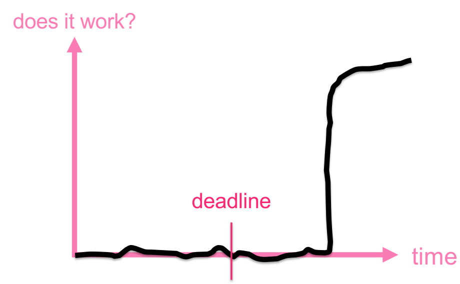
⚠️ picking something really, really hard



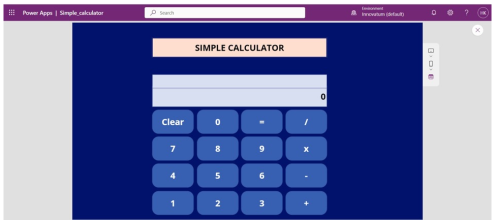
In all, reports state that Grand Theft Auto 5 cost a whopping $265 million to develop and publish. This includes the game's core budget of around **$140 million**, plus some staggering marketing costs befitting of such a release. Nov 9, 2023

Vintage is The New Old
https://www.vintageisthenewold.com › game-pedia › wh...
What was GTA 5 original budget? - Vintage is The New Old

⚠️ picking something with a spooky progress curve



⚠️ picking something you aren't really interested in
(or can't really be **extended**)



note: though if you love calculators, go for it!

---

who (doesn't) have a final project idea?

discuss amongst each other

---

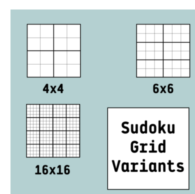
focus

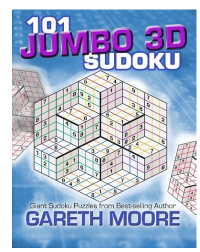
(just make sudoku)

⚠️ solving a more general problem than you need to

don't support arbitrary board sizes!

just make sudoku!



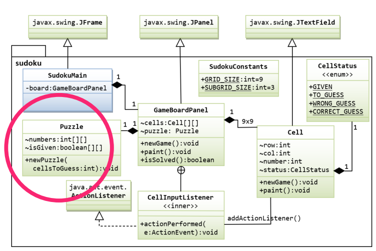


what?!

⚠️ writing more general code than you need to

don't support multiple different kinds of puzzles

just make sudoku!

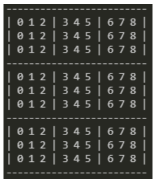


⚠️ doing the fun part first

don't make a fun animated menu screen

just make sudoku!

⚠️ skipping development steps



consider making a board print to the terminal before trying to get it working in Cow.java

consider just filling it with nonsense before filling it with a real board

consider hardcoding a starting board by hand before implementing automatic board generation

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then again, sometimes you just gotta go for it.

YOLO!

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who (doesn't) know what they're going to implement first?

discuss amongst each other

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final thoughts

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final thoughts

- don't be afraid to write code
- don't be afraid to delete code
- don't be (too) afraid to fail
- you will be graded primarily on effort

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how are we feeling?