# 14 2SLS. Control Function. Endogeneity and overidentification tests.

## 14.1 Under, just and overidentification

Consider again the linear regression model, with $\vec{x}_{1i}$ exogenous and $\vec{x}_{2i}$ endogenous.

$$y_i = \beta_0 + x'_{1i}\beta_1 + x'_{2i}\beta_2 + u_i$$

Then take instrument:

$$w_i = \begin{pmatrix} x_{1i} \\ z_i \end{pmatrix}$$

with $x_{1i}$ instrumenting for themselves (included exogenous variables) and $z_i$ instrumenting for $x_{2i}$ (excluded exogenous variables).

If $w_i$ $l$-dimensional and $x_i$ $k$-dimensional:

$$\underbrace{E[w_i y_i]}_{l\times 1} = \underbrace{E[w_i x'_i]}_{l\times k} \underbrace{\beta}_{k\times 1}$$

- If $l < k$, then we have **underidentification**
- If $l = k$, then we have **just identification**
- If $l > k$, then we have **overidentification**

The relevance condition, $E[w_i x'_i]$ full column rank, rules out underidentification. This is because now l rows will be fewer than k columns, and since column rank = row rank, we must have deficient column rank.
If $l < k$ we have more equations than unknowns and $E[w_i x'_i]$ is no longer invertible. We could throw away extra variables but better instead to use 2SLS, since we want to extract as much exogenous variation from our endogenous variables as possible.

## 14.2 2SLS

For now assume $E[\varepsilon_i|w_i] = 0$. Then:

$$0 = E[\varepsilon_i|w_i] = E[y_i - x'_i\beta|w_i] = E[y_i|w_i] - E[x'_i|w_i]\beta$$

$$\Rightarrow E[y_i|w_i] = E[x'_i|w_i]\beta$$

Suppose we also know

$$E[x'_i|w_i] = w'_i\pi$$

Then we have:

$$E[y_i|w_i] = (w'_i\pi)\beta$$

This suggest the following procedure:

**Definition 14.2.1**

**2SLS**

Stage 1:

- Regress $X_{n\times k}$ on $W_{n\times l}$ to get $\hat{\pi} = (W'W)^{-1}W'X$
- Use the results to form $\hat{X} = W\hat{\pi}$

Note: $\hat{X} = W\hat{\pi} = W(W'W)^{-1}W'X = P_W X$
For the exogenous variables columns in $\hat{X}$ this will correspond exactly to the original values, but for the endogenous variables columns, they will be formed as a linear combination of both the relevant instruments and exogenous variables.

Stage 2:

Regress $Y_{n\times 1}$ on $\hat{X}_{n\times k}$ to find:

$$\begin{aligned}\hat{\beta}_{2SLS} &= (\hat{X}'\hat{X})^{-1}\hat{X}'Y = (X'P_W'P_WX)^{-1}X'P_W'Y \\ &= (X'P_WX)^{-1}X'P_WY\end{aligned}$$

Consider the following **IV assumptions** for the model $y_i = x_i'\beta + \varepsilon_i$:

- (IV0) $y_i, x_i, w_i$ is an i.i.d sequence
- (IV1) $E[w_iw_i'] < \infty$ non-singular; $E[w_ix_i']$ has full column rank (relevance)
- (IV2) $E[\varepsilon_i|w_i] = 0$ $(\Rightarrow)$ (IV2') $E(w_i\varepsilon_i) = 0$ (exogeneity)
- (IV3) $E[\varepsilon_i^2|w_i] = \sigma^2$ (homoskedasticity) or (IV3') $V = Var(w_i\varepsilon_i)$ is finite non singular (Under IV(3): $V = E[w_iw_i'\varepsilon_i^2] - 0 = E[E[w_iw_i'\varepsilon_i^2|w_i]] = \sigma^2 E[w_iw_i']$)

**Theorem 14.2.1. 2SLS consistency**

Under IV(0) IV(1) IV(2')

$$\hat{\beta}_{2SLS} \xrightarrow{p} \beta$$

**Proof.**

$$\begin{aligned}\hat{\beta}_{2SLS} &= (\hat{X}'\hat{X})^{-1}(\hat{X}Y) = (X'P_WX)^{-1}X'P_WY \\ &= \beta + (X'P_WX)^{-1}X'P_W\varepsilon\end{aligned}$$

$$\hat{\beta}_{2SLS} - \beta = [X'W(W'W)^{-1}W'X]^{-1}X'W(W'W)^{-1}W'\varepsilon$$

$$= \left[\frac{1}{n}\sum x_iw_i'(\frac{1}{n}\sum w_iw_i')^{-1}\frac{1}{n}\sum w_ix_i'\right]^{-1}\frac{1}{n}\sum x_iw_i'(\frac{1}{n}\sum w_iw_i')^{-1}(\frac{1}{n}\sum w_i\varepsilon_i)$$

$$\xrightarrow{p} [E(x_iw_i')E(w_iw_i')^{-1}E(w_ix_i')]^{-1}E(x_iw_i')E(w_iw_i')^{-1}E(w_i\varepsilon_i)$$

By IV(2'), $E(w_i\varepsilon_i) = 0$ and by IV(1) $E(w_iw_i')$ is non-singular to a finite constant matrix (also assume $E(x_iw_i') < \infty$). Thus

$$\hat{\beta}_{2SLS} - \beta \xrightarrow{p} 0$$

□

In general $dimW \neq dimX$. In the case where they do: $\hat{\beta}_{2SLS} \equiv \hat{\beta}_{IV}$, since $W'X$ now invertible. The 2SLS procedure ensures that $dim\hat{X} = dimX$, so that $\hat{\beta}_{2SLS} \equiv \hat{\beta}_{IV}$, using $\hat{X}$ as an instrument. Explicitly: $(X'P_W X)^{-1}X'P_W Y = (X'P_W' X)^{-1}X'P_W' Y = (\hat{X}'X)^{-1}(\hat{X}'Y) = \hat{\beta}_{IV}$

**Theorem 14.2.2. 2SLS asymptotic distribution**

Under IV0-1-2'-3':

$$\sqrt{n}(\hat{\beta}_{2SLS} - \beta) \xrightarrow{d} N(0, (D'C^{-1}D)^{-1}D'C^{-1}VC^{-1}D(D'C^{-1}D)^{-1})$$

where $V = Var(w_i\varepsilon_i)$, $C = E[w_i w_i']$ and $D = E[w_i x_i']$.

**Proof.**

$$\sqrt{n}(\hat{\beta}_{2SLS} - \beta) =$$

$$\left[\frac{1}{n}\sum x_i w_i'(\frac{1}{n}\sum w_i w_i')^{-1}\frac{1}{n}\sum w_i x_i'\right]^{-1}\frac{1}{n}\sum x_i w_i'(\frac{1}{n}\sum w_i w_i')^{-1}(\frac{1}{\sqrt{n}}\sum w_i\varepsilon_i)$$

By Lindeberg-Levy CLT:

$$\frac{1}{\sqrt{n}}\sum w_i\varepsilon_i \xrightarrow{d} N(0, V)$$

By Slutsky's theorem:

$$\xrightarrow{d} [D'C^{-1}D]^{-1}D'C^{-1}N(0, V)$$

$$= N(0, (D'C^{-1}D)^{-1}D'C^{-1}VC^{-1}D(D'C^{-1}D)^{-1})$$

Under (IV3) (homoskedasticity):

$$V = Var(w_i\varepsilon_i) = E[w_i w_i'\varepsilon_i^2] - 0 = \sigma^2 E[w_i w_i'] = \sigma^2 C$$

Thus much of the asymptotic variance cancels, leaving

$$\sqrt{n}(\hat{\beta}_{2SLS} - \beta) \xrightarrow{d} N(0, \sigma^2(D'C^{-1}D)^{-1})$$

□

**Note:-**

In general for two full column rank conformable matrices $A, B$:
We have $AB$ full column rank.

Proof: Suppose $AB$ not full column rank.
Then $\exists\ x \neq 0$ such that $ABx = 0$ (by the rank-nullity theorem).
$\Rightarrow Bx \neq 0$ as $B$ full rank implies its null space is only $\{0\}$.
$\Rightarrow A(Bx) \neq 0$ as $A$ also full rank with only trivial null space.
Contradiction

We apply this proof to argue $D'C^{-1}D$ is full column rank, and hence invertible.

### 14.2.1 Linear Hypothesis Testing with $\beta_{2SLS}$

We can estimate the asymptotic variance of $\sqrt{n}(\hat{\beta}_{2SLS} - \beta)$ by:

$$\hat{V} = \hat{\sigma}^2(\tfrac{1}{n}\hat{X}'\hat{X})^{-1}$$

where $\hat{\sigma}^2 = \frac{1}{n}\hat{\varepsilon}'\hat{\varepsilon}$ and $\hat{\varepsilon} = Y - \hat{X}'\hat{\beta}_{2SLS}$

Homoskedasticity or robust variance estimates of $\hat{\beta}_{2SLS}$ can be used to form F-statistics for testing linear hypotheses in the usual way. Asymptotically, such F-statistics would be distributed as $\chi^2(p)/p$, where $p$ is the number of restrictions. However, finite sample distribution of the F-statistics would not be $F(p, n-k)$ even if $\varepsilon_i$ is normally distributed.

Asymptotically the Wald statistic for testing $H_0 : R\beta = r$ is:

$$W = (R\hat{\beta}_{2SLS} - r)'[R\hat{V}_{2SLS}R']^{-1}(R\hat{\beta}_{2SLS} - r) \xrightarrow{d} \chi^2(p)$$

where $p$ is the number of restrictions.

**Note:-**

Under (IV3') (hetereoskedasticity) we can use White's estimate as in earlier discussions:

$$\hat{V}_{het} = \frac{1}{n}\sum_{i=1}^{n} \hat{\varepsilon}_i^2 w_i w_i'$$

## 14.3 Control function approach

This is an alternative approach to 2SLS, which is useful when we have multiple endogenous variables.

Consider again the model:

$$y_i = x_{1i}'\beta_1 + x_{2i}'\beta_2 + \varepsilon_i$$

where $x_{1i}$ is exogenous and $x_{2i}$ is endogenous.

Instead of extracting the *exogenous* part $w_i'\pi$ of $x_i$ to use in the second stage, we could instead extract the **endogenous part** of $x_i$ (the control function) and add it to the regression as an additional regressor.

**Theorem 14.3.1.** The two approaches are equivalent in a linear model (but not in non-linear) $\hat{\beta}_{CF} \equiv \hat{\beta}_{2SLS}$

**Proof.** The exogenous part $w_i'\pi$ of $x_i$ is simply the best linear predictor of $x_i$ given $w_i$.

The first stage regression:

$$x_i' = w_i'\pi + u_i', \text{ where } \pi \text{ is } l \times k$$

is called a *reduced form* regression, because it does not have any structural interpretation. We just want to predict $x_i$ by a linear function of $w_i$ in the best possible way (thus exogeneity is not required). Recall $w_i$ contains components of both included exogenous variables $x_{1i}$ and excluded exogenous variables $z_i$.

Thus we partition the reduced form equations into:

$$x_{1i}' = x_{1i}'\pi_{11} + z_i'\pi_{12} + u_{1i}'$$

$$x_{2i}' = x_{1i}'\pi_{21} + z_i'\pi_{22} + u_{2i}'$$

where $\pi_{ij}$ is a $k_j \times k_i$ matrix.

Of course the BLP of $x_{1i}$ given $x_{1i}$ and $z_{1i}$ is just $x_{1i}$ so the first of the above equations is trivial $x_{1i}' = x_{1i}'$ For the second equation we drop the first subscript and rewrite as:

$$x_{2i}' = x_{1i}'\pi_1 + z_i'\pi_2 + u_i'$$

In 2SLS this regression would be estimated, obtain $\hat{x}'_{2i}$, form $\hat{x}_i$ by combining $x_{1i}$, with $\hat{x}_{2i}$ and proceeding to second stage.

But note $x_{2i}$ can only be endogenous if $E(u_i\varepsilon_i) \neq 0$, that is, the error of the first stage $u_i$ is correlated with the structural error $\varepsilon_i$. Alternatively, note $x_2 i$ can only be endogenous if $E(\vec{u}_i\varepsilon_i) \neq \vec{0}$*

That is, the error of the first stage regression, $u_i$ is correlated with the structural error $\varepsilon_i$. The error $u_i$ has *soaked up* the endogeneity in $x_{2i}$ thus adding it to the structural equation would control for the endogeneity and so get consistent estimates for the other structural parameters.

Consdier the BLP of $\varepsilon_i$ given $u_i$:

$$\varepsilon_i = u_i'\alpha + e_i$$

By definition the error of the BLP is uncorrelated to the dependent $\varepsilon_i$, else it would have been taken into account in the regression.

Substituting this into the strucutural equation, we obtain

$$y_i = x_{1i}'\beta_1 + x_{2i}'\beta_2 + u_i'\alpha + e_i$$

where:

$$E(u_ie_i) = 0$$

$$E(x_{1i}e_i) = E(x_{1i}(\varepsilon_i - u_i'\alpha)) = 0$$

$$E(x_{2i}e_i) = E((\pi_1'x_{1i} + \pi_2'z_i + u_i)e_i) = E(\pi_2'z_ie_i) = \pi_2E(z_i(\varepsilon - u_i'\alpha)) = 0$$

Thus OLS2' satisfied and the OLS estimates of $\beta_1, \beta_2$, and $\alpha$ should be consistent. But we do not observe $u_i$ so it must first be estimated from the first stage regression before insertion.

Let $\hat{U}$ be the matrix with rows $\hat{u}_i'$. Then by the partitioned regression formula (FW - theorem):

$$\hat{\beta}_{CF} \equiv (X'M_{\hat{U}}X)^{-1}X'M_{\hat{U}}X$$

But $\hat{U} = M_WX_2$ so that:

$$M_{\hat{U}} = I - \hat{U}(\hat{U}'\hat{U})^{-1}\hat{U}' = I - M_WX_2(X_2'M_WX_2)^{-1}X_2'M_W$$

Since $X_1$ is a part of $W$, $M_WX_1 = 0$, and

$$M_{\hat{U}}X_1 = X_1 = P_WX_1$$

Further

$$M_{\hat{U}}X_2 = X_2 - M_WX_2(X_2'M_WX_2)^{-1}X_2'M_WX_2 = P_WX_2$$

Therefore

$$M_{\hat{U}}X = P_WX$$

and so

$$\hat{\beta}_{CF} \equiv (X'M_{\hat{U}}X)^{-1}X'M_{\hat{U}}Y = (X'P_WX)^{-1}X'P_WY = \hat{\beta}_{2SLS}$$

*$E(x_{2i}\varepsilon_i) \neq \vec{0} \Rightarrow E[(w_i'\pi + u_i')'\varepsilon] \neq 0 \Rightarrow \pi\cancel{E[w_i\varepsilon]} + E[u_i\varepsilon_i] \neq 0$

□

## 14.4 Endogeneity and Overidentification test

*Endogeneity test:* If $x_{2i}$ is not endogenous, then OLS is efficient (BLUE) and 2SLS is not.

Test

$$H_0 : E(x_{2i}\varepsilon_i) = 0 \text{ against } H_1 : E(x_{2i}\varepsilon_i) \neq 0$$

Recall the CF regression:

$$y_i = x'_{1i}\beta_1 + x'_{2i}\beta_2 + u'_i\alpha + e_i$$

where

$$\alpha = E(u_iu'_i)^{-1}E(u_i\varepsilon_i)(\text{ the coefficient of BLP} for \varepsilon_i given u_i)$$

We have $E(x_{2i}\varepsilon_i \neq 0)$ if and only if $E(u_i\varepsilon_i) \neq 0$. Therefore hypothesis test equivalent to:

$$H_0 : \alpha = 0 \text{ against } H_1 : \alpha \neq 0$$

Therefore a natural test would be the Wald statistic for testing linear restrictions $\alpha = 0$ in the control function regression, with $u_i$ replaced with $\hat{u}_i$. It turns out this replacement des not affect the asymptotic distribution of the test stastistic under the null, and remains $\chi^2(k_2)$ where $k_2$ is the $dim(\alpha) = dim(x_{2i})$ This follows from a general result on the asymptotic distribution of the OLS estimates of regression coefficients with 'generated' regressions (i.e. the hats consistently estimating the true) H(12-26,12-27) In stata this occurs after estat endoggy WUFF WUFF after ivregress. Het robust s.e. then reported as 'robust regression F' otherwise if default daniel homoskedasticity then reported as 'Wu-Hausman F'

*Overidentification test:* With $l > k$ (instruments ¿ endoggy regressors) we can test the hypothesis that instruments are exogenous, that is

$$H_0 : E(w_i\varepsilon_i) = 0$$

Let us assume the homoskedasticity, so that $E(\varepsilon_i^2|w_i) = \sigma^2$. Then consider a reduced form regression:

$$\varepsilon_i = w'_i\alpha + e_i$$

, where

$$\alpha = (E(w_iw'_i))^{-1}E(w_i\varepsilon_i)$$

We see that $E(w_i\varepsilon_i) \neq 0$ if and only if $\alpha \neq 0$. We cannot regress $\varepsilon_i$ on $w_i$ because we do not observe $\varepsilon_i$. But we can try to replace $\varepsilon_i$ with $\hat{\varepsilon}_i$, (the residuals from the 2SLS esimtate of $\beta$ NOTE this is not the same as the second stage residuals).

Sargan proposed to use $nR^2$ from this regression as the test stat for $H_0$ vs $H_1$:

$$S = nR^2 = n\frac{SSE}{SST} = n\frac{\hat{\varepsilon}'W(W'W)^{-1}W'\hat{\varepsilon}}{\hat{\varepsilon}'\hat{\varepsilon}}$$

Asymptotic Distribution of S: Note S is invariant wrt transformations $W \to W \times A$ where $A$ is any invertible matrix. Therefore wlog we assume $W$ rotated and scaled so that $W(w_iw'_i) = I_l$ As $n \to \infty$:

$$\frac{1}{\sqrt{n}}W'\varepsilon = \frac{1}{\sqrt{n}}\sum_{i=1}^{n} w_i\varepsilon_i \xrightarrow{d} N(0, Var(w_i\varepsilon_i)) = N(0, \sigma^2 I_l) = \sigma N(0, I_l)$$

$$\frac{1}{n}W'W \xrightarrow{p} E(w_iw'_i)^{-1} = I_l$$

and $\frac{1}{n}W'X \xrightarrow{p} E(w_ix'_i) = Q$ where $Q$ is some full column rank matrix. On the other hand:

$$\frac{1}{\sqrt{n}}W'\hat{\varepsilon} = \frac{1}{\sqrt{n}}W'(Y - X\hat{\beta}_{2SLS}) = \frac{1}{\sqrt{n}}W'(Y - X(X'P_WX)^{-1}X'P_WY)$$

$$= \frac{1}{\sqrt{n}} W'(\varepsilon + X(X'P_W X)^{-1} X' P_W \varepsilon)$$

$$= (I - W'X(X'P_W X)^{-1} X' P_W) \frac{1}{\sqrt{n}} W'\varepsilon$$

$$\xrightarrow{d} (I - Q(Q'Q)^{-1}Q')\sigma N(0, I_l)$$

Therefore,

$$\hat{\varepsilon}' W(W'W)^{-1} W'\hat{\varepsilon} = \frac{1}{\sqrt{n}} \hat{\varepsilon}' W(\frac{1}{n} W'W)^{-1} \frac{1}{\sqrt{n}} W'\hat{\varepsilon}$$

$$\xrightarrow{d} \sigma^2 N'(I - Q(Q'Q)^{-1}Q')N$$

**Lemma 14.4.1.** $N'(I - Q(Q'Q)^{-1}Q')N \sim \chi^2(l - k)$

**Proof.** We have $Q'Q = I_k$ and $Q : l \times k$ where $l > k$ We define $Q_c$ as the $l \times (l-k)$ orthonormal complement matrix such that $[Q\, Q_c]$ together form an $l \times l$ complete orthogonal matrix. Thus $[Q\, Q_c][Q\, Q_c]' = I_l$

$$\Rightarrow QQ' + Q_c Q_c' = I_l$$

$$\Rightarrow Q_c Q_c' = I_l - QQ'$$

Thus

$$N'(I - Q(Q'Q)^{-1}Q')N = N'Q_c Q_c' N$$

$$= (Q_c' N)'(Q_c' N)$$

But $Q_c' N \sim N(0, Q_c' I_l Q_c) = N(0, I_{l-k})$ Thus

$$(Q_c' N)'(Q_c' N) = \sum_{i=1}^{l-k} (z_i)^2 \sim \chi^2(l - k)$$

□

Thus:

$$\hat{\varepsilon}' W(W'W)^{-1} W'\hat{\varepsilon} \xrightarrow{d} \sigma^2 \chi^2(l - k)$$

Finally, $\frac{\hat{\varepsilon}'\hat{\varepsilon}}{n} \xrightarrow{p} \sigma^2$ (sim to lec 8 proof) Therefore:

$$S = n \frac{\hat{\varepsilon}' W(W'W)^{-1} W'\hat{\varepsilon}}{\hat{\varepsilon}'\hat{\varepsilon}} \xrightarrow{d} \chi^2(l - k)$$

We reject the null of the instrument exogeneity when $s$ is larger than a critical value of $\chi^2(l - k)$

**Note:-**

The test cannot be performed in the just-identified situation ($l = k$). Then $W'X$ has full rank and so is thus invertible.

$$\frac{1}{\sqrt{n}} W'\hat{\varepsilon} = (I - W'X(X'P_W X)^{-1} X'W(W'W)^{-1}) \frac{1}{\sqrt{n}} W'\varepsilon$$

$$= (I - W'X(X'W(W'W)^{-1}W'X)^{-1} X'W(W'W)^{-1}) \frac{1}{\sqrt{n}} W'\varepsilon$$

$$= (I - W'X(W'X)^{-1} W'W(X'W)^{-1} X'W(W'W)^{-1}) \frac{1}{\sqrt{n}} W'\varepsilon$$

$$(I - I) \frac{1}{\sqrt{n}} W'\varepsilon = 0$$

## 14.5 Appendix

### 14.5.1 Chi-squared asymptotic result

**Lemma 14.5.1.** For $\vec{z} \sim N(0, V)$ We have

$$z'V^{-1}z \xrightarrow{d} \chi^2(p)$$

where $p$ is the number of elements in $z$.

**Proof.** As $V$ symmetric we can write its spectral decomposition:

$$V = Q\Lambda Q' = Q\Lambda^{1/2}\Lambda^{1/2}Q'$$

where $Q$ orthogonal and $\Lambda$ diagonal with eigenvalues $\lambda_1, ..., \lambda_p$.

$$\therefore z'V^{-1}z = z'(Q\Lambda^{1/2}\Lambda^{1/2}Q')^{-1}z$$

$$= ((\Lambda^{1/2}Q)^{-1}z)'((\Lambda^{1/2}Q)^{-1}z)$$

But

$$(\Lambda^{1/2}Q)^{-1}z \sim N(0, (\Lambda^{1/2}Q)^{-1}V(Q'\Lambda^{1/2})^{-1})$$
$$= N(0, (\Lambda^{1/2}Q)^{-1}Q\Lambda^{1/2}\Lambda^{1/2}Q'(Q'\Lambda^{1/2})^{-1})$$
$$= N(0, I_p)$$

Therefore $(\Lambda^{1/2}Q)^{-1}z$ is a vector of $p$ independent standard normals.
Therefore $((\Lambda^{1/2}Q)^{-1}z)'((\Lambda^{1/2}Q)^{-1}z)$ is the sum of $p$ independent standard normals squared, which is $\chi^2(p)$. □

### 14.5.2 Limited Info Maximum Likelihood

- no finite sample moments the same as 2SLS (so will have outliers)
- but better than 2sls with weak instruments

Recall the same linear regression model:

$$y_i = x_i'\beta + \varepsilon_i$$
$$x_i' = w_i'\pi + u_i'$$
$$\Rightarrow y_i = w_i'\pi\beta + u_i'\beta + \varepsilon_i$$

Let $(y_i, x_i) = Y_i'$

$$\Rightarrow Y_i' = w_i'(\pi\beta, \pi) + (u_i'\beta + \varepsilon_i, u_i')$$

Transposing

$$Y_i = \underbrace{\begin{pmatrix} \beta'\pi' \\ \pi' \end{pmatrix}}_{\Gamma(\beta,\pi)} w_i + \underbrace{\begin{pmatrix} \beta'u_i + \varepsilon_i \\ u_i \end{pmatrix}}_{e_i}$$

$$\Rightarrow Y_i = \Gamma(\beta, \pi)w_i + e_i$$

Assume:

$$e_i | w_i \sim N(0, \Omega)$$

We can then write likelihood function, and maximise wrt parameters to find $\hat{\beta}_{ML} = \hat{\beta}_{LIML}, \hat{\pi}_{ML}$ and $\hat{\Omega}_{ML}$