Geometric Unity: Author's Working Draft, v 1.0

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Abstract

An attempt is made to address a stylized question posed to Ernst Strauss by Albert Einstein regarding the amount of freedom present in the construction of our field theoretic universe.

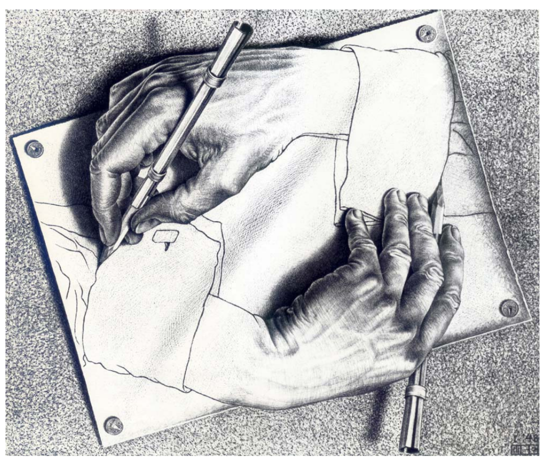

Figure 1: Drawing Hands.

Contents

*The Author is not a physicist and is no longer an active academician, but is an Entertainer and host of The Portal podcast. This work of entertainment is a draft of work in progress which is the property of the author and thus may not be built upon, renamed, or profited from without express permission of the author.

1 Introduction: The Problem

> "What really interests me is whether god had any choice in the creation of the world." -Albert Einstein to Ernst Strauss

In the beginning we will let X^4 be a 4-dimensional C^∞ manifold with a chosen orientation and unique spin structure. It is otherwise considered to be without a geometry in that no metric, symplectic, complex, volume, quaternionic or other structure is yet imposed upon it. It was the original intention of [8] to consider to what extent the opening epigrammatic question of Einstein could be considered a scientific program, rather than a philosophical one, by asking whether the observed world could be extracted from little more than such initial data as above.

Because Einstein did not specify what he meant exactly, we have taken the liberty of reformulating his question as follows:

> "Starting from X^4, as the topological structure underlying the Space-Time construction, to what extent can the observed universe together with stylized contents and laws mirroring its own be generated without further assumptions?"

so that what we are effectively asking is whether there is a plausible map:

$$X^4 \longrightarrow \begin{cases} +s_{\text{Ricci Scalar}} & \text{Einstein} \\ -\frac{1}{4} < F_A, F_A > & \text{Yang-Mills-Maxwell} \\ +i < \bar{\psi}, \not{\partial}_A \psi > & \text{Dirac} \\ + < \phi, d_A^* d_A \phi > -V(\phi) & \text{Higgs} \\ + < \hat{\psi}, \phi\psi > & \text{Yukawa} \\ \text{L} = \text{Sl}(2, \mathbb{C}) & \text{Lorentz Group} \\ \text{SU}(3) \times \text{SU}(2) \times \text{U}(1) & \text{Internal Symmetries} \\ Q = \mathbb{C}_H^{16} & \text{Family Quantum Numbers} \\ \psi = \oplus_a^3 \ \psi_a & \text{Three Families of Matter} \\ M_{CKM} & \text{Cabibo Kobayashi Maskawa Matrix} \end{cases} \tag{1.1}$$

which recovers the fundamental or seemingly fundamental stylized aspects of the observed universe from which we appear to have emerged.

To be sure, this does not seek to address the well known eternal question of "Why is there something rather than nothing?" as we are not aware of any available reformulation of that question which renders it scientific rather than religious or philosophical. Thus, as close as we will come to that question is

> "Why might we expect a world of the richness we have found through science and observation, to arise out of something which is minimally determined beyond being a low dimensional arena for the rules of calculus to collide with those of linear algebra."

From our perspective, a fundamental theory is simply a theory that effectively discourages further *scientific* search for a more foundational layer. Were the world recoverable from a manifold X^4 (with little more than minor additional data), it would not bring either physics or theology to an end by any means. It would, however move the technically minded at last away from the search for fundamental law and focus them instead upon the consequences of the rules encoded leaving the search for an explanation illuminating the initial input as a purely philosophical or, perhaps, religious question.

1.1 Strategy: Ideas over Instantiation (following Dirac)

This work arises from a particular orientation which should be shared with the reader up front. At its heart, it's belief is that most advances that are held up for years or even decades are blocked because of confounding factors particular to instantiations of the needed idea confused for being problems with the idea itself. This perspective animates Dirac's 1963 Scientific American article [3] where he argues that experiment can only be used to check agreement with the instantiation of an idea, rather than the idea itself. Dirac references Schrodinger's failure to take spin into account leading to a superficial failure to agree with experiment. He could just as easily, however, been writing about his own superficial mistake in viewing the electron and proton as anti-particles to each other despite the obvious mass asymmetry as pointed out by Heisenberg.

In all such cases, the initial instantiations of radical physical ideas were either flawed in a way the underlying ideas were not, or the presentation was such that it caused the wrong pictures to form in the minds of those who heard it. Thus our belief is that we should be following Dirac at this juncture and looking for natural theories and not over-indexing on their initial instantiations.

Further, we have noticed something exceedingly interesting and no less odd. The tiny minority of theorists who have contributed directly to physical law all appear to share a common quixotic focus on beauty and internal coherence rather than an immediate emphasis on formulae, instantiation and experiment. We take from this that Einstein, Dirac and Yang were not giving general advice as to how to do physics but rather very specific advice as to how to seek new physical law to the almost negligible subset of working theorists who might follow.

As such it is our contention that one should search for a theory that is geometrically and algebraically natural and quite close to our world at a stylistic level (e.g. chiral, three generations, etc...). If such a theory can be found, then, given the seemingly idiosyncratic nature of the various peculiarities of the Standard Model, it is our (historically well motivated but partially unjustified) belief that it will quite likely be that initial instantiations will be confounded by difficulties that are likely to prove inessential and thus surmountable.

1.2 About the Present Work

This work was begun while the author was finishing a combined Bachelor's and Master's degree program at the University of Pennsylvania which ended in 1985. At the time neutrinos were not yet claimed to posses mass and there were quite possibly 15 particles in a generation of matter under grand unified ideas like versions of SU(5). The present theory began on the narrow hope or 'joke' that in some sense if:

$$15 \text{ `` = '' } 2^{\frac{(\frac{4^2+4}{2})}{2}-1} \tag{1.2}$$

as the dimension of internal Fermionic quantum numbers, then we would be in the unique case where peculiar spinorial methods could unify the auxiliary fiber bundle Geometry of Ehresmann with the intrinsic geometry of Riemann.

1.3 From Unified Field Theory to Quantum Gravity and Back

Starting in 1984 it became common to hear from leaders of the theoretical physics community that theoretical or fundamental physics was not about traditional unification of the kind sought for by the like of Albert Einstein. Terms like "String Theory", "Theory of Everything" and "Quantum Gravity" replaced unification as the driving force behind the field.

The shift was a profound one. Students went from working mostly in physical Lorentzian Signature with hyberbolic equations to Euclidean signature to take advantage of the Atiyah-Singer index theorem. The number of dimensions considered typically dropped from the physical 4 dimensions to the toy dimensions of 2 and 3 to take advantage of complex methods and so-called Chern-Simons like theories respectively, or to ad hoc choice of 10, 11, 12 or 26 dimensions to access 'Calabi-Yau' manifolds, Super-Strings and Super-gravity and other exotic mechanisms. The physically relevant reductive or even semi-simple symmetries related to $SU(3) \times SU(2) \times U(1)$ were generally replaced with simple groups like a single $SU(2)$ or even $U(1)$ in isolation. The observed family structure of three generations of Fermions increasingly faded from interest or was pushed onto the index theory of Calabi-Yau three-folds, while investigations began to assume the presence of space-time Super-symmetry despite the existence of zero experimental observation for the phenomena.

In short, interest in the direct investigation of the physical world went from the core of physics research to a quaint backwater, as what might be termed the "Toy-Physics era" of String Theory inspired geometric physics began.

To understand how profound the shift truly was, it is helpful to understand what a major keynote address sounded like in theoretical physics in 1983, just before the anomaly cancellation and its embrace by Edward Witten and other String Theorists changed the entire nature of what it meant to be a theoretician working on fundamental physics. Today, it reads almost as an epitaph for the views of a bygone era. Here is Murray Gell-Mann addressing assembled leaders of the Theoretical Physics community at the 1983 Shelter Island II conference just before the split in the community mediated by the anomaly cancellation:

> From Renormalizability to Calculability?
>
> "As usual, solving the problems of one era has shown up the critical questions of the next era. The very first ones that come to mind, looking at the standard theory of today, are
>
> - Why this particular structure for the families? In particular, why flavor chiral with the left- and right-handed particles being treated differently, rather than, say, vectorlike, in which left and right are transformable into being treated the same?
> - Why 3 families? That's a generalization of Rabi's famous question about the muon, which I'll never forget: "Who ordered that?") The astrophysicists don't want us to have more than 3 families. Maybe they would tolerate a 4th, but no more, with massless or nearly massless neutrinos; it would upset them in their calculations of the hydrogen and helium isotope abundances. Of course, if the neutrinos suddenly jumped to some huge mass in going from known families to a new one, then they would be less upset.
> - How many sets of Higgs bosons are there in the standard theory? Well, the Peccei-Quinn symmetry, which I'll mention later, requires at least 2, if you believe in that approach. If there's a family symmetry group, there may be more, because we may want a representation of the family symmetry group: maybe there are 6 sets of 4 Higgs bosons; nobody knows.
> - Why SU(3) x SU(2) x U(1) in the first place? Here, of course, there have been suggestions. We note that the trace of the charge is zero in each family, and that suggests unification with a simple Yang-Mills group at some high energy, or at least a product of simple groups with no arbitrary U(1) factors. If the group is simple or a product of identical simple factors, then we can have a single Yang-Mills coupling constant."
>
> -Murray Gell-Mann, 1983

Sadly, the questions raised in this keynote have not really been answered at the time of this writing, nor have they been a particularly strong motivating force over the nearly 40 years that have followed this address. And with this stagnation came a desire to change the key problems in the theory just cited to ones that would allow for great deals of activity and elaboration. Thus, the emphasis shifted to 'Quantum Gravity' in the following year and the search for a compelling version of quantized string theories that would recover our world in short order. In fact, it never arrived.

1.4 The Twin Origins Problem

The problem of unification in physics is reasonably well appreciated by most members of the research community ostensibly working on this problem. What is less well appreciated is the number of distinct ways unification may be interpreted outside the dominant narrative within the field.

One way that seems to get comparatively less attention is to make use of the revolution in geometric physics that happened principally between Jim Simons and C.N. Yang at Stony Brook in the mid 1970s where it was discovered that classical Ehresmannian bundle theoretic geometry was playing the same role beneath classical and Quantum Field Theory that Riemannian geometry was playing in under-girding General Relativity.

Viewed in this way, it is possibly the difference in geometric frameworks of Einstein and Bohr that are more important than the issue of quantization. Oddly, perhaps the most succinct synopsis of the main ideas in fundamental physics was given in 1986 by Edward Witten in a way that laid bare that it has been the geometry of physical law rather than the quantum which has constituted our three greatest insights:

280 EDWARD WITTEN

If one wants to summarize our knowledge of physics in the briefest possible terms, there are three really fundamental observations: (i) Space-time is a pseudo-Riemannian manifold M, endowed with a metric tensor and governed by geometrical laws. (ii) Over M is a vector bundle X with a nonabelian gauge group G. (iii) Fermions are sections of $(\hat{S}_+ \otimes V_R) \oplus (\hat{S}_- \otimes V_{\tilde{R}})$. R and $\tilde{R}$ are not isomorphic; their failure to be isomorphic explains why the light fermions are light and presumably has its origins in a representation difference Δ in some underlying theory. All of this must be supplemented with the understanding that the geometrical laws obeyed by the metric tensor, the gauge fields, and the fermions are to be interpreted in quantum mechanical terms.

Figure 2: Edward Witten Synopsis.

Seen from this perspective (i) corresponds to the Einstein Field Equations of Semi-Riemannian geometry, (ii) to the Yang-Mills generalization of Maxwell's equations to Non-Abelian Ehresmannian gauge theory and (iii) to the Dirac equation which mixes the bundle structures of both frameworks. The Klein-Gordon equation for the Higgs Fields with its iconic quartic potential is not mentioned and the quantum is clearly featured not as a rival insight, but as a method of viewing the three main discoveries.

This raises the question: Why did we become focused on quantizing gravity when the underlying data given in (i) and (ii) are themselves of geometrically different origins? So long as auxiliary principal G-bundles are invoked without compelling justification, there is no sense in which theoretical physics will have a satisfying origin story for the universe. Why then are we not more focused on the 'Twin Origins Problem' of why we have separate inexplicable origins for $M^{1,3}$ and $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$?

Ostensibly this is likely to be due to the pessimism that grew up around the lack of progress in Kaluza Klein type theories that sought a single origin. But the fact remains that what we need is not necessarily 'quantum gravity' but rather a means of harmonizing General Relativity with the Standard Model in a theory free of paradox. To our mind, this suggests an emphasis on tying the auxiliary data in (ii) to the fundamental substrate of (i) at the level not of the quantum but at the level of geometry to resolve the tensions between intrinsic and auxiliary bundle geometries.

1.4.1 Ehresmannian Geometry Advantages

The major advantages that Ehresmannian geometry offers to theoretical physics is the freedom to choose the internal symmetries of a physical theory without being confined to symmetries tied to the tangent bundle of Space-time. This decoupling allows physics to account for forces like the Strong and Electro-Weak forces responsible for atomic nuclei, the electron orbitals that surround them, and their decay through beta radiation respectively.

Secondarily, the ability to remove redundancy in the description of nature by restricting our attention to Gauge Invariant quantities has turned out to be a powerful tool for reasons that, at least to this author, seem not yet completely clear. Yet some benefit is clearly gained by being able to write down expressions where they are most natural and then reducing them via gauge symmetry to where they are most economical and least repetitive.

1.4.2 Riemannian Geometry Advantages

In the case of Riemannian and Pseudo-Riemannian geometries, the freedom to consider ad hoc candidates for physical symmetries is radically restricted. Yet there is again a major and a minor advantage.

While the full Riemann curvature tensor is a specific example of the more general Ehresmannian Curvature tensor construction, its decomposition into sub-components has no general analog in Ehresmannian geometry. Thus the greatest advantage of Einstein and Grossman's choice of Riemannian geometry is almost certainly to replace the broad freedom lost in bundle choice with the much more restricted freedom to play separately with the Weyl, (Traceless) Ricci, and Scalar components of the full curvature tensor as Einstein did in 1915.

While that may not seem to modern field theoretic tastes like a sensible trade-off given the loss in choice of structure, it appears sufficient for one very particular application of great importance: gravity. Further, properly abstracted, the projection operators onto curvature sub-components may be seen as tensor product decompositions in such a way as to include the Dirac operator on Spinors with its contraction on 1-form valued Spinors. Viewed in this fashion, the ability to decompose tensor products of representations involving the tangent and cotangent bundles is the clear enticement to work within the metric paradigm.

The way this discussion has been framed by the above is designed to suggest the search for a parallel in the form of a second minor advantage. And here there is room for debate. The other main attribute of Riemannian geometry that suggests itself hides in plain sight under the label of the 'The Fundamental Theorem' of Riemannian geometry. The choice of a Levi-Civita connection made by the metric is certainly convenient, but its use can always be seen as a choice to be made dynamically by allowing more general connections as suggested by Palatini. Thus its main value is that it allows the torsion tensor of any other connection to be calculated by converting the affine space of connections into an honest vector space with the Levi-Civita connection at the origin.

However, this has always led to a puzzle: what good is being able to define the torsion tensor of a metric connection if the Levi-Civita connection is the only one that appears to matter in practice? Thus the torsion tensor appears, potentially, to be the solution to a problem which no one has yet thought to ask. As such, its potential advantages have so far been minor with its realized ones closer to non-existent. But it must be considered to be a potential advantage as the natural answer to a question which may some day arise.

1.5 Notes On The Present Draft Document

There is something about the nature of LaTeX that is likely to confuse the professional reader. It is as if a typeset document constitutes agreement to participate in some academic social contract. No such consent is intended by this. The author functions totally divorced from the professional research context which is oddly automatically inferred from typeset mathematics by nearly every capable reader, perhaps due to the rarity of such research programs.

Without wishing to dwell on this unduly, there is no way around the fact that the author has been working in near total isolation from the community for over 25 years, does not know the current state of the literature, and has few, if any, colleagues to regularly consult.

As such this document is an attempt to begin recovering a rather more complete theory which is at this point only partially remembered and stiched together from old computer files, notebooks, recordings and the like dating back as far as 1983-4 when the author began the present line of investigation. This is the first time the author has attempted to assemble the major components of the story and has discovered in the process how much variation there has been across matters of notation, convention, and methodology[1]. Every effort has been made to standardize notation but what you are reading is stitched together from entirely heterogeneous sources and inaccuracies and discrepancies are regularly encountered as well as missing components when old work is located.

The author notes many academicians find this unprofessional and therefore irritating. This is quite literally unprofessional as the author is not employed

[1]The biggest issue of heterodox methodology appears to have been shifts between representation theoretic and indicial methods of tensorial and spinorial products and contractions.

within the profession and has not worked professionally on such material since the fall of 1994. If you find this disagreeable, please feel free to take your professional assumptions elsewhere. This document comes from a context totally different from the world of grants, citations, research metrics, lectures, awards and positions. In fact, the author claims that if there is any merit to be found here, it is unlikely that it could be worked out in such a context due to the author's direct experience of the political economy of modern academic research. This work stands apart from that context and does so proudly, intentionally, and without apology.

With that said, the author welcomes constructive technical feedback at: technicalfeedback@geometricunity.org and non-technical constructive feedback at generalfeedback@geometricunity.org.

Lastly, the author notes that academicians repeatedly fail to cite, acknowledge or name work that does not come from within the circle of leading research institutions. The author alerts the reader that these practices are widespread within the academy but are generally not acceptable in the world outside academe. Constructive feedback is welcome but this should be considered as work in progress of the author and not an invitation for cherry topping, denigration, theft, scooping or other widespread anti-social academic practices broadly tolerated within the profession. Consult the author if this is any way confusing.

2 Incompatibility and Incompleteness Blocking Geometric Unification

It has become familiar to hear that Einstein's theory of gravity cannot be unified with the Standard Model of quantum field theory because there is no known way to renormalize a quantum theory of metrics. However, this betrays a focus on making the work of Einstein submit to the viewpoint of Bohr, that is the signature of a group of quantum theorists who view holdouts refusing to acknowledged the supremacy of the received quantum viewpoint as inviting quantum domination. We hold with Einstein that the Quantum is indisputable, but that its instantiations and their interpretations do not carry the same infallibility. So, for us, there are many incompatibilities between General Relativity and the classical field theory whose quantization defines the standard model. Our gambit is that if there is a natural classical field theory that strongly resembles the standard model together with General Relativity, then if it can be shown to emerge naturally from minimal assumptions, it is likely to be correct or close to correct and may well suggest its own preferred quantization. This cannot of course be proven but it is stated here as it is the governing philosophy of the work at hand.

2.1 Failure of Gauge Covariance: Einsteinian Projection

One of the curious failures of modern gauge theory is the uncomfortable accommodation it gives to Einstein's geometric general relativity. At first blush, a theory of gravity built on curvature should be a natural fit for the concept of bundle symmetry. For any curvature tensor of a connection on a principal-G bundle P_G, it always lives as a Lie Algebra valued 2-form:

$$F_A \in \Omega^2(\mathrm{ad}(P_G)) \tag{2.1}$$

and the Riemannian curvature ${R_{ijk}}^l$ is no different, lying as it does as a 2-form valued in the so called adjoint to Vierbein bundle. In fact,

$${R_{ijk}}^l = F_{\nabla^{g_{\mu\nu}}} \in \Omega^2(\Lambda^2(T^*(X))) \tag{2.2}$$

for the Levi-Civita connection $\nabla^{g_{\mu\nu}}$ of a metric tensor $g_{\mu\nu}$ as

$$\Lambda^2(T^*(X)) = \mathrm{ad}(P_{\mathrm{Fr}}) \tag{2.3}$$

for the given metric. Further, the Riemannian curvature is extremely well behaved under gauge transformations

$$F_{A\cdot h} = h^{-1} \cdot F_A \cdot h \tag{2.4}$$

just as if it were any other curvature tensor.

The incompatibility with gauge theory is in fact entirely due to the fact that Einstein made essential use of a linear algebraic projection of his curvature tensor which treated the twin appearances of $\Lambda^2(T^*X)$ in the definition of the Riemannian Curvature $R^l_{ijk} \in \Omega^2(X, \Lambda^2(T^*X))$ on a common footing, while the gauge transformations *strongly* distinguish the two by acting trivially on the copy corresponding to $\Omega^2(T^*X) = \Gamma^\infty(\Lambda^2(T^*X))$ but non-trivially on the copy corresponding to $\mathrm{ad}(P_{\mathrm{Fr}}) = \Lambda^2(T^*X)$. This leads to the simple fact that Einstein contraction or projection

$$P_E : \Lambda^2(TX) \otimes \Lambda^2(TX) \longrightarrow S^2(TX) \tag{2.5}$$

fails to commute

$$P_E(F_{A\cdot h}) \neq (P_E(F_A)) \cdot h \tag{2.6}$$

with gauge transformations h.

Curiously, this failure is often covered over by many wishing to shoehorn General Relativity into the gauge theoretic paradigm by asserting that the diffeomorphism group of 'General Coordinate Transformations' is somehow the Gauge Group '$\mathcal{H}$' of GR, while the space of Metrics parameterizes the relevant space of Levi-Civita Connections '$\mathcal{A}$' resulting in a weak version of $\mathcal{A}/\mathcal{H}$. Given what we take to be the self-evident artificiality of such claims, we merely note them here and will not dwell on them further.

2.2 Failure of Gauge Covariance: Torsion

In an explicitly trivialized coordinate patch, the covariant derivative ∇^A corresponding to a connection A can be given by specifying the connection as an ad-valued 1-form A relative to the trivial connection d coming from the trivialization by abuse of language. This leads to the transformation formula:

$$h^{-1} \circ (d + A) \circ h = d + \underbrace{h^{-1} \cdot A \cdot h}_{\text{Gauge Covariant}} + \underbrace{h^{-1} \cdot (dh)}_{\text{Gauge Non-Covariant}} \tag{2.7}$$

highlighting that the portion of A which is peculiar to the connection is in fact gauge covariant while the portion peculiar to the trivialization lends a disease of gauge non-covariance to all connections which has no dependence on the connection A at hand.

In the absence of a trivialization, but the presence of a prefered connection A^0 such as the Levi-Civita connection $A^g = A^0$, we can write:

$$h^{-1} \circ \nabla^A \circ h - \nabla^0 = h^{-1} \circ (\nabla^0 + A) \circ h - \nabla^0 \tag{2.8}$$

$$= h^{-1} \circ A \circ h \quad + h^{-1} \circ \nabla^0 \circ h = \underbrace{h^{-1}Ah}_{\text{Connection Specific}} + \underbrace{h^{-1}(d_0 h)}_{\text{Common Disease}}$$

to see that again part of the transformation law is well behaved while the diseased term is dependent only on the exterior derivative $d_0 = d_{A^0}$ coupled to the distinguished connection.

The problem here can be seen instantly if one tries to naively include the gauge potential A directly in a Lagrangian or action $\mathcal{I}$:

$$\mathcal{I}(A) = ||A||^2 =< A, A > \tag{2.9}$$

so that under a gauge transformation h we have:

$$\mathcal{I}(A \cdot h) = ||A \cdot h||^2 =< A \cdot h, A \cdot h > \tag{2.10}$$

$$=< h^{-1}Ah + h^{-1}dh, h^{-1}Ah + h^{-1}dh >$$

$$= \underbrace{||h^{-1}Ah||^2}_{\text{Gauge Invariant}} + \underbrace{||h^{-1}dh||^2}_{\text{Common Disease}} + \underbrace{2 < h^{-1}Ah, h^{-1}dh >}_{\text{Gauge Potential-Specific Disease}}$$

leading to a failure of gauge invariance due to both a shared and idiosyncratic term spoiling the action under symmetry. Typically, the response to this is to avoid putting the potential into the action directly and to work instead with the better behaved curvature tensor derived from the 1-forms in the case of the Standard Model, or to give up on gauge invariance at the bundle level (i.e. Einstein's compression of Riemann's curvature tensor) and use the action to penalize the torsion into vanishing (i.e. the Palatini action).

2.3 Higgs Sector Remains Geometrically Unmotivated

For most of the 20th century, fundamental physics was split into two halves, only one of which was geometric. Then, in the mid 1970s, the quantum sector was discovered to have a basis in differential geometry with the advent of the Wu-Yang dictionary of Simons, Wu and Yang. Gauge potentials corresponded to the geometer's notion of a connection and Fermi fields fit with Atiyah and Singer's rediscovery of the Dirac operator in a bundle theoretic context.

After that point, fields of fractional spin, spin 1 and spin 2 were all well motivated in either Ehresmannian or Riemannian geometry. This left the curious case of the spin 0 Higgs sector with its so-called Mexican Hat potential. While the Higgs sector could be described bundle theoretically, it was not fully natural for geometers to consider a spinless field valued in a Lie Algebra and governed by a quartic potential, despite the seemingly geometric nature of the Mexican Hat shape.

2.4 Geometric Unity

This leads us finally to the question of the motivating concept(s) behind Geometric Unity.

In essence, there is both a scientific basis and a human basis rooted in political economy. The scientific basis is the more important of the two and proceeds as follows.

Mapping out the various reasons that Riemannian and Ehresmannian geometry have continued to progress side by side reveals that there is a trade-off to be had between the two. In the case of Ehresmannian geometry, the leading advantage has to be the freedom to accommodate any observed field content found through experiment through the use of auxiliary bundles structures. As a compensating secondary advantage, the use of the gauge group to simultaneously transform the field content together with the derivative structures which furnish the differential equations that govern propagation allows us to consider a greatly reduced set of truly distinct configurations without being overwhelmed by unnecessary redundancies that we would face in the group's absence.

The power of these advantages are so central to the massive edifice that is modern Quantum Field Theory that the advantages of classical Einsteinian gravity based on Riemann's geometry theory seem highly restrictive and almost provincial or perhaps quaint by comparison.

Geometry \ Advantage	Primary	Secondary
Riemannian	Projection Operators	Distinguished Connection
Ehresmannian	Content Freedom	Gauge Group

(2.11)

The leading advantage of Einstein's theory would most likely be thought by most to be the ability to 'Project' or contract the full Riemann Curvature tensor back onto a subspace of Symmetric 2-tensors which can be put in correspondence with the tangent space to the parameter space of metrics. This gives the Einstein theory the flavor of having the matter and energy warp space directly,

$$\underbrace{\overbrace{d_A^*}^{\text{Analytic}}\ (F_A)}_{\text{Yang-Mills}} = \underbrace{\kappa_a J}_{\text{Source Term}} \qquad \text{vs} \qquad \underbrace{\overbrace{P_E}^{\text{Algebraic}}\ (F_{A^0})}_{\text{Einstein Tensor}} = \underbrace{\kappa_b T_{\mu\nu}}_{\text{Source Term}} \tag{2.12}$$

rather than a change in the curvature represented by a differential operator providing the link. This however pays a high price in that it treats the two copies of Λ^2 in $\Lambda^2 \otimes \Lambda^2$ where the Riemann curvature tensor lives symmetrically while the gauge group acts only on the second factor. This seemingly negates much of the advantage of working within the Riemannian paradigm if one is interested in harmonizing GR with QFT.

There is however a small hope. If one can give up the freedom of being able to choose auxiliary gauge content as needed in the hopes of avoiding the double origin problem, then it may be possible to work within an extremely narrow class of theories which are amenable to the advantages of both geometries. But such geometric frameworks are likely to be extremely restrictive. Thus one must hope that the Standard Model and General Relativity would be of a highly unusual and non-generic geometric variety. And the main hope here is that observed quantum numbers of (ii) and (iii) that have no explanation within Witten's point (i) seem to us to be highly suggestive that the Standard Model with its three generation and 16 particle per generation structure is of exactly the non-generic theory that carries both attributes.

This brings us to the less scientific and more human point. Let us ask the question whether the incentive structures of physics select for or against the search for a highly specific and restrictive class of geometries on which to focus. Attempts to find such a class are likely to fail, be considered as numerological and quixotic, and be career limiting. As such, leading physicists would likely avoid such theories in favor of flexible frameworks which do not paint the investigator into a research corner. To a seasoned investigator, trading gauge invariance of the action and the freedom to choose field content to fit the needs of a problem for mere Einstein projection maps and a distinguished connection, is likely to seem akin to a naive Magic Beans trade where something of great value is bartered away for something of obviously lesser or even dubious importance. Here that trade is auxiliary content freedom and gauge covariance for contraction operators and a distinguished choice of connection.

It is the assertion here that this is not only an advantageous trade but likely a necessary one. That is, we do not need auxiliary freedom because of our good fortune in the Standard Model, and we can buy back the gauge invariance after exploiting the riches of projection operators and the choice of a distinguished connection.

2.5 Geometric Harmony vs Quantum Gravity

It is frequently stated that Gravity has to be quantized because of the paradox that since every particle creates a gravitational field, a classical localization would ultimately have to be as uncertain as the quantum particle's location under observation.

In fact, this presupposes that the answer will be found in the simple Einsteinian space-time paradigm where the argument is maximally persuasive. However, even here, there is a significant issue that is often glossed over.

Because the group $\widetilde{\mathrm{Gl}}(4,\mathbb{R})$ does not carry a finite dimensional copy of the fractional spin representations, if we allow the space time metric to become uncertain, we find a profound puzzle in the fractional spin fields. While the Spin-1 force particles and Spin-0 Higgs fields may be difficult to measure between metric observations, the bundles in which they live are well defined in the absence of a definite choice of metric. This however is not true for fractional spin fields. Should the metric ever successfully be 'quantized' in a manner similar to the other fields, *there will be no finite dimensional bundle for the hadrons and leptons during the period between observations.* Not only do the waves become uncertain but the media in which the waves would live would appear to be uncertain or even said to vanish. One of the goals of GU is to ensure that the bundles for spinors and Rarita-Schwinger matter do not vanish in the absence of a metric.

By the above reasoning, We find the argument that gravity must be somehow harmonized with quantum fields in an as yet unspecified way more persuasive than the argument that metric gravity must be quantized on the same footing as the other fields. Thus, whether gravity is to be harmonized or quantized, it is the goal of GU to decouple the existence of the fractional spin bundles as the medium for matter waves from the assumption of a metric in the ultimate quantum theory.

3 The Observerse Recovers Space-Time

In order to make progress beyond modern General Relativity and Quantum Field Theory, it is a contention of the author that Space-Time itself should be sacrificed from the outset as being fundamental. As such, in Geometric Unity we will proceed without loss of generality to consider not a single space, but pairs of spaces linked by maps. The three main cases will be defined according to:

Definition 3.1 *An* **Observerse is defined to be a triple $(X^n, Y^d, \{\iota\})$ such that**

$$\iota : U_x^n \longrightarrow Y_g^d \tag{3.1}$$

are maps for local open sets $U_x^n \subset X^n$ about some points $x \in X^n$ constituting local Riemannian embeddings for neighborhoods U^n into a Riemannian manifold Y^d of equal or higher dimension inducing a pull back metric $g_X = \iota^*(g_Y)$ and defining a normal bundle $N_\iota^{d-n} \subset T(Y)$ with metric and its pull back $\iota^*(N_\iota^{d-n})$ over X^n. The main cases of this construction correspond to:

- **TRIVIAL: The manifold $Y = X$ and the map ι is the identity.**
- **EINSTEINIAN: The manifold $Y = \mathrm{Met}(X)$ is the bundle of point-wise metric tensors over X and the maps $\iota = g$ under scrutiny are sections of this bundle representing Riemannian or Semi-Riemannian metric fields.**

- **AMBIENT: The manifold Y^d_g is unconstrained beyond the condition of being an immersion.**

where the first is presented to cover the case of ordinary space-times without additional structure, and the last is included so as to allow for full generality. In this work we will consider the strongest assumption beyond those of Einstein by working within the Einsteinian Observerse to search for new physics. Note that in this case, Y doesn't usually have a pre-existing metric. Instead the choice of a section ι of the metric bundle induces a metric on Y in the portion of Y that lies above $U \subset X$.

One reason for introducing the concept of the Observerse is to allow a more fundamental role for observation and measurement. Both of these issues have been at the heart of confusions in relativistic observation and Quantum Measurement and their as yet unsatisfactory treatment has lead some physicists to claim that paradoxes can only be solved with a quantized gravity on par with the other fields. Our approach is different. By working with two spaces bridged by maps, we allow for the idea that not all fields are on par with each other and may need to be treated differently in the fully quantized theory.

To this end we will distinguish fields according to the following:

Definition 3.2 *A field χ that originates as the section of a bundle over X^n or Y^d will be called* **NATIVE to X^n or Y^d respectively. Fields on X of the form $\iota^*(\chi)$ will be called INVASIVE to X if they are pulled back from fields or jet bundles χ that are native to Y.**

It is our contention in this investigation that physics may actually be happening mostly on Y^d but that it is widely interpreted by physicists via metric pull-back as if it were occurring natively on X^n leading to confusion. In fact, in this investigation, there will only be one independent field that is truly native to X^d. This allows for the possibility that in a complete theory one could treat the observing field on X differently from the more readily quantized fields on Y without immediately leading to paradox. Different observations via different sections ι would pull back different quantized values from Y onto X.

3.1 Proto-Riemannian Geometry

The bundle of point-wise metrics has some curious elementary properties that may be well known to others, but which the author did not encounter while working in mathematics. As might be expected, there is a tension between the fact that the bundle is filled with metric information by construction, but erected without reference to any metric in particular. Thus it is chimeric in that it is in tension between both its topological and geometrical natures.

One way of noting this intermediate state is to notice that true geometries of both Riemannian and Symplectic type induce isomorphisms between the tangent and cotangent bundles.

In the presence of either a metric or symplectic form on a vector space W, we inherit a pair of canonical vector-space isomorphisms that are not canonically determined in the absence of this geometric structure:

$$W \longrightarrow W^* \quad W^* \longrightarrow W \tag{3.2}$$

leading to two exact sequences:

$$0 \leftrightarrows T \leftrightarrows T^* \leftrightarrows 0 \tag{3.3}$$

which, while rather trivial, suggest generalization in our case.

For the Einsteinian Observerse and its Tangent and Co-Tangent bundles TY, T^*Y we likewise have natural and non-trivial maps:

$$T(Y) \longrightarrow T^*(Y) \quad T^*(Y) \longrightarrow T(Y) \tag{3.4}$$

However, neither of these maps is an isomorphism as they have non-trivial kernels. These can be fit into a repeating long exact sequences

$$\dots \quad \longrightarrow \quad T \quad \longrightarrow \quad T^* \quad \longrightarrow \quad T \quad \longrightarrow \quad T^* \quad \longrightarrow \quad \dots \tag{3.5}$$

evidencing the relationship of a metric Chimeric bundle $C(Y) = C^*(Y)$ to the partial isomorphisms between $T(Y)$ and $T^*(Y)$ as we shall now explain.

There exists a commutative diagram

$$\begin{array}{ccccccccccc}
 & & 0 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \uparrow & & \downarrow & & \uparrow & & \\
\dots & \longrightarrow & V & & V^* & \longrightarrow & V & & V^* & \longrightarrow & \dots \\
 & & \downarrow & & \uparrow & & \downarrow & & \uparrow & & \\
\dots & \longrightarrow & T & \longrightarrow & T^* & \longrightarrow & T & \longrightarrow & T^* & \longrightarrow & \dots \\
 & & \downarrow & & \uparrow & & \downarrow & & \uparrow & & \\
\dots & & H & \longrightarrow & H^* & & H & \longrightarrow & H^* & & \dots \\
 & & \downarrow & & \uparrow & & \downarrow & & \uparrow & & \\
 & & 0 & & 0 & & 0 & & 0 & &
\end{array} \tag{3.6}$$

where all vertical sequences are short exact and the repeating central horizontal sequence is long exact with all other maps not covered in the preceding are metric isomorphisms.

The bundle H^* is defined to be

$$H^* = \pi^*(T^*(X)) \quad H^* \hookrightarrow T^*(Y) \tag{3.7}$$

which at the point $g \in Y$ carries an induced metric g via $H_{\pi^*(\pi_*(g))}$ as the pull back of the push-forward metric. We will refer to the dual bundle $(H^*)^*$ simply as H.

Conversely, as a fiber bundle, the total space Y carries a vertical sub-bundle $V \subset T(Y)$ as the subspace of vectors pointing along the fibers of Y over X. Here again, the structure of Y as a space of metrics becomes germane.

At a point $y \in Y$, lying in the fiber $\pi^{-1}(\pi(y))$ let two symmetric two tensors $A,\ B \in T^V_y(X)$ represent a random choice of two elements in the vertical tangent space along two different smooth paths of non-degenerate metrics $g_A(t),\ g_B(t)$ where at time $t = 0$ we have $g_A(0) = g_B(0) = y$ with $\dot{g}_A(t) = A$ and $\dot{g}_B(t) = B$. Then, by the symmetry of the metrics, the Frobenius inner product is defined by the double contraction against the metric represented by the point y which, in an orthonormal basis would look like:

$$< A, B >_y = \mathrm{Tr}_y(A^T \cdot B) = \mathrm{Tr}_y(A \cdot B) \tag{3.8}$$

were the tensors A, B represented as matrices in that basis. Here the space of metrics breaks into a Trace and Traceless component. The Traceless component has signature $(i \cdot n - i^2, \frac{n^2+2i^2-2i\cdot n+n-2}{2})$ with a freedom to assign either a (0,1) or (1,0) signature to the trace component. In this exposition we choose the later so as to assume our Frobenius metric can be taken to be of signature

$$(i \cdot n - i^2 + 1, \frac{n^2 + 2i^2 - 2i \cdot n + n - 2}{2}) = (4, 6) \qquad \text{for} \quad i = 1,\ n = 4 \tag{3.9}$$

for the four dimensional $(1, 3)$ metric of greatest physical interest with one temporal and three spatial dimensions. Naturally, this would work equally well for a cosmetic shift to the $(3, 1)$ sector, but we are treating the 3-spatial and 1-temporal dimension as anthropically determined in either case and would imagine that the other sectors carry physical reality disconnected from our sector. [2]

3.2 The Chimeric Bundle

We now define two metric bundles which, with the above assignments, are canonically isomorphic:

$$C(Y) = C = V \oplus H^* \qquad C^*(Y) = C^* = V^* \oplus H \tag{3.10}$$

where each of these Chimeric bundles may be thought of as 'semi-canonically' equivalent to the Tangent and Co-Tangent bundles according to:

$$\begin{array}{ccccc} & & C^* & & \\ & \nearrow & & \searrow & \\ T & & & & T \\ \uparrow & & & & \downarrow \\ T^* & & & & T^* \\ & \nwarrow & & \swarrow & \\ & & C & & \end{array} \tag{3.11}$$

as one way of formally linking two short exact sequences.

These bundles $C = C(Y)$ and C^* are distinguished by the fact that they are (semi-canonically) isomorphic to the tangent bundle in the case of C and

[2] NB: This choice is one of the few non-forced choices allowed in the strong form of GU.

the co-tangent bundle in the case of C^* and possess natural metrics. Because the fibers at a point $g \in Y$ impart metrics to to the vertical V_g and horizontal sub-bundles H_g^*, taking those sub-spaces to be orthogonal to each other allows for two different choices of metric to be inherited by their direct sum according to choices of sign.

3.3 Topological and Metric Spinors

What then is the purpose of defining bundles that are isomorphic to T and T^* but with only part of that isomorphism being canonical?

The answer lies in the paradox of Spinors. While the Dirac operator may well carry topological information that is not metric, it is usually impossible to define a natural finite dimensional bundle of spinors on X^n in the absence of a metric, even when one can lift the General Linear group to its connected two-fold double cover $\widetilde{\mathrm{GL}}(N, \mathbb{R})$, due to the absence of finite dimensional fractional spin representations.

By passing to Y^d however, we can do somewhat better by virtue of the fact that spinor representations carry an exponential property. That is, the spinor functor converts direct sums of vector spaces as input into tensor products of spinor representations as output:

$$\slashed{S}(W_a \oplus W_b) \cong \slashed{S}(W_a) \otimes \slashed{S}(W_b) \tag{3.12}$$

At the bundle level, we apply this to the metric Chimeric Bundle at a point $g \in Y$ to obtain:

$$\slashed{S}_g(C) \cong \slashed{S}_g(V \oplus H^*) \cong \slashed{S}_{\mathrm{Frobenius}_g}(V) \otimes \slashed{S}_g(H^*_{\pi^*(\pi_*(g))}) \tag{3.13}$$

which gives spinors defined for the metric endowed $C = C^*$ without making a choice of metric on Y.

This is important for several reasons. In the first place, we have defined a Spin Bundle for the chimeric bundle $C(Y)$ which is semi-canonically isomorphic to metric bundles of spinors on TY. Thus at the cost of replacing X with the total space of a natural bundle over it, we have come rather close to defining spinors without a choice of metric via maps:

$$\begin{array}{ccccccccc} & & & & TY & & & & \\ & & & \nearrow & & \searrow & & & \\ 0 & \longrightarrow & C & & \oplus & & C^* & \longrightarrow & 0 \\ & & & \searrow & & \nearrow & & & \\ & & & & T^*Y & & & & \end{array} \tag{3.14}$$

More importantly, we must have in the back of our minds that we are ultimately going to have to harmonize gravitation and the metric with fractional spin Fermi fields which depend on that tensor for their existence. This construction allows us to work with one single bundle of spinors even when there is no choice of metric. When a metric is chosen below on X compatibly with V and H, the

Chimeric bundle and Tangent are given a natural isomorphism via the Levi-Civita connection

$$\underbrace{\not{S}(V) \otimes \not{S}(H)}_{\text{Topological Spinors}} \longrightarrow \underbrace{\not{S}(T) = \not{S}(T^*)}_{\text{Metric Spinors}} \tag{3.15}$$

telling us how the Horizontal bundle H maps into TY. This in turn clears the way to allow us to later consider quantum Fermi fields which usually carry metric bundle dependence between invocations of metric tensors.

3.4 Reversing The Fundamental Theorem: The Zorro Construction

According to the fundamental theorem of Riemannian Geometry, every Riemannian or (semi-Riemannian metric) induces a unique metric compatible torsion free Levi-Civita connection on its tangent bundle.

As we have two spaces, X, Y, this is equally true both above and below in our construction of the Observerse:

$$\begin{array}{c|ccc} & \text{Metrics} & & \text{Connections} \\ \hline \text{On } X & \gimel & \longrightarrow & \aleph_{\gimel} \\ \text{On } Y & g & \longrightarrow & A_g \end{array} \tag{3.16}$$

However, what is necessary to split the cyclic long exact sequence of maps between T and T^* is in fact a connection on the space Y viewed as a bundle over X. By reversing the usual logic somewhat and linking these two maps, we get a train of transmission where a metric choice below on X leads ultimately to the choice of a connection above over Y:

$$\begin{array}{c|ccc} & \text{Metrics} & & \text{Connections} \\ \hline \text{On } X & \gimel & \longrightarrow & \aleph_{\gimel} \\ & & \swarrow & \\ \text{On } Y & g_{\aleph} & \longrightarrow & A_g \end{array} \tag{3.17}$$

The importance of this is that each observation of Y via a choice of metric $\gimel$ on X actually induces a metric and connection on Y identifying the Topological spinors with the metric spinors. This allows us somewhat more confidence to explore the idea that metrics are not even necessarily present on Y unless when observed where the act of observing under pull back $\gimel^*$. Further, if the problem with much of quantum gravity turns out to be the difficulty of quantizing metrics relative to other fields such as vector potentials, this excercise allows us to move to more conducive variables for those who harbor dreams of quantizing gravitation.

It should be pointed out here that most possible metric on Y are never in play. The subset of metrics that we are considering are incredibly tightly constrained and are equivalent to the space of connections that can arrise on X as Levi-Civita connections, as opposed to the space of all metrics over Y. Thus the issue of what upstairs gravity waves are possible doesn't arise as the space of relevant fields is downstairs on X.

3.5 Chimeric Spinors and Heterogeneous Spin Bundles.

Now that we have abstractly defined spinor bundles on Y, it behoves us to understand what kind of spinors we may have coaxed into existence.

We should begin by noting that the choice of $(1,3)$ metric signature (which can be mirrored by choice of a $(3,1)$ and thus is not meaningfully distinguished from it) is treated by us as anthropic data. If it were otherwise, there would likely be no life to evolve to observe these structures. This recognition, forces a $(3,6)$ signature on the space of traceless symmetric two tensors which, when combined with the trace portion seen as either a $(0,1)$ or $(1,0)$ space, can become either a $(3,7)$ or the $(4,6)$ metric respectively. Here again, we make an anthropic choice and select the latter of the two for two reasons. In the first place, we believe that $(3,7)$ will not be compatible with the observed forces and symmetries of the Standard Model. Secondly, we want to leave ourselves room for complex techniques and view a real $(4,6)_{\mathbb{R}}$ structure as holding the door open for some models we have been playing with favoring reduction to $(2,3)_{\mathbb{C}}$.

Lastly we must combine these choices of fiber metrics in Y with metrics pulled up from the base space of signature $(1,3)$. These combinations in turn could lead to $(4,10)$ or $(8,6)$ in the discarded former cases of $(7,3)$ or $(3,7)$, or $(5,9)$ or $(7,7)$ in the case of the latter $(4,6)$ or $(6,4)$. We do not know how to choose between these however as this is one of the few places in the model where choices are made that are not yet forced.

To begin with, we will assume that the metric on Y is split with signature $(7,7)$ (rather than $(9,5)$ which can be worked out by the interested reader) as it is more balanced and appears to lend itself better to both some complex and Clifford Algebra techniques.

Here the split signature Clifford algebra is of Real type and is equivalent to Real 128 square matrices with some additional structure:

$$\mathrm{Cl}_{7,7} \cong \mathbb{R}(128) \tag{3.18}$$

This in turn leads to representations for the Spin group of the form:

$$\mathrm{Spin}(7,7) \longrightarrow \mathrm{SO}(64,64) \longrightarrow \mathrm{U}(64,64) \tag{3.19}$$

so in full generality, all spin representations in $(7,7)$ signature are subsumed by:

$$\rho_{\mathrm{Spin}} : \mathrm{Spin}(14,\mathbb{C}) \longrightarrow \mathrm{U}(128,\mathbb{C}) \tag{3.20}$$

The additional structure on these matrix algebras gives them a transpose operation from combining the canonical automorphism of the Clifford algebras (by replacing the generating tetrad with its negative) and the canonical anti-automorphism gotten from reversing the order of basis factors within the various Clifford products.

This leads to concrete models for a number of Lie algebras at the level of Clifford Algebras as in:

$$\mathfrak{so}(7,7) \text{ ``}=\text{''}\ \Lambda^2 \subset \mathrm{Cl}_{\mathbb{R}(7,7)} \tag{3.21}$$

$$\mathfrak{gl}(64,\mathbb{R}) \text{ ``}=\text{''}\ (\Lambda^2 \oplus \Lambda^6 \oplus \Lambda^{10} \oplus \Lambda^{14}) \subset \mathrm{Cl}_{\mathbb{R}(7,7)} \tag{3.22}$$

As the Left and Right real Weyl spinors transform as 64-dimensional dual defining representations under the General Linear group $\mathrm{GL}(64,\mathbb{R})$, we can form a metric on the space of total Majorana Spinors

$$\mathbb{S}_{\mathbb{R}} = \mathbb{S}^L_{\mathbb{R}} \oplus \mathbb{S}^R_{\mathbb{R}} = \mathbb{S}^L_{\mathbb{R}} \oplus (\mathbb{S}^L_{\mathbb{R}})^* \tag{3.23}$$

according to

$$< \psi^L + \psi^R, \sigma^L + \sigma^R >= \psi^L(\sigma^R) + \sigma^L(\psi^R) \tag{3.24}$$

giving us a split signature metric in which both spaces of Weyls spinors are null as for Weyl Spinors we have:

$$\overbrace{\mathrm{Spin}(7,7)}^{\text{Structure Group}} \longrightarrow \overbrace{\mathrm{Gl}(64,\mathbb{R})_+}^{\text{Weyl-Left}} \times \overbrace{\mathrm{Gl}(64,\mathbb{R})_-}^{\text{Weyl-Right}} \longrightarrow \overbrace{\mathrm{Gl}(128,\mathbb{C})}^{\text{Dirac}} \tag{3.25}$$

The metric for the total Majorana spinors corresponds to the Lie Algebra:

$$\mathfrak{so}(64,64) \text{ ``}=\text{''}\ \underbrace{(\Lambda^2 \oplus \Lambda^6 \oplus \Lambda^{10} \oplus \Lambda^{14})}_{\mathbb{S}_L \otimes \mathbb{S}_R = \mathbb{S}_L \otimes \mathbb{S}^*_L = \mathbb{S}^*_R \otimes \mathbb{S}_R} \oplus \underbrace{(\Lambda^1 \oplus \Lambda^5 \oplus \Lambda^9 \oplus \Lambda^{13})}_{\Lambda^2(\mathbb{S}_L) \oplus \Lambda^2(\mathbb{S}_R)} \subset \mathrm{Cl}_{\mathbb{R}(7,7)} \tag{3.26}$$

corresponding to the non-compact real group $\mathrm{Spin}(64,64) \subset \mathrm{Cl}_{\mathbb{R}}(7,7)$.

In order to move from Orthogonal to Unitary representations[3], we pass to the Clifford Algebra $\mathrm{Cl}_{\mathbb{C}}(7,7)$

$$\mathfrak{u}(64,64) \text{``}=\text{''} \bigoplus_{i=0}^{3} \Lambda^{4i+2} \bigoplus_{i=0}^{3} i\Lambda^{4i} \bigoplus_{i=0}^{3} \Lambda^{4i+1} \bigoplus_{i=0}^{2} i\Lambda^{4i+3} \subset \mathrm{Cl}_{\mathbb{R}(7,7)} \otimes \mathbb{C} \tag{3.27}$$

corresponding to the Lie Group $H = \mathrm{U}(64,64)$, where the 'adjoint' operation in the Clifford Algebra is given by the operation of composing the canonical automorphism with the reversal of order of all basis vectors.

By exponentiation we arrive at a unitary representation

$$\rho_{\mathrm{Dirac}_{\mathbb{C}}} : \mathrm{Spin}(7,7) \hookrightarrow \mathrm{U}(64,64) \tag{3.28}$$

of the Spin group into a space of unitary Dirac Spinors whose corresponding Lie Algebra is in canonical correspondence with the exterior algebra inside $\mathrm{Cl}_{\mathbb{C}}(7,7)$.

[3]This decomposition was taken from a decades old file and should be checked by someone more current on their "Clifford Checkerboard" yoga as it did not come with a description.

3.6 The Main Principal Bundle

We have always found the byzantine intricacies of Clifford Algebras confusing and an attempt to recollect the various containments just discussed is offered here:

$$
\begin{tikzcd}
& \mathbf{Gl}(128,\mathbb{C}) & & \\
\mathbf{U}(64,64) \arrow[ur] & & \mathrm{GL}(128,\mathbb{R}) \arrow[ul] & \\
& \mathrm{O}(64,64) \arrow[ul] \arrow[ur] & & \mathrm{GL}(64,\mathbb{R})_L \times \mathrm{GL}(64,\mathbb{R})_R \arrow[ul] \\
& & \mathrm{Gl}(64,\mathbb{R}) \arrow[ul] \arrow[ur] & \\
& & \mathrm{Spin}(7,7) \arrow[u] &
\end{tikzcd}
\tag{3.29}
$$

where we are privileging one particular path up the diagram that contains the metric and unitary representations:

$$\mathrm{Spin}(7,7) \longrightarrow \mathrm{SL}(64,\mathbb{R}) \longrightarrow \mathrm{SO}(64,64) \longrightarrow \mathrm{U}(64,64) \longrightarrow \mathrm{GL}(128,\mathbb{C}) \tag{3.30}$$

It is now our aim to specify the main principal bundle in finite dimensions with which we will be working.

For the rest of this exposition, we will let $\varrho_{\mathrm{Dirac}} = \varrho_D$ be the representation

$$\varrho_{\mathrm{Dirac}} = \varrho_D : \mathrm{Spin}(7,7) \longrightarrow \mathrm{U}(64,64) \tag{3.31}$$

on complex Dirac Spinors using the notation $H = \mathrm{U}(64,64)$ in what follows.[4]

Our main object of focus, will be taken to be:

$$P_H = P_{\widetilde{\mathrm{Fr}}(C^{7,7})} \times_{\varrho_D} H \tag{3.32}$$

where $P_{\widetilde{\mathrm{Fr}}(C^{7,7})}$ is the double cover of the frame bundle of the Chimeric bundle.

$$
\begin{array}{rcl}
P_H & \hookleftarrow & \mathrm{U}(64,64) \\
\pi \downarrow & & \\
Y^{7,7} & \hookleftarrow & \widetilde{\mathrm{GL}}(4,\mathbb{R})/\mathrm{Spin}(1,3) \\
\pi \downarrow & & \\
X^4 & &
\end{array}
\tag{3.33}
$$

with the associated bundles:

$$\mathrm{ad} = \mathrm{ad}(P_H) = P_H \times_{\mathrm{ad}} \mathfrak{u}(64,64) = P_H \times_{\mathrm{ad}} \mathfrak{h} \tag{3.34}$$

$$\mathrm{Ad} = \mathrm{Ad}(P_H) = P_H \times_{\mathrm{Ad}} \mathrm{U}(64,64) = P_H \times_{\mathrm{Ad}} H \tag{3.35}$$

$$\$ = P_H \times_{\varrho_D} \mathbb{C}^{64,64} = P_H \times_{\varrho_D} \$ \tag{3.36}$$

[4]Note: The symbol H is being used to denote two different objects. A group and a horizontal vector space. This is unfortunate and may be rectified in future drafts.

Now the important thing to notice about this construction is that while it looks geometric in nature due to the presence of Spin groups and representations, it is in fact purely Topological as no metric has been chosen. In fact, all that has been chosen is the signature of a $(1,3)$-metric and one can avoid even this by working over all 5 distinct sectors $\{(i, 4-i)\}_{i=0}^{4}$ on X^4 and merely noting that we happen to appear to exist within one of them carrying a Lorentz signature by anthropic reasoning.

We may fairly ask what a topological $\slashed{S}(C_Y)$ Spinor looks like when 'observed' from X^4. The concept of an observation is of course built into the definition of the Observerse as a map ι observing Y from X via pullback.

$$\iota : U \longrightarrow Y \quad U \subset X^4 \tag{3.37}$$

and so here is naturally given by a section over a local neighborhood $U \subset X$ when Y fibers over X. Thus we have:

$$\begin{array}{ccc} \slashed{S}_{P_H} & \hookleftarrow & \mathbb{C}^{64,64} \\ \pi \downarrow & & \\ Y^{7,7} & \hookleftarrow & \widetilde{\mathrm{GL}}(4,\mathbb{R})/\mathrm{Spin}(1,3) \\ \pi \downarrow\uparrow \iota_U & & \\ U^4 & & \end{array} \tag{3.38}$$

where the key to understanding the topological spinors from the perspective of $U \subset X$ observing Y via ι is worth disentangling due to the multiplicity of roles played by ι_U.

In the first place ι_U is a local immersion as well as a section so it constitutes an embedding of X into Y seen as an ambient space. As such, it pulls back all bundles over Y just as it pushes forward the tangent bundle of $U \subset X$ in non-degenerate fashion. When it pulls back the Tangent Bundle TY as an embedding, it splits via:

$$\iota^*(TY) = TU \oplus N_\iota = TU \oplus TY/\iota_*(TU) \tag{3.39}$$

where N_ι is the normal bundle of the observation. But the act of observation also gives a splitting of the long exact sequence from before over $U \subset X$:

$$\ldots \quad \leftrightarrows \quad T \quad \leftrightarrows \quad T^* \quad \leftrightarrows \quad T \quad \leftrightarrows \quad T^* \quad \leftrightarrows \quad \ldots \tag{3.40}$$

Including this splitting into the earlier large commutative diagram

$$\begin{array}{ccccccccccc} & & 0 & & 0 & & 0 & & 0 & & \\ & & \downarrow & & \uparrow & & \downarrow & & \uparrow & & \\ \ldots & \leftrightarrow & V & & V^* & \leftrightarrow & V & & V^* & \leftrightarrow & \ldots \\ & & \downarrow & & \uparrow & & \downarrow & & \uparrow & & \\ \ldots & \leftrightarrows & T & \leftrightarrows & T^* & \leftrightarrows & T & \leftrightarrows & T^* & \leftrightarrows & \ldots \\ & & \downarrow & & \uparrow & & \downarrow & & \uparrow & & \\ \ldots & & H & \leftrightarrow & H^* & & H & \leftrightarrow & H^* & & \ldots \\ & & \downarrow & & \uparrow & & \downarrow & & \uparrow & & \\ & & 0 & & 0 & & 0 & & 0 & & \end{array} \tag{3.41}$$

we start to see the full effect of a choice of ι, as it simultaneously plays the following roles:

- **Observer** as a generator of pullback data via ι^*.
- **Ambient Embedding** generating a Normal Bundle N_ι.
- **(Downstairs) Metric** as a section of the bundle of metrics.
- **Splitting** of the long repeating sequence.
- **Connection** as can be seen as being determined via either the Zorro diagram, the Levi-Civita Construction, or the splitting diagram above.
- **(Upstairs) Metric** Via the Zorro diagram.
- **Isomorphism** generator:

$$C = C^* = TY = T^*Y \quad V = V^* = N_\iota = N_\iota^* \quad H = H^* = TX = T^*X \tag{3.42}$$

so that its introduction has a fairly violent effect in moving us from topology to true geometry. Curiously, it is the only primary field in the theory that is truly native to X. As such we will use Hebrew letters gimel ($\gimel$) and aleph $\aleph$ and denote our immersion ι in the Einsteinian Observerse by $\iota = \gimel$ and its associated connections by $\aleph_\gimel$ to remind ourselves of the separation between fields native to X and those arising naturally on Y.

4 Topological Spinors and Their Observation

The appearance of topological spinors may then finally be interrogated under observation by $\gimel^*$. Let $x \in U \subset X$ be a point in the neighborhood of a local observation $\gimel_U$.

If $\Psi_{\gimel_U(x)} \in \not{S}(C)$ is a topological spinor, then under an observation by $\gimel^*$ on X it will appear as

$$\Psi_{\gimel_U(x)} \in \underbrace{\not{S}_x(TX)}_{\text{Space-Time Spinors}} \otimes \underbrace{\not{S}_x(N_{\gimel(x)})}_{\text{`Internal' Quantum Numbers}} \tag{4.1}$$

so that an observer may be lead into error. While the topological spinor under observation is not generated by any algebraic auxiliary data unconnected to X, it is quite likely to appear as if it contains auxiliary internal quantum numbers if the observer is unaware of the Observerse structure involving Y, as the pull back fields via $\gimel^*$ would have the false appearance of being native to X. A tell tale sign that one might be looking at such a unified structure with a single origin in X would be the presence of a power of 2 in the dimension count of the auxiliary quantum numbers for $\not{S}_x(N_{\gimel(x)})$. Specifically, on an even dimensional manifold of signature $(i, j) = n$ we would expect to see internal quantum numbers of dimension

$$\mathrm{Dim}(\underbrace{\not{S}_x(N_{\gimel(x)})}_{\text{`Internal' QN}}) \quad = \quad \underbrace{2^{\frac{i^2+j^2+2ij+i+j}{4}}}_{\text{Dirac}} \qquad \text{or} \qquad \underbrace{2^{\frac{i^2+j^2+2ij+i+j-4}{4}}}_{\text{Weyl}} \tag{4.2}$$

depending on whether the theory was non-chiral (Dirac) or either effectively or fundamentally chiral (Weyl). In particular, we might expect the later in regions $U \subset X^{i,j}$ where gravity and curvature are weak and the former where they are strong and resistant to effective decoupling. And, to our way of thinking,

$$\underbrace{2^{\frac{i^2+j^2+2ij+i+j-4}{4}}}_{\text{Weyl}} = 2^{\frac{1^2+3^2+2*1*3+1+3-4}{4}} = 16_{\mathbb{C}} \tag{4.3}$$

this, with a doublet of charged and uncharged leptons together with a tri-colored positively charged quark and its negatively charged weak isospin doublet partners along with the anti-particles of all the preceding, appears to be exactly what we see repeated over three apparent low energy families.

Lastly, we note from personal communication that Frank Wilczek appears to have wondered about the spinorial coincidence (even in print), but did not find it compelling enough to pursue beyond noting its existence and the lack of incorporation within a physical framework. It is our hope that recognizing that

> "A particularly intriguing feature of SO(10) is its spinor representation, used to house the quarks and leptons, in which the states have a simple representations in terms of basis states labeled by a set of "+" and "-" signs. Perhaps this suggests composite structure. Alternatively, one could wonder whether the occurrence of spinors both in internal space and in space-time is more than a coincidence. These are just intriguing facts; they are not presently incorporated in any compelling theoretical framework as far as I know." -Frank Wilczek, 1997

Figure 3: Wilczek On Internal Spinors.

the '10' implicit within both the Georgi-Glashow and Pati-Salam theories may be tied to the 10 coupled equations of General Relativity may be considered compelling.

4.1 Maximal Compact and Complex Subgroup Reductions of Structure Group

Assume for the moment that we have a global observation:

$$\gimel : X \longrightarrow Y \tag{4.4}$$

or, in our case, simply that we have a given metric on X. The push-forward map induced on the tangent bundle TX given by

$$D\daleth : TX \longrightarrow TY \tag{4.5}$$

splits the tangent bundle on Y along $\daleth(X) \subset Y$ into the image of $D\daleth$ and its 10 dimensional complement V^{10} along the fiber of metrics for every $x \in X$. This creates an identification with the chimeric bundle $C = V_{10} \oplus H_4^*$ as the vertical piece is already a subset of both TY and C by construction, and $\mathrm{Im}(D\daleth) = (H^4_{\daleth x})^*$

Having seen that the natural break down of our Chimeric $\mathrm{Spin}(7,7)$ bundle under observation leads to a decomposition into tangent and normal components of dimensions 4 and 10 respectively, it is natural to ask what reductions of structure group are most natural to expect. Two immediately suggest themselves.

Given any normal bundle that is even dimensional, there is a natural question as to whether it admits a complex or quaternionic structure. Here the dimension 10 being equal to 2 mod 4 suggests seeking a complex structure through reduction of structure group to $\mathrm{U}(3,2) \subset \mathrm{Spin}(6,4)$.

Conversely, given the non-compact nature of $\mathrm{Spin}(6,4)$ it is natural to wonder whether the structure group breaking to a maximal compact subgroup with better stability behavior is advantaged. Thus while non-compact groups are certainly considered from time to time, if the subgroup was broken to a maximal compact sub-group, we would be anthropically screened from seeing how nature accommodates non-compact symmetry and thus without guidance as to how to find a theory which extends to the general case.

To this end, we might also consider both reductions simultaneously and ask how a reduction to a maximal compact subgroup would appear if it were accompanied by a simultaneous reduction to accommodate a 5 complex dimensional normal bundle. Starting from $\mathrm{Spin}(1,3) \times \mathrm{Spin}(6,4)$

$$\begin{array}{c}
\overbrace{\overbrace{\overbrace{\mathrm{Spin}(1,3)}^{\text{Space-Time}}}^{\text{Einstein}} \times \overbrace{\overbrace{\mathrm{Spin}(6,3)}^{\text{Traceless}} \times \overbrace{\mathrm{Spin}(0,1)}^{\text{Trace}}}^{\text{Frobenius}}}^{\text{Observerse}} \longrightarrow \overbrace{\mathrm{Spin}(1,3)}^{\text{Horizontal}} \times \overbrace{\mathrm{Spin}(6,4)}^{\text{Vertical}} \longrightarrow \overbrace{\mathrm{Spin}(7,7)}^{\text{Chimeric}} \\
\cup \\
\mathrm{Spin}(1,3) \times \underbrace{\mathrm{Spin}(6) \times \mathrm{Spin}(4)}_{\text{Maximal Compact}} \\
\wr\Vert \\
\mathrm{Sl}(2,\mathbb{C}) \times \underbrace{\mathrm{SU}(4) \times (\mathrm{SU}(2) \times \mathrm{SU}(2))}_{\substack{\text{Pati-Salam} \\ \text{(Via Low Dimensional} \\ \text{Isomorphism)}}} \\
\cup \\
\underbrace{\mathrm{Sl}(2,\mathbb{C})}_{\text{GR}} \times \underbrace{\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)}_{\text{Standard Model}}
\end{array} \tag{4.6}$$

where the Standard Model group is found within the intersection of the simultaneous reductions up to a reductive factor of $\mathrm{U}(1)$ if the special unitary group $\mathrm{SU}(3,2)$ is not privileged over the full unitary group $\mathrm{U}(3,2)$ to begin with.

4.2 Pati-Salam

One way of looking at all of this is as a geometric setting for Grand Unified theories. While the Georgi-Glashow model of $SU(5)$ and its associated $\mathrm{Spin}(10)$ enlargement may be more popular, the Pati-Salam model is no less attractive when presented differently. In the usual presentation of the Pati-Salam grand unified theory, the groups are given as $SU(4) \times SU(2) \times SU(2)$ which suffers from ambiguities. To begin with, there are no fewer simple factors in Pati-Salam theory than there are reductive factors in the Standard Model. Further, there is the naming ambiguity as we have many names for the same objects:

$$\mathrm{SU}(2) = \mathrm{Sp}(1) = S^3_{\mathbb{H}} = \mathrm{Spin}(3) \tag{4.7}$$

However, if we accept that non-compactness is the price generally to be paid in any unified theory that incorporates both space and time, we should expect reduction to non-compact groups[5] whose maximal compact subgroups

[5]We, years ago, remember following such reductions along the lines of Bar-Natan and Witten which involve incorporating an endomorphism of the non-compact complements into to the Hodge Star operators but have yet to successfully resurrect the technique, nor have we found our notes for this period. Such problems of reconstruction over nearly 40 years are, lamentably, found throughout this document but they are likely to get worse rather than better by waiting to fix them. For those sensitive to errors of this type we recommend waiting for a future draft.

will always be Semi-simple or at least reductive. Thus, we view the semi-simple nature of Pati-Salam paradoxically as *more* of a blessing than curse given that we are trying to generate quantum numbers from the anthropic choice of $\mathrm{Spin}(1,3)$ which ensures the existence of hyperbolic PDE dynamics unspoiled by ellipticity.

To this end we have:

$$(h_{\mathrm{SU}(3)_{\mathrm{QCD}}}, g_{WI}, \alpha_{WH}) \longrightarrow \tag{4.8}$$

$$\overbrace{\Bigg(\underbrace{\begin{pmatrix} \alpha \cdot h & 0 \\ 0 & \alpha^{-3} \end{pmatrix}_{\mathrm{SU}(4)}}_{\mathrm{Spin}(6)}, \underbrace{\begin{pmatrix} & g & \end{pmatrix}_{\mathrm{SU}(2)_L}, \begin{pmatrix} \alpha^{3} & 0 \\ 0 & \alpha^{-3} \end{pmatrix}_{\mathrm{SU}(2)_R}}_{\mathrm{Spin}(4)}\Bigg)}^{\text{Pati-Salam Grand Unified Group}}$$

where we have made use of the low dimensional isomorphism $\mathrm{SU}(4) = \mathrm{Spin}(6)$ which can be best understood through the study of sphere transitive Weyl spin representations via Clifford Algebras.

As for the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ factors, we see that it is most advantageous to view this via low dimensional isomorphism with $\mathrm{Spin}(4)$ so as to obtain the following diagram:

$$\begin{array}{ccccc}
 & & \mathrm{Spin}(10, \mathbb{C}) & & \\
 & \nearrow & & \nwarrow & \\
\mathrm{Spin}(6,4) & & & & \mathrm{Spin}(10) \\
\uparrow & \nwarrow & & \nearrow & \uparrow \\
\mathrm{SU}(3,2) & & \overbrace{\mathrm{Spin}(6) \times \mathrm{Spin}(4)}^{\text{Pati-Salam}} & & \overbrace{\mathrm{SU}(5)}^{\text{Georgi-Glashow}} \\
 & \nwarrow & \uparrow & \nearrow & \\
 & & \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1) & &
\end{array} \tag{4.9}$$

indicating that the appearance of a phantom 10-dimensional representation is common to both the Georgi-Glashow and Pati-Salam theories and strongly suggests looking for a fundamental explanation. One disadvantage on finding oneself on the Pati-Salam branch of the above tree, is that it suggests non-compact groups bigger than itself which are difficult to accommodate in unitary Bosonic theories with bounded energy. An advantage however is that it does not lead immediately to proton decay like the original $\mathrm{SU}(5)$ model of Georgi-Glashow. Further, by privileging both a compact structure group within $\mathrm{Spin}(6,4)$ and a complex structure on the phantom 10-dimensional representation, the Standard Model group appears to be very close to being at the intersection of those requirements.

5 Unified Field Content: The Inhomogeneous Gauge Group and Fermionic Extension

Up until now we have been dealing with finite-dimensional constructions that superficially appear to be geometric (e.g. spinorial), but are actually mostly topological in nature. In this section we focus on the field content of GU which brings us to infinite dimensional algebraic constructions.

5.1 Field Content

In what follows, we endeavor to list the field content of Geometric Unity. The major nuance here is that the fields live on separate spaces which is kept track of by having only two fields on X with Hebrew orthography. All other fields live on Y and carry Greek orthography.

5.1.1 Field Content Native to X

In everything that follows, we will endeavor to separate out all field content native to X by having it appear with Hebrew Letters. There is a single primary field $\gimel$ native to X which is a section of the metric bundle $Y(X)$ as well as a derived field $\aleph = \aleph_{\gimel}$ representing the Levi-Civita connection across all bundles on which it is induced from the metric $\gimel$.

5.1.2 Field Content Native to Y

There is a single unified field ω native to Y. Here our interpretation of Unified field is interpreted to mean unified in an algebraic sense of indecomposible.

With that said, we should say at the outset that ω is comprised of interlocking sub-sectors:

$$\omega = (\beta, \chi) = (\overbrace{\underbrace{(\overbrace{\varepsilon}^{0}, (\overbrace{\varpi}^{1})}_{\text{Bosons } \beta}, \underbrace{(\overbrace{\nu}^{\frac{1}{2}}, \overbrace{\zeta}^{\frac{1}{2},\frac{3}{2}})}_{\text{Fermions } \chi}}^{(\text{Naive})^6 \text{ spins on } Y}) \tag{5.1}$$

Of the four sub-component fields, only one, ε is non-linear at the level of topological spaces. Letting $\bar{\varepsilon}$ denote its linearization, the four components fit neatly within the following simple table of tensor products:

(Linearized) Field Content on Y: (5.2)

$\otimes$	ad	$\$$
Ω^0_Y	$\bar{\varepsilon}$	ν
Ω^1_Y	ϖ	ζ

It is the contention of the author that since the introduction of Special Relativity in 1905, Physicists have become dependent on affine space techniques for their understanding of relativistic mechanics as well as both classical and quantum field theories. While space-time has obviously not been considered flat since Einstein and Grossman first introduced General Relativity in 1913, we are rather more sympathetic to the emphasis on affine space than our frequent irritation with excuse making for Minkowski space techniques might suggest.

Simply put, we see affine physics as being central to our understanding of the world and requiring no excuse making, but believe that the culture has chosen the wrong affine space and dimensionality for its emphasis given the presence of gravity.

The simple principle we follow here is that we should implement on the affine space of connections what we are otherwise tempted to do on flattened space-time. To this end we set notation.

5.2 Infinite Dimensional Function Spaces: $\mathcal{A}, \mathcal{H}, \mathcal{N}$.

The so-called Gauge Group of automorphisms of P_H is defined to be:

$$\mathcal{H} = \Gamma^\infty(P_H \times_{\mathrm{Ad}} H) \tag{5.3}$$

where the space of connections for P_H

$$\mathcal{A} = \mathrm{Conn}(P_H) \tag{5.4}$$

is an affine space modeled on the right $\mathcal{H}$-module

$$\mathcal{N} = \Omega^1(Y, \mathrm{ad}(P_H)) \tag{5.5}$$

which carries a right action of the group

$$\mathcal{A} \times \mathcal{H} \longrightarrow \mathcal{A} \tag{5.6}$$

so that the affine difference map

$$\delta : \mathcal{A} \times \mathcal{A} \longrightarrow \mathcal{N} \qquad \delta(A, B) = A - B \in \mathcal{N} \tag{5.7}$$

is an $\mathcal{H}$-equivariant map of right $\mathcal{H}$-spaces.

5.3 Inhomogeneous Gauge Group: $\mathcal{G}$

The reliance on affine Minkowski Space together with its Lorentz and Poincare symmetry groups is somewhat curious in the presence of General Relativity. Yet given the success of analysis on affine space we are given to speculate that fundamental physics may in fact be reliant on an affine space as more than an approximation or pedagogical aid.

The gauge group $\mathcal{H}$ can be augmented (in analogy to the Lorentz Group $\mathrm{SL}(2, \mathbb{C})$ to become a subgroup of its own natural inhomogeneous extension.

Definition 5.1 *The* **Inhomogeneous Gauge Group** $\mathcal{G}$ *is defined (in analogy to the Poincare group) to be the semi-direct product*

$$\mathcal{G} = \mathcal{H} \ltimes \mathcal{N} \tag{5.8}$$

of the gauge group $\mathcal{H}$ *with the space of ad-valued one-forms* $\mathcal{N} = \Omega^1(ad(P_H))$ *viewed as a right* $\mathcal{H}$*-module, so that the explicit group multiplication rule:*

$$g_1 \cdot g_2 = (\varepsilon_1, \varpi_1) \cdot (\varepsilon_2, \varpi_2) = (\varepsilon_1 \cdot \varepsilon_2, Aut(\varepsilon_2^{-1}, \varpi_1) + \varpi_2) \tag{5.9}$$

for all $\varepsilon_i \in \mathcal{H}$ *and* $\varpi_j \in \mathcal{N}$ *defines the semi-direct product structure.*

5.4 Natural Actions of $\mathcal{G}$ on $\mathcal{A}$.

Having defined a new group augmenting the usual gauge group, it is worth noting that the actions of $\mathcal{H}$ on the space of connections $\mathcal{A}$ extend naturally to the new group $\mathcal{G}$ incorporating the additional inhomogeneous affine translations $\mathcal{N}$.

5.4.1 Right Action by $\mathcal{G}$ on $\mathcal{A}$

This inhomogeneous gauge group $\mathcal{G}$ can be seen as acting naturally on the right on the space of gauge potentials or connections $\mathcal{A}$ via

$$A \cdot g = A \cdot (\varepsilon, \varpi) = A \cdot \varepsilon + \varpi \tag{5.10}$$

extending the usual right action $A \rightsquigarrow A \cdot \varepsilon$ of an element of the gauge group $\varepsilon \in \mathcal{H}$ on an arbitrary connection $A \in \mathcal{A}$.

5.4.2 Left Action by $\mathcal{G}$ on $\mathcal{A}$

We also have a left action of $\mathcal{G}$ on $\mathcal{A}$ via

$$g \cdot A = (\varepsilon, \varpi) \cdot A = (A + \varpi) \cdot \varepsilon^{-1} \tag{5.11}$$

extending the left action $A \rightsquigarrow A \cdot \varepsilon^{-1}$ gotten from the usual right action of $\mathcal{H}$ applied to the inverse element $\varepsilon^{-1} \in \mathcal{H}$.

5.5 Fermions and SUSY

Super-symmetry has a curious status within both Mathematics and Physics. It is both incredibly natural by some measures, as well as being rather artificial by others. It is not a true symmetry, its 'integrals' are not real integrals, and its 'dimensions' are not true dimensions. Nevertheless, the dictionary between Bosonic and Fermionic constructions is astounding (at least to us). This is interpreted by the author as consistent with a signature of a problem where Super-symmetry is likely very important but somehow thoroughly misinstantiated by its often fanatical proponents compensating for its failure to materialize in any physical experiment.

If super-symmetry is taken to be natural, then surely so-called super-space is its most natural representation by an action. Here, it seems that affine nature of Galilean transformations gives us an interpretation for super-symmetric charges as the square roots of affine translations.

Yet, knowing that Minkowski space is simply an approximation to non-affine spaces is somewhat discouraging if we are to build an entire theory that leans so heavily on flat translations when both non-trivial geometry and topology threaten to significantly complicate the picture. By contrast, the affine space of gauge potentials is intrinsically affine in nature no matter what the geometry and topology are on which those potentials live. This is why we find it more appealing to see Fermionic ν and ζ as potential square roots of the as-if Galilean group $\mathcal{N} = \Omega^1(Y, \mathrm{ad})$. This is also theoretically appealing as the concept of electrons and positrons being some kind of square roots of photons is potentially very appealing given the role of Feynman diagrams in perturbation theory. The

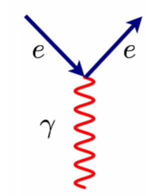

Figure 4: "Is the Electron a Square Root of a Photon?"

general rubric here is that expressions like $(\bar{\nu} \cdot \zeta)$ can be given meaning directly as elements of $\mathcal{N}$ while expressions like $\bar{\zeta}_a \cdot \zeta_b$ are harder to directly interpret as translations without more machinery as they do not initially land in the proper space $\mathcal{N}$ and would have to be moved in gauge-covariant fashion.

We may return to this in future work but do not wish to say much more as the subject of modern SUSY is rather delicate given the steadfast failure of its predicted space-time superpartners to materialize. We note however that the zoo of sleptons, squarks, gluinos and the like are all based on internal symmetry remaining the same and space-time symmetry changing spin. In a theory such as GU, there is no internal symmetry. Ergo, the (infinite dimensional) superpartners would not need to have the same internal quantum numbers anymore than they would need to carry the same space-time spins. This asks the question: if the universe is GU like rather than Standard-Model like, are the superpartners we seek already here and based on an affine space different from space-time with no-internal symmetry of which to speak?

5.6 Relationship of the Inhomogeneous Gauge Group to Standard Analysis

A brief digression is in order to relate what we are doing to the standard analysis of gauge potentials under gauge symmetry. To begin with, the usual object of study in quantum field theory is the space of connections modulo gauge equivalence, or:

$$\mathcal{A}/\mathcal{H} \tag{5.12}$$

in our notation.

This is equivalent, at an initially irrelevant level, to the expression

$$\mathcal{A}/\mathcal{H} = (\mathcal{H} \times \mathcal{A})/(\mathcal{H} \times \mathcal{H}) \tag{5.13}$$

by Cartesian product with a second copy of the gauge group in both numerator and denominator.

But by the choice of a connection A_0 we have $\mathcal{A} = \mathcal{N}$ as our affine space now has an origin while we note that the Semi-direct product of topological groups is set theoretically a product of the underlying spaces if not exactly at the algebraic level. Thus we have:

$$\begin{aligned}\mathcal{A}/\mathcal{H} = (\mathcal{H} \times \mathcal{A})/(\mathcal{H} \times \mathcal{H}) = (\mathcal{H} \ltimes \mathcal{N})/(\mathcal{H} \times \mathcal{H}) = \mathcal{G}/(\mathcal{H} \times \mathcal{H}) \\ = (\mathcal{G}/\mathcal{H})/\mathcal{H}\end{aligned} \tag{5.14}$$

and our investigations in this area are an analysis of the numerator quotient in this context.

6 The Distinguished connection A_0 and its consequences.

All of the preceding is general and did not depend on the choice of any particular connection. However, besides the ability to contract and project within the Einstein-Riemann paradigm, the secondary benefit we have discussed is the existence of a distinguished Levi-Civita connection.

It is worth briefly recalling how this connection comes into being by summoned from the choice of metric. Let $g \in \Gamma^2(S^2(T^*))$ be a metric on the tangent bundle of a manifold M. Then if $\beta \in \Omega^1(T^*)$ is a one form on M and $\tilde{g} \in \Gamma^2(S^2(T))$ is the dual metric, then

$$\nabla^g \beta = \underbrace{\underbrace{d\beta}_{\Lambda^2(T^*M)} \oplus \underbrace{L_{\tilde{g}\beta}(g)}_{S^2(T^*M)}}_{T^*M \otimes T^*M} \in \Omega^1(M, T^*) \tag{6.1}$$

or, in other words, the exterior derivative is already half of a connection. To get the other half, we use the remaining naturally occurring derivative (the Lie derivative $L(g)$) to Lie differentiate a symmetric two tensor. The main trick, however is that the metric is actually used twice, as the Lie derivative requires a vector field to pick a direction. And since we are operating on a 1-form β. we must turn it into a vector field using the same metric that is to be differentiated. As such, the Levi-Civita connection is summoned via the Lie derivative's ability to complement the exterior derivative and provide its missing half.

We now assume, however, that there is a distinguished choice of connection A_0 (such as that which we have just constructed for a given metric), which has been made and can be utilized in what follows. In our case, we begin with a metric $\gimel$, which determines a connection Aleph $\aleph_\gimel$ on the bundle TX. At that point, the connection $\aleph$ determines a metric $g_\aleph$ on TY given the metric data already present on the horizontal and vertical tangent sub-bundles H_Y and V_Y. This, in turn, determines a Levi-Civita connection $\nabla^{g_\aleph} = \nabla^0$ which, by abuse of notation, we will consider as determining a spin connection on the structure bundle P_H of Dirac spinors over Y. When we have temporarily fixed a given metric $\gimel$ on X giving rise to this spin connection on Y, we will refer to the connection as $A_0 \approx \nabla^0 \approx d_0$ as needed and when the possibility of confusion is likely resolved by context.

As a preliminary note, we remind ourselves that the choice of a distinguished connection results in a canonical isomorphism $\mathcal{A} \overset{A_0}{\cong} \mathcal{N}$ giving us an action of $\mathcal{G}$ on $\mathcal{N}$ induced from the right action of $\mathcal{G}$ on $\mathcal{A}$. Thus, if a connection $A \in \mathcal{A}$ is expressed relative to a base connection A_0 as $A = A_0 + \alpha$, we now have a right action of $\mathcal{G}$ on $\mathcal{N}$ which can be expressed via the explicit formula:

$$\alpha \cdot g = \alpha \cdot (\varepsilon, \varpi) = \varepsilon^{-1}(\alpha)\varepsilon + \varepsilon^{-1}(d_{A_0}\varepsilon) + \varpi \tag{6.2}$$

on the space of connections $\mathcal{A}$.

6.1 The Tilted Map τ into $\mathcal{G}$: the Homomorphism Rule

Lemma 6.1 *Given a base connection $A_0 \in \mathcal{A}$, the map*

$$\tau_{A_0} : \mathcal{H} \longrightarrow \mathcal{G} \tag{6.3}$$

given explicitly by

$$\tau_{A_0}(h) = (h, \delta(A_0, A_0 \cdot h)) = (h, A_0 - A_0 \cdot h) = (h, -h^{-1}(d_{A_0}h)) \tag{6.4}$$

is an injective Lie Group homomorphism. [7]

Proof: The map is clearly injective because it is the identity map onto and into the first factor in the semi-direct product. To see that it is a Lie-group homomorphism, we argue that:

$$\tau_{A_0}(h_1 \cdot h_2) = (h_1 \cdot h_2, -(h_1 \cdot h_2)^{-1}(d_{A_0}(h_1 \cdot h_2))) \tag{6.5}$$

$$= (h_1 \cdot h_2, -((h_2^{-1} \cdot h_1^{-1})((d_{A_0}h_1) \cdot h_2 + h_1 \cdot (d_{A_0}h_2)))$$

$$= (h_1 \cdot h_2, -(h_2^{-1} \cdot (h_1^{-1}(d_{A_0}h_1)) \cdot h_2 + h_2^{-1} \cdot (d_{A_0}h_2))$$

$$(h_1, -h_1^{-1}(d_{A_0}h_1)) \cdot (h_2, -h_2^{-1}(d_{A_0}h_2)) = \tau_{A_0}(h_1) \cdot \tau_{A_0}(h_2)$$

[7] There appears to have been multiple sign conventions and some notational shifts within the files from which this section was reassembled. We will endeavor to sort this out but there may be 2 or more conflicting convetions in this section.

QED.

This subgroup $\tau_{A_0}(\mathcal{H}) \subset \mathcal{G}$ is important to our constructions and thus warrants a name to distinguish it from the more obvious $\mathcal{H}$ subgroup.

Definition 6.2 *The image of $\mathcal{H}$ inside $\mathcal{G}$ under τ_{A_0} will be referred to as the* **The Tedha**[8] **Gauge Group** $\mathcal{H}_{\tau_{A_0}}$ *inside $\mathcal{G}$, with the image of $\mathcal{H}$ simply included onto the first factor of $\mathcal{G}$ and sent to 0 in the second factor being referred to as the* **Trivial or Seedhe Gauge Group** *to avoid confusion.*

6.2 Stabilizer Subgroup

If we act on the space of connections using the natural right action of the inhomogeneous gauge group $\mathcal{G}$ we may ask what the stabilizer subgroup is for the Levi-Civita spin connection A_0. To this end, solving for $g \in \mathcal{G}$ stabilizing A_0 we have:

$$0 = A_0 \cdot g - A_0 = A_0 \cdot \varepsilon + \varpi - A_0 = (A_0 + \varepsilon^{-1} \cdot d_0 \varepsilon + \varpi) - A_0 \tag{6.6}$$

implying

$$\varpi = -\varepsilon^{-1} d_{A_0} \varepsilon \tag{6.7}$$

so that the flipped $\hat{\tau}$ map:

$$\hat{\tau}_0 : \mathcal{H} \longrightarrow \mathcal{G} \tag{6.8}$$

given by:

$$\hat{\tau}_0(h) = (h, -h^{-1} d_0 h) \tag{6.9}$$

provides a parameterization.

6.3 $\mathcal{G}$ as Principal bundle: Action by Tilted Gauge Transformations Rule

The choice of a base connection A_0 determines a surjection

$$\pi_{A_0} : \mathcal{G} \longrightarrow \mathcal{N} \tag{6.10}$$

given by

$$\pi_{A_0}(g) = \pi_{A_0}((\varepsilon, \varpi)) = \varepsilon \varpi \varepsilon^{-1} + (d_{A_0} \varepsilon)\varepsilon^{-1} \tag{6.11}$$

which can be taken to be the projection map in the homogeneous principal $\mathcal{H}$-fibration:

$$\begin{array}{ccc} \mathcal{H} & \hookrightarrow & \mathcal{P}_{\mathcal{H}} = \mathcal{G} \\ & & \downarrow \pi_{A_0} \\ & & \mathcal{B} = \mathcal{G}/\mathcal{H}_{\tau_{A_0}} = \mathcal{N} \end{array} \tag{6.12}$$

determined by the right action of $\mathcal{H}$ as tilted subgroup on $\mathcal{G}$

$$g \cdot \tau_{A_0}(h) = (\varepsilon, \varpi) \cdot \tau_{A_0}(h) = (\varepsilon \cdot h, h^{-1} \varpi h - h^{-1}(d_{A_0} h)) \tag{6.13}$$

via the τ_{A_0} homomorphism.

[8] This is transliterated Hindi for slanted or crooked and got stuck in the author's head many years ago via his wife's usage. Seedhe means straight by the same token.

Lemma 6.3 *The map π_{A_0}, is the projection map for the natural right action of $\tau_{A_0}(\mathcal{H})$ on $\mathcal{G}$.*

Proof:

$$\pi_{A_0}(g\cdot\tau_{A_0}(h)) = \pi_{A_0}((\varepsilon,\varpi)\cdot(h,-h^{-1}d_{A_0}h)) = \pi_{A_0}((\varepsilon\cdot h, h^{-1}\cdot\varpi\cdot h - h^{-1}d_{A_0}h)) \tag{6.14}$$

$$= (\varepsilon\cdot h)(h^{-1}\varpi h - h^{-1}d_{A_)}h)(\varepsilon\cdot h)^{-1} + (d_{A_0}(\varepsilon\cdot h))(\varepsilon\cdot h)^{-1}$$

$$= (\varepsilon\cdot h\cdot h^{-1}\varpi h\cdot h^{-1}\cdot\varepsilon^{-1} - \varepsilon\cdot h\cdot h^{-1}d_{A_)}h)\cdot h^{-1}\cdot\varepsilon^{-1} + ((d_{A_0}\varepsilon)\cdot h))\cdot h^{-1}\cdot\varepsilon^{-1} + (\varepsilon\cdot d_{A_0}(h)))\cdot h^{-1}\cdot\varepsilon^{-1}$$

$$= \varepsilon\cdot\varpi\cdot\varepsilon^{-1} - \varepsilon\cdot(d_{A_)}h)\cdot h^{-1}\cdot\varepsilon^{-1} + (d_{A_0}\varepsilon)\cdot\varepsilon^{-1} + \varepsilon\cdot(d_{A_0}h)\cdot h^{-1}\cdot\varepsilon^{-1}$$

$$= \varepsilon\cdot\varpi\cdot\varepsilon^{-1} + (d_{A_0}\varepsilon)\cdot\varepsilon^{-1} = \pi_{A_0}((\varepsilon,\varpi)) = \pi_{A_0}(g)$$

As a benefit of this homogeneous principal fibration, we also gain a left action of $\mathcal{G}$ on $\mathcal{B} = \mathcal{N}$ via

Lemma 6.4 *The rule*

$$g\cdot\gamma = (\varepsilon,\varpi)\cdot\gamma = \varepsilon(\varpi+\gamma)\varepsilon^{-1} + (d_{A_0}\varepsilon)\varepsilon^{-1} \tag{6.15}$$

determines a left group action of $\mathcal{G}$ on $\mathcal{N}$.

Proof: The lemma can be seen from direct application of the preceding discussion and rules:

$$(g_1\cdot g_2)\cdot\gamma = ((\varepsilon_1,\varpi_1)\cdot(\varepsilon_2,\varpi_2))\cdot\gamma = (\varepsilon_1\cdot\varepsilon_2, \mathrm{Aut}(\varepsilon_2^{-1},\varpi_1)+\varpi_2)\cdot\gamma \tag{6.16}$$

$$+(\varepsilon_1\cdot\varepsilon_2)(\mathrm{Aut}(\varepsilon_2^{-1},\varpi_1)+\varpi_2+\gamma)(\varepsilon_1\cdot\varepsilon_2)^{-1} + (d_{A_0}(\varepsilon_1\cdot\varepsilon_2))(\varepsilon_1\cdot\varepsilon_2)^{-1}$$

$$= \varepsilon_1\cdot\varepsilon_2\cdot((\varepsilon_2^{-1}\cdot\varpi_1\cdot\varepsilon_2)+\varpi_2+\gamma)\cdot\varepsilon_2^{-1}\cdot\varepsilon_1^{-1} + ((d_{A_0}\varepsilon_1)\cdot\varepsilon_2 + \varepsilon_1\cdot(d_{A_0}\varepsilon_2))\cdot\varepsilon_2^{-1}\cdot\varepsilon_1^{-1}$$

$$= ((\varepsilon_1\cdot\varpi_1\cdot\varepsilon_1^{-1}) + \varepsilon_1\cdot(\varepsilon_2\cdot\varpi_2\cdot\varepsilon_2^{-1})\cdot\varepsilon_1^{-1} + \varepsilon_1\cdot(\varepsilon_2\cdot\gamma\cdot\varepsilon_2^{-1})\cdot\varepsilon_1^{-1} - (d_{A_0}\varepsilon_1)\cdot\varepsilon_1^{-1} - \varepsilon_1\cdot((d_{A_0}\varepsilon_2)\cdot\varepsilon_2^{-1})\cdot\varepsilon_1^{-1}$$

$$= \varepsilon_1\cdot(\varepsilon_2\cdot(\varpi_2+\gamma)\cdot\varepsilon_2^{-1} + (d_{A_0}\varepsilon_2)\cdot\varepsilon_2^{-1} + \varpi_1)\cdot\varepsilon_1^{-1} + (d_{A_0}\varepsilon_1)\cdot\varepsilon_1^{-1}$$

$$= (\varepsilon_1,\varpi_1)\cdot((\varepsilon_2,\varpi_2)\cdot\gamma) = g_1\cdot(g_2\cdot\gamma)$$

6.4 Map out of $\mathcal{G}$ to spaces of connection: Gauge Equivariance

Lemma 6.5 *The choice of a base connection A_0 also determines a map*

$$\mu_{A_0} : \mathcal{G} \longrightarrow \mathcal{A}\times\mathcal{A} \tag{6.17}$$

into the space of "bi-connections" $\mathcal{A}\times\mathcal{A}$ according to:

$$\mu_{A_0}(g) = \mu_{A_0}((\varepsilon,\ \varpi)) = (A_0+\varpi,\ A_0\cdot\varepsilon) \in \mathcal{A}\times\mathcal{A} \tag{6.18}$$

$$= (A_0+\varpi,\ A_0+\varepsilon^{-1}(d_{A_0}\varepsilon)) \tag{6.19}$$

so that the map μ_{A_0} of right $\mathcal{H}$ spaces is $\mathcal{H}_{\tau_{A_0}}$-equivariant.

Proof: The proof is immediate according to the following:

$$\mu_{A_0}(g \cdot \tau_{A_0}(h)) = \mu_{A_0}((\varepsilon,\ \varpi) \cdot \tau_{A_0}(h)) = \mu_{A_0}((\varepsilon,\ \varpi) \cdot (h, -h^{-1}d_{A_0}h)) \quad (6.20)$$

$$= (d_{A_0} + h^{-1}\varpi h + h^{-1}(d_{A_0}h),\ d_{A_0} + h^{-1}(\varepsilon^{-1}(d_{A_0}\varepsilon))h + h^{-1}(d_{A_0}h)) \in \mathcal{A} \times \mathcal{A}$$

$$= (A_0 + \varpi, A_0 + \varepsilon^{-1}(d_{A_0}\varepsilon)) \cdot h = \mu_{A_0}(g) \cdot h$$

QED

Definition 6.6 *The map μ_{A_0} will be called the* **Bi-Connection** *map in what follows.*

If we use the natural action of $\mathcal{H}_{\tau_{A_0}}$ on $\mathcal{A}$ to form an associated bundle of affine spaces with total space $\mathcal{T}_{\mathcal{A}}$, the bi-connection can be seen as determining two natural sections σ_1, σ_2 depicted below as:

$$\begin{array}{ccc} \mathcal{A} & \hookrightarrow & \mathcal{T}_{\mathcal{A}} \\ & & \pi \downarrow\uparrow \sigma_{1,2} \\ & & \mathcal{B} \end{array} \quad (6.21)$$

The values of these two sections $\sigma_{1,2}$ will be known as the A and B connections respectively, written as

$$A_\omega = \nabla^0 + \varpi_\omega \qquad B_\omega = \nabla^0 + \varepsilon_\omega^{-1}(\nabla^0 \varepsilon_\omega) \quad (6.22)$$

when needed separately.

7 The Augmented or Displaced Torsion

The Torsion Tensor has always presented a puzzle. It is traditionally introduced very early on in the study of Riemannian geometry and is almost never heard from thereafter except in niche explorations. One modern interpretation of this stylized fact could be that the torsion of a connection is afflicted with a disease that keeps it from being 'gauge covariant' and thus useful to the mainstream of modern theory.

An important principal of some potential relevance here is seldom stated explicitly. It is that when faced with a mathematical disease, it is often advantageous to seek a second disease in the hopes that an even number of diseases might be poised to kill each other in pairs.

7.1 Augmented Torsion and its Transformations

Our strategy is to think as follows. There are effectively two different ways of transforming a connection. One is by gauge transformation and the second is by 'translation' by adding to it an *ad*-valued 1-form on Y. Because these two group actions have been intertwined within the inhomogeneous gauge group, there are effectively two separate ways to transform the spin connection inherited from the Levi-Civita connection of the metric. Taking their difference is meaningful because both transformed connections carry exactly the same global disease that renders them non-gauge covariant. Therefore the ability to generate two separate connections with a common disease from a single group element in $\mathcal{G}$ which is still a set theoretic if not algebraic product is meaningful.

Definition 7.1 *There is a well defined map from the inhomogeneous gauge group $\mathcal{G}$ to the space of generalized torsion tensors $\mathcal{N}$ viewed as a right $\mathcal{H}$-module, given by:*

$$\delta \circ \mu_{A_0} : \mathcal{G} \longrightarrow \mathcal{N} \tag{7.1}$$

so that we can define **The Augmented Torsion Tensor** $T_g = T_{\varepsilon,\varpi} = T((\varepsilon, \varpi))$ *according to*

$$T_g = \delta(\mu_{A_0}((\varepsilon, \varpi))) \tag{7.2}$$

which is given explicitly by:

$$T_g = \varpi - \varepsilon^{-1}(d_{A_0}\varepsilon). \tag{7.3}$$

The Augmented Torsion is distinguished in that it is very well behaved under $\mathcal{H}_{\tau_{A_0}}$ transformations. This can be seen explicitly as:

Lemma 7.2 *The augmented torsion tensor as a map from $\mathcal{G}$ to $\mathcal{N}$ is equivariant as a map of right $\mathcal{H}_{\tau_{A_0}}$-spaces.*

Proof:

$$\begin{aligned} T_{g\cdot\tau_{A_0}(h)} = T_{(\varepsilon,\varpi)\cdot\tau_{A_0}(h)} = T_{(\varepsilon\cdot h, h^{-1}\varpi h - h^{-1}(d_{A_0}h))} \\ = h^{-1}\varpi h + h^{-1}(d_{A_0}h) - (\varepsilon \cdot h)^{-1}(d_{A_0}(\varepsilon \cdot h)) \\ = h^{-1}\varpi h + h^{-1}(d_{A_0}h) - (h^{-1} \cdot \varepsilon^{-1})((d_{A_0}\varepsilon) \cdot h + h^{-1} \cdot (d_{A_0}h)) \\ = h^{-1}(\varpi - \varepsilon^{-1}(d_{A_0}\varepsilon))h = \mathrm{Ad}(h^{-1}, T_g) \end{aligned} \tag{7.4}$$

QED

In essence, the group $\mathcal{G}$ is topologically the Cartesian product of two separate groups acting on the common space $\mathcal{A}$. The presence of a single connection ∇^0 and a single element $\beta = (\varepsilon_\beta, \varpi_\beta) \in \mathcal{G}$ leads to two separate new connections $A = (\nabla^0) \cdot \varepsilon, B = (\nabla^0) \cdot \varpi$ which in an affine space suggests taking a difference $A - B \in \mathcal{N}$ which diagrammatically appears as

$$\begin{array}{ccccc} & & \mathcal{A} & & \\ & \varepsilon \nearrow & & \searrow & \\ \nabla_0 & & \times & & \mathcal{N} \\ & \varpi \searrow & & \nearrow & \\ & & \mathcal{A} & & \end{array} \tag{7.5}$$

7.2 Summary of spaces and maps

The following diagram summarizes much of what we have just discussed:

$$\begin{array}{ccccccc} \mathcal{H} & \overset{\tau_{A_0}}{\hookrightarrow} & \mathcal{G} & \overset{\mu_{A_0}}{\longrightarrow} & \mathcal{A} \times \mathcal{A} & \overset{\delta}{\longrightarrow} & \mathcal{N}_{\mathcal{H}_R} \\ & & \downarrow \pi_{A_0} & & & & \\ & & \mathcal{A}_{A_0} \cong \mathcal{N}_{\mathcal{G}_L} & & & & \end{array} \tag{7.6}$$

with respect to the integral spin fields. The addition of the half-integral spin fields leads to a sort of super-space like version of the above.

8 The Family of Shiab Operators

With this machinery notated and established, we can turn to the second major advantage of working in the Meta-Riemannian paradigm.

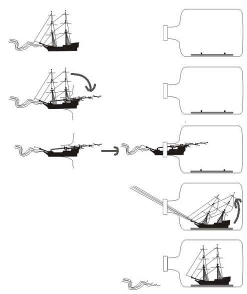

Figure 5: Ship In a Bottle Construction.

In essence, gauge theory and relativity have been disconnected because of the incompatibility of contraction and gauge covariance of terms within the action. The former typically contracts between a differential form and some other bundle associated to the tangent bundle where the differential form is valued, while gauge rotation typically acts on the latter bundle without touching the forms.

The 'Ship in a Bottle' construction attempts to get around this difficulty. By incorporating the gauge group into the contraction operator, the gauge group rotates only the bundle valued portion of a collection contracting forms $\{\Phi_i\}$ in which these special invariant differential forms are valued in such a way that it exactly compensates for the a symmetry of treatment in the form being contracted. Suppose for example that η is a gauge covariant ad$-$valued differential form. Then a Shiab contraction operator might look like:

$$\circledcirc_{\varepsilon}\eta = *^n i^{\frac{1\pm1}{2}}[(\varepsilon^{-1}\Phi_r\varepsilon)\wedge *^m\eta]_{\pm} \tag{8.1}$$

or perhaps a sum of such terms, where typically Φ_r is a normed Lie Algebra Valued r-form valued in an invariant subspace of the structure group of the tangent bundle and the bracket is either a commutator or an anti-commutator with a factor of i out front in the case of the latter.

There is most likely a byzantine taxonomy of such objects along the lines of what Reese Harvey detailed for the Clifford Algebras in his book on Spinors and Calibrations.

The author is no longer in a position to go chasing after the complete picture and simply details some of the available tools for customizing such operators.

8.1 Pure Trace Elements

As vector spaces over the real numbers,

$$\Lambda^*(T^*X) = \mathrm{Cl}(T^*X) \tag{8.2}$$

but where the algebraic structures are somewhat different. In our case, when Y inherits a $(7,7)$ metric, we have an equivalence at the level of vector spaces:

$$\vartheta : \Lambda^*(T^*Y) \longrightarrow \mathrm{Cl}(T^*X) = \mathbb{R}(128). \tag{8.3}$$

If $\mathrm{Cl}_{\mathbb{R}}(T^*Y)$ is included in its own own complexification it can be seen within the matrix algebras

$$\begin{array}{ccc} \mathfrak{spin}(64,64) & \longrightarrow & \mathfrak{u}(64,64) \\ \downarrow & & \downarrow \\ \vartheta : \mathfrak{gl}(128,\mathbb{R}) & \longrightarrow & \mathfrak{gl}(128,\mathbb{C}) \\ |\,| & & |\,| \\ \vartheta : \mathrm{Cl}_{\mathbb{R}}(T^*Y) & \longrightarrow & \mathrm{Cl}_{\mathbb{C}}(T^*Y) \end{array} \tag{8.4}$$

which commute on:

$$\mathfrak{so}(64,64) \text{ “ = ” } \underbrace{(\Lambda^2 \oplus \Lambda^6 \oplus \Lambda^{10} \oplus \Lambda^{14})}_{\$_L \otimes \$_R = \$_L \otimes \$_L^* = \$_R^* \otimes \$_R} \oplus \underbrace{(\Lambda^1 \oplus \Lambda^5 \oplus \Lambda^9 \oplus \Lambda^{13})}_{\Lambda^2(\$_L) \oplus \Lambda^2(\$_R)} \subset \mathrm{Cl}_{\mathbb{R}(7,7)}. \tag{8.5}$$

What differs instead is what happens on

$$\mathfrak{u}(64,64)/\mathfrak{so}(64,64) \text{ “ = ” } k_0\Lambda^0 \oplus k_3\Lambda^3 \oplus k_4\Lambda^4 \oplus k_7\Lambda^7 \oplus k_8\Lambda^8 \oplus k_{11}\Lambda^{11} \oplus k_{12}\Lambda^{12} \tag{8.6}$$

which must be multiplied by various factors of i inside the complexification.

Definition 8.1 *Let $\{\Phi_i\}_{i=0}^{14}$ be a basis for the invariant subspaces of*

$$[\Lambda^i(\mathbb{R}^{7,7}) \otimes \mathfrak{u}(64,64)]_{Spin(7,7)} \tag{8.7}$$

seen as a $Spin(7,7)$ representation.

8.2 Thoughts on Operator Choice

The particulars of the Shiab operators are workmanlike and not of much interest. The interesting aspect of them, is that they all essentially look like contracting over indices in the fashion familiar from Riemannian tensor geometry, but with some aspect of conjugation by the gauge group element $\varepsilon \in \mathcal{H} \subset \mathcal{G}$ living inside the inhomogeneous gauge group $\mathcal{G}$ as a non-linear sigma-field of sorts.

The author remembers choosing them years ago via representation theory techniques involving highest weight representations rather than by the more indicial methods presented here with invariant elements Φ_i. The advantage was that the Bianchi identity was able to pick the best and most appropriate operator in different circumstances. Unfortunately, the author is no longer conversant in that language and has been unable to locate the notes from decades ago that originally picked out the operator of choice to play the role of the Swerve here ◉. The author either hopes to find the original calculations or to get back to the point where he can reconstruct this argument based on using the Bianchi identity to guarantee gauge perpendicularity and/or use the Bianchi identity to guarantee automatic solution of a differential equation in the curvature.

Note: A brief discussion of additional tools for Shiab construction has been moved to a a short technical appendix.

9 Lagrangians

There are multiple considerations in putting forward Lagrangians in this context. In particular there are issues of redundant equations, Bianchi Identities, cohomological considerations for deformation complexes, so-called Supersymmetry, agreement with prior physical equations, and the issue of Dirac-Pairs where one set of more restrictive (usually first order) equations guarantees the solution of the equations of a different related (usually second order) Lagrangians.
[9]

9.1 The 1st Order Bosonic Sector

The purely Bosonic portion of the action is a real valued function:

$$\mathcal{I}_1^{\mathrm{B}} : \mathcal{G} \times \mathcal{MET}(X^{1,3}) :\longrightarrow \mathbb{R} \tag{9.1}$$

While there are other possibilities to explore for the choice of the Shiab operator, Let

$$\circledcirc_\varepsilon : \Omega^2(Y^{7,7}, \mathrm{ad}) \longrightarrow \Omega^{d-1}(Y^{7,7}, \mathrm{ad}) \tag{9.2}$$

we will begin with one[10] which makes the parallel to Einstein's contraction of the full Riemannian curvature explicit:

$$\circledcirc_\varepsilon \xi = \underbrace{[(\varepsilon^{-1}\Phi^1\varepsilon) \wedge (*\xi)]}_{\text{Ricci Like}} - \frac{*}{2}[\underbrace{(\varepsilon^{-1}\Phi^1\varepsilon)}_{g_{\mu\nu}\text{- like.}} \wedge * \underbrace{[(\varepsilon^{-1}\Phi^2\varepsilon) \wedge (*\xi)]]}_{\text{Ricci Scalar Like}} \tag{9.3}$$

for a gauge covariant ad-valued 2-form $\xi \in \Omega^2(Y, \mathrm{ad})$.

Here, as in Einstein's case, the Weyl curvature tensor is annihilated by the contraction operator above so the operator preserves and mixes only the analogues of the Ricci and Scalar curvature components.

The puzzle of how to kill off the Weyl curvature contribution to recover Riemannian geometry's ability to form Einstein tensors for gravity in such a way as to preserve Ehresmannian gauge covariance is part of what is meant by Geometric Unity. This leads to a model that abstracts the Einstein-Hilbert and Chern-Simons actions to generate linear field equations in the Riemannian and

[9]We have closely followed the history referred to by Dirac in his 1963 Scientific American Article discussing Schrodinger and the Multiple iterations of Einstein's (and Grossman's) introduction of General Relativity and taken from them that an author will have to fine tune the instantiation of a new idea. What we take away from this is that the tiny minority of authors who put forward new physical law have to have the right to explore instantiations of new ideas without the community over indexing on the instance put forward. As with any release, the interested community is welcome to send bug fixes.

[10]The author used to construct such objects from representation theory concepts like highest weights. The Shiab operator that he settled on (but cannot yet now locate) was chosen for its properties relative to the Bianchi identity. Even if it can be located, it will be in a different language with which the author no longer feels entirely familiar. So it is.

Ehresmannian curvature tensors via an action $\mathcal{I}_1^{\mathrm{B}}$ which for only the Bosonic fields of integral spin (ε, ϖ on Y and $\daleth$ on X) looks like:

$$\mathcal{I}_1^{\mathrm{B}}(\omega_Y, \daleth_X) = \mathcal{I}_1^{\mathrm{B}}((\varepsilon_Y, \varpi_Y), \daleth_X) \tag{9.4}$$

$$= < \overbrace{T_\omega}^{\substack{\text{Shifted}\\ \text{Torsion}}}, \underbrace{*}_{\substack{\text{Hodge}\\ \text{Star}}} (\overbrace{\circledcirc_\omega}^{\substack{\text{Einstein}\\ \text{Ricci}\\ \text{Shiab}}} (\underbrace{F_{B_\omega}}_{\substack{\text{Metric}\\ \text{Curvature}}} + \overbrace{\frac{1}{2}\, d_{B_\omega} T_\omega + \frac{1}{3}[T_\omega, T_\omega]}^{\text{C-S Like Terms}}) + \frac{\kappa_1}{2} \underbrace{T_\omega}_{\substack{\text{Shifted}\\ \text{Torsion}}}) > \overbrace{g_\daleth}^{\substack{\text{Zorro}\\ \text{Metric}}}$$

$$= \int_Y \mathrm{Tr}((\varpi - \varepsilon^{-1} d_0 \varepsilon) \wedge * (\underbrace{[(\varepsilon^{-1}\Phi^1\varepsilon) \wedge (*(\cdot))]}_{\text{Ricci Like}} - \frac{*}{2}[(\varepsilon^{-1}\Phi^1\varepsilon) \wedge * \underbrace{[(\varepsilon^{-1}\Phi^2\varepsilon) \wedge (*(\cdot))]}_{\text{Ricci Scalar Like}}])$$

$$(\varepsilon^{-1}\widetilde{R^\theta_{ij\sigma}}\varepsilon + \frac{1}{2}(d_0 + \varepsilon^{-1} d_0 \varepsilon)(\varpi - \varepsilon^{-1} d_0 \varepsilon) + \frac{1}{3}[\varpi - \varepsilon^{-1} d_0 \varepsilon, \varpi - \varepsilon^{-1} d_0 \varepsilon]))$$

$$+ \kappa_1 \int_Y \mathrm{Tr}((\varpi - \varepsilon^{-1} d_0 \varepsilon) \wedge * (\varpi - \varepsilon^{-1} d_0 \varepsilon))$$

where:

1. $\widetilde{R^\theta_{ij\sigma}}$ is the Spinor bundle's Riemannian curvature induced from the Levi-Civita connection.
2. $T_\omega = \varpi - \varepsilon^{-1} d_0 \varepsilon \in \Omega^1(Y, \mathrm{ad})$ is the augmented torsion tensor.
3. The Shiab Operator $\circledcirc_\omega$ on an ad-valued 2-form $\xi \in \Omega^2(Y, \mathrm{ad})$is given in accordance with the Einsteinian contraction

$$\circledcirc_\varepsilon \xi = \underbrace{[(\varepsilon^{-1}\Phi^1\varepsilon) \wedge (*\xi)]}_{\text{Ricci Like}} - \frac{*}{2}[(\varepsilon^{-1}\Phi^1\varepsilon) \wedge * \underbrace{[(\varepsilon^{-1}\Phi^2\varepsilon) \wedge (*\xi)]}_{\text{Ricci Scalar Like}}]$$

4. The B_ω connection is the gauge rotation of the Levi-Civita Spin Connection: $B_\omega = \nabla^0 + \varepsilon^{-1} d_0 \varepsilon$

By the calculus of variations we obtain Euler-Lagrange Equations of the form:

$$d\mathcal{I}_1^B(\daleth, \omega)|_{Y^{14}} = \begin{pmatrix} \Upsilon_\omega \\ \oplus \\ \Xi_\omega \end{pmatrix} \in \begin{matrix} \Omega^{d-1}(\mathrm{ad}) \\ \oplus \\ \Omega^d(\mathrm{ad}) \end{matrix} \tag{9.5}$$

where generally

$$\Xi = \mathfrak{D}_\omega \Upsilon_\omega \tag{9.6}$$

for some operator differential $\mathfrak{D}_\omega$ so that the vanishing of $\Xi_\omega = 0$ need not be considered if $\Upsilon_\omega = 0$ by the redundant nature of the second equation.

And by gathering up terms we can express them in a fashion that is closer to the more familiar equations of General Relativity. Thus we have:

$$\frac{\partial \mathcal{I}_1^{\mathrm{B}}((\varepsilon_Y, \varpi_Y + s\alpha), \daleth_X)}{\partial s} =< \alpha, \circledcirc_\omega F_{A_\omega} + *\kappa_1 T_\omega >=< \alpha, \Upsilon_\omega^B > \tag{9.7}$$

$$\underbrace{\underbrace{\circledcirc F_\omega}_{\text{Swerved Curvature}}}_{= \text{ Swervature} = \mathcal{S}_\omega} = \underbrace{- * \kappa_1 \underbrace{T_\omega}_{\text{Displaced Torsion}}}_{\text{Displasion}=\mathcal{T}_\omega} \tag{9.8}$$

So that if one wanted to locate and recover from a GU equation:

$$\Upsilon_\omega = \mathcal{S}_\omega - \mathcal{T}_\omega = 0 \tag{9.9}$$

the more familiar terms of the Einstein Field Equations for Gravity, it would generate an annotated equation along the lines of:

$$\underbrace{\underbrace{\mathcal{S}_\omega}_{R_{\mu\nu} - \frac{s}{2} g_{\mu\nu}} = \underbrace{\mathcal{T}_\omega}_{\Lambda g_{\mu\nu}}}_{T_{\mu\nu}} \tag{9.10}$$

9.2 Second Order Euler-Lagrange Equations

One of the Claims of Geometric Unity is that we have been unsuccessful in Unifying the four basic equations for Gravity, Non-Gravitational force, Matter and Higgs phenomena because they belong to a Dirac Pair. That is, we believe that the Einstein and Dirac equations belong to a unifying equation which is in the sense of Dirac something of a square root of a different equation or Lagrangian related to the Yang-Mills-Maxwell equation and the Higgs version of the Klein-Gordon equation. Thus we should seek to unifying our equations and Lagrangians much the way Dirac unified first and second order equations with his masterstroke To this end we focus on second Lagrangian of 2nd order:

$$\mathcal{I}_2^{\mathrm{B}}((\varepsilon_Y, \varpi_Y), \daleth_X) = ||\Upsilon_\omega^B||^2 \tag{9.11}$$

$$\frac{\partial \mathcal{I}_2^{\mathrm{B}}((\varepsilon_Y, \varpi_Y + s\alpha), \daleth_X)}{\partial s} =< \alpha, 2(d_{A_\omega}^* \circledcirc_\omega^* + \ \kappa_1 \mathrm{Id}) \Upsilon_\omega^B) > \tag{9.12}$$

$$= 2 < \alpha, \quad \underbrace{d_{A_\omega}^* \circledcirc_\omega^* \circledcirc_\omega F_{A_\omega}}_{\text{New 'Yang-Mills' Term}} \quad + \kappa_1 d_{A_\omega}^* \circledcirc_\omega^* T_\omega + \kappa_1 * \circledcirc_\omega F_{A_\omega} + \kappa_1^2 T_\omega > \tag{9.13}$$

yielding

$$\underbrace{\mathbf{D}_\omega^* F_{A_\omega} = J_\omega^B}_{\text{Yang-Mills-Maxwell like equation with Bosonic source.}} \tag{9.14}$$

but more efficiently as

$$\mathcal{D}_\omega^* \Upsilon_\omega = 0 \tag{9.15}$$

which is more natural within Geometric Unity.

9.3 The Fermionic Sector

In the case of the Fermionic content we can at the classical level take fields $\bar{\nu}, \bar{\zeta}, \nu, \zeta$ on Y to be four distinct fields when ultimately we will wish to integrate out the Dirac like operator to take a Berezinian 'integral' in the quantum theory.

For $\nu, \bar{\nu} \in \Omega^0(Y, \not{S})$ and $\zeta, \bar{\zeta} \in \Omega^1(Y, \not{S})$ we can begin with operators like:

$$\left(\begin{array}{cccc} \bar{\zeta}_- & \bar{\zeta}_+ & \bar{\nu}_- & \bar{\nu}_+ \end{array} \right)_{\rho(\varepsilon)} \cdot \underbrace{\left(\begin{array}{cccc} *\circledcirc\varpi_{++} & *\circledcirc(d_0 + \varpi_{+-}) & \varpi_{++} & d_0 + \varpi_{+-} \\ *\circledcirc(d_0 + \varpi_{-+}) & *\circledcirc\varpi_{--} & d_0 + \varpi_{-+} & \varpi_{--} \\ -\bar{\varpi}^*_{++} & -d_0^* - \bar{\varpi}^*_{+-} & 0 & 0 \\ -d_0^* - \bar{\varpi}^*_{-+} & -\bar{\varpi}^*_{--} & 0 & 0 \end{array} \right)}_{\not{D}_\omega} \cdot \left(\begin{array}{c} \zeta_+ \\ \zeta_- \\ \nu_+ \\ \nu_- \end{array} \right)_{\rho(\varepsilon^{-1})} \tag{9.16}$$

noting that other versions of the theory exist including one with a non-trivial map in the lower right quadrant of the operator. This two can be made to look closer to the Dirac Theory of Spinorial Fermions:

$$\not{D}^F_\omega \left(\begin{array}{c} \zeta \\ \nu \end{array} \right)_{\rho(\varepsilon^{-1})} = \not{D}_\omega \chi_{\varepsilon^{-1}} = 0 \tag{9.17}$$

where χ contains three generations of observed Fermions as well as Looking-Glass matter, dark Spinorial Matter, Rarita-Schwinger matter and more while $\mathcal{D}$ subsumes the Dirac Operators, and the various subfields of ω accomodate the functionings of the CKM matrix, the Higgs-Like soft mass fields, the Yukawa couplings, Gauge Potentials and the like.

Let us compile the Bosonic and Fermionic variations of the Spinorial Lagrangian terms in a single term:

$$\Upsilon^F = * \left(\begin{array}{c} d_{A_\omega}\nu + *\circledcirc d_{A_\omega}\zeta \\ \oplus \\ d^*_{A_\omega}\zeta \\ \oplus \\ \bar{\nu}\zeta + \bar{\zeta}\nu + \circledcirc\bar{\zeta}\zeta \end{array} \right) \tag{9.18}$$

as a kind of 1-form on some SuperSpace-like structure over $\mathcal{A}$:

$$\Upsilon^F_\omega \in \left(\begin{array}{c} \Omega^{d-1}(Y, \not{S}) \\ \oplus \\ \Omega^d(Y, \not{S}) \\ \oplus \\ \Omega^{d-1}(Y, \mathrm{ad}) \end{array} \right) \tag{9.19}$$

which can be combined with the variation of either a first or second order purely Bosonic Lagrangian so as to form:

$$\Upsilon_\omega = \Upsilon^B_\omega + \Upsilon^F_\omega = 0 \quad \text{or} \quad \mathcal{D}\omega^* \Upsilon^B_\omega = \Upsilon^F_\omega \tag{9.20}$$

At this point, we wish to take this mixed spinorial-tensorial Υ_ω and ask whether we are attempting to penalize this expression in our extremization because it is actually the obstruction term for a cohomology theory. That is, we choose to view 'wedging' with Υ_ω as the application of a zeroth order operator and ask if it possesses a non-trivial square root the in form of a first order differential operator δ^ω so that:

$$\Upsilon_\omega = 0 \ \text{ and } \ \sqrt{\Upsilon_\omega} = \delta^\omega \quad \rightsquigarrow \quad (\delta^\omega)^2 = 0 \quad \rightsquigarrow \quad \text{Cohomology} \tag{9.21}$$

to get a Lagrangian Cohomology theory of at least two steps:

$$\Upsilon_\omega = (\delta^\omega)^2 = \delta_2^\omega \circ \delta_1^\omega = 0 \tag{9.22}$$

of a geometrically meaningful cohomology complex.

10 Deformation Complex

The expected way to have our field equations arise naturally as the obstruction to a cohomology theory, is to first ask about the moduli space of solutions to the equations. That is, if ω^* represents a solution to the equations of motion with $\Upsilon_{\omega^*} = 0$, in what are essentially different directions, may we perturb ω^* to obtain new solutions? To that end we begin first with the purely Bosonic fields on Y and linearize our Tedhe action of the gauge group $\mathcal{H}$ on the group $\mathcal{G} = \mathcal{H} \ltimes \mathcal{N}$ via the τ_0 homomorphism by linearizing both the groups and the exponential map of the action:

$$0 \longrightarrow \underbrace{\Omega^0(\mathrm{ad})}_{\substack{\text{Symmetries:}\\ T_e\mathcal{H}}} \overbrace{\longrightarrow}^{\delta_1^\omega} \underbrace{\begin{matrix}\Omega^1(\mathrm{ad})\\ \oplus \\ \Omega^0(\mathrm{ad})\end{matrix}}_{\substack{\text{Fields:}\\ T_\omega\mathcal{G}}} \overbrace{\longrightarrow}^{\delta_2^\omega} \underbrace{\Omega^{d-1}(\mathrm{ad})}_{\substack{\text{Equations:}\\ T^*_\omega\mathcal{G}_{d-1}}} \longrightarrow 0 \tag{10.1}$$

In a certain sense, one can view the usual (twisted) DeRahm complex as the square root of the of the curvature as $d_A \circ d_A\phi = [F_A \wedge \phi]$. The same is true for the Υ-Spinor-Tensor so we may ask if there is a complex with a co-chain operator such that:

$$\sqrt{\Upsilon}_\omega = \delta^\omega \tag{10.2}$$

where the individual operators are configured so that:

$$\delta_1^\omega : \Omega^0(\mathrm{ad}) \longrightarrow \begin{matrix}\Omega^1(\mathrm{ad})\\ \oplus \\ \Omega^0(\mathrm{ad})\end{matrix} \qquad\qquad \delta_2^\omega : \begin{matrix}\Omega^1(\mathrm{ad})\\ \oplus \\ \Omega^0(\mathrm{ad})\end{matrix} \longrightarrow \Omega^{d-1}(\mathrm{ad}) \tag{10.3}$$

and in fact this will give our assemblage[11] the structure of a (Bosonic) deformation complex.

[11] Many years ago, while thinking about this, the author passed through Iceland and was amused to find that a 'Thing' in Icelandic was a 'Governing Assembly' and as such took to referring to this governing assemblage for deformations as a Thing with operators Things 1 and 2.

To expand the concept from purely integral spin fields to include those of fractional spin, we are led to linearize equations of the form:

$$\chi = \begin{pmatrix} \zeta \\ \nu \end{pmatrix} \tag{10.4}$$

$$\delta_2^\omega \circ \delta_1^\omega = \Upsilon_\omega = \begin{pmatrix} \not{D}_\omega \chi \\ \circledcirc_\omega F_{A_\omega} + *kT_\omega + \sigma(\circledcirc_\omega, \Psi, \bar{\Psi}) \end{pmatrix} = 0 \tag{10.5}$$

The first step then for us is to examine what we mean by 'essentially different directions' of perturbation as regards the symmetry built into the problem. As we have endeavored to keep our metric theory gauge theoretic, let us first try to remove the uninteresting redundancy that is merely due to the gauge symmetry of the $\mathcal{H}$ action.

The effect of an infinitesimal gauge transformation $\gamma \in T_e\mathcal{H}$ on a point $g = (\varepsilon, \varpi) \in \mathcal{G} = P_{\mathcal{H}}$ is given by:

$$\frac{d}{ds} g \cdot \tau_{A_0}(\exp(s\gamma)) = \frac{d}{ds}(\varepsilon, \varpi) \cdot \tau_{A_0}(\exp(s\gamma)) \tag{10.6}$$

$$= \frac{d}{ds}((\varepsilon, \varpi) \cdot (\exp(s\gamma),\ \exp(s\gamma)^{-1} d_{A_0} \exp(s\gamma)))$$

$$= (DL_\varepsilon \gamma, d_{A_0}\gamma - [\gamma, \varpi]) = (DL_\varepsilon \gamma, d_{A_\varpi}\gamma)$$

so that we have:

$$\delta_1^\omega = \begin{pmatrix} \delta_{1,a}^\omega \\ \oplus \\ \delta_{1,b}^\omega \end{pmatrix} = \begin{pmatrix} d_{A_\omega} \\ \oplus \\ DL_{\varepsilon_\omega} \end{pmatrix} \tag{10.7}$$

As for the second operator, we can search for it in the linearization of the equations of motion. To this end we posit:

$$\delta_2^\omega = \begin{pmatrix} \delta_{2,a}^\omega & \oplus & \delta_{2,b}^\omega \end{pmatrix} \tag{10.8}$$

$$\delta_{2,a}^\omega = \circledcirc_\omega \circ d_{A_\omega}(\cdot) + \kappa_1 * (\cdot) \tag{10.9}$$

$$\delta_{2,b}^\omega = F_{A_\omega} \wedge \circledcirc(\cdot) - \kappa_1 * d_{B_\omega}(\cdot)$$

Putting this Bosonic piece together with the Spinor deformations gives a diagram that looks something like:

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
 & \Omega^1\begin{pmatrix} \$ \\ \oplus \\ \mathrm{ad} \end{pmatrix} \arrow[rr, "\circledcirc_\varepsilon\begin{pmatrix} d_{A_\omega} & \bar{\zeta} \\ \bar{\zeta} & d_{A_\omega} \end{pmatrix}"'] \arrow[rr, dashed, bend left, "*\begin{pmatrix} 0 & \bar{\nu} \\ \bar{\nu} & 0 \end{pmatrix}"] \arrow[ddrr, "\begin{pmatrix} *d_A^* & \zeta^*\rfloor \end{pmatrix}"] \arrow[ddrr, dashed] & & \Omega^{d-1}\begin{pmatrix} \$ \\ \oplus \\ \mathrm{ad} \end{pmatrix} \\
 & \oplus & & \oplus \\
\Omega^0(\mathrm{ad}) \arrow[r, dashed, "\begin{pmatrix} \nu \\ \mathrm{Ad}_\varepsilon \end{pmatrix}"'] \arrow[uur, "\begin{pmatrix} \zeta \\ d_{A_\omega} \end{pmatrix}"] \arrow[uur, dashed] & \Omega^0\begin{pmatrix} \$ \\ \oplus \\ \mathrm{ad} \end{pmatrix} \arrow[rr, dashed, "\begin{pmatrix} *\kappa_2 & 0 \end{pmatrix}"'] \arrow[uurr, "\begin{pmatrix} -*d_{A_\omega} & 0 \\ *\bar{\zeta} & -*\varepsilon^{-1}d_0 \end{pmatrix}"'] \arrow[uurr, dashed] & & \Omega^d(\$)
\end{tikzcd}
\tag{10.10}
$$

[Note: This diagram is carried over from an older version and may contain some inconsistancies until it is stabilized. Caveat Emptor.]

11 Observed Field Content

One of the features that arises when doing away with the primary nature of space-time and replacing a single metric space with a tension between two separate but related spaces linked by metrics, is that we find ourselves in the novel situation where must relate fields that are native to different spaces. The principal means of doing this is via the pull back operation. To interpret differential equations governing fields native to Y back on X means pulling back not only bundles but so-called Jets or Sprays of sections on Y. But, to begin with, we can simply analyze the zeroth order of the activity on Y by pulling back bundles via the $\beth^*$ operation derived from making a metric observation of Y by X.

To begin with, let us detach from the spaces T_xX to general metric vector spaces W in even dimensions to say a few words about direct sums and tensor products of defining and spin representations for $\mathrm{Spin}(W)$.

11.1 Fermionic Quantum Numbers as Reply to Rabi's question.

To begin with, there is a simple rule for tensor products of defining representations and spinors whereby the tensor product

$$W \otimes \slashed{S}_W = \slashed{S}_W \oplus \slashed{R}_W \tag{11.1}$$

breaks into a piece representing the action of gamma matrices as spinor endomorphisms and a second piece giving the pure Rarita-Schwinger spin 3/2 representation corresponding to the sum of the highest weights of the factors.

We note further, that spinor representations carry the property of the exponential in that they take in direct sums as input and return products of the spinors of the summands as output.

$$W = U \oplus V \quad \rightsquigarrow \quad \slashed{S}(W) = \slashed{S}(U \oplus V) = \slashed{S}(U) \otimes \slashed{S}(V) \tag{11.2}$$

Both of these are likely to be well known to physicists. Some what less familiar is that the Rarita-Schwinger representation has slightly odd behavior when applied to direct sums of vector spaces.

The rule here is that

$$\slashed{R}(W) = \slashed{R}(U \oplus V) = \begin{pmatrix} \slashed{R}(U) \otimes \slashed{S}(V) \\ \oplus \\ \slashed{S}(U) \otimes \slashed{R}(V) \\ \oplus \\ \slashed{S}(U) \otimes \slashed{S}(V) \end{pmatrix} \tag{11.3}$$

with an odd re-appearance of a final term which has purely spinorial with no 3/2 spin Rarita-Schwinger component.

To apply the above to our situation we recognize that ζ represents a spinor valued 1-form and ν a spinor on Y with U representing the Horizontal and V the Vertical normal bundle $N_\gimel$ to the metric as an embedding

$$\gimel : X \longrightarrow Y. \tag{11.4}$$

Even at zero-level before the introduction of higher Jets, the pull back of ν, ζ and their host bundles is potentially of considerable interest.

11.2 The Three Family Problem in GU and Imposter Generations.

'Who ordered that?' -Isidore Rabi on the Muon

We have had to restrict ourselves to a world without auxiliary internal quantum numbers as essentially everything has been generated endogenously from X^4. This leaves the question of why we appear to see a rich offering of repeating internal Fermionic quantum numbers.

In fact we will make two likely to be controversial claims in this section that may appear to fly in the face of experimental observation. The first is that we do not believe that nature has simply repeated herself three times albeit at different mass scales. While we do believe that a second copy of Fermionic matter matches this description, we believe that a third family is merely effectively identical to the other two and, presumably, only at low energy.

Secondly, while we are often told that the discovery of parity violation in beta decay found in the 1950s by Chien-Shiung Wu following theories of Yang and Lee, proves that nature is intrinsically chiral, we will again hazard the guess that it is merely effectively chiral so that at a deeper level it remains intrinsically balanced between left and right. To see this more clearly we will decompose our Fermionic sector under the decompositions of $\gimel^*(\not{S}(T^*Y))$ and $\gimel^*(\not{R}(T^*Y))$ under

$$\gimel^*(T^*Y) = T^*X \oplus N_\gimel \tag{11.5}$$

To this, our rolled up Fermionic complex looks quite different under the above tangent space decomposition:

$$
\begin{array}{ccccccc}
 & \left(\begin{array}{c} \left(\begin{array}{c} \mathcal{Z}^-_{\frac{1}{2}} \\ \oplus \\ \mathcal{Q}^+_{\frac{3}{2}} \\ --\oplus-- \\ \boldsymbol{\mathcal{F}}^-_{\frac{1}{2}} \end{array} \right)^{\mathrm{Spin}(7,7)^+}_{832_-} \\ \oplus \\ \left(\boldsymbol{\mathcal{F}}^-_{\frac{1}{2}} \right)^{\mathrm{Spin}(7,7)^+}_{64_-} \end{array} \right)^{\Omega^1(\not{S}_-, Y^{14})}_{\zeta_-} & \longrightarrow & \left(\begin{array}{c} \left(\begin{array}{c} \mathcal{Z}^+_{\frac{1}{2}} \\ \oplus \\ \mathcal{Q}^-_{\frac{3}{2}} \\ --\oplus-- \\ \mathcal{F}^+_{\frac{1}{2}} \end{array} \right)^{\mathrm{Spin}(7,7)^+}_{832_+} \\ \oplus \\ \left(\mathcal{F}^+_{\frac{1}{2}} \right)^{\mathrm{Spin}(7,7)^+}_{64_+} \end{array} \right)^{\Omega^{13}(\not{S}_+, Y^{14})} & & \\
 & \oplus & \times & \oplus & & \\
\nearrow & \left(\left(\boldsymbol{\mathcal{F}}^-_{\frac{1}{2}} \right)^{\mathrm{Spin}(7,7)^+}_{64_-} \right)^{\Omega^0(\not{S}_+, Y^{14})}_{\nu_+} & & \left(\left(\mathcal{F}^+_{\frac{1}{2}} \right)^{\mathrm{Spin}(7,7)^+}_{64_+} \right)^{\Omega^{14}(\not{S}_-, Y^{14})} & \searrow & \\
0 \quad -- & ------\oplus------ & -- & ----\oplus---- & -- \quad 0 & \\
\searrow & \left(\left(\mathcal{F}^+_{\frac{1}{2}} \right)^{\mathrm{Spin}(7,7)^-}_{64_+} \right)^{\Omega^0(\not{S}_-, Y^{14})}_{\nu_-} & & \left(\left(\mathcal{F}^-_{\frac{1}{2}} \right)^{\mathrm{Spin}(7,7)^-}_{64_-} \right)^{\Omega^{14}(\not{S}_+, Y^{14})} & \nearrow & \\
 & \oplus & \times & \oplus & & \\
 & \left(\begin{array}{c} \left(\mathcal{F}^+_{\frac{1}{2}} \right)^{\mathrm{Spin}(7,7)^-}_{64_+} \\ \oplus \\ \left(\begin{array}{c} \mathcal{F}^+_{\frac{1}{2}} \\ \oplus \\ \mathcal{Q}^-_{\frac{3}{2}} \\ \oplus \\ \mathcal{Z}^+_{\frac{1}{2}} \end{array} \right)^{\mathrm{Spin}(7,7)^-}_{832_+} \end{array} \right)^{\Omega^1(\not{S}_+, Y^{14})}_{\zeta_+} & \longrightarrow & \left(\begin{array}{c} \left(\mathcal{F}^-_{\frac{1}{2}} \right)^{\mathrm{Spin}(7,7)^-}_{64_-} \\ \oplus \\ \left(\begin{array}{c} \mathcal{F}^-_{\frac{1}{2}} \\ \oplus \\ \mathcal{Q}^+_{\frac{3}{2}} \\ \oplus \\ \mathcal{Z}^-_{\frac{1}{2}} \end{array} \right)^{\mathrm{Spin}(7,7)^-}_{832_-} \end{array} \right)^{\Omega^{13}(\not{S}_-, Y^{14})} & &
\end{array}
$$

where we have used the notation:

$$\mathcal{F}^{\pm}_{\frac{1}{2}} = \begin{pmatrix} 2_{\mp} \otimes 16_{+} \\ \oplus \\ 2_{\pm} \otimes 16_{-} \end{pmatrix} \qquad \mathcal{Q}^{\pm}_{\frac{3}{2}} = \begin{pmatrix} 6_{\mp} \otimes 16_{+} \\ \oplus \\ 6_{\pm} \otimes 16_{-} \end{pmatrix} \qquad \mathcal{Z}^{\pm}_{\frac{1}{2}} = \begin{pmatrix} 2_{\mp} \otimes 144_{+} \\ \oplus \\ 2_{\pm} \otimes 144_{-} \end{pmatrix} \tag{11.6}$$

for $\text{Spin}(1,3) \times \text{Spin}(6,4)$. The idea being explored here is that the full operator depicted decouples effectively into two separate Dirac like operators, when there is no vacuum expectation value pulling the various sub-fields of ϖ to values significantly above zero. Thus we assert that a non-chiral total theory splits at the emergent level into two separate chiral theories and that the one above the dashed line corresponds to matter in our world with the other sectors not labeled by $\mathcal{F}$ to the left and above the line are currently dark to us.

11.3 Explict Values: Predicting the Rest of Rabi's Order.

With all that said above, we can now predict what the internal quantum numbers will likely be if GU is correct as per the following:

Names	Multiplicity	Dimension	Structure	Notation	Name(s)
Left Quarks	1	6	$[\mathbf{3} \times \mathbf{2}]_L^{n=1}$		
Left Anti-Quarks	1	3	$[\bar{\mathbf{3}} \times \mathbf{1}]_L^{n=2}$		
Left Anti-Quarks	1	3	$[\bar{\mathbf{3}} \times \mathbf{1}]_L^{n=-4}$		
Left Leptons	1	2	$[\mathbf{1} \times \mathbf{2}]_L^{n=-3}$		
Left Anti-Lepton	1	1	$[\mathbf{1} \times \mathbf{1}]_L^{n=6}$		
Left Anti-Lepton	1	1	$[\mathbf{1} \times \mathbf{1}]_L^{n=0}$		

after reductions of the structure groups. A more violent regime would be expected to reveal differences that are more profound than mere mass discrepancies.

Another surprise would be a new cousin spin-$\frac{3}{2}$ 'generation' $\mathcal{Q}^{+}_{\frac{3}{2}}$, in which the logic of the known matters is reversed in the sense that it is right handed matter and left handed anti-matter that feel the effects of Weak-Isospin.

Names	Multiplicity	Dimension	Structure	Notation	Name(s)
	1	6	$[\bar{\mathbf{3}} \times \mathbf{2}]_L^{n=-1}$		
	1	3	$[\mathbf{3} \times \mathbf{1}]_L^{n=-2}$		
	1	3	$[\mathbf{3} \times \mathbf{1}]_L^{n=+4}$		
	1	2	$[\mathbf{1} \times \mathbf{2}]_L^{n=+3}$		
	1	1	$[\mathbf{1} \times \mathbf{1}]_L^{n=-6}$		
	1	1	$[\mathbf{1} \times \mathbf{1}]_L^{n=0}$		

Number	Multiplicity	Dimension	Structure	Electric Charge	Name(s)
1	1	16	$[\mathbf{8}\times\mathbf{2}]_L^{n=-3}$	$-1,0$	
2	1	8	$[\mathbf{8}\times\mathbf{1}]_L^{n=0}$	0	
3	1	8	$[\mathbf{8}\times\mathbf{1}]_L^{n=6}$	1	
6	1	12	$[\bar{\mathbf{6}}\times\mathbf{2}]_L^{n=1}$	$+\frac{2}{3},-\frac{1}{3}$	
5	1	6	$[\mathbf{6}\times\mathbf{1}]_L^{n=2}$	$+\frac{1}{3}$	
4	1	6	$[\mathbf{6}\times\mathbf{1}]_L^{n=-4}$	$-\frac{2}{3}$	
12	1	9	$[\bar{\mathbf{3}}\times\mathbf{3}]_L^{n=2}$	$+\frac{4}{3},+\frac{1}{3},-\frac{2}{3}$	
11	1	9	$[\bar{\mathbf{3}}\times\mathbf{3}]_L^{n=-4}$	$+\frac{1}{3},-\frac{2}{3},-\frac{5}{3}$	
8	1	6	$[\mathbf{3}\times\mathbf{2}]_L^{n=7}$	$+\frac{5}{3},+\frac{2}{3}$	
7	1	6	$[\mathbf{3}\times\mathbf{2}]_L^{n=-5}$	$-\frac{1}{3},-\frac{4}{3}$	
20	*1	6	$[\mathbf{3}\times\mathbf{2}]_L^{n=1}$	$+\frac{2}{3},-\frac{1}{3}$	
13	1	6	$[\bar{\mathbf{3}}\times\mathbf{2}]_L^{n=5}$	$+\frac{4}{3},+\frac{1}{3}$	
10	1	3	$[\mathbf{3}\times\mathbf{1}]_L^{n=-2}$	$-\frac{1}{3}$	
9	1	3	$[\mathbf{3}\times\mathbf{1}]_L^{n=-8}$	$-\frac{4}{3}$	
20	*2	6	$[\mathbf{3}\times\mathbf{2}]_L^{n=1}$	$+\frac{2}{3},-\frac{1}{3}$	Imposter Quarks
18	2	3	$[\bar{\mathbf{3}}\times\mathbf{1}]_L^{n=2}$	$+\frac{1}{3}$	Imposter Anti-Quarks
19	2	3	$[\bar{\mathbf{3}}\times\mathbf{1}]_L^{n=-4}$	$-\frac{2}{3}$	Imposter Anti-Quarks
15	1	3	$[\mathbf{1}\times\mathbf{3}]_L^{n=6}$	$+2,+1,0$	
16	1	2	$[\mathbf{1}\times\mathbf{2}]_L^{n=-9}$	$-2,-1$	
23	1	1	$[\mathbf{1}\times\mathbf{1}]_L^{n=0}$	0	Imposter Anti-Neutrino
14	1	3	$[\mathbf{1}\times\mathbf{3}]_L^{n=0}$	$+1,0,-1$	
21	*1	2	$[\mathbf{1}\times\mathbf{2}]_L^{n=-3}$	$-1,0$	Imposter Leptons
22	1	1	$[\mathbf{1}\times\mathbf{1}]_L^{n=6}$	$+1$	Imposter Anti-Electron
17	1	2	$[\mathbf{1}\times\mathbf{2}]_L^{n=3}$	$+1,0$	
21	*1	2	$[\mathbf{1}\times\mathbf{2}]_L^{n=-3}$	$-1,0$	

11.4 Bosonic Decompositions

As we have argued previously, the observed standard model appears to be consistent with a reduction of structure group of the full Dirac Spinors on Y in three stages

1. First: to a splitting of T^*Y to T^*X together with the normal bundle $N_\daleth$ of the metric seen as an embedding, according to $\daleth^*(T^*Y) = T^*X \oplus N_\daleth$.

2. Second to a Maximal Compact Subgroup of structure group of the Normal bundle $N_\daleth$ from Spin(6, 4) to Spin(6) × Spin(4) ≅ SU(4) × SU(2) × SU(2) in accordance with the Pati-Salam theory.

3. Lastly to a complex structure on the Normal bundle from Spin(6)×Spin(4) to U(3) × U(2) where a final reductive factor may be removed or not by privileging a complex volume form.

As far as understanding the picture of the force carrying gauge potentials, we are really looking to understand what parts of $\Omega^1(Y, \mathrm{ad})$ carries meaning for us already within the standard model. To that end we can see that at the level of vector spaces if not as algebras, the ad$-$bundle is equivalent to the exterior bundle on Y. Both the branching of the 1-forms $\Omega^1(Y)$ and the ad-bundle are straightforward under pull back of $\daleth^*$ to X.

$$\daleth^*(\mathrm{ad}(Y)) = \Lambda^*(TX^4 \oplus N_{\daleth}^{10}) = \bigoplus_{i=0}^{4}\bigoplus_{j=0}^{10}(\Lambda^i(TX^4) \otimes \Lambda^j(N_{\daleth}^{10})) \tag{11.7}$$

$$= \Lambda^*(TX^4) \oplus \Lambda^*(N_{\daleth}^{10}) \oplus (\bigoplus_{i=1}^{4}\bigoplus_{j=1}^{10}(\Lambda^i(TX^4) \otimes \Lambda^j(N_{\daleth}^{10}))) \tag{11.8}$$

so

$$\Lambda^1(TX^4 \oplus N_{\daleth}^{10}) \otimes \Lambda^*(TX^4 \oplus N_{\daleth}^{10}) = (\Lambda^1(TX^4) \oplus \Lambda^1(N_{\daleth}^{10})) \otimes (\bigoplus_{i=0}^{4}\bigoplus_{j=0}^{10}(\Lambda^i(TX^4) \otimes \Lambda^j(N_{\daleth}^{10}))) \tag{11.9}$$

$$\supset \underbrace{\Lambda^1(TX^4) \otimes (\Lambda^2(N_{\daleth}^{10}) \otimes \Lambda^0(TX^4))}_{\text{Spin(6,4) GUT Gauge Potentials}} \oplus \underbrace{\Lambda^1(TX^4) \otimes (\Lambda^2(TX^4) \otimes \Lambda^0(N_{\daleth}^{10}))}_{\text{Spin(1,3) GR Torsion Tensors}} \tag{11.10}$$

$$\oplus \quad \underbrace{\Lambda^1(TX^4) \otimes (\Lambda^1(TX^4) \otimes \Lambda^0(N_{\daleth}^{10}))}_{\text{Space-Time Cosmological Constant and Dirac Mass}} \quad \oplus \quad \underbrace{\Lambda^1(N_{\daleth}^{10}) \otimes (\Lambda^1(N_{\daleth}^{10}) \otimes \Lambda^0(TX^4))}_{\text{Fiber Cosmological Constant and Dirac Mass}}$$

12 Summary

The approach of Geometric Unity as a candidate physical theory of our world is to work with a variety of different bundles in both finite and infinite dimensions which are all generated from a single space X^4. The structure of the relationships may be summarized here:

$$\underbrace{\begin{array}{lll} P_H & \hookleftarrow & \mathrm{U}(64,64) \\ \pi\downarrow & & \\ Y^{7,7} & \hookleftarrow & \widetilde{\mathrm{GL}}(4,\mathbb{R})/\mathrm{Spin}(1,3) \\ \pi\downarrow & & \\ X^4 & & \end{array}}_{\text{Finite Dimensions}} \underbrace{\begin{array}{lllllll} \mathcal{H} & \overset{\tau_{A_0}}{\hookrightarrow} & \mathcal{G} & \overset{\mu_{A_0}}{\longrightarrow} & \mathcal{A}\times\mathcal{A} & \overset{\delta}{\longrightarrow} & \mathcal{N}_{\mathcal{H}_R} \\ & & \downarrow \pi_{A_0} & & & & \\ & & \mathcal{A}_{A_0} \cong \mathcal{N}_{\mathcal{G}_L} & & & & \end{array}}_{\text{Infinite Dimensions}} \tag{12.1}$$

We recall that there is a theory of sections above the infinite dimensional constructions on the right hand side involved with superspaces and so-called 'induced representations', but at this point cannot remember even the standard theory and so have not entered into it here and may do so in further work if there is sufficient interest and ability to recall.

12.1 Equations

In Geometric Unity, we believe that the Einstein, Dirac, Yang-Mills and Klein-Gordon equations for the metric, Fermions, internal forces and Higgs sector respectively are not to be unified directly. Instead, the Einstein and Dirac equations are to be replaced by the reduced Euler Lagrange equations

$$\Pi(d\mathcal{I}^1_\omega) = (\delta_\omega)^2 = \Upsilon_\omega = 0 \tag{12.2}$$

for a first-order Lagrangian after removal of redundancy through projections Π

Then the Yang-Mills-Maxwell equations and Klein-Gordon equation for the Higgs follow from a second related Lagrangian

$$\Pi(d\mathcal{I}^2_\omega) = \mathcal{D}^*_\omega \Upsilon_\omega = 0 \tag{12.3}$$

whose Euler Lagrange equation are automatically satisfied if the 1st order theory is satisfied.

12.2 Space-time is not Fundamental and is to be Recovered from Observerse.

There has always been something troubling about the concept of Space-Time as the substrate for a dynamic world. In a certain sense, space-time is born as a frozen and lifeless corpse where the past is immutable and the quantum mechanically unknowable future hovers above it probabilistically waiting to be frozen in the trailing wake of four-dimensional amber which is our geometric past.

To have a hope of contributing insight, GU must, it must recover this established structure as an approximation within the theory. But at its deepest level, it seeks to break free of the tyranny of the Einsteinian prison built on the bedrock of a single space with a common past.

There is something very special about the arrow of time mathematically. Only in dimension $n = 1$ is $\mathbb{R}^n$ always well ordered. For every dimension $n > 1$ there is no such concept without additional structure chosen (e.g. indifference curves and surfaces foliating the space of baskets in consumer choice theory). In our case we have moved to a world X^4 in which we believe all signatures are in some sense 'physically' real, with $X^{1,3}$ and $X^{3,1}$ being the only two to be provably anthropic, and the others being disconnected and unreachable by the condition of non-degeneracy.

Yet in Geometric Unity, hovering above the world we see, there is always a second structure Y^{14} looming with multiple spatial and temporal dimensions beyond our own. This capacious augmentation of non-metric X^4 as proto-space-time allows us to wonder about the nature of time without a clear arrow being interpreted on a different space where the arrow is enforced by anthropics.

However, we have found it quite challenging to think through the tension between two such worlds related by a bridge $\daleth$ which must measure in order to observe. Thus, the idea of measurement and observation are forced to be intrinsically tied and the concept of multi-dimensional arrowless time above is shielded from us living as if in Plato's cave below.

12.3 Metric and Other Field Content are Native to Different Spaces.

If the metric on X^4 and the observed Bosonic and Fermionic fields are native to the same space, then there is likely a need to put both of them in the same quantum system. However, if they originate intrinsically from different spaces, then the possibilities for harmonizing them without putting them into the exact same framework increase. It may fairly be pointed out that we have a metric in the derived space Y that will have to be put in a common framework with the other fields, but even there we have a new twist. In this work we have not been considering unrestricted metrics on Y. In fact, almost all of the 'metric' information is built into the construction of Y, so that our subset of 'metrics' under consideration is really equivalent to the space of connections that split the long repeating exact sequence we have discussed between TY and T^*Y. This is not accidental but desired, as one of the goals of GU as connections, unlike metrics, have an adequate quantization theory as exhibited by QED, QCD and other theories. Hence the Zorro construction puts the only true metric field $\gimel$ on a separate space from the main quantized structures, but uses a connection to derive the highly restricted metrics on Y.

12.4 The Modified Yang-Mills Equation Analog has a Dirac Square Root in a Mutant Einstein-Chern-Simons like Equation

Without the quadratic potential term in the earlier example of a GU Bosonic Lagrangian, we are left with an expression of the form:

$$=< \overbrace{T_\omega}^{\substack{\text{Shifted}\\ \text{Torsion}}} , \underbrace{*}_{\substack{\text{Hodge}\\ \text{Star}}} (\overbrace{\circledcirc_\omega}^{\substack{\text{Einstein}\\ \text{Ricci}\\ \text{Shiab}}} (\underbrace{F_{B_\omega}}_{\substack{\text{Metric}\\ \text{Curvature}}} + \overbrace{\frac{1}{2} d_{B_\omega} T_\omega + \frac{1}{3}[T_\omega, T_\omega]}^{\text{C-S Like Terms}}) >_g$$

If we were to attempt to compare it to other Lagrangians, it would be seen as having some aspects of both the Einstein-Hilbert and Chern-Simons Lagrangians. The Einsteinian character comes from the fact that it produces a linear expression in the curvature tensor making use of Riemannian Projection via $\circledcirc_\omega$. The Chern-Simons-Palatini like properties come from the fact that it is a Lagrangian that takes connections and ad-valued 1-forms as its natural parameter space.

A comparison of the two expressions may be helpful to motivate some readers more familiar with one than the other:

$$\begin{array}{cccccccccccc}
S_{CS}(\overbrace{\nabla^0}^{\text{Trivial}}, \nabla^A, & \overbrace{\varepsilon = \mathrm{Id})}^{\text{Gauge Trans}} & = & \frac{1}{2} < A, & * & \mathrm{Id} & (0 & + & & d & A & + \frac{2}{3} A \wedge A) & >_{M^3_g} \\
 & & & \updownarrow & & \updownarrow & \updownarrow & & & \updownarrow & \updownarrow & \updownarrow & \\
S_{GU}(\underbrace{\nabla^g}_{\text{L-C}}, \nabla^{\varpi}, & \varepsilon) & = & < T_\omega, & * & \underbrace{\circledcirc_\omega}_{\text{Einstein}} & (\underbrace{F_{B_\omega}}_{R^{\nu}_{ij\mu}} & + & \frac{1}{2} & d_{B_\omega} & T_\omega & + \frac{1}{3} T_\omega \wedge T_\omega) & >_{Y^{14}_g}
\end{array} \tag{12.4}$$

Where $\omega = (\varepsilon, \varpi) \in \mathcal{G} = \mathcal{H} \ltimes \mathcal{N}$ and the connection 1-forms

$$A = \nabla^A - \nabla^0 \qquad \varpi = \nabla^{\varpi} - \nabla^g \tag{12.5}$$

are measured relative to the trivial connection in the usual Chern-Simons theory, while Geometric Unity is inclined to use the spin Levi-Civita connection. The displaced torsion on the other hand

$$T_\omega = \nabla^{\varpi} - \nabla^{g_\aleph} \cdot \varepsilon = \varpi - \varepsilon^{-1}(d_{\nabla^g} \varepsilon) \tag{12.6}$$

is measured relative to the gauge transformed Levi-Civita spin-connection $\nabla^{B_\omega} = \nabla^g \cdot \varepsilon$. The operator $\circledcirc_\omega$ depends on the gauge transformation and, like the Einstein-Ricci projection, always kills off the Weyl curvature. Unlike the Einstein-Ricci projection map, however, it does so in a gauge covariant fashion.

In the Chern-Simons case, the ad-valued 1-form A is differentiated by the exterior derivative coupled to the trivial connection. Within Geometric Unity, it is differentiated by the exterior derivative coupled to ∇^{B_ω}, the Levi-Civita spin connection gauge transformed by ε.

12.5 The Failure of Unification May Be Solved by Dirac Square Roots.

If we accept the colloquial description of the Dirac equation as the square root of the Klein-Gordon equation, we see that solutions of a first order operator can guarantee solutions of a more general second order equation.

This was oddly at the fore when the so-called 'Self-Dual' Yang-Mills equation burst onto the scene in that

$$F_A^{\pm} = 0 \rightsquigarrow d_A^* F_A = 0 \tag{12.7}$$

indicating that an equation linear in the curvature was powerful enough to guarantee the solution of a differential equation in the curvature via the Bianchi identity. This suggested to the author in the early 1980s in a seminar taught at the University of Pennsylvania that the Self-Dual equations were actually not so much meant to be Instanton equations, but were somehow more accurately the Einstein Field Equations in disguise as the square root of the Yang-Mills-Maxwell equations. Confusing this picture was the fact that the Einstein equations are usually viewed as equations for a metric rather than a connection, and the fact that the self-duality operator does not work for signatures other than $(4,0),(2,2)$ and $(0,4)$, all of which are non-physical. However, we have now attacked both of these issues in the construction of the Observerse so as to be able to address the viability of the idea that the Einstein and Yang-Mills curvature equations are so related.

Thus, in a Dirac pair, the Yang-Mills and Klein-Gordon equations would be assigned to a second order strata and the Einstein and Dirac equations to a first order strata, with a relationship between the two understood as above.

In our case of fundamental physics, there are so far four basic equations for each of the known fundamental fields,

Spin	Name	Field	Order
0	Klein-Gordon	Higgs Field ϕ	2
$\frac{1}{2}$	Dirac	Lepton and Hadron Fields ψ	1
1	Yang-Mills	Gauge Bosons A	2
2	Einstein	Gravitons g	2

(12.8)

In some sense, this can be replaced in GU by

Naive Spin	Name	Field	Order
0	Klein-Gordon w Potential	Yang-Mills-Higgs Field ϕ	2
$\frac{1}{2}, \frac{3}{2}$	'Dirac-Rarita-Schwinger'	Lepton and Hadron Fields ν, ζ	1
'1′	Yang-Mills	Gauge Bosons ϖ	2
'1′	'Chern-Simons-Einstein'	T_ω	1

(12.9)

suggests a Dirac Square Root Unification. That is, the two first order equations live inside a square root structure of a different equation that contains the two second order equations. In an extreme abuse of notation we might write

$$\text{Einstein-Dirac} = \sqrt{\text{Yang-Mills-Higgs-Klein-Gordon}} \tag{12.10}$$

to be maximally suggestive of the kind of Dirac Square Root unification we have in mind.

12.6 Metric Data Transfer under Pull Back Operation is Engine of Observation.

The metric tensor has traditionally been seen as an instrument of measurement of length and angle. This of course, is purely classical, arising as it does in both Special and General Relativity. The puzzle of Quantum measurement is, however, rather different, as it involves the application of Hermitian operators on Hilbert spaces to find eigenvectors as the possible post-measurement states, with their corresponding Eigenvalues as the experimental results.

But in GU a different picture is possible. Consider X as if it were an old fashioned Victrola and the Metric as analogous to an old fashioned stylus with Y being a phonograph. What appears to be happening on the Victrola is largely a

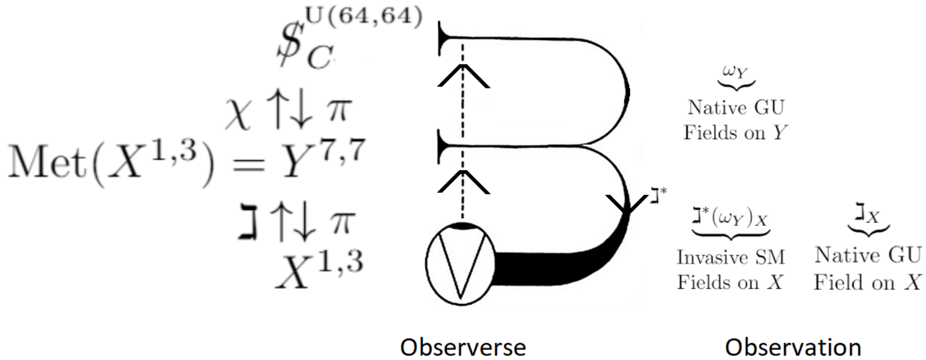

Figure 6: Observation and The Observerse.

function of where the stylus alights on the phonograph. From the point of view of the listener, each track or location on the phonograph is a different world, while from the perspective of the record manufacturer the album is a single unified release. In this way, the world of states of Y is merely being sampled and displayed as if it were the only thing happening on X.

12.7 Spinors are Taken Chimeric and Topological to Allow Pre-metric Considerations.

It has been very difficult to get upstream from Einstein's concept of space-time for a variety of reasons. In particular, the dependence of Fermions on the choice of a metric in fact appears to doom us to beginning with the assumption of a metric if we wish to consider leptonic or hadronic matter. Yet this dependence must be partially broken if we are to harmonize metric-generated gravity from within metric-dependent Quantum Field Theory.

Many years ago, Nigel Hitchin demonstrated that, while the elliptic index of a Dirac operator in Euclidean signature was an invariant by the Atiyah-Singer index theorem, the dimension of the Kernel and Co-Kernel could jump under metric variation. Since that time Jean-Pierre Bourguignon and others have expended a great deal of work tracking Spinors under continuous variation of the metric.

Given the odd way in which Spinors appear to be both intrinsically topological (e.g. the topological $\hat{A}$-genus) but confoundingly tied to the metric, we have sought to search for the natural space over which the topological nature of spinors is most clearly manifest. In essence, this has lead us to attempt to absorb the metric structure into a new base space made of pure measuring devices but constructed from the purely topological representation of $\widetilde{\mathrm{GL}}(r+s,\mathbb{R})$ on the homogeneous spaces $\widetilde{\mathrm{GL}}(r+s,\mathbb{R})/\mathrm{Spin}(r,s)$

12.8 Affine Space Emphasis Should Shift to $\mathcal{A}$ from Minkowski Space.

There appear to be many difficulties when attempting to do Quantum Field Theory in curved space. Thus there has always been a question in the author's mind as to whether the emphasis on affine Minkowski space $M^{1,3}$ is a linearized crutch to make the theorist's model building easier, or whether there is something actually fundamental about affine space analysis.

In some sense, GU attempts to split the difference here. We find the emphasis on Minkowski space misplaced, but *not* the focus on affine theory, as no matter how curved Space-time may be, there is always an affine space that is natural and available with a powerful dictionary of analogies to relate it to ordinary and super-symmetric Quantum Field Theory:

Special Relativity/QFT to GU	Relativity, QFT	GU Analog
Affine Space	$M^{1,3}$	$\mathcal{A}$
Model Space	$\mathbb{R}^{1,3}$	$\mathcal{N} = \Omega^1(\text{ad})$
Core Symmetries	$\text{Spin}(1,3)$	$\mathcal{H} = \Gamma^\infty(P_H \times_{\text{Ad}} H)$
Inhomogeneous Extension	Poincare Group $= \text{Spin}(1,3) \ltimes \mathbb{R}^{1,3}$	$\mathcal{G}$ $= \mathcal{H} \ltimes \mathcal{N}$
Fermionic Extension	Space-Time SUSY	$(\nu, \zeta) \in \Omega^0(\not{S}) \oplus \Omega^1(\not{S})$

(12.11)

This also makes more sense from the so-called super-symmetric perspective. If, historically, supercharges are to be thought of as square roots of translations, then in the context of a 'superspace' built not on $M^{1,3}$ but on $\mathcal{A}$, supercharges would have an honest affine space to act and translate where they would appear as square roots of operators or gauge potentials. This would also allow a framework where Supersymmetry[12] could be formally active without the introduction of artificial superpartners which have been remarkable in their failure to materialize at expected energies. In this framework, the supercharges may *already be here* in the form of the ν and ζ fields as this would not be space-time supersymmetry.

12.9 Chirality Is Merely Effective and Results From Decoupling a Fundamentally Non-Chiral Theory

Consider a stylized system of equations for a world Y with metric g, having scalar curvature $R(y)$, and endowed with a non-chiral Dirac operator operating on full Dirac Spinors,

$$\begin{pmatrix} -\Lambda(y) & \not{\partial}_A \\ \not{\partial}_A & -\Lambda(y) \end{pmatrix} \begin{pmatrix} \psi_L(y) \\ \psi_R(y) \end{pmatrix} = 0 \qquad R(y) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 4 \begin{pmatrix} \Lambda(y) & 0 \\ 0 & \Lambda(y) \end{pmatrix} \tag{12.12}$$

[12]The author finds supersymmetry unnecessarily confusing as an as-if symmetry and is uncomfortable saying much more about it.

which are nonetheless decomposed into chiral Weyl component-spinors. Solving both of these equations together yields a system of coupled equations:

$$\slashed{\partial}_A \psi_L(y) = \frac{R(y)}{4}\psi_R(y) \tag{12.13}$$

$$\slashed{\partial}_A \psi_R(y) = \frac{R(y)}{4}\psi_L(y) \tag{12.14}$$

leading to a stylized massive Dirac Equation with mass $m = \frac{R(y)}{4}$ for any fixed background metric for which the scalar curvature $R(y)$ is approximately constant in a region under study.

However, in any region where the scalar curvature was zero or sufficiently close to zero,

$$R(y) \approx 0 \tag{12.15}$$

the differential equations would decouple as they are only linked by the scalar curvature term of order zero.

$$\slashed{\partial}_A \psi_L(x) \approx 0 \tag{12.16}$$

$$\slashed{\partial}_A \psi_R(y) \approx 0 \tag{12.17}$$

This however, is not the end of the story when the tangent bundle has further structure. In the neighborhood of an embedding such as we have in:

$$\gimel : X^{1,3} \longrightarrow Y^{7,7} \tag{12.18}$$

we have

$$\gimel^*(TY^{7,7}) = TX^{1,3} \oplus N_{\gimel}^{6,4} \tag{12.19}$$

from our previous discussion.

However at the level of the chiral Weyl halves of the total Dirac Spinor we have two decompositions:

$$\begin{aligned}
\gimel^*(\slashed{S}^{64}_L(TY)) &= \overbrace{(\slashed{S}^2_L(TX) \otimes \slashed{S}^{16}_L(N_\gimel)) \oplus (\slashed{S}^2_R(TX) \otimes \slashed{S}^{16}_R(N_\gimel))}^{\text{Luminous Light Standard Model Family Matter}} \\
\gimel^*(\slashed{S}^{64}_R(TY)) &= \underbrace{(\slashed{S}^2_L(TX) \otimes \slashed{S}^{16}_R(N_\gimel)) \oplus (\slashed{S}^2_R(TX) \otimes \slashed{S}^{16}_L(N_\gimel))}_{\text{Dark Decoupled Looking Glass Matter}}
\end{aligned} \tag{12.20}$$

requiring a different view of chirality as both Left and Right handed spinors emerge from the branching rules of both Weyl halves confusing the picture. Left handed spinors on Y do not remain exclusively Left handed on X.

It may be asked what the relevance of the above stylized toy example is to the model under discussion. Quite simply, for every field on Y in the Observerse, there is both a naive spin and a true spin. The naive spin of a differential form valued in another bundle is taken to be the spin of the form field if the tensored bundle were taken formally to be purely auxiliary. Thus, for example, an ad-valued one form would carry naive spin 1 whether or not the ad bundle was derived from the structure bundle of the base space on which it lives.

Thus, for example, our bundle $\Omega^1(Y, \mathrm{ad})$ of ad-valued 1-forms has naive spin one, but this disguises the fact that it also contains an invariant subspace that derives from $\Lambda^1 \otimes \Lambda^1 \subset \Lambda^1 \otimes \Lambda^*$. This space of naive spin 1 would appear to be truly spinless from the point of view of Y. Thus, in some sense, the field playing the role of the fundamental mass for the generalized Dirac equations is actually part of the gauge potential. This sets up a three way linkage:

$$\text{Cosmological `Constant' } \Lambda \leftrightarrow \text{Spinless Gauge Field} \leftrightarrow \text{Fermion Mass} \tag{12.21}$$

and it is in such ways that GU seeks to attack non-anthropic fine tuning problems by having the same fields do multiple service.

In essence here, a fundamentally non-chiral world of Dirac Spinors in this simplified example would appear chiral in regions of low scalar gravity. From beings made of such chiral matter, they would naturally view the universe as being mildly chiral much the way each of the two hands in Escher's drawing is separately approximately symmetric about its middle digit. But raised high, the symmetry breaks down as digits two and four are only approximately symmetric in most people, and one and five are undeniably different. Yet it is not only the two middle fingers which are beautiful and symmetric about themselves, because the proper symmetry is left pinky to right pinky, left thumb to right thumb etc. and not left pinky to left thumb, right pinky to right thumb which is not broken as a symmetry, but simply accidental as well as being false.

12.10 Three Generations Should be Replaced by 2+1 model of two True Generations and one Effective Imposter Generation

At the time of this writing, the author is not convinced that we have three true generations of matter which differ only by mass. We instead posited here that the so-called third generation of matter is instead part of pure Rarita-Schwinger Spin$-\frac{3}{2}$ matter on Y and its Spin$-\frac{1}{2}$ appearance on X is the result of branching rules under pull back from Y where it is native:

$$\gimel^*(\slashed{R}(TY)) = \slashed{R}(\gimel^*(TY)) = \slashed{R}(TX \oplus N_\gimel) = \left(\begin{array}{c} \slashed{R}(TX) \otimes \$(N_\gimel) \\ \oplus \\ \$(TX) \otimes \slashed{R}(N_\gimel) \\ \oplus \\ \underbrace{\$(\mathbf{TX}) \otimes \$(\mathbf{N}_\gimel)}_{\text{Imposter Third Generation}} \end{array} \right) \tag{12.22}$$

Thus, part of the field $\zeta \in \Omega^1(Y, \$_R)$ is an ordinary second generation spinor in $\Omega^0(Y, \$_L)$ via the Dirac gamma matrix contraction while the complement $\slashed{R}_R(TY)$ corresponding to the sum of the highest weights contains the imposter third generation which is only revealed under decomposition as in the above. Thus, it is not a true generation as it has a different representation structure than the other two beyond its obvious mass difference.[13]

12.11 Final Thoughts

To sum up, let us revisit the Witten synopsis to see what GU has to say about it:

280 EDWARD WITTEN

If one wants to summarize our knowledge of physics in the briefest possible terms, there are three really fundamental observations: (i) Space-time is a pseudo-Riemannian manifold M, endowed with a metric tensor and governed by geometrical laws. (ii) Over M is a vector bundle X with a nonabelian gauge group G. (iii) Fermions are sections of $(\hat{S}_+ \otimes V_R) \oplus (\hat{S}_- \otimes V_{\tilde{R}})$. R and $\tilde{R}$ are not isomorphic; their failure to be isomorphic explains why the light fermions are light and presumably has its origins in a representation difference Δ in some underlying theory. All of this must be supplemented with the understanding that the geometrical laws obeyed by the metric tensor, the gauge fields, and the fermions are to be interpreted in quantum mechanical terms.

Figure 7: Edward Witten Synopsis.

As we have seen, Geometric Unity may be considered an alternative narrative that tweaks familiar concepts in various ways. As the author sees it, it is really a collection of interconnected ideas about shifting our various perspectives. Given the apparent stagnation in the major programs, GU has sought an alternate interpretation of either or both of the two incompatible models for fundamental physics of the Standard Model or General Relativity. In our opinion this represents a rather general perspective on the likely reasons for the impasse in fundamental physics encountered over the five decades since the early 1970s. At almost every level, it appears to us as if the instantiations of the most important general ideas and insights hardened prematurely into assumptions that now block progress. In most cases, our shift in perspective is usually not a rejection of the current models at the level of ideas so much as a rejection of the pressure to communicate ideas concretely through instantiations. In essence, we see an intellectual disagreement between the tiny group of physicists who have sought to discover physical law and the vast majority of theorists who attempted to work out its consequences. What good is a beauty principle that works only in the hands of Einstein, Dirac, Yang, and a handful of others, while leading to failure and madness for others? Yet, in these matters, we have come to side with Dirac's widely misunderstood perspective on the relationship between, instantiation, beauty, theory and experiment. In essence, a beautiful

[13]Note: we are speaking loosely here as if mass eigenstates and flavor eigenstates were one and the same.

theory is not its instantiation, but those who do not seek physical law cannot be forced to accept this critical issue.

To rephrase Witten's paragraph then in light of Geometric Unity, it might be rewritten as follows:

> "To Summarize the strongest claims of the strongest form of Geometric Unity, the basic assertions would be: i) Space-time $X^{1,3}$ arises as a pseudo-Riemannian manifold from maps $\gimel$ between two spaces X^n and $Y^{\frac{n^2+3n}{2}}(X) = \mathrm{Met}(X)$ where Y is constructed from X at a topological level. ii) Over Y is a bundle C with a natural metric which is (semi-canonically) isomorphic to TY, and one whose structure bundle carries a complex representation
>
> $$\mathrm{Spin}(\frac{n^2+3n}{2}, \mathbb{C}) \longrightarrow U(2^{\frac{n^2+3n}{4}}, \mathbb{C}) \qquad (12.23)$$
>
> on Dirac spinors with structure bundle P_H for H a real form of U, with no internal symmetry groups. There is an inhomogeneous extension $\mathcal{G}$ of the gauge group $\mathcal{H}$ of P_H acting on the space of connections $\mathcal{A}(P_H)$ where the stabilizer of any point $A_0 \in \mathcal{A}$ gives rise to a non-trivial endomorphism $\tau_{A_0} : \mathcal{H} \longrightarrow \mathcal{G}$. iii) Fermions on X^4 are pullbacks $\gimel^*(\nu)$ and $\gimel^*(\zeta)$ of unadorned non-chiral Dirac spinors $\nu \in \Omega^0(Y, \slashed{S})$ and 1-form valued spinors, $\zeta \in \Omega^1(Y, \slashed{S})$ on Y. In low gravitational regimes, the equations governing the fractional spin fields decouple leading to emergent effective chirality that disguises the non-chiral fundamental theory, and leading to Witten's representations R and $\bar{R}$ which are not isomorphic exactly as according to the branching rules for Spinors. The cosmological constant is actually the Vacuum Expectation Value (VEV) of a Field which plays the role of a fundamental mass, leading to the light Fermions being light in low gravity regimes. iv) If Super-symmetry is considered, it lives on the inhomogeneous gauge group and not the inhomogeneous Lorentz or Poincare group where gauge potentials take over from Galilean transformations and the affine space $\mathcal{A}$ plays the role of the Minkowski space $M^{1,3}$. The lack of internal symmetries indicates why naive super-partners have not been seen as space-time SUSY may be implementing over the wrong group. v) Gravity $\gimel$ lives on X while the fields of the standard model are native to Y leading to a reason for General Relativity to appear classical on X in contrast to the Quantum nature of the SM fields ω tied to Y. vi) Gravity is the engine of observation, so that where gravity is localized in different sections $\gimel_a, \gimel_b$, it pulls back different content while vii) Gravity on Y is replaced by a cohomological theory involving an obstruction $\delta_\omega^2 = \Upsilon$ combining elements of Einstein-Grossman, Dirac and Chern-Simons theories, while there is a new 2nd order theory replacing Yang-Mills, Higgs and Klein-Gordon theories so that the cohomological theory $\delta_\omega^2 = \Upsilon = 0$ is a 'Dirac square root' of the

second order theory. viii) The branching rules of ν leads to the appearance of one family of Fermions. ix) ζ branches as a second family due to gamma matrix multiplication on Y as $TY \otimes \not{\$}_Y = \not{\$}_Y \oplus \not{R}_Y$ with a Rarita-Schwinger remainder. The Spin$\frac{3}{2}$ portion of ζ breaks down under pull back to reveal a third 'imposter generation' that is merely effective, as it has different representation behavior in the full theory. x) The first order theory has a rich moduli of classical solutions and $\Upsilon = 0$ carries an elliptic deformation complex in Euclidean signature once the redundant Euler-Lagrange equations are discarded.[14]"

We would like to end this speculative foray with a quote from the man whose question provided the impetus for this excursion.

> "The relativity principle in connection with the basic Maxwellian equations demands that the mass should be a direct measure of the energy contained in a body; light transfers mass. With radium there should be a noticeable diminution of mass.The idea is amusing and enticing; but whether the almighty is laughing at it and is leading me up the garden path — that i cannot know." -Albert Einstein

While we believe in the story of Geometric Unity, we find the above, now as then, to be sage words in all such endeavors.

Appendix: Other Elements of Shiab Constructions

Continuing on from our earlier discussion of Shiab operator construction, the author simply wanted to note some of the gadgetry that has come up in the construction of these operators in past years. Most of this is obvious, but the fact that there are two products on the Unitary group Lie algebras given by matrix commutators, and anti-commutators multiplied by i, is an example of something that can be easily forgotten. The author may have forgotten other tools in the Shiab workshop over the years as well.

Wedge

The wedge product passes to bundle valued forms from the usual DeRham complex.

Hodge Star

As we have assumed our manifold to be oriented from the beginning, every time a metric g on Y is chosen it induces a non-vanishing volume form $dvol$

[14] The so-called Seiberg-Witten equations were first found this way around 1987 as the simplest toy model to proxy this moduli problem.

compatible with the metric and orientation. This in turn induces a Hodge Star operator

$$* : \Omega^i(B) \longrightarrow \Omega^{d-i}(B) \tag{12.24}$$

which passes to forms valued in arbitrary bundles B over Y.

Contraction

Various forms of contraction can be defined either with co-variant against contravariant tensors in the obvious way or via the wedge and star operations between forms as in:

$$\phi \vee \mu = *(\phi \wedge *\mu) \tag{12.25}$$

Adjoint Bundle

Bracket

As with any Lie Group, $\mathrm{U}(64, 64)$ carries a Lie Bracket structure. Given that it lives embedded within the Clifford Algebra $\mathrm{Cl}_{\mathbb{C}}(7, 7) = \mathbb{C}(128)$, it can be constructed from the matrix algebra product in the usual fashion:

$$[a, b] = a \cdot b - b \cdot a \tag{12.26}$$

Symmetric Product

Unlike most Lie Algebras, there is a second symmetric product on $\mathfrak{u}(n)$ gotten from taking:

$$\{a, b\} = i(a \cdot b + b \cdot a) \tag{12.27}$$

Volume Form

The analog of the Hodge Star operator is multiplication with the Clifford Volume form λ.

Appendix: Thoughts on Method

A few words are in order about what the author sees as unbridgeable differences with the mainstream of the community of professional physicists.

Experiment and The Scientific Method

The author understands the scientific method differently from many others and particularly from many within the world of String Theory. In essence there are general ideas and multiple instantiations of those ideas. The author believes that many who put their faith in the scientific method do not understand the danger of being pressured to discard ideas because one of their instantiations was invalidated by experiment. This is, in essence, the very point Dirac raised in his 1963 Scientific American Article where he warned that beauty rather than the scientific method should be used as a guide to progress:

> "It seems that if one is working from the point of view of getting beauty in one's equations, and if one has really a sound insight, one is on a sure line of progress. If there is not complete agreement between the results of one's work and experiment, one should not allow oneself to be too discouraged, because the discrepancy may well be due to minor features that are not properly taken into account and that will get cleared up with further developments of the theory."

It is the misinterpretation of this very clear point that the author finds chilling. Dirac was clearly not saying that if a theory is beautiful, it need not agree with experiment, and yet he is frequently lampooned as such. Why is this?

The author believes there is a principle, by which the scientific communities push most members for hyper explicit claims so as to learn the general idea and to wed the author to a prediction that can be easily falsified. Should the author succumb to associating her or his more general idea with a particular instantiation that fails to be confirmed, that idea is now 'up for grabs'. Further, established players can speak more generally allowing different members different privileges.

The author is proud to be able to offer algebraic predictions as to the 'internal quantum numbers' of new particles but would need the help of Quantum Field Theorists to see whether these can be sharpened further to include energy scales. The author is not equipped to undertake that effort alone but considers the predictions already offered to be considerably more explicit than many of the current contenders for a theory of everything on a relative basis. The author's experience is that in calling such quantum numbers 'predictions' is that those farthest away from making such predictions are paradoxically the most likely to complain viciously about the lack of an energy threshold so as to deflect criticism from their own theory's failure to be able to make such claims.

Isolation

It is the experience of this author that almost no professional mathematicians and physicists have any concept what it is like to be isolated from the community for 20 years or more at a time. Geometry and field theory are languages that in this author's experience, decay exceedingly rapidly when there is no one with which to speak them, and it is nearly impossible to find it actively maintained anywhere outside of the profession.

It has been over 25 years since the current author was in a professional environment where anyone else was conversant in the topics discussed here. My apologies are offered for any inconvenience caused, but the author's ability to converse with the professional community, but, in full candor, the ability to communicate was likely to get even further degraded via additional years of isolation.

Appendix: Locations Within GU

We collect here for convenience the usual ingredients that constitute fundamental physics and give their intended address within the framework of Geometric Unity.

Usual Name	GU Location
Higgs Field	$\gimel^*(\varpi)$
SM 1st Generation Fermions	$\gimel^*(\nu_L)$
SM 2nd Generation Fermions	$\gimel^*(\zeta_R^{\$})$
SM 3rd Generation Fermions	$\gimel^*(\zeta_R^{\not{R}})$
Gluons	$\gimel^*\big(\varpi_{\mathrm{Spin}(6)(N_\gimel)}\big)$
Weak Isospin	$\gimel^*\big(\varpi_{\mathrm{Spin}(4)(N_\gimel)}\big)$
Weak Hypercharge	$\gimel^*\big(\varpi_{\mathrm{Spin}(6)\times\mathrm{Spin}(4)(N_\gimel)}\big)$
Space-time Metric	$\gimel$
Higgs Potential	$< \Upsilon_\omega, \Upsilon_\omega >$
CKM Matrix	$\gimel^*(\varpi)$
Einstein Field Equations	$\Upsilon_\omega = 0$
Dirac Equations	$\Upsilon_\omega = 0$
Yang-Mills-Maxwell Equations	$\mathcal{D}_\omega^*\Upsilon_\omega = 0$
Higgs Klein-Gordon	$\mathcal{D}_\omega^*\Upsilon_\omega = 0$
Cosmological Constant	$\gimel^*(\varpi)$ as a VEV
Yukawa Couplings	$\gimel^*(\bar{\chi}\not{D}_\omega\chi)$ as a VEV

(12.28)

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