# *Infrared surprises in gravity & celestial CFT*

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UNIVERSITY OF AMSTERDAM

CERN, 15 MAY 2025

European Research Council
Established by the European Commission

ERC STARTING GRANT HOLOHAIR 852386

SIMONS FOUNDATION

SIMONS COLLABORATION ON CELESTIAL HOLOGRAPHY

# Overview: celestial CFT

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# Plan: infrared surprises

& their connection to ``celestial CFT''

Infrared triangles

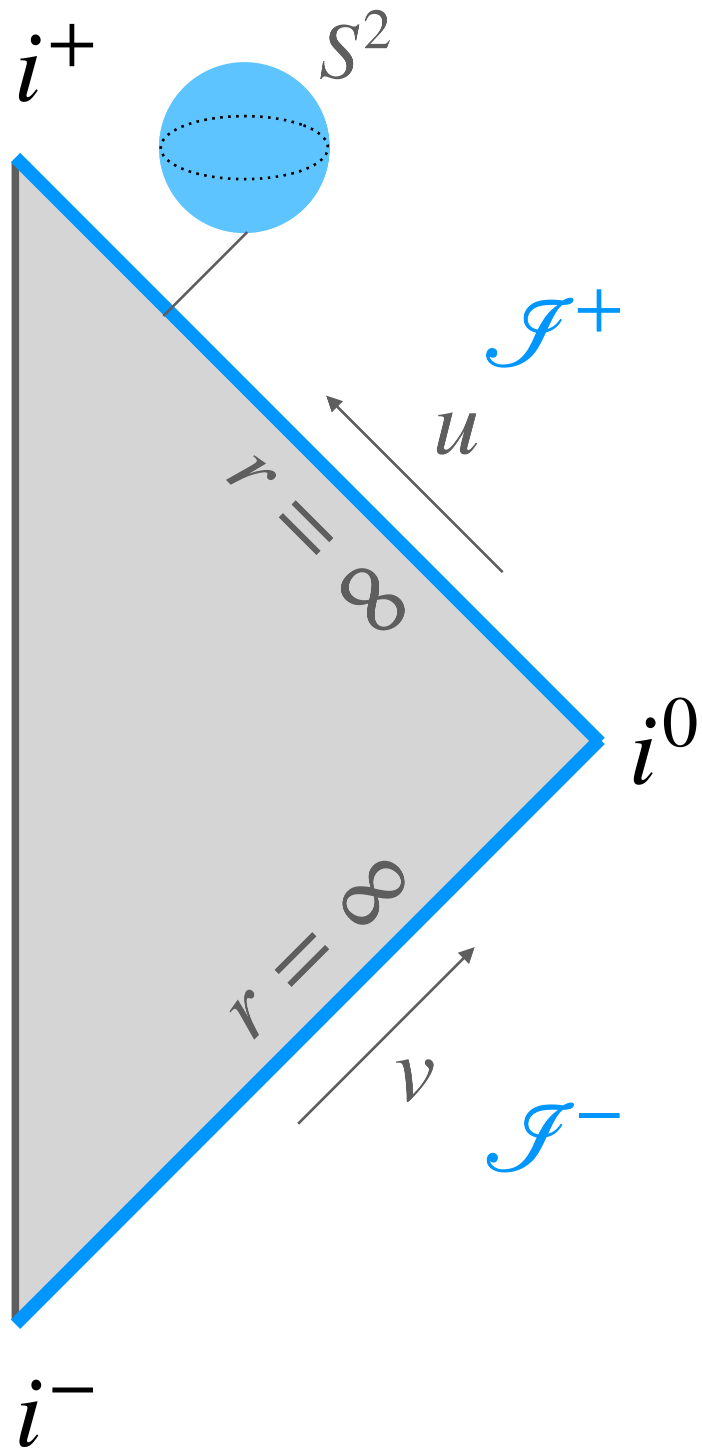
4D = 2D

Towers of ∞ symmetries

Long-range effects

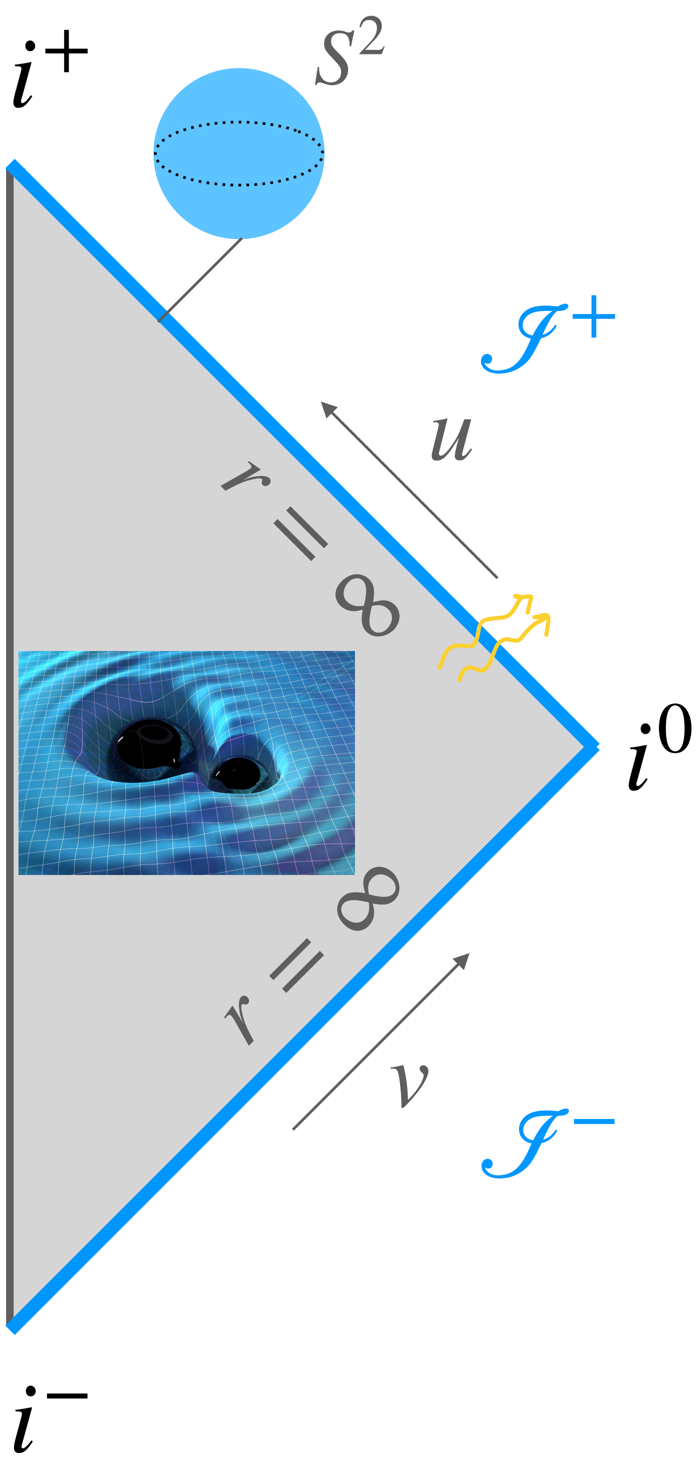
# Infrared triangles

# Asymptotic symmetries




"Large" gauge/diffeo transformations that preserve the boundary conditions of the fields, i.e. their large-distance fall-offs.

# Asymptotic symmetries




"Large" gauge/diffeo transformations that
preserve the boundary conditions of the fields,
i.e. their large-distance fall-offs.

**gravity:**

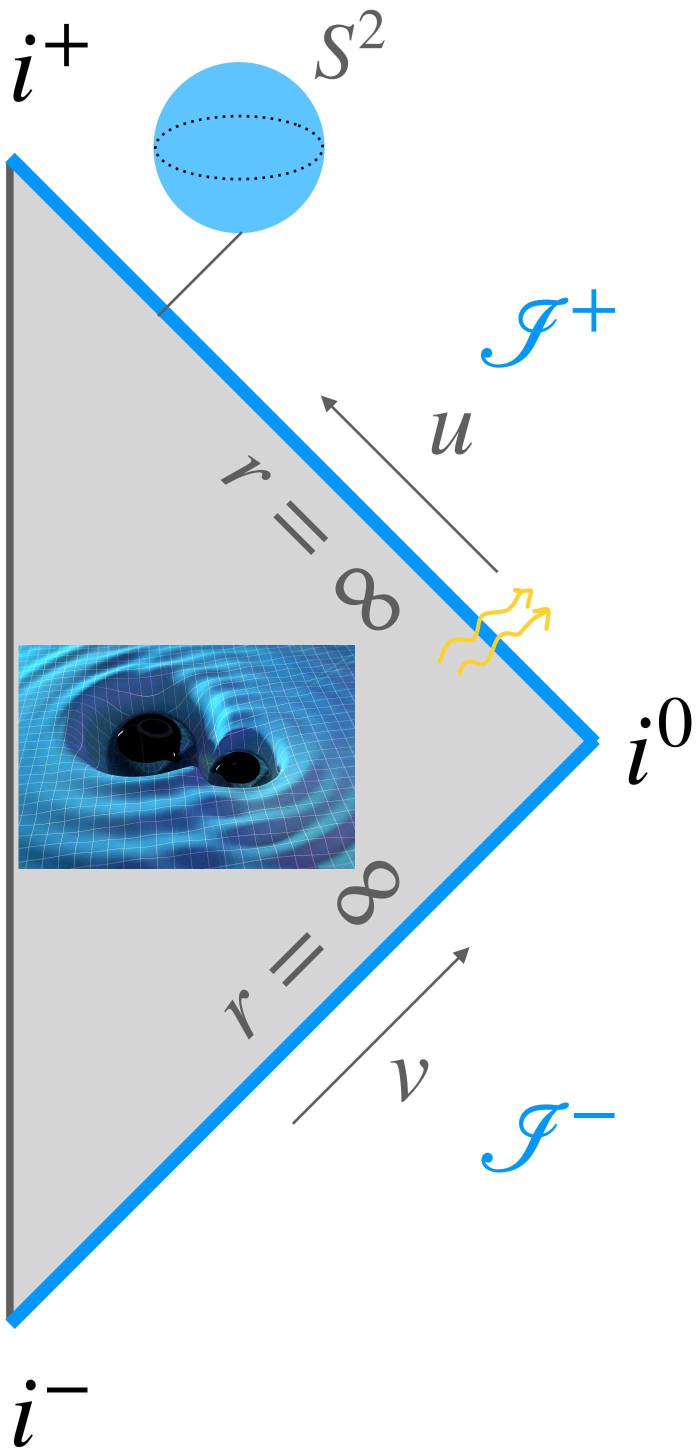
$$ds^2 = -(1+\underset{\text{mass}}{\overset{}{\dots}})du^2 - (2+\dots)dudr$$

$$+(\underset{\text{angular momentum}}{\dots})dudx^A$$

$$+(r^2\gamma_{AB} + r\,\underset{\text{shear: gravitational waves}}{C_{AB}} + \dots)dx^A dx^B$$

⇒ Bondi news $N_{AB} = \partial_u C_{AB}$

# Asymptotic symmetries




"Large" gauge/diffeo transformations that preserve the boundary conditions of the fields, i.e. their large-distance fall-offs.

**gravity:**

Find $\xi$ such that $\mathscr{L}_\xi g_{\mu\nu} \approx$ "$0$" as $r \to \infty$ .

$\updownarrow$

$O(1/r^{\#})$

Unlike gauge redundancies, asymptotic symmetries act non-trivially on the physical data → non-zero charges.

# IR triangle

[He,Lysov,Mitra,Strominger'14]

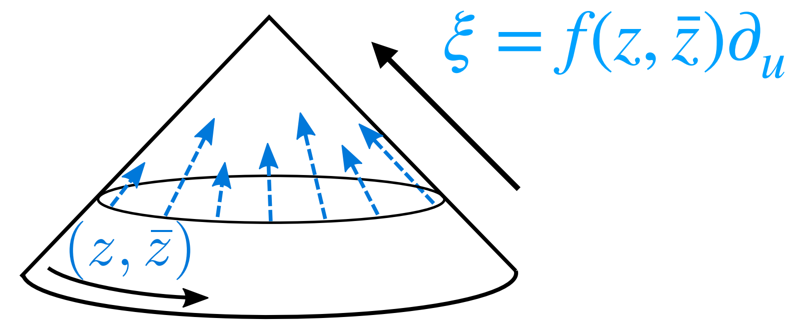
[Strominger,Zhiboedov'14]

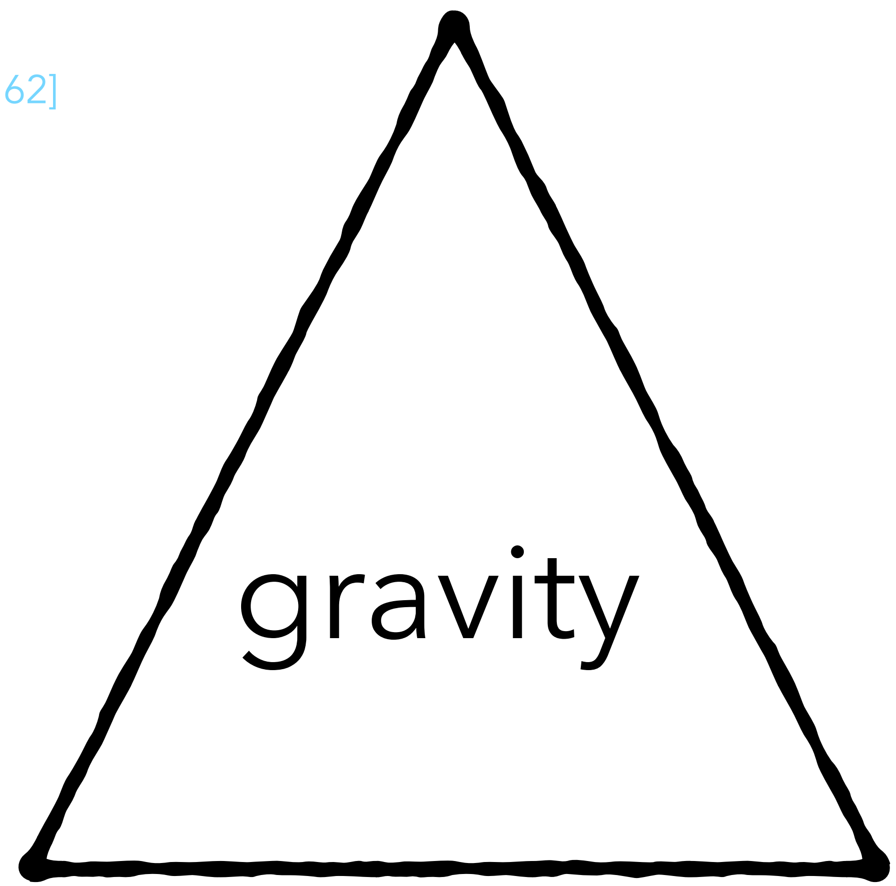
The symmetries of asymptotically flat space are not just Poincaré but an infinite extension!

[Bondi,van der Burg,Metzner'62] [Sachs'62]

BMS group

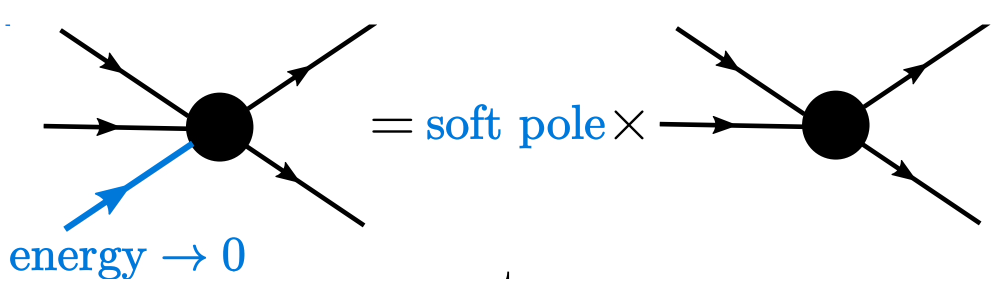
**supertranslations**



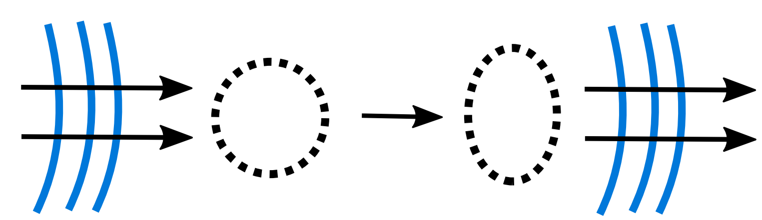

[Weinberg'65]

**soft graviton theorem**




[Zel'dovich,Polnarev'74 [Braginsky,Thorne'87]

**displacement memory effect**



# IR triangle

on $\mathscr{I}^+$ :

supertranslations

superrotations

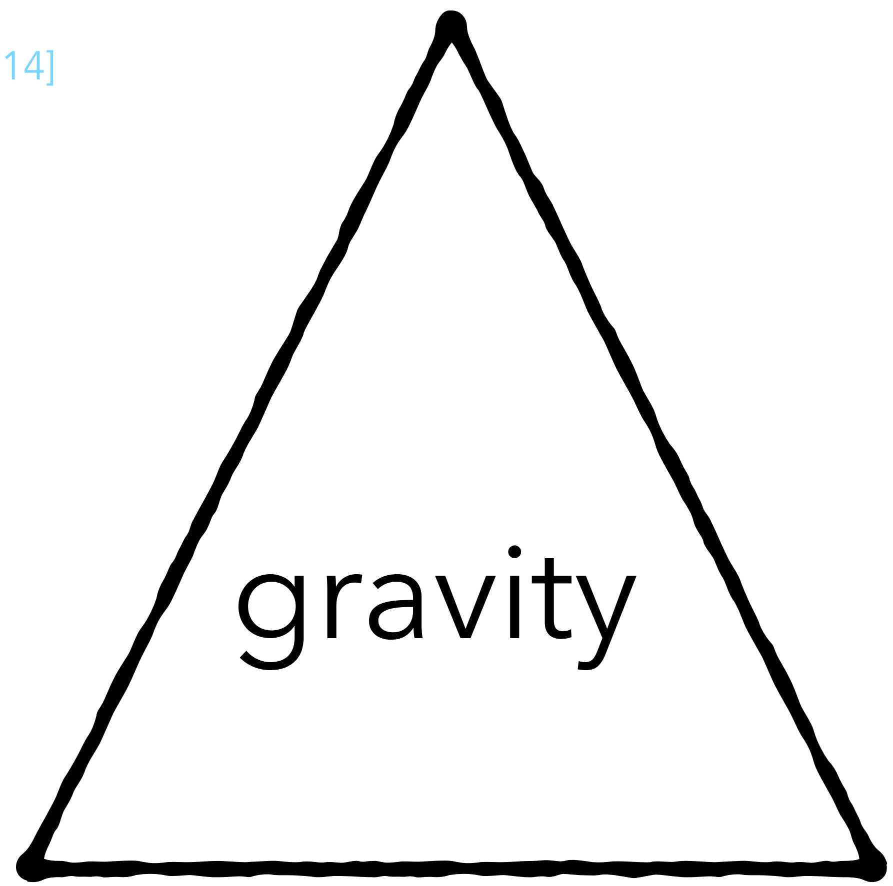
**asymptotic symmetry**

$\xi^u = f \qquad \xi^A = \frac{1}{r} D_A f$

$\xi^u = -\frac{1}{2} D_A Y^A \qquad \xi^A = Y^A$

[Barnich,Troessaert'11] [Campiglia,Laddha'14]

extended / generalized
BMS group




**soft theorem**

$\omega^{-1}$ leading soft graviton

$\omega^{0}$ subleading soft graviton

[Cachazo,Strominger'14]

**memory effect**

displacement

spin

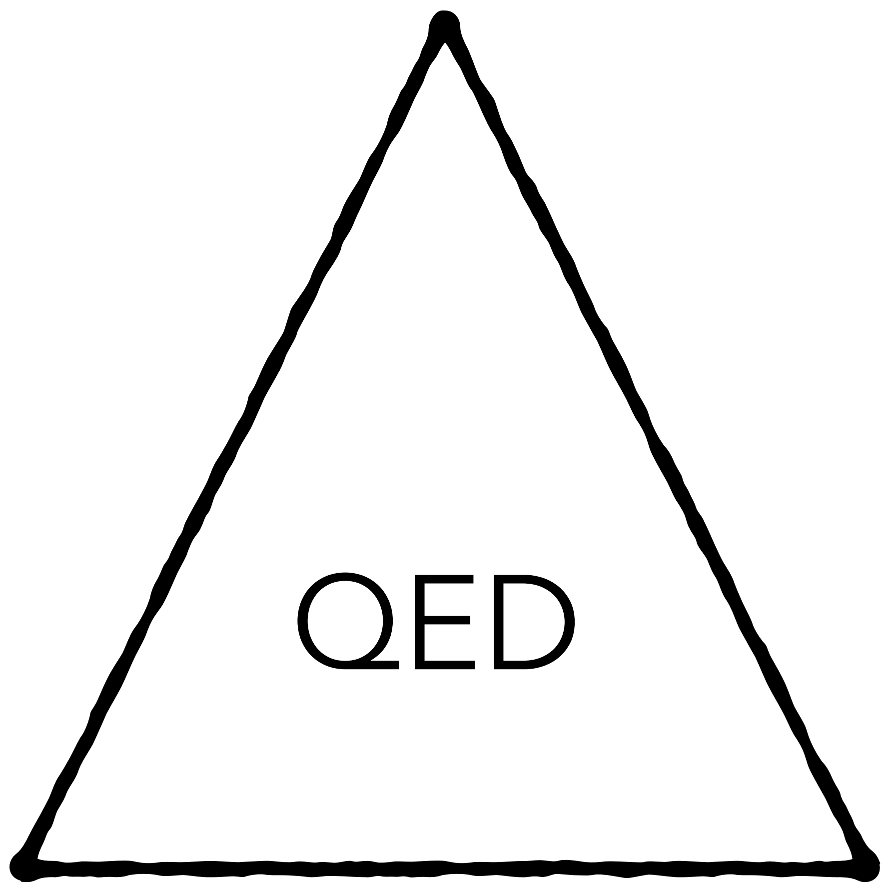
[Pasterski,Strominger,Zhiboedov'15]

# IR triangle

superphaserotation

[He,Mitra,Porfyriadis,Strominger’14]
[Kapec,Pate,Strominger’15]
[Campiglia,Laddha’15]

**asymptotic symmetry**




**soft theorem**

$\omega^{-1}$ leading soft photon

[Weinberg'65]

**memory effect**

electromagnetic kick

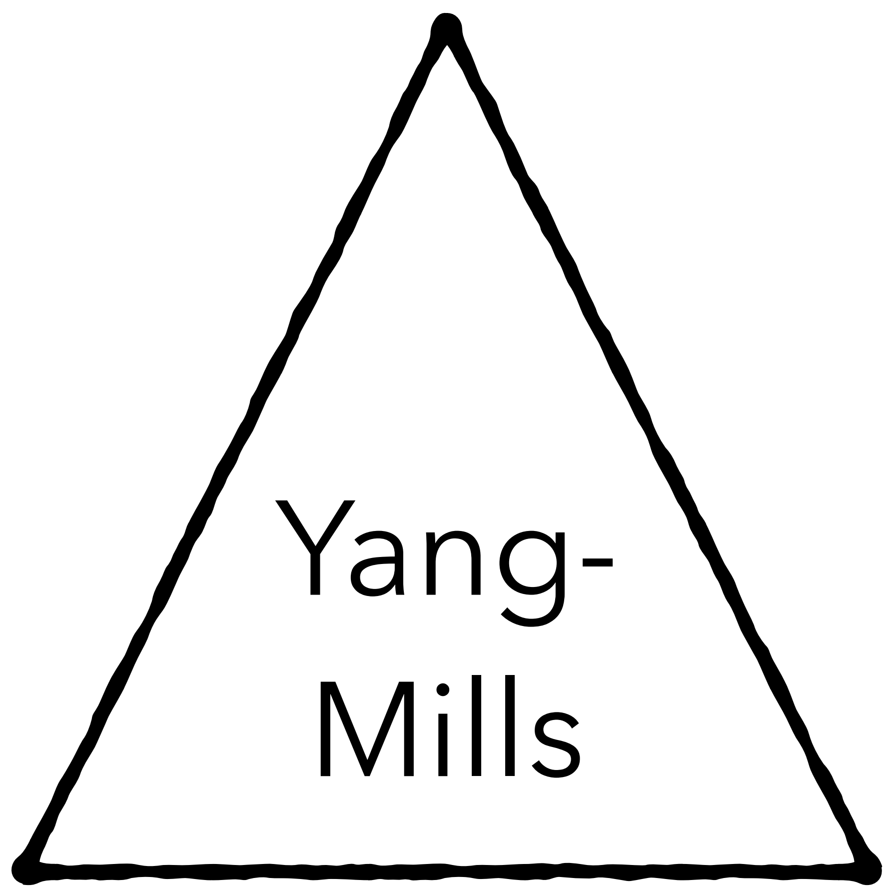
[Bieri,Garfinkle’13]
[Pasterski’15]

# IR triangle

superphaserotation

[He,Mitra,Strominger'14]

**asymptotic symmetry**




**soft theorem**

$\omega^{-1}$ leading soft gluon

[Berends,Giele'89]

**memory effect**

color

[Pate,Raclariu,Strominger'17]

# 3 bases for the IR

boost weight

$\Delta$

**asymptotic symmetry**

$\mathscr{M}$ellin

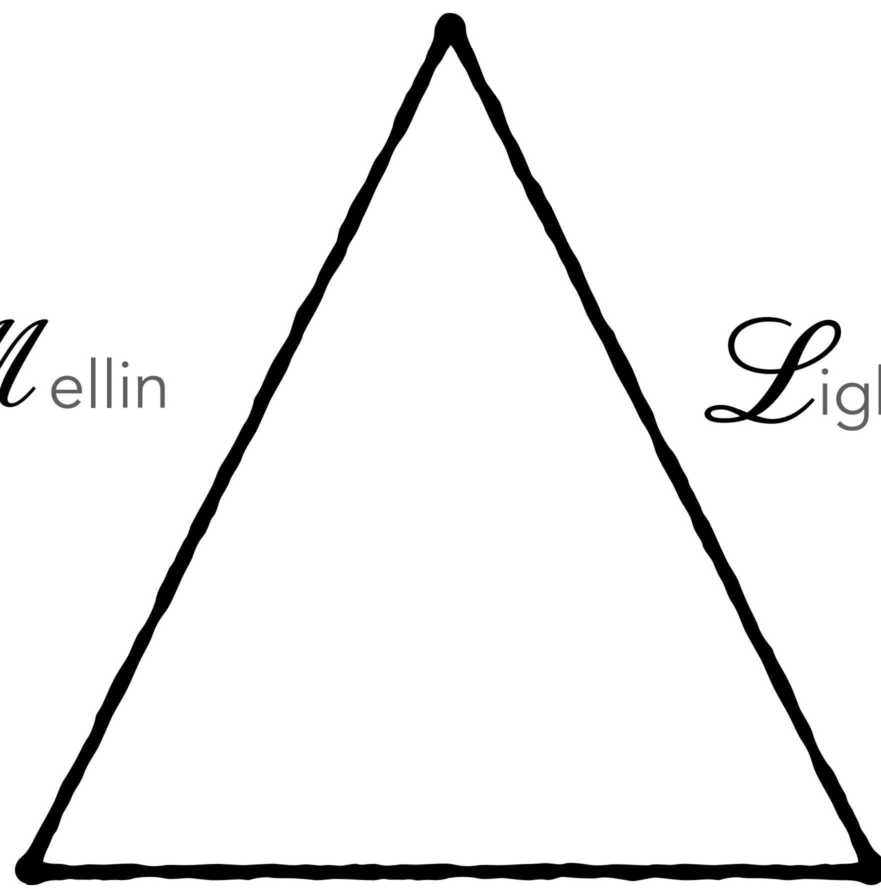
$$\mathscr{M}(\,.\,) = \int_0^\infty d\omega \omega^{\Delta-1}(\,.\,)$$

[Pasterski,Shao,Strominger'17]

$\mathscr{L}$ight-ray

$$\mathscr{L}(\,.\,) = \int_{-\infty}^{+\infty} du u^{-\Delta}(\,.\,)$$

[Pasterski,AP,Trevisani'21]



**soft theorem**

energy

$\omega$

$\mathscr{F}$ourier

$$\mathscr{F}(\,.\,) = \int_{-\infty}^{+\infty} du e^{i\omega u}(\,.\,)$$

**memory effect**

null time

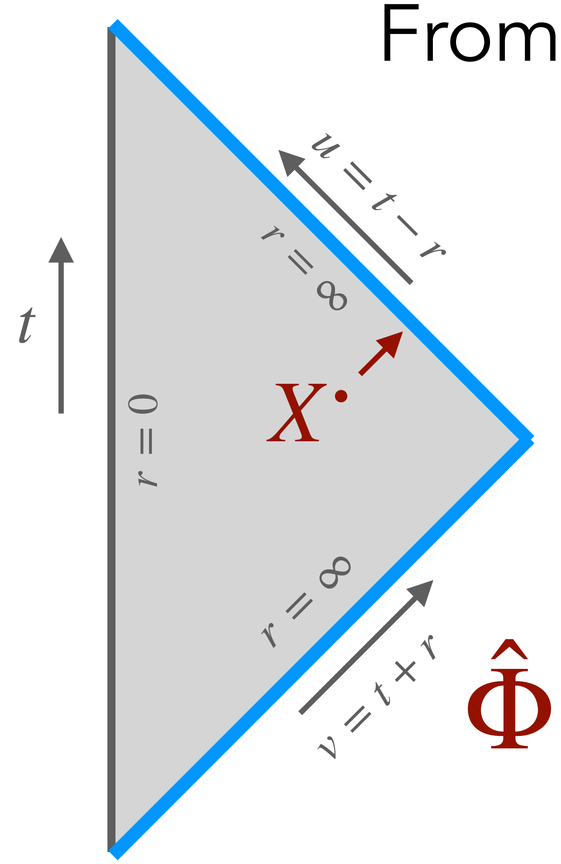
$u$

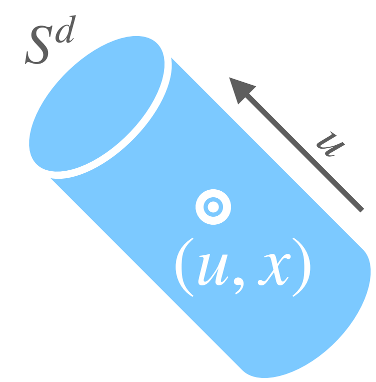
4D = 2D

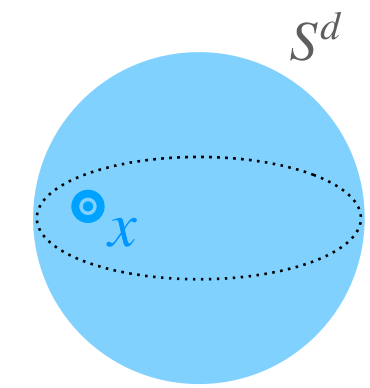
# Flat space holography

``Extrapolate" dictionary for flat holography. [Paserski,AP,Trevisani'21]

From bulk operators to boundary operators:




$$\hat{\Phi}(X) \xrightarrow[\lim_{r\to\infty} r^{1-|\ell|}]{\text{large } r \text{ limit}} \Phi_\ell(u,x) \xrightarrow[\int_{-\infty}^{+\infty} du\, u^{-\Delta}]{\mathscr{L}\text{ight-ray}} \mathcal{O}_{\Delta,\ell}(x)$$

boost-weight spin

conformal primaries

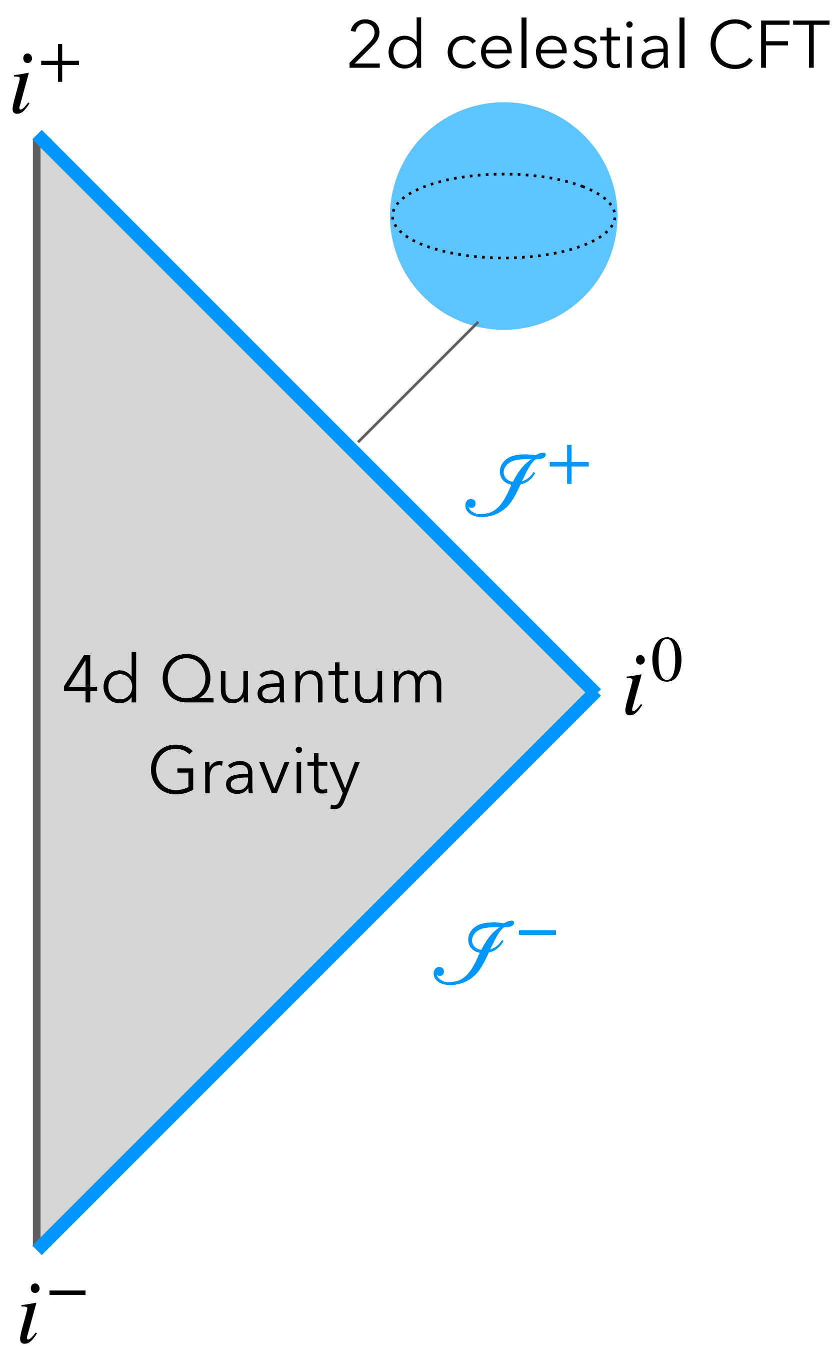
degenerate metric, $c \to 0$ field theory

Carroll weights $(k,\bar{k}) = \frac{1}{2}(1+\ell, 1-\ell)$ [Donnay,Herfray, Fiorucci,Ruzziconi'22]

bulk Lorentz acts as boundary global conformal

[Pasterski,Shao'17]

# Celestial holography




4d Quantum Gravity

in asymptotically flat spacetimes

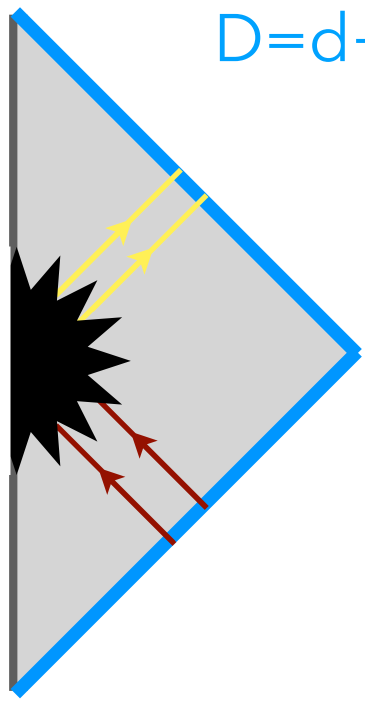
=

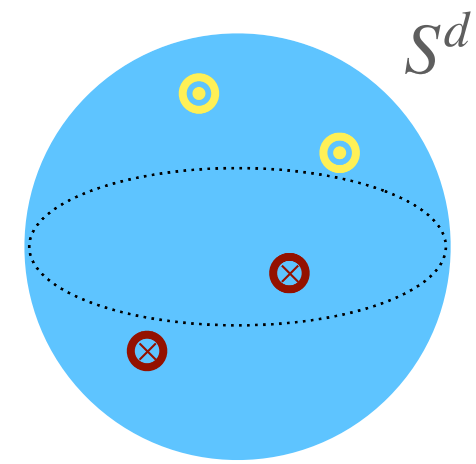
2d ''Celestial CFT''

# Symmetry & observables

symmetry:

Lorentz group in D=d+2 dimensions $\cong$ Euclidean conformal group in d dimensions

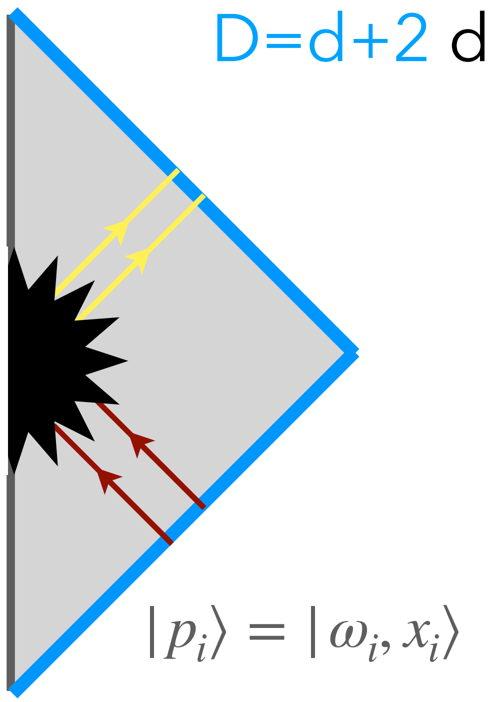




# Symmetry & observables

symmetry:

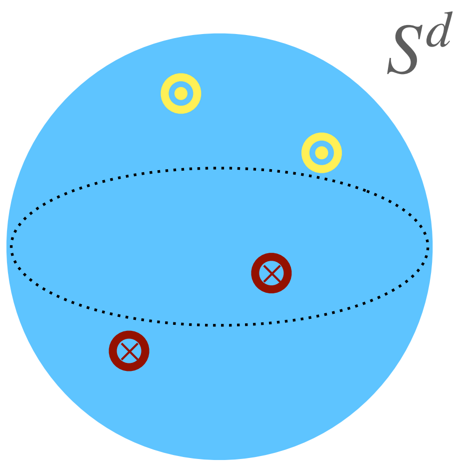
Lorentz group in D=d+2 dimensions $\cong$ Euclidean conformal group in d dimensions




energy basis

basic observables in flat space:

**S-matrix**




**Standard amplitudes**

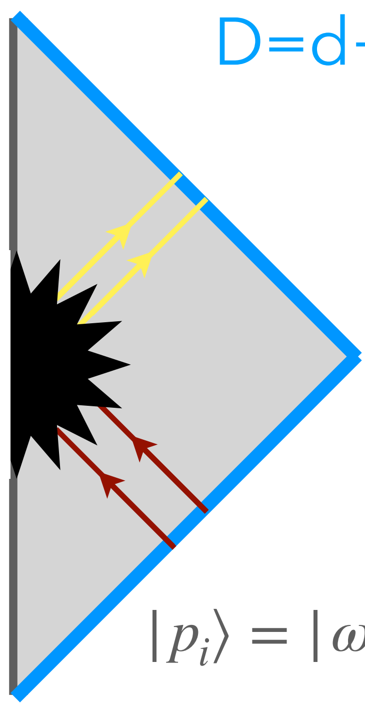
$$\langle p_n^{\text{out}} \ldots | \mathcal{S} | \ldots p_1^{\text{in}} \rangle$$

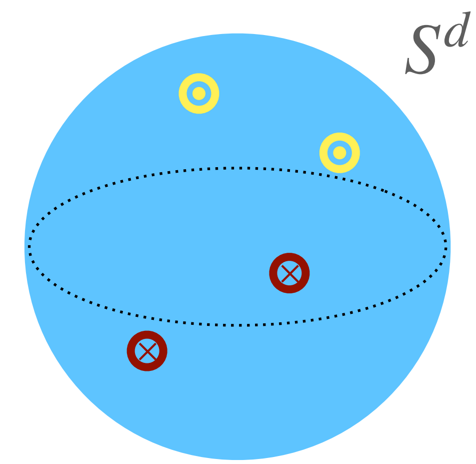
translation symmetry

# Symmetry & observables

symmetry:

Lorentz group in D=d+2 dimensions $\cong$ Euclidean conformal group in d dimensions






basic observables in flat space:

**S-matrix**

$|p_i\rangle = |\omega_i, x_i\rangle$

energy basis

$$\xrightarrow[\int_0^\infty d\omega\,\omega^{\Delta-1}]{\mathscr{M}\text{ellin}}$$

$|\Delta_i, x_i\rangle$

boost-weight basis

**Standard amplitudes**

$$\langle p_n^{\text{out}} \ldots | \mathcal{S} | \ldots p_1^{\text{in}} \rangle$$

translation symmetry

**Celestial amplitudes**

$$\langle \mathcal{O}^-_{\Delta_1}(x_1) \ldots \mathcal{O}^+_{\Delta_n}(x_n) \rangle$$

Lorentz symmetry

$\updownarrow$

global conformal symmetry

[de Boer, Solodukhin'03] [Pasterski,Shao,Strominger'17] [Pasterski,Shao'17]

# From global to local conformal

on $\mathscr{I}^+$ :

$\xi^u = f$ $\xi^A = \frac{1}{r} D_A f$

$\xi^u = -\frac{1}{2} D_A Y^A$ $\xi^A = Y^A$

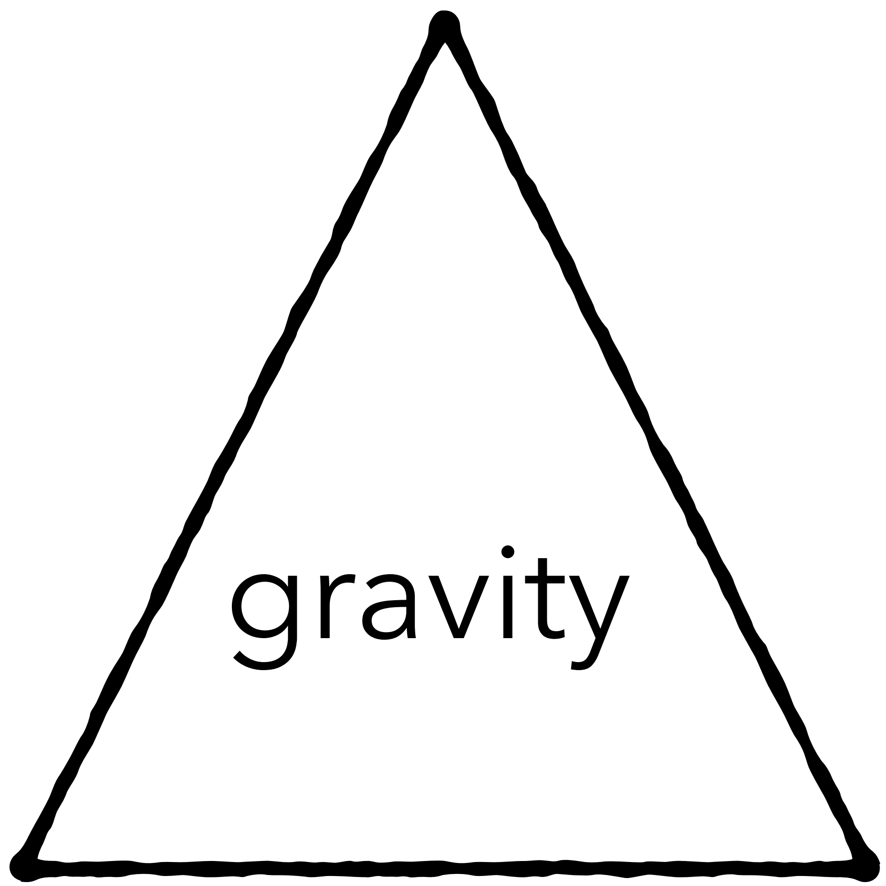
supertranslations,
superrotations, ...

[Barnich,Troessaert'11]

local conformal
symmetry on $S^2$ !

just what we need
for CCFT :-)

**asymptotic symmetry**




**soft theorem**

$\omega^{-1}$ leading soft graviton,
$\omega^{0}$ subleading soft graviton, ...

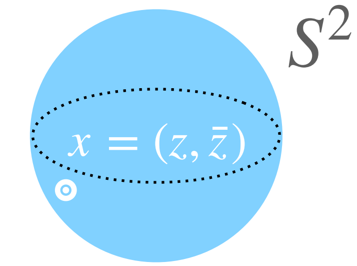
[Cachazo,Strominger'14]

**memory effect**

displacement,
spin, ...

[Pasterski,Strominger,Zhiboedov'15]

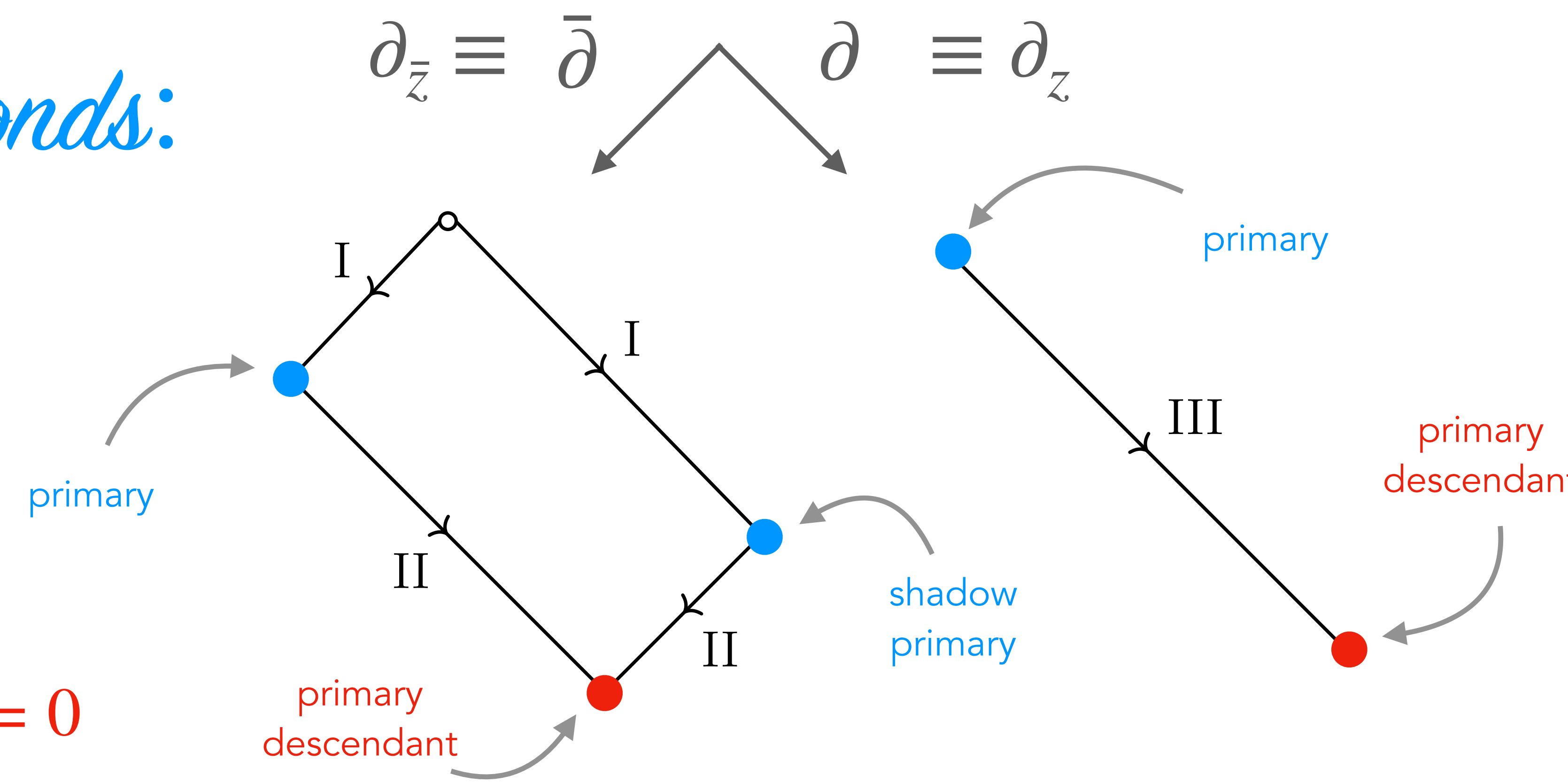
# Celestial symmetries




**celestial diamonds:**

[Pasterski,AP,Trevisani'21]

also: [Banerjee,Pandey,Paul'19]



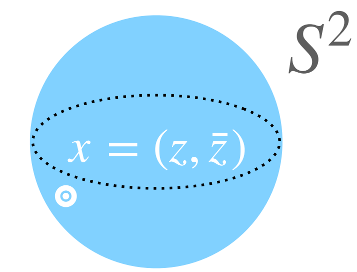

conservation equation $\bar{\partial}^{\#}\mathcal{O}(z,\bar{z}) = 0$

\# depends on types I, II, III: spin of descendant >,<,= spin of parent primary

Conformally soft operator = primary $\xrightarrow{\bar{\partial}^{\#}}$ primary descendant = conservation equation.

Noether current: $\mathcal{J} = \sum_{m=0}^{\#-1} (-1)^m \bar{\partial}^m \epsilon(z,\bar{z}) \bar{\partial}^{\#-m-1}\mathcal{O}(z,\bar{z})$

# Celestial symmetries




*celestial diamonds:*

[Pasterski,AP,Trevisani'21]

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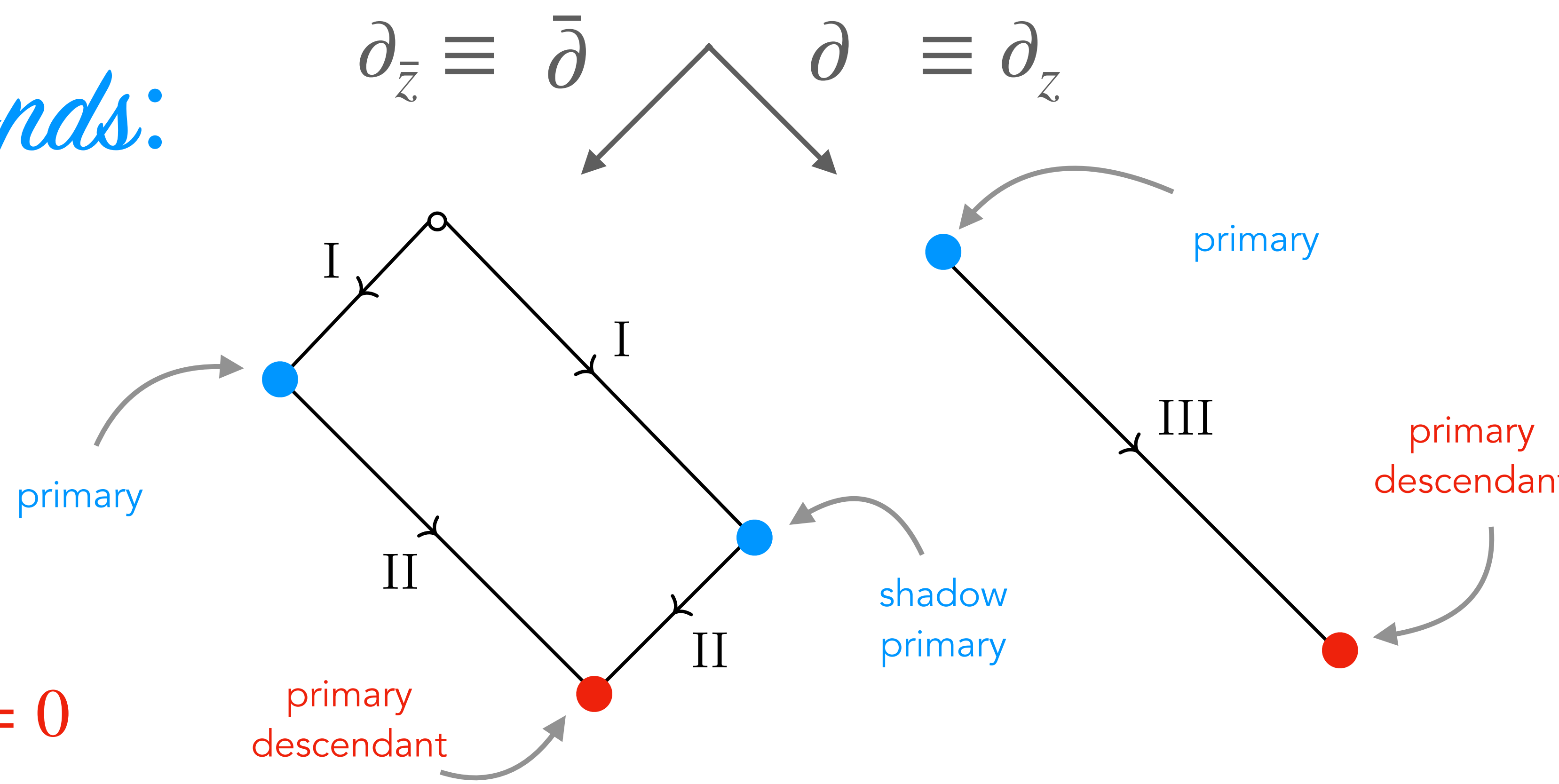



conservation equation $\bar{\partial}^{\#}\mathcal{O}(z,\bar{z}) = 0$

\# depends on types I, II, III: spin of descendant >,<,= spin of parent primary

QED: $\mathcal{O}_\Delta(x)$ with $\Delta = 1$ generates superphaserotation symmetry.

gravity: $\mathcal{O}_\Delta(x)$ with $\Delta = 1,0$ generate extended BMS symmetries.

supertranslations

action: $O_{\Delta_k} \to O_{\Delta_k+1}$

superrotations: shadow of $\mathcal{O}_{\Delta=0}$ is celestial $\mathrm{CFT}_2$ stress tensor

action: global conformal

[He,Mitra,Strominger'15]

[Kapec,Mitra,Raclariu,Strominger'16]

[Donnay,AP,Strominger'18]

[Donnay,Pasterski,AP'20]

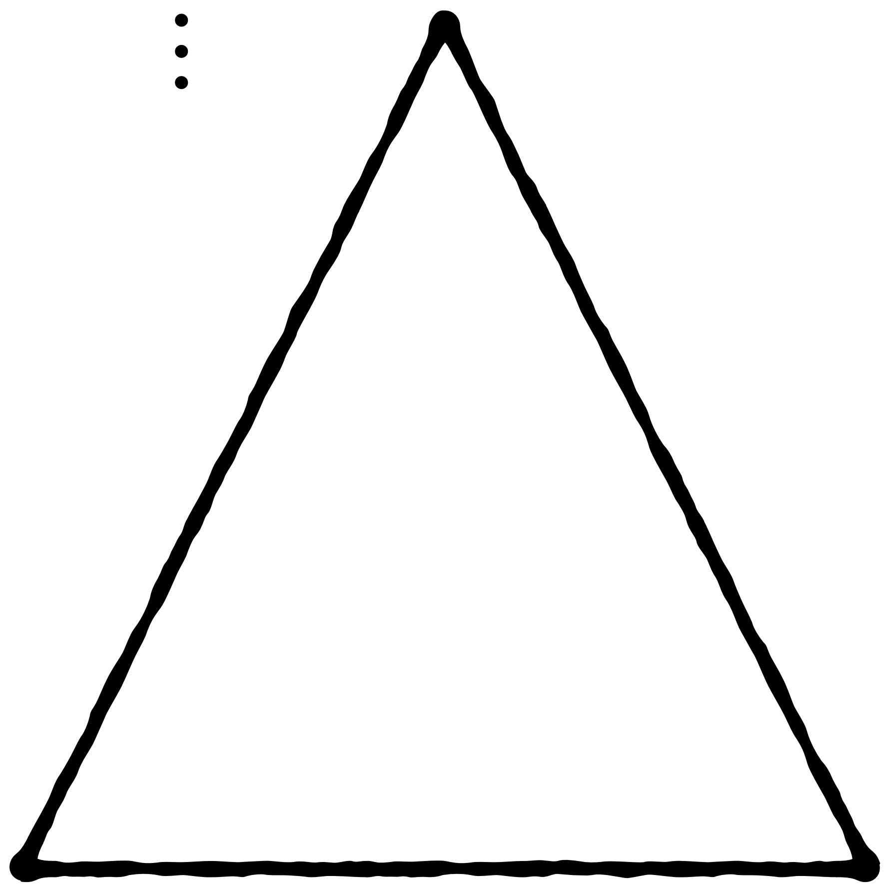
# Tower of ∞ symmetries

# ∞ IR triangle

(for projected S-matrix)

**asymptotic symmetry**

⋮



[Hamada,Shiu'18]
[Li,Lin,Zhang'18]

**soft theorem**

$\omega^n$ ⋮

$n = -1, 0, 1, \ldots$

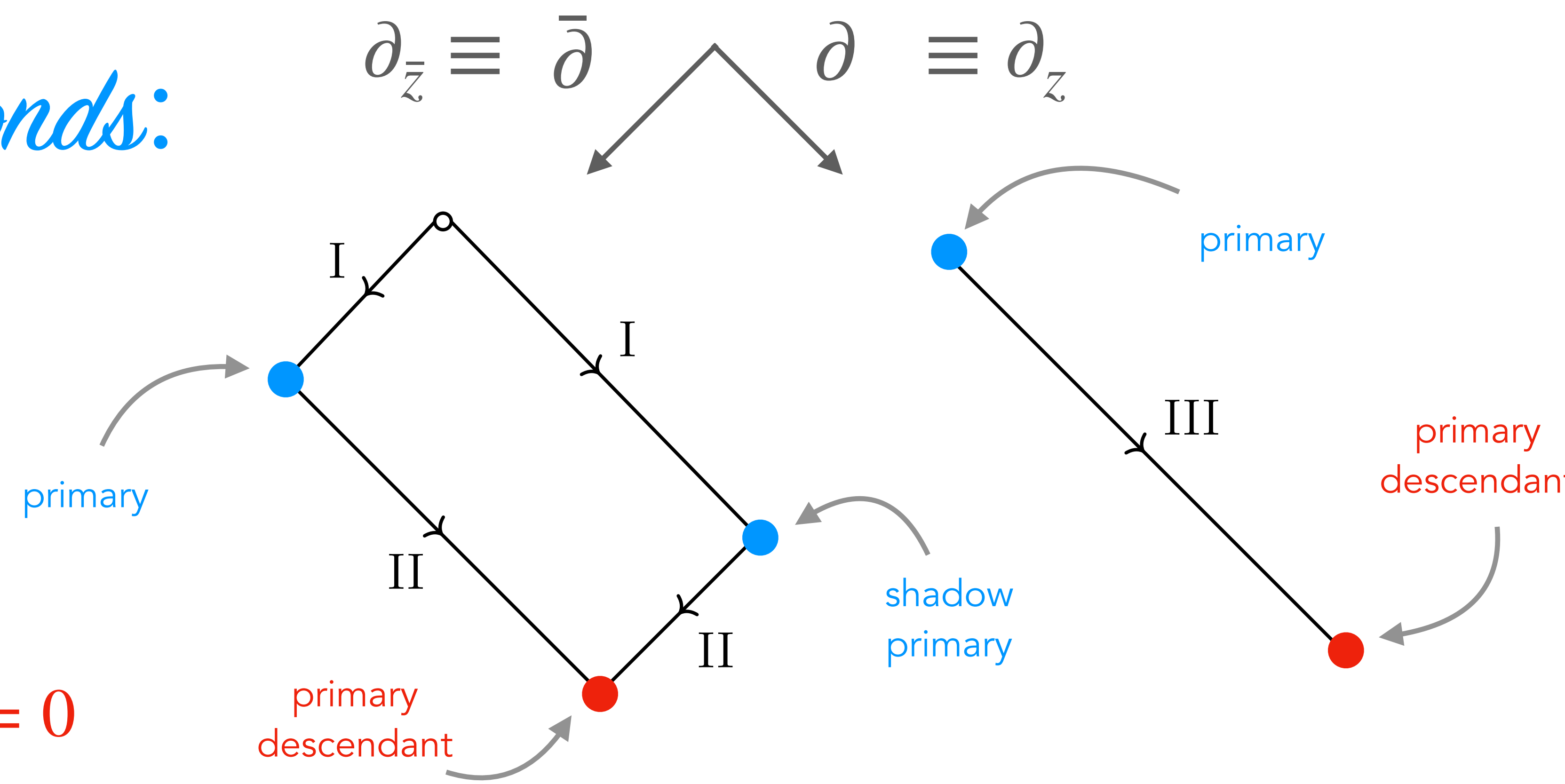
**memory effect**

⋮

# Towers of ∞ symmetries

*celestial diamonds:*

[Pasterski,AP,Trevisani'21]




conservation equation $\bar{\partial}^{\#}\mathcal{O}(z,\bar{z}) = 0$

\# depends on types I, II, III: spin of descendant >,<,= spin of parent primary

Classification of all symmetries in celestial CFT$_{d\geq 2}$.

[Pasterski,AP,Trevisani'21] $d = 2$

[Pano,AP,Trevisani'23] $d > 2$

$d = 2:$ $\mathcal{O}_\Delta(x)$ with $\Delta = 1, 0, -1, \ldots$ satisfy ∞ dimensional algebra!

[Guevara,Himwich,Pate,Strominger'21]

[Strominger'21]

# Towers of ∞ symmetries

Define a discrete family of conformally soft positive-helicity gravitons

$$H^k = \lim_{\varepsilon \to 0} \varepsilon \mathcal{O}_{k+\epsilon,+2} \qquad k = 2,1,0,-1,-2,\ldots$$

supertranslations

superrotation

with weights $(h, \bar{h}) = \left( \frac{k+2}{2}, \frac{k-2}{2} \right)$

and a consistently-truncated antiholomorphic mode expansion

$$H^k(z,\bar{z}) = \sum_{n=\frac{k-2}{2}}^{\frac{2-k}{2}} \frac{H^k_n(z)}{\bar{z}^{n+\frac{k-2}{2}}}$$

$$w^p_n = \frac{1}{\kappa}(p-n-1)!(p+n-1)!H_n^{-2p+4} \qquad p,q = 1,\frac{3}{2},2,\frac{5}{2},\ldots$$

$$[w^p_m, w^q_n] = [m(q-1) - n(p-1)]w^{p+q-2}_{m+n}$$

Arises already in Penrose's twistor construction!

This is the $w_{1+\infty}$ algebra.

[Guevara,Himwich,Pate,Strominger'21]

[Strominger'21]

# 2D soft actions

A toy model that captures the features of the diamond structure is the higher derivative Gaussian theory with action [Pasterski,AP,Trevisani'21]

$$S = \int d^2 z \left[ \partial_z^k \mathcal{O}^s_{\Delta,J} \partial_{\bar{z}}^{\bar{k}} \mathcal{O}^s_{\Delta,J} + \partial_z^k \mathcal{O}^s_{\Delta,-J} \partial_{\bar{z}}^{\bar{k}} \mathcal{O}^s_{\Delta,-J} \right]$$

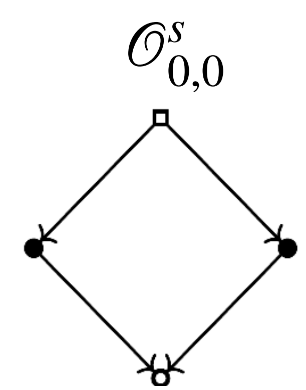
with conservation equation

$$\partial_z^k \partial_{\bar{z}}^{\bar{k}} \mathcal{O}^s_{\Delta,J} = 0$$

$$\Delta = 1 - \frac{k+\bar{k}}{2} \qquad J = \frac{k-\bar{k}}{2}$$

$$k, \bar{k} \in \mathbb{Z}_>$$

Simplest example:
free boson ($k = \bar{k} = 1$)




QED

Yang-Mills [Cheung,de La Fuente, Sundrum'15] [Magnea'21] [Gonzáles,Rojas'21]

gravity [Nguyen,Salzer'20] [Nguyen'21] [Kalyanapuram'20+'21]

# Soft effective action

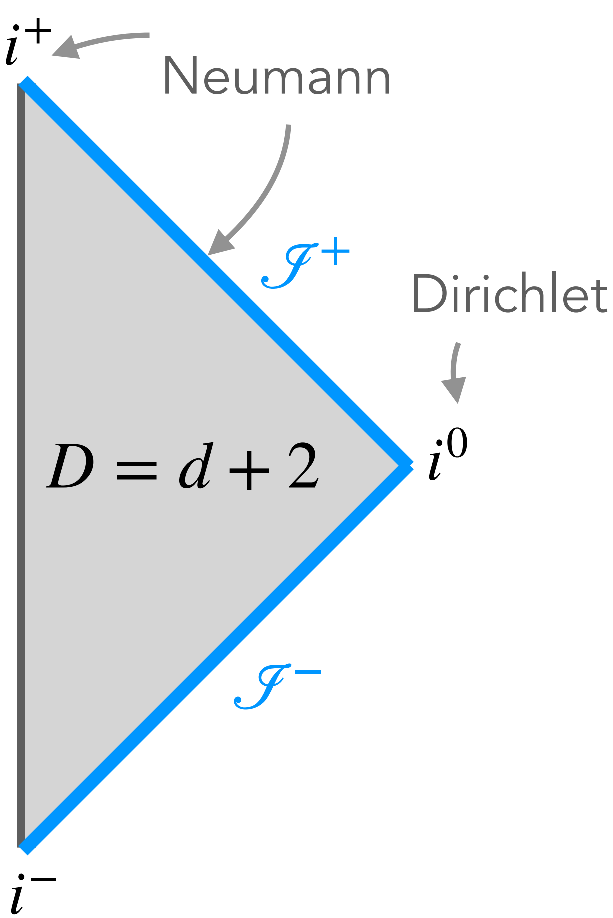
captures soft exchanges and IR divergences [Kapec,Mitra'21]

soft, hard

$$\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_{\Lambda_{\mathrm{IR}}} = \int [d\varphi_s][d\varphi_h] e^{iS_{\mathrm{bulk}}[\varphi_s,\varphi_h]} \mathcal{O}_1 \cdots \mathcal{O}_n$$

$$= \underset{\text{hard}}{\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_{\Lambda_{\mathrm{soft/hard}}}} \underbrace{\int [d\varphi_s] e^{-S_{\mathrm{soft}}[\varphi_s]-S_{\mathrm{int}}[\varphi_s,j]}}_{e^{-\Gamma(\Lambda_{\mathrm{IR}},\Lambda_{\mathrm{soft/hard}})}}$$

source




This model captures: abelian infrared divergences in $d = 2$, the re-summed (infrared finite) soft exchange in $d > 2$, reproduces the leading soft theorems in gauge and gravitational theories in all $d$.

The effective dynamics of Goldstone "edge" mode is d-dim.

[Kapec,Mitra,Sivaramakrishnan,Zurek'24]

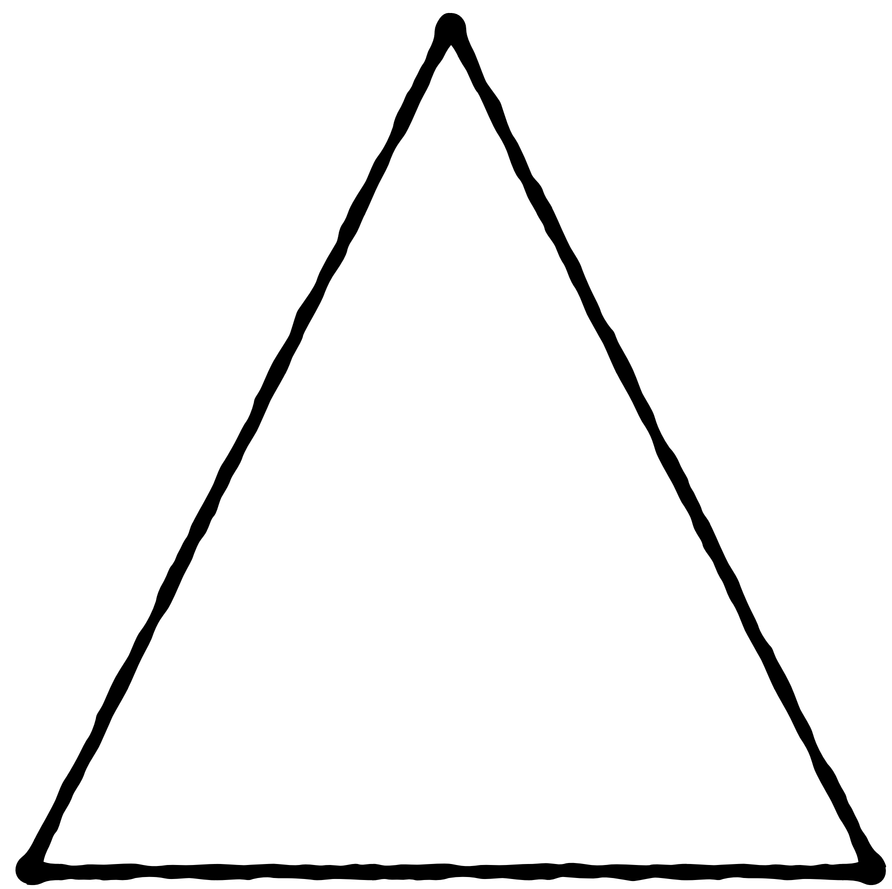
# Long-range effects

# IR triangle **@ tree !**

∞-dimensional
symmetry algebra

⊃ local conformal
symmetry on $S^2$ **!**

**asymptotic symmetry**



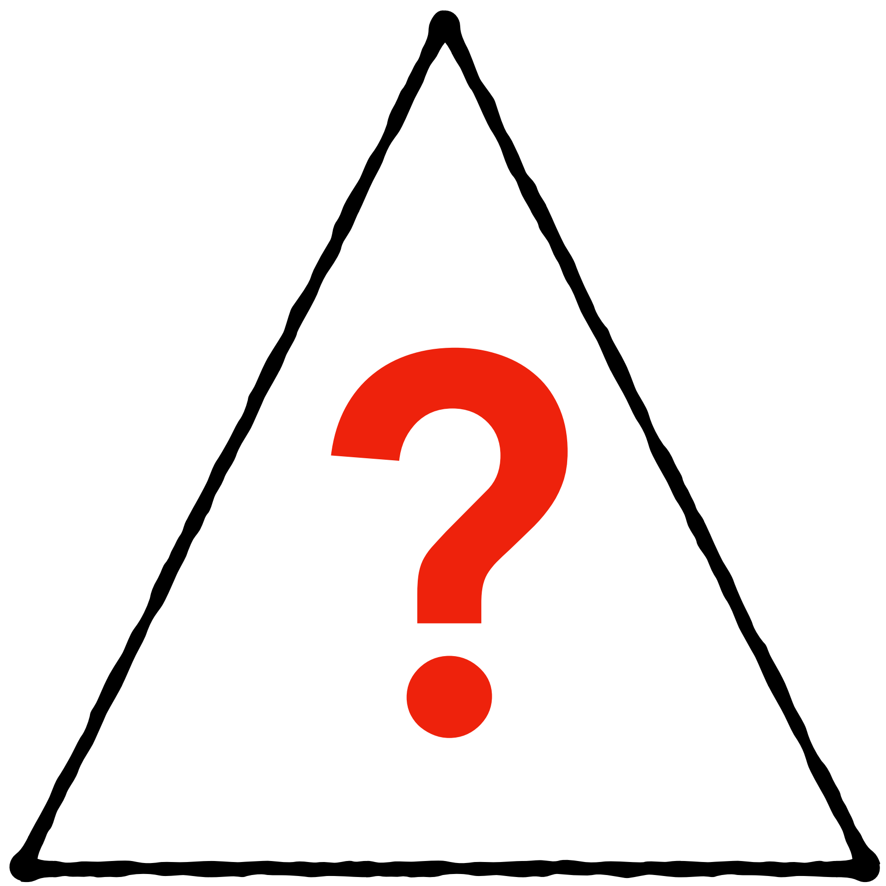
**soft theorem** **memory effect**

# IR triangle @ loop ?

$\infty$-dimensional
symmetry algebra

$\supset$ local conformal
symmetry on $S^2$ **?**

**asymptotic symmetry**




**soft theorem** **memory effect**

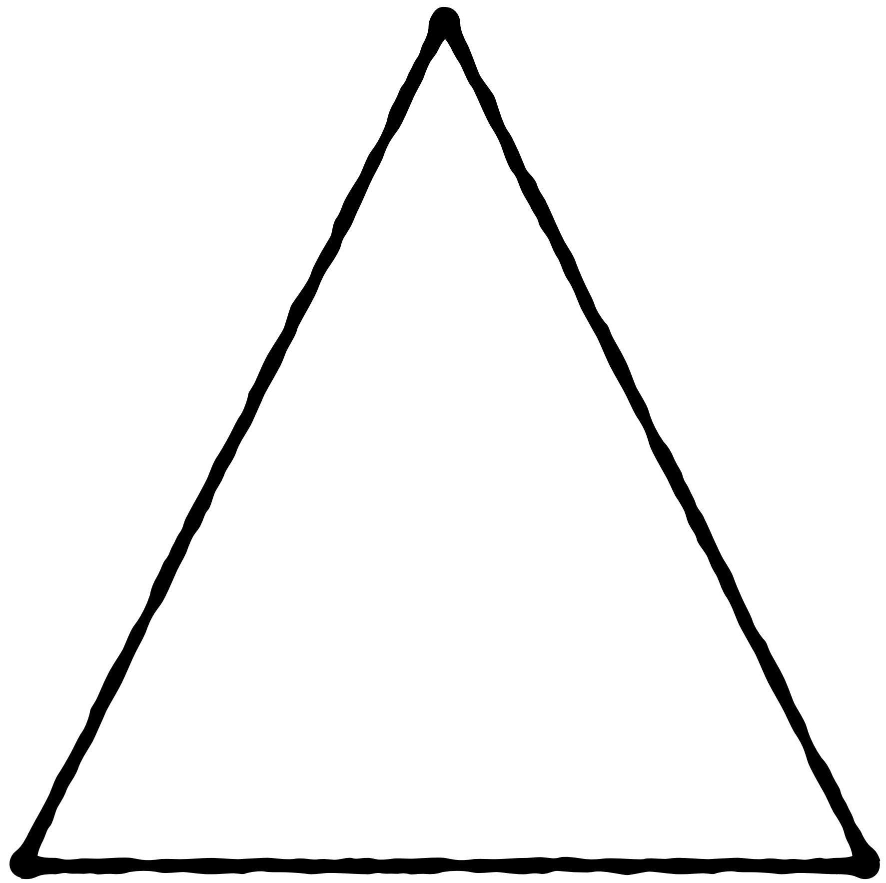
# IR triangle @ long-range IR effects

classical and quantum

$\infty$-dimensional symmetry algebra

$\supset$ local conformal symmetry on $S^2$ ?

**asymptotic ? symmetry**



**logarithmic soft theorem**

**tail memory effect**

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

[Sahoo'20]
[Sahoo,Sen'21]
[Sahoo,Krishna'23]

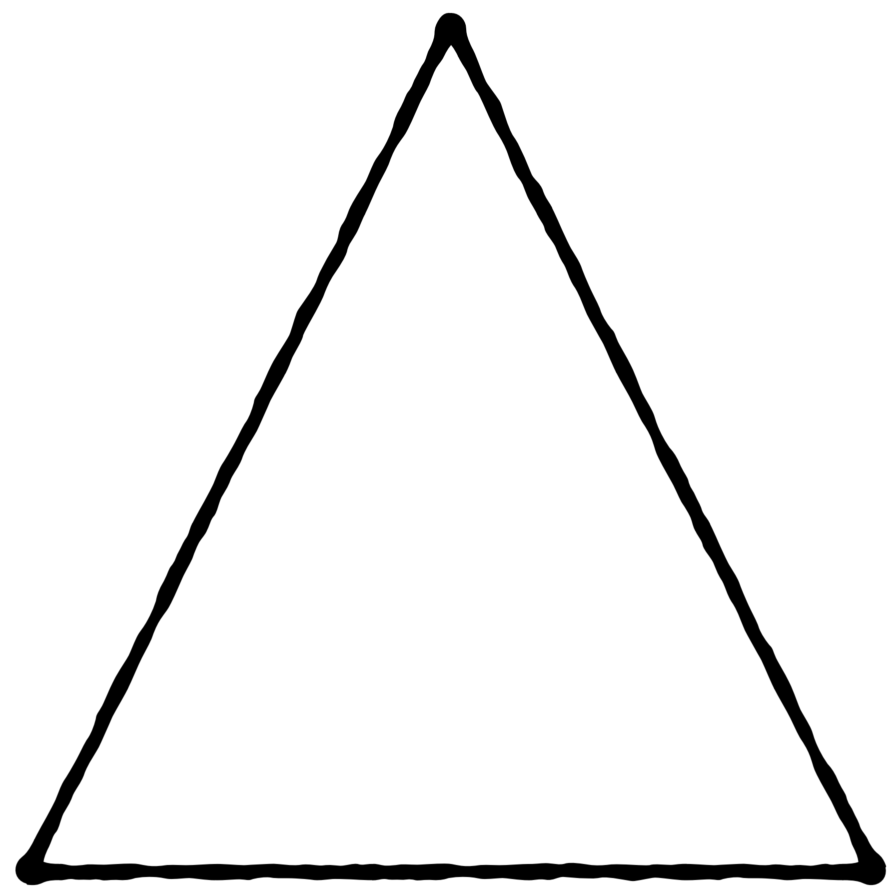
# IR triangle @ long-range IR effects

classical and quantum

$\infty$-dimensional symmetry algebra

$\supset$ local conformal symmetry on $S^2$ **?**

**asymptotic ? symmetry**



logs render ambiguous all subleading tree-level soft theorems

↑

**logarithmic soft theorem**

**tail memory effect**

$\omega^{-1}$ leading soft

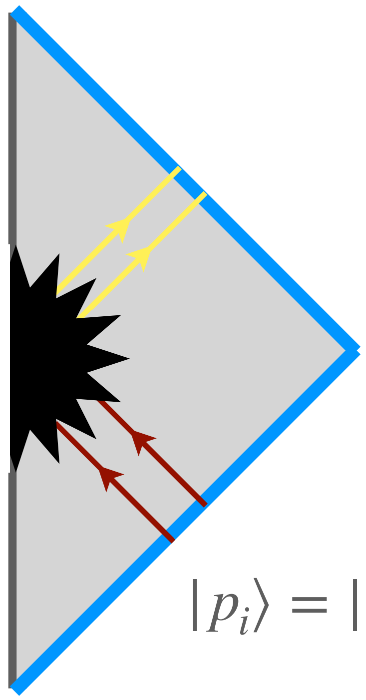
$\log \omega$ subleading soft

$\vdots$


[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

[Sahoo'20]
[Sahoo,Sen'21]
[Sahoo,Krishna'23]

# Soft tower



$|p_i\rangle = |\omega_i, x_i\rangle$

energy basis

$\langle p_n^{\text{out}} \dots | \mathcal{S} | \dots p_1^{\text{in}} \rangle$

translation symmetry

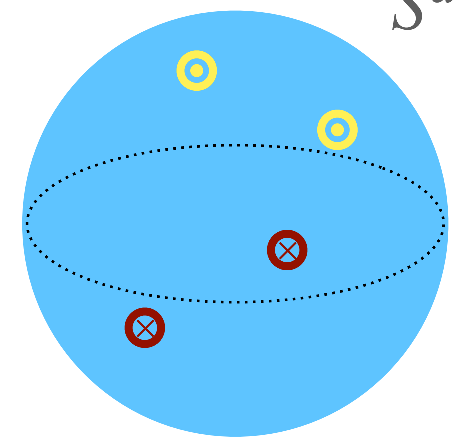
**S-matrix**

$\mathscr{M}$ellin

$\xrightarrow{\hspace{3em}}$

$\int_0^\infty d\omega\, \omega^{\Delta-1}$

$S^d$



$|\Delta_i, x_i\rangle$

boost-weight basis

$\langle \mathcal{O}^-_{\Delta_1}(x_1) \dots \mathcal{O}^+_{\Delta_n}(x_n) \rangle$

Lorentz symmetry

| energetically soft expansion: | | ``conformally soft" poles: | |
|---|---|---|---|
| $\frac{1}{\omega}, 1, \omega, ..$ | **tree-level** | $\Delta = 1, 0, -1, \dots$ | simple poles |
| $\log \omega, \dots$ | **loops** | $\Delta = 0, \dots$ | higher poles |

Same as tree-level subleading symmetry operators!

# Power-law soft theorems

**Tree-level** amplitudes admit a soft expansion:

[Low'58] [Weinberg'65] [Cachazo,Strominger'14]
[Hamada,Shiu'18] [Li,Lin,Zhang'18]

hard momenta helicity

$$\mathcal{M}_{N+1}(p_1, \ldots, p_N; (\omega, q, \ell)) = \sum_{n=-1}^{\infty} \omega^n S_n(\{p_i\}, (q, \ell))\mathcal{M}_N(p_1, \ldots, p_N) + \ldots$$

soft momentum

$$p^\mu = \omega q^\mu(z, \bar{z})$$

⊃ non-universal *

| | | | |
|---|---|---|---|
| $n = -1$ | $S_{-1}$ | Weinberg (leading) soft factor | tree exact & universal |
| $n = 0$ | $S_0$ | subleading tree soft factor | |
| $n > 0$ | $S_{n>0}$ | sub$^{n+1}$ leading tree soft factors | |

* see [Elvang,Jones,Naculich'16] for classification in EFT of massless particles

# Logarithmic Soft Theorems

**Long-range effects** yield **novel soft theorems**:

[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

hard momenta · helicity · soft momentum

$$\mathcal{M}_{N+1}(p_1, \ldots, p_N; (\omega, q, \ell)) = \sum_{n=-1}^{\infty} \boxed{\omega^n (\ln \omega)^{n+1}\, S_n^{(\ln \omega)}}(\{p_i\}, (q, \ell))\mathcal{M}_N(p_1, \ldots, p_N) + \ldots$$

$$p^\mu = \omega q^\mu(z, \bar{z})$$

$\supset$ non-universal $\sim \omega^n (\ln \omega)^{m \neq n+1}$

| | | | |
|---|---|---|---|
| $n = -1$ | $S_{-1}^{(\ln \omega)} \equiv S_{-1}$ | Weinberg (leading) soft factor | tree exact & universal |
| $n = 0$ | $S_0^{(\ln \omega)} \neq S_0$ | leading log soft factor | 1-loop exact & universal |
| $n > 0$ | $S_{n>0}^{(\ln \omega)} \neq S_{n>0}$ | sub$^n$ leading log soft factor | |

Is the **universality** of the loop exact logarithmic soft theorems a consequence of **asymptotic symmetries of the S-matrix** ?

# Conservation laws

To establish a symmetry interpretation for a soft theorem from *first principles*: for asymptotic symmetry transformations $\delta$ compute charges $Q^{\pm}$ from the symplectic structure

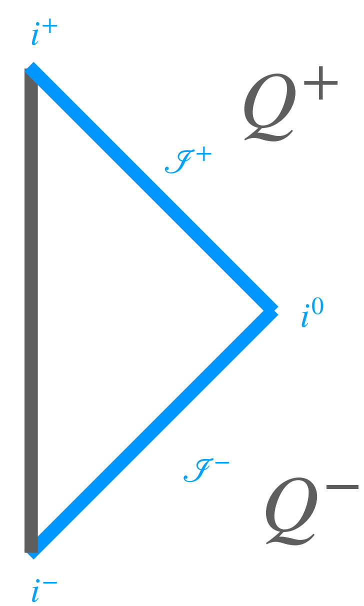
$$\Omega_{i^{\pm} \cup \mathscr{I}^{\pm}}(\delta, \delta') = \delta' Q^{\pm}$$

in the **covariant phase space formalism** and show that the

**charge conservation law**

Upon identifying the fields and symmetry parameter antipodally:

$$Q^{+} = Q^{-}$$




corresponds to the **soft theorem**.

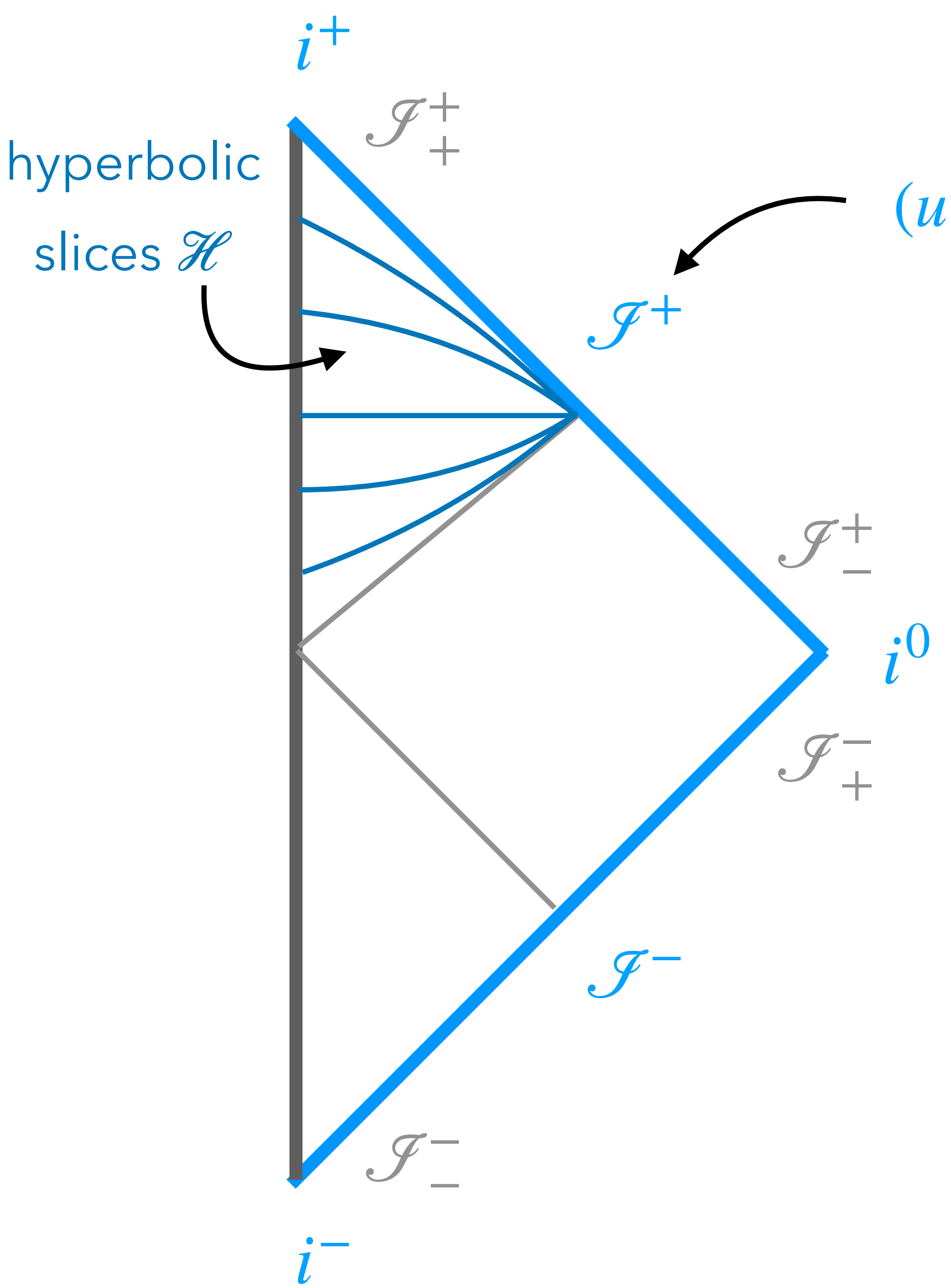
# Structure at $\infty$

blow-up of $i^+$:

$(\tau, \rho, x)$

Euclidean $\mathrm{AdS}_3$ coordinates

$$\tau = \sqrt{t^2 - r^2} \qquad \rho = \frac{r}{\sqrt{t^2 - r^2}}$$




$(u, r, x)$

retarded Bondi coordinates

$$u = t - r$$

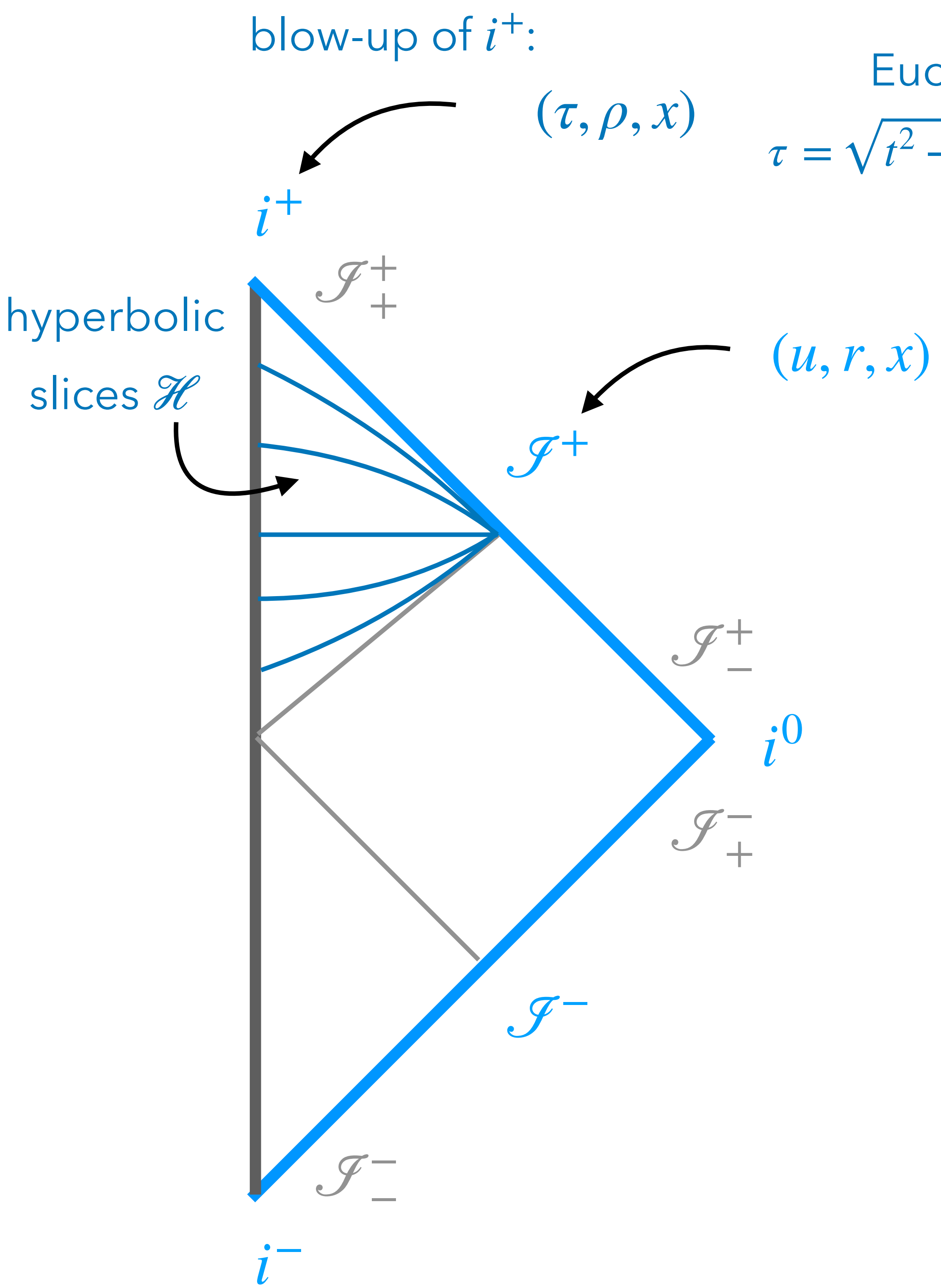
$$\Omega_{i^+ \cup \mathscr{I}^+}(\delta, \delta') = \delta' Q^+$$

limit $\tau \to \infty$ at fixed $\rho$ — limit $r \to \infty$ at fixed u

$$Q_H^+ \quad + \quad Q_S^+ \quad = \quad Q^+$$

hard charge — soft charge

# Structure at $\infty$




Euclidean $AdS_3$ coordinates

$$\tau = \sqrt{t^2 - r^2} \qquad \rho = \frac{r}{\sqrt{t^2 - r^2}}$$

retarded Bondi coordinates

$$u = t - r$$

$$\Omega_{i^+ \cup \mathscr{I}^+}(\delta, \delta') = \delta' Q^+$$

limit $\tau \to \infty$ at fixed $\rho$ — limit $r \to \infty$ at fixed u

$$Q_H^+ + Q_S^+ = Q^+$$

hard charge — soft charge

in soft theorem

action on hard matter — soft insertion

# Soft graviton factor

[Weinberg'65]

[Cachazo,Strominger'14]

Leading soft factor $\sim \frac{1}{\omega}$: $$S_{-1} = \frac{\kappa}{2}\sum_{i=1}^{N}\frac{\varepsilon_{\mu\nu}p_i^{\mu}p_i^{\nu}}{q\cdot p_i}$$

$p_i^{\mu}$... hard momenta

$p^{\mu} = \omega q^{\mu}$... soft momentum

$\varepsilon^{\mu\nu}$...soft graviton polarization

Subleading soft factor $\sim \omega^0$: $$S_0 = -\frac{i\kappa}{2}\sum_{i=1}^{N}\frac{\varepsilon_{\mu\nu}p_i^{\mu}q_{\lambda}J_i^{\lambda\nu}}{q\cdot p_i}$$

ambiguous if long-range IR effects

# Soft graviton factor

[Weinberg'65]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

Leading soft factor $\sim \dfrac{1}{\omega}$:

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$\varepsilon^{\mu\nu}$...soft graviton polarization

Subleading soft factor $\sim \ln \omega$:

$$S_0^{(\ln \omega)} = S_{0,\mathrm{classical}}^{(\ln \omega)} + S_{0,\mathrm{quantum}}^{(\ln \omega)}$$

classical: late time gravitational radiation from particle acceleration via long-range gravitational interaction

$$S_{0,\mathrm{classical}}^{(\ln \omega)} = \frac{i(\frac{\kappa}{2})^3}{8\pi} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\nu} p_i^\nu q_\rho}{q \cdot p_i} \sum_{j \neq i, \eta_i \eta_j = 1} \frac{(p_i \cdot p_j)\left[p_i^\mu p_j^\rho - p_j^\mu p_i^\rho\right]\left[2(p_j \cdot p_j)^2 - 3 p_i^2 p_j^2\right]}{\left[(p_i \cdot p_j)^2 - p_i^2 p_j^2\right]^{3/2}}$$

quantum: $\omega_{\mathrm{soft}} \ll \omega_{\mathrm{loop}} \ll \omega_{\mathrm{hard}}$
(1-loop)

$$S_{0,\mathrm{quantum}}^{(\ln \omega)} = -\frac{(\frac{\kappa}{2})^3}{16\pi^2} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\rho} p_i^\rho q_\nu}{q \cdot p_i} \left(p_i^\mu \partial_{p_i}^\nu - p_i^\nu \partial_{p_i}^\mu\right) \sum_{j \neq i} \frac{2(p_i \cdot p_j)^2 - p_i^2 p_j^2}{\sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2)}} \ln \left( \frac{p_i \cdot p_j + \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}}{p_i \cdot p_j - \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}} \right)$$

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Leading soft factor $\sim \frac{1}{\omega}$:

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quantum: $\omega_{\text{soft}} \ll \omega_{\text{loop}} \ll \omega_{\text{hard}}$ $\omega_{\text{IR}} \ll \omega_{\text{loop}} \ll \omega_{\text{soft}}$

(1-loop)

$$S_{0,\text{quantum}}^{(\ln \omega)} = -\frac{(\frac{\kappa}{2})^3}{16\pi^2} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\rho} p_i^\rho q_\nu}{q \cdot p_i} \left(p_i^\mu \partial_{p_i}^\nu - p_i^\nu \partial_{p_i}^\mu\right) \sum_{j \neq i} \frac{2(p_i \cdot p_j)^2 - p_i^2 p_j^2}{\sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2)}} \ln \left( \frac{p_i \cdot p_j + \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}}{p_i \cdot p_j - \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}} \right)$$ + drag

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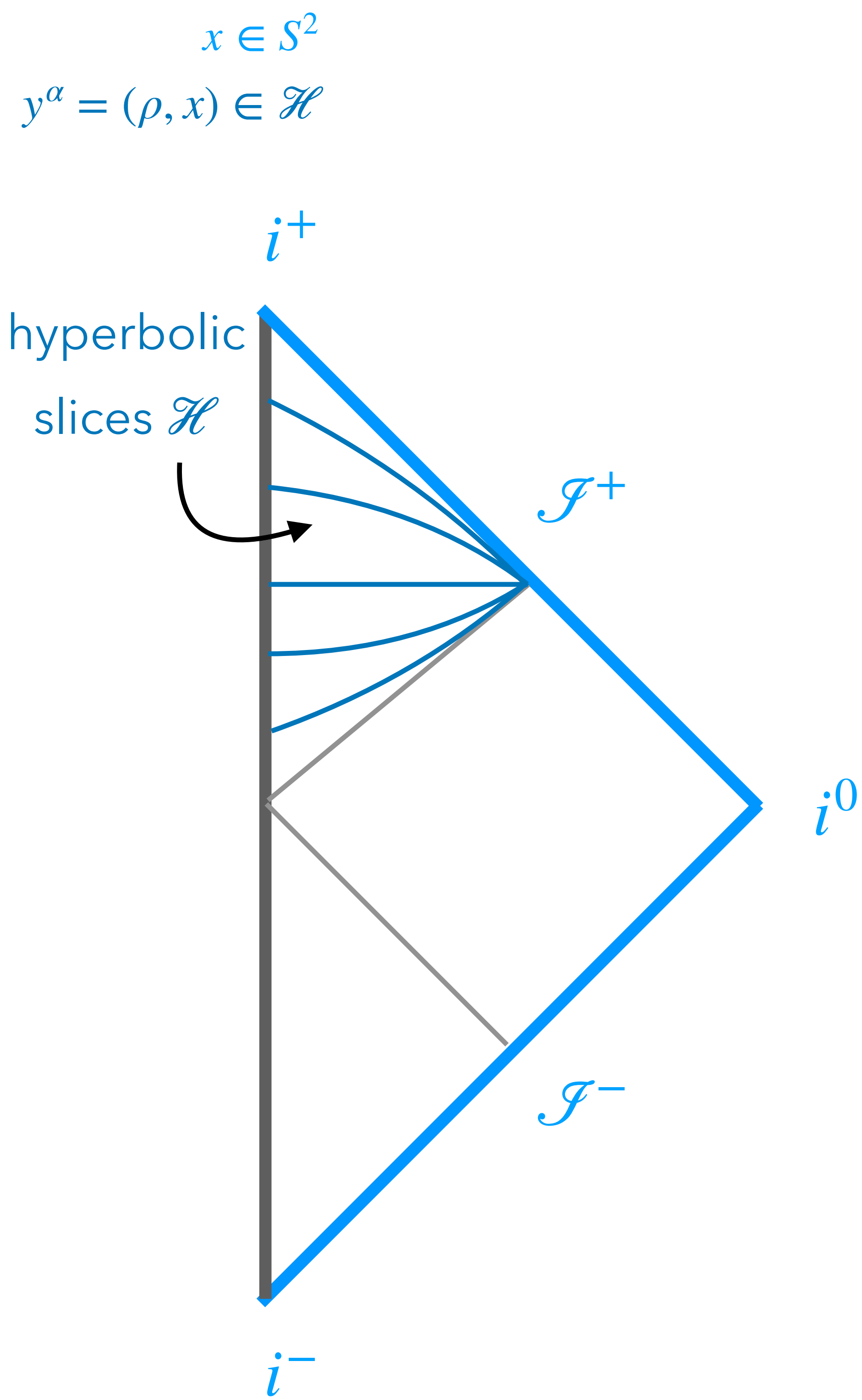
# Symmetry @ $i^+ \cup \mathscr{I}^+$




**Superrotation across $i^+ \cup \mathscr{I}^+$**

$\mathscr{I}^+$ : $Y^A(x)$

$$\delta_Y \gamma_{AB} = 2D_{(A}Y_{B)} - D\cdot Y\,\gamma_{AB}$$

$$\delta_Y C_{AB} = \left[\mathscr{L}_Y - \tfrac{1}{2}D\cdot Y(1-u\partial_u)\right] C_{AB}$$
$$+u\left[D_{(A}(D^2+1)Y_{B)} - D_A D_B D\cdot Y - \tfrac{1}{2}\gamma_{AB}(D^2+4)D\cdot Y\right]$$

$i^+$ : $\bar{Y}^\alpha(y)$

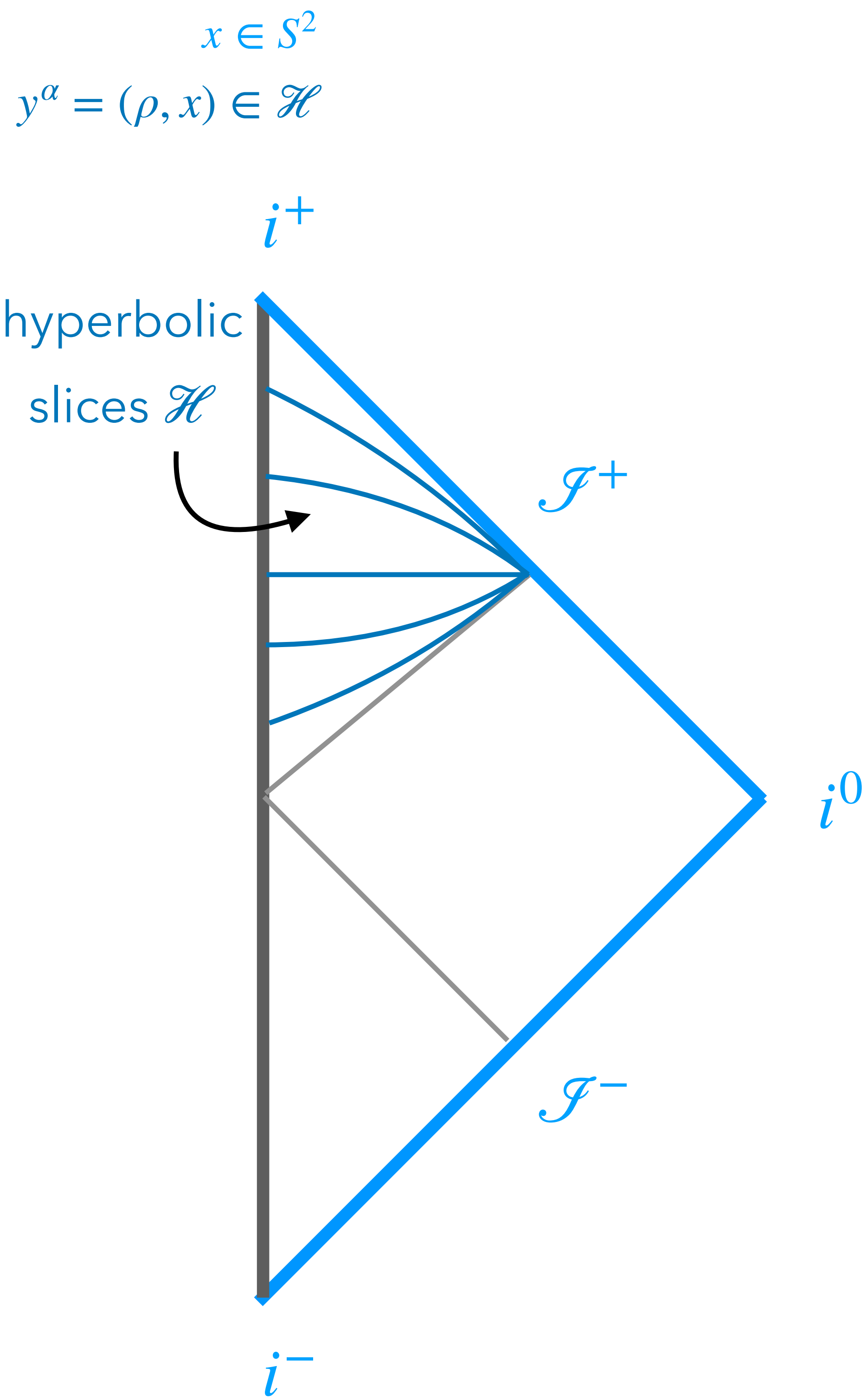
$$\delta\varphi = \bar{Y}^\alpha \partial_\alpha \varphi$$

$i^+ \cup \mathscr{I}^+$ : 

$$\bar{Y}^\alpha(y) = \int_{S^2} d^2x\, \underbrace{G^\alpha_A(y;x)}_{\text{vector Green's function}} Y^A(x)$$

$\Rightarrow$ Superrotation vector field extends smoothly across $i^+ \cup \mathscr{I}^+$ .

# Phase space @ $i^+ \cup \mathscr{I}^+$




**Asymptotic phase space on $i^+ \cup \mathscr{I}^+$**

$$i^+ : h_{\tau\tau}(\tau, y) \overset{\tau\to\infty}{=} \frac{1}{\tau}\overset{1}{h}_{\tau\tau}(y) + \dots$$

\`Coulombic' mode sourced by matter stress tensor

$$\varphi(\tau, y) = \frac{\sqrt{m}}{2(2\pi)^{3/2}} \sum_{n=0}^{\infty} \frac{e^{-im\tau}}{\tau^{\frac{3}{2}+n}} \left( \overset{\ln}{b}_n(y) \ln\tau + b_n(y) \right) + \text{c.c.} + \dots$$

$\overset{\ln}{b}_n$ and $b_{n+1}$ for $n \geq 0$ fixed by $b \equiv b_0$

$$\mathscr{I}^+ : \; C_{AB}(u, x) \overset{u\to+\infty}{=} C^{(0),+}_{AB}(x) + \frac{1}{u} C^{(1),+}_{AB}(x) + \dots$$

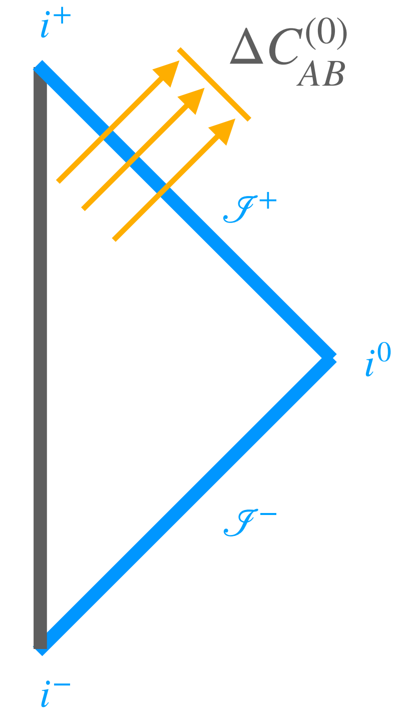
$= \lim_{r\to\infty} \frac{1}{r} h_{AB}(u, r, x)$

displacement memory

tail

$\Rightarrow$ Tails and logs from long-range interactions.

# Memory and its tail




Shear @ late times: $C_{AB}(u,x) \overset{u\to\pm\infty}{=} C_{AB}^{(0),+}(x) + \frac{1}{u} C_{AB}^{(1),+}(x) + \dots$

linear displacement memory sourced by matter field: $\Delta C_{AB}^{(0)} = C_{AB}^{(0),+} - C_{AB}^{(0),-}$

$$C_{AB}^{(0),\pm} = -\frac{\kappa^2}{8\pi}\int_{i^\pm} d^3y \frac{(\partial_A q\cdot \mathscr{Y})(\partial_B q\cdot\mathscr{Y}) + \frac{1}{2}\gamma_{AB}}{q\cdot\mathscr{Y}} {}^3T^{\mathrm{matt}}_{\tau\tau}(y)$$

$x^\mu = \tau \mathscr{Y}(y)$

unit vector in Minkowski

# Memory and its tail

Shear @ late times: $C_{AB}(u,x) \overset{u\to+\infty}{=} C^{(0),+}_{AB}(x) + \frac{1}{u}C^{(1),+}_{AB}(x) + \dots$

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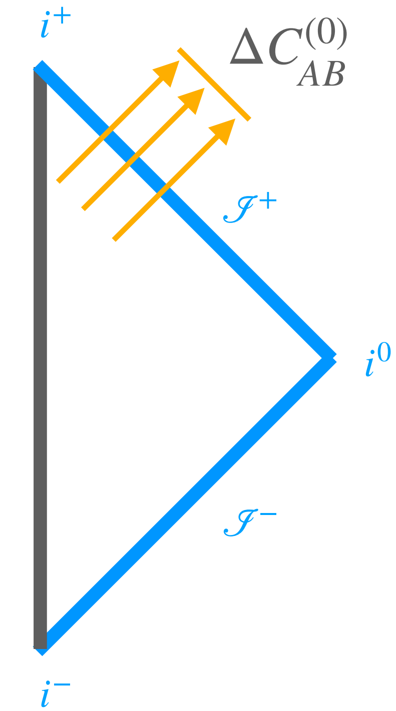
$$\mathscr{P}^\alpha(y) = \overset{3}{T}{}^{\text{matt}}_{\tau\tau}(y)\mathscr{Y}^\alpha$$

linear momentum density of massive scalar

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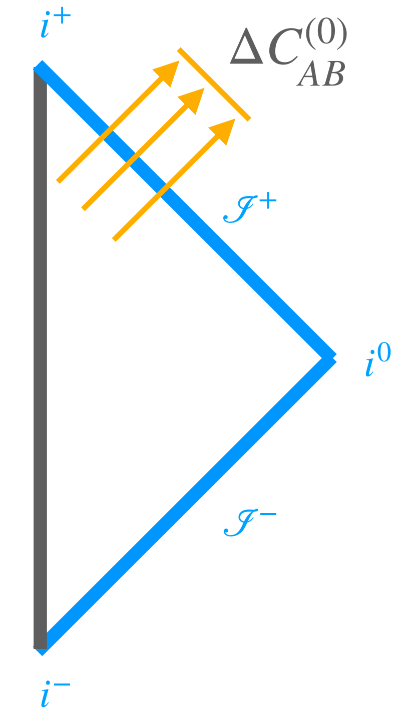
# Memory and its tail

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$$\mathscr{P}^\alpha(y) = \overset{3}{T}{}^{\text{matt}}_{\tau\tau}(y)\mathscr{Y}^\alpha$$

linear momentum density of massive scalar

$$x^\mu = \tau\mathscr{Y}(y)$$

unit vector in Minkowski

tail to the memory sourced by matter field: $\Delta C^{(1)}_{AB} = C^{(1),+}_{AB} - C^{(1),-}_{AB}$

$$C^{(1),\pm}_{AB} = -\frac{\kappa^2}{8\pi}(\partial_A q^\mu)(\partial_B q^\nu)\int_{i^\pm} d^3y\Big[\frac{(q\cdot\mathscr{Y})\mathscr{D}^\alpha(\mathscr{Y}_\mu\mathscr{Y}_\nu) - (\mathscr{Y}_\mu\mathscr{Y}_\nu + \frac{1}{2}\eta_{\mu\nu})\mathscr{D}^\alpha(q\cdot\mathscr{Y})}{q\cdot\mathscr{Y}}\overset{3,\ln}{T}{}^{\text{matt}}_{\tau\alpha}$$
$$-\left((\mathscr{Y}_\mu\mathscr{Y}_\nu + \frac{1}{2}\eta_{\mu\nu})k^{\alpha\beta} + (\mathscr{D}^\alpha\mathscr{Y}_\mu)(\mathscr{D}^\beta\mathscr{Y}_\nu) - \frac{1}{2}\eta_{\mu\nu}(\mathscr{D}^\alpha\mathscr{Y}_\sigma)(\mathscr{D}^\beta\mathscr{Y}^\sigma)\right)\overset{2}{T}{}^h_{\alpha\beta}\Big]$$

# Gravity: hard and soft charges

[Choi,Laddha,AP'24]

$$\Omega_{i^+ \cup \mathscr{I}^+} = \underbrace{\Omega_{i^+}}_{\downarrow} + \underbrace{\Omega_{\mathscr{I}^+}}_{\downarrow}$$

$$Q = Q_H + Q_S$$

hard charge soft charge

$$\Omega_{i^\pm \cup \mathscr{I}^\pm}(\delta\,, \boxed{\delta_Y}) = \delta Q_\pm$$

**superrotation**

# Gravity: hard and soft charges

[Choi,Laddha,AP'24]

diverges logarithmically as $\tau \to \infty, u \to \infty$

$$\Omega_{i^+ \cup \mathscr{I}^+} = \Omega_{i^+} + \Omega_{\mathscr{I}^+}$$

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hard charge    soft charge

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**superrotation**

# Gravity: hard and soft charges

[Choi,Laddha,AP'24]

diverges logarithmically as $\tau \to \infty, u \to \infty$ → $\Omega_{i^+ \cup \mathscr{I}^+} = \Omega_{i^+} + \Omega_{\mathscr{I}^+}$

regulate via late-time cutoff $\Lambda^{-1}$

$$Q = Q_H + Q_S$$

hard charge soft charge

$$\Omega_{i^\pm \cup \mathscr{I}^\pm}(\delta\,, \delta_Y) = \delta Q_\pm$$

**superrotation**

# Gravity: hard and soft charges

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$$\Omega_{i^+\cup\mathscr{I}^+} = \Omega_{i^+} + \Omega_{\mathscr{I}^+}$$

$$\Omega_{i^\pm\cup\mathscr{I}^\pm}(\delta\,,\delta_Y) = \delta Q_\pm$$

superrotation

$$Q = Q_H + Q_S$$

hard charge soft charge

Regularized Noether charge:

$$\boxed{Q^\Lambda = \ln\Lambda^{-1}\left(Q_H^{(\ln)} + Q_S^{(\ln)}\right) + \left(Q_H^{(0)} + Q_S^{(0)}\right) + \ldots}$$

$\overset{1}{h}_{\tau\tau}$ `Coulombic'

$$Q_{H,+}^{(\ln)}[\bar{Y}] = \int_{i^+} d^3y\,\bar{Y}^\alpha\,\overset{3,\ln}{T}{}^{\text{matt}}_{\tau\alpha}$$

leading interacting stress tensor

$$Q_S^{(\ln)}[Y] = -\frac{2}{\kappa}\int_{\mathscr{I}^+} du\,d^2x\,D_z^3Y^z\,\partial_u(u^2\partial_u C^{zz}) + \text{c.c.}$$

log soft projector

$$Q_{H,+}^{(0)}[\bar{Y}] = \int_{i^+} d^3y\,\bar{Y}^\alpha(y)\overset{3}{T}_{\tau\alpha}(y)$$

free stress tensor

$$Q_S^{(0)}[Y] = -\frac{2}{\kappa}\int_{\mathscr{I}^+} du\,d^2x\,D_z^3Y^z\,u\partial_u C^{zz} + \text{c.c.}$$

$D_A$ ... covariant derivative on $S^2$

sub tree soft projector

# $Q^{(0)}$ charge

$$Q^{\Lambda} = \ln \Lambda^{-1} \left( Q_H^{(\ln)} + Q_S^{(\ln)} \right) + \left( Q_H^{(0)} + Q_S^{(0)} \right) + \ldots$$

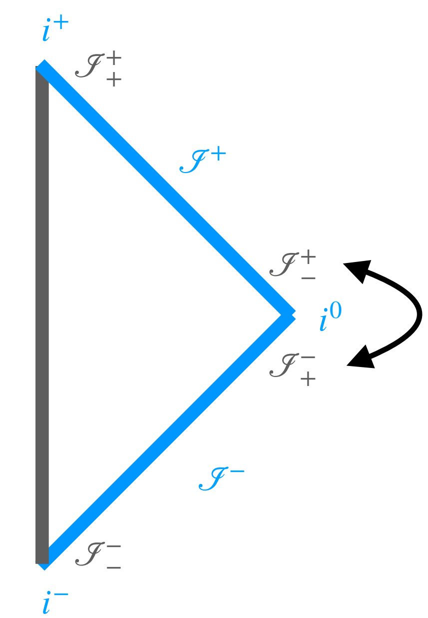
$$\equiv Q^{(0)}$$

Conservation law:

[Campiglia,Laddha'15]

$$Q_+^{(0)} = Q_-^{(0)}$$

Upon identifying the fields and gauge parameter antipodally:



# $Q^{(0)}$ charge

$$Q^{\Lambda} = \ln \Lambda^{-1} \left( Q_H^{(\ln)} + Q_S^{(\ln)} \right) + \underbrace{\left( Q_H^{(0)} + Q_S^{(0)} \right)}_{\equiv Q^{(0)}} + \ldots$$

Conservation law:

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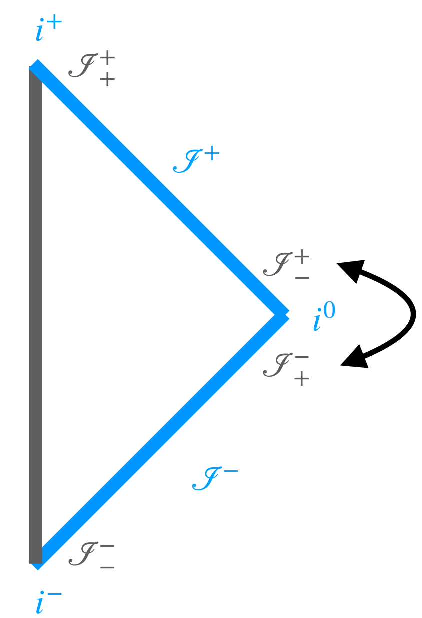
Upon identifying the fields and gauge parameter antipodally:

[Kapec,Lysov,Pasterski,Strominger'14]

judicious choice of **superrotation** $Y^A(x)$




Tree-level subleading soft **graviton** theorem:

$$\mathscr{M}_{N+1} = \left( \omega^{-1} S_{-1} + \omega^{0} S_0 \right) \mathscr{M}_N + \ldots$$

tree-level soft expansion

[Cachazo,Strominger'14]

# $Q^{(0)}$ charge

$$Q^{\Lambda} = \ln \Lambda^{-1} \left( Q_H^{(\ln)} + Q_S^{(\ln)} \right) + \left( Q_H^{(0)} + Q_S^{(0)} \right) + \ldots$$

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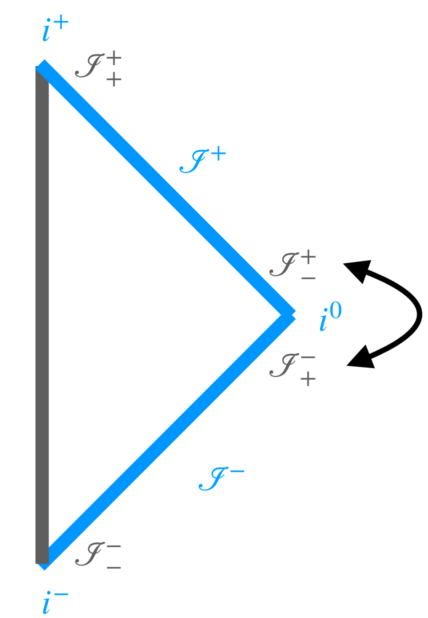



judicious choice of **superrotation** $Y^A(x)$

Tree-level subleading soft **graviton** theorem:

$$\mathscr{M}_{N+1} = \left( \omega^{-1} S_{-1} + \omega^0 S_0 \right) \mathscr{M}_N + \ldots$$

tree-level soft expansion

[Cachazo,Strominger'14]

Recall: IR effects render subleading soft theorem at tree-level ambiguous.

# $Q^{(\ln)}$ charge

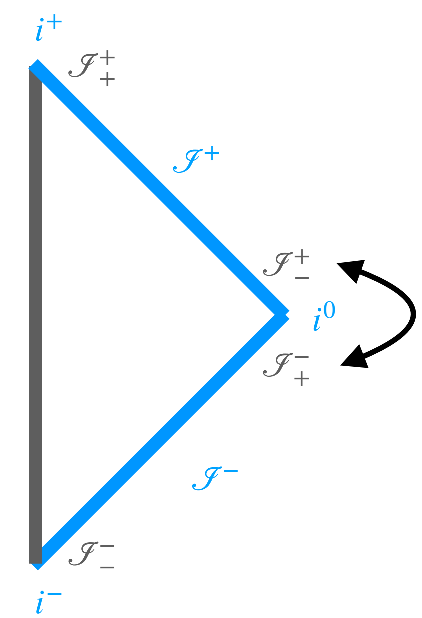
$$Q^{\Lambda} = \ln \Lambda^{-1} \underbrace{\left( Q_H^{(\ln)} + Q_S^{(\ln)} \right)}_{\equiv\, Q^{(\ln)}} + \left( Q_H^{(0)} + Q_S^{(0)} \right) + \ldots$$

Conservation law:

[Choi,Laddha,AP'24]

$$Q_+^{(\ln)} = Q_-^{(\ln)}$$

Upon identifying the fields and gauge parameter antipodally:




This is **exact** in the **coupling** $\kappa$ !

# $Q^{(\ln)}$ charge

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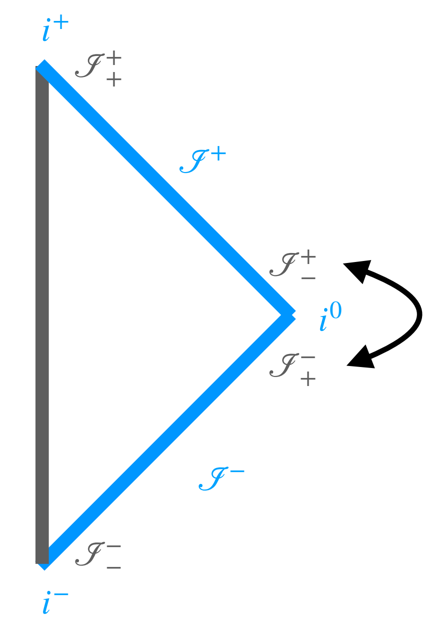


judicious choice of **superrotation** $Y^A(x)$

Logarithmic soft **graviton** theorem:

$$\mathscr{M}_{N+1} = \left( \omega^{-1} S_{-1} + \omega^0 \ln \omega \, S_0^{(\ln \omega)} \right) \mathscr{M}_N + \ldots$$

log soft expansion

[Sahoo,Sen'18] [Saha,Sahoo,Sen'19]

This establishes the symmetry interpretation of the classical logarithmic soft graviton theorem.* [Choi,Laddha,AP'24]

* up to drag terms

# Gravity is a drag

Due to the spacetime curvature caused by the matter the soft graviton experiences a gravitational drag at late times:

$$\Delta_{\text{drag}} S^{(\ln \omega)}_{0,\text{classical}} = -\frac{i}{4\pi} \log \omega \sum_{j,\eta_j=-1} (q \cdot p_j) \underbrace{\frac{\kappa}{2} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\nu} p_i^\mu p_i^\nu}{q \cdot p_i}}_{\text{leading soft factor } S_{-1}}$$

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The resulting time delay to reach a detector at distance $r$ can be captured by defining the retarded time at the detector:

$$u = t - r + \log r \times f_{\text{drag}}$$

effect of the long range gravitational force on the gravitational wave as it travels from the scattering center to the detector

$$f_{\text{drag}} = 2G\sum_{j=1}^{N} q\cdot p_j$$

$$q^{\mu} = (1, \frac{\vec{x}}{r})$$

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How does the drag show up in the charge?

# Gravity is a drag

The time-delay results in a logarithmically divergent phase which effectively shifts the shear:

$u^0$ term in $u \to \pm\infty$ expansion:

depends on matter stress tensor at $i^+$

$$C_{AB}(u,x) \to C_{AB}(u,x) - \frac{\kappa}{2} \ln r \, \overset{0}{h}_{rr}(x) \partial_u C_{AB}(u,x)$$

sQED: [Bhatkar'19]

This results in an additional contribution to the log soft charge:

$$Q^{\text{drag}}_{S,+}[Y] = \ln \Lambda^{-1} \left[ -\frac{1}{2\kappa} \int_{\mathcal{J}^+} du d^2x \, D_z^3 Y^z \, \overset{0}{h}_{rr} \, \underbrace{\partial_u C^{zz}} \right]$$

judicious choice of **superrotation** $Y^A(x)$

action on Fock state

leading soft projector

This yields the drag term in the soft theorem:

$$S_{-1} = \frac{\kappa}{2} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\nu} p_i^\mu p_i^\nu}{q \cdot p_i}$$

leading soft factor

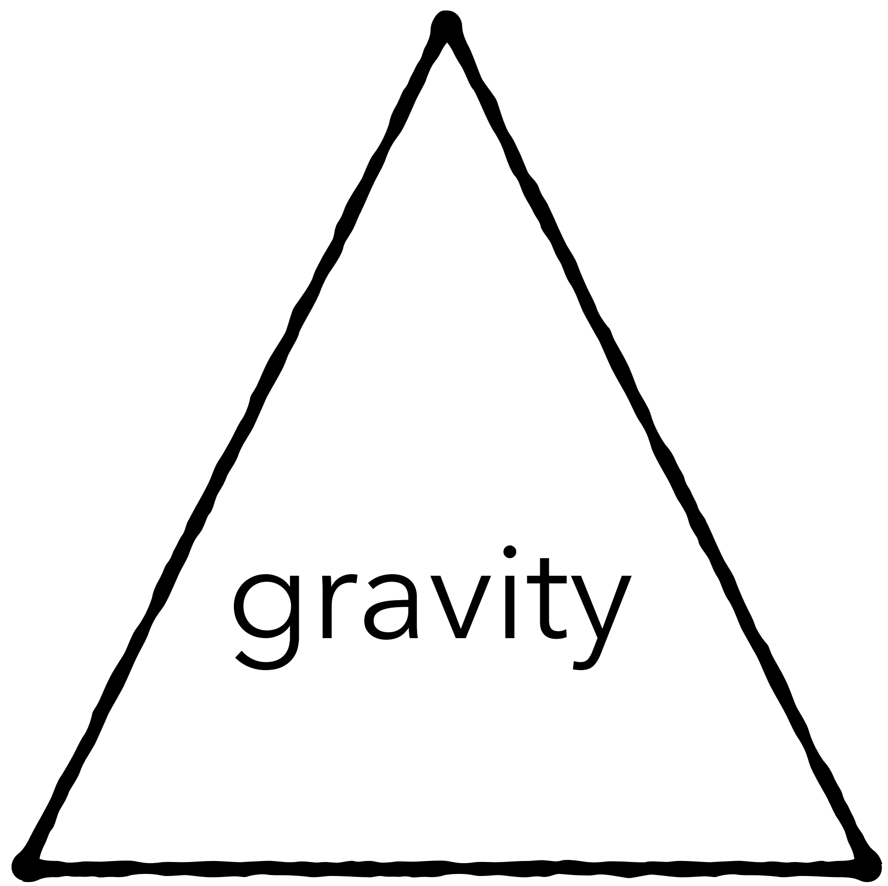
$$\Delta_{\text{drag}} S^{(\ln \omega)}_{0,\text{classical}} = -\frac{i}{4\pi} \log \omega \sum_{j, \eta_j = -1} (q \cdot p_j) \, S_{-1}$$

[Choi,Laddha,AP'24]

# Classical superrotation IR triangle

[Choi,Laddha,AP’24]

**superrotation**




**classical log soft theorem**

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

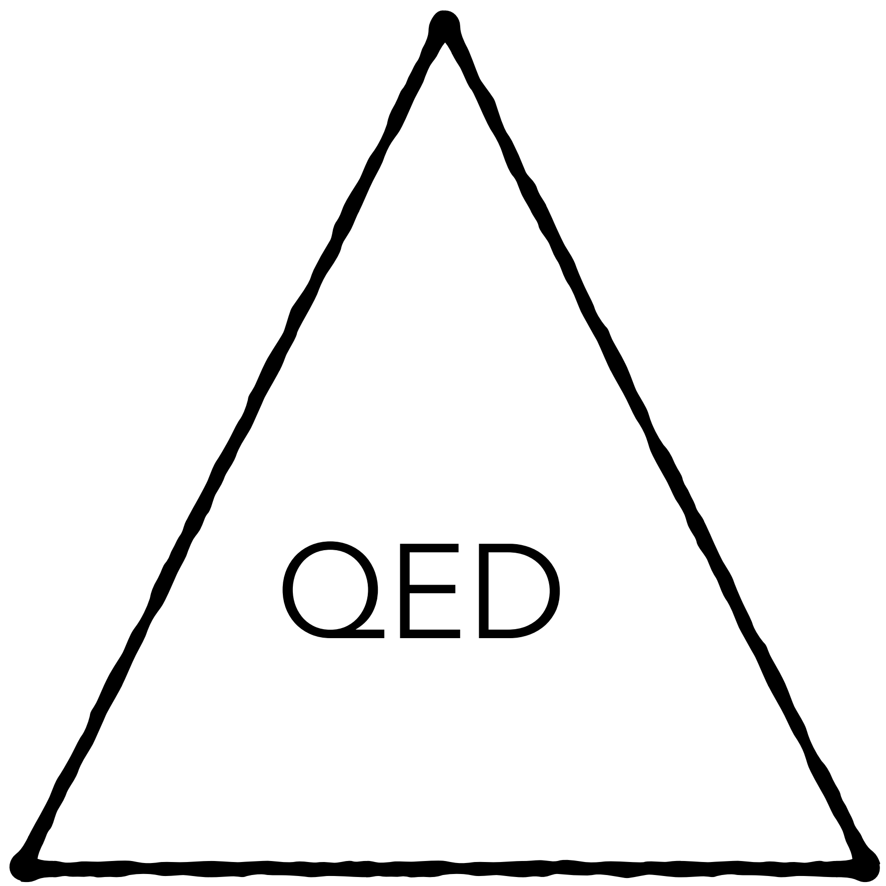
**tail to the memory effect**

[Saha,Sahoo,Sen'19] particles
[Choi,Laddha,AP’24] fields

# Classical super*phase*rotation IR triangle

[Choi,Laddha,AP’24]

**superphaserotation**




**classical log soft theorem**

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

**tail to the memory effect**

[Saha,Sahoo,Sen'19] particles
[Choi,Laddha,AP’24] fields

# Infrared surprises

## Infrared triangles

Complete?

## 4D = 2D

What are the axioms of celestial CFT?

Exact celestial duals? [Costello,Paquette'22]

[Costello,Paquette,Sharma'22]

## Towers of ∞ symmetries

Symmetries of what theories?

How powerful constraints?

## Long-range effects

Beyond gravity & QED ? Beyond leading log ?

Quantum log soft factor?

$\log(u)$? [Campiglia,Laddha'19]

logarithmic CFT? [Bissi,Donnay,Valsesia'24]

see also [Donnay,Nguyen,Ruzziconi'22],
[Agrawal,Donnay,Nguyen,Ruzziconi'23]

# More infrared surprises ?