Equations Found in our Code

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Change of Coordinate System

The Cartesian system uses x, y, z, and is most commonly used. We use spherical variables r, θ, ϕ in the Spherical system. Converting from Cartesian to Spherical requires use of these equations.

Cartesian to Spherical system conversion

Typically both θ and ϕ come in degrees, but conversion is simple.

$$x = r\cos(\theta)\sin(\phi)$$

$$y = r\sin(\theta)\sin(\phi)$$

$$z = r\cos(\phi)$$

Spherical to Cartesian system conversion

These conversions are used most in lines 116-120 of our Sphere3D code.

$$r = \sqrt{(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2}$$

$$\theta = \arctan\left(\frac{y - y_c}{x - x_c}\right)$$

$$\phi = \arccos\left(\frac{(z - z_c)}{\sqrt{(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2}}\right)$$

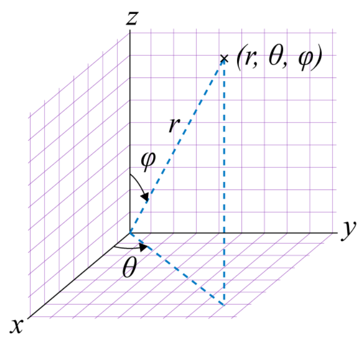

Figure 1: https://byjus.com/maths/spherical-coordinates/

New Velocity Terms

Our velocity terms come from Stanford AA200 - Applied Aerodynamics. It is in the Cartesian system and defines uniform flow past a sphere.

Direct terms from textbook

Equation 10.75

$$U_x = U_\infty(1 - \frac{3(R_{sphere})^3 x^2}{2r^5} + \frac{(R_{sphere})^3}{2r^3})$$

$$U_y = -U_\infty \frac{3(R_{sphere})^3 xy}{2r^5})$$

$$U_z = -U_\infty \frac{3(R_{sphere})^3 xz}{2r^5}$$

For our specific case

We used diameter as opposed to radius in our declaration of variables, giving a mathematically identical equation but one that looks slightly different.

$$U_x = U_\infty(1 - \frac{3(D_{sphere})^3 x^2}{16r^5} + \frac{(D_{sphere})^3}{16r^3})$$

$$U_y = -U_\infty \frac{3(D_{sphere})^3 xy}{16r^5})$$

$$U_z = -U_\infty \frac{3(D_{sphere})^3 xz}{16r^5}$$

These equations are declared, defined, and used in lines 193-195 of file UniformFlow3D.