A proposed model of commodity derivatives markets with heterogenous traders — BEM114: SP 2016 —

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Abstract

In this proposal, we outline a heterogeneous agent model for a commodities market. We introduce a highly simplified model to highlight basic implications. We consider a hedging trader, who suffers from a false-consensus bias and mis-estimates the market price of risk by anchoring it to his own inventory level. The hedger trades against an arbitrageur, who is sophisticated and fully rational. We show how this simple difference of opinion model allows for trading and contango in the market for a single period. We then consider methods to make necessary improvements in the model, as well as present some broader implications of the research.

1 Introduction and literature review

Commodity markets are some of the oldest forms of financial markets. Indeed, futures contracts believed to be over 3000 years written on stone tablets from ancient Mesopotamia have been recovered (Sinha 2011). The classical commodities literature began with Keynes in the 1920s. Keynes essentially viewed commodity markets consisting of industrial hedgers and speculators, and posited that, in general, industrial hedgers prefer to go net short on futures, which in turn means that speculators are generally net long (Keynes 1930). This generally leads to a market condition known as normal backwardation, in which spot prices trade above futures prices. More recently, Hirshleifer developed an equilibrium model of a spot and futures market involving primary growers and intermediate processors, and showed that this two-stage production system can lead to downward-biases in futures prices (normal backwardation) for sufficiently low transaction costs (Hirshleifer 1988).

Behavioral heterogeneous agent models have gained traction in the financial literature, including the seminal work done by DeLong et al. (1990) and Hong and Stein (1999). Scheinkman and Xiong, in particular, develop a heterogeneous model with short sell constraints to generate trading and even buying of security when it is above an agent's fundamental valuation due to speculation that it can be resold (Scheinkman and Xiong, 2003). More recently in the commodities literature, Ellen and Zwinkels develop a heterogeneous agent model that incorporates two types of boundedly rational agents to predict oil prices, finding that it outperformed random walk and vector-autoregression models in out-of-sample forecasting (Ellen and Zwinkels 2010)

In this proposal, we outline a simple, two-agent model in a spot and futures market. We consider a Keynesian-style market with industrial hedgers and speculators. Both agents rationally price securities, however, we assume that one agent in particular mis-estimates the market price of risk when formulating his strategy via an experience effect and overconfidence. In particular, he is confident his own inventory of the commodity is representative for overall industrial supply. We then show how this allows for the market to enter contango, as well as present some broader implications and routes for future research.

2 Outline of model

We present a highly simplified version of the model and present some intuitive implications.

We consider a model in which two traders trade a commodity with spot price S, and a futures contract on that commodity, F. Parties enter the futures contract at time 0 with settlement at time T. In particular, we call one trader a *hedger*. We can think of this agent as someone that deals directly in the consumption and production of the commodity and usually looks to position himself in the commodities market to hedge his first-order risk. We call the other trader the *arbitrageur*. This trader does not deal with the commercial production or sale of the commodity directly. These traders observe the market identically, with one primary difference. The hedger determines the market price of risk—which we will denote ψ — as a function of *his* inventory of the commodity. He essentially suffers from a false-consensus bias, in that he assumes his behavior and inventory match those of the overall market. Naturally, when he carries a high inventory, he will assume a high market price of risk. The arbitrageur, who we take to be a sophisticated investor, can accurately predict ψ.

Thus, both agents behave rationally given their beliefs, however, the hedger utilizes a simple heuristic to formulate a—potentially incorrect—belief about the level of market risk.

We build on basic models developed in Ross (1995). Consider a commodity spot price, S, following the stochastic process:

$$\frac{dS}{S} = \kappa(\mu - logS)dt + \sigma dW \tag{1}$$

where W is a one-dimensional Brownian motion.

We impose the behavioral component at time 0. Before deciding how to trade, each agent formulates beliefs about the value of market risk. The hedger estimates the level of risk as $\hat{\psi}_H$. He does so according to

$$\hat{\psi}_H = \theta(I_H), \tag{2}$$

where I_H is his personal inventory of the commodity. At this point, we do not specify the functional form of $\theta(\cdot)$ beyond that it is increasing in I_H:

$$\theta'(I_H) > 0, \;\; \forall \, I_H \in \mathbb{R}$$

Naturally, the hedger believes it risky to carry and store excessive supplies of the commodity. Most notably, however, he assumes the overall inventories and storage strain on the market as a whole mirror that of his own. Indeed, we expect the true value of ψ to increase in overall inventory and correlate with the business cycle (Schwarz, 1997).

Let us continue by considering how the fully-rational arbitrageur prices the futures contract.

We define $X = log(S)$ and apply Ito's Lemma, recovering

$$dX = \kappa(\alpha - X)dt + \sigma dW \tag{3}$$

We recognize this as the well-studied Ornstein-Uhlenbeck stochastic process in finance and physics. Indeed, from Girsanov's Theorem, we know that there exists an equivalent martingale probability measure. We transform this process under this risk-neutral probability

$$dX = \kappa(\alpha^* - X)dt + \sigma dW^*. \tag{4}$$

From this transformation, we have

$$\alpha^* = \alpha - \psi \tag{5}$$

and where W^* is the Brownian motion under the risk-neutral probability measure.

Under the risk-neutral probability measure, the time T distribution of the process given in equation 4 is normal. The mean and variance are given by:

$$\mathbf{E}_0[X(T)] = X(0)e^{-\kappa T} + \alpha^*(1 - e^{-\kappa T}) \tag{6}$$

$$Var_0[X(T)] = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa T}) \tag{7}$$

Recall $X = log(S)$. Therefore, the spot price of the commodity at time T is distributed log-normally.

From a rational perspective, the futures price agreed upon at time 0 must be the expected spot price of the commodity at time T. Recognize that this is not the price to *enter* into the futures contract at time 0 (that is of course zero), but rather this is the agreed price to pay for the commodity upon settlement.

$$F_A(S, 0, T) = \mathbf{E}_0[S(T)] = e^{\mathbf{E}_0[X(T)] + \frac{1}{2}Var_0[X(T)]} \tag{8}$$

The arbitrageur therefore arrives at a time 0 futures price of

$$F_A(S, 0, T) = \mathbf{E}_0[S(T)] = e^{log(S(0))e^{-\kappa T} + \alpha^*(1-e^{-\kappa T}) + \frac{1}{4}\frac{\sigma^2}{\kappa}(1-e^{-2\kappa T})} \tag{9}$$

$$\rightarrow log(F_A(S, 0, T)) = log(S(0))e^{-\kappa T} + \alpha^*(1 - e^{-\kappa T}) + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa T}) \tag{10}$$

Recall that the hedger himself rationally prices the futures contract. Therefore, his futures price takes the same form, except, we must include a term to capture his heuristic-driven belief.

$$log(F_H(S, 0, T)) = log(S(0))e^{-\kappa T} + (\alpha - \hat{\psi_H})(1 - e^{-\kappa T}) + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa T}) \tag{11}$$

$$log(F_H(S, 0, T)) = log(S(0))e^{-\kappa T} + \alpha(1 - e^{-\kappa T}) - \theta(I_H)(1 - e^{-\kappa T}) + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa T}) \tag{12}$$

2.1 A simple implication: Contango

We show that the heterogeneous trading model intuitively leads to an interesting regime in the market.

Definition 2.1. Contango. If the futures price is trading above the arbitrageur's expected future spot price of the commodity such that the arbitrageur prefers to short futures, then the futures market is in *contango.* In our model, contango occurs when $\alpha - \alpha^* > 0$.

Clearly, contango occurs in this model when the hedger is overestimating storage strain in the market.

Intuitively, what happens in contango? The hedger himself carries high inventory. If he were to buy spot to take on more inventory, he would need to utilize more expensive storage locations. He believes that this is true across the whole market. As such, he views spot as relatively expensive. He thus will prefer to short spot and long the futures contract. The arbitrageur, who can more accurately predict the true levels of inventory and costs of storage will short the futures and long spot for arbitrage profit.

3 Future Research and Broader Implications

This proposed research looks to provide a starting point for incorporating behavioral foundations in models of commodities markets. Clearly, the model we outlined here is very simple and quite limited in scope. The proposed research looks to build on this type of approach.

For example, we can look to jointly verify psychological effects in commodities experimentally and look to see if the theoretical results hold empirically. Here we only presented a single period, two-agent model and showed that we can generate trading and contango. It would be compelling to examine multi-period effects, in particular looking to find the extent to which arbitrage is limited in commodity markets and how pricing is affected over time.

Commodity markets reflect a global participation and we can also look to develop robust equilibrium models with endogenous pricing, as well as look to take a more behavioral-macro approach to commodities markets. These latter approaches are the primary plans for the research. Developing a robust equilibrium model with endogenous price determination and regime switching (between backwardation and contango) is the ultimate goal of this research and would provide a significant improvement to current approaches in the literature. Such a model would help reconcile the Keynesian foundations of commodity trading with the phenomena observed in markets, past, present, and future.

We end with a brief discussion of broader implications. In 1995, Metallgesellschaft, one of Germany's largest industrial firms, reported over $1.3 billion loss on positions in energy futures and swaps. They needed another $1 billion to buy themselves out of expensive contracts with other energy corporations (Edwards 1995). Metallgesellschaft essentially was trying to hedge their short *forward* positions for long-term delivery of energy by rolling over long positions in short-horizon *futures*. This hedge would in general be effective if the market were to remain in normal backwardation. However, the market went into contango and essentially lead to the firm's undoing. It took a $2 billion rescue effort from other German firms to save Metallgesellschaft from bankruptcy and left a massive wake in markets around the globe and for Metallgesellschaft's 50,000 employees. Underlying it all were human biases, most notably overextrapolation and overconfidence. To put it simply, Metallgesellschaft management strongly believed that markets would behave in the future as they had in the past, and the results were catastrophic. Therefore, understanding the behavioral underpinnings of commodity trading are critical to prevent another Metallgesellschaft from occuring again.

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