

Algebraic Topology Problems

Part III preparatory notes based on Algebraic Topology Problems by Many Authors

Notes taken by James Arthur

Summer 2021

Part III requires a load of complex and interesting Mathematics that I have not covered thus far in my course and will not. These collections of notes are my thoughts on topics and needed intuition to bits of Mathematics. They aren't here to teach or to be used a stand alone text.

Contents

0 Problem 0

2

0 Problem 0

Exercise. Let $a : S^n \rightarrow S^n$ be the antipodal map, $a(x) = -x$. Show that a is homotopic to the identity map when n is odd. [Try $n = 1$ first.]¹

Solution. Consider a homotopy of $H_1(z, t) = e^{\pi i t} z$. This is the only homotopy as others i.e. $H_2(x, t) = 2xt - 1$ don't lie in the circle. We know H_1 lies in the circle as,

$$|H_1(z, t)| = |ze^{\pi i t}| = |z| = 1$$

and also note that $H_1(z, t) \in \mathbb{C}^n$ and \mathbb{C}^n is a '2n-dimensional' space and so this homotopy won't work for odd dimensional systems and so it only maps to the identity map when n is odd.

¹Cambridge Algebraic Topology Part ii Example Sheet I.I