January Group Meeting: Topological defects

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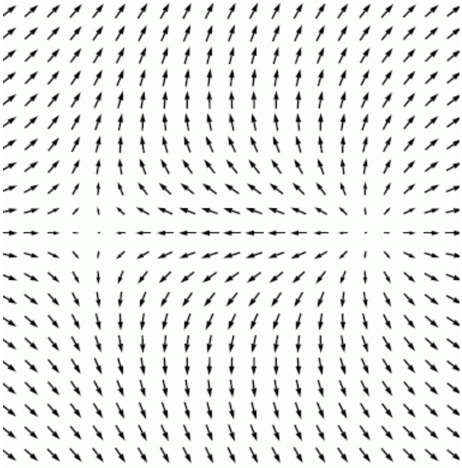
Simplest topological defects: 2D magnetic systems

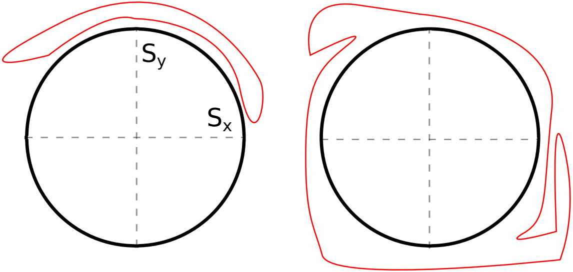
- 2-component spins on a 2D lattice
- $\mathcal{H} = -J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$
- More complex than the Z_2 Ising model, Hamiltonian has $U(1)$ symmetry

KT theory: phase transition via *topological* excitations



Topological defect pairs pop in and out of the system and eventually unbind, leading to exponentially decaying correlations

Crucial idea: order parameter space



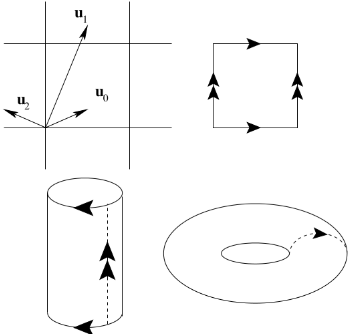
Vortices are characterized by nontrivial winding around OP space.

Conservation of topological charge

- A lone vortex can be detected arbitrarily far away from the vortex core.
- One single vortex has an energy that goes logarithmically with the size of the system: *probability 0 in the thermodynamic limit.*
- One $+1$ vortex is indistinguishable from two $+1$ vortices and a -1 vortex (from sufficiently far from the cores).
- Therefore: *there is a topological charge neutrality condition.* For every vortex spontaneously created, there is an anti vortex.

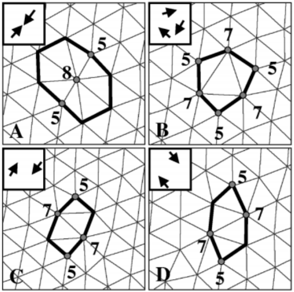
HNY theory: Crystals

- Crystals are characterized by broken translational and rotational symmetry
- Hexatic order parameter ψ lives on a circle (1D torus) just like the XY model's order parameter.
- Crystal displacement field $\mathbf{u}(\mathbf{r}_i)$ lives on a 2D torus: this is its order parameter space.

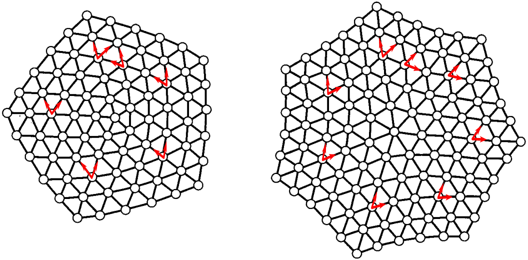


Two types of topological defects in 2D crystals

- XY vortex $\rightarrow$ winding number: disrupts spin-phase order
- Dislocation $\rightarrow$ Burgers vector: disrupts translational order
- Disclination $\rightarrow$ winding number: disrupts bond-orientational order



(a) Dislocations



(b) Disclinations

Phase transitions in 2D *rely* on topological defects

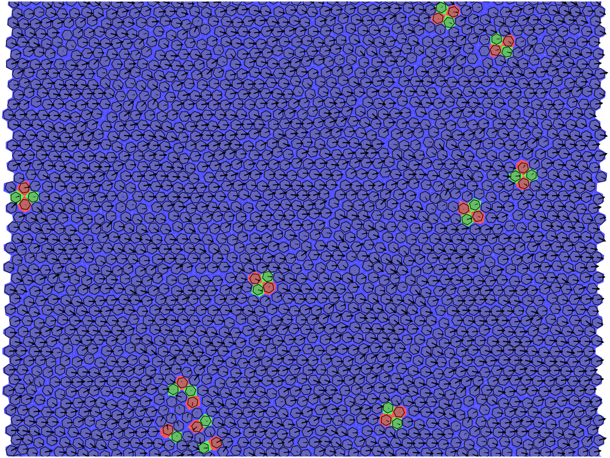
- What distinguishes a disordered system from one that hasn't "melted" in two dimensions?
  - Stiffness (non-infinite susceptibility for XY, non-zero elastic constants for crystals)
  - Stiffnesses are renormalized by the presence of defects
  - Power-law correlations (as opposed to exponential correlations)
- They won't show up in the Hamiltonian unless you:
  - understand the symmetries of the original, unapproximated Hamiltonian
  - account for the topology of the order parameter space

Interactions of defects

- KTHNY theory says defects show Coulombic attraction.
  - In 2D, Coulomb energy is not $1/r$ but $\log r$ instead.
  - Dislocations are vector charges, disclinations are scalar charges.
- Costs energy to create the core of a defect/anti-defect pair, then costs more energy to separate them.

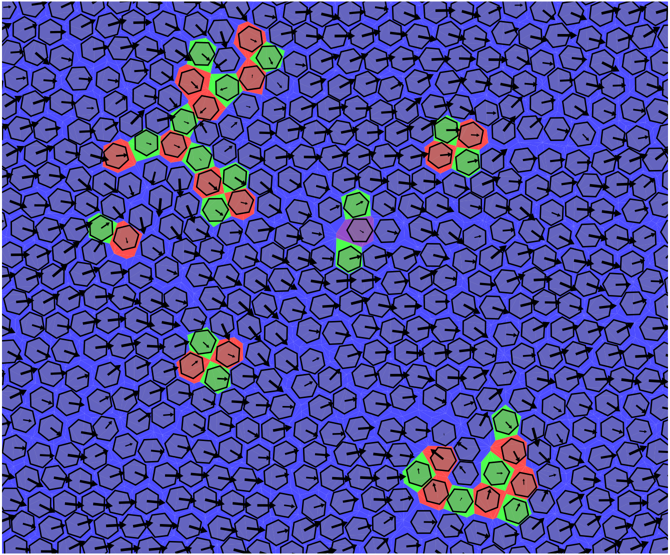
Now for some actual data...

$\phi = 0.710$, $N = 50^2$

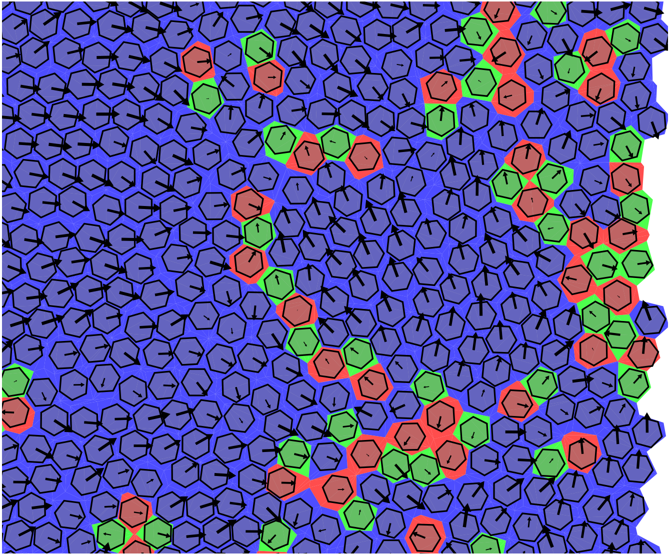


$\phi = 0.710$, a few dislocation/anti-dislocation pairs. One dislocation triplet. Unbinding is extremely unfavorable.

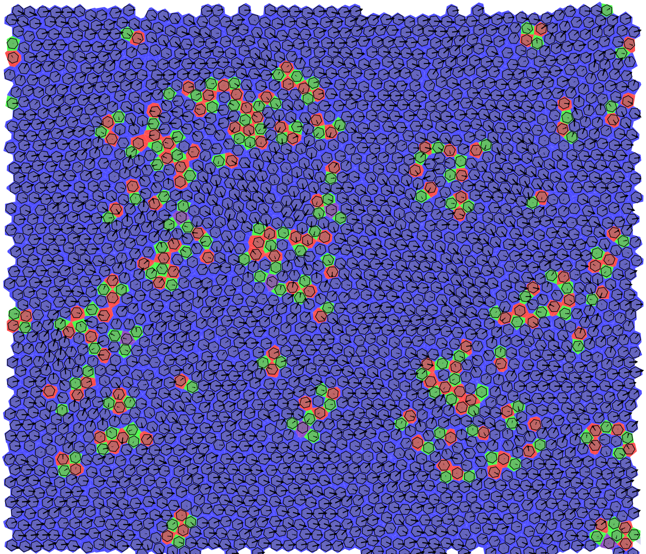
$\phi = 0.700$, clustering, dislocation unbinding



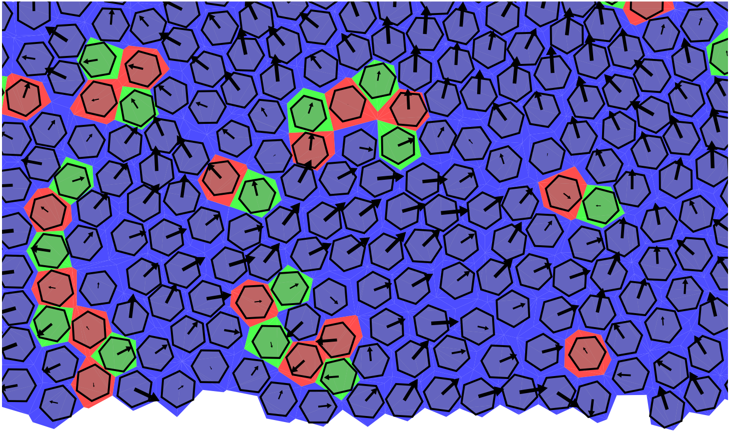
$\phi = 0.685$, hexatic domain walls



$\phi = 0.685$, well in the hexatic phase



$\phi = 0.680$, disclination unbinding



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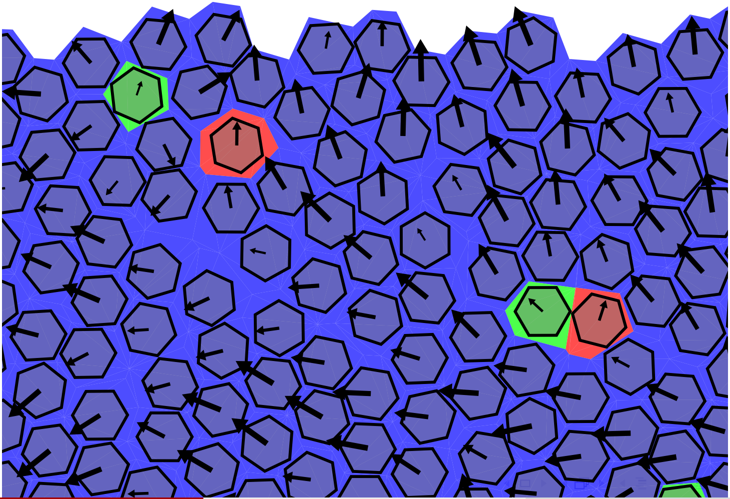

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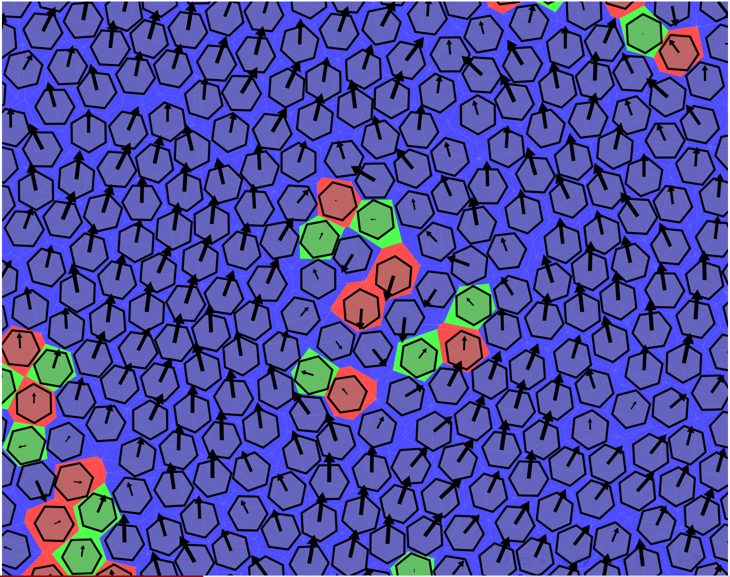
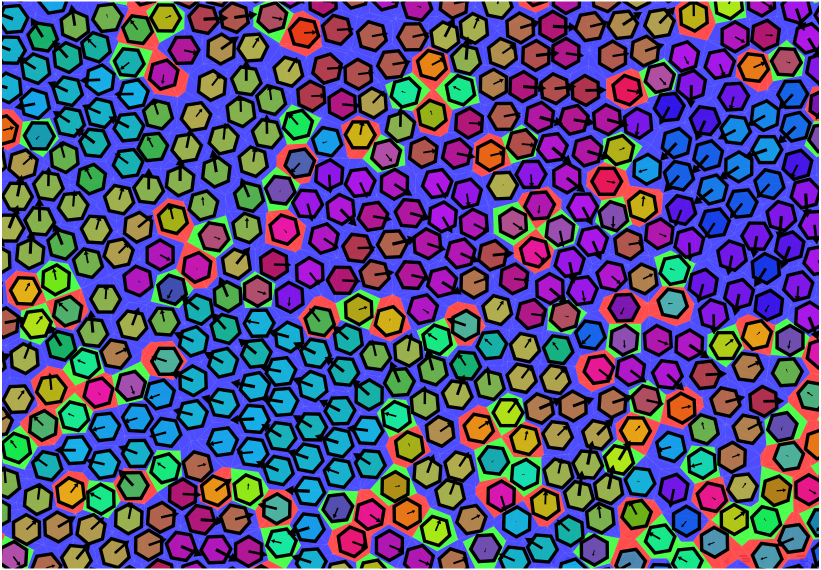


$\phi = 0.670,\ N = 128^2$



Phonon/libron modes: defects affect stiffness constants

Light blue: phonon elastic stiffness. Red: libron elastic stiffness. Blue: libron mass. Green: libron-phonon coupling (0 by symmetry).