

Yeah, I see exactly what happens now. What you have is totally fine until you sub out $\frac{\partial E}{\partial x}\bigg|_T$ for J , which isn't exactly right. We can get the right expression by expressing E as a function of x and S :

$$E(x, T) = E(x, S(x, T)).$$

$$\frac{\partial E}{\partial x}\bigg|_T = \frac{\partial E}{\partial x}\bigg|_S + \frac{\partial E}{\partial S}\bigg|_x \frac{\partial S}{\partial x}\bigg|_T.$$

We can use a Maxwell relation to swap out the last partial derivative:

$$\frac{\partial S}{\partial x}\bigg|_T = - \frac{\partial J}{\partial T}\bigg|_x.$$

Now we can use

$$dE = TdS + Jdx., \quad \frac{\partial E}{\partial x}\bigg|_S = J, \quad \frac{\partial E}{\partial S}\bigg|_x = T.$$

So all in all:

$$\frac{\partial E}{\partial x}\bigg|_T = J - T \frac{\partial J}{\partial T}\bigg|_x.$$

$$\frac{\partial E}{\partial x}\bigg|_T = ax - bT + cTx - T(-b + cx) = ax.$$

And this is all preceded by $\frac{\partial}{\partial T}\bigg|_x$, so then we get 0!