**Physics 212 Project Proposal: Ising Model in Higher Dimensions**

This paper proposal seeks to replicate one of the proposed project ideas in simulating the Ising model numerically using Monte Carlo techniques.

1. Critical Exponents of the Ising Model in various dimensions

Specifically, we will reproduce Critical exponents and critical temperatures. The critical exponents are essentially power relations for the order parameters and other properties of the system such as heat capacity. For example, for heat capacity, which is typically defined as: , where is the free energy and is the temperature, we usually see that near the critical point: , where here is the order parameter. Similar relations are expected for the magnetization.

1. Description of Metropolis Hastings Monte Carlo

A crucial aspect of simulating the Ising model is finding a way to avoid computations of the partition function of system, which is essentially intractable to calculate for even relatively small system sizes. We will provide a brief description of how Monte Carlo simulations overcome this issue by effectively analyzing only differences in energy from flipping one spin (probability ratio then cancels out the partition function).

1. Determining Critical Temperature and Exponents

To achieve the goals using Monte Carlo, we will run the simulation for a sufficient number of epochs and the measure the usual gauge of order: the magnetization. We will repeat such simulations across a different spectrum of and should see a kink in the magnetization indicating the phase transition. The location of the kink determines the critical temperature.

Afterwards, we will densely sample magnetizations close to put past the critical point and perform a non-linear fit of the form which should provide the final determination of the critical exponent

1. Mean field behavior at high dimensions (d = 5), we should be able to compare our numerical results with the predictions for mean field theory. For this project, we will analyze d = 5. Already in just 5 dimensions, every site has 10 nearest neighbors so the computational expense starts mounting when simulating a 5D cubic volume of these spins (volume is proportional to a fourth power instead of a 3rd power in 3D, 2nd power in 2D), so total computation is