KR&R: Assignment 2

1. Implement a decision procedure for deciding consistency of an ABox in the description logic $ALCQ$ (i.e., for the description logic ALC extended with qualified number restrictions). The input of your algorithm should consist of an ABox and a TBox, the output should be *true* or *false* together with the model of the ABox (20 points).

2. Implement a decision procedure for deciding subsumption in the description logic $ALCQ$. (10 points).

3. Use your implementation to proof that the following subsumptions are valid with respect to the empty TBox (10 points):

   - $\forall r.\forall s.A \sqcap \exists r.\forall s.B \sqcap \forall r.\exists s.C \sqsubseteq \exists r.\exists s.(A \sqcap B \sqcap C)$
   - $\forall r.\forall s.A \sqcap (\exists r.\forall s.\neg A \sqcup \forall r.\exists s.B) \sqsubseteq \forall r.\exists s.(A \sqcap B) \sqcup \exists r.\forall s.\neg B$

4. Given the TBox $T = \{ParentWithMax2Children \equiv \leq 2HasChild.\top\}$, use your implementation to determine if the following ABox is consistent with respect to T (10 points):

   - $HasChild(joe, ann)$
   - $HasChild(joe, eva)$
   - $HasChild(joe, mary)$
   - $ParentWithMax2Children(joe)$

   If the ABox is consistent, show an interpretation/model that satisfies the ABox using your implementation.

5. (Bonus) The Web Ontology Language (OWL) is a formal language based on description logics used in the Semantic Web. The OWLAPI (`http://owlapi.sourceforge.net/`) provides a comprehensive library for creating and manipulating theories in OWL. Implement a decision procedure for the description logic $ALCQ$ using the OWLAPI's `OWLReasoner` interface. Use the OWLAPI to formulate the problems in (3) and (4), and use your implementation of the OWLReasoner interface in the OWLAPI to proof the subsumptions (10 points).

The completed assignment should consist of a short report and the source code of your implementation.