Problem Set 7

Written by James Bang for the NSW AMOC Correspondence Program

21th December 2019

Instructions

- This problem set is based off the notes "*Inequalities*".
- They are in roughly difficulty order and get quite difficult, so you **not** expected to be able to solve every problem.
- However, please attempt as many questions as you can and submit your solutions to your mentor for marking and feedback.
- You may type your solutions or take a **clear** scan/photo of **legible** written solutions.
- Feel free to discuss these problems with your peers and on the forum but the solutions you submit must be written by yourself.

Problems

1. For all real x, show that $\dfrac{x^2+2}{\sqrt{x^2+1}} \geqslant 2$.
2. Prove that for all real numbers x, we have $x^4+6x^2+1 \geqslant 4x(x^2+1)$. When does inequality hold?
3. Prove that for all positive reals a, b, c, we have $\dfrac{a^2}{b}+\dfrac{b^2}{c}+\dfrac{c^2}{a} \geqslant a+b+c$.
4. What is the maximum value of $a^5(1-a)$ for $0<a<1$?
5. Let x, y be nonnegative reals with sum 2. Prove that $x^2y^2(x^2+y^2) \leqslant 2$.