

Consider a square of dimensions $L \times L$, pierced by $2N$ vortices. Assuming there are an approximately equal number of $+1$ and -1 vortices piercing this square, the probability of n $+1$ or -1 vortices piercing some area $A \in L^2$ is

$$P_N(n) = \binom{N}{n} \left(\frac{A}{L^2} \right)^n \left(1 - \frac{A}{L^2} \right)^{N-n}. \quad (1)$$

Therefore, by assuming the different vortex phases are uncorrelated, the expectation value of the Wilson loop can be written as

$$\langle W(\partial A) \rangle = \sum_{n=0}^N \left(\exp \left(\frac{2\pi i}{3} \right) \right)^n P_N(n) \sum_{m=0}^N \left(\exp \left(\frac{-2\pi i}{3} \right) \right)^m P_N(m). \quad (2)$$

Taking just the first sum, it evaluates to

$$\left(1 - \frac{A}{L^2} \right)^N \sum_{n=0}^N \binom{N}{n} \left(\exp \left(\frac{2\pi i}{3} \right) \frac{A}{L^2} \left(1 - \frac{A}{L^2} \right)^{-1} \right)^n = \left(1 + \left(\exp \left(\frac{2\pi i}{3} \right) - 1 \right) \frac{A}{L^2} \right)^N.$$

So the total expectation value is

$$\begin{aligned} \langle W(\partial A) \rangle &= \left(1 + \left(\exp \left(\frac{2\pi i}{3} \right) - 1 \right) \frac{A}{L^2} \right)^N \left(1 + \left(\exp \left(\frac{-2\pi i}{3} \right) - 1 \right) \frac{A}{L^2} \right)^N \\ &= \left(1 - 3 \frac{A}{L^2} + 3 \left(\frac{A}{L^2} \right)^2 \right)^N \\ &= \left(\left(\frac{A}{L^2} \right)^3 + \left(1 - \frac{A}{L^2} \right)^3 \right)^N. \end{aligned}$$

Rewriting this in terms of the vortex density $\rho = \frac{N}{L^2}$, we have

$$\langle W(\partial A) \rangle = \left(\left(\frac{A\rho}{N} \right)^3 + \left(1 - \frac{A\rho}{N} \right)^3 \right)^N. \quad (3)$$

Taking the limit as $N, L^2 \rightarrow \infty$, keeping ρ constant, this becomes

$$\langle W(\partial A) \rangle = \exp(-3\rho A). \quad (4)$$

Hence we have a simple model for area-law confinement from the vortex model.