Factors Affecting the Damping Constant of a Spring System

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Abstract

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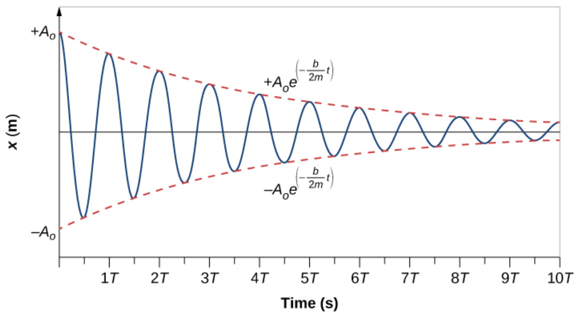

Figure 1: Displacement–time graph illustrating the characteristics of an underdamped harmonic oscillator.

1. Introduction

A vertical mass–spring system provides a simple model for studying oscillatory motion and energy dissipation in mechanical systems. An ideal mass–spring system undergoes simple harmonic motion with constant amplitude, but in real conditions mechanical energy is dissipated through a variety of means, causing oscillations to gradually decrease in amplitude. This phenomenon is known as damping, defined as "the loss of energy of an oscillating system by dissipation."

The degree of damping can be characterised by the decay constant γ, which specifies the rate at which oscillations diminish; larger values correspond to faster energy loss and more rapid amplitude reduction. The value of this constant depends on both the properties of the oscillating mass and the restoring system.

1.1. Aim and Hypothesis

The aim of this investigation is to determine how spring configuration and attached mass influence the rate of damping, quantified by the exponential decay constant γ, in a vertically oscillating mass-spring system.

It is hypothesised that if the attached mass is increased, there will be a decrease in the decay constant γ proportional to m^{-1}, and if the spring system is configured in series, there will also be a decrease in the decay constant.

The primary sources of damping are expected to be internal friction within the spring in conjunction with resistive forces such as air drag on the mass. For simplicity, it will be assumed that these factors can be modelled by a damping force which is linearly dependent on velocity, as with a standard viscous dampening model.

1.2. Variables

Independent Variables: The mass attached to the end of the spring, and the configuration of the spring system.

Dependent Variable: The curve-fitting coefficients of the damped harmonic motion displacement-time graph.

Controlled Variables: The positioning of the retort stand and clamps, motion sensor alignment and sampling frequency, mass cross-sectional geometry, and ambient air conditions (temperature, pressure, humidity).

2. Theory

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$$\begin{aligned}
\sum \overrightarrow{F} = m\overrightarrow{a} &= \overrightarrow{F}_{\text{restoring}} + \overrightarrow{F}_{\text{damping}} \\
m\overrightarrow{g} &= -k\overrightarrow{x} - b\overrightarrow{v} \\
0 &= k\overrightarrow{x} + b\frac{\mathrm{d}\overrightarrow{x}}{\mathrm{d}t} + m\frac{\mathrm{d}^2\overrightarrow{x}}{\mathrm{d}t^2} \\
0 &= k\overrightarrow{x} + b\frac{\mathrm{d}\overrightarrow{x}}{\mathrm{d}t} + m\frac{\mathrm{d}^2\overrightarrow{x}}{\mathrm{d}t^2} \\
0 &= \frac{k}{m}\overrightarrow{x} + \frac{b}{m}\frac{\mathrm{d}\overrightarrow{x}}{\mathrm{d}t} + \frac{\mathrm{d}^2\overrightarrow{x}}{\mathrm{d}t^2} \\
0 &= \frac{k}{m}e^{ut} + \frac{b}{m}\frac{\mathrm{d}(e^{ut})}{\mathrm{d}t} + \frac{\mathrm{d}^2(e^{ut})}{\mathrm{d}t^2} \\
0 &= k + bu + mu^2 \\
u &= \frac{-b \pm \sqrt{b^2 - 4km}}{2m}
\end{aligned} \tag{1}$$

Assume the spring is underdamped (*i.e.* $b^2 - 4km < 0$)

$$\begin{aligned}
\overrightarrow{x} &= \alpha e^{\left(\frac{-b+\sqrt{b^2-4km}}{2m}\right)t} + \beta e^{\left(\frac{-b-\sqrt{b^2-4km}}{2m}\right)t} \\
&= e^{-\frac{bt}{2m}}\left[\alpha e^{\left(\frac{\sqrt{b^2-4km}}{2m}\right)t} + \beta e^{\left(\frac{-\sqrt{b^2-4km}}{2m}\right)t}\right] \\
&= e^{-\frac{bt}{2m}}\left[\alpha e^{\left(\frac{i\sqrt{4km-b^2}}{2m}\right)t} + \beta e^{\left(\frac{-i\sqrt{4km-b^2}}{2m}\right)t}\right] \\
&= e^{-\frac{bt}{2m}}\left[(\alpha+\beta)\cos\left(\frac{\sqrt{4km-b^2}}{2m}t\right) + i(\alpha-\beta)\sin\left(\frac{t\sqrt{4km-b^2}}{2m}t\right)\right] \\
&= 2\sqrt{\alpha\beta}\,e^{\left(-\frac{b}{2m}\right)t}\cos\left(\frac{\sqrt{4km-b^2}}{2m}t + \phi\right)
\end{aligned} \tag{2}$$

$$\begin{aligned}
\text{Let } A &= 2\sqrt{\alpha\beta} \\
\gamma &= \frac{b}{2m} \\
\omega' &= \frac{\sqrt{4km-b^2}}{2m}
\end{aligned}$$

$$\therefore \overrightarrow{x} = Ae^{-\gamma t}\cos\left(\omega' t + \phi\right) \tag{3}$$

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$$\begin{aligned}
\omega' &= \frac{\sqrt{4km - b^2}}{2m} \\
\omega'^2 &= \frac{4km - b^2}{4m^2} \\
\omega'^2 &= \frac{k}{m} - \frac{b^2}{4m^2} \\
\omega'^2 &= \frac{k}{m} - \gamma^2 \qquad \text{with } \gamma = \frac{b}{2m} \\
\omega'^2 + \gamma^2 &= k\left(\frac{1}{m}\right)
\end{aligned} \tag{4}$$

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3. Equipment and Method

3.1. Equipment

- 1 × LabQuest 2 and cables
- 1 × Vernier Motion Detector (±1 mm)
- 2 × Stiff springs
- 1 × Light spring
- 1 × Vernier Hanging Mass Set (250 g/50 g)
- 1 × Retort stand
- 1 × Bosshead clamp, spring hanger, and rod
- 1 × Scientific scale (±0.01 g)
- 1 × Computer with Logger Pro 3

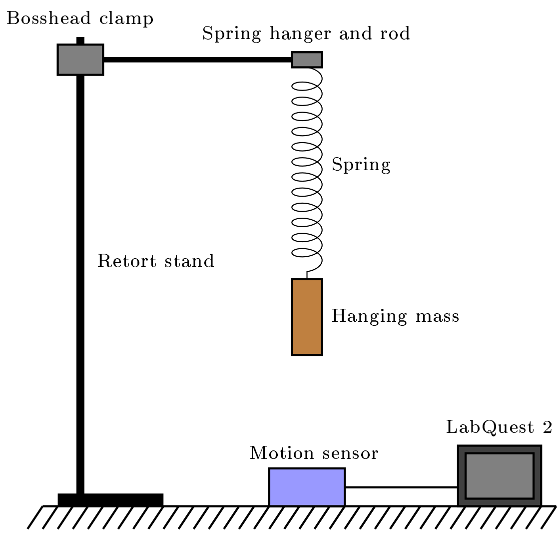

Figure 2: Experimental setup diagram for a vertical mass–spring system utilising a motion sensor.

3.2. Method

1. Set up equipment as depicted in Fig. 2, with the light spring hooked onto the spring hanger and base of the retort stand towards the mass.
2. Initialise the LabQuest 2 and Motion Detector and configure the mode to be displacement–time, sampling period to be 60 s, and sample rate to be 60 Hz.
3. Connect the LabQuest 2 wirelessly to a computer with Logger Pro running.
4. Measure and record the mass of the hanger from the Hanging Mass Set on a scientific scale.
5. Attach the mass to the end of the spring and release it in a controlled manner, ensuring there is no horizontal displacement of the system before starting the data recording from the computer.

6. Allow the 60 s data collection period to finish without disturbing the system before removing the mass from the spring system.
7. Apply a 'Damped Harmonic' curve fit in Logger Pro and record coefficients A, B, and C, and the R^2 value.
8. Place a 50 g mass from the Hanging Mass set onto the hanger and measure and record the new total mass.
9. Repeat steps 5 to 8 until at least 5 different masses have been trialled.
10. Replace the light spring with a stiff spring and repeat steps 5 to 9.
11. Replace the stiff spring with two stiff springs in series and repeat steps 5 to 9.

4. Results

Table 1: Regression parameters for the light spring.

m_{load} (kg)	A (m)	γ (s^{-1})	ω' (rad s^{-1})	R^2
0.050 25	0.011 33	0.024 24	21.47	0.9990
0.100 14	0.012 20	0.019 52	15.43	0.8385
0.150 44	0.024 08	0.015 78	12.53	0.9947
0.200 11	0.043 02	0.012 86	10.89	0.9996
0.250 19	0.074 20	0.012 24	9.75	0.9894
0.300 12	0.112 10	0.011 83	8.92	0.9999
0.349 91	0.140 50	0.011 38	8.28	0.9999

Table 2: Regression parameters for the stiff spring.

m_{load} (kg)	A (m)	γ (s^{-1})	ω' (rad s^{-1})	R^2
0.300 10	0.030 17	0.007 77	13.41	0.9971
0.350 14	0.045 13	0.006 54	12.45	0.9955
0.400 14	0.054 04	0.005 95	11.67	0.9993
0.450 04	0.064 17	0.005 35	11.02	0.9993
0.499 84	0.069 48	0.004 91	10.47	0.9995

Table 3: Regression parameters for two stiff springs in series.

m_{load} (kg)	A (m)	γ (s^{-1})	ω' (rad s^{-1})	R^2
0.300 10	0.071 61	0.004 96	9.581	0.9996
0.350 14	0.096 85	0.004 32	8.907	0.9995
0.400 14	0.1114	0.003 47	8.363	0.9991
0.450 04	0.1214	0.003 11	7.907	0.9995
0.499 84	0.1385	0.002 79	7.517	0.9996

5. Analysis of Results

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6. Conclusion

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