MATH4063   SCIENTIFIC COMPUTING AND C++

Suggested Submission Date: Monday 23rd November 2020, 2pm   **Coursework 1**

Your results to the assessed coursework may be submitted using this template. Please cut and paste the subsequent output into the correct parts of this file and replace the placeholder solution.jpg *plots with your own. Once this template has been completed, you must then create a pdf file for submission. Under Windows or Mac you can use Texmaker + a LaTeX compiler; from the Windows Virtual Desktop this may be accessed as follows:*

Start > UoN Applications > (UoN) Texmaker 5

Open this file under File; *to build the pdf file, click the arrow next to* Quick Build; *this will then generate the file Coursework1_submission.pdf.*

A single zip file containing your solution should be submitted *on Moodle.* **Note:** *All parameters and values should be set within your codes: do NOT use inputs such as those obtained with* std::cin.

Your code should be separated into a folder main, with subfolders source and include as follows:

File checklist:
Coursework1_submission.pdf

main:	source:	include:
• Q1c.cpp	• general.cpp	• general.hpp
• Q2b.cpp	• quadrature.cpp	• quadrature.hpp
• Q3b.cpp	• linear_algebra.cpp	• linear_algebra.hpp
• Q3c.cpp	• fem.cpp	• fem.hpp

1(a) No output required.

1(b) No output required.

1(c) Enter your output here:

```
%%%%%%%%%%%%%%%%%% Output for 1c %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
n=5, Approximation=0.318327, Error=1.74302e-05
n=10, Approximation=0.318311, Error=1.07978e-06
n=20, Approximation=0.31831, Error=6.73374e-08
All estimates are very accurate with the worst estimate only differing from
the true solution at the 5th decimal place. Increasing the number of nodal
points decreases the error as expected.
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2(a) No output required.

2(b) Enter your output here:

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%%%%%%%%%%%%%%%%% Output for 2b %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
n = 10
iteration: 1; residual error (2-norm) = 1.152
iteration: 2; residual error (2-norm) = 0.642276
iteration: 3; residual error (2-norm) = 0.391081
iteration: 4; residual error (2-norm) = 0.252924
iteration: 5; residual error (2-norm) = 0.58257
iteration: 6; residual error (2-norm) = 0.0256928
iteration: 7; residual error (2-norm) = 0.00347694
iteration: 8; residual error (2-norm) = 0.00051216
iteration: 9; residual error (2-norm) = 3.70198e-05
iteration: 10; residual error (2-norm) = 7.66929e-07
Error = 2.65962e-15

n = 100
iteration: 1; residual error (2-norm) = 4.25918
iteration: 2; residual error (2-norm) = 1.79642
iteration: 3; residual error (2-norm) = 1.37188
iteration: 4; residual error (2-norm) = 1.2324
iteration: 5; residual error (2-norm) = 1.68831
iteration: 6; residual error (2-norm) = 0.311173
iteration: 7; residual error (2-norm) = 0.58385
iteration: 8; residual error (2-norm) = 0.167844
iteration: 9; residual error (2-norm) = 0.195396
iteration: 10; residual error (2-norm) = 0.112775
iteration: 11; residual error (2-norm) = 0.106103
iteration: 12; residual error (2-norm) = 0.031898
iteration: 13; residual error (2-norm) = 0.736631
iteration: 14; residual error (2-norm) = 0.00945753
iteration: 15; residual error (2-norm) = 0.00700703
iteration: 16; residual error (2-norm) = 0.00538015
iteration: 17; residual error (2-norm) = 0.00715339
iteration: 18; residual error (2-norm) = 0.00273113
iteration: 19; residual error (2-norm) = 0.00382707
iteration: 20; residual error (2-norm) = 0.00221626
iteration: 21; residual error (2-norm) = 0.0033948
iteration: 22; residual error (2-norm) = 0.000804512
iteration: 23; residual error (2-norm) = 0.000750367
iteration: 24; residual error (2-norm) = 0.00128371
iteration: 25; residual error (2-norm) = 0.00133822
iteration: 26; residual error (2-norm) = 0.000152233
iteration: 27; residual error (2-norm) = 0.000148985
iteration: 28; residual error (2-norm) = 0.000147229
iteration: 29; residual error (2-norm) = 5.55349e-05
iteration: 30; residual error (2-norm) = 9.66297e-05
iteration: 31; residual error (2-norm) = 8.29717e-05
iteration: 32; residual error (2-norm) = 0.000124191
iteration: 33; residual error (2-norm) = 0.000115666
iteration: 34; residual error (2-norm) = 8.08047e-05
iteration: 35; residual error (2-norm) = 0.00163127
iteration: 36; residual error (2-norm) = 1.05093e-05
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iteration: 37; residual error (2-norm) = 1.29011e-05
iteration: 38; residual error (2-norm) = 7.30893e-06
iteration: 39; residual error (2-norm) = 9.76712e-06
iteration: 40; residual error (2-norm) = 4.19222e-06
iteration: 41; residual error (2-norm) = 1.40744e-06
iteration: 42; residual error (2-norm) = 9.53199e-07
iteration: 43; residual error (2-norm) = 6.07069e-07
iteration: 44; residual error (2-norm) = 3.65412e-07
iteration: 45; residual error (2-norm) = 4.58794e-07
iteration: 46; residual error (2-norm) = 4.1725e-07
iteration: 47; residual error (2-norm) = 2.1758e-07
iteration: 48; residual error (2-norm) = 7.15185e-07
iteration: 49; residual error (2-norm) = 1.71245e-06
iteration: 50; residual error (2-norm) = 9.42237e-07
iteration: 51; residual error (2-norm) = 1.44173e-07
iteration: 52; residual error (2-norm) = 4.84518e-08
iteration: 53; residual error (2-norm) = 5.97884e-08
iteration: 54; residual error (2-norm) = 2.70195e-08
iteration: 55; residual error (2-norm) = 8.83673e-09
iteration: 56; residual error (2-norm) = 5.05872e-09
iteration: 57; residual error (2-norm) = 4.25598e-09
iteration: 58; residual error (2-norm) = 1.81265e-07
iteration: 59; residual error (2-norm) = 4.3946e-09
iteration: 60; residual error (2-norm) = 2.77405e-09
iteration: 61; residual error (2-norm) = 2.37884e-09
iteration: 62; residual error (2-norm) = 2.76022e-09
iteration: 63; residual error (2-norm) = 1.38406e-09
Error = 4.88284e-10

n = 1000
iteration: 1; residual error (2-norm) = 14.5393
iteration: 2; residual error (2-norm) = 6.26704
iteration: 3; residual error (2-norm) = 4.98474
iteration: 4; residual error (2-norm) = 5.25953
iteration: 5; residual error (2-norm) = 3.82577
iteration: 6; residual error (2-norm) = 1.23627
iteration: 7; residual error (2-norm) = 0.777525
iteration: 8; residual error (2-norm) = 0.527208
iteration: 9; residual error (2-norm) = 33.3121
iteration: 10; residual error (2-norm) = 0.350714
iteration: 11; residual error (2-norm) = 0.204132
iteration: 12; residual error (2-norm) = 0.28147
iteration: 13; residual error (2-norm) = 0.241043
iteration: 14; residual error (2-norm) = 0.0489266
iteration: 15; residual error (2-norm) = 0.234266
iteration: 16; residual error (2-norm) = 0.0225857
iteration: 17; residual error (2-norm) = 0.0175472
iteration: 18; residual error (2-norm) = 0.014308
iteration: 19; residual error (2-norm) = 0.00905981
iteration: 20; residual error (2-norm) = 0.0115016
iteration: 21; residual error (2-norm) = 0.00559174
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iteration: 22; residual error (2-norm) = 0.0111293
iteration: 23; residual error (2-norm) = 0.0353535
iteration: 24; residual error (2-norm) = 0.00982559
iteration: 25; residual error (2-norm) = 0.00457825
iteration: 26; residual error (2-norm) = 0.00382223
iteration: 27; residual error (2-norm) = 0.00515819
iteration: 28; residual error (2-norm) = 0.0040293
iteration: 29; residual error (2-norm) = 0.0353884
iteration: 30; residual error (2-norm) = 0.00211422
iteration: 31; residual error (2-norm) = 0.000754401
iteration: 32; residual error (2-norm) = 0.00122434
iteration: 33; residual error (2-norm) = 7.69348e-05
iteration: 34; residual error (2-norm) = 7.58592e-05
iteration: 35; residual error (2-norm) = 4.40298e-05
iteration: 36; residual error (2-norm) = 3.03618e-05
iteration: 37; residual error (2-norm) = 2.31608e-05
iteration: 38; residual error (2-norm) = 1.42165e-05
iteration: 39; residual error (2-norm) = 1.08436e-05
iteration: 40; residual error (2-norm) = 8.92174e-06
iteration: 41; residual error (2-norm) = 6.6983e-06
iteration: 42; residual error (2-norm) = 4.49581e-06
iteration: 43; residual error (2-norm) = 4.92695e-06
iteration: 44; residual error (2-norm) = 6.42071e-05
iteration: 45; residual error (2-norm) = 7.11553e-06
iteration: 46; residual error (2-norm) = 6.77257e-06
iteration: 47; residual error (2-norm) = 2.16462e-06
iteration: 48; residual error (2-norm) = 2.17129e-06
iteration: 49; residual error (2-norm) = 2.44789e-06
iteration: 50; residual error (2-norm) = 5.42881e-06
iteration: 51; residual error (2-norm) = 5.92228e-07
iteration: 52; residual error (2-norm) = 4.41521e-07
iteration: 53; residual error (2-norm) = 4.25034e-07
iteration: 54; residual error (2-norm) = 5.4717e-07
iteration: 55; residual error (2-norm) = 3.83706e-06
iteration: 56; residual error (2-norm) = 1.70124e-07
iteration: 57; residual error (2-norm) = 6.35091e-07
iteration: 58; residual error (2-norm) = 9.9054e-08
iteration: 59; residual error (2-norm) = 4.06689e-08
iteration: 60; residual error (2-norm) = 4.13456e-08
iteration: 61; residual error (2-norm) = 2.99777e-08
iteration: 62; residual error (2-norm) = 2.81929e-08
iteration: 63; residual error (2-norm) = 4.05764e-08
iteration: 64; residual error (2-norm) = 3.60161e-08
iteration: 65; residual error (2-norm) = 9.06074e-08
iteration: 66; residual error (2-norm) = 1.27503e-07
iteration: 67; residual error (2-norm) = 2.13842e-08
iteration: 68; residual error (2-norm) = 2.14881e-08
iteration: 69; residual error (2-norm) = 7.69668e-09
iteration: 70; residual error (2-norm) = 2.72483e-09
iteration: 71; residual error (2-norm) = 2.11882e-09
iteration: 72; residual error (2-norm) = 5.34008e-09
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Error = 7.60178e-10

n = 10000
iteration: 1; residual error (2-norm) = 47.073
iteration: 2; residual error (2-norm) = 19.1304
iteration: 3; residual error (2-norm) = 15.4883
iteration: 4; residual error (2-norm) = 15.4906
iteration: 5; residual error (2-norm) = 772.079
iteration: 6; residual error (2-norm) = 40.9779
iteration: 7; residual error (2-norm) = 2.253
iteration: 8; residual error (2-norm) = 1.556
iteration: 9; residual error (2-norm) = 1.48444
iteration: 10; residual error (2-norm) = 0.986761
iteration: 11; residual error (2-norm) = 1.01669
iteration: 12; residual error (2-norm) = 0.301942
iteration: 13; residual error (2-norm) = 0.243955
iteration: 14; residual error (2-norm) = 0.188601
iteration: 15; residual error (2-norm) = 0.110684
iteration: 16; residual error (2-norm) = 0.0640378
iteration: 17; residual error (2-norm) = 0.0544683
iteration: 18; residual error (2-norm) = 0.0641564
iteration: 19; residual error (2-norm) = 0.0243827
iteration: 20; residual error (2-norm) = 0.0249781
iteration: 21; residual error (2-norm) = 0.091121
iteration: 22; residual error (2-norm) = 0.012303
iteration: 23; residual error (2-norm) = 0.00759624
iteration: 24; residual error (2-norm) = 0.0226155
iteration: 25; residual error (2-norm) = 0.0170232
iteration: 26; residual error (2-norm) = 0.0515311
iteration: 27; residual error (2-norm) = 0.00286028
iteration: 28; residual error (2-norm) = 0.00262664
iteration: 29; residual error (2-norm) = 0.0023839
iteration: 30; residual error (2-norm) = 0.00363721
iteration: 31; residual error (2-norm) = 0.00103248
iteration: 32; residual error (2-norm) = 0.000822597
iteration: 33; residual error (2-norm) = 0.000790028
iteration: 34; residual error (2-norm) = 0.000525879
iteration: 35; residual error (2-norm) = 0.000576319
iteration: 36; residual error (2-norm) = 0.000353832
iteration: 37; residual error (2-norm) = 0.00016547
iteration: 38; residual error (2-norm) = 0.000127889
iteration: 39; residual error (2-norm) = 7.68296e-05
iteration: 40; residual error (2-norm) = 6.74593e-05
iteration: 41; residual error (2-norm) = 5.80462e-05
iteration: 42; residual error (2-norm) = 2.87246e-05
iteration: 43; residual error (2-norm) = 9.34091e-05
iteration: 44; residual error (2-norm) = 1.4789e-05
iteration: 45; residual error (2-norm) = 1.46049e-05
iteration: 46; residual error (2-norm) = 3.58976e-05
iteration: 47; residual error (2-norm) = 9.93242e-06
iteration: 48; residual error (2-norm) = 1.25248e-05
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iteration: 49; residual error (2-norm) = 1.11508e-05
iteration: 50; residual error (2-norm) = 2.52323e-06
iteration: 51; residual error (2-norm) = 3.03099e-06
iteration: 52; residual error (2-norm) = 7.9925e-07
iteration: 53; residual error (2-norm) = 1.09825e-06
iteration: 54; residual error (2-norm) = 8.17451e-07
iteration: 55; residual error (2-norm) = 7.05452e-07
iteration: 56; residual error (2-norm) = 7.89603e-07
iteration: 57; residual error (2-norm) = 5.44073e-07
iteration: 58; residual error (2-norm) = 1.47492e-07
iteration: 59; residual error (2-norm) = 1.27076e-07
iteration: 60; residual error (2-norm) = 7.34426e-08
iteration: 61; residual error (2-norm) = 4.10214e-08
iteration: 62; residual error (2-norm) = 3.44667e-08
iteration: 63; residual error (2-norm) = 2.42802e-08
iteration: 64; residual error (2-norm) = 2.34321e-08
iteration: 65; residual error (2-norm) = 6.12079e-08
iteration: 66; residual error (2-norm) = 1.67799e-08
iteration: 67; residual error (2-norm) = 8.35694e-08
iteration: 68; residual error (2-norm) = 6.10168e-09
iteration: 69; residual error (2-norm) = 1.03563e-08
iteration: 70; residual error (2-norm) = 4.58102e-09
iteration: 71; residual error (2-norm) = 1.97085e-08
iteration: 72; residual error (2-norm) = 1.37691e-09
iteration: 73; residual error (2-norm) = 1.15699e-09
iteration: 74; residual error (2-norm) = 4.72373e-09
iteration: 75; residual error (2-norm) = 2.78421e-09
iteration: 76; residual error (2-norm) = 2.58069e-09
iteration: 77; residual error (2-norm) = 1.3077e-09
iteration: 78; residual error (2-norm) = 3.26359e-09
iteration: 79; residual error (2-norm) = 2.81598e-09
iteration: 80; residual error (2-norm) = 3.34953e-09
Error = 4.05661e-10


n = 100000
iteration: 1; residual error (2-norm) = 148.784
iteration: 2; residual error (2-norm) = 60.4022
iteration: 3; residual error (2-norm) = 48.5083
iteration: 4; residual error (2-norm) = 43.7371
iteration: 5; residual error (2-norm) = 31.1283
iteration: 6; residual error (2-norm) = 14.7114
iteration: 7; residual error (2-norm) = 17.3843
iteration: 8; residual error (2-norm) = 5.22481
iteration: 9; residual error (2-norm) = 5.76455
iteration: 10; residual error (2-norm) = 2.49911
iteration: 11; residual error (2-norm) = 1.68848
iteration: 12; residual error (2-norm) = 1.62722
iteration: 13; residual error (2-norm) = 2.89263
iteration: 14; residual error (2-norm) = 1.56
iteration: 15; residual error (2-norm) = 0.838873
iteration: 16; residual error (2-norm) = 1.33243
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iteration: 17; residual error (2-norm) = 0.281499
iteration: 18; residual error (2-norm) = 2.82645
iteration: 19; residual error (2-norm) = 0.0730303
iteration: 20; residual error (2-norm) = 0.0528448
iteration: 21; residual error (2-norm) = 0.0448622
iteration: 22; residual error (2-norm) = 0.0522311
iteration: 23; residual error (2-norm) = 0.0290698
iteration: 24; residual error (2-norm) = 0.0241026
iteration: 25; residual error (2-norm) = 0.0328832
iteration: 26; residual error (2-norm) = 0.0188748
iteration: 27; residual error (2-norm) = 0.0185721
iteration: 28; residual error (2-norm) = 0.0163888
iteration: 29; residual error (2-norm) = 0.0960766
iteration: 30; residual error (2-norm) = 0.0021644
iteration: 31; residual error (2-norm) = 0.00181138
iteration: 32; residual error (2-norm) = 0.00162975
iteration: 33; residual error (2-norm) = 0.000771803
iteration: 34; residual error (2-norm) = 0.00057319
iteration: 35; residual error (2-norm) = 0.00133238
iteration: 36; residual error (2-norm) = 0.000862693
iteration: 37; residual error (2-norm) = 0.000891432
iteration: 38; residual error (2-norm) = 0.0041218
iteration: 39; residual error (2-norm) = 0.000598373
iteration: 40; residual error (2-norm) = 0.000506321
iteration: 41; residual error (2-norm) = 0.000224038
iteration: 42; residual error (2-norm) = 0.000125212
iteration: 43; residual error (2-norm) = 0.000411787
iteration: 44; residual error (2-norm) = 9.32729e-05
iteration: 45; residual error (2-norm) = 9.83699e-05
iteration: 46; residual error (2-norm) = 0.000772774
iteration: 47; residual error (2-norm) = 5.06508e-05
iteration: 48; residual error (2-norm) = 3.10185e-05
iteration: 49; residual error (2-norm) = 2.37452e-05
iteration: 50; residual error (2-norm) = 1.65912e-05
iteration: 51; residual error (2-norm) = 3.38159e-05
iteration: 52; residual error (2-norm) = 1.43055e-05
iteration: 53; residual error (2-norm) = 1.82146e-05
iteration: 54; residual error (2-norm) = 3.94857e-06
iteration: 55; residual error (2-norm) = 3.63189e-06
iteration: 56; residual error (2-norm) = 1.35737e-05
iteration: 57; residual error (2-norm) = 1.02001e-05
iteration: 58; residual error (2-norm) = 1.13636e-05
iteration: 59; residual error (2-norm) = 2.6658e-05
iteration: 60; residual error (2-norm) = 5.36755e-06
iteration: 61; residual error (2-norm) = 8.22744e-07
iteration: 62; residual error (2-norm) = 2.37886e-07
iteration: 63; residual error (2-norm) = 1.4778e-07
iteration: 64; residual error (2-norm) = 5.60661e-07
iteration: 65; residual error (2-norm) = 8.87624e-08
iteration: 66; residual error (2-norm) = 5.36576e-07
iteration: 67; residual error (2-norm) = 1.42091e-07
```
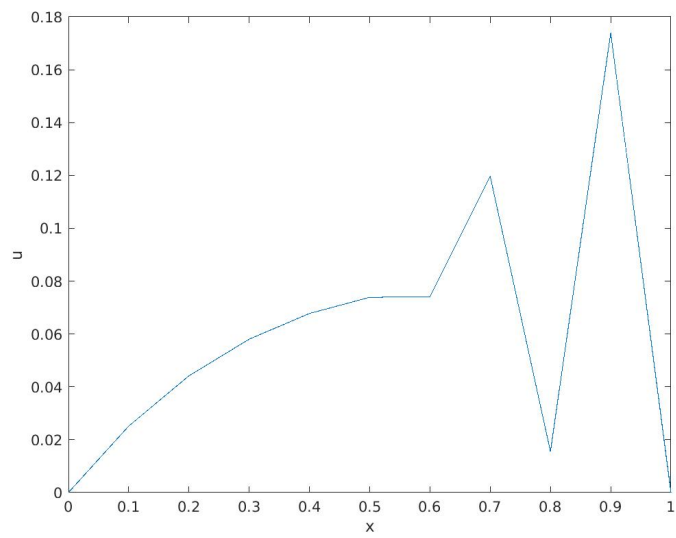
```
iteration: 68; residual error (2-norm) = 1.24805e-07
iteration: 69; residual error (2-norm) = 1.26026e-07
iteration: 70; residual error (2-norm) = 1.14082e-07
iteration: 71; residual error (2-norm) = 2.4536e-08
iteration: 72; residual error (2-norm) = 1.83921e-08
iteration: 73; residual error (2-norm) = 1.36042e-08
iteration: 74; residual error (2-norm) = 9.6058e-09
iteration: 75; residual error (2-norm) = 8.02852e-09
iteration: 76; residual error (2-norm) = 6.9082e-09
iteration: 77; residual error (2-norm) = 2.47164e-09
iteration: 78; residual error (2-norm) = 5.29004e-08
iteration: 79; residual error (2-norm) = 1.12738e-09
iteration: 80; residual error (2-norm) = 5.92773e-08
iteration: 81; residual error (2-norm) = 1.01854e-09
iteration: 82; residual error (2-norm) = 9.23733e-10
Error = 7.41467e-10
```

3(a) No output required.

3(b) Enter your your plot for $N = 100$ below:
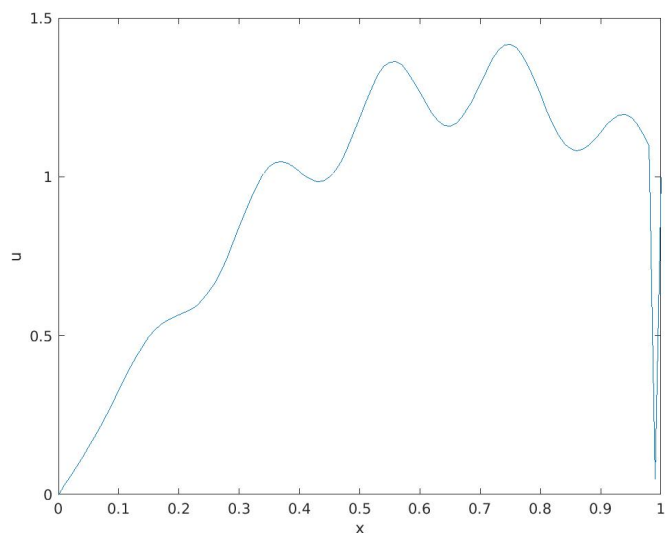
3(c) Enter your your plot for $N = 10$ below:

Enter your your plot for $N = 100$ below: