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Stock Price Prediction Using Dynamic Mode Decomposition

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***Abstract*—Stock price prediction is a challenging problem as the market is quite unpredictable. We propose a method for price prediction using Dynamic Mode Decomposition assuming stock market as a dynamic system. DMD is an equation free, data-driven, spatio-temporal algorithm which decomposes a system to modes that have predetermined temporal behaviour associated with them. These modes help us determine how the system evolves and the future state of the system can be predicted. We have used these modes for the predictive assessment of the stock market. We worked with the time series data of the companies listed in National Stock Exchange. The granularity of time was minute. We have sampled a few companies across sectors listed in National Stock Exchange and used the minute-wise stock price to predict their price in next few minutes. The obtained price prediction results were compared with actual stock prices. We used Mean Absolute Percentage Error to calculate the deviation of predicted price from actual price for each company. Price prediction for each company was made in three different ways. In the first, we sampled companies belonging to the same sector to predict the future price. In the latter, we considered sampled companies from all sectors for prediction. In the first and second method, the sampling as well as the prediction window size were fixed. In the third method the sampling of companies was done from all sectors considered. The sampling window was kept fixed, but predictions were made until it crossed a threshold error. Prediction was found to be more accurate when samples were taken from all the sectors, than from a single sector. When sampling window alone was fixed; the predictions could be made for longer period for certain instances of sampling.**

***Index Terms*—Dynamic Mode Decomposition,Proper Orthogonal Decomposition, Mean Absolute Percentage Error**

I. Introduction

Stock price prediction is a challenging problem as the market is quite unpredictable. Arguments prevail,regarding the possibility of stock price prediction. Efficient market hypothesis states that the current price reflects the current state of the market and nothing further can be inferred from it [1] [2]. Fundamental analysts believe that stock price predictions can be made by analyzing the current financial situations and by taking into account the financial statements of a company. Technical analysis involves studying the past data to predict future prices. Though there are a number of studies supporting efficient market hypothesis, recently there are studies that support technical analysis [3] [4] [5] [6]. Technical analysis uses statistical analysis to predict the future price. With the advent of automated trading, technical analysis is gaining further popularity. In technical analysis time series analysis and neural networks are most widely used methods.AR (Autoregressive),MA (Moving Average),ARMA (Autoregressive Moving Average),ARIMA(Autoregressive Integrated Moving Average) are some of the models used for time series analysis [7] [8].In AR the future values is determined using past values,MA uses past error values to determine future values and ARMA uses both for prediction.Changing dynamics of the financial system adds to the non-stationarity of the data which makes long term predictions inaccurate. Also time series analysis is computationally costly. As neural networks are efficient in pattern learning, these models can be used to learn patterns in time series data. Predictions using neural networks are also widely done in stock market [9].One major drawback of neural network is its requirement of large amount of training data. Each time there is a pattern change in financial market, the model has to be trained for new parameters [10] [11]. This pattern change can happen quite often in financial market. In our proposed method we assume stock market to be a dynamical system and use DMD (Dynamic Mode Decomposition), a data driven, spatial-temporal coherent algorithm to identify the evolutionary patterns of this system [12] [13]. DMD is computationally very efficient as it exploits the low-dimensional structure of the data. Due to this advantageous property of DMD, it finds its application in many fields. A dynamical system can be decomposed to modes and these modes can be used to identify the evolutionary patterns of the system [14] [15]. Few decomposition methods are, wavelet decomposition, fourier decomposition,proper orthogonal decomposition. Wavelet modes has been used to identify business cycles that China shared with the rest of the world economy [16]. However the system had to be decomposed into large number of wavelet modes. Using DFT we cannot estimate the growth and decay of the system, as DFT eigen values are always 1. The difference between POD and DMD is that POD modes are orthogonal in space with multi-frequency time signals, whereas DMD modes are non-orthogonal in space with single frequency time signal [14].DMD decomposes a system to a set of modes that have prescribed time dynamics

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[12] [14] [15] [13]. Since DMD modes are dynamic modes, they capture the trend of market in them. Dynamic systems are usually described using equation that describe how their trajectories evolve in finite dimensional space. An alternative description of this can be made using an operator that acts on infinite dimensional space of functions.Koopman operator is one such infinite dimensional,linear operator [17] [18].The evolution of a dynamic system from time t to t+1 can be captured using koopman operator. This koopman operator captures the dynamics of the system. DMD decomposes this matrix into modes. These are the dynamic modes of the system. DMD eigen values are not always 1 unlike that of DFT. This means that DMD can estimate the rate of growth or decay of the system [15]. In the context of analyzing stock market, DMD modes can be thought of as coherent structures in the financial activity. DMD has been used in finance to extract cyclic nature in market as well as for price prediction [19]. In our work, we use DMD for short term prediction of price.

DMD framework and proposed method is detailed in section[II].Section[III] contains Results. In Section [IV] the obtained results are discussed in detail. Section[V] draws out the conclusions and discusses the scope for future work.

II. Methodology

A dynamic system is defined using a set of governing equations.We consider financial system as a non-linear dynamic system whose governing equations are not known to us.The snapshots we take corresponds to a state of the system. Snapshot of a system consists of observed measurements of that system at time *t*. In our case the observed measurements are the stock price of each company at the time *t*. A dynamic system is described using a governing set of differential equations.

$$\frac{dx}{dt} = F(x,t) \tag{1}$$

At each state we can make different kinds of measurements of the observables.The measurement function can be denoted as

$$G(x, t_k) = 0 \tag{2}$$

where $k = 1, 2, \ldots, M$ where M is measurement time.The initial condition is stated as

$$x(0) = x_0 \tag{3}$$

The non-linear function F that defines the set of governing equations is unknown. All we have is the initial conditions and the measurements taken.In our case the measurement is the stock price. In DMD procedure, we construct an approximate linear evolution of the system.

$$\frac{d\tilde{x}}{dt} = A\tilde{x} \tag{4}$$

The well-known solution for equation(4) is,

$$\tilde{x}(t) = \sum_{k=1}^{K} b_k \psi_k \exp(\omega_k t) \tag{5}$$

Hereψ_k andω_k are the eigenvectors and eigenvalues of matrix A. If real part of eigenvalues are positive and greater than one, then it means a growing mode and growing money. If eigenvalues are negative, then it means decaying modes and losing money. The data we used was the minute-wise data of transactions from 1 July 2014 to 30 June 2015.The data was structured as date,transaction id,time,company name,price of the stock and volume of the stock. For our analysis,we selected 57 companies across sectors IT,financial services,pharma,automobiles. These 4 sectors combined hold 63.94 %market share.Our sampling interval is one minute.Not all companies have transactions in every minute. In such cases we substitute the previous stock price in the current minute. The reasoning behind this is that, even though no transaction was done in that minute,the stock price at that time is the previously transacted price.The data matrix given for DMD algorithm is snapshots of the system taken at equispaced time intervals.Each snapshot consists of the stock price of all the companies considered at that particular time instant.DMD decomposes data matrix to modes that spans spatially and has temporal frequency associated with them [12].The data matrix will be an $n \times m$ matrix where n=number of companies considered m=number of snapshots taken The matrix will be of the form,

$$X = [x_1, x_2, x_3, \ldots x_m] \tag{6}$$

From this matrix we need to capture the underlying dynamics of the system.For this we splits the data matrix into,

$$X_1 = [x_1, x_2, x_3, \ldots x_{m-1}] \tag{7}$$

$$X_2 = [x_2, x_3, x_4, \ldots x_m] \tag{8}$$

X_2 is one time slot shifted from X_1.Let A be a linear operator that maps X_i to x_{i+1}.Then X_2 can be expressed as

$$X_2 = [Ax_1, Ax_2, Ax_3, \ldots Ax_{m-1}] \tag{9}$$

$$X_2 = AX_1 \tag{10}$$

DMD expects X_1 to have low-rank structure. If not DMD fails immediately. Since$N \gg M$,the matrix X_1 is sure to have a low rank structure.At some point addition of a new snapshot to matrix X_1 will not add on to the vector space spanned by X_1. We can express x_m as combination of previous columns of X_1.

$$x_m = \sum_{i=1}^{m-1} a_i x_i + r \tag{11}$$

Here r is the residual vector.DMD algorithm does the minimization of r.

$$X_2 = X_1 S + r e_{m-1}^* \tag{12}$$

Here S is the companion matrix.e_{m-1} is (m-1)th unit vector.It can be seen that

$AX_1 = X_2 \approx X_1 S$ Eigen values of A are approximately that of S.To utilize the low rank structure of X_1 we apply SVD to it.Singular Value Decomposition decomposes X_1 to $U\Sigma V^*$where $U \in C^{n\times r}$,

$\Sigma \in C^{r \times r}$and
$V \in C^{(m-1)\times r}$.Here Σ is diagonal matrix,whereas U and V are unitary matrix.Decomposing X_1 using SVD and substituting it,we get

$$X_2 \approx U\Sigma V^* S \tag{13}$$

$$S \approx V\Sigma^{-1}U^* X_2 \tag{14}$$

For robust implementation of the algorithm, instead of finding the eigen values and eigen vectors of S,we find the same for a similar matrix $\tilde{S}$

$$\tilde{S} \approx U^* X_2 V \Sigma^{-1} \tag{15}$$

The eigen values of similar matrices are same.Eigen vector can be found using similar matrix property.DMD eigen v are,

$$\tilde{S}v_j = \lambda_j v_j$$

DMD eigen vectors are,

$$\phi_j = U v_j$$

For ease of representation DMD modes can be conv to fourier modes as $\omega_j = \ln(\lambda_j)$.So now using these vectors and eigen values we can reconstruct the syste follows

$$X_{DMD}(t) = \sum_{j=1}^{r} b_j \phi_j e^{\omega_j t} = \phi diag(e^{\omega t})b$$

$X_{DMD}(t)$ defines the state of the system at time t.I case,as we had taken stock prices to represent the sta the system,$X_{DMD}(t)$ gives stock prices at time t.Equ (10)is basically a regression equation.It tries to find a least square fit for the points considered. In our method we considered 6 snapsots of the system to make future predictions. In one approach the prediction window was kept fixed at the size of sampling window. In another approach the sampling wi was kept same, but prediction was made until it cros threshold value. MAPE was used to validate the result predicted values by DMD was also compared with AR model. We gave an input of 2000 data points, to fix the m For this we used forecasting package in tool R. ARIMA r is of the form ARIMA(p,d,q). Here p is the lag or AR the number of previous data points on which the current point is dependent. The number of times the series had differenced to obtain a stationary signal is denoted by d lag of MA is denoted by q.

III. Results

We approached our problem in three different ways DMD algorithm.In the first method we used, the snap consisted of the companies sampled from same sector.I second method the snapshots consisted of the comp sampled across different sectors. In both these approaches,the size of the sampling window and prediction window was kept same.The sampling window size chosen for our experiment was 6. On comparison with the actual values, it was observed that the predicted result using second approach was more accurate. In the third method sampling window size was kept fixed at 6,but prediction was made until it crossed a threshold error of 3.5%. It was observed that for certain time instances, the prediction could be made for longer time periods. But for other time instances, the prediction could be made only for shorter time period than that of fixed window approach. MAPE was used on the results obtained by all the three approaches. MAPE values for second and third method was significantly lower than the first method. The comparison between the results obtained by DMD and ARIMA for a pharma company CIPLA is shown in table 1. The ARIMA model that fit the data was ARIMA(1,1,0).

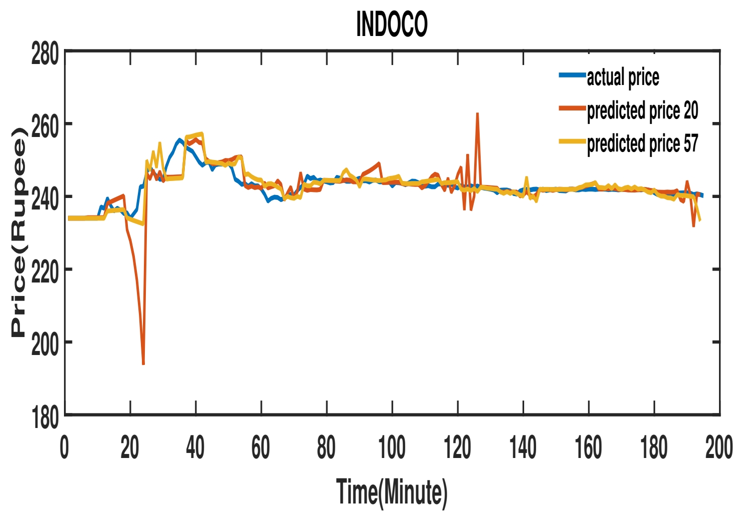


Fig. 1: Plot between the actual, predicted values using DMD for 20 companies and DMD for 57 companies for pharma company INDOCO

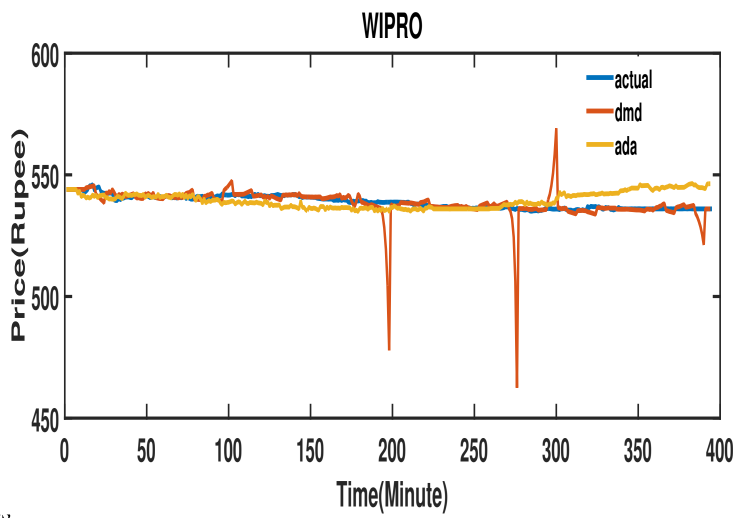


Fig. 2: Plot between the actual, predicted values using DMD and prediction using adaptive DMD for IT company WIPRO.

From fig.1,it is clear that,there exists inter sectoral dependence.DMD captures these inter-sector dependencies.MAPE measure was used on the predicted output.

$$M = \frac{100}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right| \tag{19}$$

TABLE I: Price prediction for pharma company CIPLA

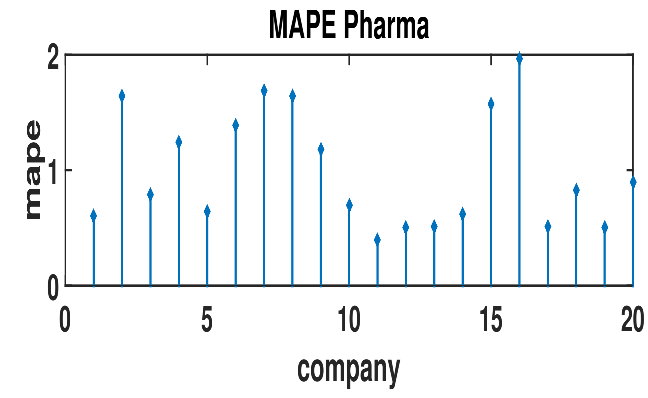
Sl:no	Actual	Arima(1,1,0) predicted	DMD predicted	Arima error percent	DMD error percent
1	463.2	462.76	463.15	0.09	0.01
2	463.3	462.76	463.12	0.11	0.03
3	463.5	462.76	463.19	0.15	0.066
4	463.45	462.76	463.19	0.14	0.05
5	463.2	462.76	463.02	0.09	0.03

IV. Discussion

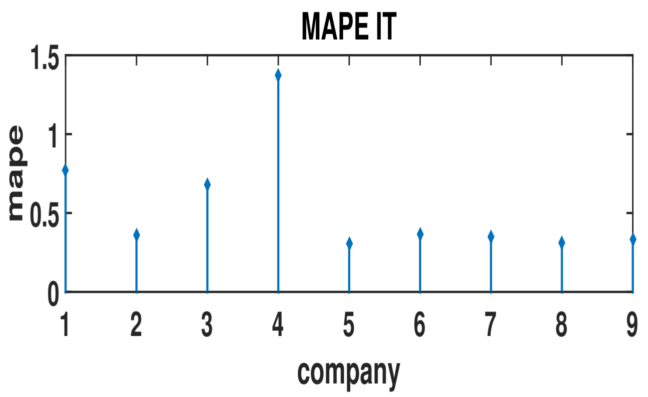
From figure(1) it is clear that prediction by taking into account, different companies from across sectors gives better prediction. This means that there is an inter-sector dependency in stock market. So prediction can be made better by observing the whole market dynamics than exclusively observing a particular stock. From figure(2) it is observed that simple DMD gives predictions close to actual values,but at times shows sharp variations. Adaptive DMD was conditioned to mitigate these sharp variations by setting a threshold error. From figure(3) and (4), it can be observed that the MAPE values for second and third method are more or less the same. In adaptive DMD at certain instances, the predictions could be made for longer period of time than simple DMD, but not so in other time periods. This is because, at certain times, a particular market dynamics holds for a longer period. Technical analysis can be most effectively used at this period of time. It can also be seen that for certain companies simple DMD gives better results than adaptive DMD. This is because when, more companies are added, the predictions become better. Since a threshold is set for adaptive DMD, these improvements are not reflected in adaptive DMD. We set a common threshold for all companies. Lowering the threshold when number of companies are more, would mean that prediction duration will be cut shorter. This is because if error of any single company exceeds threshold, the algorithm again takes actual data to make predictions. It can be seen from table 1 that DMD predictions are better than ARIMA predictions. Also for ARIMA we had to give 2000 data points, for it to fix a model. The model that fit was ARIMA(1,1,0). Here DMD only required previous 6 minutes data. This signifies the efficiency of DMD algorithm. Comparing the fitted ARIMA model with the form ARIMA(p,d,q), we can see that p and d are 1. This means the current values of CIPLA is dependent on its previous value and also the series was differenced once, to obtain stationary signal.

V. Conclusion

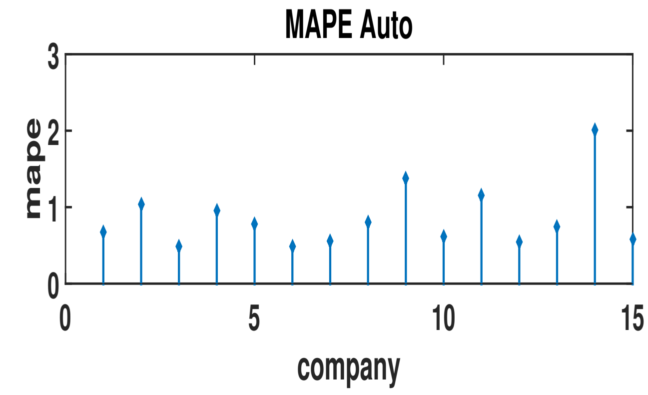
DMD is an efficient,computationally fast algorithm that can predict the future states of the system,without any prior knowledge of the underlying equations. Also while using



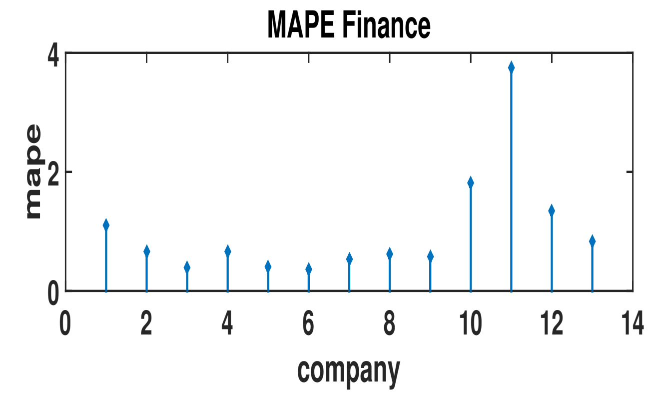
(a) MAPE using DMD



(b) MAPE using DMD

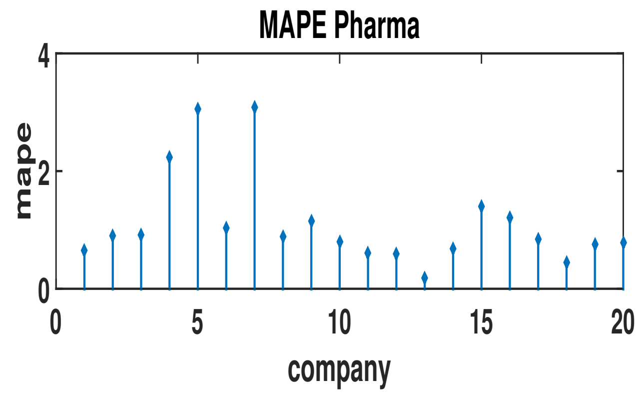


(c) MAPE using DMD

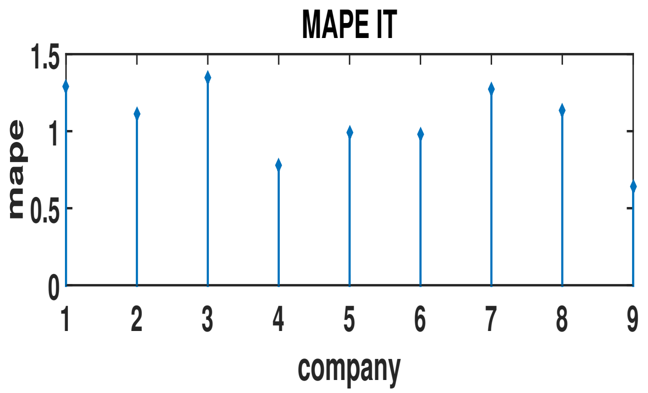


(d) MAPE using DMD

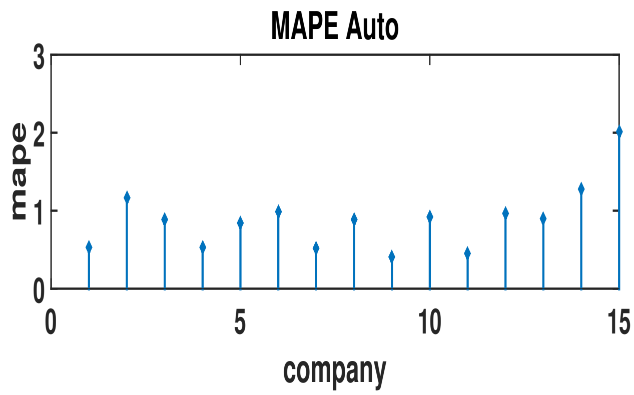
Fig. 3: DMD Result without using adaptive (a) shows the plot for MAPE values for each company taken in pharma sector (b) shows the plot for MAPE values for each company taken in IT sector (c) shows the plot for MAPE values for each company taken in Automobiles sector (d) shows the plot for MAPE values for each company taken in Financial Services sector



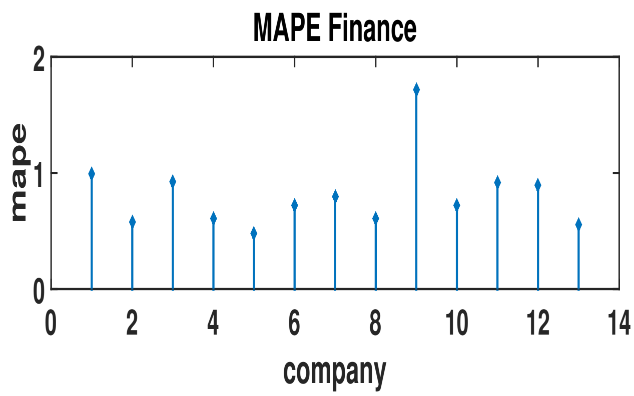
(a) MAPE using adaptive DMD



(b) MAPE using adaptive DMD



(c) MAPE using adaptive DMD



(d) MAPE using adaptive DMD

Fig. 4: Adaptive DMD Result (a) shows the plot for MAPE values for each company taken in pharma sector (b) shows the plot for MAPE values for each company taken in IT sector (c) shows the plot for MAPE values for each company taken in Automobiles sector (d) shows the plot for MAPE values for each company taken in Financial Services sector

adaptive DMD, it was observed that at some instances, the predictions could be made for a longer time period but not so in other instances. This happens because for certain time periods,the market dynamics doesn't change. So our study supports technical analysis, but limited to certain times. In our work, we also observed an interdependence among sectors. Prediction improves when companies from various sectors were taken into account. We believe this is because of the existence of inter-sectoral dynamics. So predictions considering the whole of market dynamics is more accurate than predictions made by focusing only on a single stock, or a small subset of stocks. Our method fails to predict accurately, when there is a sudden change in the dynamics,like when stock split is announced or when a new product is launched in the market. In short our method doesn't take care of the exogenous influences. Input-output DMD can be used to take care of exogenous factors [20]. Stock split identification can be done by identifying variation point. Since DMD is computationally efficient we can apply it for large scale stock data to obtain profitable results. DMD unlike univariate methods like ARIMA can make predictions for more number of companies at a time. These predictions are made taking into account the interdependencies between the companies. As this approach produced better result than univariate ARIMA, we can conclude that considering interdependencies can improve prediction. We also observed as more number of companies were considered for prediction, the predictions improves. This implies that considering whole of market dynamics to make predictions is better than making predictions by monitoring only a particular stock or a subset of stocks.

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