Buffalo Trail Public Schools

SESSION 6
Trig. Equations and Identities

KEY

Math 30-1

R^3

(Revisit, Review and Revive)

Mathematics 30–1 Formula Sheet

For $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Relations and Functions

Graphing Calculator Window Format

x: $[x_{\min}, x_{\max}, x_{\text{scl}}]$

y: $[y_{\min}, y_{\max}, y_{\text{scl}}]$

Laws of Logarithms

$$\log_b(M \times N) = \log_b M + \log_b N$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$$\log_b(M^n) = n \log_b M$$

$$\log_b c = \frac{\log_a c}{\log_a b}$$

Growth/Decay Formula

$$y = ab^{\frac{t}{p}}$$

General Form of a Transformed Function

$$y = af\left[b(x - h)\right] + k$$

Permutations, Combinations, and the Binomial Theorem

$n! = n(n-1)(n-2)\ldots 3 \times 2 \times 1$,
where $n \in N$ and $0! = 1$

$$_nP_r = \frac{n!}{(n-r)!}$$

$$_nC_r = \frac{n!}{(n-r)!r!} \qquad _nC_r = \binom{n}{r}$$

In the expansion of $(x + y)^n$,
the general term is $t_{k+1} = {}_nC_k x^{n-k} y^k$.

Trigonometry

$$\theta = \frac{a}{r}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

$$\cos(2\alpha) = 2\cos^2\alpha - 1$$

$$\cos(2\alpha) = 1 - 2\sin^2\alpha$$

$$\tan(2\alpha) = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$y = a\sin\left[b(x - c)\right] + d$$

$$y = a\cos\left[b(x - c)\right] + d$$

Mathematics 30-1 Learning Outcomes

Specific Outcome 5: Solve, algebraically and graphically, first and second degree trigonometric equations with the domain expressed in degrees and radians.

- Verify, with or without technology, that a given value is a solution to a trigonometric equation.
- Determine, algebraically, the solution of a trigonometric equation, stating the solution in exact form when possible.
- Determine, using technology, the approximate solution of a trigonometric equation in a restricted domain
- Relate the general solution of a trigonometric equation to the zeros of the corresponding trigonometric function (restricted to sine and cosine functions).
- Determine, using technology, the general solution of a given trigonometric equation
- Identify and correct errors in a solution for a trigonometric equation.

Key ideas:

Verify means to see if it works for a certain value of the variable. If you put in a value and its false then it cannot be an identity. If you put in a value and it is true then it **may** be an identity. You may also verify a trig identity graphically in your calculator (both graphs should appear the same). Verifying an equation is not enough to conclude that the equation is an identity.

Prove: means to show that the left side is equal to the right side for **all** values of the variable in the domain. When proving you may not move anything from one side of the equation to the other. You have to work with each side of the equation independently.

- Try to simplify the more complicated side
- Use identities from your formula sheet to make substitutions
- Rewrite in terms of sine and cosine only
- factor

Solve: find the particular value of the variable for which the equation is true. When solving you may do the same thing to both sides of the equation to isolate the variable.

- Isolate trig ratio
- Factor with one side of the equation being equal to zero
- Simplify using trig identities then solve
- Remember to check for non-permissible values.

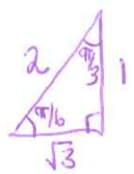
Example 1: Verify that $\theta = \frac{7\pi}{6}$ is a solution to the trigonometric equation

$5\sin\theta + 2 = 1 + 3\sin\theta$ in the domain $\pi < \theta < \frac{3\pi}{2}$.

→ quad 3, sine is negative

ref angle is $\pi/6$

LS	RS
$-5\sin\frac{\pi}{6} + 2$	$1 + -3\sin\frac{\pi}{6}$
$-5\left(\frac{1}{2}\right) + 2$	$1 + -3\left(\frac{1}{2}\right)$
$-\frac{5}{2} + \frac{4}{2}$	$\frac{2}{2} - \frac{3}{2}$
$= -\frac{1}{2}$	$= -\frac{1}{2}$



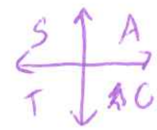
$\sin\frac{\pi}{6} = \frac{1}{2}$

LS = RS ∴ verified

Example 2: Determine the exact roots for the trigonometric equation $2\cos\theta - \sqrt{3} = 0$ in the domain $0 \le \theta < 2\pi$.

$2\cos\theta - \sqrt{3} = 0$

$\cos\theta = \frac{\sqrt{3}}{2} = \frac{adj}{hyp}$



$\theta = \frac{\pi}{6}$

Cosine is also positive in quad 4 so $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\theta = \frac{11\pi}{6}$

The exact roots are $\pi/6$ and $\frac{11\pi}{6}$.

Example 3: Solve the trigonometric equation in the specified domain.

$2\sec\theta + 6 = 0,\ 0° \le \theta < 360°$

$2\sec\theta + 6 = 0$

$\sec\theta = \frac{-6}{2}$

$\sec\theta = -3$

$\frac{1}{\cos\theta} = -3$

$\cos\theta = -\frac{1}{3}$

$\cos^{-1}\left(\frac{1}{3}\right) = 70.5°$ ← reference angle

Cos and secant are negative in quadrants 2 and 3.

Quad 2: $180° - 70.5° = 109.5°$

Quad 3: $180° + 70.5° = 250.5°$

Example 4: How is the general solution of ① $2\cos^2\theta - 3\cos\theta + 1 = 0$ related to the zeros of ② $y = 2\cos^2\theta - 3\cos\theta + 1$?

The general solution and the zeros are the same.

Example 5: Determine the general solution for $3\csc\theta - 6 = 0$.

$3\csc\theta - 6 = 0$
$\csc\theta = \frac{6}{3}$
$\csc\theta = 2$
$\frac{1}{\sin\theta} = 2$
$\sin\theta = \frac{1}{2}$

find θ_R: $\sin^{-1}\left(\frac{1}{2}\right) = 30°$ or $\frac{\pi}{6}$
$\sin\theta$ is positive in quad 1 and 2. $\frac{\pi}{6} + \pi$
Quad 2 solution: $180° + 30° = 210°$
$\theta = 30° + 360°n,\ n \in I$ and $\theta = 210° + 360°n$
OR
$\theta = \pi/6 + 2\pi n$

Example 6: Solve the following trigonometric equation.

$$9\sin^2\theta + 12\sin\theta + 4 = 0,\ \theta \in [0°, 360°)$$

graph: $y_1 = 9\sin^2\theta + 12\sin\theta + 4$
$y_2 = 0$
find intersection
$x = 221.8°$ $x = 318.2°$

Practice Questions:

1.

The solutions to the equation $\cos^2 x = \cos x$, where $0 \le x < 2\pi$ are

A. $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

B. $\frac{\pi}{2}, \frac{3\pi}{2}$

C. $0, \frac{\pi}{2}, \frac{3\pi}{2}$

D. $0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$

$\cos^2 x - \cos x = 0$
$\cos x(\cos x - 1) = 0$
$\cos x = 0$ OR $\cos x - 1 = 0$
$x = \pi/2, \frac{3\pi}{2}$ $\cos x = 1$
$x = 0$

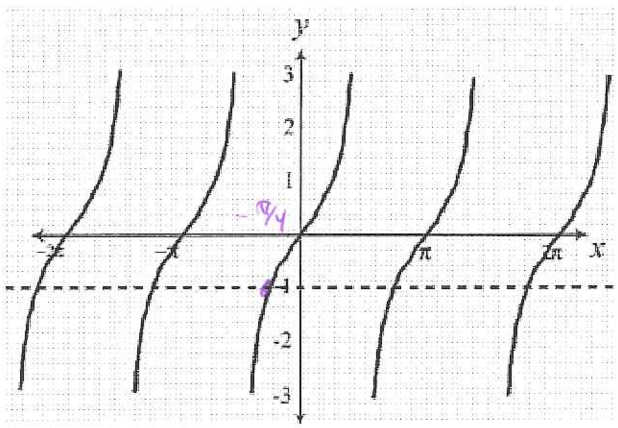
Numerical Response

1. The identity $\cot^2 x + \csc x = \frac{\cos^2 x + \sin x}{\sin^2 x}$ may be verified by substituting 2.1 rad for x on each side. When this substitution is made, the numerical value of each side, to the nearest hundredth, is 1.50 .

2.

Use the following information to answer the next question.

A student solves the equation $\tan x = -1$ in their graphing calculator as shown in the diagram below.



The student determines the general solution of this graph is $-\dfrac{\pi}{4} + n\pi,\ n \in I$

The general solution to the equation $\tan(5x) = -1$ is

horizontal stretch by factor of $\frac{1}{5}$

A. $-\dfrac{\pi}{3} + \dfrac{n\pi}{10},\ n \in I$

B. $-\dfrac{\pi}{4} + n\pi,\ n \in I$

C. $-\dfrac{5\pi}{4} + 5n\pi,\ n \in I$

D. $-\dfrac{\pi}{20} + \dfrac{n\pi}{5},\ n \in I$ ✓

$-\frac{\pi}{4}\left(\frac{1}{5}\right) = -\frac{\pi}{20}$

$n\pi\left(\frac{1}{5}\right) = \frac{\pi n}{5}$

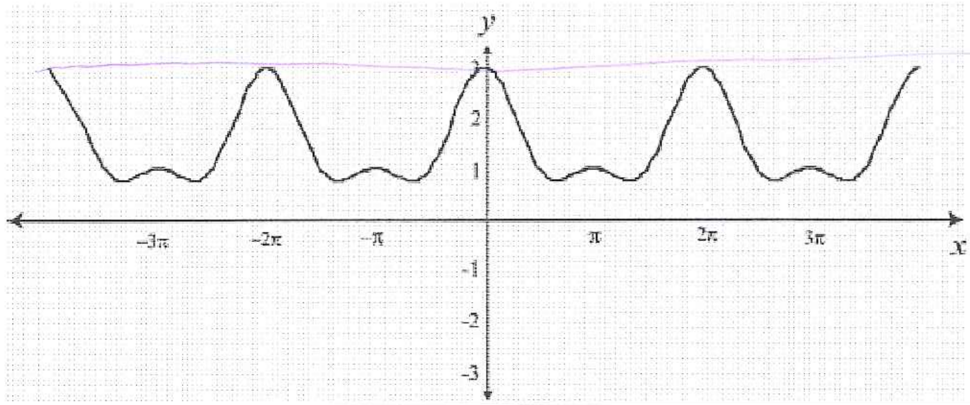
$-\frac{\pi}{4}$

$-\frac{\pi}{20} + \frac{\pi n}{5}$

3.

Use the following information to answer the next question.

The graph of $f(x) = \cos^2 x + \cos x + 1$ is shown below



y = 3

If the equation $\cos^2 x + \cos x + 1 = 3$ has the general solution $2n\pi,\ n \in I$,

then a possible solution to the equation $\cos^2\left(\frac{x}{3}\right) + \cos\left(\frac{x}{3}\right) + 1 = 3$ is

A. 2π
B. 3π
C. 9π
D. 12π

horizontal stretch by factor of 3.

$0, 6\pi, 12\pi.$

4.

The expression $\dfrac{\cos x}{1 - 2\sin x}$ is undefined when the values of x are

A. $\dfrac{\pi}{6} \pm 2n\pi,\ \dfrac{5\pi}{6} \pm 2n\pi$

B. $\dfrac{\pi}{6} \pm n\pi$

C. $\dfrac{\pi}{6} \pm 2n\pi\ ,\ \dfrac{5\pi}{6} \pm 2n\pi\ ,\ \dfrac{\pi}{2} \pm n\pi$

D. $\dfrac{n\pi}{2}$

$1 - 2\sin x = 0$

$\sin x = \frac{-1}{-2}$

$\sin x = \frac{1}{2}$

$x = \pi/6$ Quad 1

$x = \frac{5\pi}{6}$ Quad 2

5.

The line $y=\frac{1}{2}$ intersects the graph of $\cos^2 x-\sin x$ twice in the interval $0\le x<2\pi$. An equation that can be used to solve for x is

$\cos^2 x - \sin x = \frac{1}{2}$ mult by 2, sub 1.

$2\cos^2 x - 2\sin x - 1 = 0$

A. $\cos^2 x=\sin x$
B. $2\cos^2 x-2\sin x-1=0$
C. $\sin x-\cos^2 x=2$
D. $2\cos^2 x+2\sin x-1=0$

6.

$2\pi \div 4 = \frac{\pi}{2}$

The general solution to the equation $\sin 4x=-\frac{1}{2}$ is

$y = 4x$

$\sin y = -\frac{1}{2}$

ref. $y = \pi/6$

Quad 3: $\frac{7\pi}{6}$ Quad 4: $\frac{11\pi}{6}$

$4x = \frac{7\pi}{6}$ and $4x = \frac{11\pi}{6}$

$x = \frac{7\pi}{24}$ $x = \frac{11\pi}{24}$

A. $\frac{7\pi}{24}\pm\frac{n\pi}{2}$
B. $\frac{5\pi}{12}\pm\frac{n\pi}{4}, \frac{3\pi}{12}\pm\frac{n\pi}{4}$
C. $\frac{7\pi}{24}\pm\frac{n\pi}{2}, \frac{11\pi}{24}\pm\frac{n\pi}{2}$
D. $\frac{3\pi}{12}\pm\frac{n\pi}{4}$

Numerical Response

2. If $\frac{1}{1+\cot^2 x}=0.43$, and $0\le x<\frac{\pi}{2}$, then the value of x in radians, to the nearest tenth, is 0.7.

$y_1 = 1/(1+\cot^2 x)$

$y_2 = 0.43$

find intersection

$x = 0.7$

7.

When the following pairs of functions are graphed, the pair that could **not** be used to solve the equation $4 \sin x - 1 = 0$ is

A. $y = \sin x$ and $y = 1$

B. $y = \sin x$ and $y = \frac{1}{4}$ ✓

C. $y = 4 \sin x$ and $y = 1$ ✓

D. $y = 4 \sin x - 1$ and $y = 0$ ✓

8.

Use the following information to answer the next question.

Refraction describes the bending of light rays. Refraction can be calculated using the formula

$$n = \frac{\sin(\theta + \alpha)}{\sin \theta},$$

where n represents refraction.

If $\alpha = 30°$, then an equivalent expression for n is

A. $\frac{\sqrt{3}}{2} + \cos \theta$

B. $\frac{\sqrt{3}}{2} + \frac{1}{2} \cot \theta$

C. $\frac{\sqrt{3}}{2} + \frac{1}{2} \cos \theta$

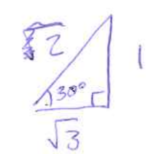
D. $\frac{1}{2} + \frac{\sqrt{3}}{2} \cot \theta$

$$n = \frac{\sin(\theta + 30°)}{\sin\theta}$$

$$= \frac{\sin\theta\cos 30° + \sin 30° \cos\theta}{\sin\theta}$$

$$= \cos 30° + \frac{\sin 30° \cos\theta}{\sin\theta}$$

$$\frac{\sqrt{3}}{2} + \frac{1}{2}\cot\theta$$



Specific Outcome 6: Prove trigonometric identities, using:
• reciprocal identities
• quotient identities
• Pythagorean identities
• sum or difference identities (restricted to sine, cosine and tangent)
• double-angle identities (restricted to sine, cosine and tangent).

- Explain the difference between a trigonometric identity and a trigonometric equation
- Verify a trigonometric identity numerically for a given value in either degrees or radians.
- Explain why verifying that the two sides of a trigonometric identity are equal for given values is insufficient to conclude that the identity is valid.
- Determine, graphically, the potential validity of a trigonometric identity, using technology.
- Determine the non-permissible values of a trigonometric identity
- Prove, algebraically, that a trigonometric identity is valid.
- Determine, using the sum, difference and double-angle identities, the exact value of a trigonometric ratio.

<u>**Key ideas:**</u>

trigonometric equation An equation involving trigonometric ratios.

trigonometric identity A trigonometric equation that is true for all permissible values of the variable in the expression on both sides of the equation.

Note: Sometimes the symbol $\equiv$ is used instead of $=$ to indicate that a statement is an identity.

$a = r\sigma$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin(A+B) = \sin A\cos B + \cos$$

$$\sin(A-B) = \sin A\cos B - \cos$$

$$\cos(A+B) = \cos A\cos B - \sin$$

$$\cos(A-B) = \cos A\cos B + \sin$$

$$\sin(2A) = 2\sin A\cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

Examples:

Example 7: Verify that the equation $\dfrac{\sec\theta}{\tan\theta+\cot\theta}=\sin\theta$ is true for $\theta=30°$ and for $\theta=\dfrac{\pi}{4}$.

LS	RS
$\dfrac{\sec 30°}{\tan 30° + \cot 30°}$	$\sin 30°$
$\dfrac{\frac{2}{\sqrt{3}}}{\frac{1}{\sqrt{3}}+\frac{\sqrt{3}}{1}\frac{\sqrt{3}}{\sqrt{3}}}$	$=\dfrac{1}{2}$
$=\dfrac{2}{\sqrt{3}}\times\dfrac{\sqrt{3}}{4}$	
$=\dfrac{1}{2}$	

RS	LS
$\dfrac{\sec \pi/4}{\tan \pi/4 + \cot \pi/4}$	$\sin \pi/4$
$=\dfrac{\frac{\sqrt{2}}{1}}{1+1}$	$=\dfrac{1}{\sqrt{2}}$
$=\dfrac{\sqrt{2}}{1}\times\dfrac{1}{2}$	$=\dfrac{\sqrt{2}}{2}$
$=\dfrac{\sqrt{2}}{2}$	

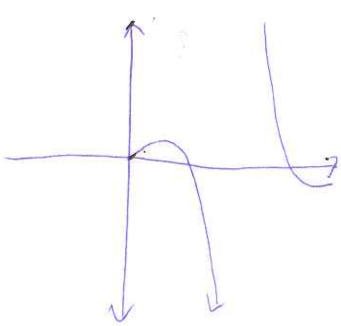
LS = RS.

Example 8: Explain why verifying that the two sides of a trigonometric identity are equal for given values is insufficient to conclude that the identity is valid.

Verifying shows that it is true for that particular value of the variable. It does not show that its true for all values.

Showing that a identity ~~value~~ is false by showing one value that is not true is enough to show its not a valid identity.

Example 9: Consider the equation $\dfrac{\sin\theta\cos\theta}{1+\cos\theta}=\dfrac{1-\cos\theta}{\tan\theta}$. Graph the two sides of the equation using technology, over the domain $0\le\theta<2\pi$. Could it be an identity?



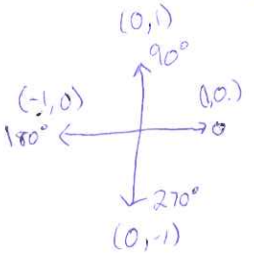
yes it could be an identity because the graphs are the same.

(I animated the second graph)

Example 10: Consider the identity $\frac{1-\cos\theta}{\sin\theta}=\frac{\sin\theta}{1+\cos\theta}$. What are the non-permissible values for the identity over the domain $0° \le \theta < 360°$.

non permissible values come from attempting to divide by zero

so $\sin\theta \neq 0$ and $1+\cos\theta \neq 0$



$\sin\theta = 0$ when $\theta = 0°, 180°$

$1+\cos\theta = 0$ when $\cos\theta = -1$ $\quad \theta = 180°$

$\therefore \theta \neq 0°$ or $180°$

Example 11: Prove each identity.

a) $\frac{\cos\theta+\cot\theta}{\sec\theta+\tan\theta}=\cos\theta\cot\theta$

LS	RS
$\dfrac{\cos\theta + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$	$\cos\theta \frac{\cos\theta}{\sin\theta}$
$\dfrac{\frac{\cos\theta\sin\theta + \cos\theta}{\sin\theta}}{\frac{1+\sin\theta}{\cos\theta}}$	$= \frac{\cos^2\theta}{\sin\theta}$

LS

$\frac{\cos\theta\sin\theta + \cos\theta}{\sin\theta} \times \frac{\cos\theta}{1+\sin\theta}$

$\frac{\cos\theta(\cos\theta)\cancel{(\sin\theta+1)}}{\sin\theta\cancel{(\sin\theta+1)}}$

$= \frac{\cos^2\theta}{\sin\theta}$ $\quad$ LS = RS

Non permissible values.

$\theta \neq 0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$

because $\sin\theta \neq 0$ and $\cos\theta \neq 0$

b) $\sec\theta+\tan\theta=\frac{\cos\theta}{1-\sin\theta}$

LS	RS
$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$	$\frac{\cos\theta}{1-\sin\theta}$
$\frac{1+\sin\theta}{\cos\theta}$	

c) Prove the identity sin2x= 2sinx cosx

LS	RS
$\sin 2x$ $= \sin(x+x)$ $= \sin x\cos x + \cos x \sin x$ $= 2\sin x \cos x$	$2\sin x \cos x$

LS=RS

Example 12: Determine the exact value for each expression.

↳ special angles

$\frac{\pi}{3} = \frac{4\pi}{12}$ $\quad \frac{\pi}{4} = \frac{3\pi}{12}$ $\quad \frac{\pi}{6} = \frac{2\pi}{12}$

a) $\sin\dfrac{\pi}{12}$

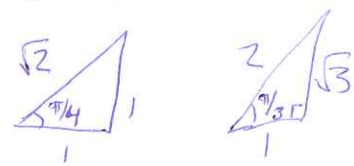
I could use $\frac{\pi}{3} - \frac{\pi}{4}$ OR $\frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$

$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$



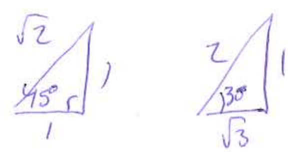
b) cos 75° $= \cos(45° + 30°)$ $\qquad 30°, 60°, 45°$

$= \cos 45° \cos 30° - \sin 45° \sin 30°$

$\frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$



Practice:

9.

The exact value of sin 75° can be determined using the expression

$\sin(45° + 30°)$

A. $\sin 90° - \sin 15°$
B. $\sin 45° + \sin 30°$
C. $\sin 45°\cos 30° + \cos 45°\sin 30°$
D. $\cos 45°\cos 30° + \cos 45°\cos 30°$

10.

The expression $\cot^2 x + \csc x - 4$ is equivalent to

A. $\csc^2 x + \csc x - 5$
B. $\csc^2 x + \csc x - 3$
C. $\dfrac{\cos^2 x}{\sin^2 x} + \dfrac{1}{\cos x} - 4$
D. -4

$\left(\frac{\cos x}{\sin x}\right)^2 + \frac{1}{\sin x} - 4$

$\frac{\cos^2 x}{\sin^2 x} + \frac{\sin x}{\sin^2 x} - \frac{4\sin^2 x}{\sin^2 x}$

$\frac{\cos^2 x + \sin x - 4\sin^2 x}{\sin^2 x}$

$\frac{1 - \sin^2 x + \sin x - 4\sin^2 x}{\sin^2 x}$

$= \frac{-5\sin^2 x + \sin x + 1}{\sin^2 x}$

put into graphing calculator. Find which graphs are equal

11.

The expression $\sqrt{\dfrac{1+\tan^2 x}{1-\sin^2 x}}$, is equivalent to

A. $\sqrt{\dfrac{(1+\tan x)(1-\tan x)}{(1+\sin x)(1-\sin x)}}$
B. 1
C. $\sec x$
D. $\sec^2 x$

tech solution

$y_1 = \sqrt{\frac{1+\tan^2 x}{1-\sin^2 x}}$

$y_2 =$ answers

$\sqrt{\frac{\sec^2 x}{\cos^2 x}}$

$= \frac{\sec x}{\cos x}$

$= \frac{1}{\cos x} \times \frac{1}{\cos x}$

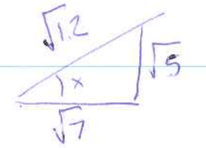
$= \frac{1}{\cos^2 x} = \sec^2 x$

12.

If $\tan^2 x = \frac{5}{7}$, then $\sec^2 x$ is equivalent to

A. $\frac{12}{7}$

$\tan x = \pm \frac{\sqrt{5}}{\sqrt{7}}$ $\frac{opp}{adj}$



B. $\frac{7}{5}$

$\sec^2 x = \left(\frac{\sqrt{12}}{\sqrt{7}}\right)^2 = \frac{12}{7}$

C. $\frac{5\sqrt{74}}{74}$

D. $\frac{\sqrt{74}}{7}$

13.

The expression $\cos^2(4\pi) - \sin^2(4\pi)$ is equivalent to

$1 - \sin^2(4\pi) - \sin^2(4\pi)$

$= 1 - 2\sin^2(4\pi)$

$= \cos(2(4\pi))$

$= \cos 8\pi$

$\cos^2 4\pi - (1 - \cos^2(4\pi))$

$= \cos^2(4\pi) + \cos^2 4\pi - 1$

$2\cos^2(4\pi) - 1$

A. $\cos^2(4\pi)$

B. $\sin^2(8\pi)$

C. $\cos(8\pi)$

D. $\cos(4\pi)\sin(4\pi)$

Use the following information to answer the next question.

The steps used by a student to simplify the expression $(\sin x + \cos x)^2$ are shown below

Step 1: $\sin^2 x + \cos^2 x$ $(\sin x + \cos x)(\sin x + \cos x)$

Step 2: $\sin^2 x + (1 - \sin^2 x)$

Step 3: $(1 - \cos^2 x) + (1 - \sin^2 x)$

Step 4: $2 - \sin^2 x - \cos^2 x$

Numerical Response

3. The step which contains a mathematical error is step 1.

14.

The expression $\dfrac{\sin x}{\tan x}+\dfrac{1}{\sec x}$ is equivalent to

A. $2\cos x$

B. $2\sec x$

C. $\dfrac{\sin x+1}{\tan x+\sec x}$

D. $\dfrac{\sin x}{\tan x \sec x}$

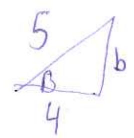
$\dfrac{\sin x}{\frac{\sin x}{\cos x}} + \cos x$

$\sin x \cdot \dfrac{\cos x}{\sin x} + \cos x$

$= 2\cos x$

15.

Given $\sin A=\dfrac{7}{8}$ and $\cos B=\dfrac{4}{5}$, where A and B are acute angles, the value of $\cos(A-B)$ is equal to



A. $\dfrac{3}{8}$

B. $\dfrac{16}{25}+\dfrac{49}{64}$

C. $\dfrac{4\sqrt{15}+21}{40}$

D. $\dfrac{28-3\sqrt{15}}{40}$

$8^2-7^2=a^2$ $\quad$ $5^2-4^2=b^2$

$a=\sqrt{15}$ $\quad$ $b=3$

$\cos A-B = \cos A\cos B + \sin A \sin B$

$\dfrac{\sqrt{15}}{8}\left(\dfrac{4}{5}\right)+\left(\dfrac{7}{8}\right)\left(\dfrac{3}{5}\right)$

$\dfrac{4\sqrt{15}}{40}+\dfrac{21}{40} = \dfrac{4\sqrt{15}+21}{40}$

16.

The expression $\sin\left(\dfrac{\theta}{5}\right)\cos\left(\dfrac{2\theta}{7}\right)-\cos\left(\dfrac{\theta}{5}\right)\sin\left(\dfrac{2\theta}{7}\right)$ is equivalent to

A. $\cos\left(\dfrac{17\theta}{35}\right)$

B. $\sin\left(\dfrac{17\theta}{35}\right)$

C. $\sin\left(\dfrac{3\theta}{35}\right)$

D. $\sin\left(\dfrac{-3\theta}{35}\right)$

$\sin\left(\dfrac{\theta}{5}-\dfrac{2\theta}{7}\right) = \sin\left(\dfrac{7\theta}{5(7)}-\dfrac{10\theta}{(5)(7)}\right)$

$= \sin\left(\dfrac{-3\theta}{35}\right)$

17.

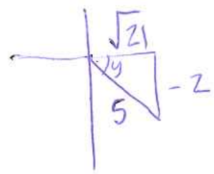
If $\tan x = -\dfrac{6}{7}$ and $\sin y = -\dfrac{2}{5}$, the exact value of $\sec(x+y)$, given that $\dfrac{3\pi}{2} \le x < 2\pi$, $\dfrac{3\pi}{2} \le y < 2\pi$, is

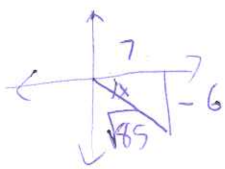
$\dfrac{1}{\cos(x+y)}$

$= \dfrac{1}{\cos x\cos y - \sin x \sin y}$

$= \dfrac{1}{\left(\frac{7}{\sqrt{85}}\right)\left(\frac{\sqrt{21}}{5}\right) - \left(\frac{-6}{\sqrt{85}}\right)\left(\frac{-2}{5}\right)}$

$\dfrac{1}{\frac{7\sqrt{21}}{5\sqrt{85}} - \frac{12}{5\sqrt{85}}} =$





A. $\sqrt{21} - 5$
B. $5 - \sqrt{21}$
C. $\dfrac{7\sqrt{21}}{12\sqrt{85} - 5}$
D. $\dfrac{5\sqrt{85}}{7\sqrt{21} - 12}$

18.

The expression $\dfrac{1+\cot\theta}{\csc\theta}$ is equivalent to

A. $\sin\theta + \cos\theta$

B. $\dfrac{\sin\theta + \cos\theta}{\sin\theta}$

C. $\dfrac{1+\cos\theta}{\sin\theta}$

D. $1 + \cos\theta$

$\dfrac{1 + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$

$= \dfrac{\sin\theta + \cos\theta}{\cancel{\sin\theta}} \times \dfrac{\cancel{\sin\theta}}{1}$

$= \sin\theta + \cos\theta$ $\quad \sin\theta \neq 0$

19.

To create an identity (a statement that is true for all x in the domain) for the equation $\cos^2 x(1 + \cot^2 x) = A$, the value of A would need to be

A. $\sin^2 x$
B. $\cos^2 x$
C. $\cot^2 x$
D. $\sec^2 x$

$\cos^2 x\left(1 + \dfrac{\cos^2 x}{\sin^2 x}\right)$

$\cos^2 x + \dfrac{\cos^2 x \cos^2 x}{\sin^2 x}$

$\dfrac{\cos^2 x \sin^2 x + \cos^2 x \cos^2 x}{\sin^2 x}$

$= \dfrac{\cos^2 x(\sin^2 x + \cos^2 x)}{\sin^2 x}$

$= \dfrac{\cos^2 x}{\sin^2 x} = \cot^2 x$

20.

The value of $\cos^2\left(\frac{\pi}{6}\right) - \sin^2\left(\frac{\pi}{6}\right)$ is equal to the value of

A. $\cos\frac{\pi}{12}$

B. $\cos\frac{\pi}{6}$

C. $\cos\frac{\pi}{3}$

D. $\cos\frac{\pi}{2}$

$\cos^2\left(\frac{\pi}{6}\right) - \left(1 - \cos^2\left(\pi/6\right)\right)$

$= \cos^2(\pi/6) - 1 + \cos^2(\pi/6)$

$= 2\cos^2(\pi/6) - 1$

$= \cos(2(\pi/6))$

$= \cos \pi/3$