Description of the Pandemic Game

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1 Two Players' Strategies

The government provides a suggested level of social distance μ to the general public, i.e., from 1 to 10. However, this level of social distance is just a suggestion rather than command. The general public as a whole can choose a standard deviation σ from 1 to 5 such that the distribution of the general public in social distancing level follows $N(\mu, \sigma^2)$. Then the situation can be modelled by a two-person non-zero-sum game whose payoff matrix is of size 10×5. Both players' payoff will be related to the distribution of the social distance $N(\mu, \sigma^2)$, which is jointly determined by the government and the general public. We treat the case in which $\sigma = 1$ as "obedient", the case in which $\sigma = 3$ as "neutral" and the case in which $\sigma = 5$ as "disobedient".

2 Game Setting

The game is categorized into three modes,i.e., easy, medium and hard. Each mode corresponds to the difficulty level of virus the player is going to face. Viruses of different difficulty levels have different parameters, such as infection rate and recovery rate, etc. Moreover, we simplify general public's strategies into three cases, i.e., standard deviation equals 1, 3 or 5. The system will randomly choose a number from 1, 3, and 5 to simulate the decision made by your opponent (general public).

3 Notations

Base infection rate of the virus:

$$\beta_{base}$$

Infection rate of the virus adjusted by $N(\mu, \sigma^2)$:

$$\beta_{real}$$

Recovery rate:

$$\gamma$$

Discount factor function:

$$g$$

Average discount factor:

$$\bar{g}$$

Probability density function of $S_d \sim N(\mu, \sigma^2)$:

$$f_{s_d}$$

Duration of the epidemic:

$$T$$

A row vector of the proportion of the infected population recorded by day:

$$i$$

4 Formula

1. Effect of social distancing on the infection rate of the virus:

$$g = \frac{1}{1+s_d^2}$$

$$\bar{g} = \int_0^{10} \frac{1}{1+s_d^2} f_{S_d}(s_d) ds_d$$

$$\beta_{real} = \bar{g}\beta_{base}$$

2. Utility of the general public(Average utility per day during the period of the epidemic):

$$u = \int_0^{10} (\frac{10}{s_d} - (\frac{1}{s_d})^2)^{0.25} f_{Sd}(s_d) ds_d - 0.05\sigma - \frac{1}{T}\sum_{k=1}^{T} 1.826 i[k]$$

3. Utility of Government(Average percentage change of GDP per quarter):

$$\Delta GDP = 100\Big(\prod_{k=1}^{T} \frac{100 - 1.8 - 2.877 i[k] + \frac{27.66}{(2.8+s_d)^2}}{100}\Big)^{\frac{1}{T}} - 100$$

5 Epidemic Model

We used the famous SIR model to simulate the development of the epidemic in our project. Since we are using discrete time unit (day) in this project, we applied the explicit Euler's method to approximate the record of the proportion of the Susceptible population, the infectious population and the recovered population. Please refer to our script "game_theory.py" for more detailed algorithms.

6 Results and Discussions

6.1 Payoff Matrices and Equilibrium Pairs

$$M_{Hard} = \begin{bmatrix} (-1.03, 0.608) & (-4.303, 0.9) & (-5.844, 1.042) & (-6.625, 1.025) & (-7.085, 0.833) \\ (-0.6, 0.978) & (-3.207, 1.276) & (-5.596, 1.368) & (-6.787, 1.3) & (-7.49, 1.058) \\ (-0.978, 0.981) & (-1.55, 1.497) & (-4.538, 1.607) & (-6.421, 1.505) & (-7.387, 1.224) \\ (-1.202, 0.859) & (-1.202, 1.523) & (-2.959, 1.734) & (-5.518, 1.624) & (-6.995, 1.32) \\ (-1.346, 0.757) & (-1.346, 1.427) & (-1.677, 1.735) & (-4.204, 1.649) & (-6.27, 1.34) \\ (-1.443, 0.683) & (-1.443, 1.284) & (-1.443, 1.611) & (-2.715, 1.58) & (-5.211, 1.286) \\ (-1.512, 0.626) & (-1.512, 1.118) & (-1.512, 1.4) & (-1.619, 1.42) & (-3.908, 1.165) \\ (-1.563, 0.57) & (-1.563, 0.921) & (-1.563, 1.135) & (-1.563, 1.174) & (-2.606, 0.986) \\ (-1.602, 0.458) & (-1.602, 0.682) & (-1.602, 0.84) & (-1.602, 0.887) & (-1.686, 0.756) \\ (-1.632, 0.244) & (-1.632, 0.419) & (-1.632, 0.541) & (-1.632, 0.585) & (-1.632, 0.484) \end{bmatrix}$$

The equilibrium pairs are (I_6, II_3), (I_8, II_4) and $(\mathbf{X}, \mathbf{Y})$ where $\mathbf{X} = [0, 0, 0, 0, 0, 0.39, 0.61, 0, 0, 0]$ and $\mathbf{Y} = [0, 0, 0.94, 0.06, 0]$.

$$M_{Medium} = \begin{bmatrix} (0.115, 0.631) & (-1.398, 0.952) & (-3.351, 1.081) & (-4.657, 1.052) & (-5.599, 0.85) \\ (-0.6, 0.978) & (-0.833, 1.322) & (-2.76, 1.417) & (-4.492, 1.334) & (-5.656, 1.082) \\ (-0.978, 0.981) & (-0.978, 1.509) & (-1.745, 1.659) & (-3.67, 1.55) & (-5.231, 1.255) \\ (-1.202, 0.859) & (-1.202, 1.523) & (-1.202, 1.769) & (-2.619, 1.676) & (-4.411, 1.361) \\ (-1.346, 0.757) & (-1.346, 1.427) & (-1.346, 1.742) & (-1.651, 1.698) & (-3.399, 1.39) \\ (-1.443, 0.683) & (-1.443, 1.284) & (-1.443, 1.611) & (-1.443, 1.606) & (-2.361, 1.339) \\ (-1.512, 0.626) & (-1.512, 1.118) & (-1.512, 1.4) & (-1.512, 1.422) & (-1.674, 1.209) \\ (-1.563, 0.57) & (-1.563, 0.921) & (-1.563, 1.135) & (-1.563, 1.174) & (-1.563, 1.007) \\ (-1.602, 0.458) & (-1.602, 0.682) & (-1.602, 0.84) & (-1.602, 0.887) & (-1.602, 0.758) \\ (-1.632, 0.244) & (-1.632, 0.419) & (-1.632, 0.541) & (-1.632, 0.585) & (-1.632, 0.484) \end{bmatrix}$$

The only equilibrium pair is (I_4, II_3).

$$M_{Easy} = \begin{bmatrix} (0.115, 0.631) & (0.115, 0.982) & (0.115, 1.147) & (0.115, 1.139) & (-0.072, 0.948) \\ (-0.6, 0.978) & (-0.6, 1.326) & (-0.6, 1.459) & (-0.6, 1.408) & (-0.6, 1.174) \\ (-0.978, 0.981) & (-0.978, 1.509) & (-0.978, 1.674) & (-0.978, 1.602) & (-0.978, 1.335) \\ (-1.202, 0.859) & (-1.202, 1.523) & (-1.202, 1.769) & (-1.202, 1.704) & (-1.202, 1.422) \\ (-1.346, 0.757) & (-1.346, 1.427) & (-1.346, 1.742) & (-1.346, 1.704) & (-1.346, 1.43) \\ (-1.443, 0.683) & (-1.443, 1.284) & (-1.443, 1.611) & (-1.443, 1.606) & (-1.443, 1.357) \\ (-1.512, 0.626) & (-1.512, 1.118) & (-1.512, 1.4) & (-1.512, 1.422) & (-1.512, 1.212) \\ (-1.563, 0.57) & (-1.563, 0.921) & (-1.563, 1.135) & (-1.563, 1.174) & (-1.563, 1.007) \\ (-1.602, 0.458) & (-1.602, 0.682) & (-1.602, 0.84) & (-1.602, 0.887) & (-1.602, 0.758) \\ (-1.632, 0.244) & (-1.632, 0.419) & (-1.632, 0.541) & (-1.632, 0.585) & (-1.632, 0.484) \end{bmatrix}$$

The only equilibrium pair is (I_1, II_3)

From the equilibrium pairs of the three types of viruses, we can see that for the strong virus, the smallest μ that is worthwhile is 6, while that is 4 in the medium case and 1 in the weak case. Hence we may conclude that generally, the government should set a larger μ for a more fierce virus which is consistent with the common sense.

For the weak case, we can see that II_5 is strictly dominated by II_4, and after deleting II_5, I_1 becomes a dominant strategy for the government which implies that when the virus is weak, for the sake of maximizing economic growth, the government should just relax and there's no need to impose restrictions to people.

6.2 Duration matrices

We count the duration when the public play strategies are 1, 3, 5. Denote the matrices by D

$$D_{Hard} = \begin{bmatrix} 260 & 91 & 81 \\ 61 & 103 & 83 \\ 20 & 130 & 87 \\ 15 & 203 & 93 \\ 14 & 365 & 104 \\ 14 & 46 & 125 \\ 13 & 23 & 168 \\ 13 & 17 & 274 \\ 13 & 15 & 365 \\ 13 & 14 & 59 \end{bmatrix}$$

$$D_{Medium} = \begin{bmatrix} 55 & 93 & 66 \\ 15 & 126 & 72 \\ 11 & 228 & 81 \\ 10 & 271 & 98 \\ 9 & 24 & 130 \\ 9 & 15 & 207 \\ 9 & 12 & 365 \\ 9 & 10 & 49 \\ 9 & 10 & 22 \\ 9 & 10 & 15 \end{bmatrix}$$

$$D_{Easy} \begin{bmatrix} 9 & 24 & 365 \\ 8 & 16 & 365 \\ 8 & 12 & 40 \\ 7 & 10 & 22 \\ 7 & 9 & 15 \\ 7 & 8 & 12 \\ 7 & 8 & 10 \\ 7 & 8 & 9 \\ 7 & 7 & 9 \\ 7 & 7 & 8 \end{bmatrix}$$

From the duration matrices we observe something interesting. In the case of strong virus, if the government plays I_1, then, the bigger σ that general public chooses, the shorter the duration is, which is actually the opposite from most of other rows.

We may interpret this by considering that choosing $\mu = 1$ is too small for a strong virus. Increasing σ when $\mu = 1$ can lead to higher level of social distancing of the general public which is conducive to the control of the epidemic. This explains the strange phenomenon.

7 Nash Solvablility

From the Nash equilibrium pairs above, we can observe that the game is Nash solvable in the weak virus and medium virus cases since both have only one Nash equilibrium pair.

However, the game becomes not solvable in the strong virus case since neither pure equilibrium pair is interchangeable with the mixed equilibrium pair.

8 Other Explanations

8.1 Time limit

For the SIR model, if the stopping condition is when the proportion of recovered people converges, then the epidemic may last for thousands of days in some cases. Thus, we set an upper bound, 365 days, which presumes that vaccines can come out in one year, as the maximum number of days in the SIR model.

8.2 Special cases when T suddenly changes to 365

We may observe from the duration matrix D_{strong} that there are some sudden changes in the duration of the epidemic. For example, note that both the (5,1)-th entry and the (5,3)-th entry are much less than 365 days but the (5,2)-th entry suddenly increased to 365 days, which is the time limit set in section 7.1.

The reason is as follows. Since the recovery rate γ remains unchanged for different μ and σ, the sudden change must be attributed to the infection rate β_{real}. If β_{real} is large, then the infected population,i, will increase very fast. But on the other hand, since we assume constant γ, the number of people recovered everyday will also be large because of the large basis, i. As a consequence, the total duration of the epidemic will not be too long.If β_{real} is small, then the virus will extinct in a short period since the infected population,i, will decrease at a fast pace. Hence the only case in which the epidemic will last for a long period is that at some (μ, σ), β_{real} is of the same magnitude as γ, where the infected people will neither increase too fast nor decrease too fast. This explains the sudden change in the duration of the epidemic.

9 Bibliography

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