Simplified CreditMetrics^{TM} Model Using Monte Carlo Simulation

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About

CreditMetrics™ is a widely used framework developed by J.P. Morgan & Co. in 1997 for assessing potential credit risk in a bond portfolio due to obligor's failure to repay their debts. Due to the model's complex mathematical derivations, it is not sensible to find a closed form formula for a well-diversified bond's portfolio. Thus, this project aims to approximate the Value-at-Risk (VaR) and the Expected Shortfall (ES) of a bond portfolio using Monte Carlo simulation, in order to provide a quantified assessment of a bond portfolio's risk profile.

Project GitHub Link: https://github.com/jamesckcc/CreditMetrics

Theoretical Background

One Bond Case:

The model is based on the analysis of migration in the credit rating transition matrix. Let's say there are seven rating categories (AAA, AA, A, BBB, BB, B, CCC). For a single BBB rated bond, in one-years' time, it's credit rating could either upgrade to AAA, AA, A, retain its rating (BBB), downgrade to BB, B, CCC, or Default. Assuming that the One-year transition matrix is known (See *Table 1.1*).

Table 1.1

One-year transition matrix (%)

Initial rating	Rating at year-end (%)							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	90.81	8.33	0.68	0.06	0.12	0	0	0
AA	0.70	90.65	7.79	0.64	0.06	0.14	0.02	0
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.95	86.93	5.30	1.17	0.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
B	0	0.11	0.24	0.43	6.48	83.46	4.07	5.20
CCC	0.22	0	0.22	1.30	2.38	11.24	64.86	19.79

Source: Standard & Poor's CreditWeek (15 April 96)

Then the credit rating migration probability for the BBB Rated Bond could be obtained, as shown in *Table 1.2.*

Table 1.2

Probability of credit rating migrations in one year for a BBB

Year-end rating	Probability (%)
AAA	0.02
AA	0.33
A	5.95
BBB	86.93
BB	5.30
B	1.17
CCC	0.12
Default	0.18

Since the value of a bond is directly related to the counterparty's credibility, the bond's value in one-years' time would be different if it falls to different rating categories. Consider we have also obtained the one-year forward zero curves by credit rating category (*Table 1.3)*, then we could find the value of the bond given what category it falls to after one-year.

Table 1.3

Example one-year forward zero curves by credit rating category (%)

Category	Year 1	Year 2	Year 3	Year 4
AAA	3.60	4.17	4.73	5.12
AA	3.65	4.22	4.78	5.17
A	3.72	4.32	4.93	5.32
BBB	4.10	4.67	5.25	5.63
BB	5.55	6.02	6.78	7.27
B	6.05	7.02	8.03	8.52
CCC	15.05	15.02	14.03	13.52

Let's say the BBB-rated bond have the following attributes:

- Face Value = 100
- Maturity = 5 years
- Coupon Rate = 6%
- Senior Secured Class

If the bond upgrades to A, the value in one year could be calculated using discounted cashflow as follow:

$$V_{BBB} = 6 + \frac{6}{1.0372} + \frac{6}{(1.0432)^2} + \frac{6}{(1.0493)^3} + \frac{106}{(1.0532)^2} = 108.66$$

By using this method, we can then find the value of the bond if it falls into other rating categories. Further assume we obtained the recovery rates given default by seniority class in *Table 1.4*, then we would have the distribution of bond value in one year's time, *Table 1.5*.

Table 1.4

Recovery rates by seniority class (% of face value, i.e., "par")

Seniority Class	Mean (%)	Standard Deviation (%)
Senior Secured	53.80	26.86
Senior Unsecured	51.13	25.45
Senior Subordinated	38.52	23.81
Subordinated	32.74	20.18
Junior Subordinated	17.09	10.90

Source: Carty & Lieberman [96a] —Moody's Investors Service

Table 1.5

Distribution of value of a BBB par bond in one year

Year-end rating	Value ($)	Probability (%)
AAA	109.37	0.02
AA	109.19	0.33
A	108.66	5.95
BBB	107.55	86.93
BB	102.02	5.30
B	98.10	1.17
CCC	83.64	0.12
Default	51.13	0.18

Then the mean and the standard deviation of the portfolio would be defined as following,

$$\mu = \sum p_i * \mu_i \ = 107.09$$

$$\sigma = \sqrt{\sum p_i * \left(\mu_i^2 + \sigma_i^2\right) - \mu^2} = 3.18$$

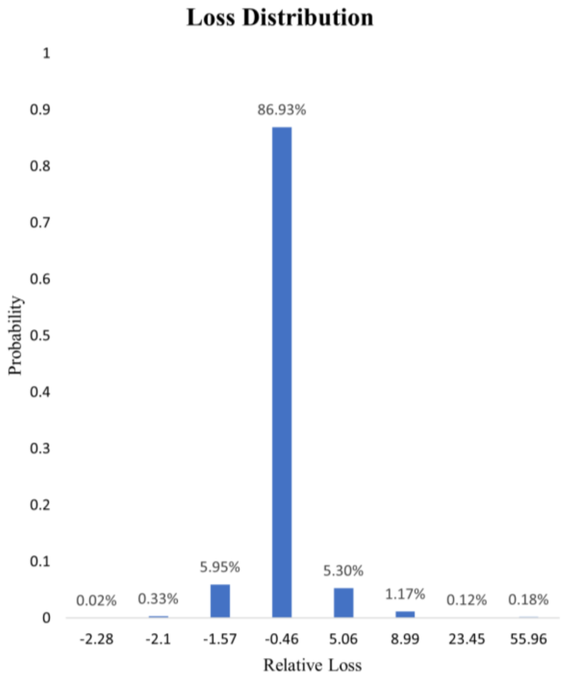
Note that the standard deviation added a component σ_i^2 to include the uncertainty in the bond value. In this case, it is only applicable to $i = 8$, the defaulted state, as this is the only situation where the value is uncertain. This result is proven in the original technical document.

By setting up the relative loss distribution as

$$\begin{aligned} Relative\ Loss &= Expected\ Portfolio\ Value - Portfolio\ Value \\ &= 107.09 - Portfolio\ Value \end{aligned}$$

We obtained *Chart 1.1:*

Chart 1.1



The 95% relative Value-at-Risk (VaR) of the portfolio would be found to be \$5.06.

The Expected Shortfall (ES) is:

$$E[L|L > 95\%\ rel.VaR] = 8.25$$

Two Bonds Case:

In order to construct the portoflio's value distribution, the joint migration probability is to be considered. Since it is impossible to obtain the real joint probaility density function of the two bonds credit migration, CreditMetrics™ proposed a clever way to estimate the distribution, by using normal approximation, that could also capture the correlation between bonds.

To make an example, assume the portfolio consist of the following two bonds with the same weight, and have a correlation $\rho = 0.3$ between them.

Bond 1

- BBB-rated
- Face Value = 100
- Maturity = 5 years
- Coupon Rate = 6%
- Senior Secured Class

Bond 2

- A-rated
- Face Value = 100
- Maturity = 5 years
- Coupon Rate = 6%
- Senior Secured Class

Bond 1, the BBB-rated bond, by using normal approximation, would have this marginal value distribution:

Graph 1.1

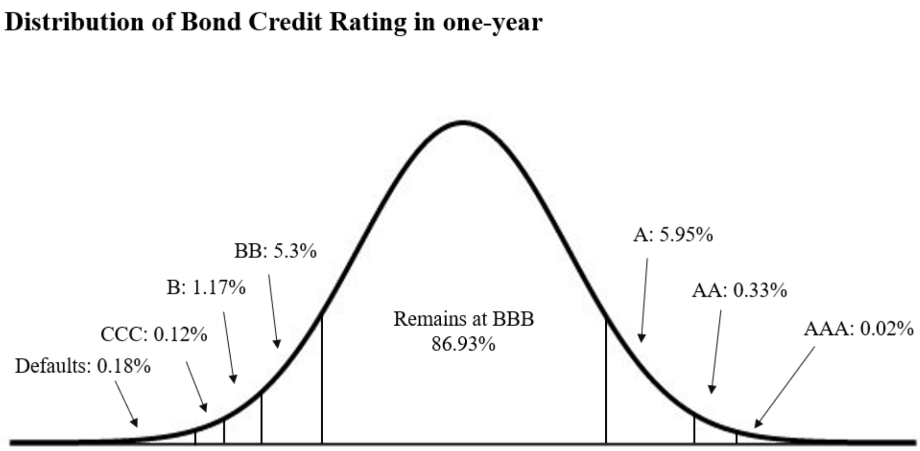


Table 1.6

Rating	Probability	Z-value range	
		Min	Max
AAA	0.02%	3.54	inf
AA	0.33%	2.70	3.54
A	5.95%	1.53	2.70
BBB	86.93%	-1.49	1.53
BB	5.3%	-2.18	-1.49
B	1.17%	-2.74	-2.18
CCC	0.12%	-2.91	-2.75
Default	0.18%	-inf	-2.91

We could validate the result mathematically, take the bond remaning at BBB as an example:

$$Pr(\text{Remains at BBB}) = \int_{-1.49}^{1.53} \frac{1}{\sqrt{2\pi}} exp(-\frac{x^2}{2})dx = 86.93\%$$

Also consider Bond 2, an A-rated bond:

Table 1.7

Rating	Probability	Z-value range	
		Min	Max
AAA	0.09%	3.12	inf
AA	2.27%	1.98	3.12
A	91.05%	-1.51	1.98
BBB	5.52%	-2.30	-1.51
BB	0.74%	-2.72	-2.30
B	0.26%	-3.19	-2.72
CCC	0.01%	-3.24	-3.19
Default	0.06%	-inf	-3.24

Assuming joint normality, we have the joint pdf:

$$f(x_1, x_2; \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} exp\left\{-\frac{1}{2(1-\rho^2)}[x_1^2 - 2\rho x_1 x_2 + x_2^2]\right\}$$

In this portfolio, it would be:

$$f(x_1, x_2; 0.3) = \frac{1}{2\pi\sqrt{1-0.3^2}} exp\left\{-\frac{1}{2(1-0.3^2)}[x_1^2 - 2 * 0.3 x_1 x_2 + x_2^2]\right\}$$

Then for example, the probability that both bonds retains their rating is

$$\int_{-1.51}^{1.98}\int_{-1.49}^{1.53}\frac{1}{2\pi\sqrt{1-0.3^2}}exp\left\{-\frac{1}{2(1-0.3^2)}[x_1^2-2*0.3x_1x_2+x_2^2]\right\}dx_1dx_2=79.69\%$$

By doing so for the remaning 63 possibilities, we would obtain the joint migration probabilites matrix, as shown in *Table 1.8.*

Table 1.8

Joint migration probabilities with 0.30 asset correlation (%)

Obligor #1 (BBB)		**Obligor #2 (single-A)**							
		AAA	AA	A	BBB	BB	B	CCC	Default
		0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
AAA	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00
AA	0.33	0.00	0.04	0.29	0.00	0.00	0.00	0.00	0.00
A	5.95	0.02	0.39	5.44	0.08	0.01	0.00	0.00	0.00
BBB	86.93	0.07	1.81	79.69	4.55	0.57	0.19	0.01	0.04
BB	5.30	0.00	0.02	4.47	0.64	0.11	0.04	0.00	0.01
B	1.17	0.00	0.00	0.92	0.18	0.04	0.02	0.00	0.00
CCC	0.12	0.00	0.00	0.09	0.02	0.00	0.00	0.00	0.00
Default	0.18	0.00	0.00	0.13	0.04	0.01	0.00	0.00	0.00

By further finding the portfolio value given any of the 64 situations, we could find the portfolio expected one-year value and it's standard deviation and further risk profile:

Table 1.9

All possible 64 year-end values for a two-bond portfolio ($)

Obligor #1 (BBB)		**Obligor #2 (single-A)**							
		AAA	AA	A	BBB	BB	B	CCC	Default
		106.59	106.49	106.30	105.64	103.15	101.39	88.71	51.13
AAA	109.37	215.96	215.86	215.67	215.01	212.52	210.76	198.08	160.50
AA	109.19	215.78	215.68	215.49	214.83	212.34	210.58	197.90	160.32
A	108.66	215.25	215.15	214.96	214.30	211.81	210.05	197.37	159.79
BBB	107.55	214.14	214.04	213.85	213.19	210.70	208.94	196.26	158.68
BB	102.02	208.61	208.51	208.33	207.66	205.17	203.41	190.73	153.15
B	98.10	204.69	204.59	204.40	203.74	201.25	199.49	186.81	149.23
CCC	83.64	190.23	190.13	189.94	189.28	186.79	185.03	172.35	134.77
Default	51.13	157.72	157.62	157.43	156.77	154.28	152.52	139.84	102.26

$$\mu=\sum_{i=1}^{64}p_i*\mu_i=213.63$$

$$\sigma=\sqrt{\sum_{i=1}^{64}p_i*(\mu_i^2+\sigma_i^2)-\mu^2}\approx 3.69$$

$$95\%\ Rel.VaR\approx 4.905$$

$$ES\approx 12.20$$

Three or more bonds case:

For three or more bonds, the calculations procedures started to be complex. We will have to find out all of the possible rating combination's price and probability within the bonds.

By normal approximation, the joint pdf of the *n* bonds would be:

$$f(\mathbf{x};\boldsymbol{\Sigma}) = \left[(2\pi)^{n/2}\sqrt{|\boldsymbol{\Sigma}|}\right]^{-1} exp\left[-\frac{1}{2}\mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x}\right]$$

The derivation of the probability of a particular state would be:

$$Pr(\text{Bond 1 rated XYZ } \cap \text{ Bond 2 rated XYZ } \cap \cdots \cap \text{ Bond n rated XYZ})$$
$$= \int_{b_1}^{a_1}\int_{b_2}^{a_2}\cdots\int_{b_n}^{a_n}\left[(2\pi)^{n/2}\sqrt{|\boldsymbol{\Sigma}|}\right]^{-1} exp\left[-\frac{1}{2}\mathbf{x}^T\boldsymbol{\Sigma}\mathbf{x}\right]dx_1dx_2\ldots dx_n$$

Since for each bond there are 8 possible states in our scenario (AAA, AA, A, BBB, BB, B, CCC, Default), for a *n* bonds portfolio, we have to compute 8^n combination's probability and price. For example, the fixed income fund by BlackRock, *iShares iBoxx $ Investment Grade Corporate Bond ETF(LQD)*, consist of over 1,000 investment grades corporate bonds in a single fund. It is absolutely impossible to evaluate all 8^{1000} combinations. Not even for computer programs, since this computation have a time complexity of $O(8^n)$, it is not sensible to derive the risk profile of the portfolio algebarically. Using Monte Carlo simulation to assess the risk profile would serve a better option, which will be discussed next.

Model Implementation

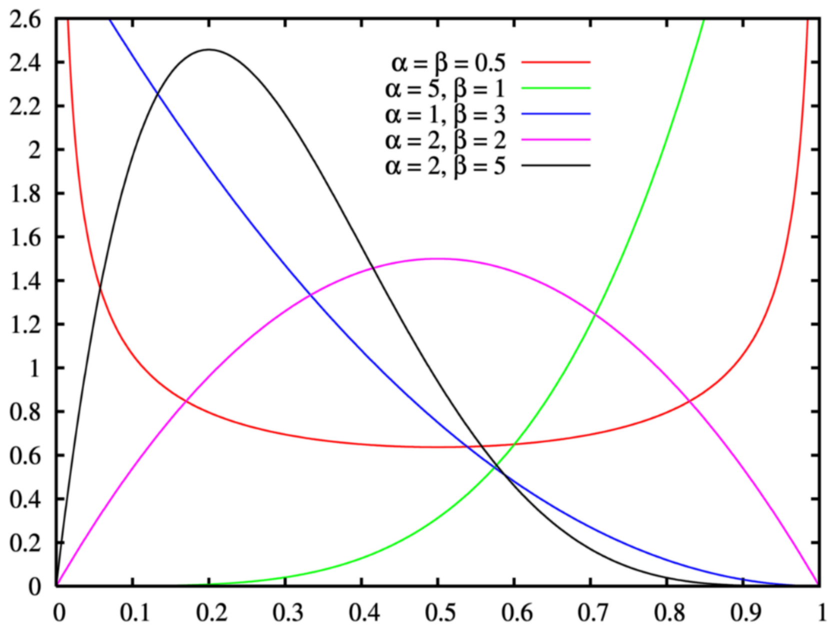
Modelling Recovery Rates

Before delving into the exact implementation of the Monte Carlo simulation in python, we will discuss the way the program models the recovery rates first. The CreditMetrics™ framework did not explicitly mentioned how to model the recovery rate since this is not its main intention. The recovery rate's induced volatility is incorporated in the equation to calculate the portfolio's standard deviation, recall the formula:

$$\sigma = \sqrt{\sum p_i * \left(\mu_i^2 + \sigma_i^2\right) - \mu^2}$$

The σ_i is only non zero iff state i consist at least one of the bonds defaulted. Since for non-defaulted bonds, the exact value of a bond could be found using discounted cash flow method as mentioned earlier. But when a bond defaulted, the Recovery Rate is a random variable with aforementioned mean and standard deviation, without specifying its actual distribution, see *Table 1.4.* As our program intends to also find the ES, the recovery rate distribution when default happened is important for us to assess tail risk. A common probability distribution used to simulate recovery rates is Beta distribution, since it is a flexible, continuous distribution in range (0,1).

Graph 2.1: Different Beta distribution's pdf



The common Beta distribution pdf is defined as follow:

$$f(x;\alpha,\beta) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} & \text{for } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Where $\Gamma(\cdot)$ is the gamma function.

We will have the following attributes after solving it algebarically, which will not be proved entirely here:

For $X \sim Beta(\alpha,\beta)$

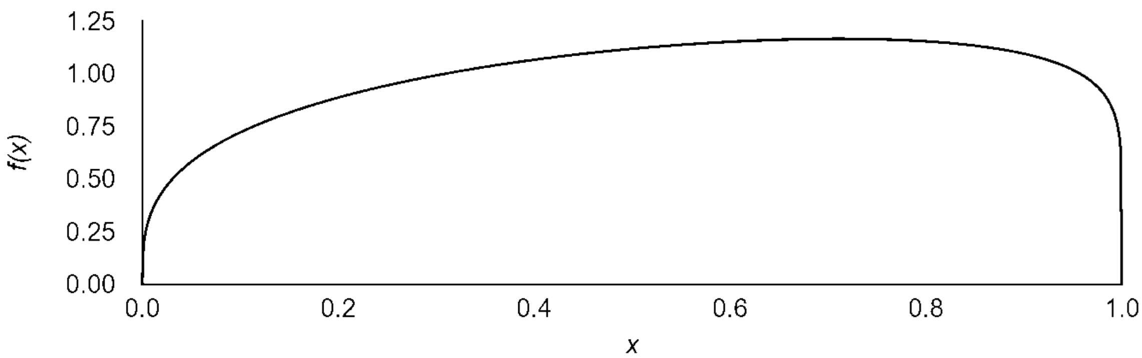
$$E(X) = \frac{\alpha}{\alpha+\beta} \qquad Var(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

If we have the mean and the variance of a Beta Distributed random variable, the maximum likelihood estimator (MLE) of the parameters would be:

$$\alpha = \left(\frac{1-\mu}{\sigma^2} - \frac{1}{\mu}\right)\mu^2 \qquad \beta = \alpha\left(\frac{1}{\mu} - 1\right)$$

For example, the seniority class of Senior Secured have a recovery rate of mean 53.8% and standard deviation of 26.86%, the MLE estimate of the parameters would be $\alpha = 1.3155$, $\beta = 1.1297$.

Graph 2.2: MLE beta distribution of Senior Secured Class's Recovery Rate



The recovery rate modelling methodology will be used throughout the simulation process for all classes of seniority.

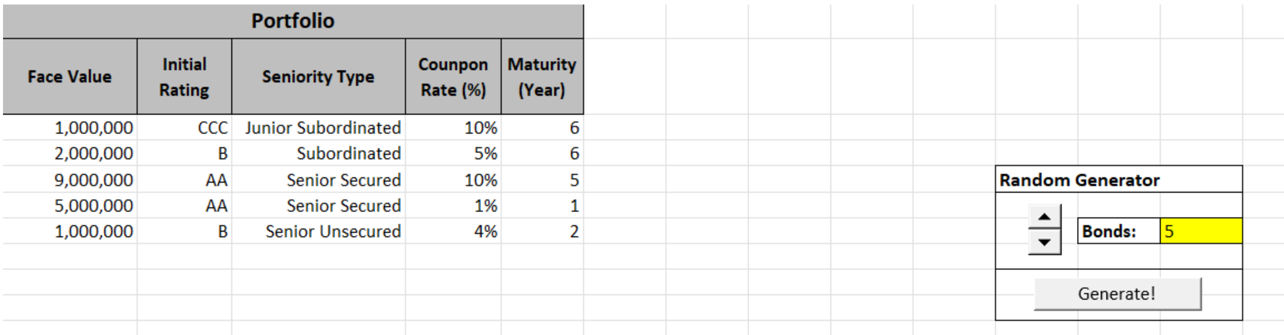
Program Walkthrough

This program utilizes Monte Carlo simulation to model the performance of the entire portfolio. It simulates the behaviour of the portfolio a lot of times, for example 150,000 iterations. The simulated results would be aggregated to get insights of the actual likelihood of different outcomes, instead of computing the exact probabilities.

1. Portfolio

The Excel file *BondPortfolio.xlsm* essentially works as the program's UI. At the first sheet, <Portfolio>, is where users input the essential data for each of the bond in the portfolio, including each bond's Face Value, Initial Rating, Seniority, Coupon Rate and Maturity.

Portfolio Example



Portfolio				
Face Value	Initial Rating	Seniority Type	Counpon Rate (%)	Maturity (Year)
1,000,000	CCC	Junior Subordinated	10%	6
2,000,000	B	Subordinated	5%	6
9,000,000	AA	Senior Secured	10%	5
5,000,000	AA	Senior Secured	1%	1
1,000,000	B	Senior Unsecured	4%	2

In <Portfolio>, for each bond's initial rating, users can only input one of the following values: AAA, AA, A, BBB, BB, B, CCC. For the seniority type, users can only input one of these options: Senior Secured, Senior Unsecured, Senior Subordinated, Subordinated, Junior Subordinated. Users could manually add new bond by inputting data in the next empty row.

To test the model, users can utilize the random generator feature. By adjusting the desired number of bonds and clicking the "Generate!" button, users can create a random portfolio. Although there is no strict limit on the number of bonds that can be added, larger portfolios will take longer to process. (Note: To run the program successfully, please enable macro for *BondPortfolio.xlsm*)

2. Correlation Matrix

After building the portfolio on the first sheet, users should construct the correlation matrix of the bonds on the second sheet titled <Correlation>. Click "Initiate" to generate an n*n empty correlation matrix, where n represents the number of bonds in the Portfolio tab. If the number of bonds in the portfolio changes or if you wish to reset the correlation matrix, click "Initiate" again to create an empty correlation matrix of the proper dimensions.

Users are advised to define the correlation matrix themselves. However, for testing purpose, users could also opt to the “Randomize” button to create a random correlation matrix, upon initiating the correlation board.

Initiated empty correlation matrix board

Correlation Matrix Initiate Randomize

	Bond1	Bond2	Bond3	Bond4	Bond5
Bond1	1	0	0	0	0
Bond2	0	1	0	0	0
Bond3	0	0	1	0	0
Bond4	0	0	0	1	0
Bond5	0	0	0	0	1

Random correlation matrix board Example

	Bond1	Bond2	Bond3	Bond4	Bond5
Bond1	1	0.182	0.149	-0.525	0.264
Bond2	0.182	1	0.374	0.485	0.194
Bond3	0.149	0.374	1	0.17	0.431
Bond4	-0.525	0.485	0.17	1	-0.41
Bond5	0.264	0.194	0.431	-0.41	1

(Note: I have delegated writing the VBA codes to generate a random positive semi-definite correlation matrix to GPT-4.)

3. Running the simulation

Once satisfied with the portfolio’s information, the user could then edit the Transition Matrix, Forward Zero Curve and Recovery Rates by Seniority Class in the sheets <Transition Matrix>, <Forward Zero Curve> and <Seniority> respectively. These tables are already pre-defined, users could skip these sheets if no adjustment is needed.

To run the program, go to the sheet <Run Program>. Here are a few points to check to run the simulation successfully.

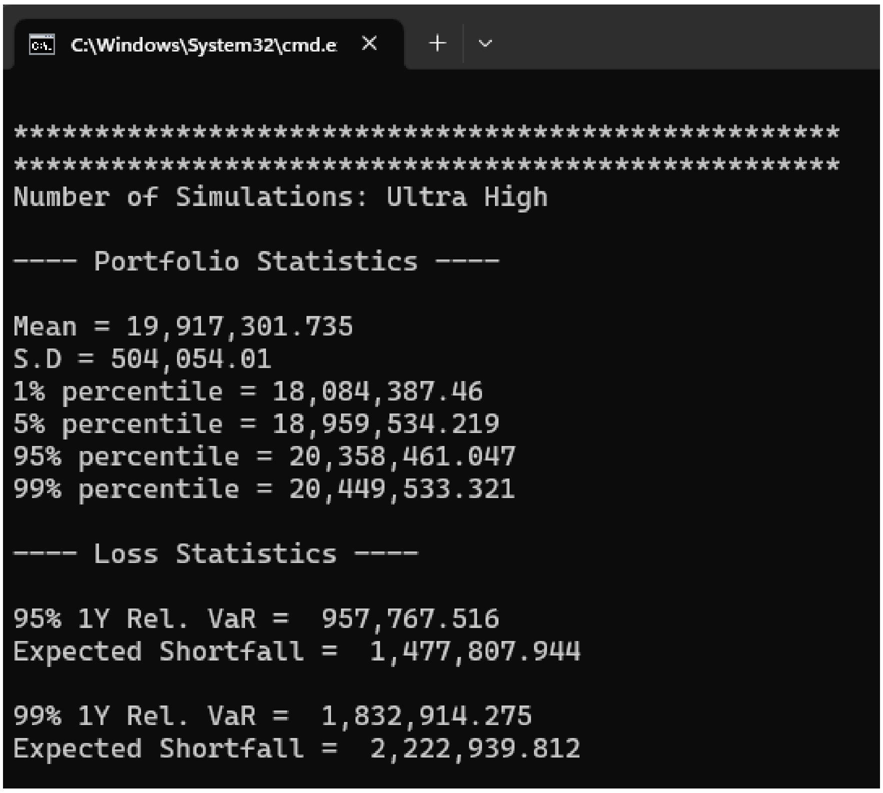
- The Excel file *BondPortfolio.xlsm* and the python program *main.py* must be in the same folder.
- Python version 3.7 or higher must be installed. The following python libraries must be installed: Pandas, NumPy, Matplotlib, SciPy and openpyxl.

After setting up the environment, user should adjust the number of simulation. By default, the level of simulation is normal. In normal, the whole simulation will run through 150,000 iterations. Users could adjust the level as needed. A higher simulation count produces more accurate results but requires more processing time. A lower count reduces processing time but decreases accuracy. After everything is set, click the “Execute Program” button. This button will run the python program at the Command Prompt.

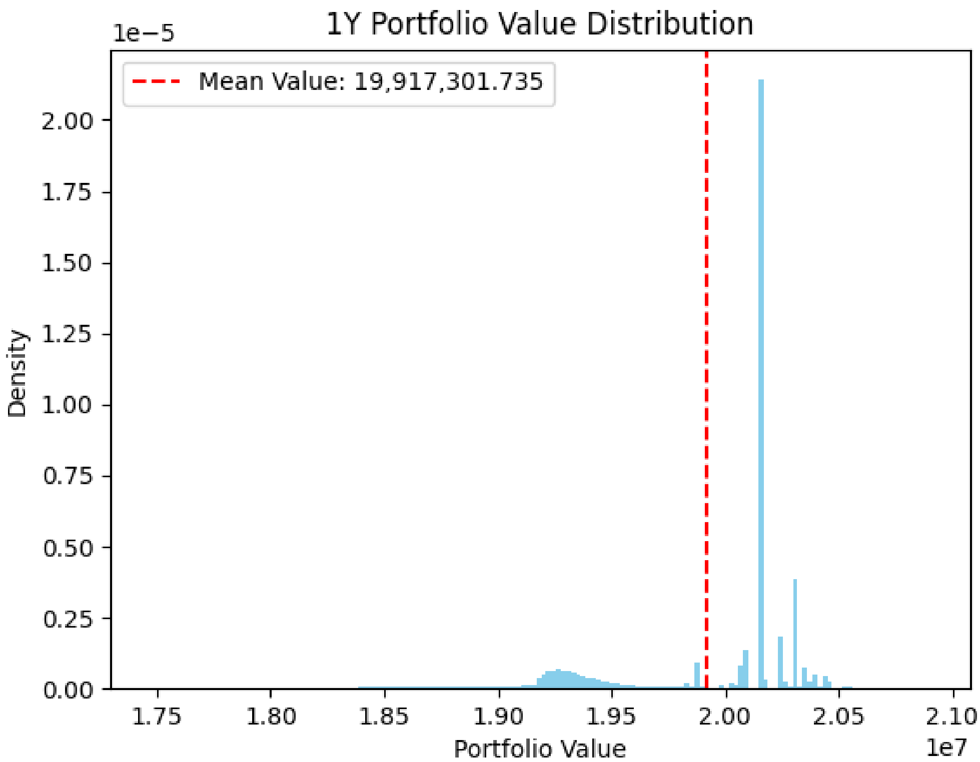
4. Output

There will be two parts of output: Command Prompt, and the matplotlib window. Statistical results of the simulated portfolios, such as portfolio mean, standard deviation, percentiles, VaR and ES are showed in the Command Prompt. While the histogram of the simulated values would be presented in the matplotlib window.

Example of statistical results output in Command Prompt



Example of matplotlib window output of portfolio distribution



Example Portfolio

1. Initiation

Consider this random generated bond portfolio consisting of 24 corporate bonds:

Portfolio Table

Portfolio				
Face Value	**Initial Rating**	**Seniority Type**	**Counpon Rate (%)**	**Maturity (Year)**
5,200,000	AAA	Senior Unsecured	11%	1
7,600,000	AAA	Senior Subordinated	9%	7
7,700,000	A	Subordinated	4%	5
9,600,000	AAA	Senior Subordinated	6%	7
2,200,000	BBB	Junior Subordinated	4%	6
8,600,000	A	Senior Unsecured	5%	3
200,000	BB	Senior Subordinated	9%	5
4,900,000	AAA	Junior Subordinated	8%	2
5,500,000	B	Senior Secured	12%	5
500,000	CCC	Senior Secured	6%	3
1,300,000	BB	Junior Subordinated	12%	5
5,100,000	CCC	Subordinated	10%	6
6,400,000	BB	Subordinated	4%	4
4,000,000	AA	Senior Secured	3%	2
6,800,000	B	Senior Unsecured	4%	6
4,300,000	A	Subordinated	7%	2
8,200,000	B	Junior Subordinated	5%	7
2,400,000	CCC	Senior Secured	3%	7
2,700,000	BB	Senior Unsecured	12%	3
6,400,000	BBB	Subordinated	12%	5
400,000	CCC	Senior Subordinated	1%	5
300,000	CCC	Subordinated	2%	4
1,200,000	B	Junior Subordinated	1%	6
9,100,000	B	Senior Subordinated	5%	3

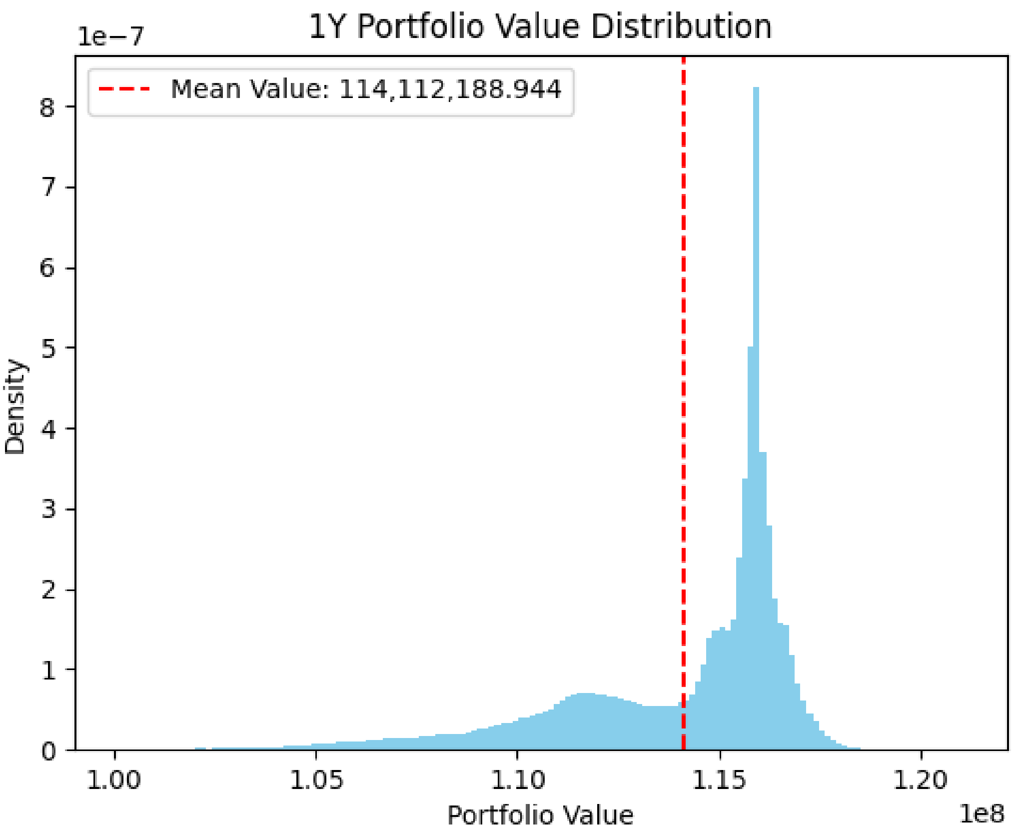
Also consider the following random generated correlation matrix between the bonds:

Portfolio Correlation Matrix

	Bond1	Bond2	Bond3	Bond4	Bond5	Bond6	Bond7	Bond8	Bond9	Bond10	Bond11	Bond12	Bond13	Bond14	Bond15	Bond16	Bond17	Bond18	Bond19	Bond20	Bond21	Bond22	Bond23	Bond24
Bond1	1	0.248	-0.37	0.315	0.436	0.37	-0.39	0.058	0.163	-0.461	0.055	-0.184	0.37	-0.314	0.026	-0.035	-0.112	0.345	-0.139	0.195	-0.046	-0.08	-0.26	0.285
Bond2	0.248	1	0.333	0.431	-0.24	0.486	-0.1	-0.41	-0.42	-0.23	-0.168	-0.052	0.421	0.285	0.049	0.082	0.403	-0.108	-0.097	-0.132	0.28	0.085	0.17	0.339
Bond3	-0.37	0.333	1	0.063	0.145	0.253	0.308	-0.3	-0.27	0.141	0.053	0.068	0.113	0.375	-0.129	0.034	0.13	-0.281	0.189	0.111	0.395	-0.042	-0.063	-0.03
Bond4	0.315	0.431	0.063	1	0.019	0.163	0.193	-0.09	0.065	-0.296	0.001	-0.37	0.428	-0.167	-0.318	0.317	0.084	0.323	0.247	0.27	0.176	-0.151	0.116	0.235
Bond5	0.436	-0.24	0.145	0.019	1	0.478	-0.1	-0.13	0.488	0.112	0.368	-0.31	0.322	-0.441	-0.095	0.095	-0.478	0.354	-0.115	0.212	0.225	-0.293	-0.191	-0.034
Bond6	0.37	0.486	0.253	0.163	0.478	1	-0.41	-0.2	-0.05	-0.119	-0.074	-0.074	0.509	-0.034	0.262	0.361	-0.172	0.014	-0.1	0.035	0.339	-0.032	0.207	0.081
Bond7	-0.39	-0.1	0.308	0.193	-0.1	-0.41	1	-0.41	-0.09	0.477	0.373	0.149	-0.37	0.014	-0.47	-0.158	0.101	-0.005	0.3	-0.076	-0.091	-0.364	-0.133	-0.095
Bond8	0.058	-0.41	-0.3	-0.09	-0.13	-0.2	-0.41	1	0.255	-0.218	-0.105	0.284	-0.024	-0.174	0.282	0.085	0.012	-0.132	0.303	0.393	-0.239	0.307	-0.113	-0.214
Bond9	0.163	-0.42	-0.27	0.065	0.488	-0.05	-0.09	0.255	1	0.256	0.111	-0.216	0.028	-0.553	0.145	0.075	-0.314	0.533	-0.273	0.34	0.07	-0.094	-0.01	-0.188
Bond10	-0.46	-0.23	0.141	-0.3	0.112	-0.12	0.477	-0.22	0.256	1	0.203	0.266	-0.258	-0.01	0.071	-0.038	-0.018	-0.126	0.039	-0.262	0.077	-0.347	0.084	-0.172
Bond11	0.055	-0.17	0.053	0.001	0.368	-0.07	0.373	-0.11	0.111	0.203	1	0.005	0.312	-0.506	-0.326	-0.008	0.035	0.179	0.074	-0.136	-0.23	-0.288	-0.22	0.037
Bond12	-0.18	-0.05	0.068	-0.37	-0.31	-0.07	0.149	0.284	-0.22	0.266	0.005	1	-0.284	0.418	0.418	-0.169	0.33	-0.426	0.301	-0.09	-0.36	0.049	-0.067	-0.094
Bond13	0.37	0.421	0.113	0.428	0.322	0.509	-0.37	-0.02	0.028	-0.258	0.312	-0.284	1	-0.166	-0.111	0.458	0.172	0.15	-0.045	-0.103	0.303	0.111	0.097	0.243
Bond14	-0.31	0.285	0.375	-0.17	-0.44	-0.03	0.014	-0.17	-0.55	-0.01	-0.506	0.418	-0.166	1	0.055	-0.258	0.368	-0.431	0.267	-0.093	0.183	0.21	0.107	0.214
Bond15	0.026	0.049	-0.13	-0.32	-0.1	0.262	-0.47	0.282	0.145	0.071	-0.326	0.418	-0.111	0.055	1	0.223	-0.229	-0.188	-0.355	0.034	-0.305	0.017	0.167	-0.14
Bond16	-0.04	0.082	0.034	0.317	0.095	0.361	-0.16	0.085	0.075	-0.038	-0.008	-0.169	0.458	-0.258	0.223	1	-0.128	0.224	-0.18	0.019	0.023	0.013	0.093	-0.044
Bond17	-0.11	0.403	0.13	0.084	-0.48	-0.17	0.101	0.012	-0.31	-0.018	0.035	0.33	0.172	0.368	-0.229	-0.128	1	0.005	0.257	-0.255	-0.068	0.267	-0.164	0.133
Bond18	0.345	-0.11	-0.28	0.323	0.354	0.014	-0.01	-0.13	0.533	-0.126	0.179	-0.426	0.15	-0.431	-0.188	0.224	0.005	1	-0.292	0.091	-0.079	-0.272	-0.268	0.133
Bond19	-0.14	-0.1	0.189	0.247	-0.12	-0.1	0.3	0.303	-0.27	0.039	0.074	0.301	-0.045	0.267	-0.355	-0.18	0.257	-0.292	1	0.304	0.054	-0.083	0.067	0.064
Bond20	0.195	-0.13	0.111	0.27	0.212	0.035	-0.08	0.393	0.34	-0.262	-0.136	-0.09	-0.103	-0.093	0.034	0.019	-0.255	0.091	0.304	1	-0.018	0.127	0.031	-0.023
Bond21	-0.05	0.28	0.395	0.176	0.225	0.339	-0.09	-0.24	0.07	0.077	-0.23	-0.36	0.303	0.183	-0.305	0.023	-0.068	-0.079	0.054	-0.018	1	0.098	0.322	0.297
Bond22	-0.08	0.085	-0.04	-0.15	-0.29	-0.03	-0.36	0.307	-0.09	-0.347	-0.288	0.049	0.111	0.21	0.017	0.013	0.267	-0.272	-0.083	0.127	0.098	1	0.181	-0.033
Bond23	-0.26	0.17	-0.06	0.116	-0.19	0.207	-0.13	-0.11	-0.01	0.084	-0.22	-0.067	0.097	0.107	0.167	0.093	-0.164	-0.268	0.067	0.031	0.322	0.181	1	0.305
Bond24	0.285	0.339	-0.03	0.235	-0.03	0.081	-0.1	-0.21	-0.19	-0.172	0.037	-0.094	0.243	0.214	-0.14	-0.044	0.133	0.133	0.064	-0.023	0.297	-0.033	0.305	1

2. Simulated Results

Portfolio Value Distribution



Portfolio Statistics

```
C:\Windows\System32\cmd.e
**************************************************
**************************************************
Number of Simulations: Ultra High

---- Portfolio Statistics ----

Mean = 114,112,188.944
S.D = 2,928,122.714
1% percentile = 104,482,402.301
5% percentile = 108,023,168.445
95% percentile = 116,824,788.324
99% percentile = 117,490,923.38

---- Loss Statistics ----

95% 1Y Rel. VaR =  6,089,020.499
Expected Shortfall =  8,394,473.821

99% 1Y Rel. VaR =  9,629,786.643
Expected Shortfall =  11,972,265.882
```

The Number of Simulations is set to Ultra High, it simulated the portfolio 1,500,000 times, we have the following results:

Key Portfolio Statistics

- Mean Portfolio Value ≈ $114.112M
- S.D. of Portfolio Value ≈ $2.928M
- 1% Percentile ≈ $104.482M
- 5% Percentile ≈ $108.023M
- 95% Percentile ≈ $116.824M
- 99% Percentile ≈ $117.490M

Key Loss Statistics

- 95% 1-year Relative VaR ≈ $6.089M
- Corresponding ES ≈ $8.394M
- 99% 1-year Relative VaR ≈ $9.629M
- Corresponding ES ≈ $11.972M

The 1,500,000 simulated portfolio prices most likely lands at the value of around $116M. This occurs most often because the most probable scenario, based on the transition matrix, is that all bonds retain their initial ratings. The $116M value reflects the 1-year market value of the portfolio if all bonds maintain their original ratings.

The graph also shows an overall decreasing trend from left to right but forms a small peak around $111M before sloping down again. This temporary uptick could stem from many factors, but it is primarily due to the high proportion of junk bonds in the portfolio. Junk bonds have a higher Probability of Default (PD), and once defaulted, they are assigned a random beta-distributed recovery rate. The left tail of the distribution depends heavily on the recovery rate distribution of the defaulted bonds, thus producing an approximate bell curve shape at the left end.

REFERENCE

- J.P. Morgan & Co. (1997). *CreditMetrics – Technical Document.*