

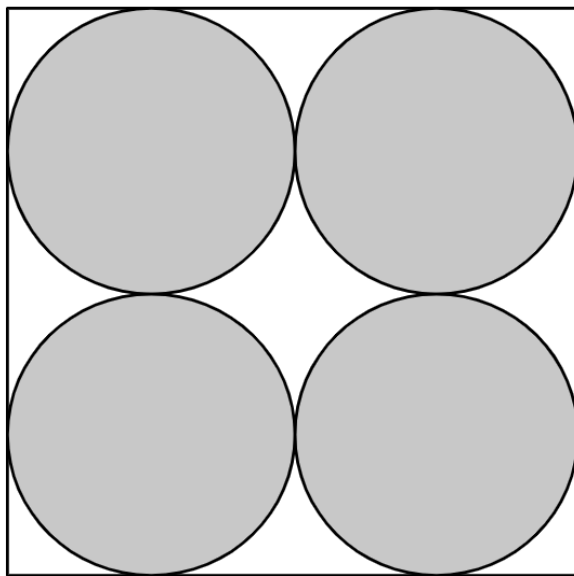
Algebra I Team Round Test Booklet

2024 James Clemens Math Tournament

1. Your team has 60 minutes to complete this exam.
2. This exam consists of 15 free-response questions, each worth 10 points if answered correctly and 0 points if answered incorrectly or left unanswered.
3. Calculators, books, and other aides are prohibited during this examination. Scratch paper will be provided for calculations. Diagrams are not necessarily drawn to scale.
4. Mark your answers to the questions in the provided team answer sheet. You may use the test booklets for scratch work, but only answers marked in the team answer sheet will be counted. If you require additional scratch paper, simply raise your hand and a volunteer will assist you.
5. Unless otherwise specified, all answers in the form of decimals should be converted to fraction form. All fractions should be fully reduced and written in improper form. There should be no negative exponents or unrationalized denominators in your answer.
6. A team's final score will be calculated by adding each team member's written test scores (excluding tie breaking questions) as well as the team round score, for a maximum of 550 points.
7. In the event of a tie, the team with the highest team round score will be favored. If team round scores result in another tie, answers will be evaluated starting backwards from question 15 to 1 to determine a winner.
8. Although this math tournament is intended to demonstrate your knowledge and skills in math, it is also a great opportunity for you to interact with your fellow peers, so be sure to enjoy yourself and have fun!

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1. A cube with side length 2 has 8 spherical balls placed into it resulting in the side view depicted below, and then filled to the top with water. The height of the water after 4 balls are removed can be written in the form $a - \frac{b\pi}{c}$, where a , b , and c are natural numbers. Find abc .



2. How many ways are there for 5 volunteers, V1 - V5, to pick from a list of 4 jobs, such that no two odd or two even numbered volunteers end up with the same job?
3. Two trains, simultaneously leaving from stations A and B, are heading towards each other in a straight line at speeds of 75 mph and 60 mph, respectively. Along the route, the train from station B breaks down, and as a result, the train from station A passes it an hour later than expected. If the stations are 450 miles apart, how far did the train from station B travel before breaking down?
4. John is purchasing some utensils for his restaurant, which come in packs of 5 and 13. What is the greatest number of utensils that he cannot purchase using these packs?
5. Find x^2 if $\sqrt{-3x} = \sqrt{120 - 27x} - 2\sqrt{21 - 3x}$.

6. Maria draws 4 times without replacement from a 52-card deck of shuffled playing cards. If the probability that these four cards are able to be put into numerical consecutive order (A, J, Q, K = 1, 11, 12, 13) can be expressed in the form

$$\frac{a!bd^d}{c(c-1)(c-2)(c-3)}, \text{ find } a + b + c + d.$$

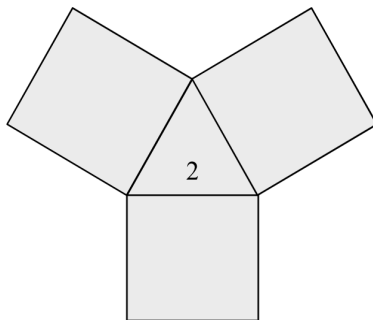
7. John and Ronald are two avid car collectors part of the same club, in which John owns 35% of the cars in the club, and Ronald owns 40%. However, the total value of John's cars make up 60% of the club's, while the total value of Ronald's make up 20%. If the ratio of the average value of John's cars to Ronald's cars can be written in the form $a : b$, where a and b are relatively prime integers, find $a + b$.

8. Find the sum of three consecutive prime numbers whose squares sum to 5739.

9. If the rational function $\frac{13x + 23}{2x^2 - x - 28}$ can be expressed as the sum of two fully reduced fractions with positive integer numerators and linear denominators, find the product of the fractions' numerators.

10. A fidget spinner is manufactured with the design below, consisting of an equilateral triangle with a square attached to each side. If the side length of the

spinner is spun can be written in the form $\frac{\pi(a + b\sqrt{c})}{d}$, find $ab - cd$.

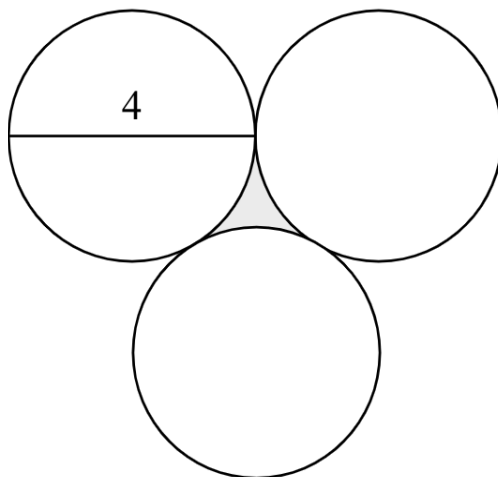


11. There are three hoses filling a pool with water simultaneously. Hose A can fill the pool in 3 hours, hose B in 4 hours, and hose C in 5 hours. However, there is now a hole in the bottom of the pool, causing 20% of the water coming in to drain out. If the difference in filling time, in hours, between the undamaged and damaged pool can be written in the form $\frac{a}{b}$, where a and b are relatively prime integers, find $a + b$.

12. Grant is trying to arrange a certain number of marbles to fill a solid large square, but ends up with a leftover marble after he finishes. He then makes two smaller squares, each with a side length 8 marbles less than the larger square, but now has 8 marbles leftover. Find the number of marbles in the larger square.

13. 20 slips of paper, numbered 1 - 20, are placed in a hat and mixed. The probability that the product of two numbers drawn without replacement is divisible by 5 or 7 can be written in the form $\frac{a}{b}$. Find $b - a$.

14. In the diagram below, 3 congruent circles with diameter 4 are connected through tangent points, in which the shaded region denotes the area enclosed by the circles. If this area can be written in the form $a\sqrt{b} - c\pi$, find abc .



15. A box in an old theater dressing room contains an assortment of identically structured hats in 5 different colors. If the number of combinations of color

patterns that can be made by placing hats on 7 mannequins in a line, provided that no two adjacent mannequins wear the same colored hat can be written in the form $a^b \cdot c^d$, where a , and b are prime numbers, find $a + b + c + d$.