Algebra I Written Test

2024 James Clemens Math Tournament

1. You have 90 minutes to complete this exam.
2. This exam consists of 25 multiple-choice questions and 3 free-response questions used as tie-breakers. The multiple-choice questions are each worth 4 points if answered correctly and no points if left unanswered. 1 point will be deducted for each incorrect answer. The free-response questions are each worth 0.1 point if answered correctly, and no points if answered incorrectly or left unanswered. The maximum score for this test is 100.3 points.
3. Calculators, books, and other aides are prohibited during this examination. Scratch paper will be provided for calculations. Diagrams are not necessarily drawn to scale.
4. Mark your answers to the questions in the provided Scantron form. You may use the test booklets for scratch work, but only answers marked in the Scantron form will be counted. If you require additional scratch paper, simply raise your hand and a volunteer will assist you.
5. In the event of a tie, answers will be evaluated starting backwards from multiple-choice question 25 to 1 to determine a winner.
6. Although this math tournament is intended to demonstrate your knowledge and skills in math, it is also a great opportunity for you to interact with your fellow peers, so be sure to enjoy yourself and have fun!

DO NOT TURN THE PAGE UNTIL TOLD TO DO SO

1. A classroom in a university has a large number of desks, which if arranged in rows of seven leave two empty spots in the last row, in rows of six leave one empty spot in the last row, and in rows of five leave no empty spots in the last row. What is the fewest possible number of desks in this classroom?

(A) 75 (B) 145 (C) 215 (D) 285 (E) NOTA

2. A scientist is testing the effects of a new alcohol-based drug and needs to make a 500 mL solution of 45% isopropyl alcohol using two stock solutions of 30% strength (Solution A) and 50% strength (Solution B). Find the difference, in mL, of the volumes of solutions A and B needed to achieve the desired strength.

(A) -250 (B) -100 (C) 100 (D) 250 (E) NOTA

3. Find the value of $\sqrt{6+\sqrt{6+\sqrt{6+....}}}$

(A) $\sqrt{3}$ (B) 2 (C) $2\sqrt{2}$ (D) 3 (E) NOTA

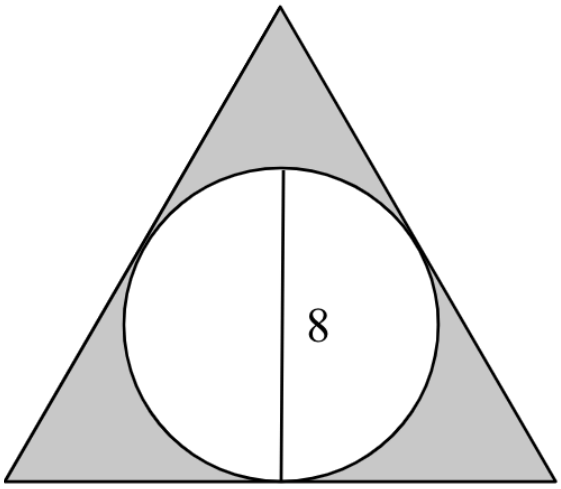
4. Find the area of the region enclosed by the graphs of $x = 0$, $y = 2$, $x = 2$, and $y = \sqrt{4-(x-2)^2}$.

(A) π (B) 2π (C) $4-\pi$ (D) $8-2\pi$ (E) NOTA

5. How many ways are there to arrange seven distinct books, B1 - B7, such that books B1 - B3 are always together?

(A) 120 (B) 360 (C) 720 (D) 5040 (E) NOTA

6. The diagram below shows a circle with diameter 8 inscribed in an equilateral triangle, with the shaded region representing the area included inside the triangle but outside the circle. This area can be written in the form $a\sqrt{3} - b\pi$, where a and b are positive integers. Find $a + b$.



(A) 8 (B) 20 (C) 32 (D) 56 (E) NOTA

7. Let S_n denote the sum of the first n terms of the arithmetic series:

$x + (x - 2) + (x - 4) + \ ...$

If the remainder of $\dfrac{S_4 S_5}{S_2}$ can be written in the form $\dfrac{a}{bx - c}$, find $\dfrac{a}{b + c}$.

(A) 20 (B) 30 (C) 40 (D) 50 (E) NOTA

8. Consider the following equation for distinct positive numbers a, b, and c : $\dfrac{b}{a - c} = \dfrac{a + b}{c} = \dfrac{a}{b}$. What is the value of $\dfrac{a}{b}$?

(A) $\dfrac{1}{2}$ (B) $\dfrac{2}{3}$ (C) 1 (D) 2 (E) NOTA

9. Grant is practicing archery on a target with 9 rings, each having an equal

thickness, that get successively smaller, and a small circle in the center with a radius equal to the thickness of the outer rings. The rings increase in point value, starting with the largest ring worth 1 point, up to the small circle worth 10 points. What is the probability that Ryan will shoot an even number for his first shot?

(A) $\frac{9}{20}$ (B) $\frac{1}{2}$ (C) $\frac{11}{20}$ (D) $\frac{6}{10}$ (E) NOTA

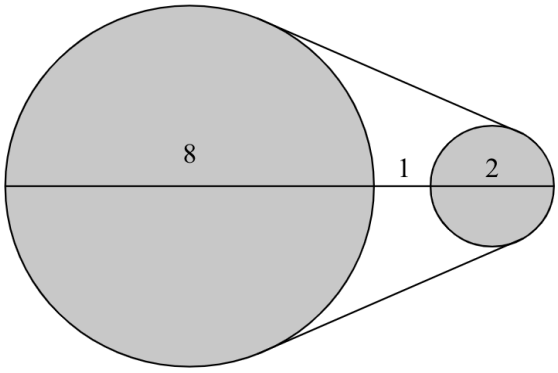
10. Ben drives to his friend James' house at a constant speed of 40 miles per hour, stays for an hour, and drives back at a constant speed of 30 miles per hour. What was Ben's average speed, in miles per hour and rounded to the nearest integer, for the entire round trip, excluding the time spent at James' house?

(A) 32 (B) 35 (C) 37 (D) 38 (E) NOTA

11. If $a =$ the GCF of 195 and 156, $b =$ the number of primes before 1000, and $c = \int_{\pi}^{3\pi} e^{x^2} dx$, find $\frac{ac}{\frac{b}{c}} \cdot \frac{b}{c^2}$.

(A) 13 (B) 39 (C) 55 (D) 72 (E) NOTA

12. Ryan has to construct a pulley system for a school project, in which a rubber band is stretched around two circles. These circles have diameters of 8 inches and 2 inches, and they are 1 inch apart. The length required of the rubber band to stretch completely around the two circles, as shown below, can be written in the form $a\sqrt{b} + c\pi$. Find $2a + b + c$.



(A) 17 (B) 19 (C) 21 (D) 23 (E) NOTA

13. What is the largest n such that 3^n divides 123!?

(A) 41 (B) 42 (C) 58 (D) 59 (E) NOTA

14. Find the sum of all solutions to the equation: $2x + \sqrt{2x-2} = 2$.

(A) $\frac{5}{2}$ (B) 3 (C) $\frac{7}{2}$ (D) 5 (E) NOTA

15. If the greatest common factor of two numbers is 17, and their product is 867, what is the least common multiple of these two numbers?

(A) 26 (B) 31 (C) 37 (D) 42 (E) NOTA

16. Greg can run x meters in y minutes. How many kilometers can he run in z hours?

(A) $\frac{60y}{xz}$ (B) $\frac{xz}{60y}$ (C) $\frac{6xz}{100y}$ (D) $\frac{10y}{6xz}$ (E) NOTA

17. If $x \ \phi \ y \ = \frac{x+y}{x-y}$, and $3 \ \phi \ x = \ 3$, find x.

(A) $\frac{3}{2}$ (B) $\frac{4}{3}$ (C) 2 (D) 3 (E) NOTA

18. A triangle and a trapezoid have equal altitude and area. Provided that the base of the triangle is 2 feet, what is the sum of the base lengths of the trapezoid in inches?

(A) 12 (B) 18 (C) 24 (D) 30 (E) NOTA

19. The probability that a basketball player makes any three point shot is $\frac{1}{3}$. What is the probability that he makes exactly 2 out of his next 4 three-pointers?

(A) $\frac{4}{81}$ (B) $\frac{2}{27}$ (C) $\frac{1}{9}$ (D) $\frac{8}{27}$ (E) NOTA

20. If the sum $\frac{1}{4+\sqrt{15}}+\frac{1}{\sqrt{15}+\sqrt{14}}+\frac{1}{\sqrt{14}+\sqrt{13}}+\frac{1}{\sqrt{13}+2\sqrt{3}}$ can be expressed in the form $a-b\sqrt{c}$, Find $a+b+c$.

(A) 8 (B) 9 (C) 10 (D) 11 (E) NOTA

21. What is the probability that two randomly selected, distinct, single digit positive integers have an even product?

(A) $\frac{5}{9}$ (B) $\frac{11}{18}$ (C) $\frac{2}{3}$ (D) $\frac{13}{18}$ (E) NOTA

22. Johnny is roller skating through a straight road in his neighborhood, with mailboxes spaced equally apart at 30 feet. Assuming a constant pace, which of the following is the best estimate of the time, in seconds, that it takes Johnny to pass x mailboxes traveling at x miles per hour?

(A) 15 (B) 20 (C) 25 (D) 30 (E) NOTA

23. Find the sum of coordinates of the intersection points of the graphs: $y = |2x| - 2$ and $y = -|2x| + 2$.

(A) -2 (B) -1 (C) 0 (D) 1 (E) NOTA

24. Find the number of integers that satisfy the following inequality:

$$x^2 - 5x - 14 \leq 0$$

(A) 2 (B) 8 (C) 9 (D) 10 (E) NOTA

25. Find the sum of all solutions to the following equation:

$$\sqrt{x - 15} + \frac{15}{\sqrt{x - 15}} = 8$$

(A) 36 (B) 38 (C) 42 (D) 44 (E) NOTA

Tiebreakers:

TB1. How many ways are there to write a four letter 'word' using only the first 10 letters of the alphabet with no repetitions such that the second and fourth letters are vowels and the first a consonant?

TB2. If it is currently four o'clock, the number of minutes that will pass before the minute hand and the hour hand first pass each other can be written in the form $\frac{a}{b}$. Find $a + b$.

TB3. In the diagram below, a circle with radius 1 is inscribed in a square, along with four smaller circles each tangent to the circle and two sides of the square. The area of the shaded region can be written in the form $a - \pi(b - c\sqrt{2})$. Find $a + b - c$.