Pre-Algebra Written Test
2024 James Clemens Math Tournament

1. You have 90 minutes to complete this test.
2. This exam consists of 25 multiple-choice questions and 3 free-response questions used as tie-breakers. The multiple-choice questions are each worth 4 points if answered correctly and no points if left unanswered. 1 point will be deducted for each incorrect answer. The free-response questions are each worth 0.1 point if answered correctly, and no points if answered incorrectly or left unanswered. The maximum score for this test is 100.3 points.
3. Calculators, books, and other aides are prohibited during this examination. Scratch paper will be provided for calculations. Diagrams are not necessarily drawn to scale.
4. Mark your answers to the questions in the provided Scantron form. You may use the test booklets for scratch work, but only answers marked in the Scantron form will be counted. If you require additional scratch paper, simply raise your hand and a volunteer will assist you.
5. In the event of a tie, answers will be evaluated starting backwards from multiple-choice question 25 to 1 to determine a winner.
6. Although this math tournament is intended to demonstrate your knowledge and skills in math, it is also a great opportunity for you to interact with your fellow peers, so be sure to enjoy yourself and have fun!

DO NOT TURN THE PAGE UNTIL TOLD TO DO SO

1. How many distinguishable arrangements are there for the word ALABAMA?

    (A) 5040   (B) 2520   (C) 210   (D) 840   (E) NOTA

2. If (x, y) is the solution to the following system of equations, find the sum $x + y$.

$$4x + 5y = 50$$
$$8x + 7y = 10$$

    (A) 5   (B) 25   (C) 25   (D) 30   (E) NOTA

3. What is the area of a regular hexagon with side length 8?

    (A) 48   (B) $184\sqrt{3}$ (C) $96\sqrt{3}$ (D) $48\sqrt{3}$ (E) NOTA

4. What is the 101st term in the arithmetic sequence $1, 4, 7, 10, 13, 16, \ldots$?

    (A) 295   (B) 292   (C) 298   (D) 301   (E) NOTA

5. Grant's mother has 5 children. The first child is named Carly. The second child is named Chritstian. The third child is named Kiley. The fourth child is named Elliot. What is the name of the fifth child?

    (A) David   (B) Leslie   (C) Walter   (D) Allison   (E) NOTA

6. Let A $=$ the number of primes less than 100. Let B $= \sqrt[3]{10^3 + 9^3 - 1}$. Let C $=$ the GCF of 91 and 49. Let D $=$ the units digit of 3^7. What holiday is $\dfrac{BD}{AC}$?

    (A) Halloween (B) July 4th   (C) Easter   (D) Christmas (E) NOTA

7. Edward and Jacob are facing opposite to each other. The distance between

their noses is 50 m. Edward is walking forward at a constant rate of 15 m/hr, and Jacob walking forward at a constant rate of 10 m/hr. There is a bee that starts from the nose of Edward, and this bee travels to the nose of Jacob, it travels back to Edward. The bee continues traveling like this until Edward and Jacob's noses touch. The bee is traveling at a constant rate of 100 m/hr. How many meters did the bee travel?

(A) 255 m (B) 150 m (C) 227.5 m (D) 250 m (E) NOTA

8. If $a \prod b = \dfrac{(b+a)(a-b)}{a!}$, and $a \circ b = \dfrac{b \prod a}{b}$. Find the reciprocal of the sum of $5 \prod 3$ and $3 \circ 5$.

(A) $\dfrac{3}{25}$ (B) $\dfrac{25}{2}$ (C) $\dfrac{25}{3}$ (D) $\dfrac{25}{4}$ (E) NOTA

9. Find the sum of the digits of the expression $12345678 \cdot 999$.

(A) 28 (B) 30 (C) 27 (D) 31 (E) NOTA

10. What is the resulting point if (6, 5) is dilated about center (2, 4) with a scale factor of 7?

(A) (42, 35) (B) (14, 35) (C) (14, 28) (D) (42, 28) (E) NOTA

11. A number is *palindromic* if its value is the same even when the digits are reversed. For example, 131, 11, and 1 are all palindromic, but 123 and 12 are not. How many 7-digit palindromic numbers are there?

(A) 9,000 (B) 100 (C) 10,000 (D) 5,040 (E) NOTA

12. Tom and Jerry are arguing about how long a piece of cheese has been aged (with the number of months aged represented using c). Tom knows that c satisfies the inequality $c^3 - 1 > 2c^2 + 2c + 2$, and Jerry knows that c satisfies the inequality $c^4 - 1 < 4c^3 + 4c^2 + 4c + 4$. Assuming that c is a positive integer and knowledge is correct, how long has the cheese been aged?

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

13. If $x + \dfrac{1}{x} = 5$, what is the value of $x^4 + \dfrac{1}{x^4}$?

   (A) 625   (B) 529   (C) 23   (D) 527   (E) NOTA

14. There are two cylindrical containers that have the same volume, but the first container's radius is 75% longer in length than the second container. What is the ratio of the first container's height to the second container's height?

   (A) 3 : 4   (B) 4 : 7   (C) 4 : 3   (D) 49 : 16   (E) NOTA

15. If the number 690_A_768 is divisible by 28, what is the value of the digit A?

   (A) 5   (B) 6   (C) 4   (D) 9   (E) NOTA

16. Simplify the expression $\dfrac{(x^2 + 4x - 21)(x - 5)}{(x^2 - 8x + 15)(x + 7)}$ assuming x is a value such that the denominator does not equal 0.

   (A) 1   (C) $x - 5$ (C) $x - 3$ (D) $x + 3$ (E) NOTA

17. There is a circle with center O. There is also a triangle inscribed in this circle with its vertices denoted A, B, and C. $\overline{AC}$ is a diameter of the circle. $\overline{AB}$ measures 6 units, and $\overline{BC}$ measures 8 units. Find the area of the circle in terms of π.

   (A) 16π (B) 9π (C) 100π (D) 25π (E) NOTA

18. A square is inscribed in a semicircle as shown below with the midpoint of the square lying on the center of the circle. The circle has a radius of length 15 units. Find the area of the inscribed square.
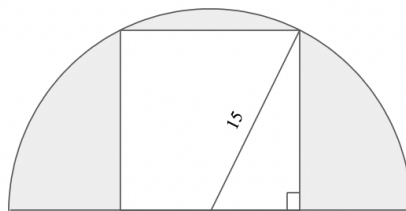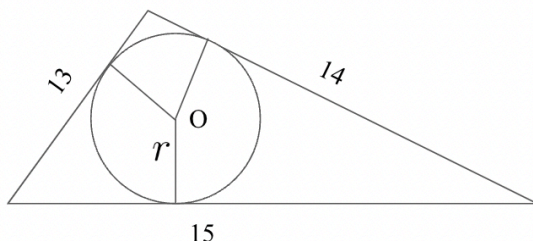

   (A) 225 units^2 (B) 45 units^2 (C) 100 units^2 (D) 180 units^2 (E) NOTA

19. What is the remainder when 2023^{2023} is divided by 10?

(A) 3   (B) 9   (C) 7   (D) 1   (E) NOTA

20. There is a circle inscribed in a scalene triangle with side lengths of 13, 14, and 15 units, as shown below. If r denotes the radius of the inscribed circle, find r.

(A) 5   (B) 3   (C) 4   (D) $2\sqrt{3}$ (E) NOTA

21. Anastasia is playing a one-person coin game. In the game, one flips five fair coins, and if there are at least 3 heads, the player wins. What is the probability that Anastasia wins this coin game?

(A) $\dfrac{1}{4}$ (B) $\dfrac{3}{16}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{7}{16}$ (E) NOTA

22. Let A = the larger of the two numbers: 3^{50} and 5^{30}. Let B = the GCF of 15^{30} and 27^{15}. Find $\dfrac{\text{B}}{\text{A}}$.

(A) $\dfrac{3^{20}}{5}$ (B) $\dfrac{1}{3^{20}}$ (C) $\dfrac{5^{30}}{3^{15}}$ (D) $\dfrac{1}{3^{45}}$ (E) NOTA

23. Erin loves to play Roblox. As a result, he also tracks how long he spends on the game to ensure an efficient grindset. On weekends, he plays 15% of the day, while on Mondays and Fridays, he spends 35% of the day playing Roblox. On Tuesdays, Wednesdays, and Thursday, he does not play Roblox to rest his fingers. What is the average amount of hours he spends playing Roblox every day?

(A) $\dfrac{4}{7}$ hours   (B) $\dfrac{24}{7}$ hours   (C) $\dfrac{7}{4}$ hours   (D) $\dfrac{6}{7}$ hours   (E) NOTA

24. What is the minimum value of the parabola given by the equation
$x^2 - 2x + 4$?

    (A) 3   (B) 8   (C) 15   (D) 24   (E) NOTA

25. Simplify the expression $\dfrac{\sqrt{3} + \sqrt{7}}{\sqrt{3} + \sqrt{2}}$ completely.

    (A) 2   (B) 3   (C) $\sqrt{3} - \sqrt{6}$ (D) $\sqrt{6} + \sqrt{21}$ (E) NOTA

Tiebreakers:

TB1. Find the remainder when $2x^4 + x^3 + x^2 + x + 1$ is divided by $x^3 + x^2 + x + 1$.

TB2. There is a triangle ABC (shown below). side $\overline{AB}$ is 8 units long, and side $\overline{AC}$ is 6 units long. There is a line segment that bisects $\angle CAB$ and intersects side $\overline{BC}$ at point D so that $\overline{CD}$ measures 3 units long. Find the length of side $\overline{BC}$.
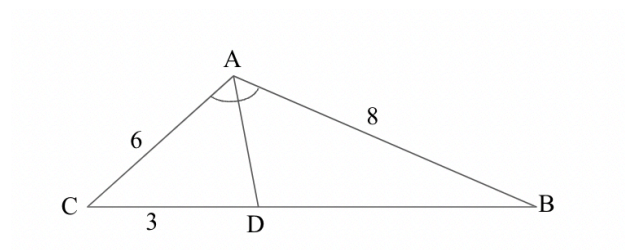
TB3. Abbie, Beatrice, Caroline, Danielle, and Emily go out to eat at a restaurant. At the restaurant, they sit at a circular table with 5 evenly spaced chairs. How many distinct ways can they sit at the circular table (with "rotated seating" being regarded the same as the original ordering)?