Pre-Algebra Team Round Test Booklet

2024 James Clemens Math Tournament

1. Your team has 60 minutes to complete this exam.
2. This exam consists of 15 free-response questions, each worth 10 points if answered correctly and 0 points if answered incorrectly or left unanswered.
3. Calculators, books, and other aides are prohibited during this examination. Scratch paper will be provided for calculations. Diagrams are not necessarily drawn to scale.
4. Mark your answers to the questions in the provided team answer sheet. You may use the test booklets for scratch work, but only answers marked in the team answer sheet will be counted. If you require additional scratch paper, simply raise your hand and a volunteer will assist you.
5. Unless otherwise specified, all answers in the form of decimals should be converted to fraction form. All fractions should be fully reduced and written in improper form. There should be no negative exponents or unrationalized denominators in your answer.
6. A team's final score will be calculated by adding each team member's written test scores (excluding tie breaking questions) as well as the team round score, for a maximum of 550 points.
7. In the event of a tie, the team with the highest team round score will be favored. If team round scores result in another tie, answers will be evaluated starting backwards from question 15 to 1 to determine a winner.
8. Although this math tournament is intended to demonstrate your knowledge and skills in math, it is also a great opportunity for you to interact with your fellow peers, so be sure to enjoy yourself and have fun!

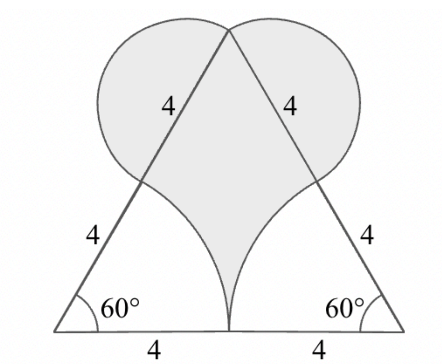
DO NOT TURN THE PAGE UNTIL TOLD TO DO SO

1. Let A = the units digit of 3^{20}. Let B = the area of a trapezoid with height 3 units and bases of length 4 and 8 units. Let C = the smallest integer greater than $\sqrt{360}$. Let D = the greatest common factor of 81 and 57. Find $\dfrac{\mathrm{D}(\mathrm{A} + \mathrm{B})}{\mathrm{C}}$.

2. How many ways are there to distribute 7 indistinguishable balls into 3 distinct baskets if each basket must contain at least one ball?

3. If $f(x) = \begin{cases} 3x+1 & \text{if } x \text{ is odd} \\ \frac{x}{2} & \text{if } x \text{ is even} \end{cases}$, what is the value of $f(f(f(f(f(f(f(f(26))))))))$?

4. The following shape is drawn with the given side lengths (note that the two shaded circular regions are both semicircles). If the shaded area may be written in the form $a\sqrt{3} - \dfrac{b\pi}{c}$, where a, b, and c are integers and b and c are relatively prime, find $a + b + c$.



5. Each side of a rhombus measures 26 units. The longer diagonal of the rhombus measures 48 units. What is the area of the rhombus?

6. If the distance from line $y = -x + 1$ to the origin can be written as $\dfrac{a}{b}\sqrt{c}$, where a and b are relatively prime integers and the radical is simplified, find $(b^c)! + a$.

7. The integers 1 through 49 are placed in a 7 x 7 grid such that the summing each number in each row or column yields the same number, x, find the value of x.

8. Jerry is playing a best of 5 sets tennis match (the first player to win 3 sets wins the match). Jerry has a $\frac{1}{3}$ probability of losing any given set. If his probability of winning his next 5 set match against John may be written as $\frac{a}{b}$, where a and b are relatively prime, find $a + b$.

9. Jared is a very interesting person in that he eats his sandwich in a particular way. He eats $\frac{1}{2}$ of his sandwich the first day, and on each subsequent day, he eats half of what's leftover from the day before. After he eats the sandwich for an infinite amount of days, he has finished $x\%$ of his sandwich. What is the value of x?

10. What is the value of x such that

$$\sqrt{5+x} = \sqrt{12\sqrt{x}+25} - 4\ ?$$

11. If (x, y, z) is the solution to the following system of equations,

$$\begin{aligned}
&12x + 15y + 13z = 226 \\
&5x + 3y + 2z = 79 \\
&x + 8y + 8z = 21
\end{aligned}$$

find $x + y + z$.

12. There are 3 fair six-sided dice that are rolled, and the sum of the rolls is calculated. If the probability that the sum was greater than 6 may be expressed as $\frac{a}{b}$, a proper fraction in simplest form, find $a + b$.

13. What is the value of $(1+3+5+\cdots+35)+(36+38+40+\cdots+70)$?

14. In the coordinate plane, how many ways are there to move from point (2, 3) to point (5, 7) if one can only move up 1 unit or right 1 unit at a time?

15. There is a square with side length 8 units. 4 semicircles are placed in the square as shown in the diagram. Then, part of the resulting figure is shaded as shown. If the shaded area may be expressed in the form $a\pi - b$, where a and b are positive integers, what is $a + b$?