

## **6th Grade Team Round Test Booklet**

### **2024 James Clemens Math Tournament**

1. Your team has 60 minutes to complete this exam.
2. This exam consists of 15 free-response questions, each worth 10 points if answered correctly and 0 points if answered incorrectly or left unanswered.
3. Calculators, books, and other aides are prohibited during this examination. Scratch paper will be provided for calculations. Diagrams are not necessarily drawn to scale.
4. Mark your answers to the questions in the provided team answer sheet. You may use the test booklets for scratch work, but only answers marked in the team answer sheet will be counted. If you require additional scratch paper, simply raise your hand and a volunteer will assist you.
5. Unless otherwise specified, all answers in the form of decimals should be converted to fraction form. All fractions should be fully reduced and written in improper form. There should be no negative exponents or unrationalized denominators in your answer.
6. A team's final score will be calculated by adding each team member's written test scores (excluding tie breaking questions) as well as the team round score, for a maximum of 550 points.
7. In the event of a tie, the team with the highest team round score will be favored. If team round scores result in another tie, answers will be evaluated starting backwards from question 15 to 1 to determine a winner.
8. Although this math tournament is intended to demonstrate your knowledge and skills in math, it is also a great opportunity for you to interact with your fellow peers, so be sure to enjoy yourself and have fun!

**DO NOT TURN THE PAGE UNTIL TOLD TO DO SO**

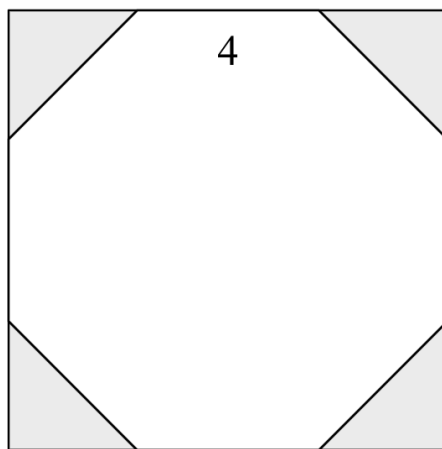
1. If the solution to the following system of equations is  $(x, y)$ , find  $x + y$ .

$$\begin{aligned}\frac{1}{a} + \frac{2}{b} &= 6 \\ \frac{5}{a} + \frac{6}{b} &= 24\end{aligned}$$

2. Two runners start moving towards each other at speeds of 12 miles per hour and 9 miles per hour. However, after half an hour, the faster runner stops for a 10 minute water break, and starts to run again at a pace of 6 miles per hour. After a runner passes the other's starting point, he rings the bell at that location. If the runners start 15 miles apart, how much time, in minutes, passes between the two dings?

3. Ava rolls 3 fair 6-sided dice. If the probability of rolling a 3 or 4 can be expressed in the form  $\frac{a}{b}$ , where  $a$  and  $b$  are relatively prime integers, find  $a + b$ .

4. A regular octagon with side length 4 is inscribed in a square, as depicted in the diagram below. Find the area of the shaded region.

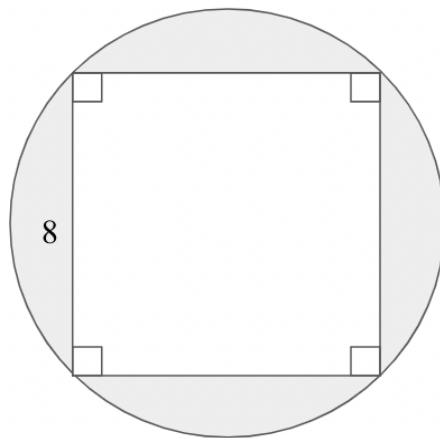


5. What is the value of the following expression?

$$\frac{\sqrt{\sqrt{256} + \sqrt{81}}}{\sqrt[3]{729 + 1000} - 1} \cdot (\sqrt{81} + \sqrt{9})$$

6. The equation  $\frac{720 - x}{4} = n^2 - 1$  holds true for  $x$ , a three-digit number, and  $n$ , a palindrome, a number that has the same value even when the digits are reversed. Find the sum of the exponents in the prime factorization of the least possible value of  $x$ .

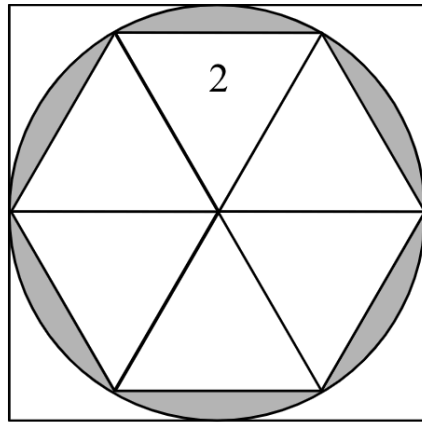
7. A square with side length 8 units is inscribed in a circle as shown in the diagram below. The area that is inside the circle but outside the square is shaded. If the area of the shaded region may be expressed as  $a\pi - b$ , where  $a$  and  $b$  are positive integers, find  $\frac{b}{a}$ .



8. If it takes 30 days for 6 workers to build 3 sheds, how much time will it take 9 workers to build 8 sheds provided that two workers put in half the normal effort?

9. Find the sum of all integer solutions to  $(x - 5)^{(x-2)} = 1$ .

10. A regular hexagon with side length 2 is inscribed in a circle, which is itself inscribed in a square. If the area of the shaded region can be written in the form  $b\pi - c\sqrt{3}$ , find  $a + b$ .



11. The student council has 7 members. For the upcoming school year, 3 officers (a president, vice-president, and treasurer) are to be chosen. Frank will not be able to be an officer due to prior commitments. With this restriction, how many ways are there to choose 3 officers from the 7 member student council?
12. Graham is using marbles to construct a triangle by forming rows, each consisting of one more marble than the last. For example, a triangle with 3 rows has  $1 + 2 + 3 = 6$  marbles. When he tries to make a triangle with a certain number of rows, he ends up with one leftover marble. He then successfully rearranges the marbles into two smaller triangles, each with three less rows than the original. Find the number of marbles that Graham is working with.
13. Let  $A$  = the least common multiple of 90 and 8. Let  $B$  = the number of ways to arrange the word SORTS. Let  $C$  = the slope of the line perpendicular to the line  $3x + y = 5$ . Find  $\frac{A}{BC}$ .

14. A local pool manager orders two large hoses, Hose A and Hose B, to refill his 3200 gallon swimming pool. However, during transportation, both hoses are damaged, with the efficiency of Hose A being reduced to 50% and Hose B to 80%. Hose A could normally fill the swimming pool alone in 50 hours and Hose B in 64 hours. If the time it will take for the damaged hoses to fill the swimming pool together can be written in the form  $\frac{a}{b}$ , where  $a$  and  $b$  are relatively prime integers, find  $a + b$ .

15. There are three spinners as shown below, and the probability of landing on a specific number for each of these spinners is  $\frac{1}{3}$ ,  $\frac{1}{4}$ , and  $\frac{1}{5}$ , respectively. Jihyo spins each of the spinners once and calculates the product of the numbers she lands on. If the probability that she gets an even product can be expressed in the form  $\frac{a}{b}$ , where  $a$  and  $b$  are relatively prime integers, find  $a + b$ .

