ME597 Assignment 1

# Motion, Measurement and Estimation

Abdul Qureshi 20388761

ChengLin Yeh 20369072

Oct 26th, 2014

# 

# **Introduction**

The purpose of this lab was to study three of fundamental components of autonomous robot, motion, measurement and estimation. First, a motion model of a three-wheeled omnidirectional model was derived. Next, sensor models for GPS and magnetometer were defined. Finally, Extending Kalman filter and multi-rate Kalman filter was used to combine the measurements from the sensors and measurements from the robot itself to estimate robot’s correct location.

# 1 Motion modeling of omnidirectional wheeled robot

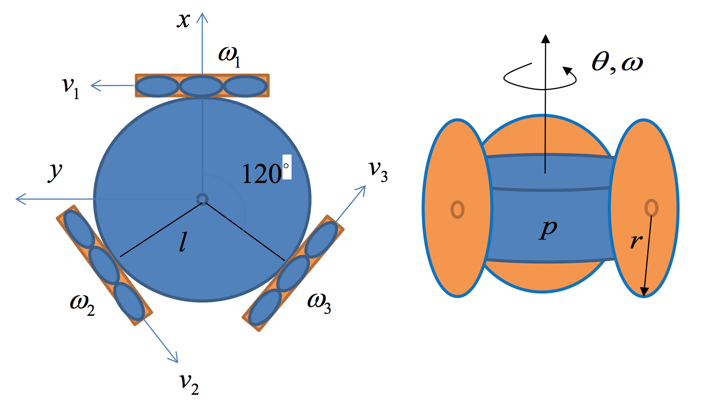


Figure 1 – Omnidirectional wheeled robot

The velocity for the robot can be expressed as:

Note that the different velocities here are all in robot’s frame.

To derive the motion model for the omnidirectional wheeled robot, we first look at the velocity decomposition of each wheel.

Next, the velocity found above is in the robot’s frame, thus, we need to convert it into the world frame. Note that the theta here is in the global frame.

# 2 Simulation of the Robot

Given the input , the following figure is what the output looks like over 15s with 10Hz update.

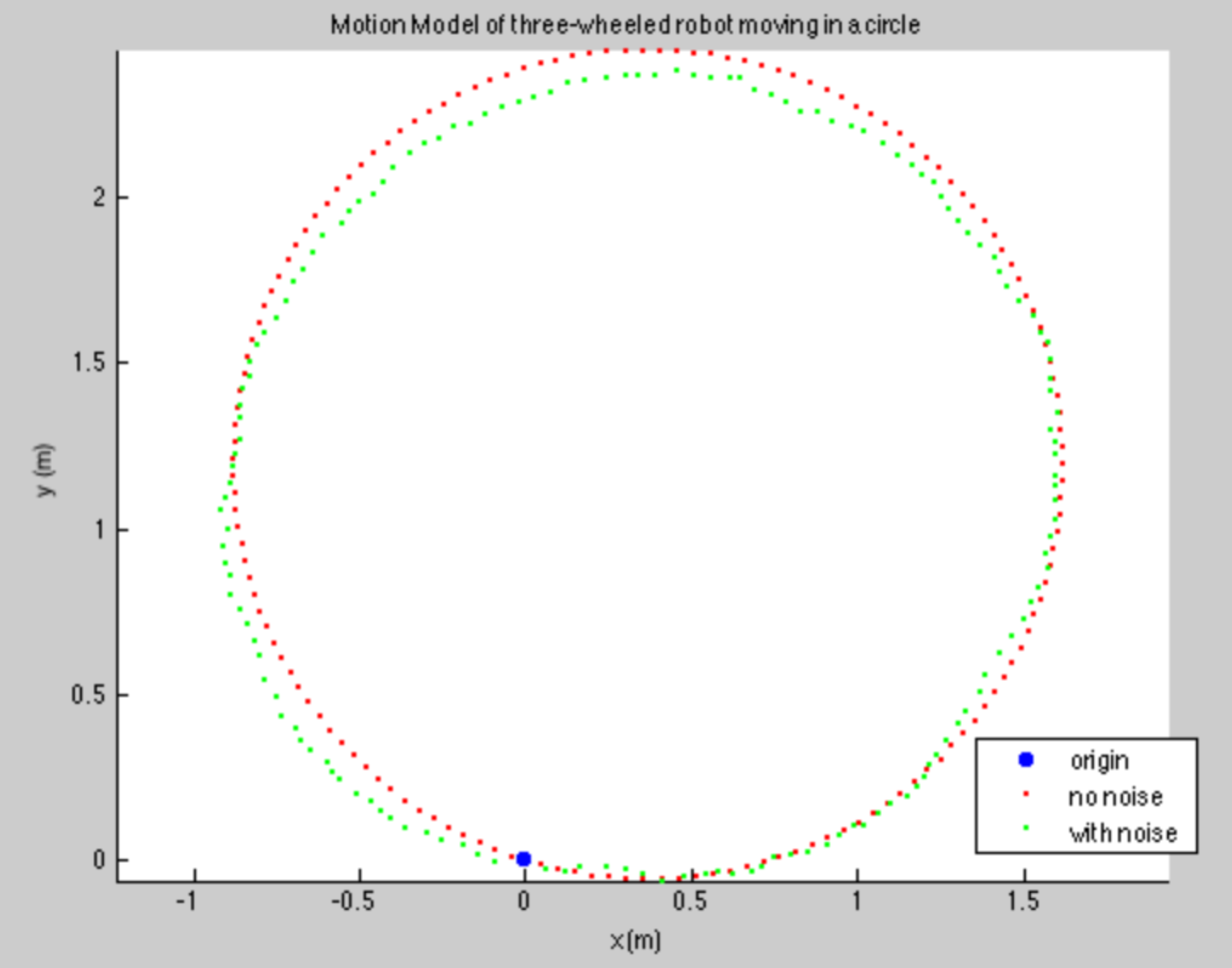


Figure 2 – Omnidirectional wheeled robot simulation given inputs

For the robot to move in a straight line, solve for the following

or

the result of solving these equation is

, to move in the x direction

or

, to move in the y direction

The result of this calculation is shown in the below figure

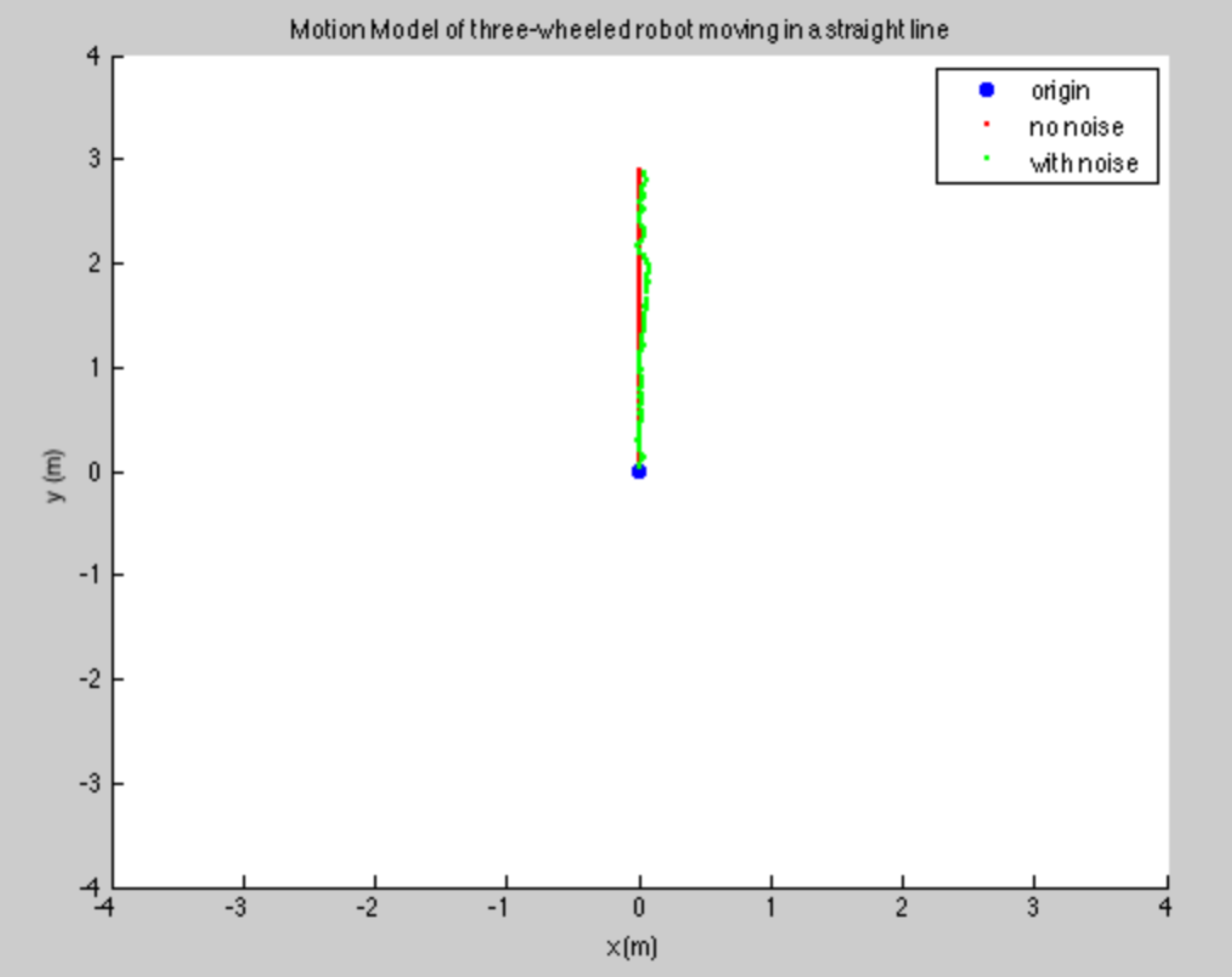


Figure 3 – Omnidirectional wheeled robot moving in straight line

To move in a circle with radius of r,

For this specific question r = 1, for the sake of simplicity, set y = 0

The result is shown in the below figure

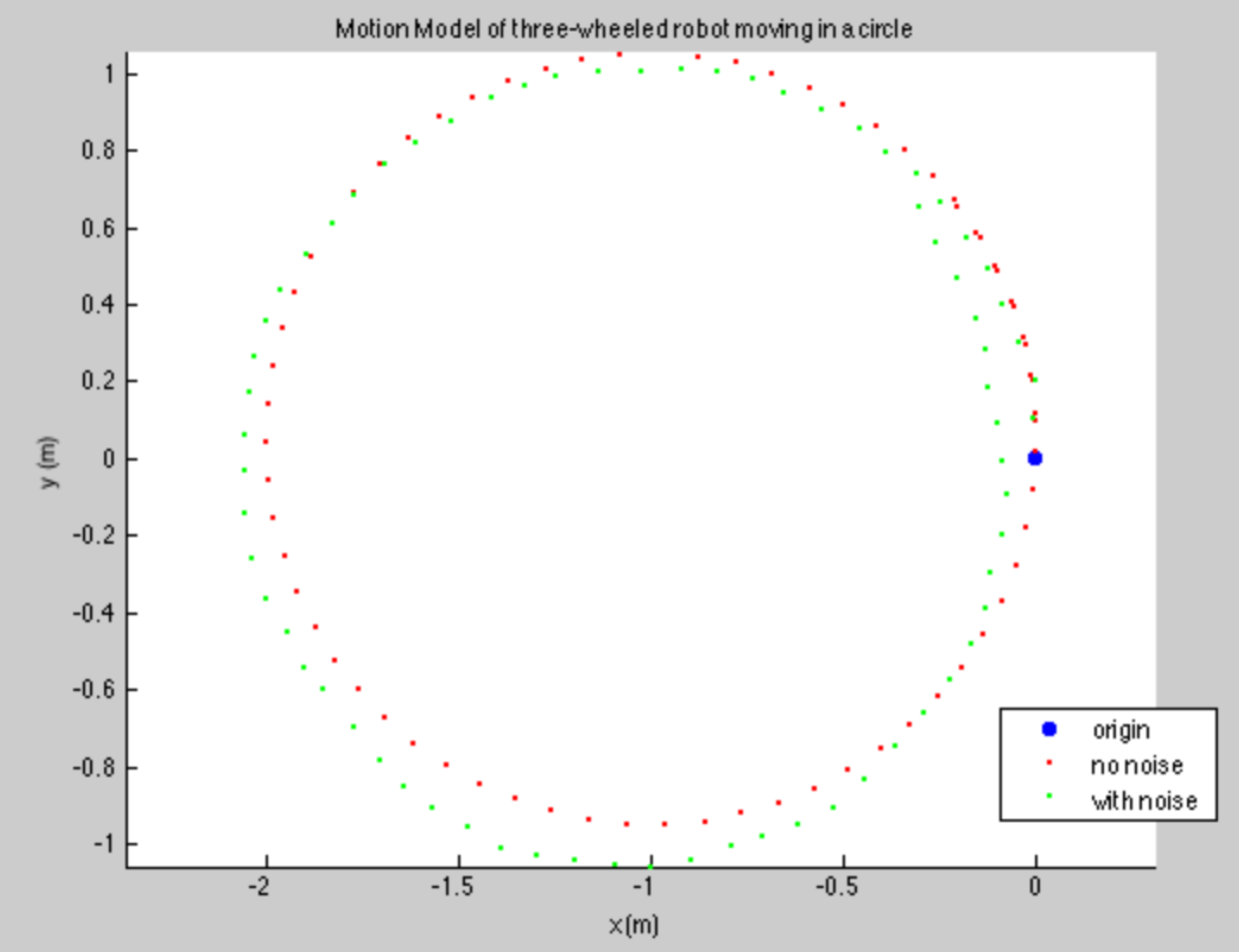


Figure 4 – Omnidirectional wheeled robot moving in circle with radius = 1m

To move in a spiral, constant velocity for the wheels will not suffice. To solve for a spiral, fix angular velocity at one and keep y = 0 but set x to a function z = x\*t

The result is in the following figure

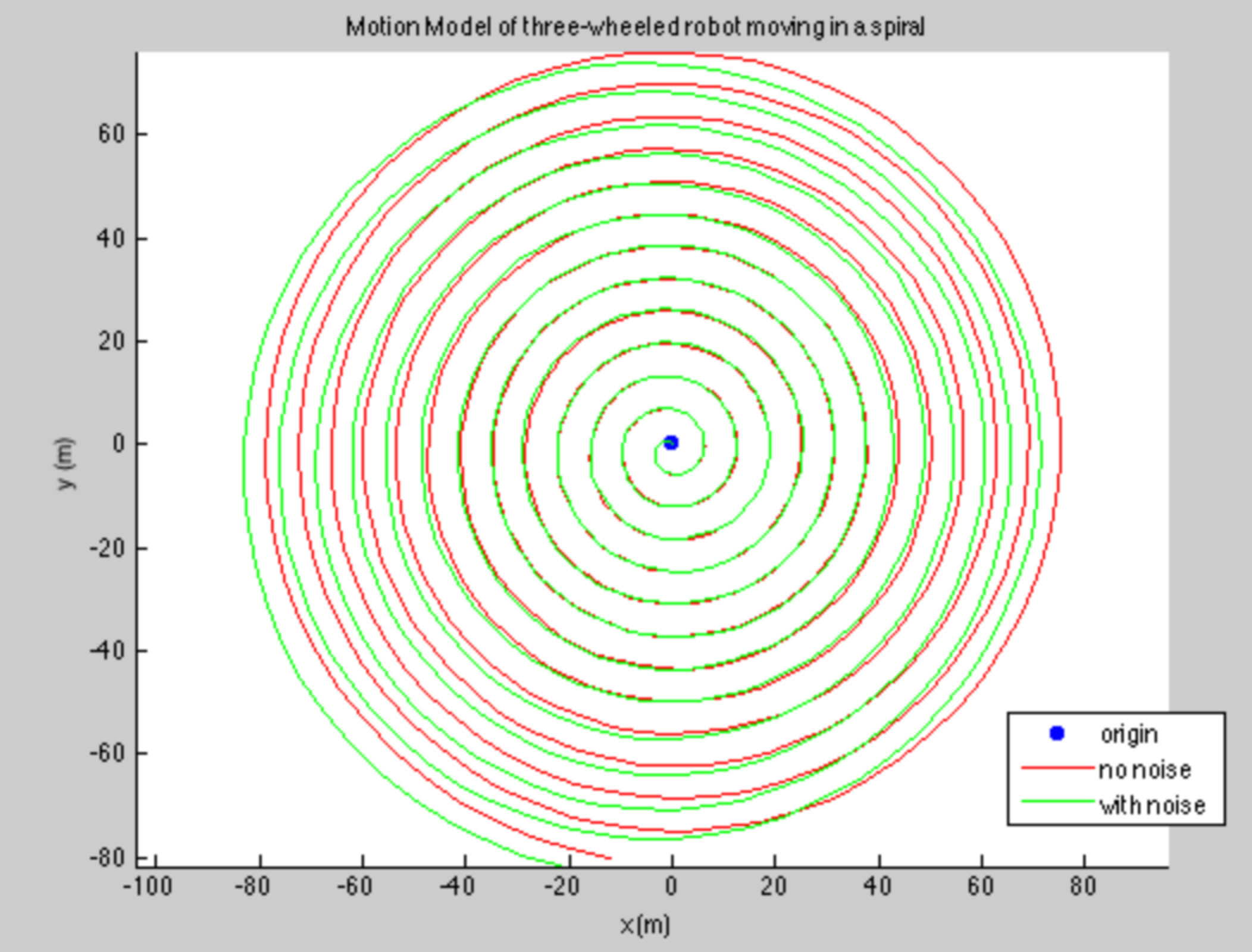


Figure 5 – Omnidirectional wheeled robot moving in spiral

# 3 Measurement Model

## GPS

The measurement model for the robot will use GPS and a magnetometer on a 2D plane. The GPS output will have error in the form of an additive Gaussian distribution with standard deviation of 0.50m in both the north and east direction. The final measurement, yg will be as follows:

Yg = CXg + dg

The matrix C represents the covariance between the different measurements, in this case just an identity matrix, and dg = N(0, 0.5).

## Magnetometer

The magnetometer also has additive Gaussian noise of 10 degrees north, with the model presented as follows:

Ym = CXm + dm, dm = N(0, )

Represented as one model, the measurements can be represented as:

,

It is also noted that at Waterloo the declination is 9.7 degrees west but since the robot is moving in a fairly small area and this will just be a constant offset so it is ignored here.

# 4 Extended Kalman Filter

The extended Kalman filter is simply a version of the Kalman filter where the measurement and motion model are linearized in order to approximate non-linear motion model. The states vector that will be used to model the robot is as follows:

One may argue a better choice of state vector will be include vx, vy and vtheta instead of the inputs. However, from the state vector above, we can simply multiply another matrix to get vx, vy and vtheta from w1, w2, w3. On top of that using w1, w2, w3 makes the modeling a lot easier. It will also be helpful later to visualize how well the filter is estimating its input velocity. Thus, the particular choice of state vector above is used.

The linearized motion model is represented as follows:

where the motion equations are broken down by the direction in the inertial frame, x,y and with respect to the angular velocities of the wheels, Since the state vector was picked to have and the position can be represented using this information as well as the previous state, the current position can be achieved without any linearization. The prediction update equations for the mean and covariance are then represented by:

Since the measurement model is already linear for the specific problem, it can be represented as:

The measurement update equations can then be written as:

The update equations were then used with the rotation inputs of w1 = -1.5 rad/s, w2 = 2.0 rad/s and w3 = 1.0 rad/s over a 15 second simulation. The measured and estimated positions of the robot in the x, y directions are illustrated in the figures below, with the estimate shown in red and the measurements in blue. The x and y position correct themselves after slight divergences at the start and the theta position is pretty accurately predicted.

|  |  |  |
| --- | --- | --- |
| Figure 6 – True x position vs. EFK predicted x position | Figure 7 – True y position vs. EFK predicted y position | Figure 8 – True theta position vs. EFK predicted theta position |
| Legend: Red is EFK prediction, Blue is true position | | |

The measured and estimated motion of the robot is illustrated in the figures directly below. As can be seen, the angular velocities of all three wheels converge back to the measured values over time.

|  |  |  |
| --- | --- | --- |
| Figure 9 – True w1 velocity vs. EFK predicted w1 velocity | Figure 10 – True w2 velocity vs. EFK predicted w2 velocity | Figure 11 – True w3 velocity vs. EFK predicted w3 velocity |
| Legend: Red is EFK prediction, Blue is true position | | |

For the purpose of better visualization, the error ellipses were screen captured at multiple points while the simulation was running over a 50 second interval instead of the 15 seconds outlined in the instructions. The illustrations are shown chronologically below. The prediction was set to a value far off of the actual position to start but quickly corrected itself. Immediately after this, in the third image, the prediction began to diverge a bit but quickly converged back to the actual measurements and after multiple circles, as seen in the last image, the prediction followed actual measurements fairly closely.

|  |  |
| --- | --- |
| EFK iterations | |
|  |  |
|  |  |
|  |  |
| Legend: Blue is EFK prediction, Red is true position | |

# 5 Multi-Rate Kalman Filter

The multi-rate Kalman filter is being used in this case to update the GPS measurements to an improved standard deviation of 0.01m at a 1Hz frequency. This means that every 10 measurements, one will be significantly more accurate than the others. The same extended Kalman filter was used to make predictions, with the only change being that a different Q was used every 10 measurements.

The predictions and measurements for the different states (position and angular velocity) are displayed in the figures below.

|  |  |  |
| --- | --- | --- |
| Figure 12 – True x position vs. EFK predicted x position velocity | Figure 13 – True y position velocity vs. EFK predicted y position | Figure 14 – True theta position vs. EFK predicted theta position |
| Figure 15 – True w1 velocity vs. EFK predicted w1 velocity | Figure 16 – True w2 velocity vs. EFK predicted w2 velocity | Figure 17 – True w3 velocity vs. EFK predicted w3 velocity |
| Legend: Red is EFK prediction, Blue is true position | | |

It can be seen that the position predictions are definitely closer than previous predictions when the multi rate measurement was not used in both the x and y directions. The change in measurement had little to no affect on the angular velocities though which is expected as the improved measurement only inspires more confidence in the position. The error ellipses illustrate the same thing, with the confidence improving and error ellipse shrinking every time the measurements improve. Overall, there is a fairly significant improvement in the prediction.

|  |  |
| --- | --- |
| Multirate EFK iterations | |
|  |  |
|  |  |
| Legend: Blue is EFK prediction, Red is true position | |

|  |  |
| --- | --- |
|  |  |
|  |  |