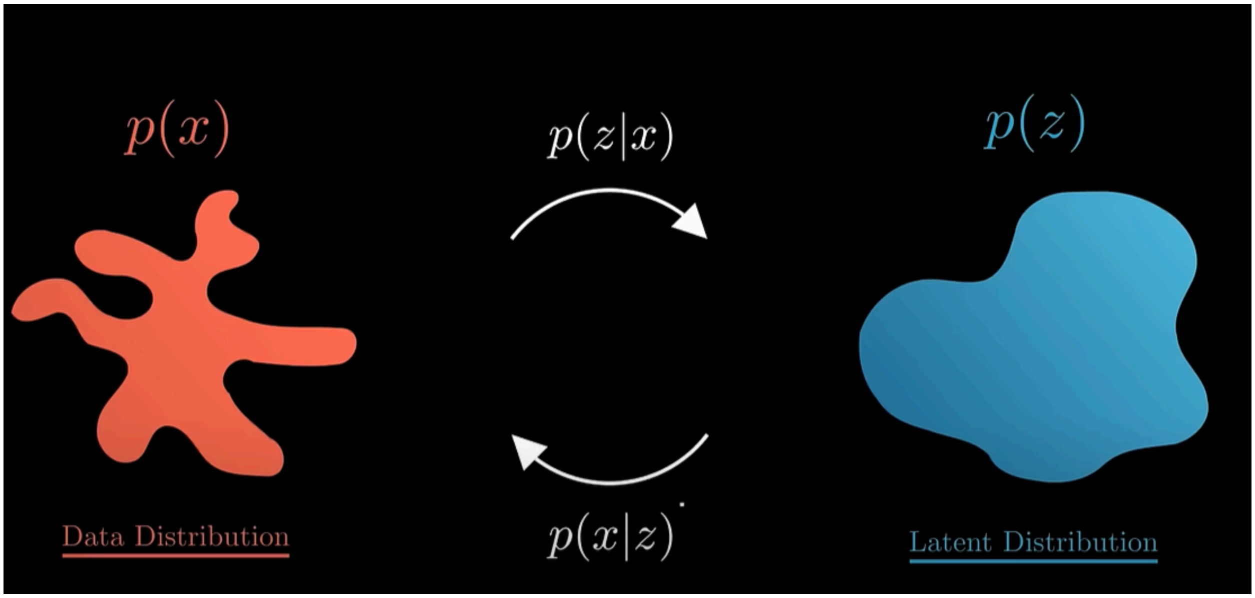
p(x): pure data
p(z): low-variable data
We need to connect the 2 by mapping
p(z|x) = posterior distribution -> Gives the probability that the latent vector z was generated by a particular image x
p(x|Z) = likelihood distribution -> gives the probability of reconstructing an image x from it



But we need to make sure p(z) = normal distribution = N(0,1)

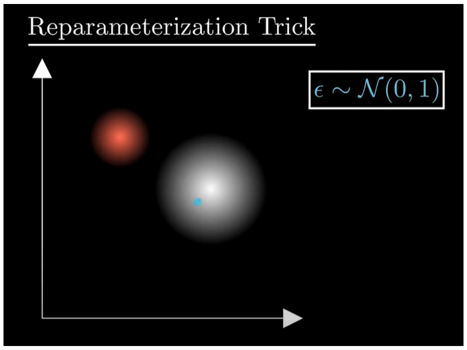
Also we don't know p(z|x), so we make Gaussian distribution q(z|x) = N(u,o) (variational Baes), making a deep encoder to estimate these parameters from the images

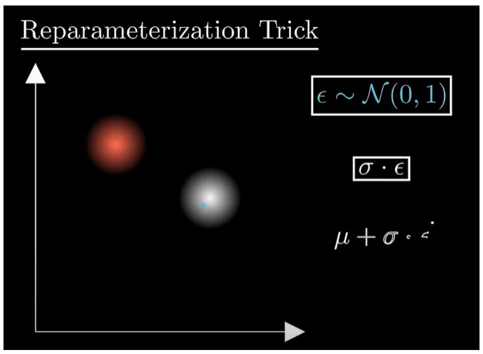
Then we will use a decoder to reconstruct images from latent variables that sampled from the approximate posterior

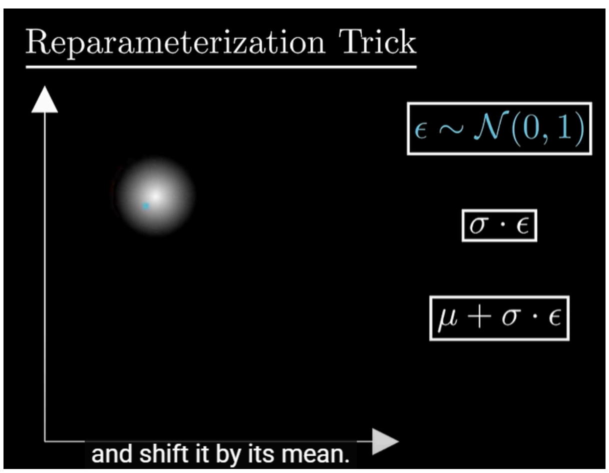
Data consistency becomes L2
Regularization becomes Latent space regularization

$$\mathcal{L}(x) = \underbrace{\mathbb{E}_{q(z|x)}\left[\log p(x|z)\right]}_{\text{Data consistency}} - \overbrace{\mathrm{KL}(q(z|x) \mid p(z))}^{\text{Regularization}}$$

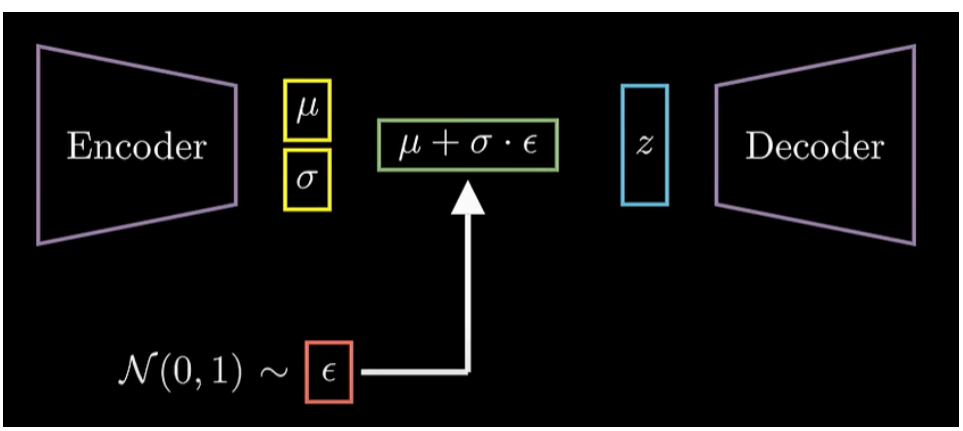
But how do we backpropagate?

- e = First Sample a random point from a normal distribution: (blue dot)
- 

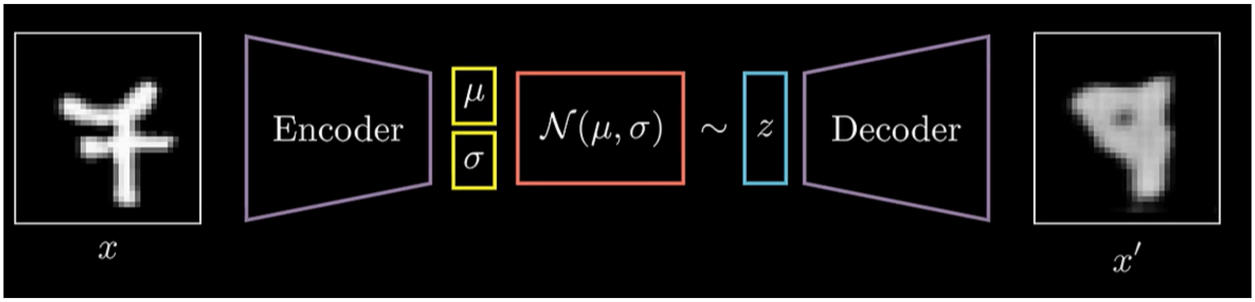
- o * e = Rescale by the variance of our approximated posterior distribution
- 

- u+o*e = Now shift it by its mean
- 

- Not it looks like we just sampled from the posterior distribution, except by using the reparameterization trick



1. Put an image through the network, it reconstructs the image



2. We now compare this new image with the original image to measure how well the network is performing

$$\mathcal{L} = \mathcal{L}_{KL}(\mathcal{N}(\mu, \sigma) \mid \mathcal{N}(0, 1)) + \boxed{\mathcal{L}_2(x, x')}$$

3. Now bc we are using a VAE, the network also output the parameters u and o of the approximate posterior distribution. So we then calculate the Kullback-Leibler divergence between this posterior distribution and the prior distribution (the standard normal distribution)

$$\mathcal{L} = \boxed{\mathcal{L}_{KL}(\mathcal{N}(\mu, \sigma) \mid \mathcal{N}(0, 1))} + \mathcal{L}_2(x, x')$$

4. For 2 Gaussian distributions, the KL divergence has the following closed Form

$$\mathcal{L}_{KL} = -\frac{1}{2}(1 + \log(\sigma^2) - \mu^2 - \sigma^2)$$

5. Compute this quantity, Backpropagate the combined loss, update the network's weights and repeat for each image in the dataset