Assessment: Individual Coursework 1

Due date: 13th December 2019, 1pm

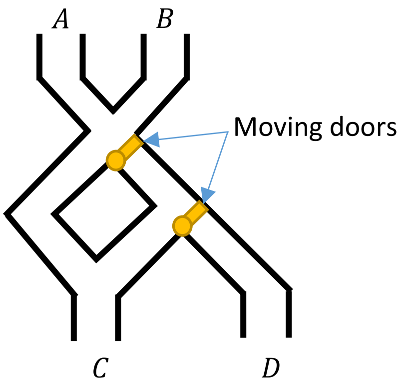
Question Sheet

Total Marks: 100

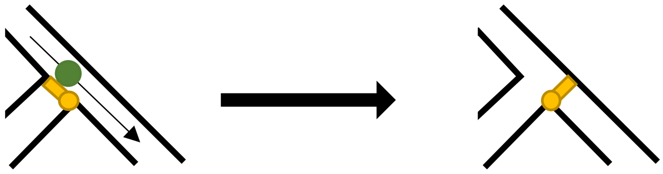
Assignment Rules:

1. *Submission and late submission:* The solutions **must be submitted in pdf** only via Canvas. You may wish to complete your coursework electronically using Word (utilising Microsoft Equation Editor where appropriate), LaTeX, or other technology which facilitates mathematical typesetting. If you complete your coursework on paper you must create a digital copy of your work using a scanner. **Photographs of paper based coursework will not be accepted.** Late submissions will be deducted 5% marks per day of delay. No submission will be accepted more than 7 days later than deadline.
2. *No plagiarism allowed:* This is an individual assignment. No plagiarism is permitted: you should not copy your solutions from each other or any resource. If you need to refer to online sources/books (e.g. for some new definition), you must refer to them appropriately. If plagiarism is detected you may lose marks and/or face other action.
3. *Collusion is not permitted:* The submitted work should be solely of your own completion in accordance of Section 2.5 of the Academic Offences guidelines. You are not permitted to work with other students or third parties e.g. using online forums or pay-for-solution websites.
4. A guide to academic offences for students can be viewed: https://www.qub.ac.uk/directorates/AcademicStudentAffairs/AcademicAffairs/AppealsComplaintsandMisconduct/AcademicOffences/Student-Guide/ .
5. This is an open book and open resource assignment. You are allowed to access books and online resources. However, you must attribute sources (see point 6) and the solutions must be in your own words.
6. *Attribution:* If at all you need to cite any sources/books (*standard definitions do not require a citation*), have a separate **references** section at the end. All the references should be present using a single standardised reference style (e.g. IEEE, APA, Harvard etc.).
7. *Show working out:* It is recommended that you show your working out throughout this assignment. This can be beneficial in case you make some mistake in which case partial marks may be awarded for showing the correct process.

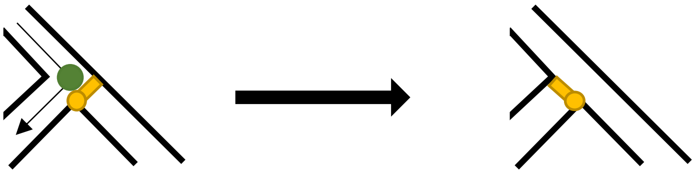
1. Consider the toy marble machine shown below:



A sequence of marbles are inserted at the top of the machine through either opening A or B. The marbles can be either green (g) or red (r). If the marble is green it is dropped through opening A and if the marble is red it is dropped through opening B. The machine contains two moving doors. Whenever a green marble interacts with a door, it will change position so that the next marble will take the alternate path. Below is an illustration of how the interaction with the green marble causes the moving doors to change:



Or



A sequence of inputs are accepted if the last marble exits the machine at D.

a) Construct a Deterministic Finite Automaton (DFA) to model the marble machine's computation by

   i. Illustrating your DFA using a finite state diagram. (10 marks)

   ii. Providing a formal, 5-tuple, definition of your DFA. (8 marks)

b) State whether the machine would either "Accept" or "Reject" the following two sequences of input marbles:

   i. gggg
   ii. rgrgrgrg

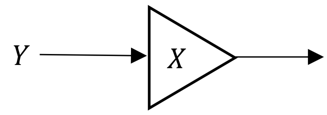
(2 marks)

c) Describe the language of the machine using a regular expression.

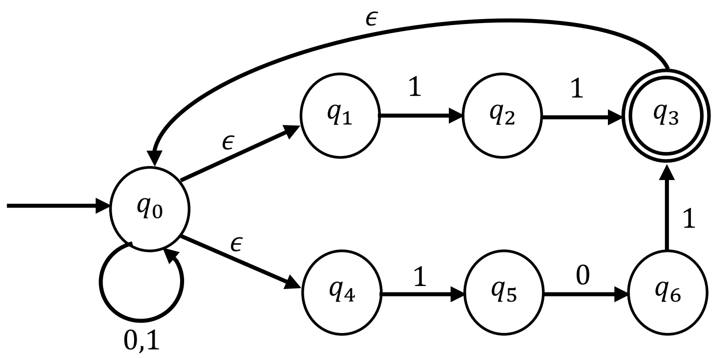
(5 marks)

2. A neuron can be considered a single processing unit within the central nervous system. A single neuron may be connected to many other neurons. A neuron will either receive a spike (1) or no spike (0) from another neuron. The sequence of inputs will determine if a neuron will activate or not activate.

a) Below is an example of a neuron (denoted X) that receives input from one other neuron (denoted Y).



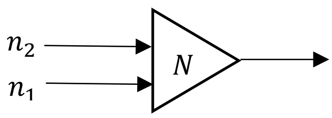
The neuron X will activate (i.e. accept) any input sequence that ends with two consecutive 1-inputs or the sequence 101. The computation of neuron X can be modelled by the non-deterministic finite automaton (NFA) illustrated in the finite state diagram below.



Convert the NFA above to an equivalent DFA to model the computation of neuron X. Illustrate your solution with a finite state diagram [You do not have to provide a formal, 5-tuple, definition of your solution. You are advised to show your working out so that partial marks may be awarded if the solution is not entirely correct].

(10 marks)

b) A different neuron, N, receives input from two neurons, n_1 and n_2, **simultaneously.** This configuration is illustrated below:



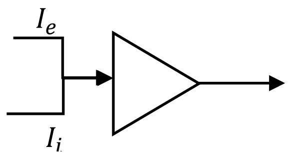
Neurons n_1 and n_2 will **each** provide an input of 1 or 0. Neuron N will activate (i.e. accept) if the input sequence ends with an input of 1 from **both** n_1 and n_2. Alternatively, N will activate (i.e. accept) if the input sequence ends with two consecutive 1's from n_1 or two consecutive 1's from n_2. Once neuron N reaches a state of activation (i.e. acceptance), its behaviour is reset before further input is received. Assume that the starting inputs occur immediately after neuron N has just reset.

Construct an NFA to model the behaviour of N by

i. Illustrating your NFA using a finite state diagram. (12 marks)

ii. Providing a formal, 5-tuple, definition of your NFA. (11 marks)

3. A neuron receives a single input which is **either** an excitatory input, I_e, or an inhibitory input, I_i, as illustrated in the diagram below:



The neuron will activate (or accept) if for every inhibitory input there exists **at least** twice as many excitatory inputs **later** in the input sequence.

For example, the following input sequences would cause the neuron to activate:

- $I_iI_eI_iI_eI_eI_eI_e$
- $I_iI_iI_eI_eI_eI_e$
- $I_eI_iI_eI_e$

Whereas, the following input sequences would **not** cause the neuron to activate:

- $I_eI_eI_i$
- $I_eI_iI_e$
- $I_iI_eI_iI_e$

Identify an appropriate automaton to model the behaviour of this neuron and illustrate your solution with a finite state diagram.

(15 marks)

4. Decide whether or not the language

$$L = \left\{ a^i b^j \mid i > 2j, \text{for } i, j \in \mathbb{N} \right\}$$

over the alphabet $\{a, b\}$ is regular. If so, design a DFA/NFA to recognise it, and if not, give a formal proof (based on the Pumping Lemma).

(10 marks)

5. Consider the following context free grammar:

$$\boldsymbol{X \rightarrow 0X2 \mid 0Y}$$
$$\boldsymbol{Y \rightarrow 1Y \mid \epsilon}$$

a) Using a parse tree show how the word 0012 is derived from the grammar above.

(4 marks)

b) Provide a set definition to describe the language generated by this grammar.

(5 marks)

c) Convert the grammar stated at the start of this question to Chomsky-Normal Form (CNF).

(8 marks)