CSC 2047 - Individual Coursework 2 - James Earls

note: Exceptional circumstances granted an extension for this assignment.

1.

1a) No- The most complex term in the expression n! + 3n 6 + 2n 3 = O(n 6) is n! Therefore the most dominant case in the equation is O(n!)

1b) Yes

1c) Yes

1d) Yes

1e) No- The most complex term in the expression $3 \cdot 2^n = O(1.5 n)$ is 2^n Therefore the most dominant case in the equation is $O(2^n)$

1f) Yes

1g) Yes

1h) Yes

1i) Yes

1j) Yes

2.

2a)

i. $(G_1, 1) \in L$ - Holds
ii. $(G_2, 2) \in L$ - Doesn't Hold
iii. $(G_3, 1) \in L$ - Doesn't Hold
iv. $(G_3, 4) \in L$ - Doesn't Hold
v. $(G_4, 3) \in L$ - Holds
vi. $(G_4, 4) \in L$ - Doesn't Hold

2b)

D = "On input<G,n> where G is an undirected graph with n connected components

1. Select the first node of G and mark it.
2. Repeat the following stage until all nodes that are connected to the marked node are considered:
3. For each node in G, mark it if it is attached by an edge to a node that is already marked.
4. when all the nodes that are connected to a marked node have been marked increment the number of components by 1.
5. Repeat steps 3 and 4 to ensure all nodes are visited from the next node.
6. End the loop
7. Accept if n is equal to the number of components, otherwise reject.

Sourced from Decidable languages slide deck

2c)

3.

3a)

i. q_0#101

ii. #$q_1$101

3b)

i. q_0#

#q_1

q_2#

#q_a

yes

ii. q_0##

#q_1#

#q_r#

no

iii. $q_0$0

q_r0

no

iv. q_0#0

#$q_1$0

#0q_1

#q_{20}

q_3#1

#$q_1$1

#1q_1

#$q_2$1

q_2#0

#q_a0

yes

v. $q_{0,}$#1

#$q_1$1

#1q_1

#$q_2$1

q_2#0

#q_a0

yes

vi. q_0#00

#$q_1$00

#0$q_1$0

#00q_1

#0$q_2$0

#$q_3$01

q_3#01

#$q_1$01

#0$q_1$1

#01q_1

#0$q_2$1

#$q_2$00

q_3#10

#$q_1$10

#1$q_1$0

#10q_1

#1$q_2$0

#$q_3$11

$q_3\#11$

$\#q_1 11$

$\#1q_1 1$

$\#11q_1$

$\#1q_2 1$

$\#q_2 10$

$q_2\#00$

$\#q_a 00$

Yes

vii. $q_0\#01$

$\#q_1 01$

$\#0q_1 1$

$\#01q_1$

$\#0q_2 1$

$\#q_2 00$

$q_3\#10$

$\#q_1 10$

$\#1q_1 0$

$\#10q_1$

$\#1q_2 0$

$\#q_3 11$

$q_3\#11$

$\#q_1 11$

$\#1q_1 1$

$\#11q_1$

$\#1q_2 1$

$\#q_2 10$

$q_2\#00$

$\#q_a 00$

Yes

viii. $q_0\#10$

$\#q_1 10$

$\#1q_1 0$

$\#10q_1$

$\#1q_2 0$

$\#q_3 11$

$q_3\#11$

$\#q_1 11$

$\#1q_1 1$

$\#11q_1$

$\#1q_2 1$

$\#q_2 10$

$q_2\#00$

$\#q_a 00$

yes

ix. $q_0\#11$

$\#q_1 11$

$\#1q_1 1$

$\#11q_1$

$\#1q_2 1$

$\#q_2 10$

$q_2\#00$

$\#q_a 00$

yes

x. $q_0\#101$

$\#q_1 101$

$\#1q_1 01$

$\#10q_1 1$

$\#101q_1$

$\#10q_21$

$\#1q_200$

$\#q_3110$

$q_3\#110$

$\#q_1110$

$\#1q_110$

$\#11q_10$

$\#110q_1$

$\#11q_20$

$\#1q_311$

$\#q_3111$

$q_3\#111$

$\#q_1111$

$\#1q_111$

$\#11q_11$

$\#111q_1$

$\#11q_21$

$\#1q_210$

$\#q_2100$

$q_2\#000$

$\#q_a000$

yes

3c)
#(0 | 1)*

3d)
#0*

3e)
$0(n^2)$

3f)
0(n)

3g)

$T = (Q, \Sigma, \Gamma, \delta, q_0, q_a, q_r)$:

$Q = \{q_0, q_1, q_2, q_3, q_r\}$

$\Sigma = \{\#, 0, 1\}$

$\Gamma = \{0, 1, \_\}$

$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$

	0	1	#	_
q_0	-	-	$(q_1,\#,R)$	-
q_1	$(q_1,0,R)$	(q_1,R)	-	$(q_2,\_,L)$
q_2	$(q_3,1,L)$	$(q_2,0,L)$	$(q_a,\#,R)$	-
q_3	$(q_3,0,L)$	$(q_3,1,L)$	$(q_3,\#,R)$	-
q_a	-	-	-	-

4.

4a)

i. $q_0,q_{0,}q_0$

$q_7,q_{7,}q_7$

$q_{a,}q_{a,}q_a$

yes

ii. $q_0abc,q_{0,}q_0$

q_1abc,aq_1,aq_1

q_2abc,aaq_2,aaq_2

$aq_0bc,aaaq_0,aaq_0$

$aq_3bc,aaaq_3,aaq_3$

aq_4bc,aaq_4a,aaq_4

abq_3c,aaq_3,aaq_3

abq_5c,aaq_5,aaq_5

abq_6c, aaq_6, aaq_6a

$abcq_5, aaq_5, aq_5$

$abcq_7, aq_7a, q_7a$

$abcq_r, aq_ra, q_ra$

no

iii. $q_0abbbcc, q_0, q_0$

$q_1abbbcc, aq_1, aq_1$

$q_2abbbcc, aaq_2, aaq_2$

$aq_0bbbcc, aaaq_0, aaq_0$

$aq_3bbbcc, aaaq_3, aaq_3$

aq_4bbbcc, aaq_4a, aaq_4

abq_3bbcc, aaq_3, aaq_3

abq_4bbcc, aq_4a, aaq_4

$abbq_3bcc, aq_3, aaq_3$

$abbq_4bcc, q_4a, aaq_4$

$abbbq_3cc, q_3, aaq_3$

$abbbq_5cc, q_5, aaq_5$

$abbbq_6cc, q_6, aq_6a$

$abbbcq_5c, q_5, aq_5$

abbbcq_6c,q_6,q_6a

abbbccq_5,q_5,q_5

abbbccq_6,q_6,q_6

yes

iv. $q_0$abbb,q_0,q_0

$q_1$abbb,aq_1,aq_1

$q_2$abbb,aaq_2,aaq_2

a$q_0$bbb,aaaq_0,aaq_0

a$q_3$bbb,aaaq_3,aaq_3

a$q_4$bbb,aaq_4a,aaq_4

ab$q_3$bb,aaq_3,aaq_3

ab$q_4$bb,aq_4a,aaq_4

abbq_3b,aq_3,aaq_3

abbq_4b,q_4a,aaq_4

abbbq_3,$q_3$aaq_3

abbb$_r$,q$_r$,aaq$_r$

no

v. $q_0$bac,q_0,q_0

$q_3$bac,q_3,q_3

$q_4$bac,q_4,q_4

$q_r$bac,q_r,q_r

no

vi. $q_0$aabbbbbbcccc,q_0,q_0

$q_1$aabbbbbbcccc,aq_1,aq_1

$q_2$aabbbbbbcccc,aaq_2,aaq_2

a$q_0$abbbbbbcccc,aaaq_0,aaq_0

a$q_1$abbbbbbcccc,aaaaq_1,aaaq_1

a$q_2$abbbbbbcccc,aaaaaq_2,aaaaq_2

aa$q_0$bbbbbbcccc,aaaaaaq_0,aaaaq_0

aa$q_3$bbbbbbcccc,aaaaaaq_3,aaaaq_3

aa$q_4$bbbbbbcccc,aaaaaq_4a,aaaaq_4

aab$q_3$bbbbbcccc,aaaaaq_3,aaaaq_3

aab$q_4$bbbbbcccc,aaaaq_4a,aaaaq_4a

aabb$q_3$bbbbcccc,aaaaq_3,aaaaq_3

aabb$q_4$bbbbcccc,aaaq_4a,aaaaq_4

aabbb$q_3$bbbcccc,aaaq_3,aaaaq_3

aabbb$q_4$bbbcccc,aaq_4a,aaaaq_4

aabbbb$q_3$bbcccc,aaq_3,aaaaq_3

$aabbbbq_4bbcccc,aq_4a,aaaaq_4$

$aabbbbbq_3bcccc,aq_3,aaaaq_3$

$aabbbbbq_4bcccc,q_4a,aaaaq_4$

$aabbbbbbq_3cccc,q_3,aaaaq_3$

$aabbbbbbq_5cccc,q_5,aaaaq_5$

$aabbbbbbq_6cccc,q_6,aaaq_6a$

$aabbbbbbcq_5ccc,q_5,aaaq_5$

$aabbbbbbcq_6ccc,q_{6,}aaq_6a$

$aabbbbbbccq_5cc,q_5,aaq_5$

$aabbbbbbccq_6cc,q_6,aq_6a$

$aabbbbbbcccq_5c,q_5,aq_5$

$aabbbbbbcccq_6c,q_6,q_6a$

$aabbbbbbccccq_5,q_5,q_5$

$aabbbbbbccccq_7,q_7,q_7$

$aabbbbbbccccq_a,q_a,q_a$

yes

4b)

$\{a^x b^{x(x3)} c^{x(x2)}\}$ $X \geq 0$ for $X \in \mathbb{N}^*$

4c)

5.

5a)

i. $q_0 \varepsilon$

$q_r \varepsilon$

No

ii. $q_0 0$

$0 q_0$

$0 q_r$

No

iii.
$$\begin{array}{ll} q_0 1 \rightarrow & 1 q_0 \\ \downarrow & q_r \\ q_1 \_ & \\ q_r & \end{array}$$

No

iv. $q_0 01$

$$\begin{array}{ll} 0 q_0 1 \rightarrow & 0 \_ q_1 \\ \downarrow & q_r \\ 01 q_0 & \\ q_r & \end{array}$$

No

v.
$$\begin{array}{ll} q_0 10 \rightarrow & q_1 \_ 0 \\ \downarrow & q_r \\ 1 q_0 0 & \\ 10 q_0 & \\ q_r & \end{array}$$

No

vi. $q_0$101 □⟶ q_1_01

↓ q_r

1$q_0$01

10$q_0$1 ⟶ 1$q_1$0

↓ 10q_a

1$q_1$0

10q_a

Yes

vii. $q_0$111000 □□□⟶ q_1_1100

| q_r

↓ q_r_1100

1$q_0$1100 □⟶ q_1__100

↓ q_r__100

11$q_0$100 ⟶ q_1___00

↓ q_r___00

111$q_0$00

1110$q_0$0

11100q_0

11100q_r

No

viii. $q_0$10100 □⟶ q_1_0100

↓ q_r_0100

1$q_0$0100

10$q_0$100 ⟶ 1$q_1$0_00

101$q_0$00 10q_a_00

1010$q_0$0

10100q_0

10100q_r

Yes

5b)
$(0 \cup 1)^* \, 01 \, (0 \cup 1)^*$

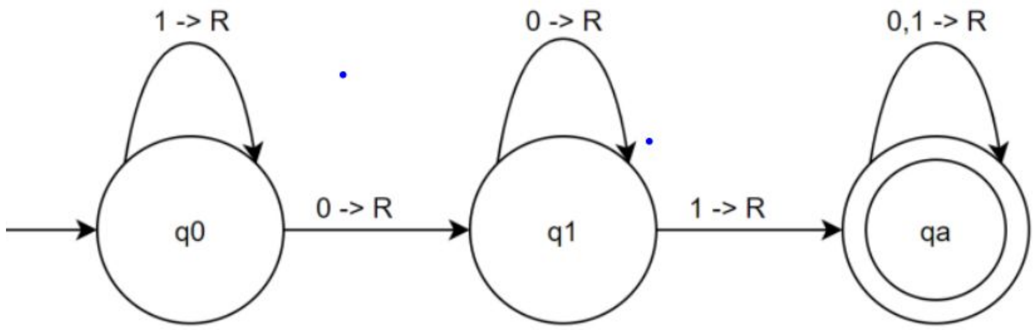
5c)
f(n) = n + n -1 + 1

f(n) = 2n

5d)
g(n) = n + 1

5e)
n + 1

5f)
1

5g)


5h)
T = (Q, Σ , Γ , δ , q_0 , q_a , q_r):

$Q = \{q_0, q_1, q_2, q_3, q_r\}$

$\Sigma = \{\#, 0, 1\}$

$\Gamma = \{0, 1, \_\}$

$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$

	0	1	_
q_0	$\{(q_0,0,R)\}$	$\{(q_0,0,R), (q_1,\_,L)\}$	-
q_1	$\{(q_a,0,R)\}$	$\{(q_1,\_,L)\}$	-
q_a	-	-	-

6.

6a)

6b)

6c)

6d)

6e)

6f)

7.

7a)

$$f(n) = \begin{cases} 1 & \text{if } n = 0 \\ f(n-1) + 2n & \text{else} \end{cases}$$

$$g(n) = \begin{cases} 1 & \text{if } n = 0 \\ 2g(n-1) + 2n & \text{else} \end{cases}.$$

i. f(0) = f(n-1) + 2n
= 1 + 0
= 1

ii. g(1) = 2g(n-1) + 2n
= 2g(1-1) + 2
= 2g(0) + 2
= 2(1) + 2
=4

iii. g(2) = 2g(n-1) + 2n
= 2(2-1) + 2 x 2
= 2(1) + 4
= 2(4) + 4
= 8 + 4
= 12

iv. f(10) = f(n-1) + 2n
= f(9) + 20
= f(8) + 18 + 20
= f(7) + 16 + 18 + 20
= f(6) + 14 + 16 + 18 + 20
= f(5) + 12 + 14 + 16 + 18 + 20
= f(4) + 10 + 12 + 14 + 16 + 18 + 20
= f(3) + 8 + 10 + 12 + 14 +16 + 18 + 20
= f(2) + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20
= f(1) + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20
= f(0) + 2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20
= 1 + 2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20
= 111

7b)

We have

f(n) = f(n-1) + 2n

f(n) = n^2 + n + 1

7c)

It seems like: $f(n) = n^2 + n + 1$

Proof By induction

Base Case: n = 0

$f(0) = 1$

$f(0) = n^2 + n + 1$ (by definition)

Induction Hypothesis:

$f(n-1) = (n-1)^2 + (n-1) + 1$

Induction step:

f(n) = f(n-1) + 2n

f(n) = (n-1) 2 + (n-1) + 1 + 2n

f(n) = n2 – 2n + 1 + n – 1 + 1 + 2n

f(n) = n2 + n + 1

By Mathematical induction, f(n) = n2 + n + 1 for all n >= 0

7d)

It seems like: g(n) = 5 · 2 n − 2n − 4

Proof by induction:

Base Case: n = 0:

g(n) = 1

g(0) = 5 · 2n-1 – 2(n-1) – 4 (by definition)

Induction Hypothesis: g(n-1) = 5 · 2n-1 – 2(n-1) – 4

Induction step:

g(n) = 2g(n-1) + 2n (by definition)

g(n) = 2(5 · 2n-1 – 2(n-1) – 4) + 2n (by induction hypothesis)

g(n) = 10 · (2n / 21) – 4n + 4n – 8 + 2n

g(n) = 10 · (2n / 2) – 2n – 4

g(n) = 5 · 2n – 2n – 4

By Mathematical induction, g(n) = 5 · 2n – 2n – 4