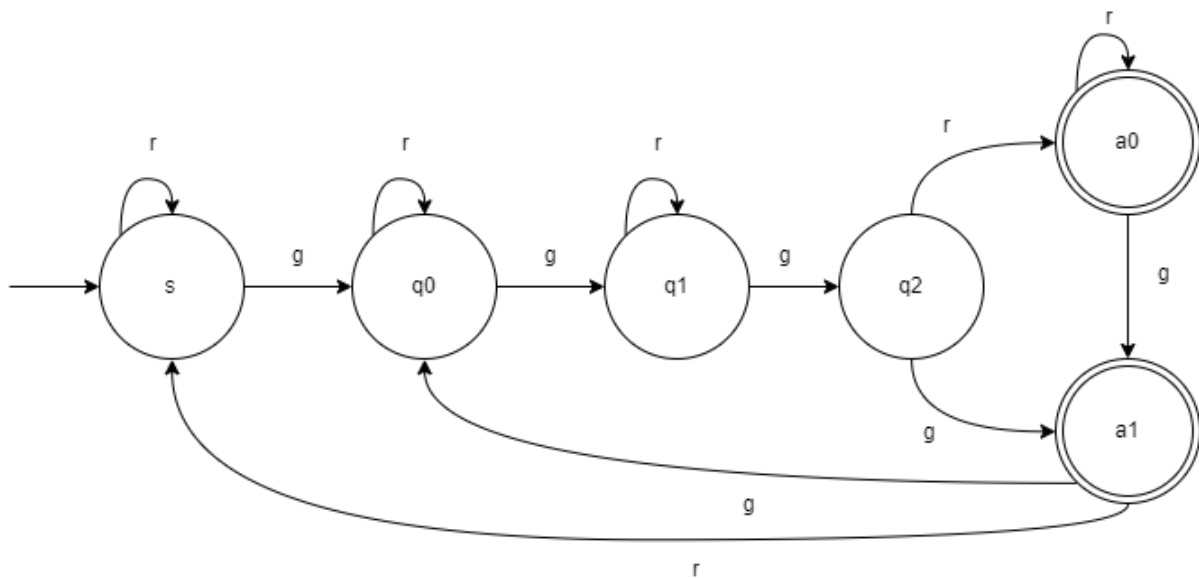


## Assessment: Individual Coursework 1

1ai)

s	q0	q1	q2
Door 1 = Right	Door 1 = Left	Door 1 = Right	Door 1 = Left
Door 2 = Right	Door 2 = Right	Door 2 = Left	Door 2 = Left
A0	A1		
Door 1 = Left	Door 1 = Right		
Door 2 = Left	Door 2 = Right		



1aii)

$$M = (Q, \Sigma, \delta, q_0, F)$$

$$Q = \{s, q_0, q_1, q_2, a_0, a_1\}$$

$$\Sigma = \{g, r\}$$

$$q_0 = s$$

$$F = \{a_0, a_1\}$$

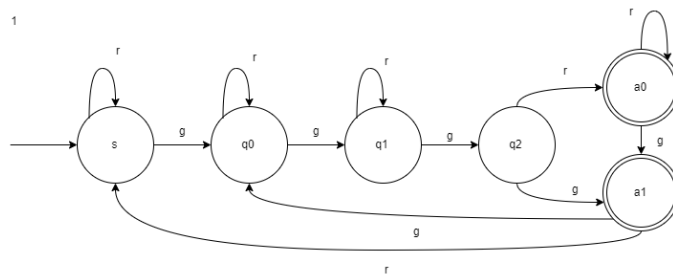
$$\delta: Q \times \Sigma \rightarrow Q \text{ is}$$

$Q \setminus \Sigma$	$g$	$r$
$s$	$q_0$	$s$
$q_0$	$q_1$	$q_0$
$q_1$	$q_2$	$q_1$
$q_2$	$a_1$	$a_0$
$a_0$	$a_1$	$a_0$
$a_1$	$q_0$	$s$

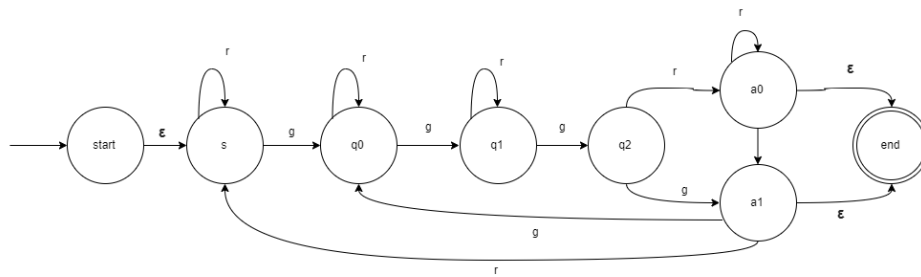
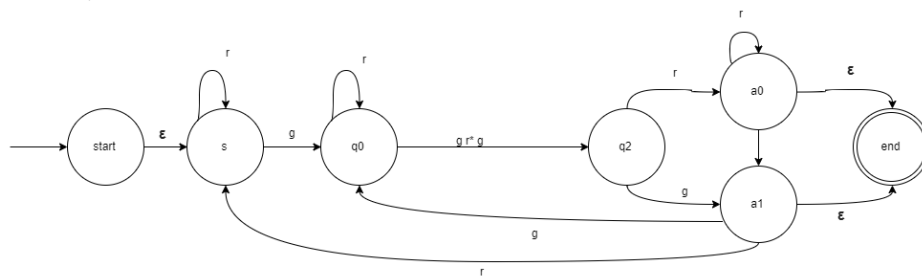
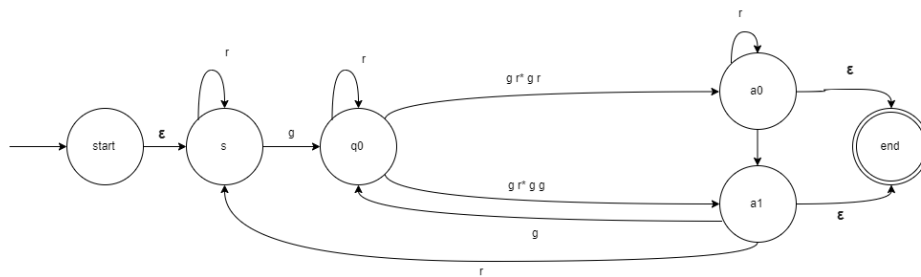
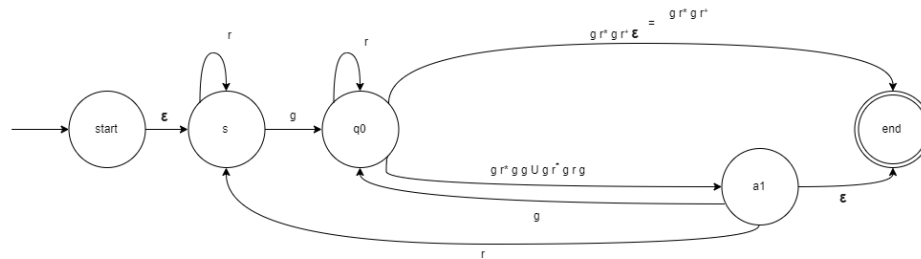
1bi) Accepted

1bii) Accepted

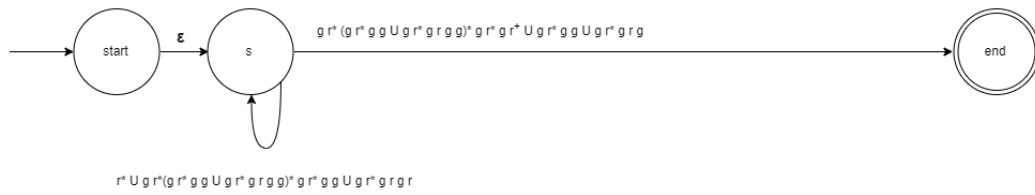
1c)



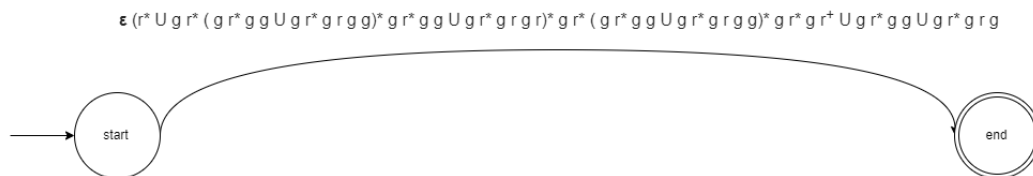
2 Add start and end states

3 Remove  $q_1$ 4 Remove  $q_2$ 5 Remove  $a_0$ 

6 Remove q0



7 Remove s



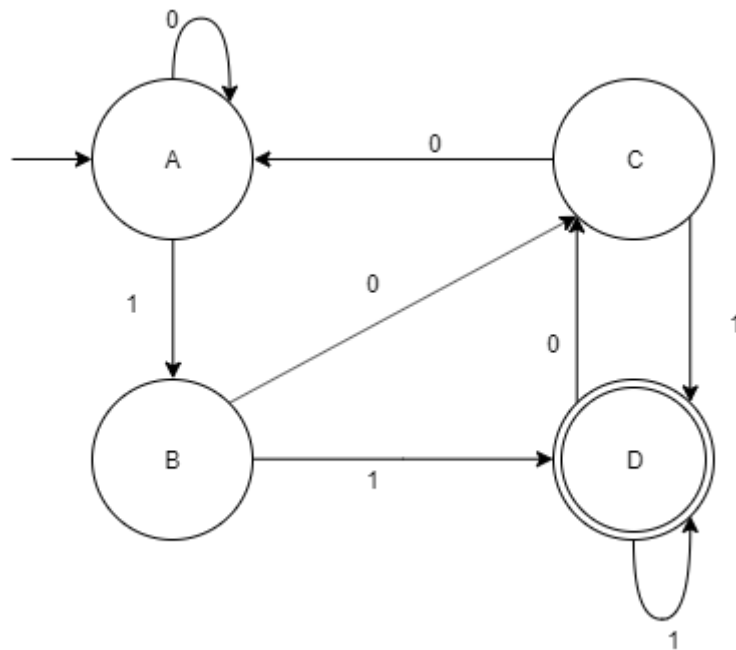
Therefore the language recognised by the automata can be described as:

$(r^* U g r^* (g r^* g g U g r^* g r g g)^* g r^* g r g r (g r^* g g U g r^* g r g g)^* g r^* g r^* U g r^* g g U g r^* g r g$

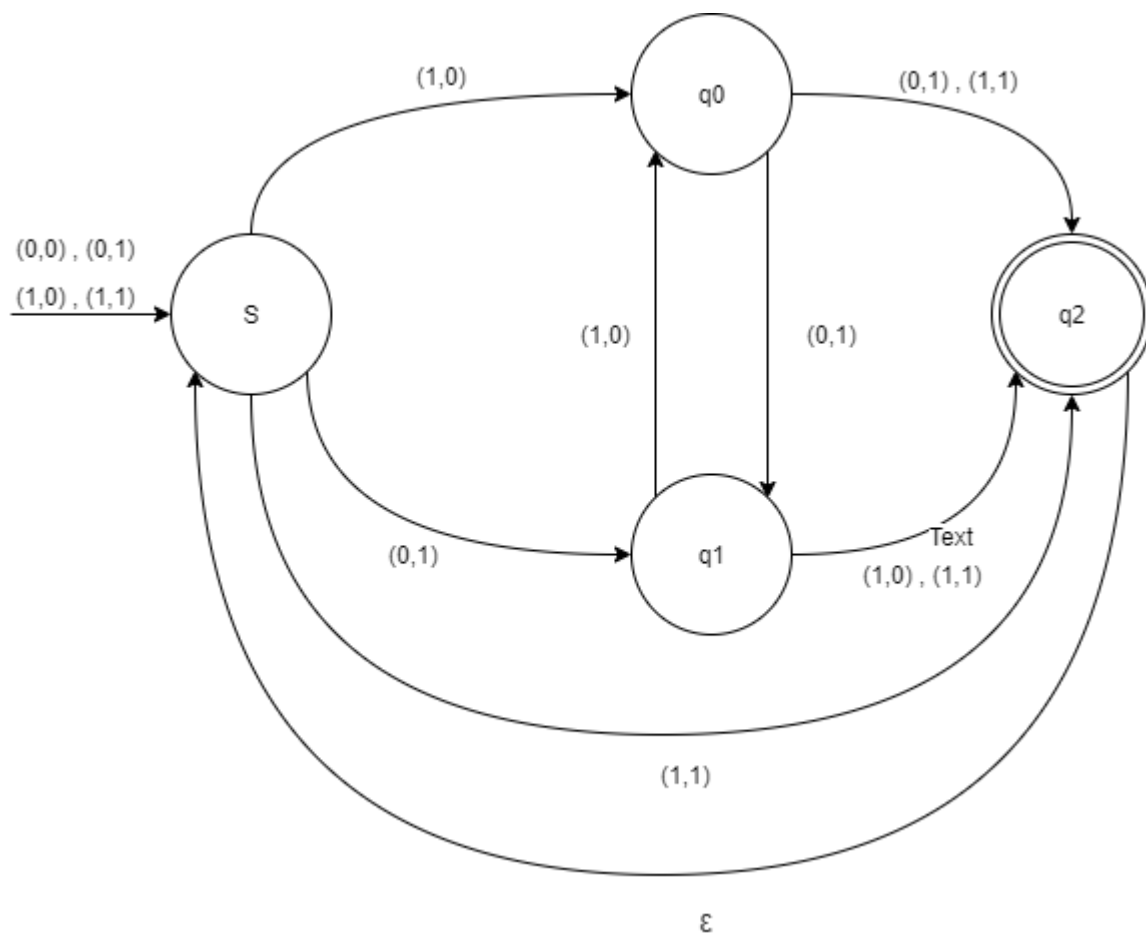
2a)

$Q \setminus \Sigma$	0	1	
$[q0]$	$[q0]$	$[q0\ q2\ q5]$	A
$[q0\ q2\ q5]$	$[q0\ q6]$	$[q0\ q2\ q5\ q3]$	B
$[q0\ q6]$	$[q0]$	$[q0\ q2\ q5\ q3]$	C
$[q0\ q2\ q5\ \underline{q3}]$	$[q0\ q6]$	$[q0\ q2\ q5\ q3]$	D

	0	1
A	A	B
B	C	D
C	A	D
D	C	D



2bi)

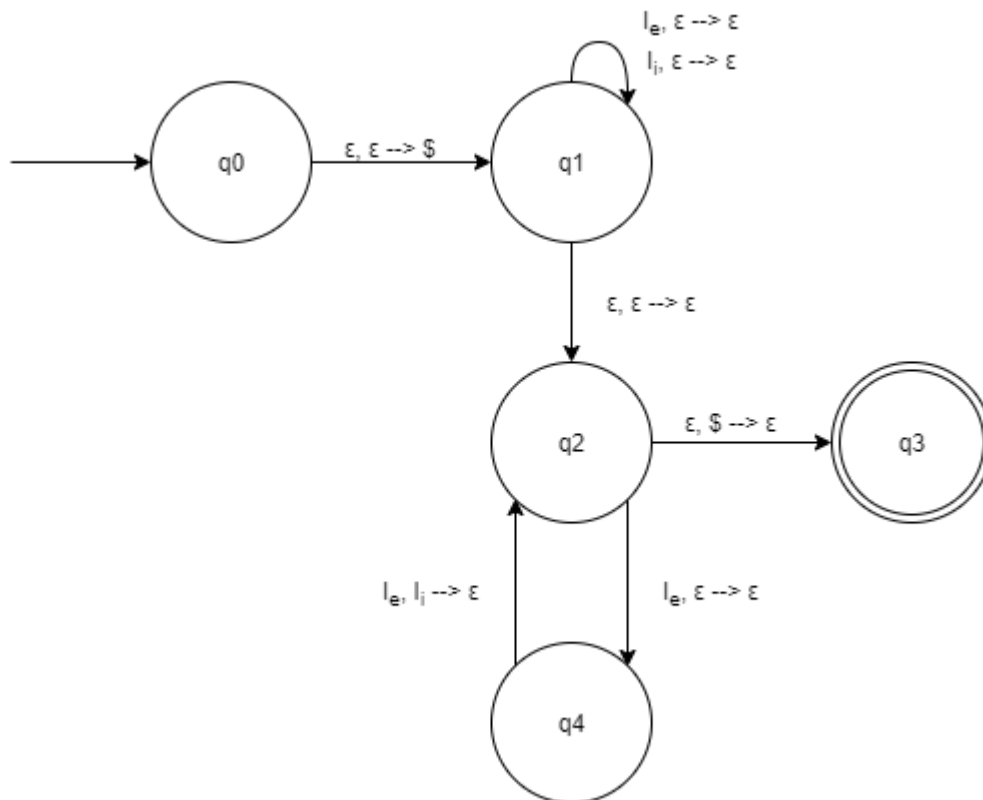


2bii)

$Q \setminus \Sigma$	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$s$	$\emptyset$	$\{q1\}$	$\{q0\}$	$\{q2\}$
$q0$	$\emptyset$	$\{q1, q2\}$	$\emptyset$	$\{q2\}$
$q1$	$\emptyset$	$\emptyset$	$\{q0, q2\}$	$\{q2\}$
$q2$	$\{s\}$	$\{s\}$	$\{s\}$	$\{s\}$

3)

$$\{(l_e^* l_i)^n l_e^{2N} \mid n > 0\}$$



4)

$$L = \{a^i b^j \mid i > 2j \text{ for } i, j \in \mathbb{N}\}$$

Assume by way of contradiction that  $L \in \text{REG}$ , then  $L$  satisfies the conditions of the pumping lemma.

Let  $P > 0$  be the pumping length

Consider the word  $S = a^{2p+1} b^p$

Clearly  $W \in L$  and  $|w| > p$  so according to the pumping lemma there exists  $x, y, z \in \Sigma$  such that  $w = x y z$ ,  $|x y| \leq p$ ,  $|y| > 0$  and for all  $i \geq 0$  it holds that  $x y^i z \in L$ .

Since  $|x y| \leq p$ , then  $x = a^n$ ,  $y = a^m$  and  $z = a^k b^p$  such that  $m + n + k = 2p + 1$  and  $m > 0$

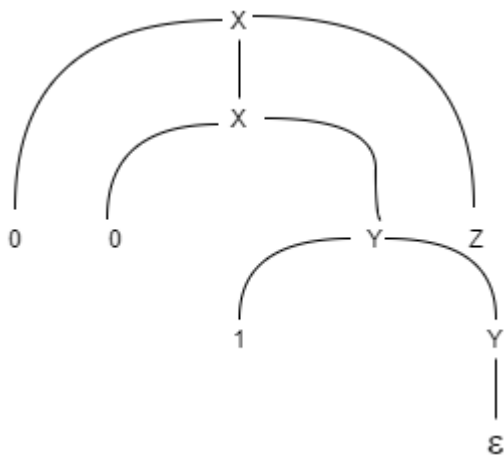
We pump with  $i = 0$  and get the word  $x z = a^n z = a^n a^k b^p$ .

Since  $m + n + k = 2p + 1$  and  $m > 0$  then  $n + k \leq 2p + 1$

Thus  $x z \notin L$  in contradiction to the pumping lemma

Therefore  $L$  is not regular.

5a)



5b)



5c)

$$X \rightarrow 0X2 \mid 0Y$$

$$Y \rightarrow 1Y \mid \epsilon$$

1: Create new start variable and ‘map’ it to the original start variable

$$S \rightarrow X$$

$$A \rightarrow 0X2 \mid 0Y$$

$$Y \rightarrow 1Y \mid \epsilon$$

2: Remove  $\epsilon$  rules

$$S \rightarrow X$$

$$A \rightarrow 0X2 \mid 0Y$$

$$Y \rightarrow 1Y \mid 1$$

3: Remove unit rules

$$S \rightarrow 0X2 \mid 0Y \mid 0$$

$$A \rightarrow 0X2 \mid 0Y \mid 0$$

$$Y \rightarrow 1Y \mid 1$$

4: “Break chains” of length  $> 2$ 

$$S \rightarrow 0X2 \qquad X \rightarrow 0X2$$

$$S \rightarrow 0Z_0 \qquad X \rightarrow 0Z_0$$

$$Z_0 \rightarrow X2$$

$$S \rightarrow T_0 Z_0 \mid T_0 Y \mid T_0$$

$$X \rightarrow T_0 Z_0 \mid T_0 Y \mid T_0$$

$$Y \rightarrow T_1 Y \mid T_1$$

$$Z_0 \rightarrow X2$$

$$T_0 \rightarrow 0$$

$$T_1 \rightarrow 1$$

## References

18-Pumping lemma example to prove a language not regular by Deeba Kannan. (n.d.).

Retrieved from <https://www.youtube.com/watch?v=SLG-LvaA6Hc>

Using pumping lemma to prove  $\{a^i b^j \mid i > j\}$  non-regular. (2018, November 3). Retrieved

December 5, 2019, from [https://cs.stackexchange.com/questions/99546/using-](https://cs.stackexchange.com/questions/99546/using-pumping-lemma-to-prove-aibj-i-j-non-regular)

[pumping-lemma-to-prove-aibj-i-j-non-regular](https://cs.stackexchange.com/questions/99546/using-pumping-lemma-to-prove-aibj-i-j-non-regular)