

## 1 实例分析

### 1.1 一些有趣的例子

#### 1.1.1 相似等腰三角形

以下示例选自精彩的 **Descartes et les Mathématiques** 网站,在此,未对原文进行任何修改,仅用 `tkz-euclide` 宏包对其进行了绘制。

<https://debart.pagesperso-orange.fr/seconde/triangle.html>

参考文献:

- Géométrie au Bac - Tangente, special issue no. 8 - Exercise 11, page 11
- Elisabeth Busser and Gilles Cohen: 200 nouveaux problèmes du “Monde” - POLE 2007 (200 new problems of “Le Monde”)
- Affaire de logique n° 364 - Le Monde February 17, 2004

构造相似等腰三角形有两种方式, 一是 *Tangente* 杂志提供的方法, 另一个是 *Le Monde* 杂志提供的方法。

“*Tangente*”杂志编辑: 两个相似等腰三角形  $AXB$  和  $BYC$  由主顶点分别是  $X$  和  $Y$ , 其中,  $A$ 、 $B$  和  $C$  共线。令  $\alpha$  为顶点的角度 ( $\widehat{AXB} = \widehat{BYC}$ ), 然后可以构造与前两个等腰三角形相似的第三个等腰三角形  $XZY$ , 其主顶点是  $Z$  点, 需要证明  $Z$  点属于直线  $(AC)$ 。

“*Le Monde*”杂志编辑: 两个相似等腰三角形  $AXB$  和  $BYC$  的主顶点分别是  $X$  和  $Y$ , 其中  $A$ 、 $B$  和  $C$  点共线。令  $\alpha$  为顶点的角度 ( $\widehat{AXB} = \widehat{BYC}$ ), 线段  $[AC]$  上的点  $Z$  与  $X$  和  $Y$  的距离相等。那么这两个顶点的角度是多少?

下面两页给出了解决这两个问题的 `tkz-euclide` 代码和绘图过程, 但在查看代码前, 建议先搜索相关文献。这些代码展示了这两种推导过程。



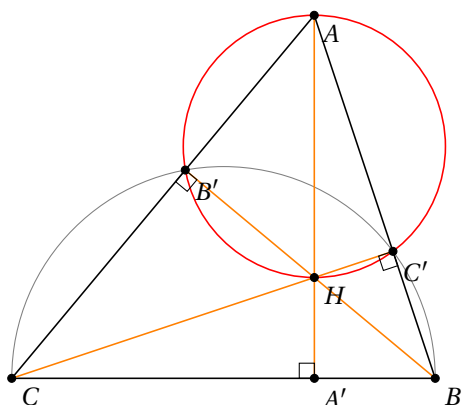


## 1.1.4 三角形的高

以下示例选自精彩的 **Descartes et les Mathématiques** 网站 (Descartes and the Mathematics)。

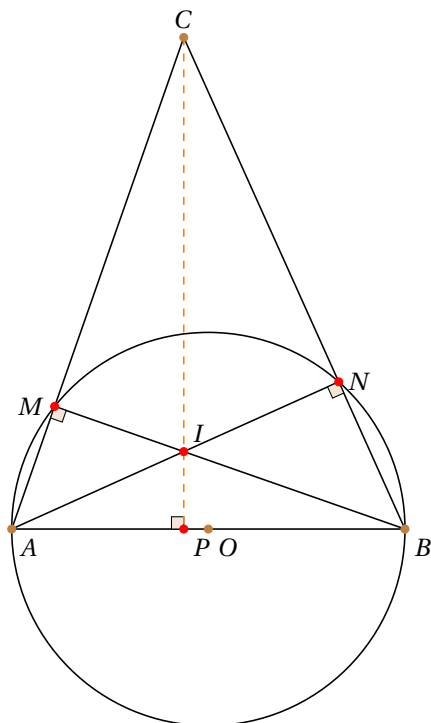
[https://debart.pagesperso-orange.fr/geoplan/geometrie\\_triangle.html](https://debart.pagesperso-orange.fr/geoplan/geometrie_triangle.html)

三条高相交于  $H$  点。



```
\begin{tikzpicture}[scale=.8]
\tkzDefPoint(0,0){C}
\tkzDefPoint(7,0){B}
\tkzDefPoint(5,6){A}
\tkzDrawPolygon(A,B,C)
\tkzDefMidPoint(C,B)
\tkzGetPoint{I}
\tkzDrawArc(I,B)(C)
\tkzInterLC(A,C)(I,B)
\tkzGetSecondPoint{B'}
\tkzInterLC(A,B)(I,B)
\tkzGetFirstPoint{C'}
\tkzInterLL(B,B')(C,C')
\tkzGetPoint{H}
\tkzInterLL(A,H)(C,B)
\tkzGetPoint{A'}
\tkzDefCircle[circum](A,B',C')
\tkzGetPoint{O}
\tkzDrawCircle[color=red](O,A)
\tkzDrawSegments[color=orange](B,B' C,C' A,A')
\tkzMarkRightAngles(C,B',B B',C',C C',A',A)
\tkzDrawPoints(A,B,C,A',B',C',H)
\tkzLabelPoints(A,B,C,A',B',C',H)
\end{tikzpicture}
```

## 1.1.5 三角形的高 - 另一种构造方式

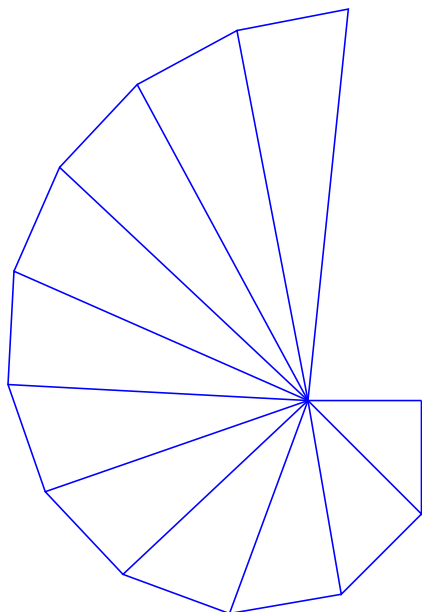


```
\begin{tikzpicture}[scale=0.65]
\tkzDefPoint(0,0){A}
\tkzDefPoint(8,0){B}
\tkzDefPoint(3.5,10){C}
\tkzDefMidPoint(A,B)
\tkzGetPoint{O}
\tkzDefPointBy[projection=onto A--B](C)
\tkzGetPoint{P}
\tkzInterLC(C,A)(O,A)
\tkzGetSecondPoint{M}
\tkzInterLC(C,B)(O,A)
\tkzGetFirstPoint{N}
\tkzInterLL(B,M)(A,N)
\tkzGetPoint{I}
\tkzDrawCircle[diameter](A,B)
\tkzDrawSegments(C,A C,B A,B B,M A,N)
\tkzMarkRightAngles[fill=brown!20](A,M,B A,N,B A,P,C)
\tkzDrawSegment[style=dashed,color=orange](C,P)
\tkzLabelPoints(O,A,B,P)
\tkzLabelPoint[left](M){M}
\tkzLabelPoint[right](N){N}
\tkzLabelPoint[above](C){C}
\tkzLabelPoint[above right](I){I}
\tkzDrawPoints[color=red](M,N,P,I)
\tkzDrawPoints[color=brown](O,A,B,C)
\end{tikzpicture}
```

## 1.2 其他作者提供的实例

### 1.2.1 整数的算术平方根

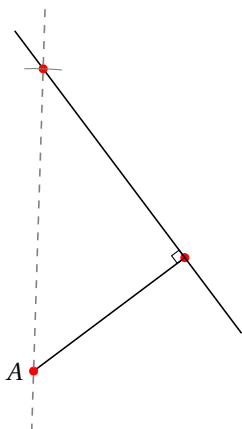
本例演示了如何用尺规求解  $1$ 、 $\sqrt{2}$  和  $\sqrt{3}$  的方法。



```
\begin{tikzpicture}[scale=1.5]
\tkzDefPoint(0,0){O}
\tkzDefPoint(1,0){a0}
\tkzDrawSegment[blue](O,a0)
\foreach \i [count=\j] in {0,...,10}{%
\tkzDefPointWith[orthogonal normed](a\i,O)
\tkzGetPoint{a\j}
\tkzDrawPolySeg[color=blue](a\i,a\j,O)}
\end{tikzpicture}
```

### 1.2.2 直角三角

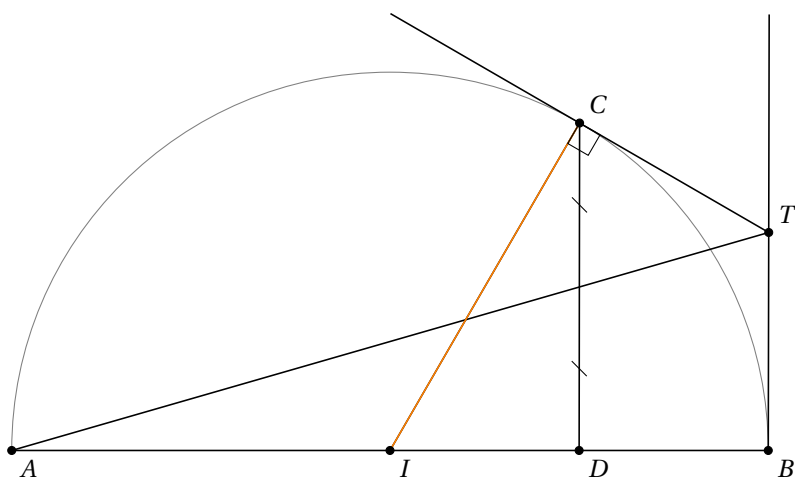
有一线段  $[AB]$ ，然后确定一点  $C$  使  $AC = 8 \text{ cm}$ ，并且  $ABC$  是以  $B$  为直角的直角三角形。



```
\begin{tikzpicture}[scale=.5]
\tkzDefPoint["$A$" left](2,1){A}
\tkzDefPoint(6,4){B}
\tkzDrawSegment(A,B)
\tkzDrawPoint[color=red](A)
\tkzDrawPoint[color=red](B)
\tkzDefPointWith[orthogonal,K=-1](B,A)
\tkzDrawLine[add = .5 and .5](B,t kzPointResult)
\tkzInterLC[R](B,t kzPointResult)(A,8 cm)
\tkzGetPoints{C}{J}
\tkzDrawPoint[color=red](C)
\tkzCompass(A,C)
\tkzMarkRightAngle(A,B,C)
\tkzDrawLine[color=gray,style=dashed](A,C)
\end{tikzpicture}
```

### 1.2.3 阿基米德等分

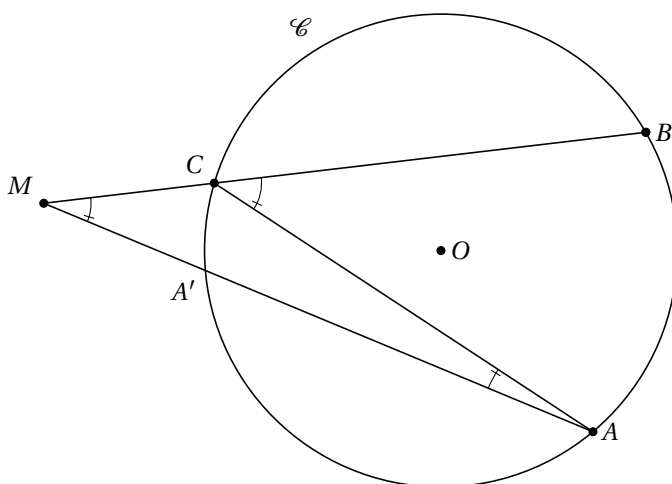
这是伟大的希腊数学家阿基米德证明的一个古老问题。下图有一个直径为  $AB$  的半圆，一条直线在  $B$  点与半圆相切，在  $C$  点有半圆的另一条切线。把  $C$  点投影到线段  $[AB]$  上的  $D$  点。两条切线相交于  $T$  点。现证明直线  $(AT)$  平分直线  $(CD)$



```
\begin{tikzpicture}[scale=1.25]
  \tkzDefPoint(0,0){A}\tkzDefPoint(6,0){D}
  \tkzDefPoint(8,0){B}\tkzDefPoint(4,0){I}
  \tkzDefLine[orthogonal=through D](A,D)
  \tkzInterLC[R](D,\tkzPointResult)(I,4 cm) \tkzGetFirstPoint{C}
  \tkzDefLine[orthogonal=through C](I,C) \tkzGetPoint{c}
  \tkzDefLine[orthogonal=through B](A,B) \tkzGetPoint{b}
  \tkzInterLL(C,c)(B,b) \tkzGetPoint{T}
  \tkzInterLL(A,T)(C,D) \tkzGetPoint{P}
  \tkzDrawArc(I,B)(A)
  \tkzDrawSegments(A,B A,T C,D I,C) \tkzDrawSegment[color=orange](I,C)
  \tkzDrawLine[add = 1 and 0](C,T) \tkzDrawLine[add = 0 and 1](B,T)
  \tkzMarkRightAngle(I,C,T)
  \tkzDrawPoints(A,B,I,D,C,T)
  \tkzLabelPoints(A,B,I,D) \tkzLabelPoints[above right](C,T)
  \tkzMarkSegment[pos=.25,mark=s|](C,D) \tkzMarkSegment[pos=.75,mark=s|](C,D)
\end{tikzpicture}
```

#### 1.2.4 示例 (Dimitris Kapeta)

在本例中，由于  $\widehat{CAM}$  过小，因此需要在 `\tkzMarkAngle` 命令中使用 `mkpos=.2` 选项。另一种解决方案是使用 `\tkzFillAngle` 命令。



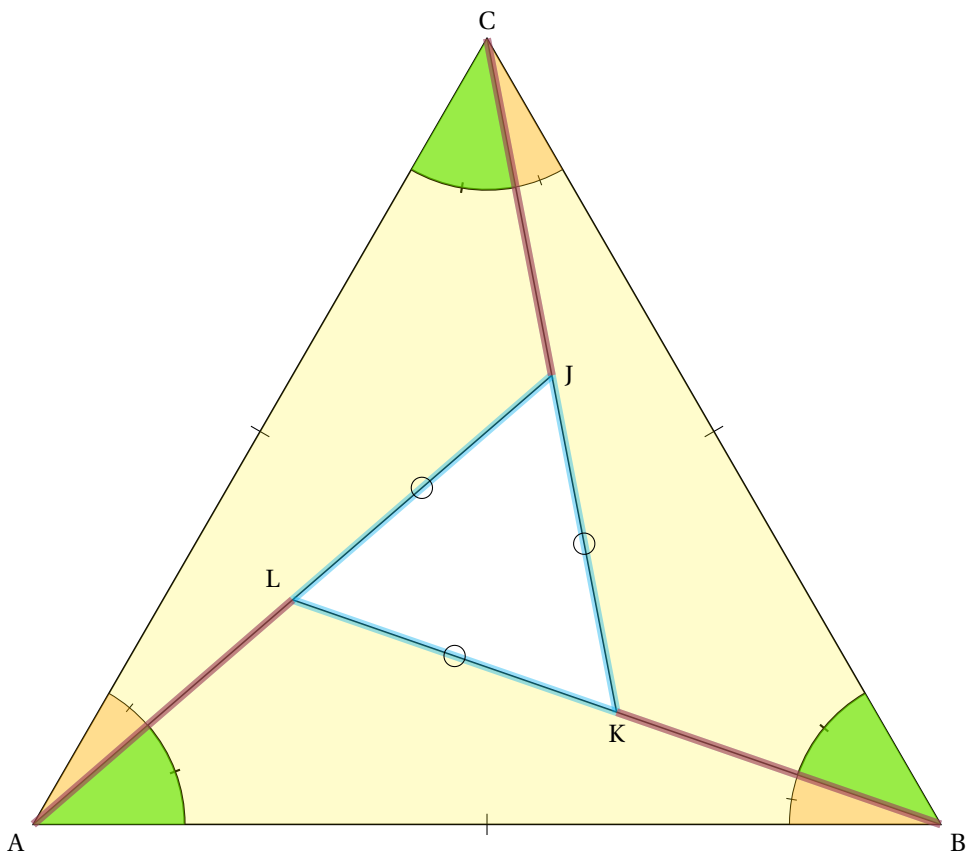
```

\begin{tikzpicture}[scale=1.25]
  \tkzDefPoint(0,0){O}
  \tkzDefPoint(2.5,0){N}
  \tkzDefPoint(-4.2,0.5){M}
  \tkzDefPointBy[rotation=center O angle 30](N)
  \tkzGetPoint{B}
  \tkzDefPointBy[rotation=center O angle -50](N)
  \tkzGetPoint{A}
  \tkzInterLC(M,B)(O,N) \tkzGetFirstPoint{C}
  \tkzInterLC(M,A)(O,N) \tkzGetSecondPoint{A'}
  \tkzMarkAngle[mkpos=.2, size=0.5](A,C,B)
  \tkzMarkAngle[mkpos=.2, size=0.5](A,M,C)
  \tkzDrawSegments(A,C M,A M,B)
  \tkzDrawCircle(O,N)
  \tkzLabelCircle[above left](O,N)(120){$\mathcal{C}$}
  \tkzMarkAngle[mkpos=.2, size=1.2](C,A,M)
  \tkzDrawPoints(O, A, B, M, B, C)
  \tkzLabelPoints[right](O,A,B)
  \tkzLabelPoints[above left](M,C)
  \tkzLabelPoint[below left](A'){$A'$}
\end{tikzpicture}

```

### 1.2.5 证明示例 1(John Kitzmiller)

证明  $\triangle LKJ$  是等边三角形。



```

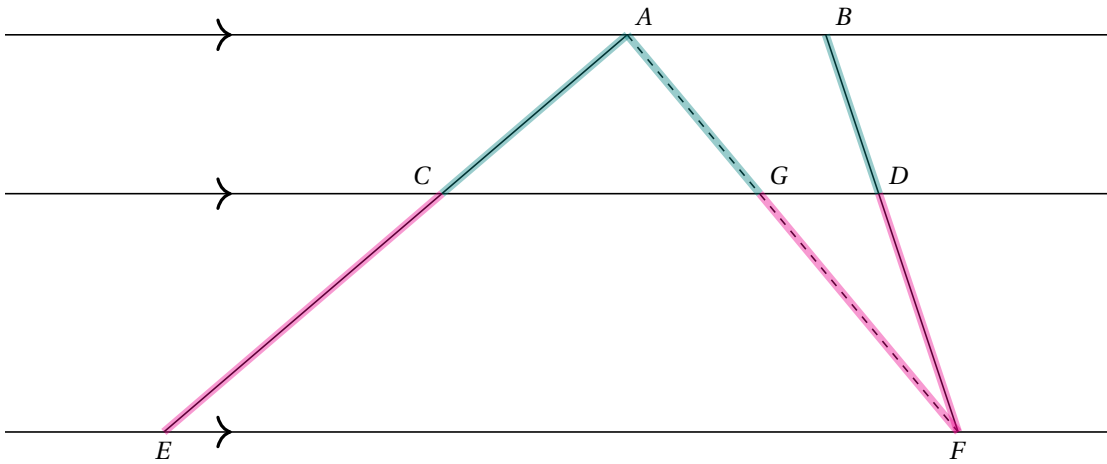
\begin{tikzpicture}[scale=2]
  \tkzDefPoint[label=below left:A](0,0){A}
  \tkzDefPoint[label=below right:B](6,0){B}
  \tkzDefTriangle[equilateral](A,B) \tkzGetPoint{C}
  \tkzMarkSegments[mark=|](A,B A,C B,C)
  \tkzDefBarycentricPoint(A=1,B=2) \tkzGetPoint{C'}
  \tkzDefBarycentricPoint(A=2,C=1) \tkzGetPoint{B'}
  \tkzDefBarycentricPoint(C=2,B=1) \tkzGetPoint{A'}
  \tkzInterLL(A,A')(C,C') \tkzGetPoint{J}
  \tkzInterLL(C,C')(B,B') \tkzGetPoint{K}
  \tkzInterLL(B,B')(A,A') \tkzGetPoint{L}
  \tkzLabelPoint[above](C){C}
  \tkzDrawPolygon(A,B,C) \tkzDrawSegments(A,J B,L C,K)
  \tkzMarkAngles[size=1 cm](J,A,C K,C,B L,B,A)
  \tkzMarkAngles[thick,size=1 cm](A,C,J C,B,K B,A,L)
  \tkzMarkAngles[opacity=.5](A,C,J C,B,K B,A,L)
  \tkzFillAngles[fill= orange,size=1 cm,opacity=.3](J,A,C K,C,B L,B,A)
  \tkzFillAngles[fill=orange, opacity=.3,thick,size=1,](A,C,J C,B,K B,A,L)
  \tkzFillAngles[fill=green, size=1, opacity=.5](A,C,J C,B,K B,A,L)
  \tkzFillPolygon[color=yellow, opacity=.2](J,A,C)
  \tkzFillPolygon[color=yellow, opacity=.2](K,B,C)
  \tkzFillPolygon[color=yellow, opacity=.2](L,A,B)
  \tkzDrawSegments[line width=3pt,color=cyan,opacity=0.4](A,J C,K B,L)
  \tkzDrawSegments[line width=3pt,color=red,opacity=0.4](A,L B,K C,J)
  \tkzMarkSegments[mark=o](J,K K,L L,J)
  \tkzLabelPoint[right](J){J}
  \tkzLabelPoint[below](K){K}
  \tkzLabelPoint[above left](L){L}
\end{tikzpicture}

```

### 1.2.6 证明示例 2(John Kitzmiller)

证明:  $\frac{AC}{CE} = \frac{BD}{DF}$

John 的另一个有趣的例子是如何在 `tkz-euclide` 宏包中使用 TikZ 的类似 `decoration` 和 `postaction` 选项。





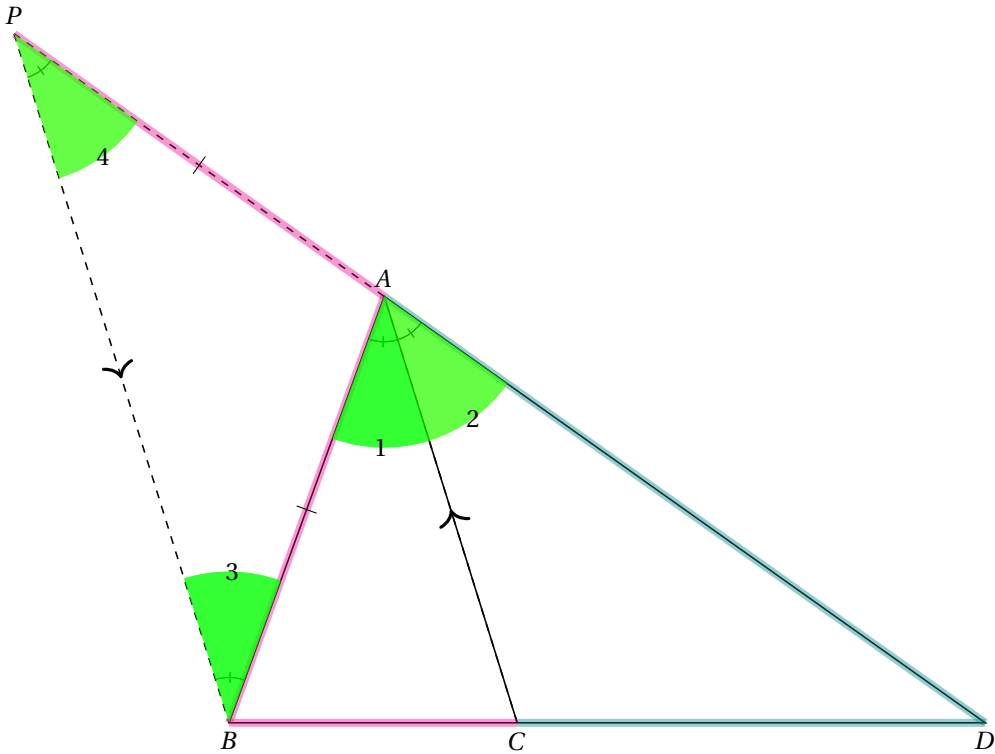
```

\begin{tikzpicture}[scale=1.75,decoration={markings,
  mark=at position 3cm with {\arrow[scale=2]{>}}}]
\tkzDefPoints{0/0/E, 6/0/F, 0/1.8/P, 6/1.8/Q, 0/3/R, 6/3/S}
\tkzDrawLines[postaction={decorate}] (E,F P,Q R,S)
\tkzDefPoints{3.5/3/A, 5/3/B}
\tkzDrawSegments(E,A F,B)
\tkzInterLL(E,A)(P,Q) \tkzGetPoint{C}
\tkzInterLL(B,F)(P,Q) \tkzGetPoint{D}
\tkzLabelPoints[above right](A,B)
\tkzLabelPoints[below](E,F)
\tkzLabelPoints[above left](C)
\tkzDrawSegments[style=dashed](A,F)
\tkzInterLL(A,F)(P,Q) \tkzGetPoint{G}
\tkzLabelPoints[above right](D,G)
\tkzDrawSegments[color=teal, line width=3pt, opacity=0.4](A,C A,G)
\tkzDrawSegments[color=magenta, line width=3pt, opacity=0.4](C,E G,F)
\tkzDrawSegments[color=teal, line width=3pt, opacity=0.4](B,D)
\tkzDrawSegments[color=magenta, line width=3pt, opacity=0.4](D,F)
\end{tikzpicture}

```

### 1.2.7 证明示例 3(John Kitzmiller)

证明:  $\frac{BC}{CD} = \frac{AB}{AD}$  (角平分线).



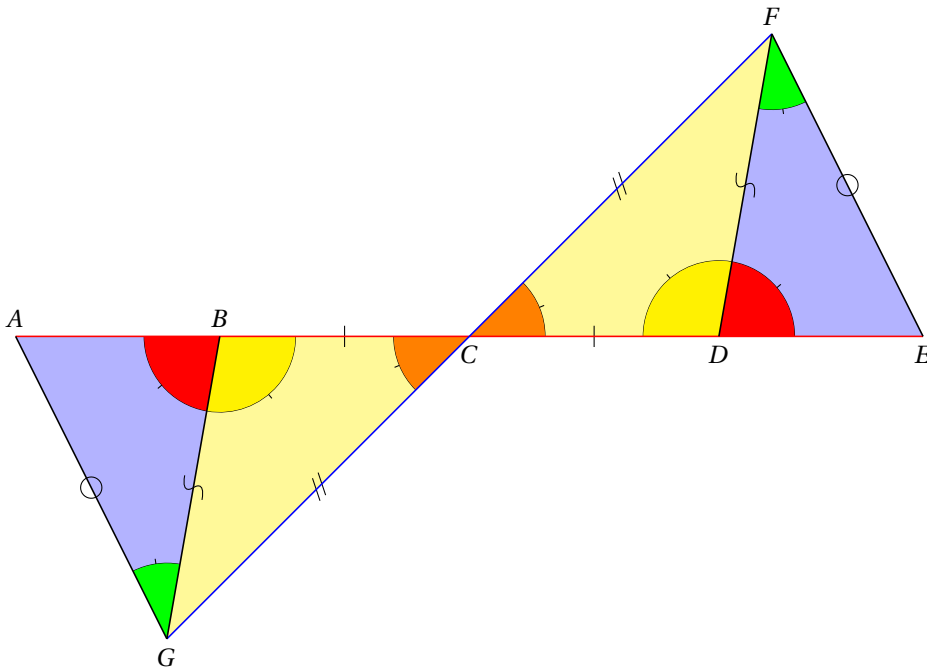
```

\begin{tikzpicture}[scale=2]
  \tkzDefPoints{0/0/B, 5/0/D} \tkzDefPoint(70:3){A}
  \tkzDrawPolygon(B,D,A)
  \tkzDefLine[bisector](B,A,D) \tkzGetPoint{a}
  \tkzInterLL(A,a)(B,D) \tkzGetPoint{C}
  \tkzDefLine[parallel=through B](A,C) \tkzGetPoint{b}
  \tkzInterLL(A,D)(B,b) \tkzGetPoint{P}
  \begin{scope}[decoration={markings, mark=at position .5 with {\arrow[scale=2]{>}}}]
    \tkzDrawSegments[postaction={decorate},dashed](C,A P,B)
  \end{scope}
  \tkzDrawSegment(A,C) \tkzDrawSegment[style=dashed](A,P)
  \tkzLabelPoints[below](B,C,D) \tkzLabelPoints[above](A,P)
  \tkzDrawSegments[color=magenta, line width=3pt, opacity=0.4](B,C P,A)
  \tkzDrawSegments[color=teal, line width=3pt, opacity=0.4](C,D A,D)
  \tkzDrawSegments[color=magenta, line width=3pt, opacity=0.4](A,B)
  \tkzMarkAngles[size=3mm](B,A,C C,A,D)
  \tkzMarkAngles[size=3mm](B,A,C A,B,P)
  \tkzMarkAngles[size=3mm](B,P,A C,A,D)
  \tkzMarkAngles[size=3mm](B,A,C A,B,P B,P,A C,A,D)
  \tkzFillAngles[fill=green, opacity=0.5](B,A,C A,B,P)
  \tkzFillAngles[fill=yellow, opacity=0.3](B,P,A C,A,D)
  \tkzFillAngles[fill=green, opacity=0.6](B,A,C A,B,P B,P,A C,A,D)
  \tkzLabelAngle[pos=1](B,A,C){1} \tkzLabelAngle[pos=1](C,A,D){2}
  \tkzLabelAngle[pos=1](A,B,P){3} \tkzLabelAngle[pos=1](B,P,A){4}
  \tkzMarkSegments[mark=|](A,B A,P)
\end{tikzpicture}

```

#### 1.2.8 证明示例 4(John Kitzmiller)

证明:  $\overline{AG} \cong \overline{EF}$  (Detour).



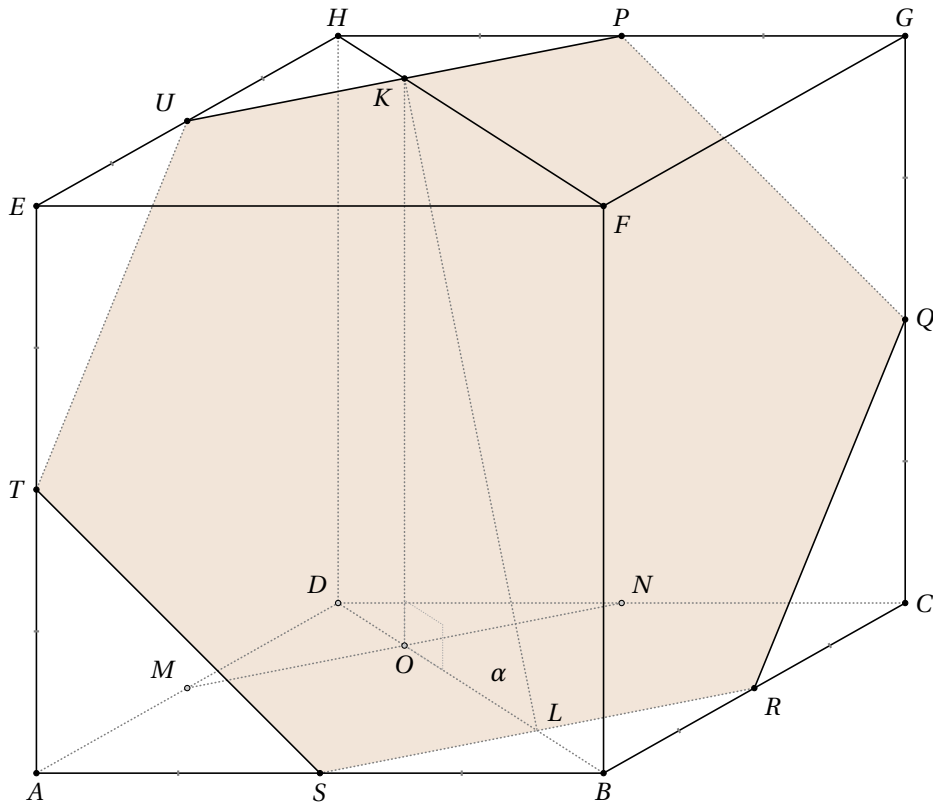


```

\begin{tikzpicture}[scale=3]
  \tkzDefPoints{0/0/A,2/0/B}
  \tkzDefSquare(A,B) \tkzGetPoints{C}{D}
  \tkzDefPointBy[rotation=center D angle 45](C)\tkzGetPoint{G}
  \tkzDefSquare(G,D)\tkzGetPoints{E}{F}
  \tkzInterLL(B,C)(E,F)\tkzGetPoint{H}
  \tkzFillPolygon[gray!10](D,E,H,C,D)
  \tkzDrawPolygon(A,...,D)\tkzDrawPolygon(D,...,G)
  \tkzDrawSegment(B,E)
  \tkzMarkSegments[mark=|,size=3pt,color=gray](A,B B,C C,D D,A E,F F,G G,D D,E)
  \tkzMarkSegments[mark=||,size=3pt,color=gray](B,E E,H)
  \tkzLabelPoints[left](A,D)
  \tkzLabelPoints[right](B,C,F,H)
  \tkzLabelPoints[above](G)\tkzLabelPoints[below](E)
  \tkzMarkRightAngles(D,A,B D,G,F)
\end{tikzpicture}

```

### 1.2.10 构图示例 2(Indonesia)

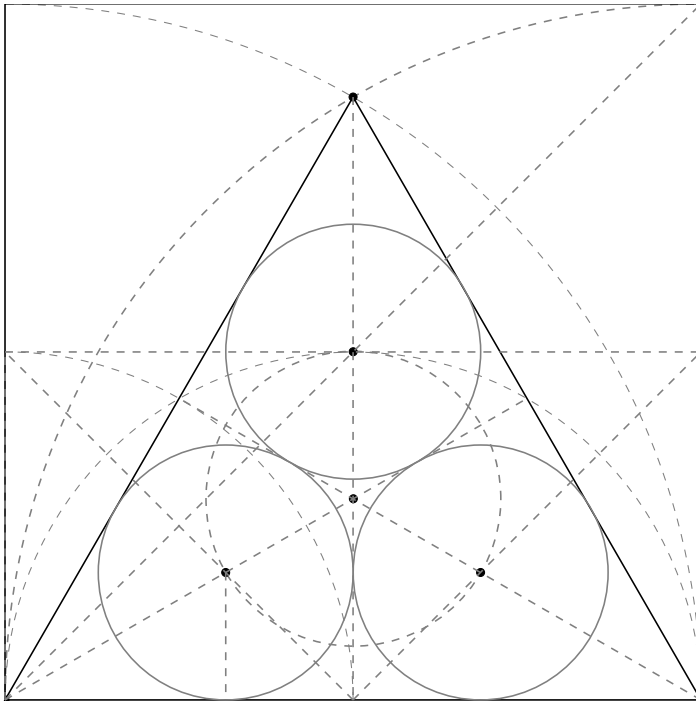


```

\begin{tikzpicture}[pol/.style={fill=brown!40,opacity=.5},
                    seg/.style={tkzdotted,color=gray},
                    hidden pt/.style={fill=gray!40},
                    mra/.style={color=gray!70,tkzdotted,/tkzrightangle/size=.2},
                    scale=3]
\tkzSetUpPoint[size=2]
\tkzDefPoints{O/0/A,2.5/0/B,1.33/0.75/D,0/2.5/E,2.5/2.5/F}
\tkzDefLine[parallel=through D](A,B) \tkzGetPoint{I1}
\tkzDefLine[parallel=through B](A,D) \tkzGetPoint{I2}
\tkzInterLL(D,I1)(B,I2) \tkzGetPoint{C}
\tkzDefLine[parallel=through E](A,D) \tkzGetPoint{I3}
\tkzDefLine[parallel=through D](A,E) \tkzGetPoint{I4}
\tkzInterLL(E,I3)(D,I4) \tkzGetPoint{H}
\tkzDefLine[parallel=through F](E,H) \tkzGetPoint{I5}
\tkzDefLine[parallel=through H](E,F) \tkzGetPoint{I6}
\tkzInterLL(F,I5)(H,I6) \tkzGetPoint{G}
\tkzDefMidPoint(G,H) \tkzGetPoint{P}
\tkzDefMidPoint(G,C) \tkzGetPoint{Q}
\tkzDefMidPoint(B,C) \tkzGetPoint{R}
\tkzDefMidPoint(A,B) \tkzGetPoint{S}
\tkzDefMidPoint(A,E) \tkzGetPoint{T}
\tkzDefMidPoint(E,H) \tkzGetPoint{U}
\tkzDefMidPoint(A,D) \tkzGetPoint{M}
\tkzDefMidPoint(D,C) \tkzGetPoint{N}
\tkzInterLL(B,D)(S,R) \tkzGetPoint{L}
\tkzInterLL(H,F)(U,P) \tkzGetPoint{K}
\tkzDefLine[parallel=through K](D,H) \tkzGetPoint{I7}
\tkzInterLL(K,I7)(B,D) \tkzGetPoint{O}
\tkzFillPolygon[pol](P,Q,R,S,T,U)
\tkzDrawSegments[seg](K,O K,L P,Q R,S T,U
                     C,D H,D A,D M,N B,D)
\tkzDrawSegments(E,H B,C G,F G,H G,C Q,R S,T U,P H,F)
\tkzDrawPolygon(A,B,F,E)
\tkzDrawPoints(A,B,C,E,F,G,H,P,Q,R,S,T,U,K)
\tkzDrawPoints[hidden pt](M,N,O,D)
\tkzMarkRightAngle[mra](L,O,K)
\tkzMarkSegments[mark=|,size=1pt,thick,color=gray](A,S B,S B,R C,R
              Q,C Q,G G,P H,P
              E,U H,U E,T A,T)
\tkzLabelAngle[pos=.3](K,L,O){$\alpha$}
\tkzLabelPoints[below](O,A,S,B)
\tkzLabelPoints[above](H,P,G)
\tkzLabelPoints[left](T,E)
\tkzLabelPoints[right](C,Q)
\tkzLabelPoints[above left](U,D,M)
\tkzLabelPoints[above right](L,N)
\tkzLabelPoints[below right](F,R)
\tkzLabelPoints[below left](K)
\end{tikzpicture}

```

## 1.2.11 三个相切圆

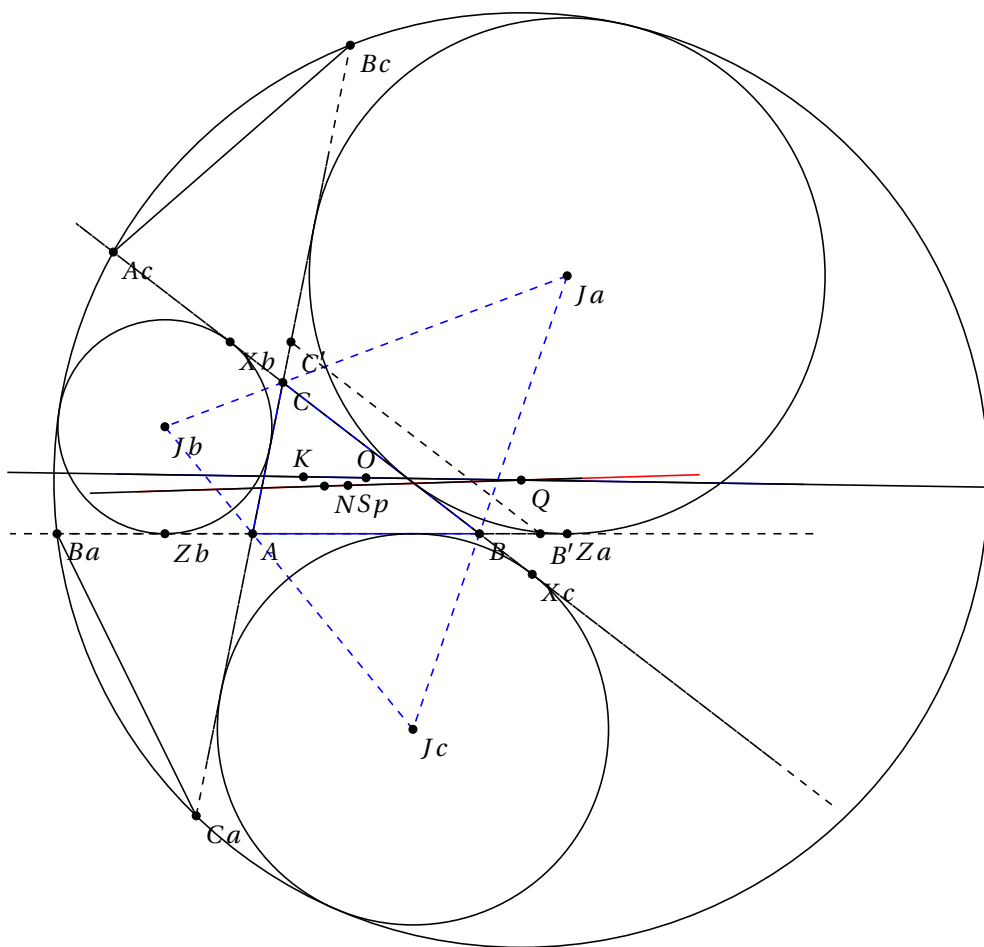


```

\begin{tikzpicture}[scale=1.15]
  \tkzDefPoints{0/0/A,8/0/B,0/4/a,8/4/b,8/8/c}
  \tkzDefTriangle[equilateral](A,B) \tkzGetPoint{C}
  \tkzDrawPolygon(A,B,C)
  \tkzDefSquare(A,B) \tkzGetPoints{D}{E}
  \tkzClipBB
  \tkzDefMidPoint(A,B) \tkzGetPoint{M}
  \tkzDefMidPoint(B,C) \tkzGetPoint{N}
  \tkzDefMidPoint(A,C) \tkzGetPoint{P}
  \tkzDrawSemiCircle[gray,dashed](M,B)
  \tkzDrawSemiCircle[gray,dashed](A,M)
  \tkzDrawSemiCircle[gray,dashed](A,B)
  \tkzDrawCircle[gray,dashed](B,A)
  \tkzInterLL(A,N)(M,a) \tkzGetPoint{Ia}
  \tkzDefPointBy[projection = onto A--B](Ia)
  \tkzGetPoint{ha}
  \tkzDrawCircle[gray](Ia,ha)
  \tkzInterLL(B,P)(M,b) \tkzGetPoint{Ib}
  \tkzDefPointBy[projection = onto A--B](Ib)
  \tkzGetPoint{hb}
  \tkzDrawCircle[gray](Ib,hb)
  \tkzInterLL(A,c)(M,C) \tkzGetPoint{Ic}
  \tkzDefPointBy[projection = onto A--C](Ic)
  \tkzGetPoint{hc}
  \tkzDrawCircle[gray](Ic,hc)
  \tkzInterLL(A,Ia)(B,Ib) \tkzGetPoint{G}
  \tkzDrawCircle[gray,dashed](G,Ia)
  \tkzDrawPolySeg(A,E,D,B)
  \tkzDrawPoints(A,B,C)
  \tkzDrawPoints(G,Ia,Ib,Ic)
  \tkzDrawSegments[gray,dashed](C,M A,N B,P M,a M,b A,a a,b b,B A,D Ia,ha)
\end{tikzpicture}

```

## 1.2.12 APOLLONIUS 圆



```

\begin{tikzpicture}[scale=.5]
  \tkzDefPoints{O/0/A,6/0/B,0.8/4/C}
  \tkzDefTriangleCenter[euler](A,B,C)      \tkzGetPoint{N}
  \tkzDefTriangleCenter[circum](A,B,C)     \tkzGetPoint{O}
  \tkzDefTriangleCenter[lemoine](A,B,C)    \tkzGetPoint{K}
  \tkzDefTriangleCenter[spieker](A,B,C)    \tkzGetPoint{Sp}
  \tkzDefExCircle(A,B,C)      \tkzGetPoint{Jb}
  \tkzDefExCircle(C,A,B)     \tkzGetPoint{Ja}
  \tkzDefExCircle(B,C,A)     \tkzGetPoint{Jc}
  \tkzDefPointBy[projection=onto B--C ](Jc) \tkzGetPoint{Xc}
  \tkzDefPointBy[projection=onto B--C ](Jb) \tkzGetPoint{Xb}
  \tkzDefPointBy[projection=onto A--B ](Ja) \tkzGetPoint{Za}
  \tkzDefPointBy[projection=onto A--B ](Jb) \tkzGetPoint{Zb}
  \tkzDefLine[parallel=through Xc](A,C)     \tkzGetPoint{X'c}
  \tkzDefLine[parallel=through Xb](A,B)     \tkzGetPoint{X'b}
  \tkzDefLine[parallel=through Za](C,A)     \tkzGetPoint{Z'a}
  \tkzDefLine[parallel=through Zb](C,B)     \tkzGetPoint{Z'b}
  \tkzInterLL(Xc,X'c)(A,B)                 \tkzGetPoint{B'}
  \tkzInterLL(Xb,X'b)(A,C)                 \tkzGetPoint{C'}
  \tkzInterLL(Za,Z'a)(C,B)                 \tkzGetPoint{A''}
  \tkzInterLL(Zb,Z'b)(C,A)                 \tkzGetPoint{B''}
  \tkzDefPointBy[reflection= over Jc--Jb](B') \tkzGetPoint{Ca}
  \tkzDefPointBy[reflection= over Jc--Jb](C') \tkzGetPoint{Ba}
  \tkzDefPointBy[reflection= over Ja--Jb](A'') \tkzGetPoint{Bc}
  \tkzDefPointBy[reflection= over Ja--Jb](B'') \tkzGetPoint{Ac}
  \tkzDefCircle[circum](Ac,Ca,Ba)           \tkzGetPoint{Q}
  \tkzDrawCircle[circum](Ac,Ca,Ba)
  \tkzDefPointWith[linear,K=1.1](Q,Ac)      \tkzGetPoint{nAc}
  \tkzClipCircle[through](Q,nAc)
  \tkzDrawLines[add=1.5 and 1.5,dashed](A,B B,C A,C)
  \tkzDrawPolygon[color=blue](A,B,C)
  \tkzDrawPolygon[dashed,color=blue](Ja,Jb,Jc)
  \tkzDrawCircles[ex](A,B,C B,C,A C,A,B)
  \tkzDrawLines[add=0 and 0,dashed](Ca,Bc B,Za A,Ba B',C')
  \tkzDrawLine[add=1 and 1,dashed](Xb,Xc)
  \tkzDrawLine[add=7 and 3,blue](O,K)
  \tkzDrawLine[add=8 and 15,red](N,Sp)
  \tkzDrawLines[add=10 and 10](K,O N,Sp)
  \tkzDrawSegments(Ba,Ca Bc,Ac)
  \tkzDrawPoints(A,B,C,N,Ja,Jb,Jc,Xb,Xc,B',C',Za,Zb,Ba,Ca,Bc,Ac,Q,Sp,K,O)
  \tkzLabelPoints(A,B,C,N,Ja,Jb,Jc,Xb,Xc,B',C',Za,Zb,Ba,Ca,Bc,Ac,Q,Sp)
  \tkzLabelPoints[above](K,O)
\end{tikzpicture}

```