GOLF BALL AERODYNAMICS

A dissertation submitted to the University of Manchester
for the degree of Master of Science
in the Faculty of Engineering and Physical Sciences

2014

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Word count xxxxx

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Abstract

In this project we work on golf balls and stuff.

Declaration

No portion of the work referred to in the dissertation has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

Intellectual Property Statement

Acknowledgements

I would like to thank my supervisor Prof. Jitesh Gajjar for his guidance during the project without which I would have been completely lost. Additionally, I would like to thank the Royal and Ancient Golf Club for their sponsorship of the project and Dr Steve Otto for his guidance and assistance.

Chapter 1

Introduction

Stuff here about the project and aims and such.

1.1 A Brief History of Golf

The origins of the game of golf are difficult to trace, with suggestions that the game originated in either Scotland, France, the Netherlands, China, or even going back as far as the Roman Empire. Golf in its more modern incarnation however, is agreed to have originated in 15th century Scotland, where the first written records of the game are (somewhat humorously) related to King James II of Scotland banning the game in 1457 for fear of a decrease in archery practice in its favour.

From the 18th century onwards golf began to take form fully in Scotland, with the founding of both The Royal and Ancient Golf Club in St Andrews and The Royal Burgess Golfing Society in Edinburgh. The oldest surviving rules of golf also date from this time and these rules have been in a state of constant revision up to the modern day.

In the 19th century the popularity of golf vastly increased, seeing larger numbers of people knowing and playing the game, and the start of the first major tournaments. Additionally, the game spread out to encompass much of the British empire, to the United States and eventually to Japan, making golf into a global sport supported by a plethora of associated manufacturers, sponsors and organisations.

In the modern day, golf is potentially one of the largest sports on earth, with golf tournaments, golf manufacturing and related industries accounting for hundreds of billions of pounds of economic activity. If successful on the golf tournament circuit, golf professionals can earn huge sums in prize money. With the players themselves and their sponsors having such a vested interest in success having a consistent and fair rule set is of paramount importance and this is dealt with jointly by The R&A (The Royal and Ancient) in most of the world and the USGA (United States Golf Association) in the Americas.

1.2 A Slightly Larger History of the Golf Ball

Golf ball technology has advanced greatly since the advent of the game. Initially, hard wooden balls were used for playing, however these were soon replaced with featherie balls which are leather pouches stuffed with feathers and then painted white.

The next major innovation in the design of golf balls came in 1848, when the gutta-percha ball was invented. This is the first ball to use a rubbery substance as continues to this day, and was easier to make into a proper sphere, unlike the previous types of ball. It was around this time that it was discovered that abrasions to the surface of the ball would improve the aerodynamic properties of the ball, making it easier to control the flight of the ball and increasing the distance at which the game could be played. This would start a series of innovations that would lead to todays dimpled balls, which we will discuss later.

After this the golf ball once again changed form with the advent of using wrapped rubber thread to help the ball to bounce better. This was coupled with the first usage of a plastic covering, in order to protect the rubber inside the ball on impact with the club. This cover also persists to this day, although the inside of the ball has seen significant development.

The modern golf ball has changed significantly from old designs. The interior of the ball is now usually a 3 piece rubber composite, with different properties in each rubber to maximize the controllability of the ball during play. The exterior is a polyurethane cover (normally white but some are in other colours) with usually between 300 to 400 dimples (though these can go as low as 200 dimples, and beyond 600 in some cases). The properties of the ball are stipulated to be within certain ranges, as set by The R&A and USGA in the rules of golf. The weight of a ball must not be greater than

(a) “Featherie” balls

(b) Pro V1 ball

Figure 1.1: In 1.1a are “Featherie” golf balls, taken from https://en.wikipedia.org/wiki/File:Featherie_golf_ball.JPG, and in 1.1b is a modern style ball, namely the Titleist Pro V1 ball.

45.93g, the diameter no less than 42.67mm and the ball must be spherically symmetric.

1.3 Aims of the Project

The aim of this project is to obtain a model for how golf balls fly based on simple physical principles. Given this model we then wish to categorise individual balls based on measurements of their flight, and use this categorisation to predict trajectories for the ball

Finally, using this model, we will attempt to use a limited set of flight data (between 20 and 30 m) to predict the full flight for the ball.

Chapter 2

Preliminary Investigations and Background

In order to devise a simple model for golf ball flight we first must understand some prerequisite physics for projectiles and fluid dynamics for the airflow over the ball. Understanding how the fluid flows over the surface of the ball is crucial to understanding the difference between the flight of a golf ball and that of a standard projectile. Quantifying this effect will be a large component of this project.

There has been significant work done previously in understanding the fluid dynamics around a golf ball and how a golf ball flies. We will attempt to review some of this literature in this chapter and summarise previous work on the topic.

First though, we must understand how normal projectiles fly without taking into account fluid dynamics effects.

2.1 Projectile Motion

A projectile is a body fired into the air by an initial impulse and then allowed to fall back to ground under the action of gravity alone. This is the most naive and simplistic model of golf ball flight, completely ignoring all aerodynamic effects, however we must understand it before building up to a more complex model.

Consider motion in a 2 dimensional plane, labelled by x along the horizontal and y along the vertical. A projectile is given an initial speed of the form $\mathbf{V}_0 = (v_x, v_y)$. We set the origin of the coordinate system to be the point at the start of the trajectory,

$(x_0, y_0) = (0, 0)$. In this problem the acceleration on the projectile, after the initial impulse, is constant and of the form

$$a_x = 0, \quad a_y = -g \tag{2.1.1}$$

where g is the acceleration due to gravity. Since the acceleration is constant we can use the standard formulas for motion under constant acceleration to derive the dynamics of the projectile Young and Freedman (2008), which are

$$v = v_0 + at \tag{2.1.2a}$$

$$x = v_0 t + \frac{1}{2}at^2 \tag{2.1.2b}$$

$$v^2 = v_0^2 + 2ax \tag{2.1.2c}$$

$$x = \left(\frac{v_0 + v}{2}\right) t. \tag{2.1.2d}$$

Where v is the speed at a time t, x is the distance from the origin of the coordinate system, a is the acceleration and v_0 is the initial speed.

We will write the equations in component form along the axes. Let $\mathbf{V}_0$ be the initial velocity. In component form these will be

$$v_{0x} = v_0 \cos\alpha$$

along the x-axis and

$$v_{0y} = v_0 \sin\alpha$$

where $v_0 = |\mathbf{V}_0|$ and α is the angle $\mathbf{V}_0$ makes with the x-axis. Now using (2.1.1) and (2.1.2a) we may find

$$v_x = v_0 \cos\alpha \tag{2.1.3a}$$

and

$$v_y = v_0 \sin\alpha - gt. \tag{2.1.3b}$$

Now, using (2.1.2b) (or by integrating (2.1.3) with respect to t), we can obtain the standard formulas for the x and y positions during the flight of the projectile:

$$x = (v_0 \cos\alpha)t \tag{2.1.4a}$$

and

$$y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2. \tag{2.1.4b}$$

Eliminating t between these equations demonstrates that projectiles follow parabolic paths, giving

$$y = x\tan\alpha - \frac{g}{2v_0^2\cos^2\alpha}x^2 \tag{2.1.5}$$

for the path of the projectile.

Finally we may use these equations to find the maximum height, range and time of flight for a projectile. The maximum height is obtained when $v_y = 0$ and solving (2.1.3b) with this condition gives

$$t_H = \frac{v_0 \sin\alpha}{g}. \tag{2.1.6}$$

The range is obtained by solving for $y = 0$ in (2.1.4b) and selecting the non trivial root for t of

$$t_F = \frac{2v_0 \sin\alpha}{g} \tag{2.1.7}$$

where t_F is the time of flight for the projectile. Inserting this into (2.1.4a) gives

$$x = \frac{2v_0^2\cos\alpha\sin\alpha}{g}$$

and recalling that $\sin 2\alpha = 2\cos\alpha\sin\alpha$ gives

$$x = \frac{v_0^2\sin 2\alpha}{g} \tag{2.1.8}$$

for the range of the projectile.

2.1.1 3D Projectile Motion

Projectile motion in 3 dimensions works in exactly the same fashion as 2D projectile motion. Here we will take the z-axis to be the vertical and x and y axes to be labelling the surface. The only component of acceleration is along the z-axis, with

$$a_z = -g$$

as before. All other equations remain the same.

2.2 Basic Aerodynamics

Of course, the flight of a golf ball is inevitably affected by aerodynamics. As such we need to have some understanding of fluid mechanics. In particular, we will need to understand how boundary layers form on and separate from the surface of the golf ball and how this effects the drag on the ball. First we will review some basic fluid mechanics.

2.2.1 Fluid Mechanics

In this project we will model the air flowing around the ball as being an incompressible fluid. This is an Eulerian description of fluid flow, viewing the fluid as though the ball is fixed in the centre of the coordinate system and the fluid moving around the ball Ruban and Gajjar (2014).

We will not concern ourselves with a full discussion of fluid mechanics from basic principles here, instead we will simply state some useful results predominantly following Ruban and Gajjar (2014) and Sears (2011).

Let $\mathbf{V}$ be the fluid velocity, which is a function of the the position $\mathbf{r}$ from the origin of the coordinate system and of time t. Let ρ be the density of the fluid and p the pressure. We define the material derivative to be (as a differential operator)

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\mathbf{V} \cdot \nabla)$$

and represents the rate of change of some quantity within the fluid, while moving with a small element of the fluid flow. That is, in a description where the fluid moves relative to the coordinate system the material derivative measures the rate of change as seen by a moving fluid element.

At all points within the fluid the mass continuity equation must apply

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{V} = 0. \tag{2.2.1}$$

This equation encodes the condition that mass is conserved within a fluid without any sources or sinks. In an incompressible fluid, as we will be primarily conserved with in this project, ρ will not change with time, and as such $D\rho/Dt = 0$. As a consequence, both terms on the left hand side of (2.2.1) must be zero everywhere within the fluid, and therefore the equation reduces to

$$\nabla \cdot \mathbf{V} = 0 \tag{2.2.2}$$

for an incompressible fluid.

The continuity equation gives one equation for the velocity components u, v, w[1] within a fluid. In order to specify the pressure and velocity everywhere we therefore require three more equations to determine the system. These three equations are supplied by considerations of energy and momentum conservation. Keeping all of these in mind, we may write a momentum equation in the form

$$\rho \frac{D\mathbf{V}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{V} + \mathbf{f} \tag{2.2.3}$$

where $\mathbf{f}$ is body force per unit volume acting on the fluid (for example a gravitational force) and μ is the viscosity of the fluid.

Equations (2.2.2) and (2.2.3) when taken together form the Navier-Stokes equations for the velocity and pressure fields within an incompressible fluid. It is well known that these equations are highly non-linear and exceedingly difficult to solve both analytically and numerically.

The Navier-Stokes equations give rise to a number of fascinating effects within fluid dynamics. In this project, we are particularly interested in boundary layer effects and turbulence both of which are resultant from considerations of the Navier-Stokes equations.

2.2.2 Non Dimensional Variables and the Reynolds Number

2.2.3 Boundary Layers

One of the fundamental ideas within fluid mechanics is that of the no slip condition. The no slip condition specifies that when a fluid encounters a solid body it must, at the surface of the body, take the velocity and temperature of that body. This means that there will be a thin layer of fluid around the body where the velocity changes

[1] The convention within fluid mechanics is that the x, y, z velocity components are called u, v, w respectively. That is

$$v_x = u, \qquad v_y = v, \qquad v_z = w.$$

from that of the overall stream to match the velocity of the solid body: this layer is referred to as the boundary layer.

The equations which govern the flow within the boundary layer are a simplified version of the Navier-Stokes equations, taking into account the order of magnitude of the boundary layer compared to the size of the body. The derivation of these equations completed by scaling the Navier-Stokes based on the assumption that the boundary layer is much smaller than the body size Anderson (1985).

In 2D these equations look as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(Continuity equation)} \tag{2.2.4a}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \qquad (x \text{ momentum}) \tag{2.2.4b}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} = 0 \qquad (y \text{ momentum}) \tag{2.2.4c}$$

2.2.4 Boundary Layer Separation and the Magnus Effect

2.2.5 Lift and Drag

2.3 Previous Work on Modelling Golf Ball Flight

Chapter 3

A Model of Golf Ball Flight

Chapter 4

Finding c_d and c_l

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