GOLF BALL AERODYNAMICS

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Contents

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Abstract

In this project we work on golf balls and stuff.

Declaration

No portion of the work referred to in the dissertation has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

Intellectual Property Statement

Acknowledgements

I would like to thank my supervisor Prof. Jitesh Gajjar for his guidance during the project without which I would have been completely lost. Additionally, I would like to thank the Royal and Ancient Golf Club for their sponsorship of the project and Dr Steve Otto for his guidance and assistance.

Chapter 1

Introduction

Stuff here about the project and aims and such.

1.1 A Brief History of Golf

The origins of the game of golf are difficult to trace, with suggestions that the game originated in either Scotland, France, the Netherlands, China, or even going back as far as the Roman Empire. Golf in its more modern incarnation however, is agreed to have originated in 15th century Scotland, where the first written records of the game are (somewhat humorously) related to King James II of Scotland banning the game in 1457 for fear of a decrease in archery practice in its favour.

From the 18th century onwards golf began to take form fully in Scotland, with the founding of both The Royal and Ancient Golf Club in St Andrews and The Royal Burgess Golfing Society in Edinburgh. The oldest surviving rules of golf also date from this time and these rules have been in a state of constant revision up to the modern day.

In the 19th century the popularity of golf vastly increased, seeing larger numbers of people knowing and playing the game, and the start of the first major tournaments. Additionally, the game spread out to encompass much of the British empire, to the United States and eventually to Japan, making golf into a global sport supported by a plethora of associated manufacturers, sponsors and organisations.

In the modern day, golf is potentially one of the largest sports on earth, with golf tournaments, golf manufacturing and related industries accounting for hundreds of billions of pounds of economic activity. If successful on the golf tournament circuit, golf professionals can earn huge sums in prize money. With the players themselves and their sponsors having such a vested interest in success having a consistent and fair rule set is of paramount importance and this is dealt with jointly by The R&A (The Royal and Ancient) in most of the world and the USGA (United States Golf Association) in the Americas.

1.2 A Slightly Larger History of the Golf Ball

Golf ball technology has advanced greatly since the advent of the game. Initially, hard wooden balls were used for playing, however these were soon replaced with featherie balls which are leather pouches stuffed with feathers and then painted white.

The next major innovation in the design of golf balls came in 1848, when the gutta-percha ball was invented. This is the first ball to use a rubbery substance as continues to this day, and was easier to make into a proper sphere, unlike the previous types of ball. It was around this time that it was discovered that abrasions to the surface of the ball would improve the aerodynamic properties of the ball, making it easier to control the flight of the ball and increasing the distance at which the game could be played. This would start a series of innovations that would lead to todays dimpled balls, which we will discuss later.

After this the golf ball once again changed form with the advent of using wrapped rubber thread to help the ball to bounce better. This was coupled with the first usage of a plastic covering, in order to protect the rubber inside the ball on impact with the club. This cover also persists to this day, although the inside of the ball has seen significant development.

The modern golf ball has changed significantly from old designs. The interior of the ball is now usually a 3 piece rubber composite, with different properties in each rubber to maximize the controllability of the ball during play. The exterior is a polyurethane cover (normally white but some are in other colours) with usually between 300 to 400 dimples (though these can go as low as 200 dimples, and beyond 600 in some cases). The properties of the ball are stipulated to be within certain ranges, as set by The R&A and USGA in the rules of golf. The weight of a ball must not be greater than

(a) "Featherie" balls

(b) Pro V1 ball

Figure 1.1: In 1.1a are "Featherie" golf balls, taken from https://en.wikipedia.org/wiki/File:Featherie_golf_ball.JPG, and in 1.1b is a modern style ball, namely the Titleist Pro V1 ball.

45.93g, the diameter no less than 42.67mm and the ball must be spherically symmetric.

1.3 Aims of the Project

The aim of this project is to obtain a model for how golf balls fly based on simple physical principles. Given this model we then wish to categorise individual balls based on measurements of their flight, and use this categorisation to predict trajectories for the ball

Finally, using this model, we will attempt to use a limited set of flight data (between 20 and 30 m) to predict the full flight for the ball.

Chapter 2

Preliminary Investigations and Background

In order to devise a simple model for golf ball flight we first must understand some prerequisite physics for projectiles and fluid dynamics for the airflow over the ball. Understanding how the fluid flows over the surface of the ball is crucial to understanding the difference between the flight of a golf ball and that of a standard projectile. Quantifying this effect will be a large component of this project.

There has been significant work done previously in understanding the fluid dynamics around a golf ball and how a golf ball flies. We will attempt to review some of this literature in this chapter and summarise previous work on the topic.

First though, we must understand how normal projectiles fly without taking into account fluid dynamics effects.

2.1 Projectile Motion

A projectile is a body fired into the air by an initial impulse and then allowed to fall back to ground under the action of gravity alone. This is the most naive and simplistic model of golf ball flight, completely ignoring all aerodynamic effects, however we must understand it before building up to a more complex model.

Consider motion in a 2 dimensional plane, labelled by x along the horizontal and y along the vertical. A projectile is given an initial speed of the form $\mathbf{V}_0 = (v_x, v_y)$. We set the origin of the coordinate system to be the point at the start of the trajectory,

$(x_0, y_0) = (0, 0)$. In this problem the acceleration on the projectile, after the initial impulse, is constant and of the form

$$a_x = 0, \quad a_y = -g \tag{2.1.1}$$

where g is the acceleration due to gravity. Since the acceleration is constant we can use the standard formulas for motion under constant acceleration to derive the dynamics of the projectile Young and Freedman (2008), which are

$$v = v_0 + at \tag{2.1.2a}$$

$$x = v_0 t + \frac{1}{2} a t^2 \tag{2.1.2b}$$

$$v^2 = v_0^2 + 2ax \tag{2.1.2c}$$

$$x = \left(\frac{v_0 + v}{2} \right) t. \tag{2.1.2d}$$

Where v is the speed at a time t, x is the distance from the origin of the coordinate system, a is the acceleration and v_0 is the initial speed.

We will write the equations in component form along the axes. Let $\mathbf{V}_0$ be the initial velocity. In component form these will be

$$v_{0x} = v_0 \cos \alpha$$

along the x-axis and

$$v_{0y} = v_0 \sin \alpha$$

where $v_0 = |\mathbf{V}_0|$ and α is the angle $\mathbf{V}_0$ makes with the x-axis. Now using (2.1.1) and (2.1.2a) we may find

$$v_x = v_0 \cos \alpha \tag{2.1.3a}$$

and

$$v_y = v_0 \sin \alpha - gt. \tag{2.1.3b}$$

Now, using (2.1.2b) (or by integrating (2.1.3) with respect to t), we can obtain the standard formulas for the x and y positions during the flight of the projectile:

$$x = (v_0 \cos \alpha) t \tag{2.1.4a}$$

and

$$y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2. \tag{2.1.4b}$$

Eliminating t between these equations demonstrates that projectiles follow parabolic paths, giving

$$y = x\tan\alpha - \frac{g}{2v_0^2\cos^2\alpha}x^2 \tag{2.1.5}$$

for the path of the projectile.

Finally we may use these equations to find the maximum height, range and time of flight for a projectile. The maximum height is obtained when $v_y = 0$ and solving (2.1.3b) with this condition gives

$$t_H = \frac{v_0\sin\alpha}{g}. \tag{2.1.6}$$

The range is obtained by solving for $y = 0$ in (2.1.4b) and selecting the non trivial root for t of

$$t_F = \frac{2v_0\sin\alpha}{g} \tag{2.1.7}$$

where t_F is the time of flight for the projectile. Inserting this into (2.1.4a) gives

$$x = \frac{2v_0^2\cos\alpha\sin\alpha}{g}$$

and recalling that $\sin 2\alpha = 2\cos\alpha\sin\alpha$ gives

$$x = \frac{v_0^2\sin 2\alpha}{g} \tag{2.1.8}$$

for the range of the projectile.

2.1.1 3D Projectile Motion

Projectile motion in 3 dimensions works in exactly the same fashion as 2D projectile motion. Here we will take the z-axis to be the vertical and x and y axes to be labelling the surface. The only component of acceleration is along the z-axis, with

$$a_z = -g$$

as before. All other equations remain the same.

2.2 Basic Aerodynamics

Of course, the flight of a golf ball is inevitably affected by aerodynamics. As such we need to have some understanding of how aerodynamic effects will . In particular, we will need to understand how boundary layers form on and separate from the surface of the golf ball and how this effects the drag on the ball. First we will review some basic fluid mechanics.

2.2.1 Fluid Mechanics

In this project we will model the air flowing around the ball as being an incompressible fluid. This is an Eulerian description of fluid flow, viewing the fluid as though the ball is fixed in the centre of the coordinate system and the fluid moving around the ball Ruban and Gajjar (2014).

We will not concern ourselves with a full discussion of fluid mechanics from basic principles here, instead we will simply state some useful results predominantly following Ruban and Gajjar (2014) and Sears (2011).

Let $\mathbf{V}$ be the fluid velocity, which is a function of the the position $\mathbf{r}$ from the origin of the coordinate system and of time t. Let ρ be the density of the fluid and p the pressure. We define the material derivative to be (as a differential operator)

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\mathbf{V} \cdot \nabla)$$

and represents the rate of change of some quantity within the fluid, while moving with a small element of the fluid flow. That is, in a description where the fluid moves relative to the coordinate system the material derivative measures the rate of change as seen by a moving fluid element.

At all points within the fluid the mass continuity equation must apply

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{V} = 0. \tag{2.2.1}$$

This equation encodes the condition that mass is conserved within a fluid without any sources or sinks. In an incompressible fluid, as we will be primarily conserved with in this project, ρ will not change with time, and as such $D\rho/Dt = 0$. As a consequence, both terms on the left hand side of (2.2.1) must be zero everywhere within the fluid, and therefore the equation reduces to

$$\nabla \cdot \mathbf{V} = 0 \tag{2.2.2}$$

for an incompressible fluid.

The continuity equation gives one equation for the velocity components u, v, w[1] within a fluid. In order to specify the pressure and velocity everywhere we therefore require three more equations to determine the system. These three equations are supplied by considerations of energy and momentum conservation. Keeping all of these in mind, we may write a momentum equation in the form

$$\rho \frac{D\mathbf{V}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{V} + \mathbf{f} \tag{2.2.3}$$

where $\mathbf{f}$ is body force per unit volume acting on the fluid (for example a gravitational force) and μ is the viscosity of the fluid.

Equations (2.2.2) and (2.2.3) when taken together form the Navier-Stokes equations for the velocity and pressure fields within an incompressible fluid. It is well known that these equations are highly non-linear and exceedingly difficult to solve both analytically and numerically, except in special circumstances.

From solutions the Navier-Stokes equations emerges a number of fascinating effects within fluid dynamics. In this project, we are particularly interested in boundary layer effects and turbulence.

2.2.2 Non Dimensional Variables and the Reynolds Number

In physics we are often interested in understanding the behaviour of a system independent of a choice of units Misic et al. (2010). Instead, we wish to have a system of measurement which does not depend intrinsically on one set on units, but on more fundamental ideas such as length or mass. We desire this for two main reasons:

- Physical statements should not have any dependency on the units they are stated in. Non-dimensionalising the problem ensures that this is the case.

[1] The convention within fluid mechanics is that the x, y, z velocity components are called u, v, w respectively. That is

$$v_x = u, \qquad v_y = v, \qquad v_z = w.$$

- Using non-dimensional variables allow problems of different scales to be compared to each other on an equal footing. Often the points when the physical behaviour of the system changes will depend on some non-dimensional parameter, as we will see with turbulence later Jensen (2013).

Additionally, analysing the fundamental dimensions of a physical problem can yield information on the functional form of quantities within the model without having to use more advanced mathematical techniques to derive such results. By simply understanding the dependency on fundamental units we often can find interesting scaling laws for functions of interest.

The fundamental dimensions (only those which will be useful in this project) are as follows:

Dimension	Symbol
Length	L
Mass	M
Time	T

Table 2.1: List of fundamental dimensions we will require in this report.

Any quantities of interest can be written using these variables: for example, an area A would have the dimensions $[A] = L^2$ and a velocity $[V] = L/T$.

Physical laws can, in general, be written as Misic et al. (2010)

$$q_0 = f(q_1, q_2, \ldots, q_n) \tag{2.2.4}$$

where q_0 is a physical quantity we are interested in obtaining, $q_1, \ldots, q_n$ are independent physical quantities and f is a functional relationship between them.

When analysing the dimensions of a problem we must ensure that the following conditions are satisfied

- Both sides of (2.2.4) must have the same fundamental dimensions.
- Any sum of q_i must have the same dimensions.
- Any function of q_i (say, exponential or trigonometric) must be dimensionless. This is as a direct result of the last condition and being able to expand these functions as a power series.

We will demonstrate this technique in section 2.2.4.

One of the aims of dimensional analysis is to find groupings of physical quantities which are dimensionless. These have useful properties of scale invariance which leads to natural parameterisations for physical problems.

Within fluid mechanics one such dimensionless grouping which manifests often is the Reynolds number, defined as

$$Re = \frac{\rho \mathbf{V} L}{\mu}. \tag{2.2.5}$$

For a sphere, $\mathbf{V}, \mu$ and ρ are defined as they have already been in this text, and the characteristic length L is defined to be the diameter of the sphere. This quantity represents the ratio of inertial forces to viscous ones, and we will make considerable use of it within the project. The Reynolds number, in some ways, can be considered to be a non dimensional analogue of the velocity, taking into account the intrinsic length scale of the problem under consideration.

2.2.3 Boundary Layers

One of the fundamental ideas within fluid mechanics is that of the no slip condition. The no slip condition specifies that when a fluid encounters a solid body it must, at the surface of the body, take the velocity and temperature of that body. This means that there will be a thin layer of fluid around the body where the velocity changes from that of the overall stream to match the velocity of the solid body: this layer is referred to as the boundary layer.

The equations which govern the flow within the boundary layer are a simplified version of the Navier-Stokes equations, taking into account the order of magnitude of the boundary layer compared to the size of the body. The derivation of these equations completed by scaling the Navier-Stokes based on the assumption that the boundary layer is much smaller than the body size Anderson (1985).

In 2D these equations look as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(Continuity equation)} \tag{2.2.6a}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \qquad (x \text{ momentum}) \tag{2.2.6b}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} = 0 \qquad (y \text{ momentum}) \tag{2.2.6c}$$

where ν is the kinematic viscosity, defined as $\nu = \mu/\rho$.

Boundary layer theory has been hugely important in the development of aerodynamics, taking previously intractable problems and facilitating a better understanding of the fluid dynamics of bodies Anderson (1985). The affect of the boundary layer will be important in the modelling of golf balls, as we will see later.

2.2.4 Lift and Drag

In order to go beyond modelling golf ball flight as simply that of a projectile we must understand how the lift and drag forces affect the flight of the ball. We can use dimensional analysis arguments to obtain a functional form for these effects. Here we follow the analysis of Jensen (2013) in order to demonstrate the analysis for the drag force. The lift is found in a similar way.

One first must ask what physical terms we would expect the drag to have a dependency on. We would expect some dependency on the velocity v of the object through the fluid, on the shape of the body and a characteristic length scale r for the body, on the density of the fluid ρ, and finally on the viscosity of the fluid ν. As noted in Jensen (2013) we have no dependency from the mass of the body on the drag. As per the rest of this project we also assume the fluid is incompressible.

Forming an equation in the style of (2.2.4) between these quantities and the drag force D we obtain

$$D = g\, v^{\alpha} r^{\beta} \rho^{\gamma} \nu^{\delta} \tag{2.2.7}$$

where g is a dimensionless constant which will take into account the shape of the body and potentially other dimensionless parameters such as the Reynolds number (2.2.5).

We attempt to obtain values of $\alpha, \beta, \gamma, \delta$ which balances (2.2.7). The dimensions of the quantities in (2.2.7) are as follows:

$$[D] = MLT^{-1}, \qquad [v] = LT^{-1}, \qquad [r] = L, \qquad [\rho] = ML^{-3}, \qquad [\nu] = ML^{-1}T^{-1}$$

and so balancing (2.2.7) implies that

$$MLT^{-2} = M^{\gamma+\delta} L^{\alpha+\beta-3\gamma-\delta} T^{-\alpha-\delta}. \tag{2.2.8}$$

Equating the powers leads to a system of 3 linear equations in 4 variables for each fundamental unit given by

$$1 = \gamma + \delta \tag{2.2.9a}$$

$$1 = \alpha + \beta - 3\gamma - \delta \tag{2.2.9b}$$

$$-2 = \alpha - \delta \tag{2.2.9c}$$

for M, L and T respectively.

Using these equations we can eliminate all but one of the parameters, and writing all these in terms of δ we can find

$$D = gv^{2-\delta}r^{2-\delta}\rho^{1-\delta}\nu^{\delta} \tag{2.2.10}$$

and rearranging these powers

$$D = g\rho r^2 v^2 \left(\frac{\nu}{\rho r v}\right)^{\delta}. \tag{2.2.11}$$

We can then recognise that the term in the brackets is $1/Re$. Following Jensen (2013) we can derive, using a similar analysis, that g must also have some functional dependency on the Reynolds number, and as such

$$D = \rho r^2 v^2 f(Re). \tag{2.2.12}$$

The form of f depends on the velocity of the body moving through the fluid.

In the low velocity limits of low and high velocity we find slightly different forms for the dependence on velocity. In the low velocity limit, the dependency on velocity is only linear, that is

$$D = g\nu r v. \tag{2.2.13}$$

where g is simply related to the shape of the body. This is known as stokes drag, corresponding to laminar flow about the body. Whereas in the high velocity limit we would expect turbulent flow behind the body. This is the case which manifests during the flight of a golf ball, and the distinction will be important later.

$$D = g\rho r^2 v^2 \tag{2.2.14}$$

On comparison with (2.2.12) we see that at low velocity, $f(Re) \propto Re^{-1}$ and in the high velocity case $f(Re)$ is independent of Re.

Within the literature related to fluid dynamics and aerodynamics (2.2.14) is normally written with different names for the variables, namely

$$F_D = \frac{1}{2}\rho \mathbf{V}^2 A c_D \tag{2.2.15}$$

where F_D is the drag force, and c_D is the dimensionless drag coefficient. Is it this form we will use from here on.

An expression for the lift can be derived using very similar analysis, and has the same form as the drag:

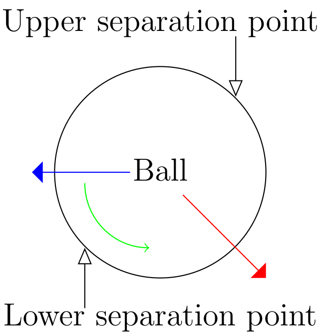
$$F_L = \frac{1}{2}\rho \mathbf{V}^2 A c_L \tag{2.2.16}$$

where here c_L is the lift coefficient.

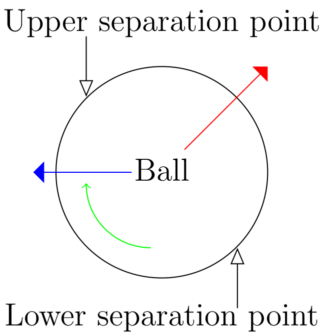
2.2.5 Boundary Layer Separation and the Magnus Effect

In addition to the drag forces which we have discussed in the previous section, there is another major effect from the motion of the fluid on golf ball trajectories: the Magnus effect. The Magnus effect is caused by the spin of the ball moving through the fluid medium, and accounts for large deviations from a trajectory not considering the effect Seifert (2012).

In the Magnus effect, the boundary layer around the spinning body separates (in some literature this is called detaching instead) from the body and forms a wake behind the body.



(a) Boundary layer separation



(b) Reversed boundary layer separation

Figure 2.1: Here we see the two scenarios for boundary layer detachment. The red arrow indicates the direction which the detached boundary layer proceeds in, the blue the direction of the ball, and the green arrow the direction of spin of the ball.

The points at which the boundary layer separates from the body at the top and bottom of the ball make a significant difference to the direction in which the wake will point: if the boundary layer at the top of the ball separates later than the bottom (see a in Figure 2.1) then we will have a positive Magnus effect, and the lift will be increased. In the opposite case, where the bottom separates later than the top, we will have a negative Magnus effect (see b in Figure 2.1) and the lift on the ball will be decreased or potentially act to push the ball further towards the ground.

The fluid behind the rotating sphere will become turbulent as the sphere leaves a gap which the wake refills as it passes. This mechanism is important while modelling the golf ball later, as the size of the turbulent wake will affect the drag on the ball.

The streamlines over the ball can be visualised as such

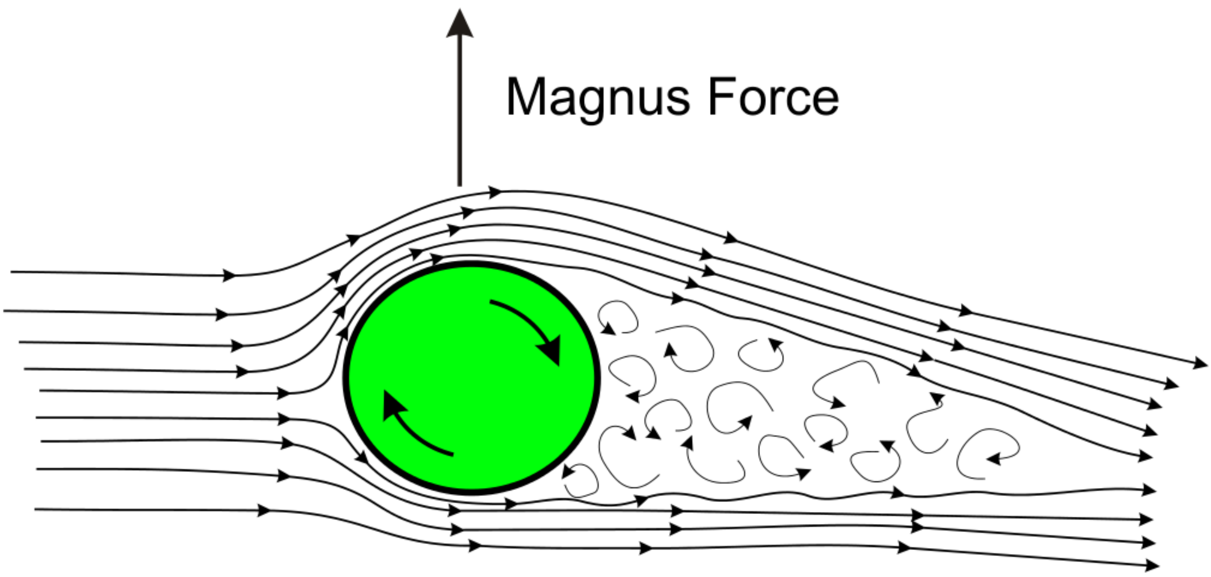


Figure 2.2: A diagram of the Magnus force on a rotating sphere. Adapted from http://en.wikipedia.org/wiki/File:Sketch_of_Magnus_effect_with_streamlines_and_turbulent_wake.svg.

In this diagram we have lamina flow in the boundary layer over the ball. However, this is not always the case: when the flow becomes turbulent the drag can change considerably.

2.2.6 The Drag Crisis

When measuring the flow around a smooth sphere, one finds a curious phenomenon. At approximately $Re = 2 \times 10^5$ the drag coefficient on the sphere suddenly decreases from around $c_D = 0.4$ to $c_D = 0.1$. This sudden change in drag is associated with the boundary layer around the sphere becoming turbulent and the wake behind the ball, as described before, becoming shorter and thinner as compared to the size of the ball.

Modelling this transition to turbulence is incredibly difficult, even for smooth spheres, and is incredibly difficult for the case of a dimpled golf ball. While there are some hints towards progress in finding analytic solutions for this transition to turbulence for a smooth sphere Assis et al. (2010), we must make do with measurements from experiments to get some idea of how this works.

In Morrison (2010) all experimental data for the drag on a smooth sphere is combined to give a formula for the drag in the Reynolds number range $Re = 1$ to $Re = 10^6$. This combined form exhibits the drag crisis drop at around $Re = 2 \times 10^5$ experiments have seen, and is given by

$$c_D = \frac{24}{Re} + \frac{2.6(Re/5)}{1 + (Re/5)^{1.52}} + \frac{0.411(Re/263000)^{-7.94}}{1 + (Re/263000)^{-8.00}} + \frac{Re^{0.8}}{461000}. \tag{2.2.17}$$

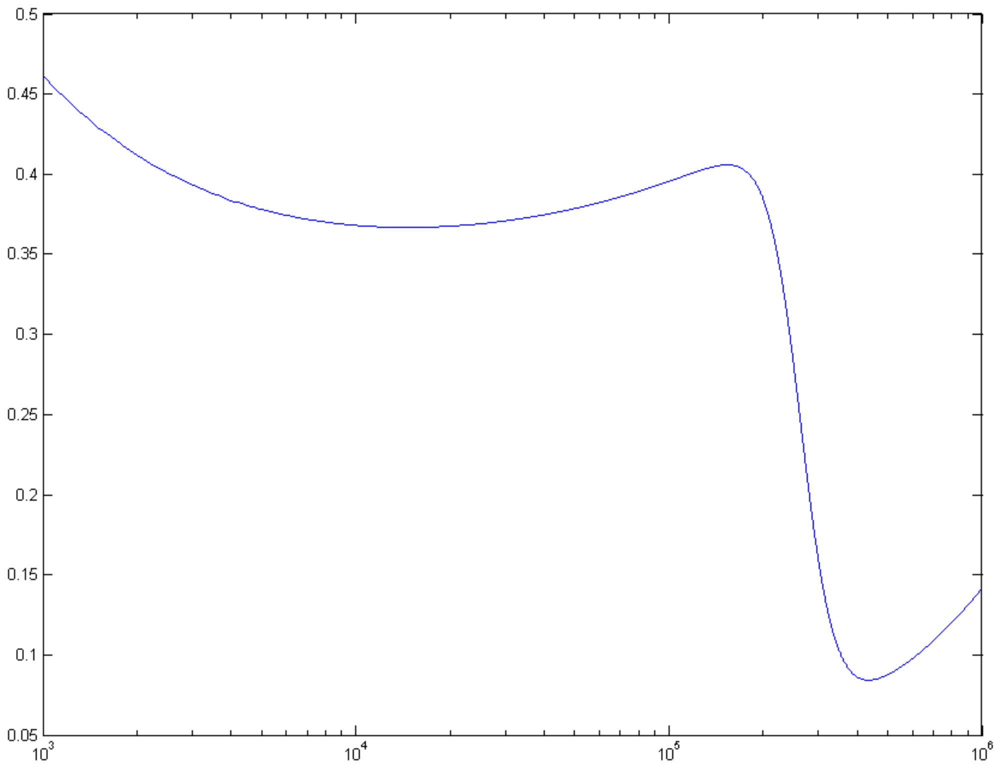


Figure 2.3: A plot of the Morrison drag formula from $Re = 10^3$ to $Re = 10^6$. Note the sudden drop in drag at around $Re = 2 \times 10^5$ as expected.

For a golf ball, the presence of dimples on the surface of the ball serves to reduce the Reynolds number at which the drag crisis occurs Alam et al. (2011), moving the range of speeds where the golf ball is in the low drag (supercritical) region to within the capability of a human golfer to hit. This vast reduction in drag makes a large difference to the flight of a ball, and we will need to account for this affect in any model we form.

The configuration of the dimples on the ball also has a large effect on the values of Re at which the drag crisis occurs, and the value of c_D at either side of the drop Naruo and Mizota (2014). There is a large variation between particular type of ball (see Figure 3 in Naruo and Mizota (2014)), which mean that individual balls can likely be characterised simply in terms of their drag function.

2.3 Previous Work on Modelling Golf Ball Flight

There has been considerable attention within the literature on the topic of modelling the flight of a golf ball, both due to the considerable industry surrounding the game and the interesting fluid dynamics which results from golf ball flight. A small selection of such papers are Smits and Ogg (2004); Bearman and Harvey (1976); Penner (2003); Alam et al. (2011); Kensrud and Smith (2010); Leong and Lin (2007) however there are many more which could be discussed.

The earliest of these papers is Bearman and Harvey (1976) which is one of the first attempts to understand the fluid dynamics over a golf ball and provide a model for the flight of a ball taking this into account. The drag crisis on a golf ball is shown in experimental data taken from an earlier paper and from measurements the authors made in a wind tunnel. These measurements were taken at a range of Re values and spin values, providing a useful set data to correlate any findings against. The paper also emphasizes the importance of the dimples on the aerodynamic characteristics of the ball, in agreement with other papers.

In Smits and Ogg (2004) the authors summarise the main effects one will find on golf ball trajectories, mentioning both the laminar and turbulent boundary layer we discussed previously, the effect of spin on the lift coefficient, and including some of the data from Bearman and Harvey (1976) to illustrate these points. The paper also suggests that much work still remains in understanding how the fluid dynamics over golf balls functions, saying that fundamentally balls are designed via empirical means, simply using the knowledge contained in the previous sections and experiments to improve the designs of balls. The authors state also state, with reference to the design of golf balls Smits and Ogg (2004, page 10):

"The fundamental design challenge in optimizing golf ball aerodynamics is achieving the lower possible drag level at high Reynolds number while ensuring a high lift coefficient at the lowest Reynolds number in the design space."

These two goals are virtually at opposite ends from each other and thus means the challenge of making a good golf ball for all ranges of speed in a typical game is very difficult.

2.3.1 Computational Simulations of Golf Ball Flight

Smith et al. (2010); Beratlis et al. (2012)

2.4 Measuring Golf Ball Trajectories

Martin et al. (2012)

Chapter 3

A Model of Golf Ball Flight

This section is about Robinson and Robinson (2013) and the comment Jensen (2014) and the reply Robinson and Robinson (2014).

3.1 A Model for Golf Ball Flight

Describe the model, provide diagrams, show basic runs.

Model is a good comprimise between simplicity and flexibility. Can plug in new drag coefficents. Some parts of the model are purely heurestic like the form for the lift, others seem just plain wrong. The overall structure is good though. Compare basic run to data, show similar shape but incorrect carry.

3.2 Limitations of the Model

Talk about the Jensen (2014) comment, dimensional analysis, lead into talking about why we need to improve this model to find a better form for c_D. Ppotentially a way to estimate the spin ratio. Mention how Robinson and Robinson (2014) addresses some of the concerns of the comment but does not give forms for c_D. Talk about how the golf ball is in the middle between the high and low reynold limits and this will require some matching modelling to find a good form.

Chapter 4

Finding c_L and c_D

This is where we discuss the main results we've had.

4.1 Estimating c_D from Experiments

- We try to find results for spheres and change them to account for the earlier drag crisis.
- Take the function from Morrison (2010) and change the coefficents to "move" the drag drop to lower values of Re.
- This works fairly well but cannot capture all of the behaviour, as other work shows that different balls should expect to see different drags Alam et al. (2011).

4.2 Parameterising c_D and c_L by Non Dimensional Variables

- We follow the idea from Lieberman and Smits (2001): form c_D and c_L from dimensionless groupings and use the data to estimate the parameters in this model.
- Use least squares to do this. See Appendix for discussion of what an inverse problem is, how to use least squares to solve them, and what numerical techniques there are to do this.

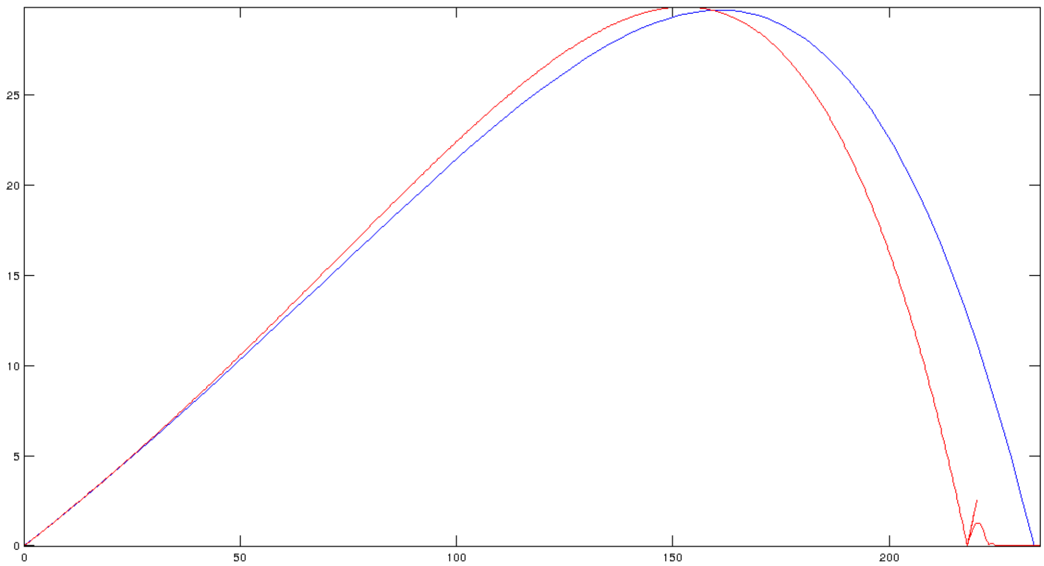


Figure 4.1: Using the modified Morrison form for c_D results in a fairly accurate profile. Here red is the data and blue is the predictions of the model.

- Find that doing this is very hard: the problem is likely not well posed and finding a minimum is difficult. Would benefit from a more through analysis of the least squares problem but unsure how this would be done.

- This inverse problem is a good way to move forwards with the problem in the future.

4.3 tanh Matching

- Take a hybrid approch between the two previous ideas: form a function which "looks" similar to Morrison (2010) and has the same behaviour, but is parameterised in such a way as to allow us to use a least squares solver to estimate parameters.

- For the drop, use a tanh function of the form

$$c_D = a + b\tanh(-c(Re - d))$$

  where a, b, c, d are constants which we can use least squreas to determine.

- Match the tanh at the top with the $24/Re$ form we know from spheres and past the drop match to the weakly linear form we see from the previous work.

- Still trying to decide on results for this.

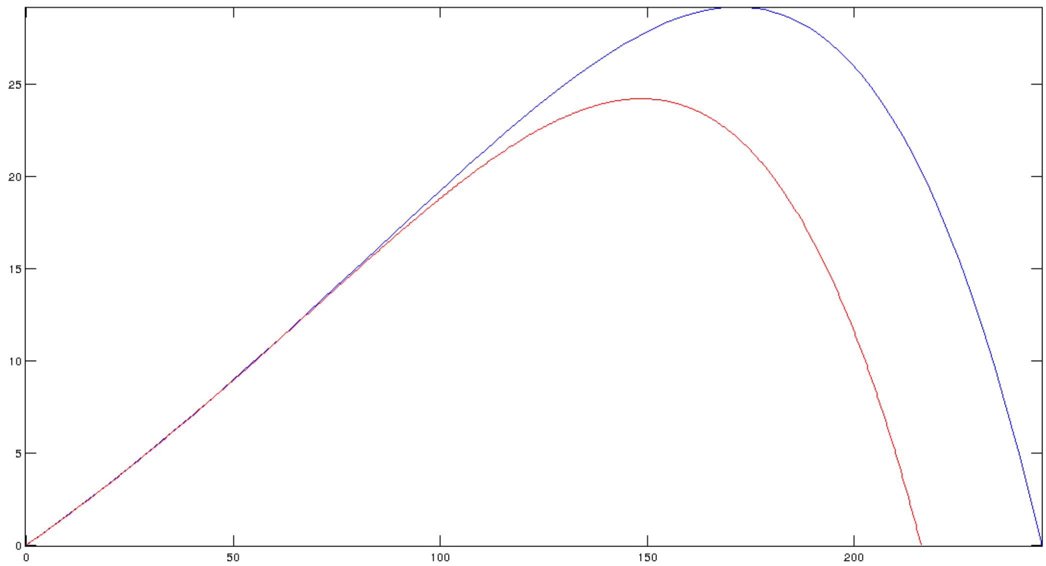


Figure 4.2: The Morrison drag form does not always produce accurate results, however we do see good agreement at the start. Here red is the data and blue is the predictions of the model.

Chapter 5

Conclusions

5.1 Possible Future Work

Appendix A

Inverse Problems and Least Squares

A.1 Inverse Problems

Tarantola (2005); Tarantola and Valette (1982)

A.2 Least Squares

A.2.1 Guass-Newton Method

A.2.2 Levenberg-Marquardt Algorithm

Pujol (2007)

A.2.3 Trust Region Method

Kelley (1999)

A.3 Well Posedness and Regularization

Fang (2004)

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