Chapter 2: The Value of Involving Vectors - Homework Problems

1 Section 2.1: Describing Vectors in the Plane

1.1 Problem 1.1

Given the vectors:

$$\mathbf{u} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 0 \\ -3 \end{bmatrix}$$

Find the terminal points when each vector is drawn in standard position.

1.2 Problem 1.2

Draw the vector $\mathbf{v} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ with endpoint at $(3, -1)$. Find the coordinates of the terminal point.

1.3 Problem 1.3

Given the three-space vector $\mathbf{a} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$, identify the terminal point when drawn in standard position.

2 Section 2.2: Scalar Multiplication

2.1 Problem 2.1

Given $\mathbf{v} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$, compute:

1. $3\mathbf{v}$
2. $-2\mathbf{v}$
3. $\frac{1}{2}\mathbf{v}$
4. $0 \cdot \mathbf{v}$

2.2 Problem 2.2

For the vector $\mathbf{u} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$, compute:

1. $5\mathbf{u}$
2. $0.4\mathbf{u}$
3. $-\frac{3}{2}\mathbf{u}$

2.3 Problem 2.3

Calculate the following scalar multiplications:

1. $7 \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$
2. $-3 \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix}$
3. $\frac{2}{3} \begin{bmatrix} 9 \\ -6 \\ 3 \end{bmatrix}$

3 Section 2.3: Adding and Subtracting Vectors

3.1 Problem 3.1

Given vectors $\mathbf{u} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$, compute:

1. $\mathbf{u} + \mathbf{v}$
2. $\mathbf{u} - \mathbf{v}$
3. $\mathbf{v} - \mathbf{u}$

3.2 Problem 3.2

For three-space vectors $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$, and $\mathbf{c} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$, compute:

1. $\mathbf{a} + \mathbf{b} + \mathbf{c}$
2. $2\mathbf{a} - 3\mathbf{b} + \mathbf{c}$
3. $\mathbf{a} - 2\mathbf{b} + 4\mathbf{c}$

3.3 Problem 3.3

Calculate the following vector operations:

1. $\begin{bmatrix} 4 \\ 7 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$
2. $\begin{bmatrix} 6 \\ -4 \end{bmatrix} - \begin{bmatrix} 2 \\ -7 \end{bmatrix}$
3. $3 \begin{bmatrix} 2 \\ -1 \end{bmatrix} + 2 \begin{bmatrix} -3 \\ 4 \end{bmatrix}$

4 Section 2.4: Vector Magnitude

4.1 Problem 4.1

Calculate the magnitude of each vector:

1. $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$
2. $\mathbf{v}_2 = \begin{bmatrix} -5 \\ 12 \end{bmatrix}$
3. $\mathbf{v}_3 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$
4. $\mathbf{v}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

4.2 Problem 4.2

Given $\mathbf{u} = \begin{bmatrix} 6 \\ -8 \end{bmatrix}$, calculate:

1. $\|\mathbf{u}\|$
2. $\|3\mathbf{u}\|$
3. $\|-2\mathbf{u}\|$
4. $\left\|\frac{1}{2}\mathbf{u}\right\|$

4.3 Problem 4.3

Find unit vectors in the direction of each given vector:

1. $\mathbf{v} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$
2. $\mathbf{w} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$
3. $\mathbf{u} = \begin{bmatrix} 5 \\ 0 \\ -12 \end{bmatrix}$

5 Section 2.5: Triangle Inequality

5.1 Problem 5.1

For vectors $\mathbf{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, calculate:

1. $\|\mathbf{u} + \mathbf{v}\|$
2. $\|\mathbf{u}\| + \|\mathbf{v}\|$
3. Verify that $\|\mathbf{u} + \mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$

5.2 Problem 5.2

For vectors $\mathbf{a} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$, calculate:

1. $\|\mathbf{a} - \mathbf{b}\|$
2. $\|\mathbf{a}\| + \|\mathbf{b}\|$
3. $\|\mathbf{a}\| - \|\mathbf{b}\|$

6 Section 2.6: Inner Product (Dot Product)

6.1 Problem 6.1

Calculate the dot product for each pair of vectors:

1. $\mathbf{u} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$
2. $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$
3. $\mathbf{p} = \begin{bmatrix} -2 \\ 3 \\ 1 \\ 4 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 0 \end{bmatrix}$

6.2 Problem 6.2

Calculate the dot product and determine if the vectors are orthogonal:

1. $\mathbf{u} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$
2. $\mathbf{p} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$
3. $\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$, $\mathbf{s} = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$

6.3 Problem 6.3

Find the value of k that makes the vectors orthogonal:

1. $\mathbf{u} = \begin{bmatrix} 2 \\ k \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$
2. $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ k \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

7 Section 2.7: Angles Between Vectors

7.1 Problem 7.1

Find the angle between the following pairs of vectors (in degrees):

1. $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
2. $\mathbf{a} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$
3. $\mathbf{p} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{q} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

7.2 Problem 7.2

Given the magnitudes and angle, find the dot product:

1. $\|\mathbf{u}\| = 5$, $\|\mathbf{v}\| = 7$, angle $= 60$
2. $\|\mathbf{a}\| = 3$, $\|\mathbf{b}\| = 4$, angle $= 90$
3. $\|\mathbf{p}\| = 2$, $\|\mathbf{q}\| = 6$, angle $= 120$

7.3 Problem 7.3

Calculate the angle between the vectors and express in both degrees and radians:

1. $\mathbf{u} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$
2. $\mathbf{a} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 2\sqrt{3} \end{bmatrix}$

8 Section 2.8: Mixed Computational Problems

8.1 Problem 8.1

Given vectors $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$, calculate:

1. $3\mathbf{u} - 2\mathbf{v}$
2. $\mathbf{u} \cdot \mathbf{v}$
3. $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$
4. The angle between $\mathbf{u}$ and $\mathbf{v}$ (in degrees)

8.2 Problem 8.2

For vectors $\mathbf{a} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$, and $\mathbf{c} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$, calculate:

1. $\mathbf{a} + 2\mathbf{b} - 3\mathbf{c}$
2. $\|\mathbf{a} + \mathbf{b}\|$
3. $(\mathbf{a} + \mathbf{b}) \cdot \mathbf{c}$
4. The unit vector in the direction of $\mathbf{a} - \mathbf{b}$

8.3 Problem 8.3

Calculate the following expressions:

1. $\left\| 2 \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} + 3 \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \right\|$
2. $\begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix} \cdot \left(3 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} - \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} \right)$
3. The angle between $2\mathbf{u}$ and $3\mathbf{v}$ where $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

9 SOLUTIONS

9.1 Section 2.1 Solutions

Problem 1.1: Terminal points: $\mathbf{u}$ at $(3, -2)$, $\mathbf{v}$ at $(-1, 4)$, $\mathbf{w}$ at $(0, -3)$

Problem 1.2: Terminal point at $(3 + 2, -1 + 5) = (5, 4)$

Problem 1.3: Terminal point at $(2, -1, 3)$

9.2 Section 2.2 Solutions

Problem 2.1:

1. $3\mathbf{v} = \begin{bmatrix} 12 \\ -9 \end{bmatrix}$
2. $-2\mathbf{v} = \begin{bmatrix} -8 \\ 6 \end{bmatrix}$
3. $\frac{1}{2}\mathbf{v} = \begin{bmatrix} 2 \\ -1.5 \end{bmatrix}$
4. $0 \cdot \mathbf{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Problem 2.2:

1. $5\mathbf{u} = \begin{bmatrix} -10 \\ 5 \\ 15 \end{bmatrix}$
2. $0.4\mathbf{u} = \begin{bmatrix} -0.8 \\ 0.4 \\ 1.2 \end{bmatrix}$
3. $-\frac{3}{2}\mathbf{u} = \begin{bmatrix} 3 \\ -1.5 \\ -4.5 \end{bmatrix}$

Problem 2.3:

1. $\begin{bmatrix} 14 \\ -7 \\ 28 \end{bmatrix}$
2. $\begin{bmatrix} -15 \\ 0 \\ 6 \end{bmatrix}$
3. $\begin{bmatrix} 6 \\ -4 \\ 2 \end{bmatrix}$

9.3 Section 2.3 Solutions

Problem 3.1:

1. $\mathbf{u} + \mathbf{v} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$
2. $\mathbf{u} - \mathbf{v} = \begin{bmatrix} 8 \\ -6 \end{bmatrix}$

3. $\mathbf{v} - \mathbf{u} = \begin{bmatrix} -8 \\ 6 \end{bmatrix}$

Problem 3.2:

1. $\mathbf{a} + \mathbf{b} + \mathbf{c} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$
2. $2\mathbf{a} - 3\mathbf{b} + \mathbf{c} = \begin{bmatrix} -9 \\ 8 \\ -5 \end{bmatrix}$
3. $\mathbf{a} - 2\mathbf{b} + 4\mathbf{c} = \begin{bmatrix} -13 \\ 8 \\ 7 \end{bmatrix}$

Problem 3.3:

1. $\begin{bmatrix} 3 \\ 10 \\ 3 \end{bmatrix}$
2. $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$
3. $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$

9.4 Section 2.4 Solutions

Problem 4.1:

1. $\|\mathbf{v}_1\| = 5$
2. $\|\mathbf{v}_2\| = 13$
3. $\|\mathbf{v}_3\| = \sqrt{14}$
4. $\|\mathbf{v}_4\| = 2$

Problem 4.2:

1. $\|\mathbf{u}\| = 10$
2. $\|3\mathbf{u}\| = 30$
3. $\| - 2\mathbf{u}\| = 20$
4. $\left\|\frac{1}{2}\mathbf{u}\right\| = 5$

Problem 4.3:

1. $\begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix}$
2. $\begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix}$
3. $\begin{bmatrix} 5/13 \\ 0 \\ -12/13 \end{bmatrix}$

9.5 Section 2.5 Solutions

Problem 5.1:

1. $\|\mathbf{u} + \mathbf{v}\| = \sqrt{10}$
2. $\|\mathbf{u}\| + \|\mathbf{v}\| = \sqrt{13} + \sqrt{5}$
3. $\sqrt{10} \leq \sqrt{13} + \sqrt{5}$ (approximately $3.16 \leq 5.85$)

Problem 5.2:

1. $\|\mathbf{a} - \mathbf{b}\| = 3$
2. $\|\mathbf{a}\| + \|\mathbf{b}\| = \sqrt{26} + 3$
3. $\|\mathbf{a}\| - \|\mathbf{b}\| = \sqrt{26} - 3$

9.6 Section 2.6 Solutions

Problem 6.1:

1. $\mathbf{u} \cdot \mathbf{v} = 1$
2. $\mathbf{a} \cdot \mathbf{b} = -4$
3. $\mathbf{p} \cdot \mathbf{q} = -3$

Problem 6.2:

1. $\mathbf{u} \cdot \mathbf{v} = 0$ (orthogonal)
2. $\mathbf{p} \cdot \mathbf{q} = 0$ (orthogonal)
3. $\mathbf{r} \cdot \mathbf{s} = 0$ (orthogonal)

Problem 6.3:

1. $k = \frac{3}{2}$
2. $k = -\frac{1}{2}$

9.7 Section 2.7 Solutions

Problem 7.1:

1. 45
2. 108.43
3. 60

Problem 7.2:

1. $\mathbf{u} \cdot \mathbf{v} = 17.5$
2. $\mathbf{a} \cdot \mathbf{b} = 0$
3. $\mathbf{p} \cdot \mathbf{q} = -6$

Problem 7.3:

1. 70.53 or 1.23 radians
2. 60 or $\frac{\pi}{3}$ radians

9.8 Section 2.8 Solutions

Problem 8.1:

1. $3\mathbf{u} - 2\mathbf{v} = \begin{bmatrix} 4 \\ -11 \\ 13 \end{bmatrix}$
2. $\mathbf{u} \cdot \mathbf{v} = -8$
3. $\|\mathbf{u}\| = \sqrt{14}$, $\|\mathbf{v}\| = \sqrt{21}$
4. 117.8

Problem 8.2:

1. $\mathbf{a} + 2\mathbf{b} - 3\mathbf{c} = \begin{bmatrix} 8 \\ -5 \\ -9 \end{bmatrix}$
2. $\|\mathbf{a} + \mathbf{b}\| = \sqrt{14}$
3. $(\mathbf{a} + \mathbf{b}) \cdot \mathbf{c} = 4$
4. $\frac{1}{\sqrt{6}} \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}$

Problem 8.3:

1. $\sqrt{70}$
2. -9
3. 90