MIDS UC Berkeley - Machine Learning at Scale

DATSCIW261 ASSIGNMENT #6

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W261-1, Spring 2016
Week 6 Homework

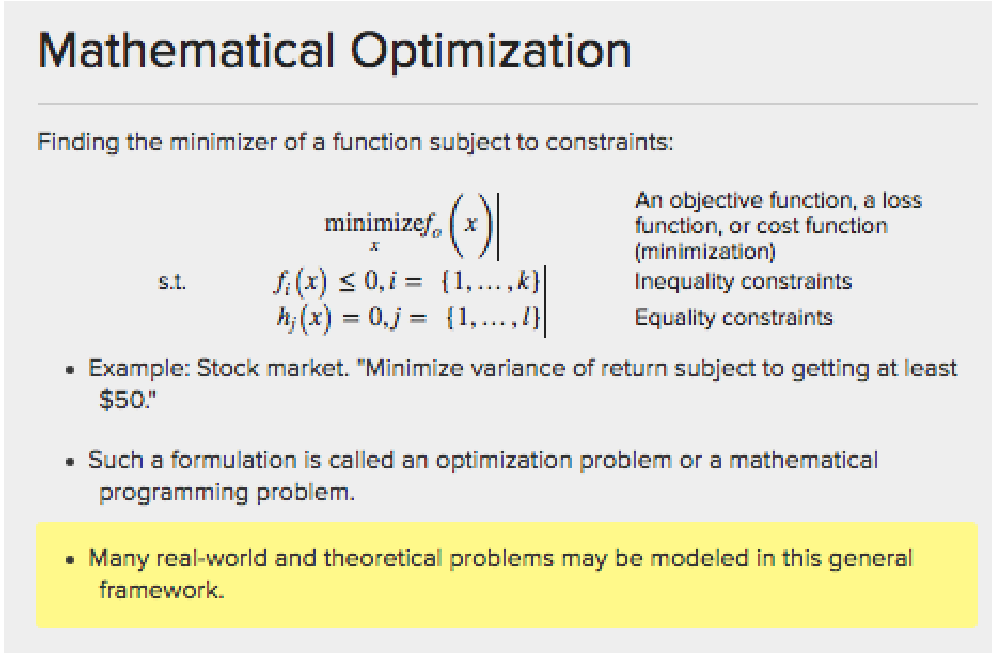
In [47]:
```
# Code modules for notebook

# the autoreload will "reload" the MRJob code with the latest changes
%load_ext autoreload
%autoreload 2

# inline will render matplotlib charts inside the notebook
%matplotlib inline

import numpy as np
import pylab as plt
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

HW6.0 - Mathematical Optimization

In mathematics, computer science, economics, or management science what is mathematical optimization? Give an example of a optimization problem that you have worked with directly or that your organization has worked on. Please describe the objective function and the decision variables. Was the project successful (deployed in the real world)? Describe.

Mathematical optimization is the selection of a best element (with regard to some criteria) from some set of available alternatives. In the simplest case, an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from within an allowed set and computing the value of the function.



While I have not worked hands-on these directly, here are a few optimization examples from previous organizations. At Chevron, the objective was to make various grades of gasoline with the components at least cost and not exceeding the octane rating. These were real-time optimization models that controlled the mixtures of various streams of components. The decisions variables where the component quantities for each material that would be combined to create gasoline at a specific octane rating. At Microsoft, we ran optimization models to determine how we would route and assign a customer support case to a specific agent queque. The objective was to minimize time by assigning the case to the agent with the right level of product knowledge, customer knowledge, language and availablity to solve the case. These form the decision variables including others such as satisfaction rating, geography and contract cost (per minute).

HW6.1 - Optimization Theory

For unconstrained univariate optimization what are the first order Necessary Conditions for Optimality (FOC)?

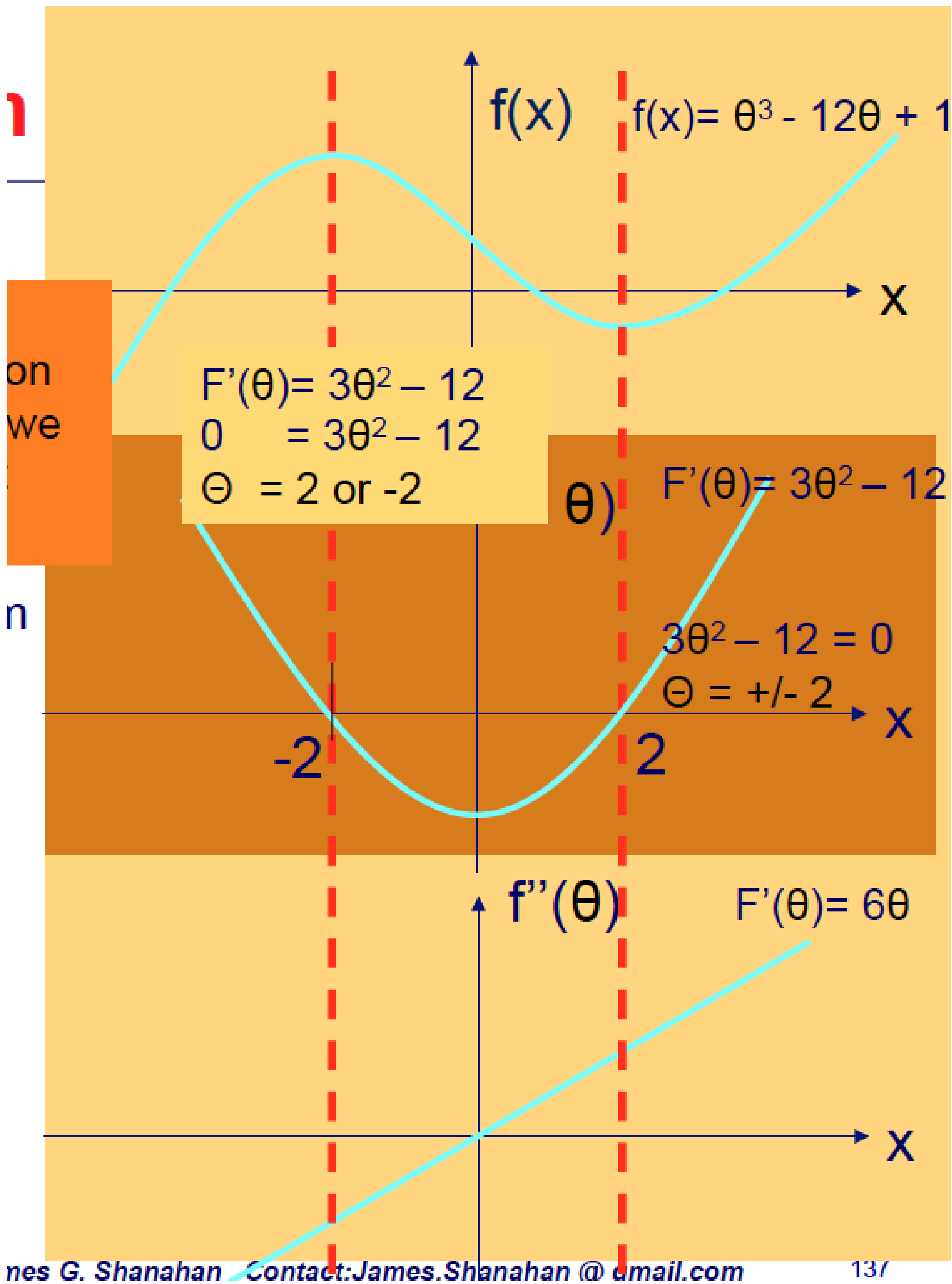
The FOC is the first derivative of the objective function will be equal to zero for the input value x and this represents the maximum and minimum values of the objective function. The following equation represents this mathematically:

$$f'(x = x*) = 0 \qquad \text{(First-order condition)}$$

The first derivative is middle graph below where x = -2, 2

What are the second order optimality conditions (SOC)? Give a mathematical defintion. Also in python, plot the univartiate function X^3 -12x^2-6 defined over the real domain -6 to +6.

The SOC conditions determine if the "roots" determined by the first order condition (first derivative) are the local maximum or minimum by taking the second derivative of the objective function. The SOC is represented in the bottom part of the picture below. The local maximum for the objective function exists when the second derivative is negative and the local minumum exists when the second derivative is positive.



Also plot its corresponding first and second derivative functions. Eyeballing these graphs, identify candidate optimal points and then classify them as local minimums or maximums. Highlight and label these points in your graphs. Justify your responses using the FOC and SOC.

The optimal points for the local minimum and maximum are when the 1st derivative (green line in plot below) passes through the x axis. We also need to look at the 2nd derivative to determine if each point is a maximum or minimum. The candidate optimal points are x = 0 and x=8. The objective function maximum is 0 given that the 2nd derivative is negative at point while the local minimum is 8 give that the 2nd derivative is positive at that point.

For unconstrained multi-variate optimization what are the first order Necessary Conditions for Optimality (FOC). What are the second order optimality conditions (SOC)? Give a mathematical defintion. What is the Hessian matrix in this context?

In this scenario we have many dimensions to determine the first and second order conditions. We set the gradient of the multivariate objective function equal to zero to determine the maximum and minimum optimal points. Here the gradient function are represented by partial derivatives of the input values as shown below.

- The gradient at a specific point $x = x'$ is the vector whose elements are the respective partial derivatives evaluated at $X = x'$, so that

$$\nabla f(X = x') = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n}\right)$$

$$\nabla(X = x') = 0; \quad \nabla f(X = x') = (0, 0, \ldots 0)$$

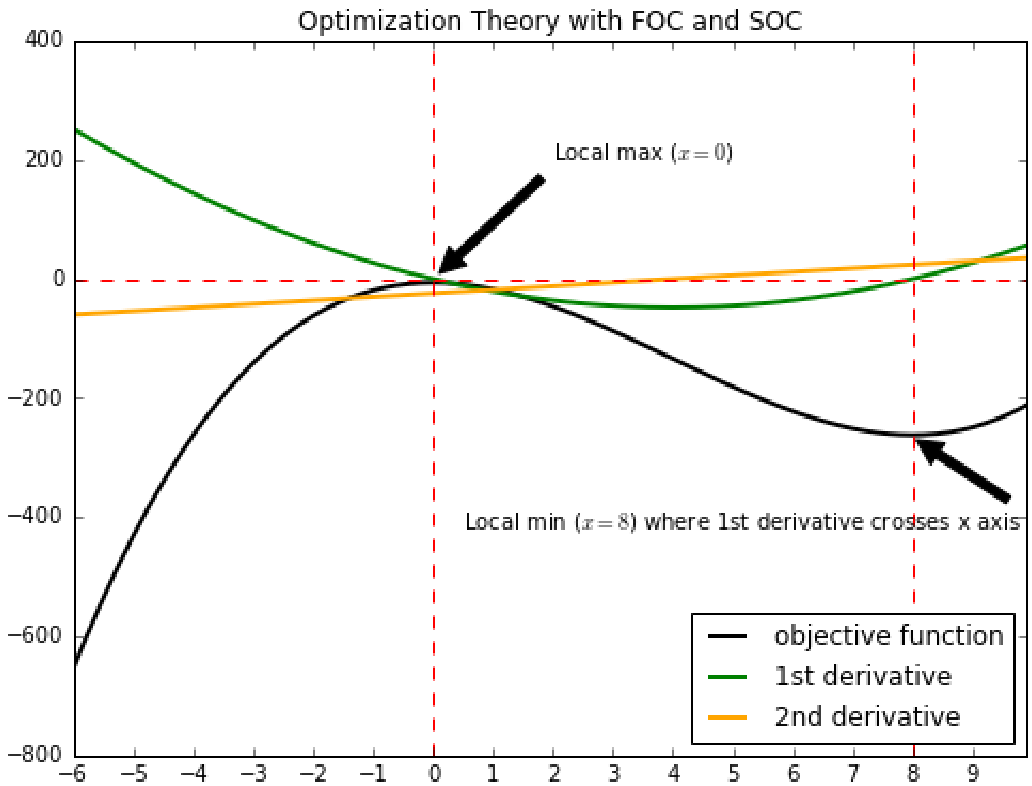
at a candidate extremum (FOC).

The Hessian matrix represents the partial second derivatives of the objective function.

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

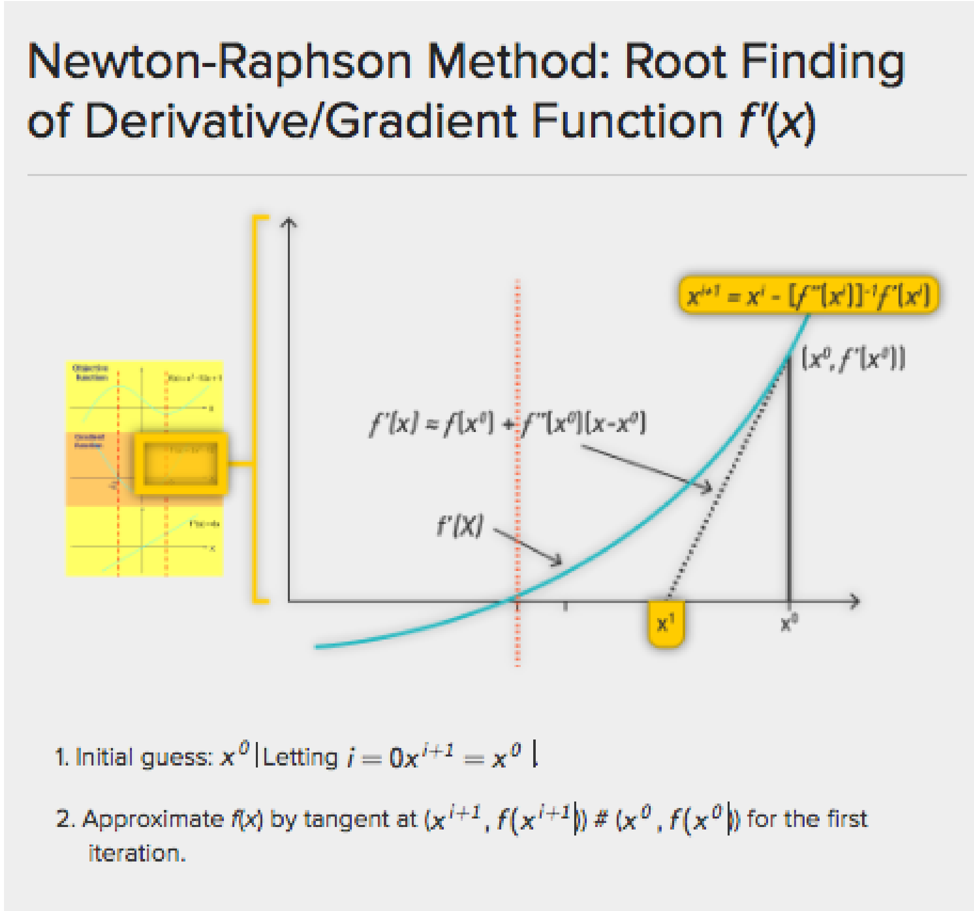
$$\nabla f(X = x') = \left(\frac{df}{dx_1}, \frac{df}{dx_2}, \ldots \frac{df}{dx_n}\right) = J(X = x')$$

In [45]:
```
# in this example we will plot an objective function f(x)=x^3-12x-6 ov

def f(x):
    """The objective function"""
    return x**3-12*x**2-6

def foc(x):
    """The first order condition is the 1st derivative of the objectiv
    return 3*x**2-24*x

def soc(x):
    """The second order condition is the 2nd derivative of the objecti
    return 6*x-24

def rootplot():

    # Vectors for x, first derivative, second derivative
    x=np.arange(-6,10,.1)
    y0=f(x)
    y1=foc(x)
    y2=soc(x)

    #Plot each vector
    fg = plt.figure(figsize=(8,6))
    plt.xticks(np.arange(min(x), max(x)+1, 1.0))
    plt.plot(x,y0, color="black", label='objective function', linewidt
    plt.plot(x,y1, color="green", label='1st derivative', linewidth =
    plt.plot(x,y2, color="orange", label='2nd derivative', linewidth =
    plt.legend(loc='lower right')
    plt.title("Optimization Theory with FOC and SOC")

    #Plot dotted lines at each root and where x=0
    plt.axhline(0, color ="red", ls = '--', )
    plt.axvline(0, color ="red", ls = '--')
    plt.axvline(8, color = "red", ls = '--')

    # comments for the local max and local min
    plt.annotate('Local max ($x=0$)', xy=(0, 0), xytext=(2, 200),arrow
    plt.annotate('Local min ($x=8$) where 1st derivative crosses x axi
        arrowprops=dict(facecolor='black', shrink=0.05),
        )

    # show the plot with the 3 curves
    plt.show()

rootplot()
```



HW6.2 - Newton-Raphson Method to Calculate Roots

Taking x=1 as the first approximation(xt1) of a root of X^3 + 2x -4 = 0, use the Newton-Raphson method to calculate the second approximation (denoted as xt2) of this root. (Hint the solution is xt2=1.2)



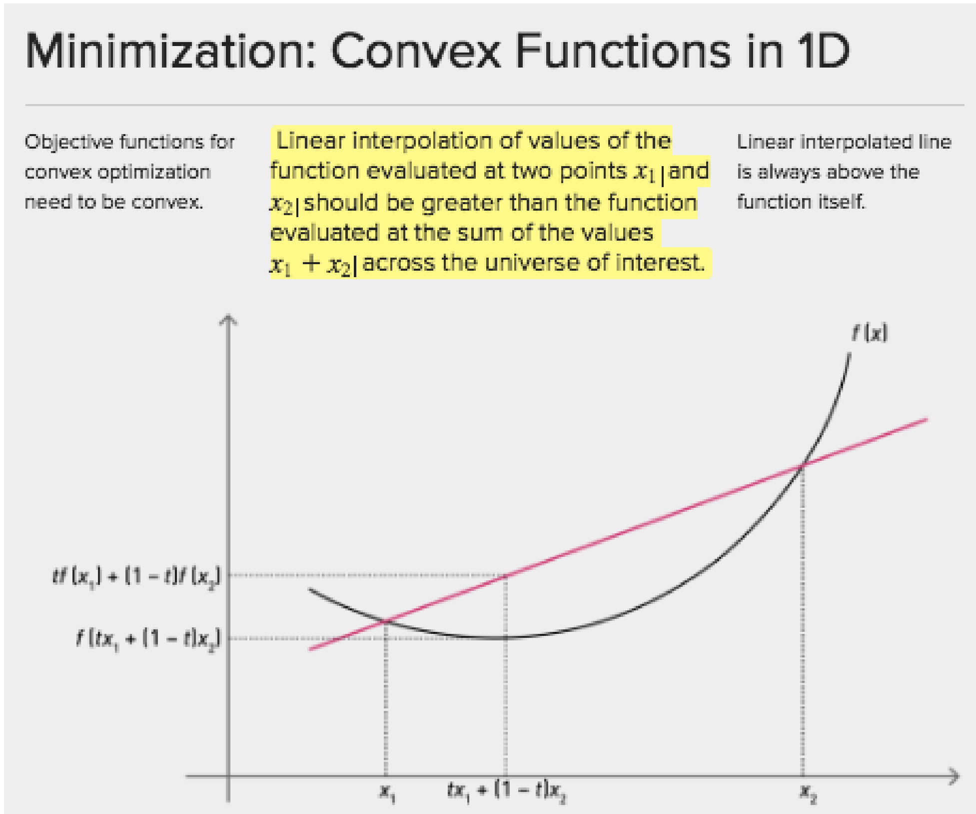
In [35]:
```
from __future__ import division

# this code will perform the Newton-Raphson method to calculate the ro
# we will start with the first approximation equal to 1 and calculate

def f(x):
    """Calculate the value of our objective function"""
    return x**3+2*x-4

def foc(x):
    """Calculate the first order condition - 1st derivative"""
    return 3*x**2+2

def iterate_newton(xt1):
    """Calculate a single iteration of the Newton-Raphson method for o
    xt2=xt1-(f(xt1)/foc(xt1))
    return xt2

# calculate the second approximation of this root using "1" as the fir
iterate_newton(1)
```

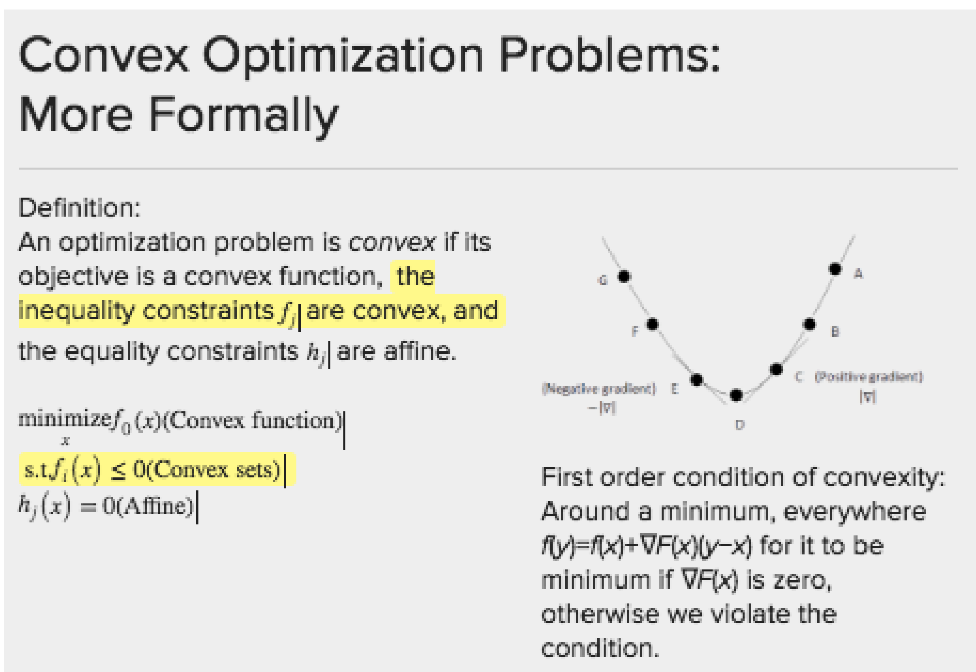
Out[35]: 1.2

HW6.3 - Convex Optimization

What makes an optimization problem convex?

An optimization problem is convex if its objective is a convex function, the inequality constraints fj are convex, and the equality constraints hj are affine.





What are the first order Necessary Conditions for Optimality in convex optimization.

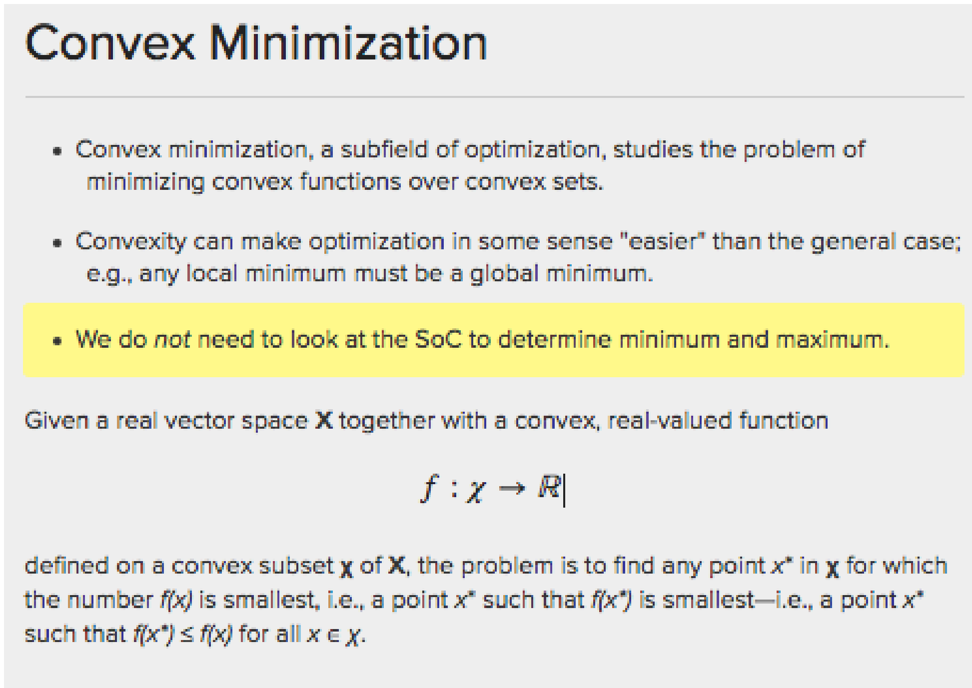
The first order condition is similar to objective as the first derivative of the objective function.

What are the second order optimality conditions for convex optimization?

If the problems is convex then we don't need to calculate the second order condition given that any local minimum will be a global minimum.

Are both necessary to determine the maximum or minimum of candidate optimal solutions?

No, as described above only the first order condition is required to determine the optimal solution.



Fill in the BLANKS here:

Convex minimization, a subfield of optimization, studies the problem of minimizing **convex** functions over **convex** sets.

The **convexity** property can make optimization in some sense "easier" than the general case - for example, any local minimum must be a global minimum.

HW 6.4 - OLS Regression

The learning objective function for weighted ordinary least squares (WOLS) (aka weight linear regression) is defined as follows:

0.5 *sumOverTrainingExample i (weight_i* (W * X_i - y_i)^2)

Where training set consists of input variables X (in vector form) and a target variable y, and W is the vector of coefficients for the linear regression model.

Derive the gradient for this weighted OLS by hand; showing each step and also explaining each step.

Step by Step Derivation for Weighted OLS Regression

X = input variable vector

y = target variable

W = model coefficients vector

We start with the learning objective function:

$$J(W) = 0.5 * \sum_i w_i (WX_i - y_i)^2 \tag{Step 1}$$

Next expand the squared term and multiple to enable the partial derivates of these terms:

$$J(W) = 0.5 * \sum_i w_i (WX_i - y_i)(WX_i - y_i) \tag{Step 2}$$

$$J(W) = 0.5 * \sum_i w_i ((WX_i)^2 - 2WX_i y_i + y_i^2) \tag{Step 3}$$

Next continuing squaring terms as needed:

$$J(W) = 0.5 * \sum_i w_i (W^2 X_i^2 - 2WX_i y_i + y_i^2) \tag{Step 4}$$

Next we can compute the gradient by taking the partial derivative with respect to W as we have a vector of inputs:

$$\frac{\partial J(W)}{\partial W} = \frac{\partial}{\partial W}(0.5 * \sum_i w_i (W^2 X_i^2 - 2WX_i y_i + y_i^2)) \tag{Step 5}$$

$$\frac{\partial J(W)}{\partial W} = 0.5 * \sum_i w_i (2WX_i^2 - 2X_i y_i) \tag{Step 6}$$

Next we can pull out the constant term of "2" to simplify the equation:

$$\frac{\partial J(W)}{\partial W} = 0.5 * 2 * \sum_i w_i (WX_i^2 - X_i y_i) \tag{Step 7}$$

Next we can also simplify by pulling out Xi:

$$\frac{\partial J(W)}{\partial W} = \sum_i w_i X_i (WX_i - y_i) \tag{Step 8}$$

Finally, we set the gradient to 0 to find the value of W that minimizes J(W):

$$\sum_i w_i X_i (WX_i - y_i) = 0 \tag{Step 9}$$

HW 6.5 - MapReduce OLS

Write a MapReduce job in MRJob to do the training at scale of a weighted OLS model using gradient descent. Generate one million datapoints just like in the following notebook:

http://nbviewer.ipython.org/urls/dl.dropbox.com/s/kritdm3mo1daolj/MrJobLinearRegressionGD
(http://nbviewer.ipython.org/urls/dl.dropbox.com/s/kritdm3mo1daolj/MrJobLinearRegressionGI

Weight each example as follows: weight(x)= abs(1/x)

Sample 1% of the data in MapReduce and use the sampled dataset to train a (weighted if available in SciKit-Learn) linear regression model locally using SciKit-Learn (http://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LinearRegression.html (http://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LinearRegression.html)) Plot the resulting weighted linear regression model versus the original model that you used to generate the data. Comment on your findings.

Data Generation - 1M Points

In [3]:
```
import numpy as np
size = 1000000 #Create 1m random points
x = np.random.uniform(-4, 4, size)
y = x * 1.0 - 4 + np.random.normal(0,0.5,size)
data = zip(y,x) # zip together x and y into one structure
np.savetxt('LinearRegression.csv',data,delimiter = ",")

x[2:]
```

Out[3]:
```
array([-0.68066303, -0.78211012,  0.47537293, ...,  1.17700515,
        2.46265522, -1.81539453])
```

OLS Regression - Gradient Descent

In [4]:

```
%%writefile MrJobBatchGDUpdate_LinearRegression.py
from __future__ import division

from mrjob.step import MRStep
from mrjob.job import MRJob

def weight_point(x):
    """Define a simple function to calculate our weight value, given a
    return abs(1/x)

# This MrJob calculates the gradient of the entire training set
class MrJobBatchGDUpdate_LinearRegression(MRJob):
    # run before the mapper processes any input
    def read_weightsfile(self):
        # Read weights file

        with open('weights.txt', 'r') as f:
            self.weights = [float(v) for v in f.readline().split(',')]
        # Initialize gradient for this iteration
        self.partial_Gradient = [0]*len(self.weights)
        self.partial_count = 0

    # Calculate partial gradient for each example
    def partial_gradient(self, _, line):
        D = (map(float,line.split(',')))
        # y_hat is the predicted value given current weights
        y_hat = self.weights[0]+self.weights[1]*D[1] #Y_hat=beta_0+bet

        self.partial_Gradient[0]+=(D[0]-y_hat)*weight_point(D[1]) #Upd
        self.partial_Gradient[1]+=(D[0]-y_hat)*D[1]*weight_point(D[1])
        self.partial_count+=1

    # Finally emit in-memory partial gradient and partial count
    def partial_gradient_emit(self):
        yield None, (self.partial_Gradient,self.partial_count)

    # Accumulate partial gradient from mapper and emit total gradient
    # Output: key = None, Value = gradient vector
    def gradient_accumulater(self, _, partial_Gradient_Record):
        total_gradient = [0]*2
        total_count = 0
        for partial_Gradient,partial_count in partial_Gradient_Record:
            total_count = total_count + partial_count
            total_gradient[0] = total_gradient[0] + partial_Gradient[0
            total_gradient[1] = total_gradient[1] + partial_Gradient[1
        yield None, [v/total_count for v in total_gradient]

    def steps(self):
        return [MRStep(mapper_init=self.read_weightsfile,
                       mapper=self.partial_gradient,
                       mapper_final=self.partial_gradient_emit,
                       reducer=self.gradient_accumulater)]

if __name__ == '__main__':
```

```
    MrJobBatchGDUpdate_LinearRegression.run()
```

```
Writing MrJobBatchGDUpdate_LinearRegression.py
```

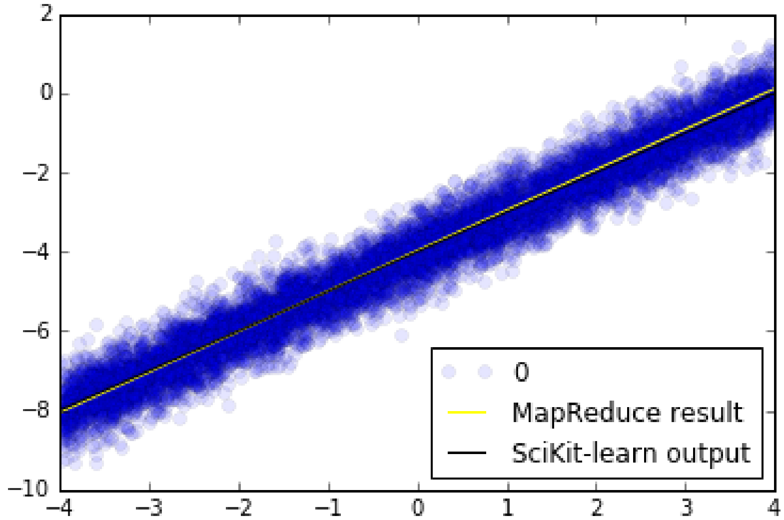
Driver Code

In [5]:
```
from numpy import random,array
from MrJobBatchGDUpdate_LinearRegression import MrJobBatchGDUpdate_Lin

learning_rate = 0.05
stop_criteria = 0.000005

# Generate random values as inital weights
weights = array([random.uniform(-3,3),random.uniform(-3,3)])
# Write the weights to the files
with open('weights.txt', 'w+') as f:
    f.writelines(','.join(str(j) for j in weights))

# create a mrjob instance for batch gradient descent update over all d
mr_job = MrJobBatchGDUpdate_LinearRegression(args=['LinearRegression.c
                                                   'weights.txt','--no
# Update centroids iteratively
i = 0
while(1):
    print "iteration ="+str(i)+"  weights =",weights
    # Save weights from previous iteration
    weights_old = weights
    with mr_job.make_runner() as runner:
        runner.run()
        # stream_output: get access of the output
        for line in runner.stream_output():
            # value is the gradient value
            key,value =  mr_job.parse_output_line(line)
            # Update weights
            weights = weights + learning_rate*array(value)
    i+=1
    # Write the updated weights to file
    with open('weights.txt', 'w+') as f:
        f.writelines(','.join(str(j) for j in weights))
    # Stop if weights get converged
    if(sum((weights_old-weights)**2)<stop_criteria):
        break

print "Final weights\n"
print weights
```

```
iteration =0  weights = [ 1.8883926   1.96833003]
iteration =1  weights = [ 0.69256042  1.8714606 ]
iteration =2  weights = [-0.25925393  1.78428982]
iteration =3  weights = [-1.01684388  1.70584557]
iteration =4  weights = [-1.6198422   1.63525339]
iteration =5  weights = [-2.09979436  1.57172662]
iteration =6  weights = [-2.48180874  1.51455755]
iteration =7  weights = [-2.78587021  1.46310949]
iteration =8  weights = [-3.0278856   1.41680956]
iteration =9  weights = [-3.22051584  1.37514236]
iteration =10  weights = [-3.37383832  1.33764411]
iteration =11  weights = [-3.49587406  1.30389753]
iteration =12  weights = [-3.59300737  1.27352712]
iteration =13  weights = [-3.67031975  1.246195  ]
iteration =14  weights = [-3.73185582  1.22159713]
iteration =15  weights = [-3.78083486  1.19945992]
iteration =16  weights = [-3.81981923  1.17953715]
iteration =17  weights = [-3.85084842  1.16160727]
iteration =18  weights = [-3.87554576  1.14547088]
iteration =19  weights = [-3.89520331  1.13094857]
iteration =20  weights = [-3.91084949  1.11787885]
iteration =21  weights = [-3.92330286  1.10611641]
iteration =22  weights = [-3.93321493  1.09553049]
iteration =23  weights = [-3.9411043   1.08600339]
iteration =24  weights = [-3.9473837   1.07742919]
iteration =25  weights = [-3.95238168  1.06971258]
iteration =26  weights = [-3.95635973  1.06276778]
iteration =27  weights = [-3.95952597  1.05651759]
iteration =28  weights = [-3.96204606  1.05089253]
iteration =29  weights = [-3.96405187  1.04583007]
iteration =30  weights = [-3.96564833  1.04127395]
iteration =31  weights = [-3.96691898  1.03717351]
iteration =32  weights = [-3.96793032  1.03348318]
iteration =33  weights = [-3.96873525  1.03016194]
iteration =34  weights = [-3.96937591  1.02717288]
iteration =35  weights = [-3.9698858   1.02448277]
iteration =36  weights = [-3.97029163  1.02206171]
Final weights

[-3.97061462  1.0198828 ]
```

Scikit-learn Linear Regression

In [41]:
```
from sklearn.linear_model import LinearRegression
import pandas as pd
import random as rand

#Randomly sample 1% of our data
n = 1000000 # this is the total number of samples in the dataset
s = int(n*0.01) # take 1% of the sample
filename = "LinearRegression.csv" # sample this out to a CSV file
skip = sorted(rand.sample(xrange(n),n-s))
df = pd.read_csv(filename, skiprows=skip, header=None) # read the data

#Set up background stuff to plot mapreduce data
floor_x=min(df[1])
ceiling_x=max(df[1])
step_x=(ceiling_x-floor_x)/s
pred_x=np.arange(floor_x,ceiling_x,step_x)

def get_line(x,m,b):
    """ calculate Y values given X, slope, and intercept"""
    return m*x+b

mr_intercept=weights[0]
mr_slope=weights[1]
pred_y_mr=get_line(pred_x,mr_slope,mr_intercept)

#Build Sklearn model
sklearn_model=LinearRegression()
sklearn_model.fit(df[1].reshape(-1,1),df[0])
sk_slope=sklearn_model.coef_
sk_intercept=sklearn_model.intercept_

pred_y_sk=get_line(pred_x,sk_slope,sk_intercept)

#Actually make the plots
plt.plot(df[1],df[0],'bo',alpha=0.1) #Plot data points, with alpha tur
plt.plot(pred_x,pred_y_mr,color="yellow", label="MapReduce result") #P
plt.plot(pred_x,pred_y_sk, color="black", label="SciKit-learn output")
plt.legend(loc="lower right")
plt.show()
```



HW6.5.1 (OPTIONAL)

Using MRJob and in Python, plot the error surface for the weighted linear regression model using a heatmap and contour plot. Also plot the current model in the original domain space. (Plot them side by side if possible) Plot the path to convergence (during training) for the weighted linear regression model in plot error space and in the original domain space. Make sure to label your plots with iteration numbers, function, model space versus original domain space, etc. Comment on convergence and on the mean squared error using your weighted OLS algorithm on the weighted dataset versus using the weighted OLS algorithm on the uniformly weighted dataset.

HW6.6 Clean up notebook for GMM via EM (OPTIONAL)

Using the following notebook as a starting point: http://nbviewer.jupyter.org/urls/dl.dropbox.com/s/0t7985e40fovlkw/EM-GMM-MapReduce%20Design%201.ipynb (http://nbviewer.jupyter.org/urls/dl.dropbox.com/s/0t7985e40fovlkw/EM-GMM-MapReduce%20Design%201.ipynb) Improve this notebook as follows:

- Add in equations into the notebook (not images of equations)
- Number the equations
- Make sure the equation notation matches the code and the code and comments refer to the equations numbers
- Comment the code
- Rename/Reorganize the code to make it more readable
- Rerun the examples similar graphics (or possibly better graphics)

HW6.7 Implement Bernoulli Mixture Model via

EM (OPTIONAL)

Implement the EM clustering algorithm to determine Bernoulli Mixture Model for discrete data in MRJob.

As a unit test use the dataset in the following slides:
https://www.dropbox.com/s/maoj9jidxj1xf5l/MIDS-Live-Lecture-06-EM-Bernouilli-MM-Systems-Test.pdf?dl=0 (https://www.dropbox.com/s/maoj9jidxj1xf5l/MIDS-Live-Lecture-06-EM-Bernouilli-MM-Systems-Test.pdf?dl=0)

Cross-check that you get the same cluster assignments and cluster Bernouilli models as presented in the slides after 25 iterations. Dont forget the smoothing.

As a full test: use the same dataset from HW 4.5, the Tweet Dataset.

Using this data, you will implement a 1000-dimensional EM-based Bernoulli Mixture Model algorithm in MrJob on the users by their 1000-dimensional word stripes/vectors using K = 4. Use the same smoothing as in the unit test. Repeat this experiment using your KMeans MRJob implementation fron HW4. Report the rand index score using the class code as ground truth label for both algorithms and comment on your findings.

Here is some more information on the Tweet Dataset. Here you will use a different dataset consisting of word-frequency distributions for 1,000 Twitter users. These

Twitter users use language in very different ways, and were classified by hand according to the criteria:

- 0: Human, where only basic human-human communication is observed.
- 1: Cyborg, where language is primarily borrowed from other sources (e.g., jobs listings, classifieds postings, advertisements, etc...).
- 2: Robot, where language is formulaically derived from unrelated sources (e.g., weather/seismology, police/fire event logs, etc...).
- 3: Spammer, where language is replicated to high multiplicity (e.g., celebrity obsessions, personal promotion, etc...)

Check out the preprints of recent research,which spawned this dataset:
http://arxiv.org/abs/1505.04342 (http://arxiv.org/abs/1505.04342)
http://arxiv.org/abs/1508.01843 (http://arxiv.org/abs/1508.01843)

The main data lie in the accompanying file: topUsers_Apr-Jul_2014_1000-words.txt and are of the form: USERID,CODE,TOTAL,WORD1_COUNT,WORD2_COUNT,.... . . where USERID = unique user identifier CODE = 0/1/2/3 class code TOTAL = sum of the word counts Using this data, you will implement a 1000-dimensional K-means algorithm in MrJob on the users by their 1000-dimensional word stripes/vectors using several centroid initializations and values of K.

In [ ]:

```
```