Experiment 1
Ratio of Specific Heat Capacities
PHYS 217

James Hartwick

May 21, 2014

Date Performed: May 20, 2014

1 Objective

To determine the ratio of the specific heats of air, carbon dioxide, and argon.

2 Theory

In this experiment the ratio of specific heat at constant pressure, C_P to the specific heat at constant volume, C_V will be measured for several gases. This ratio, γ, depends upon the molecular configuration of the gas. This experiment attempts to measure γ by observing the simple harmonic motion of a steel ball as it moves up and down in a tube separating the system within a jar and the atmosphere outside.
A ball of mass m barely makes contact with the inner surface of a tube with cross-sectional area A. When the ball is released it drops and the thus the air in the jar gets compressed. Once the ball reaches it's equilibrium position the pressure in the jar P_{eq} is given by the following where the atmospheric pressure is P_0.

$P_{eq} = P_0 + \frac{mg}{A}$

Now if the ball is displaced from it's equilibrium position a force is exerted on the ball back towards equilibrium by the pressure difference from P_{eq}. This force is equal to the product of the pressure difference and the area upon which it is applied (A) leading the equation:

$m\frac{d^2x}{dt^2} = -A\delta P$

We assume the change in the system to be adiabatic, quasistatic, and reversible. Thus we can say that

$\ln P + \gamma \ln V$ is constant.

We want to isolate δP from this equation to find it's contribution to the force equation above. Taking the derivative and isolating δP yields

$\delta P = -\frac{\gamma P \delta V}{V}$

δV is just simply the volume created between the displaced ball and it's equilibrium position or

$\delta V = -Ax$

Where x is the distance the ball is displaced from equilibrium. Plugging these equations into our force equation gives the equation of simple harmonic motion $m\frac{d^2x}{dt^2} = \frac{A^2\gamma P_{eq}x}{V}$ and corresponding period $T = 2\pi\sqrt{\frac{mV}{A^2\gamma P_{eq}}}$

Now γ can be separated easily as

$\gamma = \frac{64mV}{d^2 P_{eq} T^2}$

3 Apparatus

10L Aspirator
Precision Glass Tubing
Steel Ball
PC with Vernier Lab Pro Data acquisition system coupled with a pressure sensor
CO_2 and Argon gas cylinder with filling tube

4 Diagram

5 Procedure

Use diagram above as reference. It is vital to ensure that the steel ball and the inside of the tube are clean both before starting and throughout the experiment. The equipment should be cleaned with methanol to ensure they are not contaminated by dust or other particles.
The internal pressure of the Jar is monitored by the PC Logger Pro software while the ball completes it's oscillations. For best results the sampling rate should be set to be as high as possible. In this case we use 20 samples/second. The ball is placed at the top of the tube and a stopper is inserted to prevent the ball from dropping. Once the stopper is removed the ball drops and undergoes simple harmonic motion until it eventually stops at it's equilibrium position. The period of the oscillations is then used to find γ
This process is then repeated with the jar filled with argon and carbon dioxide.

6 Experimental Data

Mass of steel ball	$16.5 \pm 0.05\,\mathrm{g}$
Diameter of steel ball	$16.00 \pm 0.005\,\mathrm{mm}$
Cross sectional diameter A of steel ball/tube	$201.1 \pm 0.178\,\mathrm{mm}^2$
Height of mercury	$760.6 \pm 0.05\,\mathrm{mm}$
	$= 101404.993 \pm 6.66$ pascals
Period of oscillations (air)	$1.1164 \pm 0.05\,\mathrm{s}$
Period of oscillations (arogon)	$1.0317 \pm 0.05\,\mathrm{s}$
Period of oscillations (carbon dioxide)	$1.1453 \pm 0.05\,\mathrm{s}$
Volume of Aspirator	10.75L

7 Calculations

7.1 Determining the equilibrium pressure

$P_{eq} = P_0 + \frac{mg}{A} \implies P_{eq} = 101404.993Pa + \frac{(0.0165kg)(9.81m/s^2)}{0.000402123m^2} = 101807.519Pa$

Uncertainties:
$6.66Pa + \sqrt{(0.05/16.5)^2 + (.178/402.1)^2}(\frac{mg}{A})Pa = 7.90Pa$
$\implies P_{eq} = 101807.519 \pm 7.90Pa$

7.2 Determining Heat capacity ratio γ

$\gamma = \frac{64mV}{d^4 P_{eq} T^2} \implies \gamma_{Air} = \frac{64(.0165kg)(0.01037m^3)}{(0.016m)^4(101807.519Pa)(1.1164s)} = 1.317$

Uncertainties:
$\sqrt{(0.05/16.5)^2 + 4(.005/16)^2 + (7.9/101807)^2 + 2(0.05/1.1164)^2}\gamma_{Air} = \pm 0.08352$
$\implies \gamma_{Air} = 1.317 \pm 0.08352$

Similarly:
$\gamma_{Argon} = 1.542 \pm 0.09036$
$\gamma_{CO_2} = 1.251 \pm 0.08141$

Accepted values:
$\gamma_{Air} = 1.400$
$\gamma_{Argon} = 1.670$
$\gamma_{CO_2} = 1.300$

Consistency check:
$\gamma_{Air} \implies 1.317 + 0.08352 = 1.40052 > 1.400 \therefore consistent$
$\gamma_{Argon} \implies 1.542 + 0.09036 = 1.63236 < 1.670 \therefore inconsistent$
$\gamma_{CO_2} \implies 1.251 + 0.08141 = 1.33241 > 1.300 \therefore consistent$

7.3 Verify Eqn 1.13 (from lab manual) is valid

$\frac{kT_1}{2} = \frac{\ln A_n - \ln A_{n+i}}{i} \implies k = \frac{2(\ln A_n - \ln A_{n+i})}{iT_1}$
$A_n = 0.52Pa$
$A_{n+i} = 0.22Pa$
$i = 3$
$T_1 = 1.05s$
$\implies k = \frac{2(\ln .52 - \ln .22)}{3(1.05)} = 0.54616$

Now verifying eq 1.13:
$\frac{k^2 T_1^2}{4} << 4\pi^2 \implies \frac{(.546)^2(1.05)^2}{4} = 0.0822 << 39.48 = 4\pi^2$
The assumption condition is true and no noticeable change in the period was observed so the assumption appears to be valid.

8 Discussion

The most obvious source of experimental uncertainty comes from the seal between the stopper, the tube, and the Aspirator. It was observed that their was a very slow leak. After achieving it's equilibrium position the steel ball would slowly drop indicating an imperfect seal. This means it is not a closed system as we would like since mass can be transferred to the surroundings. An imperfect seal would translate to a smaller force exerted on the ball to return it to the equilibrium position which in turn would increase the period. A longer period would decrease the value of γ since T in in the denominator.
This could be the reason why all the observed values of γ are lower than their accepted values (even though two are still consistent.)

9 Conclusions

The parameters of interest determined by this experiment are the specific heat ratios of Air, Argon, and Carbon Dioxide.
γ_{Air} was determined to be 1.317 ± 0.08352 which is consistent with accepted values.
γ_{Argon} was determined to be 1.542 ± 0.09036 which is inconsistent with accepted values.
γ_{CO_2} was determined to be 1.251 ± 0.08141 which is consistent with accepted values.

10 Questions

In general the specific hear ratio of a molecule is related to the degrees of freedom (f) it has. For an ideal gas $\gamma \ \alpha \ \frac{f+2}{f}$ so for a monatomic gas with 3 degrees of freedom like Argon $\gamma = \frac{3+2}{3} = 1.67$. For air we have 3 translational and two rotational degrees of freedom. Thus $\gamma = \frac{5+2}{5} = 1.4$. For real gases the degrees of freedom that become available increase with temperature. So different gases will have different numbers of degrees of freedom depending on their chemical make-up and temperatures resulting in the various values of γ observed during this experiment.