

Whose Data Is It Anyway? A Formal Treatment of Differential Privacy for Surveys

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These Slides Are Available Online:



jameshbailie.github.io/talks/

This Presentation Is Based on Two Papers:

JB and Jörg Drechsler (2024). "Whose Data Is It Anyway? Towards a Formal Treatment of Differential Privacy for Surveys". *NBER Working Paper*.



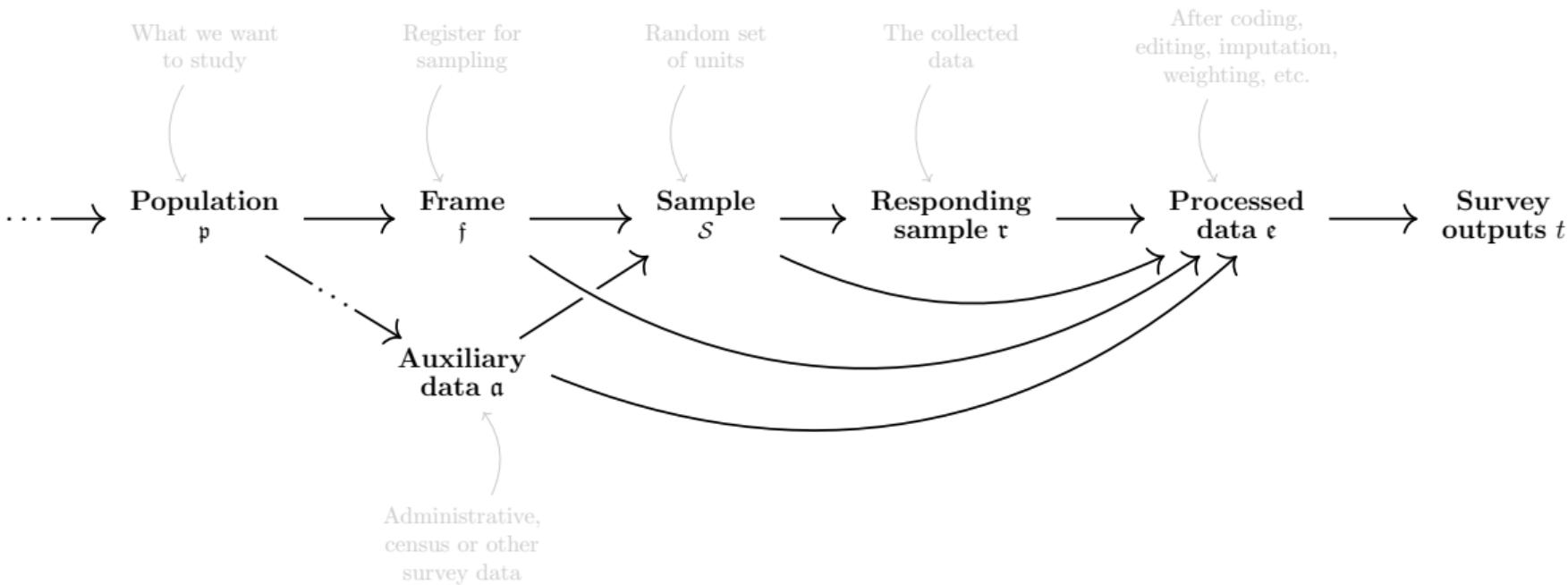
Jörg Drechsler and JB (2024). "The Complexities of Differential Privacy for Survey Data". To appear in *Data Privacy Protection and the Conduct of Applied Research: Methods, Approaches and Their Consequences*.



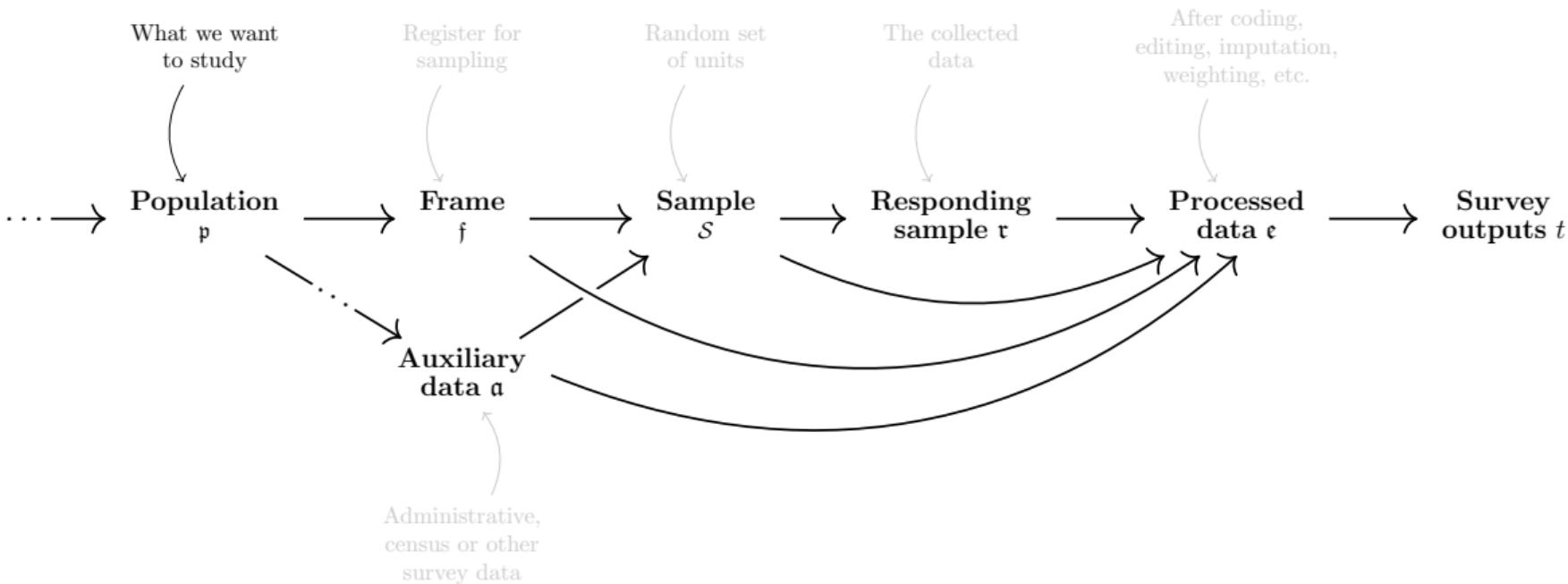
Motivation

- The US Census Bureau has committed to adopting *formal privacy* for all their data products (US Census Bureau 2022).
- Most of their collections are *surveys*.
- Yet the “*science ... does not yet exist*” for a formally private solution to the American Community Survey (for example).
- In implementing differential privacy (DP), surveys come with their own set of *unique challenges and opportunities*.

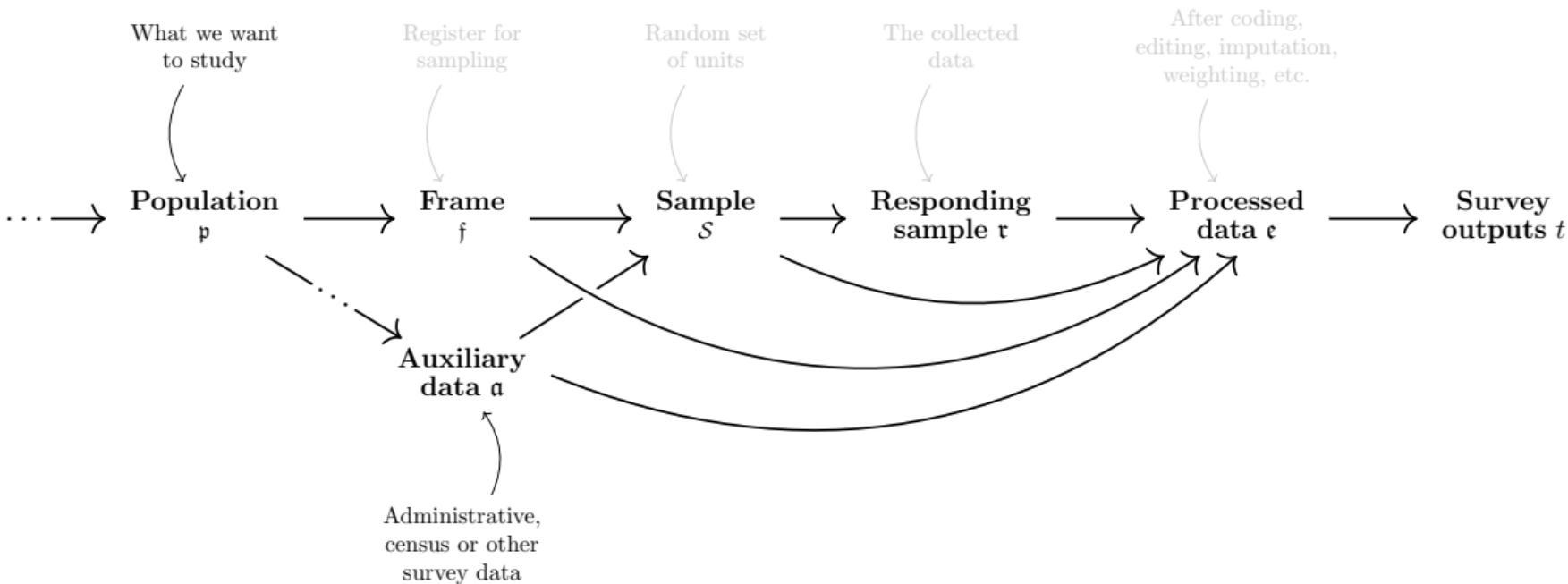
The Survey Data Pipeline



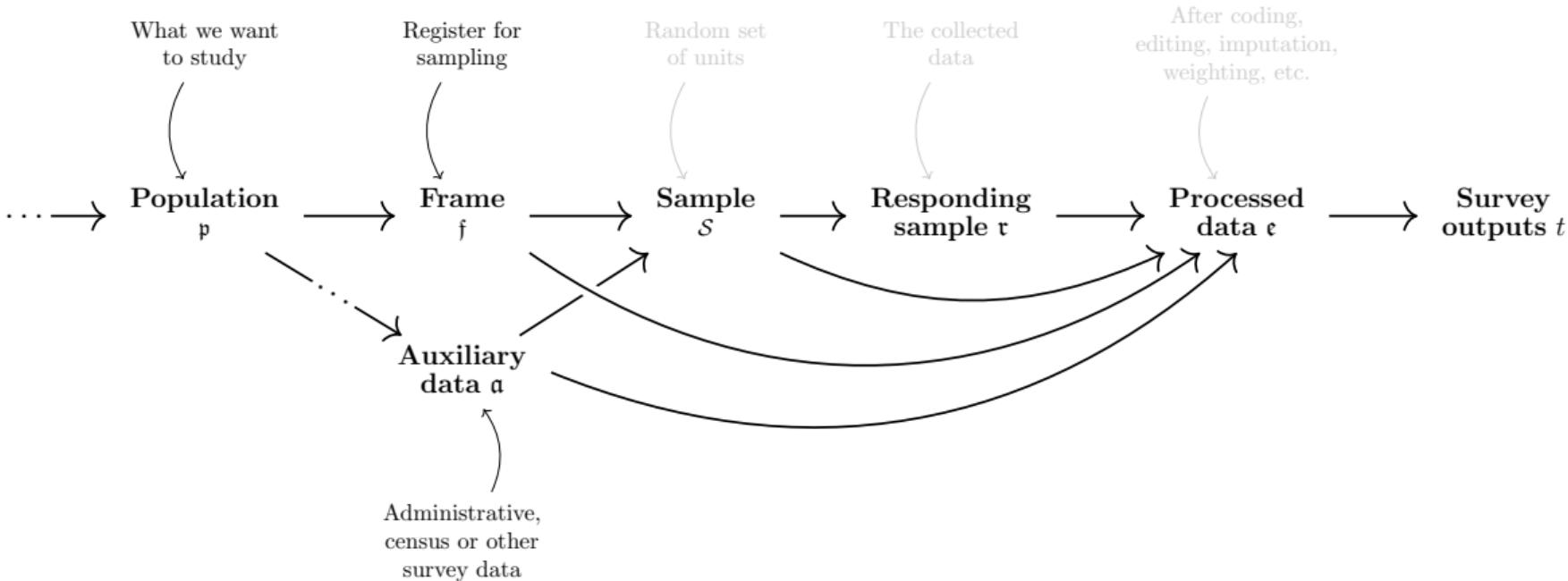
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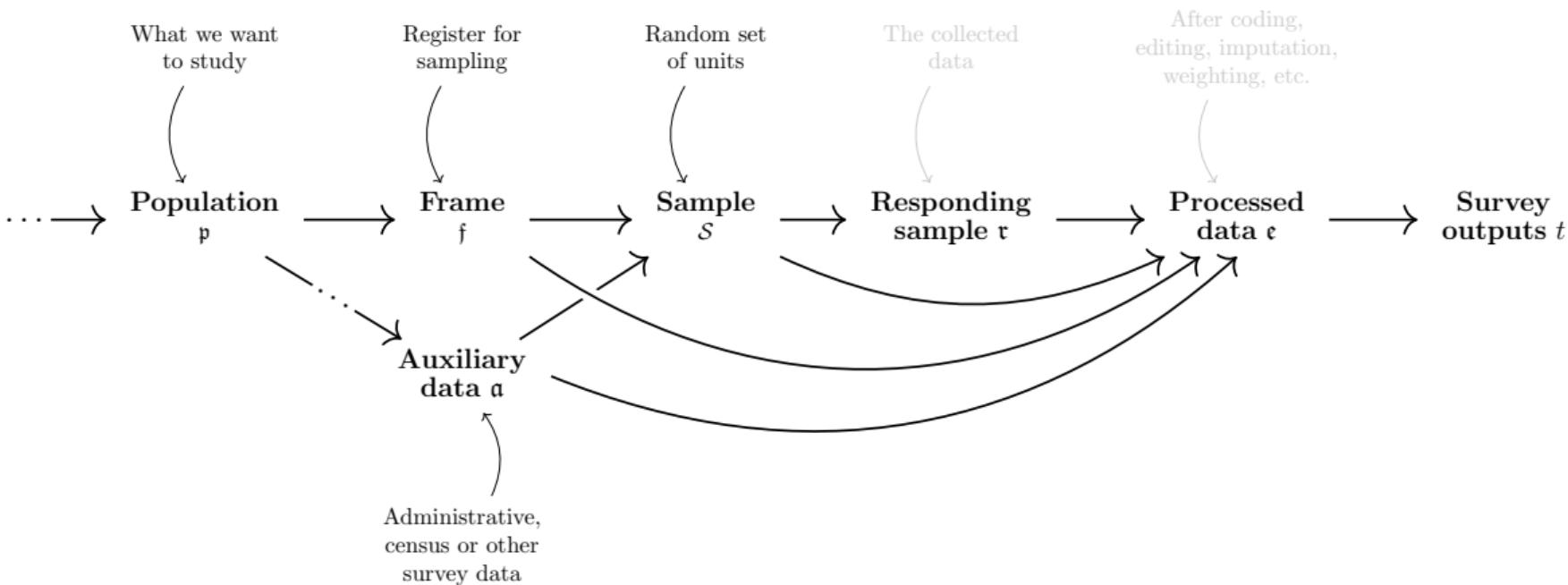
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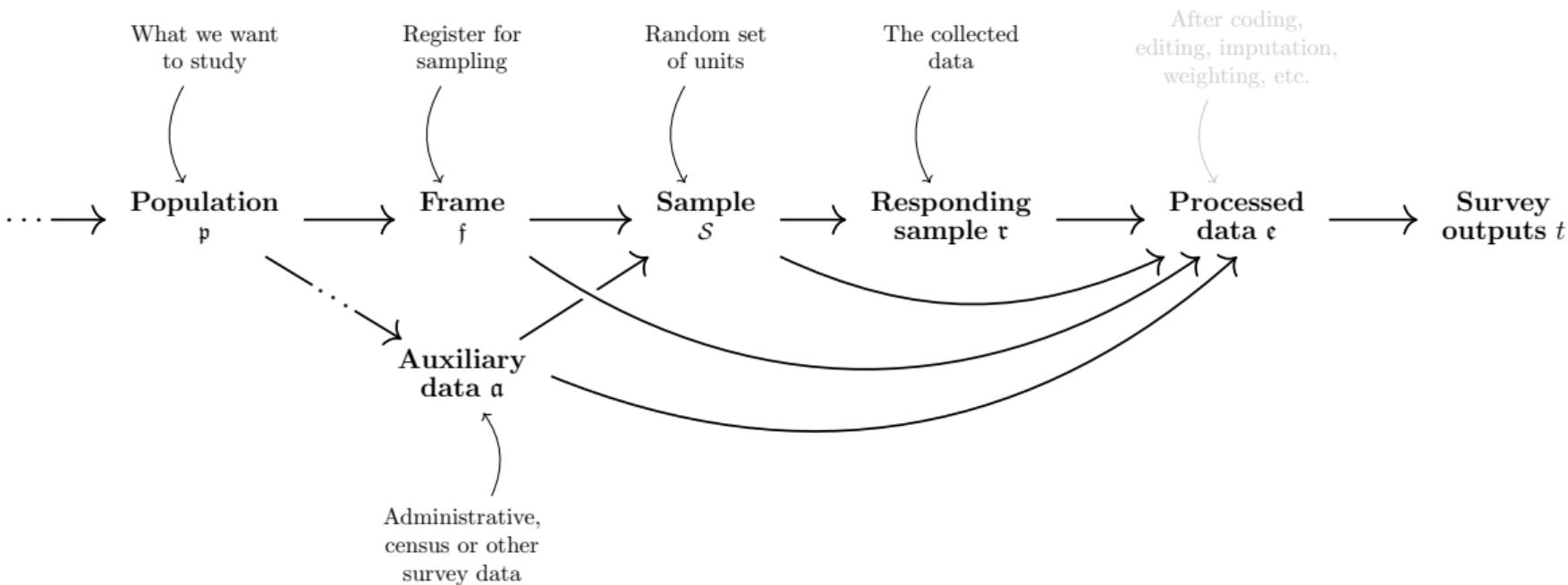
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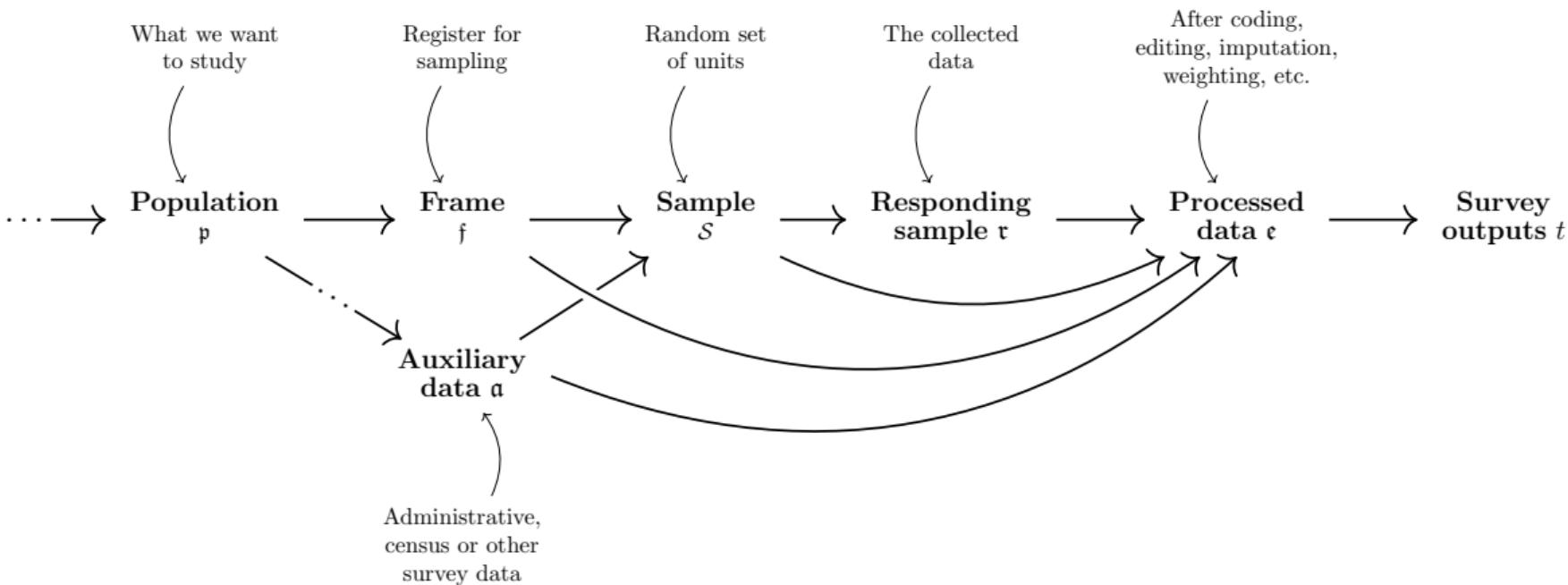
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DP Settings for Surveys

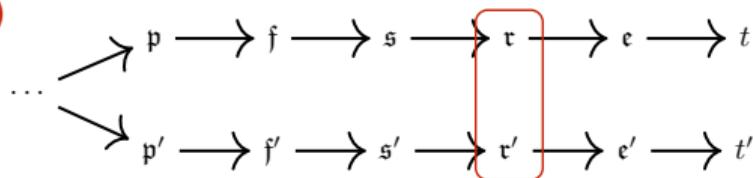
$$\dots \rightarrow p \rightarrow f \rightarrow s \rightarrow r \rightarrow e \rightarrow t$$

Two considerations

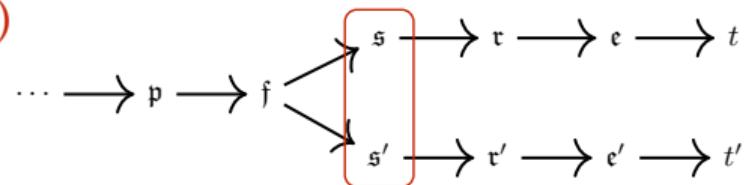
- Where does the DP mechanism *start* in the data pipeline? (What is \mathcal{X} ?)
- Which of the previous steps are kept *invariant*? (What is \mathcal{D} ?)

For example,

1)



2)



Why Does This Matter? One Example

Move \mathcal{X} from the samples \mathfrak{s} to the frames \mathfrak{f} – i.e. start the data-release mechanism one step earlier.

Privacy amplification by sampling

If $T(\mathfrak{s})$ is ε -DP and $\mathcal{S}(\mathfrak{f})$ randomly samples f fraction of the frame \mathfrak{f} , then $T' = T \circ \mathcal{S}$ is ε' -DP where $\varepsilon' \approx f\varepsilon$. (Balle et al. 2020)

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Implications for Privacy Semantics and Composition

- Some fundamental results in differential privacy:
 1. *Privacy semantics*: how does DP protect your data from any possible attacker?
 2. *Composition*: how does ϵ grow as you make more releases?
- Challenges to these results in the survey context (and beyond)

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Privacy Semantics

- DP protects against any attacker, *regardless of their auxiliary knowledge.*
 - How to formalise this? Model the attacker as a Bayesian agent with prior π .
 - Suppose the attacker wants to learn a record x_i .
 - Pure ε -DP guarantees that the attacker's prior-to-posterior ratio is bounded by e^ε :

$$e^{-\varepsilon} \leq \frac{\pi(x_i \mid T, \mathbf{x}_{-i})}{\pi(x_i \mid \mathbf{x}_{-i})} \leq e^\varepsilon.$$

- There is also posterior-to-posterior semantics, which compare the attacker's posterior to the counterfactual where i didn't contribute their data.

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Posterior-to-posterior privacy semantics

What would an attacker learn about a single record if it is included in the input dataset, relative to a counterfactual world in which it is not included?

- If T is ϵ -DP, then the posterior-to-posterior ratio is in $[e^{-\epsilon}, e^{\epsilon}]$. (Kifer et al. 2022)
- What record (in what input dataset) is being protected depends on where T starts in the data pipeline; and what counterfactual worlds are possible depends on what steps are *invariant*.

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- So the posterior-to-posterior ratio of T' should be in the interval $[e^{-\varepsilon'}, e^{\varepsilon'}]$.

Traditional statistical disclosure control attacker models

- The *nosy neighbor*: Knows that a record is in the sample.
- The *journalist*: Wants to learn about *any* record, so picks one in the sample.

- For these attackers, the posterior-to-posterior ratio (or prior-to-posterior ratio) of T' is in the interval $[e^{-\varepsilon}, e^{\varepsilon}]$, *not* the interval $[e^{-\varepsilon'}, e^{\varepsilon'}]$.

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- This also applies wherever sampling is used for privacy.

For example, consider the following attack on a differentially private mechanism:

Attack on a noisy mechanism

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For example, consider the following attack on a differentially private mechanism M :

$$\Pr[M(x) = 1] = \frac{1}{2} + \frac{1}{2} \text{erf}\left(\frac{x - \mu}{\sigma\sqrt{2}}\right)$$

where $\mu = 0$ and $\sigma = 1$. This is a Gaussian mechanism with standard deviation $\sigma = 1$.

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Composition

- How does ε grow as you make more releases?
- How to formalise this? Suppose you have two data-releases T_1 and T_2 which are both ε -DP. Then (T_1, T_2) is 2ε -DP.
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- Statistical agencies often use sample designs which are *dependent*.
 - For example, to reduce respondent burden.
- For $i \in \{1, 2\}$, suppose $T_i(\mathbf{s})$ is ε -DP, and $T'_i = T_i \circ S$.
- Privacy loss of the composition (T'_1, T'_2) is not the sum of T'_1 and T'_2 's privacy losses.
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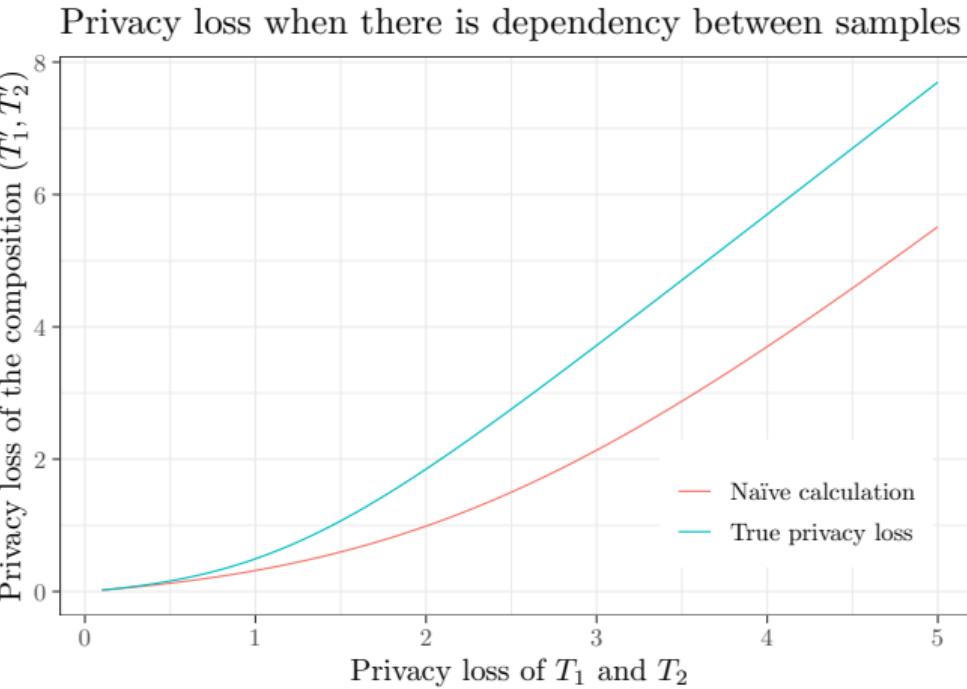
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Utility Considerations (I)

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- *Take-away:* If the sampling procedure is included, less noise is required to achieve the same privacy budget.
- *But* there is little privacy amplification when \mathcal{S} is a complex sampling design. (Bun et al. 2022)

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Utility Considerations (II)

- Surveys use weighted estimators $\sum_{i=1}^n w_i x_i$, which have increased sensitivity.
- Unweighted sums $\sum_{i=1}^n x_i$ have sensitivity $|\max x_i - \min x_i|$, where the max, min are over all possible values of x_i .
- Weighted estimators can have sensitivity

$$|\max w_i x_i - \min w_i x_i| + (n-1)(\max w_i - \min w_i)(|\max x_i| \vee |\min x_i|),$$

because changing a record can change the weights of other records.

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- Taking the frame as invariant means that the weights do not change.

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Additional Complications

- Data-dependent sampling designs are typical; but these pose a challenge unless the frame is fixed.
- Steps of the data release mechanism must be “algorithmised”.
- Nonresponse must be included in the mechanism if starting from the sample-level or earlier.
 - In order to satisfy DP, one must assume that the nonresponse indicators are independent.

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