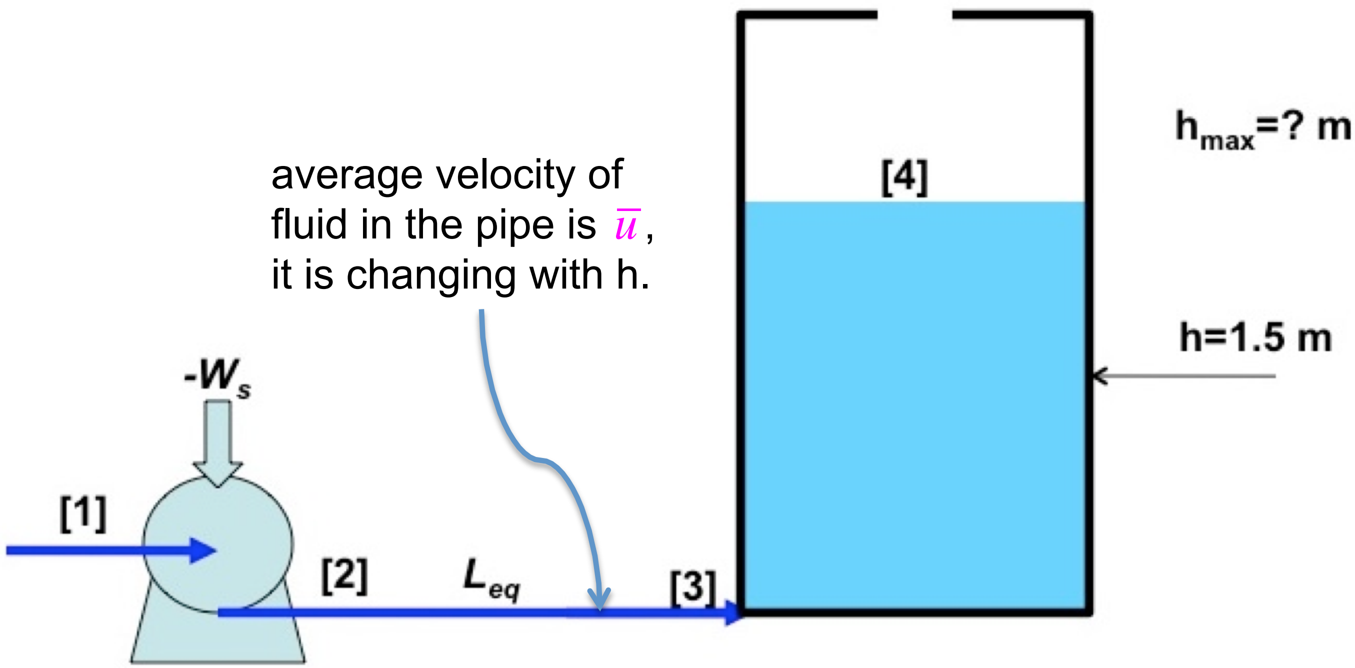
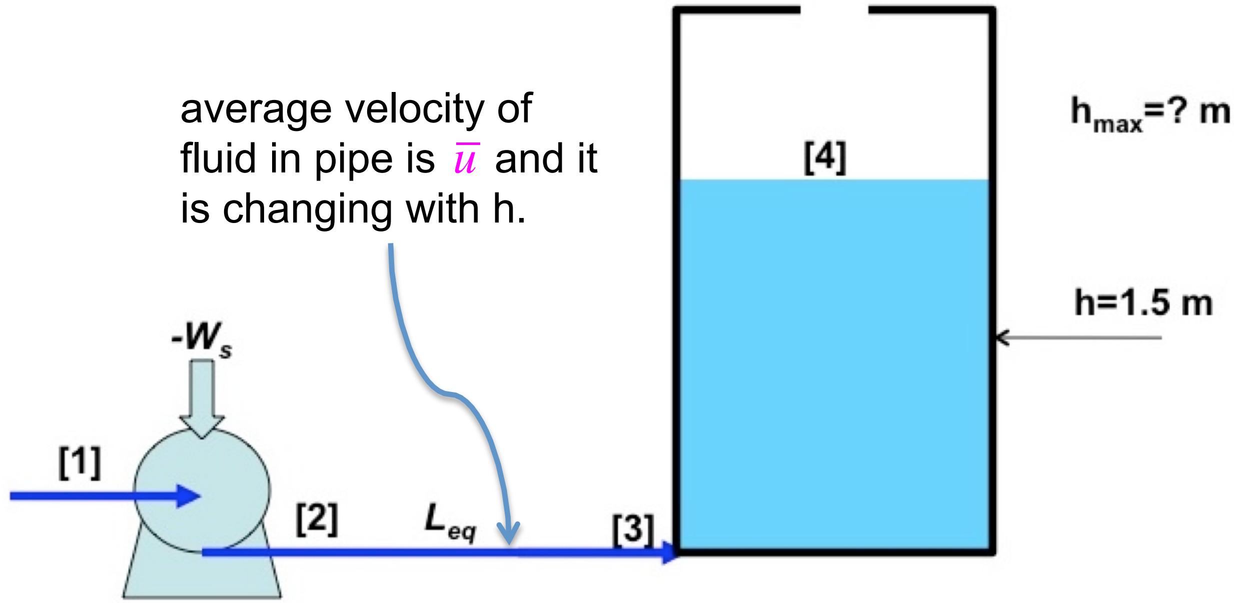
Oregon State University

An empty tank is being filled with a fluid (ρ =1000 [kg/m^3], μ = 0.0043 [kg/ms]) as shown in the illustration below. The equivalent length of the pipe is L_{eq}=25 [m]. A constant work of ($-W_{Sout}$) = 29.43 [J/kg fluid] is continuously supplied to the fluid by a centrifugal pump.



CHE 331
Fall 2022



a) What is the maximum possible height of the fluid in the tank?

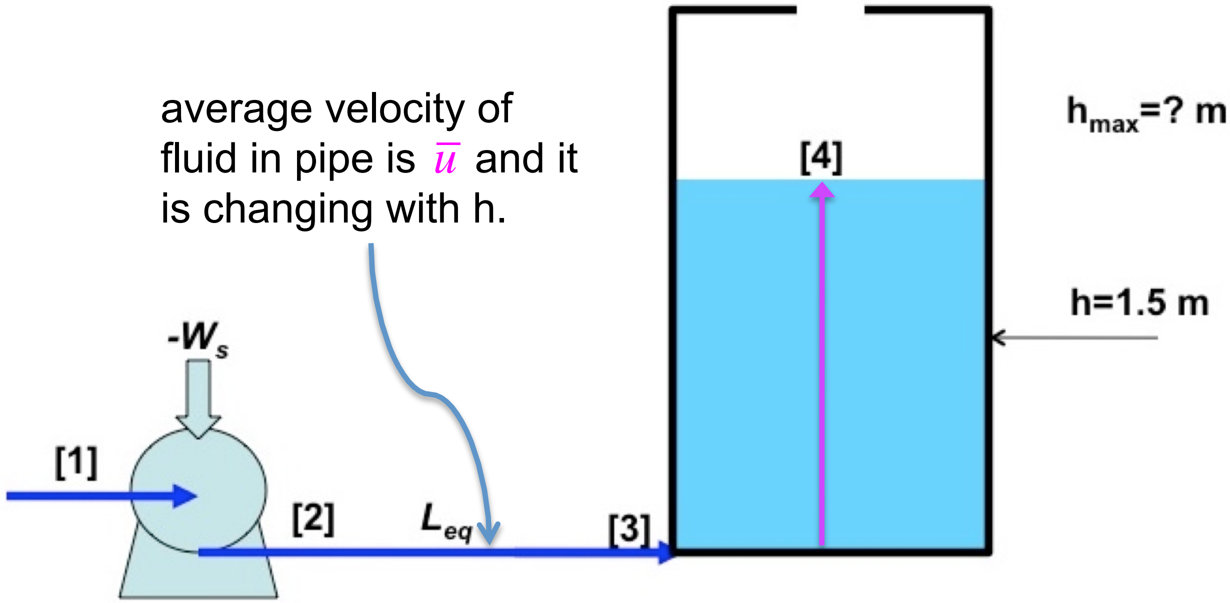
b) How long will it take to reach the maximum height of the fluid in the tank?

c) What is the maximum velocity of the fluid in the pipe?

d) How long will it take to raise the level of the fluid to h=1.5 [m]?

Data: D_{tank} = 1[m]; d_{pipe}= 0.01 [m]; ρ =1000 [kg/m^3]; μ = 0.0043 [kg/ms];

L_{eq}=25 [m]; $(-W_{Sout})$ = 29.43 [J/kg



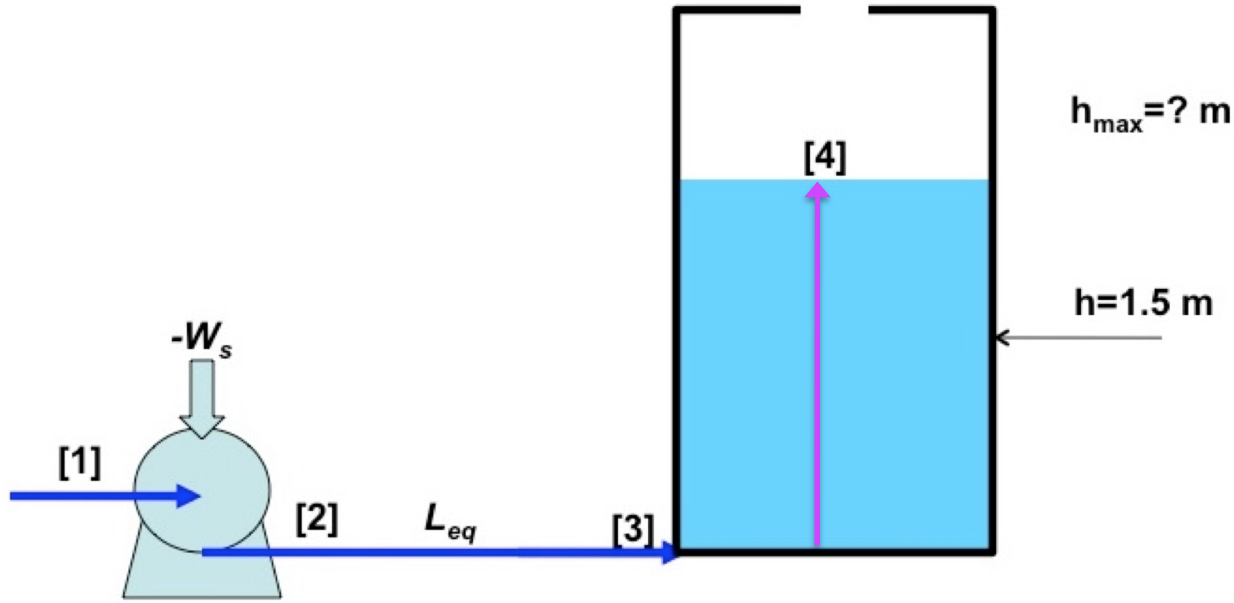
a) What is the maximum possible height of the fluid in the tank?

Set MEB Equation between points 1-4 when h_{max} is reached;

$$\text{at } h_{\max} \quad \Rightarrow \quad \bar{u} = 0 \text{ and } \sum F = 0$$

$$g\Delta Z + \cancel{\frac{\Delta \bar{u}^2}{2}} + \cancel{\frac{\Delta P}{\rho}} + W_{Sout} + \cancel{\sum F} = 0 \quad \Rightarrow \quad g\left(h_{\max} - \cancel{h_1}\right) - 29.43 = 0$$

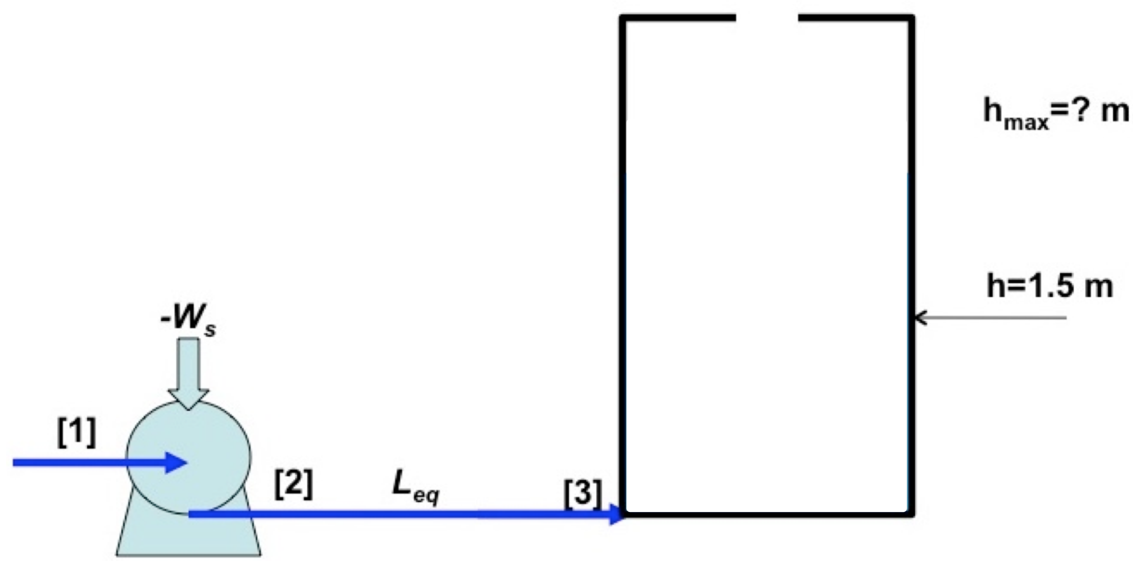
$$\boxed{h_{\max} = \frac{29.43}{9.81} = 3\ [m]}$$



b) How long will it take to reach the maximum height of water in the tank?

This problem has an exponential solution; therefore, any steady state solutions occurs at infinite time. This will be evident from the solution obtained in part 'd'. For now we can say that the maximum level in the tank will be reached at:

$$\boxed{\tau = \infty}$$

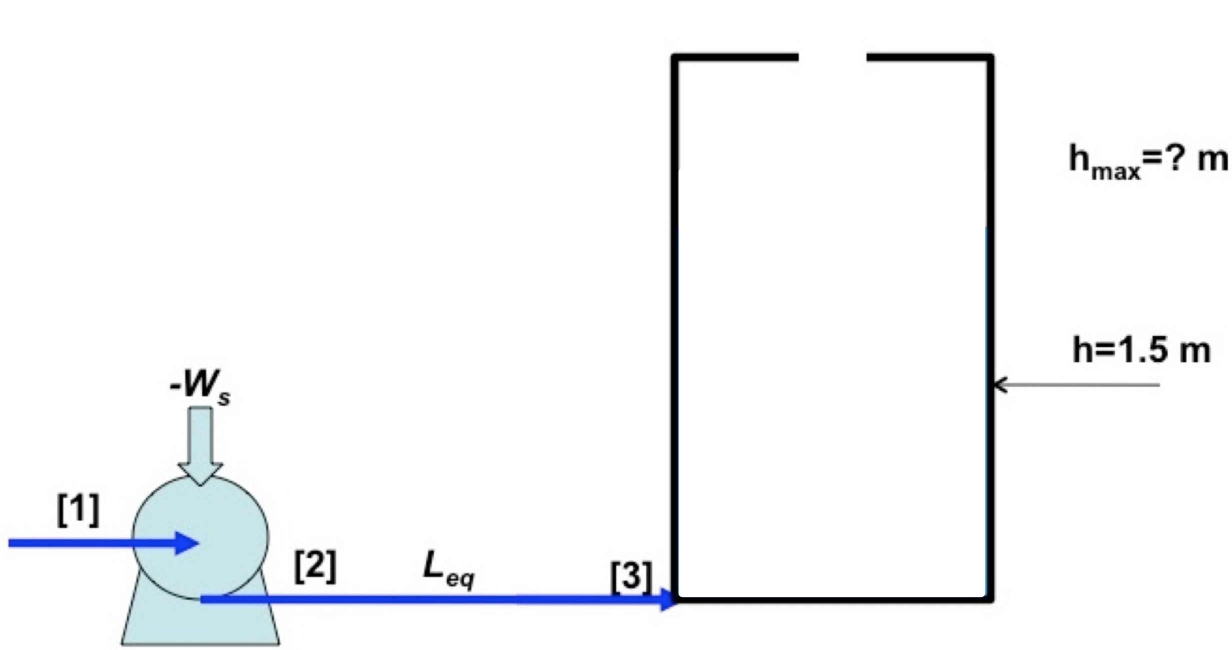


c) What is the maximum velocity of the fluid in the pipe?

Maximum velocity in the pipe occurs when the tank is empty. Therefore, set the MEB Equation between points 1-3 (tank is empty, h_3=0). We will also assume that we have laminar flow through the pipe and after the velocity is calculated we will check if Re<2100.

$$\cancel{g\Delta Z} + \cancel{\frac{\Delta \bar{u}^2}{2}} + \cancel{\frac{\Delta P}{\rho}} + W_{Sout} + \sum F = 0 \quad \Rightarrow \quad -W_{Sout} = \sum F$$

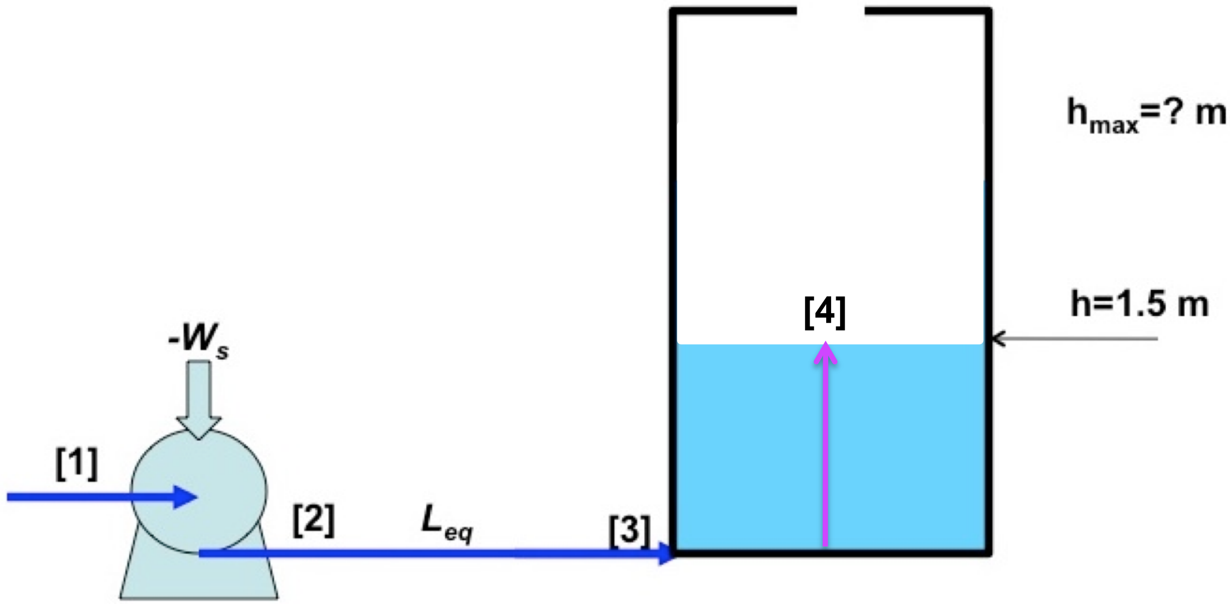
$$29.43 = \frac{32\ \bar{u}_{\max}\ \mu\ L_{eq}}{d_p^2 \rho} \quad \Rightarrow \quad \boxed{\bar{u}_{\max} = \frac{29.43 \times d_p^2 \rho}{32 \times \mu L_{eq}} = \frac{29.43 \times 0.01^2 \times 1000}{32 \times 0.0043 \times 25} = 0.855}$$



The maximum velocity of the fluid in the pipe is u_{max}=0.855 [m/s]

IMPORTANT – Check the Re number; was laminar flow assumption correct?

$$\boxed{\mathrm{Re}_{\max} = \frac{\rho \bar{u}_{\max} d_p}{\mu} = \frac{1000 \times 0.855 \times 0.01}{0.0043} = 2000}$$



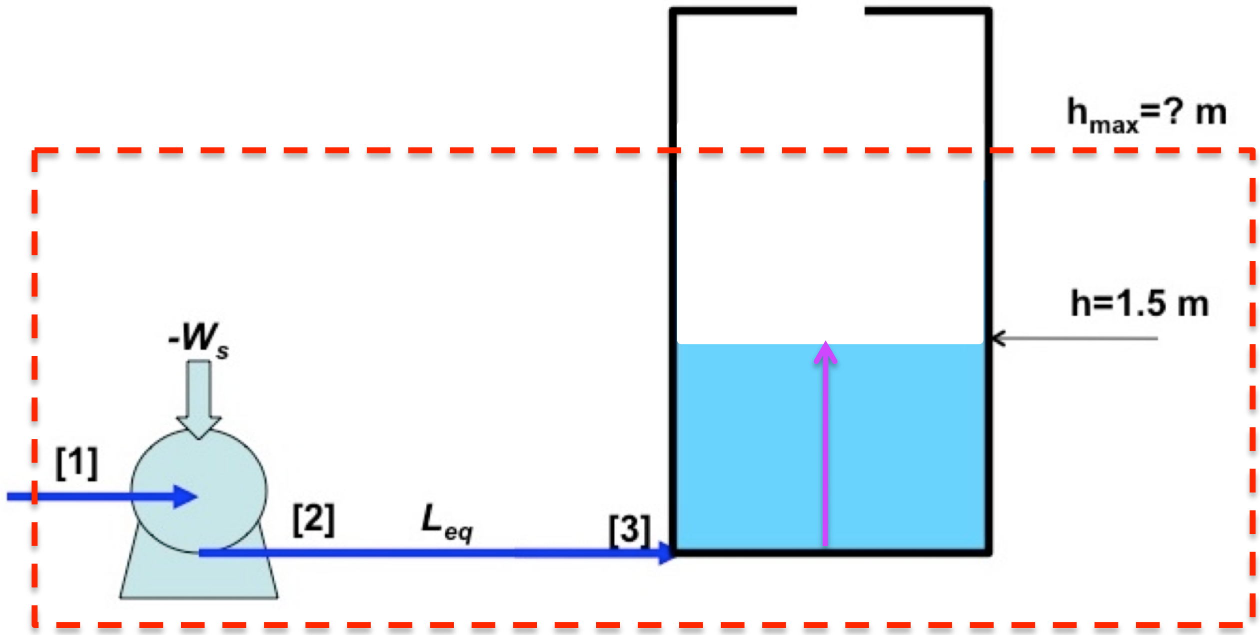
d) How long will it take to raise the level of the fluid to h=1.5 [m]?

Obtain (from MEB Equation) relationship between average velocity in the pipe $\bar{u}$, and any level of the fluid in the tank, h. (set MEB equation between points 1 and 4)

$$g\Delta Z+\frac{\Delta \cancel{\bar{u}}^2}{2}+\frac{\Delta \cancel{P}}{\rho}+W_{Sout}+\sum F=0 \Rightarrow g\left(h-\cancel{h_1}\right)-29.43+\frac{32\times\bar{u}\mu L_{eq}}{d_p^2\rho}=0$$

$$h\ -3+\frac{32\times\bar{u}\mu L_{eq}}{g\times d_p^2\rho}=0 \Rightarrow \bar{u}=(3-h)\frac{d_p^2\rho g}{32\times\mu L_{eq}}=(3-h)\frac{0.01^2\times1000\times9.81}{32\times0.0043\times25}$$

$$\boxed{\bar{u}=\frac{(3-h)}{3.5}}$$



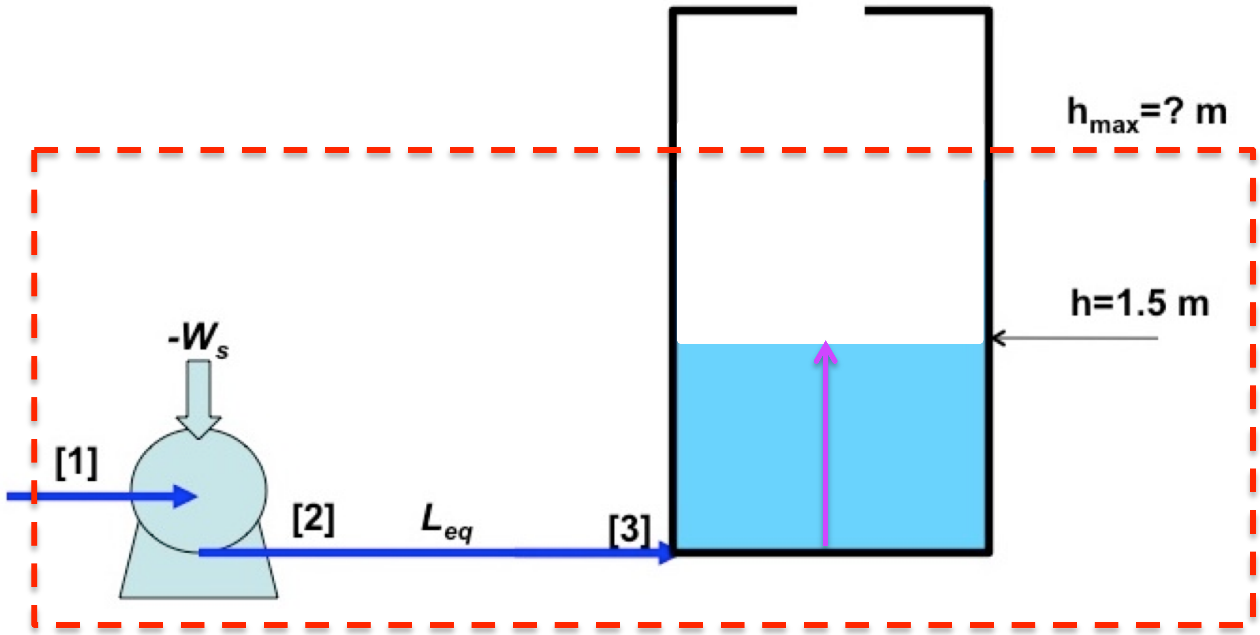
Now perform material balance on the entire tank; ***Input – Output = Accum.***

$$\boxed{\bar{u}\cdot\left[\frac{\pi d_p^2}{4}\right]\rho\cdot\Delta t \;-\; 0 = \frac{\pi D_t^2}{4}\left[h|_{t+\Delta t} - h|_t\right]\cdot\rho}$$

$$\bar{u} = \left(\frac{D_t^2}{d_p^2}\right)\left[\frac{h|_{t+\Delta t} - h|_t}{\Delta t}\right] \quad\Rightarrow\quad \bar{u} = \left(\frac{D_t^2}{d_p^2}\right)\frac{dh}{dt}$$

$$\boxed{\bar{u} = \frac{(3-h)}{3.5}}$$

$$h-3 = -3.5\left(\frac{1}{0.01}\right)^2\frac{dh}{dt} = -35000\frac{dh}{dt} \quad\Rightarrow\quad \frac{dt}{35000} = -\frac{dh}{h-3}$$



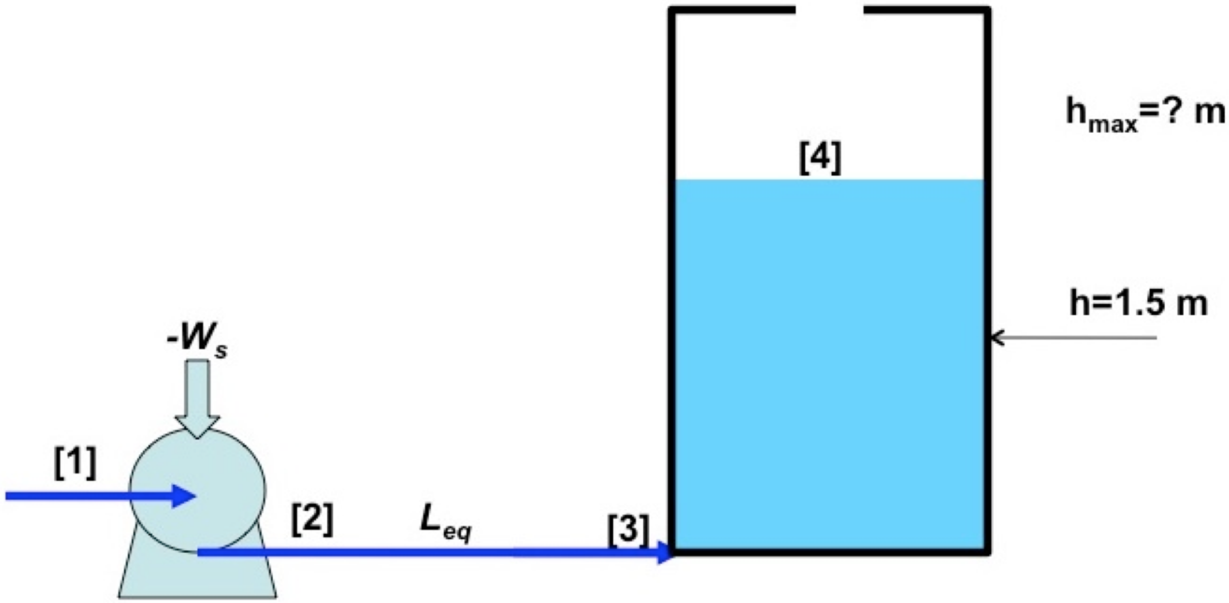
Now integrate the obtained differential equation between ***t = 0*** and some ***t = t, which*** correspond to ***h = 0*** and ***h = h:***

$$\frac{dt}{35000} = -\frac{dh}{h-3} \Rightarrow \frac{1}{35000}\int_0^t dt = -\int_0^h \frac{dh}{h-3} \Rightarrow \boxed{\frac{t}{35000} = -\ln\left(\frac{h-3}{-3}\right)}$$

$$\boxed{h = 3\times\left(1-e^{-\frac{t}{35000}}\right)} \Rightarrow t_{h=1.5} = 35000\times\left(-\ln\left(\frac{1}{2}\right)\right) = 35000\times\ln(2) = 24260$$

$$\boxed{t_{h=1.5} = 24260[s]}$$

Oregon State University

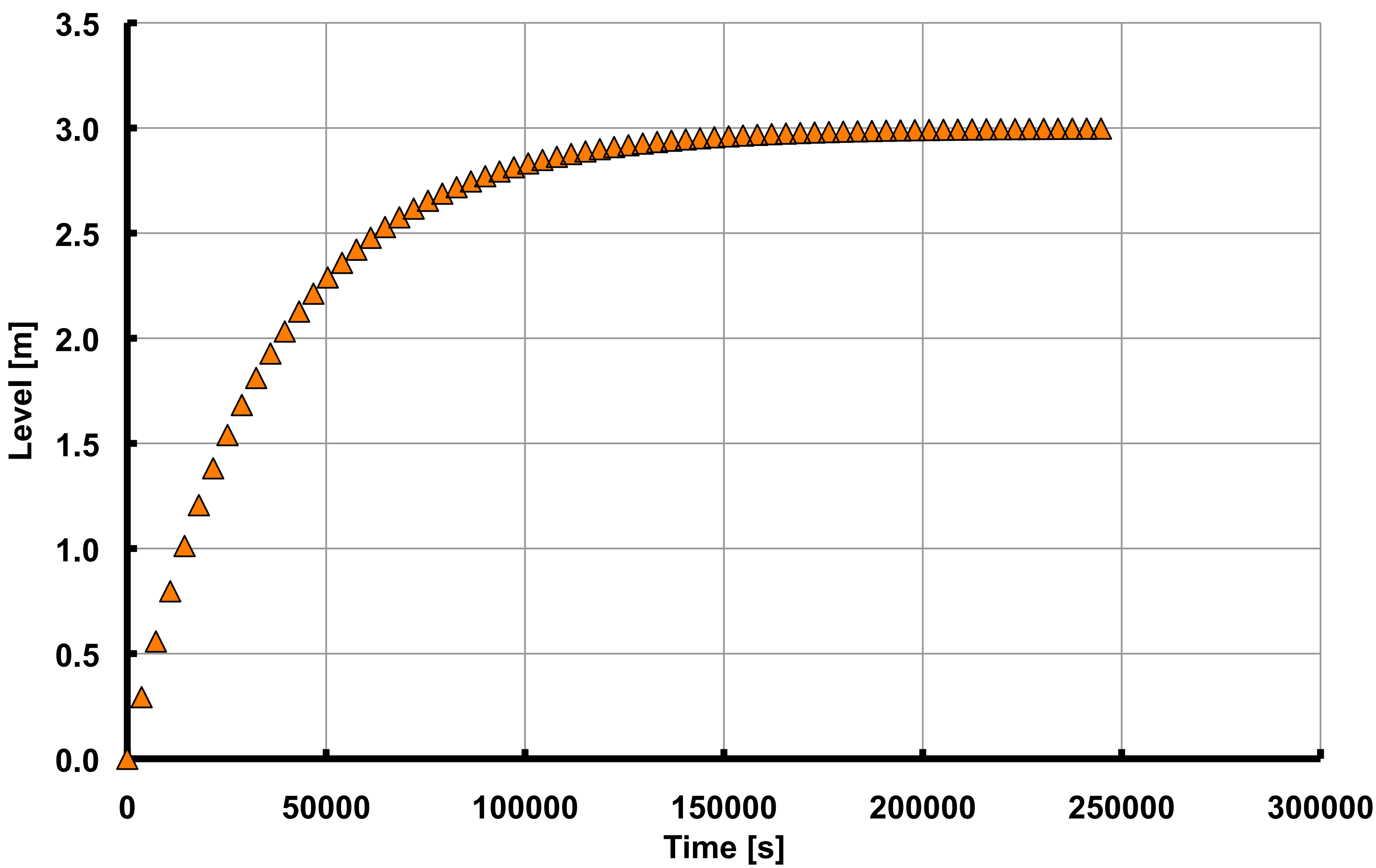


Now we can calculate at what time fluid level will reach maximum; h_{max}=3.0 [m]

$$\boxed{h = 3\times\left(1-e^{\frac{-t}{35000}}\right)} \Rightarrow h_{\max} = 3 \Rightarrow 3 = 3\times\left(1-e^{\frac{-t}{35000}}\right)$$

$$\Rightarrow \frac{3}{3} = 1 = \left(1-e^{\frac{-t}{35000}}\right) \Rightarrow \textit{this is only posible when } \boxed{t=\infty}$$

$$\boxed{t_{h=h_{maz}} = \infty}$$



Level [m]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
0
50000
100000
150000
200000
250000
300000
Time [s]

Oregon State University

Oregon State University OSU College of Engineering

People. Ideas. Innovation.

Thank you for your attention!