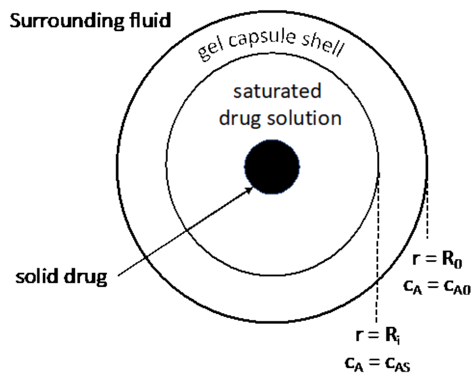
ChE 333 Transport Phenomena III, fundamentals of Mass Transfer
Pseudo-steady state

Capsule for slow drug release: The spherical gel capsule shown below is used for slow drug release. The drug capsule is made of a gel capsule filled with a saturated drug solution. The drug solution also has a lump of solid drug which maintains the concentration in the saturated drug solution over a longer time. The drug diffuses through the gel capsule into the surrounding fluid. Eventually, the solid drug is depleted and the concentration in the liquid goes down with time but as long as the solid drug exists the solution remains saturated and constant.



The diffusion coef. of the drug in the gel phase is 1.5×10^{-5} cm^2/s, and the solubility of the drug in the gel capsule material is $c^*_A = 0.01$ gmole/cm^3.
$R_i = 0.2$ cm and $R_0 = 0.35$ cm

(a) Identify the SOURCE, SINK, diffusing species, and phase for your control system.

SOURCE at R_i, diffusing species: drug
SINK at R_o, phase: gel (liquid like)

(b) State your assumptions and boundary conditions

assumptions:
steady state
1-D flux

No reactions
dilute diffusion
unimolecular diffusion

boundary conditions.

@ $r = R_i \quad c_A = c_{AS}$

@ $r = R_o \quad c_A = c_{Ao}$

(c) Simplify the general differential and flux equation (you may assume dilute diffusion so $y_A \approx 0$)

general differential eqn.

$$-\nabla \cdot N_A + \underbrace{\cancel{R_A}}_{=\emptyset} = \underbrace{\frac{\cancel{\partial c_A}}{\partial t}}_{=\emptyset}$$

$$-\frac{1}{r^2}\frac{d}{dr}(r^2 N_{A,r}) = \emptyset$$

$$\frac{d}{dr}(r^2 N_{A,r}) = \emptyset$$

Flux equation

$$N_A = -D_{AB}\underbrace{\nabla c_A}_{\frac{dc_A}{dr}} + \underbrace{\cancel{y_A}}_{\simeq \emptyset}(N_A + N_B)$$

$$N_A = -D_{AB}\frac{dc_A}{dr}$$

(d) Develop an analytical, integrated equation for the total drug release rate, (W_A) but $W_A = N_A * S$ where S is surface area.

math hint: if

$$\frac{d}{dx}N_{A,z} = 0 \qquad N_{A,z} \text{ is constant} \qquad \int (N_{A,x})dx = N_{A,x}\int dx$$

Similarly, if

$$\frac{d}{dr}(rN_{A,r}) = 0 \qquad r\,N_{A,r} \text{ is constant} \qquad \int (N_{A,r})dr = \int \left(\frac{r}{r}N_{A,r}\right)dr = rN_{A,r}\int \frac{dr}{r}$$

And $r\,N_{A,r} = R_0\,N_{A,R0} = R_i\,N_{A,Ri}$

$$\int \frac{r^2}{r^2} N_A\, dr = \int -D_{AB}\, dc_A$$

$$r^2 N_{Ar} \int_{R_i}^{R_o} \frac{dr}{r^2} = -D_{AB}\int_{C_{As}}^{C_{Ao}} dc_A \;:\; \frac{r^2 N_{Ar}}{\left(\frac{1}{R_i} - \frac{1}{R_o}\right)} = D_{AB}(C_{As} - C_{Ao})$$

$$N_{Ar} = \frac{D_{AB}}{r^2}\frac{C_{As} - C_{Ao}}{\left(\frac{1}{R_i} - \frac{1}{R_o}\right)}$$

$$W_A = N_{Ar}\, 4\pi r^2 = \frac{4\pi r^2 D_{AB}}{r^2}\frac{(C_{As} - C_{Ao})}{\left(\frac{1}{R_i} - \frac{1}{R_o}\right)}$$

(e) What is the maximum possible rate of drug release from the capsule in gmole/hr? (the maximum rate would be when $c_{A0} \approx 0$)

$$W_A = \frac{4\pi\, 1.5\times10^{-5}\,\frac{\cancel{cm^2}}{s}\cdot 0.01\,\frac{mole}{\cancel{cm^3}}}{\frac{1}{0.2\,\cancel{cm}} - \frac{1}{0.35\,\cancel{cm}}} = 8.79\times10^{-7}\, mole/s$$

$$= \boxed{3.17\cdot10^{-3}\, mole/hr}$$