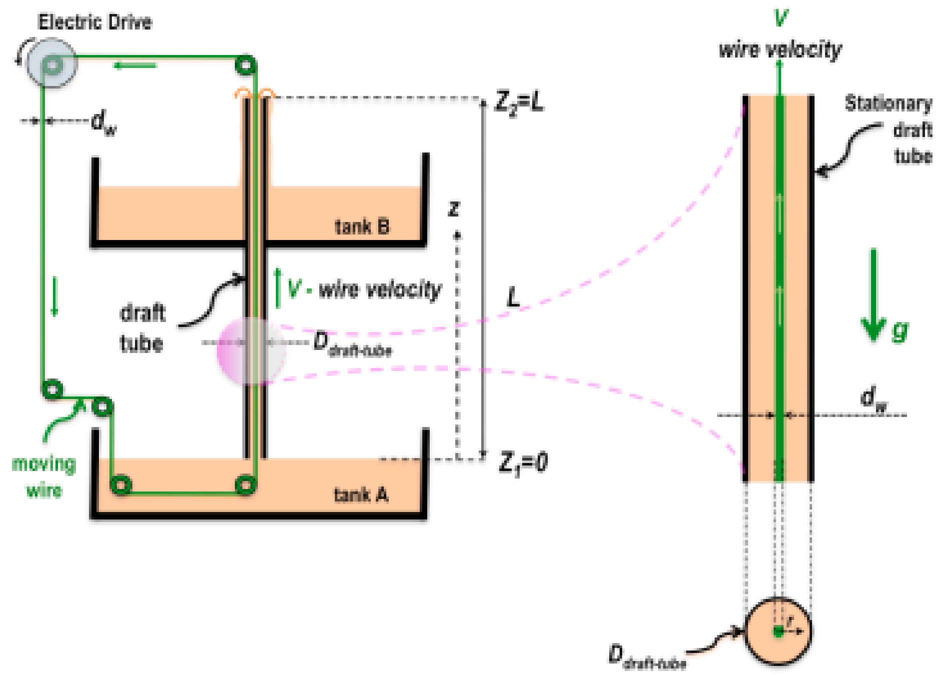
(120 points) Consider a so-called "***wire drag-pump***", which pumps a liquid Newtonian Incompressible fluid by dragging the fluid upward through a stationary ***draft tube*** from container A into container B, as illustrated below. The wire is located in the center of the circular *draft tube* and the fluid fills the tube while flowing upward. The fluid flow in the draft tube is non-slip and strictly drag-induced, working against gravity. The fluid flow is isothermal, laminar, unidirectional, fully developed, and steady state. The wire velocity, ***V***, is such that it yields a ***zero shear stress*** at $r = R_{draft-tube}$.



a) (10 points) State all assumptions that are pertinent for this problem. Provide the list of variables.

b) (35 points) Develop a mathematical model, starting from Navier-Stokes equations in cylindrical coordinate, that describes this flow situation. Do not forget the boundary conditions. Also use the continuity equation.

c) (35 points) Solve the mathematical model obtained in (b) above, and derive the velocity profile of the fluid in the draft tube.

d) (15 points) Derive the expression for the wire velocity, ***V***, that induces the developed velocity profile.

e) (25 points) Find the expression for the volumetric flow rate, $Q \; \left[m^3/s\right]$, from tank *A* to the Tank *B*.

f) **(10 points)** Does the volumetric flowrate $Q\ [m^3/s]$ depend on the length of the ***draft tube***?

g) **(15 points)** Using numerical data provided below plot the velocity profile $u_z(r)$ as a function of "r"; i.e., make an Excel graph $u_z(r)$ for $R_{draft-tube} \geq r \geq R_{wire}$. Label you graph properly.

h) **(20 points)** Calculate the Re number for the flow through the draft tube with data given below. Note, to calculate Reynolds number here, you need to use the equivalent hydraulic diameter of concentric pipes (draft tube and wire) using $d_{hydraulic} = 4\left(\frac{A}{p}\right)$, where *A* is the cross sectional area available for the fluid flow, and *p* is the wetted perimeter. Please use the data provided below:

Data

$$R_{wire} = 0.001\ [m] \qquad R_{draft-tube} = 0.006\ [m]$$

$$\rho = 1000\ [kg/m^3] \qquad \mu = 0.1\ [Pa \cdot s]$$

Continuity Equation:

$$\frac{1}{r}\frac{\partial (r\ u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

Navier-Stokes Equations:

Momentum equation in r direction:

$$\rho\left[\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z\frac{\partial u_r}{\partial z}\right] = -\frac{\partial P}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r u_r)\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2}\right] + \rho g_r$$

Momentum equation in θ direction:

$$\rho\left[\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z\frac{\partial u_\theta}{\partial z}\right] = -\frac{1}{r}\frac{\partial P}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r u_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2}\right] + \rho g_\theta$$

Momentum equation in z direction:

$$\rho\left[\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right] = -\frac{\partial P}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho g_z$$