Oregon State University

OREGON STATE UNIVERSITY
CBEE
DEPARTMENT OF CHEMICAL ENGINEERING

CHE 331
Transport Phenomena I

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Mechanical Energy Balance Equation II

Please turn-off cell phones

CHE 331
Fall 2022

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MECHANICAL ENERGY BALANCE EQUATION

$$g\Delta Z + \frac{\Delta \bar{u}^2}{2} + \frac{\Delta P}{\rho} + \sum F + \dot{W}_{Sout} = 0$$

Potential Energy

Kinetic Energy

Flow Work ρ =const

Friction Losses

Shaft Work (out)

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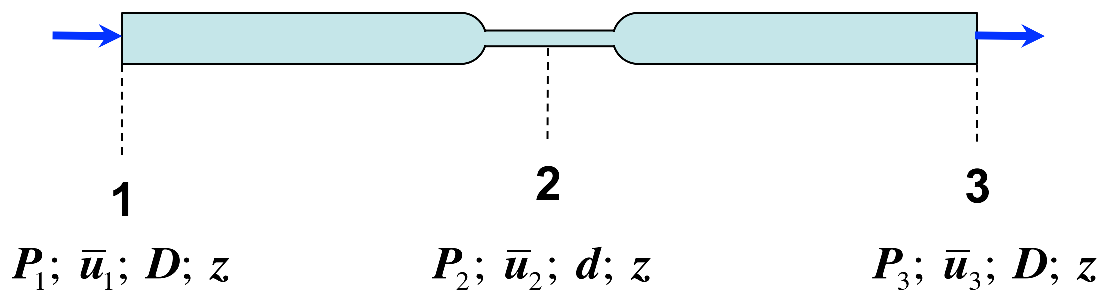
MECHANICAL ENERGY BALANCE EQUATION

$$g\Delta Z + \frac{\Delta \bar{u}^2}{2} + \frac{\Delta P}{\rho} + \sum F + \dot{W}_{Sout} = 0$$



Assumption:

Negligible frictional losses – MEB equation becomes Bernoulli equation !

$$g\Delta Z + \frac{\Delta \bar{u}^2}{2} + \frac{\Delta P}{\rho} + \cancel{\sum F} + \dot{W}_{Sout} = 0$$

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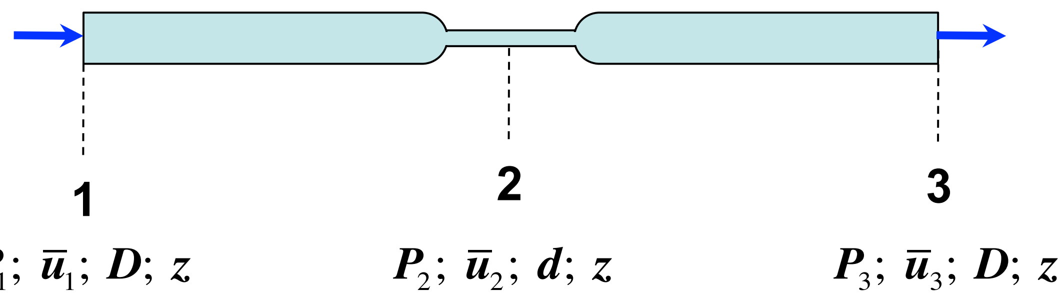
MECHANICAL ENERGY BALANCE EQUATION

$$g\Delta Z + \frac{\Delta \bar{u}^2}{2} + \frac{\Delta P}{\rho} + \dot{W}_{Sout} = 0$$



MEB between point 1 and 2:

$$g\cancel{\Delta Z} + \frac{\Delta u^2}{2} + \frac{\Delta P}{\rho} + \cancel{\dot{W}_{Sout}} = 0$$

$$\frac{\Delta u^2}{2} + \frac{\Delta P}{\rho} = 0 \quad \Rightarrow \quad \frac{\bar{u}_2^2 - \bar{u}_1^2}{2} + \frac{P_2 - P_1}{\rho} = 0 \quad \Rightarrow \quad \boxed{\frac{\bar{u}_2^2 - \bar{u}_1^2}{2} = \frac{P_1 - P_2}{\rho}}$$

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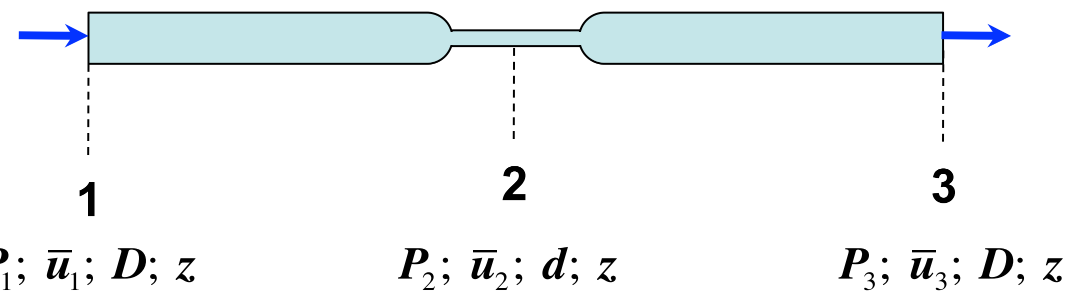
MECHANICAL ENERGY BALANCE EQUATION

$$g\Delta Z + \frac{\Delta \bar{u}^2}{2} + \frac{\Delta P}{\rho} + \dot{W}_{Sout} = 0$$



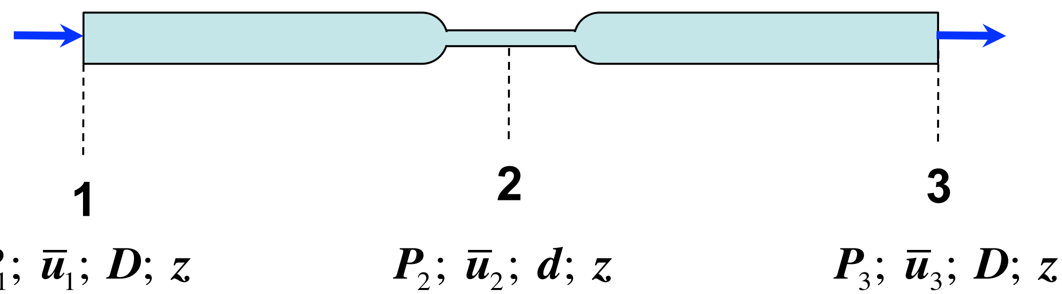
MEB between point 2 and 3:

$$\cancel{g\Delta Z} + \frac{\Delta u^2}{2} + \frac{\Delta P}{\rho} + \cancel{\dot{W}_{Sout}} = 0$$

$$\frac{\Delta u^2}{2} + \frac{\Delta P}{\rho} = 0 \quad \Rightarrow \quad \frac{u_3^2 - u_2^2}{2} + \frac{P_3 - P_2}{\rho} = 0 \quad \Rightarrow \quad \boxed{\frac{u_3^2 - u_2^2}{2} = \frac{P_2 - P_3}{\rho}}$$

MECHANICAL ENERGY BALANCE EQUATION

$$g\Delta Z + \frac{\Delta \bar{u}^2}{2} + \frac{\Delta P}{\rho} + \dot{W}_{Sout} = 0$$



MEB between point 1 and 3:

$$\cancel{g\Delta Z} + \frac{\Delta \bar{u}^2}{2} + \frac{\Delta P}{\rho} + \cancel{\dot{W}_{Sout}} = 0$$

$$\boxed{P_1 = P_3}$$

$$\frac{\Delta \bar{u}^2}{2} + \frac{\Delta P}{\rho} = 0 \quad \Rightarrow \quad \frac{\bar{u}_3^2 - \bar{u}_1^2}{2} + \frac{P_3 - P_1}{\rho} = 0 \quad \Rightarrow \quad \boxed{\cancel{\frac{\bar{u}_3^2 - \bar{u}_1^2}{2}} = \frac{P_1 - P_3}{\rho}}$$

MECHANICAL ENERGY BALANCE EQUATION

Consider a simple case where fluid flows in a horizontal tube. This time we ***will not ignore*** the energy loss due to the friction of fluid on tube walls *(assume liquid fluid ρ =const).*

$$\cancel{g\Delta Z} + \cancel{\frac{\Delta \bar{u}^2}{2}} + \frac{\Delta P}{\rho} + \sum F + \cancel{\dot{W}}_{Sout} = 0$$

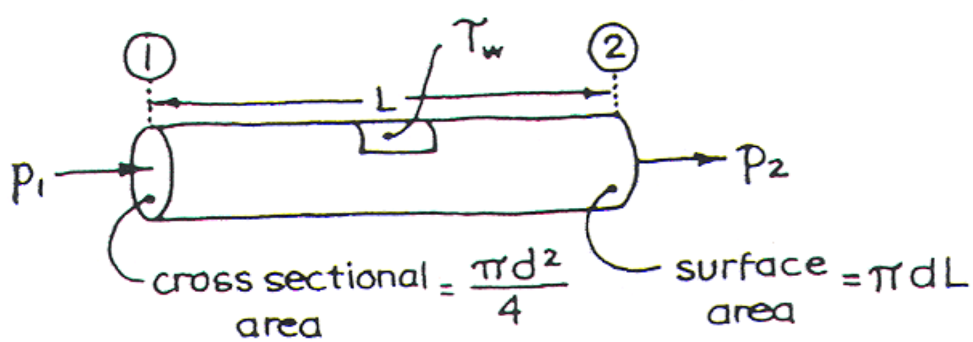
$$\sum F = -\frac{P_2 - P_1}{\rho} \Rightarrow \rho \sum F = P_1 - P_2 = \Delta P_{friction}$$

$$\rho \sum F = \Delta P_{friction}$$

MECHANICAL ENERGY BALANCE EQUATION

In order to account for the friction losses we may conveniently define a Friction Factor, f_F, as:

$$f_F = \frac{\left(\dfrac{Frictional\ Drag\ Force}{Area\ of\ Pipe\ Surface}\right)}{Kinetic\ Energy\ of\ Fluid} = \frac{\left(\dfrac{F_{Drag}}{A_{pipe}}\right)}{\dfrac{\rho u^2}{2}}$$



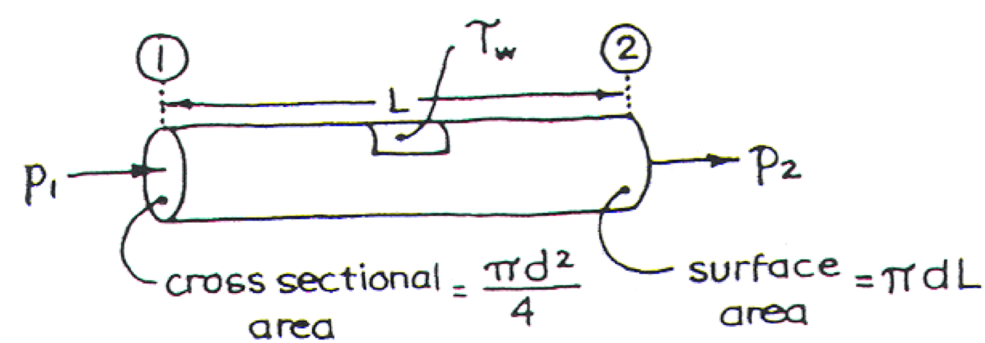
MECHANICAL ENERGY BALANCE EQUATION

$$F_{Drag} = f_F A_{pipe} \frac{\rho \bar{u}^2}{2} \Rightarrow \frac{F_{Drag}}{A_{pipe}} = \tau_w = f_F \frac{\rho \bar{u}^2}{2}$$

$$\boxed{\tau_w = f_F \frac{\rho \bar{u}^2}{2}}$$

Average velocity

Shear Stress



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$$\sum F = -\frac{P_2 - P_1}{\rho} \Rightarrow \rho \sum F = P_1 - P_2 = \Delta P_{friction}$$

$$\{Energy\ lost\ by\ the\ fluid\} = \{Energy\ transmitted\ to\ the\ wals\}$$

$$\left\{L\left(\frac{\pi d^2}{4}\right)(P_1 - P_2)\right\} = \{L * L\pi d * \tau_w\}$$

$$\Updownarrow \qquad\qquad\qquad \Updownarrow$$

$$\left\{L\left(\frac{\pi d^2}{4}\right)\rho \sum F\right\} = \left\{L * L\pi d * f_F \frac{\rho u^2}{2}\right\}$$

$$\left\{L\left(\frac{\pi d^2}{4}\right)\rho\sum F\right\} = \left\{L * L\pi d * f_F \frac{\rho u^2}{2}\right\}$$

$$\boxed{\sum F = \frac{2 f_F L u^2}{d}}$$

At this point, we still do not know anything about frictional losses because we do not know how to evaluate f_F. Researchers have found that:

$$f_F = \varphi\{\text{Reynolds number (Re); Pipe roughness } (\varepsilon)\}$$

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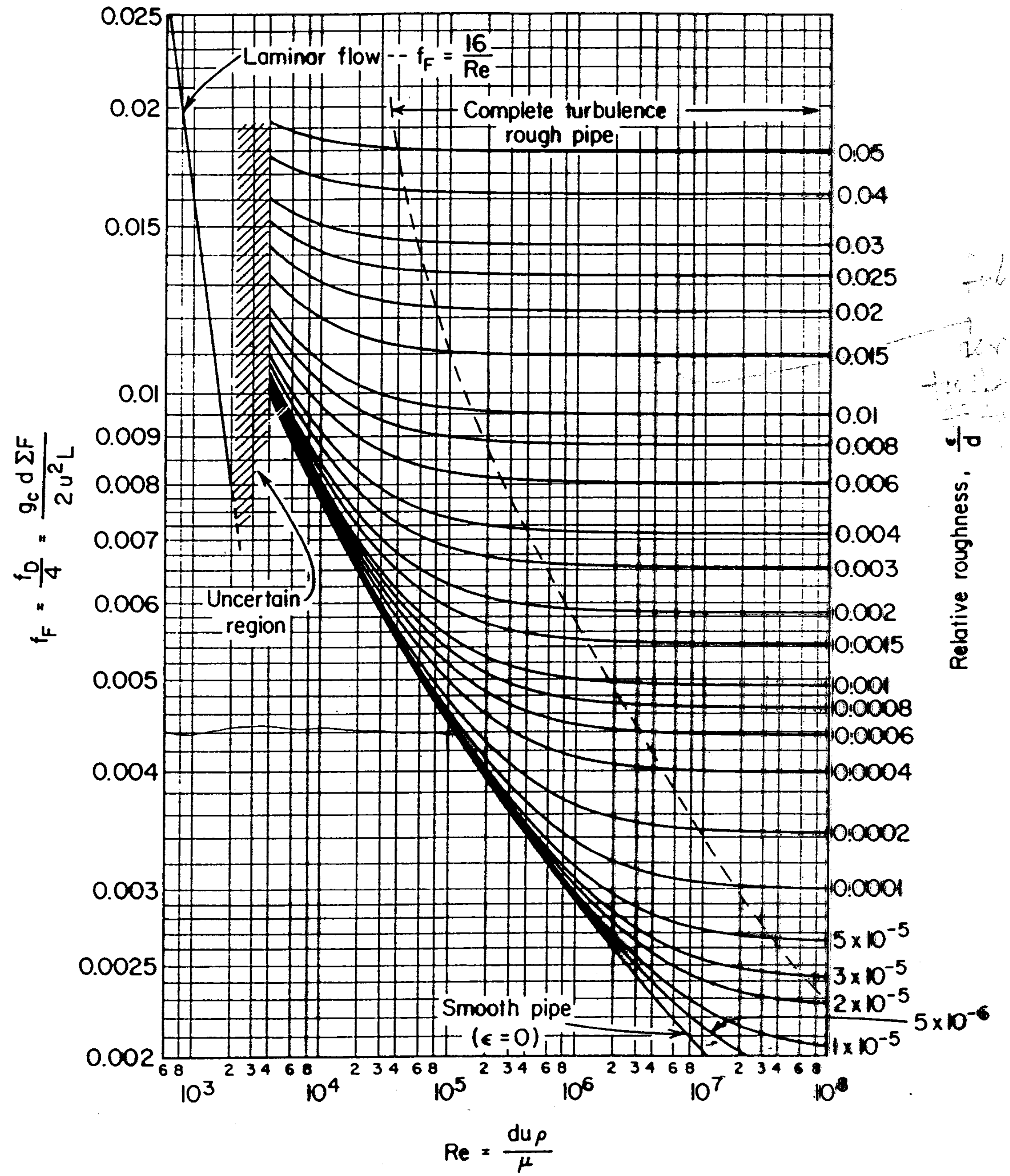

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MECHANICAL ENERGY BALANCE EQUATION

Finally:

$$g\Delta Z + \frac{\Delta \bar{u}^2}{2} + \frac{\Delta P}{\rho} + \sum F + \dot{W}_{Sout} = 0$$

$$\sum F = \frac{2 f_F L \bar{u}^2}{d}$$

$$f_F = \frac{16}{\mathrm{Re}}$$ for laminar flow (Re < 2100 for pipe)

$$\mathrm{Re} = \frac{\rho \bar{u} d}{\mu}$$

MECHANICAL ENERGY BALANCE EQUATION

Friction Factor for Turbulent flow:

$$\frac{1}{\sqrt{f_F}} = -4\log\left[\frac{\varepsilon}{3.7*d} + \frac{1.255}{\mathrm{Re}\sqrt{f_F}}\right]$$

$$\frac{1}{\sqrt{f_F}} = -4*\log\left[\frac{\varepsilon}{3.7d} + \frac{5.76}{\mathrm{Re}^{0.9}}\right]$$

$$\frac{1}{\sqrt{f_F}} = +4\log\left[\frac{3.7*d}{\varepsilon}\right]$$

For friction factor independent of Re

$$\frac{1}{\mathrm{Re}} = \frac{\sqrt{f_F}}{1.255}\left[10^{-\frac{0.25}{\sqrt{f_F}}} - \frac{\varepsilon}{3.7*d}\right]$$

MECHANICAL ENERGY BALANCE EQUATION

For Laminar flow we can develop final expression for the frictional energy loss term:

$$\sum F = \frac{2 f_F L \bar{u}^2}{d} \qquad \mathrm{Re} = \frac{\rho \bar{u} d}{\mu}$$

$$f_F = \frac{16}{\mathrm{Re}} \quad \text{for laminar flow (Re < 2100 for pipe)}$$

$$\boxed{\sum F = \frac{32 \mu L \bar{u}}{d^2 \rho}}$$

MECHANICAL ENERGY BALANCE EQUATION

Friction Losses in laminar flow (J/kg)

$$\boxed{g\Delta Z + \frac{\Delta \bar{u}^2}{2} + \frac{\Delta P}{\rho} + \frac{32\mu L\bar{u}}{d^2\rho} + \dot{W}_{Sout} = 0}$$

Potential Energy

Kinetic Energy

Flow Work With ρ=const

Shaft Work (out)

Steady State

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People. Ideas. Innovation.

Thank you for your attention!