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CBEE

CHE 331
Transport Phenomena I

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Surface Tension Forces I

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Surface Tension

There are two kinds of surface forces that create most of the liquid surface phenomena.

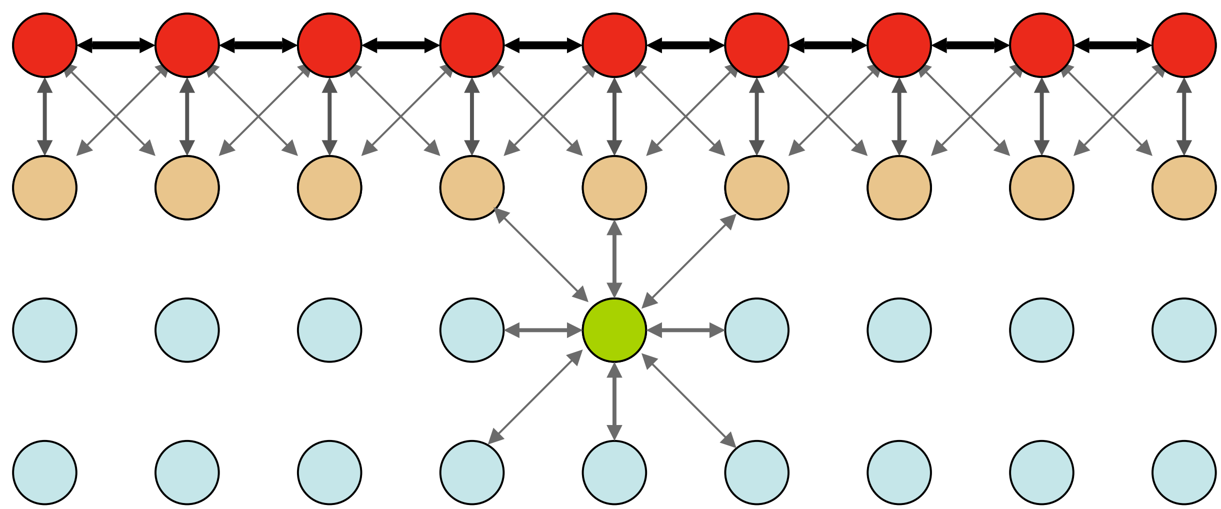
- *The **cohesive** forces are attractive forces among **alike** liquid molecules. The cohesive forces are responsible for the phenomenon known as **Surface Tension (σ)**.*

- *The **adhesive** forces are attractive forces among **unlike** molecules. The relative strength between adhesive and cohesive forces gives rise to hydrophobicity/hydrophilicity of a given liquid on a particular solid surface.*

For example, the adhesive forces between water molecules and glass tube walls are stronger than the cohesive forces of water molecules. This creates a water meniscus at the walls of the vessel and contributes to capillary action.

Cohesion Forces and Surface Tension

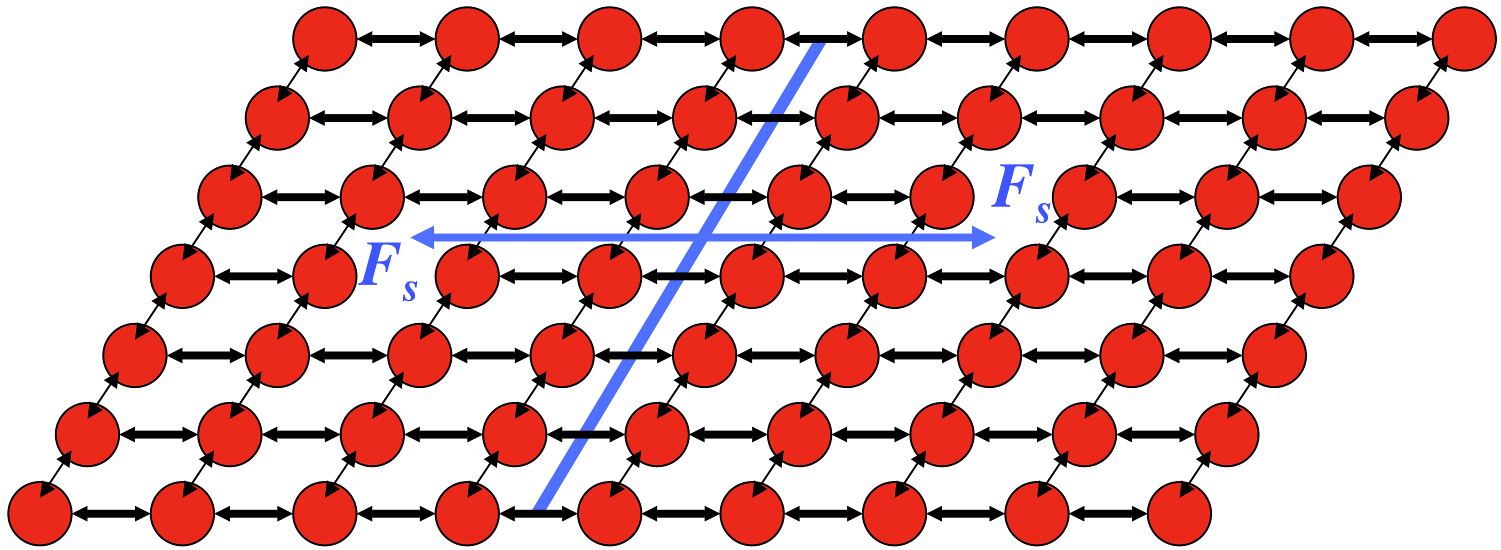
The ***cohesive forces*** between molecules inside a liquid are shared with all neighboring atoms.



Molecules on the surface have less neighbors of its own kind. They exhibit stronger attractive forces upon their nearest atoms on the surface. This enhancement of the intermolecular attractive forces at the surface contributes to ***interface pressure difference.***

Cohesion Forces and Surface Tension

Consider a neighborhood of molecules on a flat surface:



If we would like to tear (rupture) this surface along a line "*l*" we would have to apply a force F_s. ***The Surface Tension is defined as:***

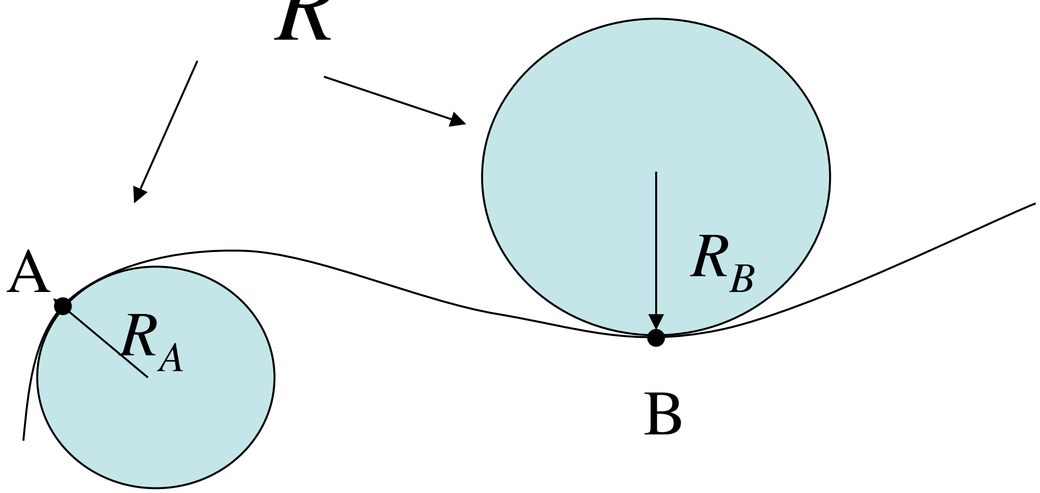
$$\sigma = \frac{F_S}{l} \left[\frac{N}{m} \right]$$

Cohesion Forces and Surface Tension

If the surface is curved the surface tension forces give rise to ***interface pressure difference*** ΔP_{int}.

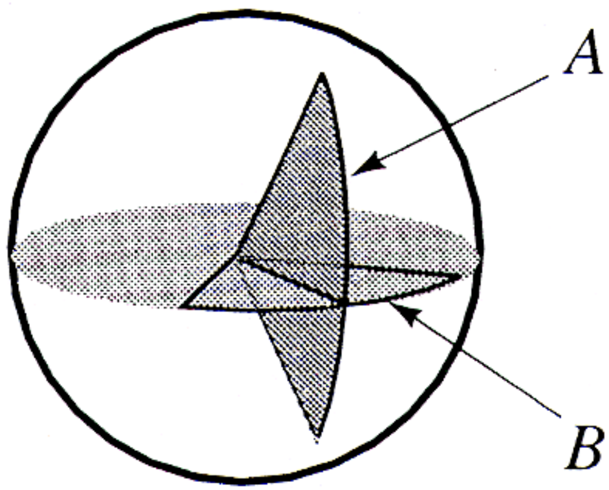
But first consider the concept of curvature. The curvature of a line at a given point, *A*, is defined as:

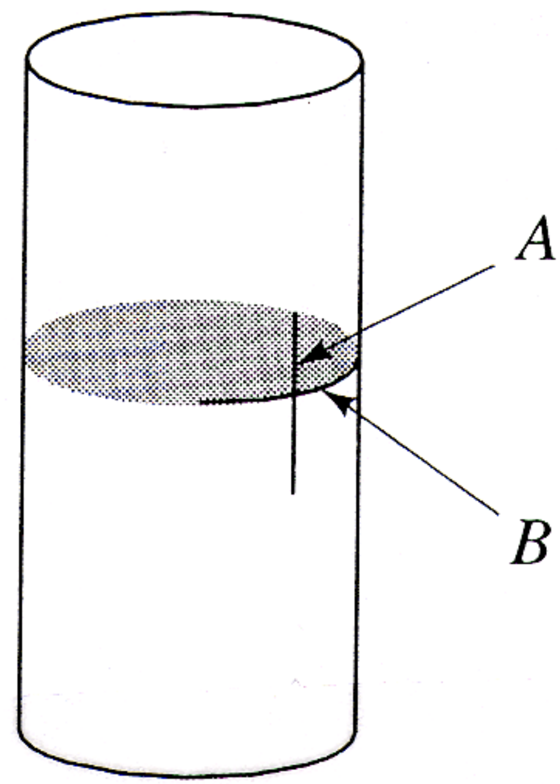
$$\kappa = \frac{1}{R}$$



Cohesion Forces and Surface Tension

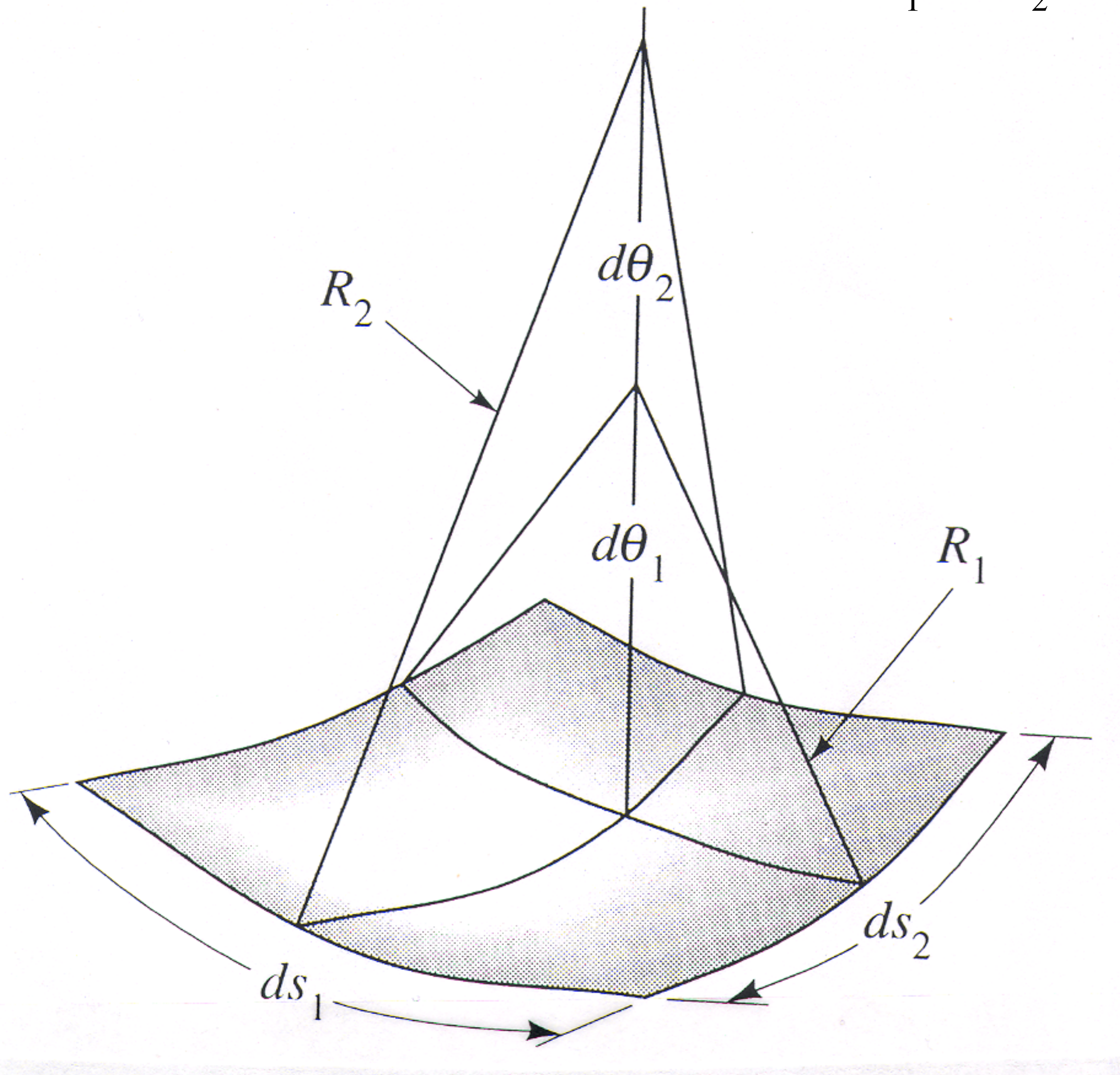
Similarly, for curved surfaces there are two characteristic radii that define the curvature of the surface at a given point.



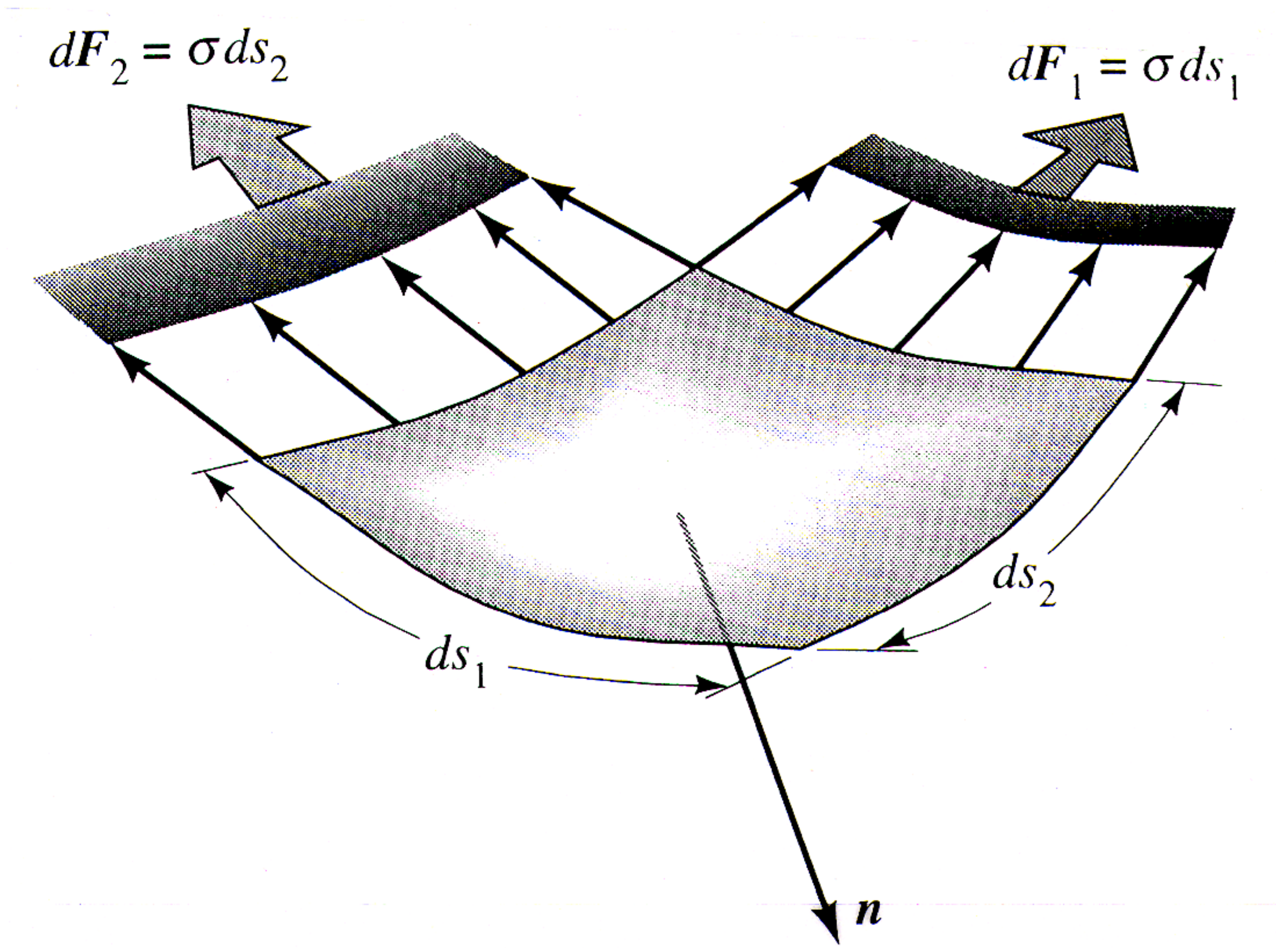


Cohesion Forces and Surface Tension

Now, consider a differential element of the surface : $ds_1 \times ds_2$



Cohesion Forces and Surface Tension



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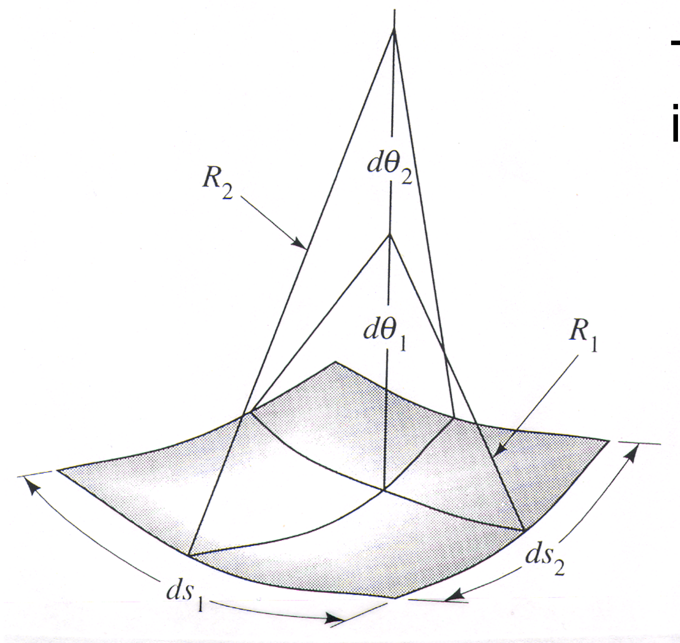
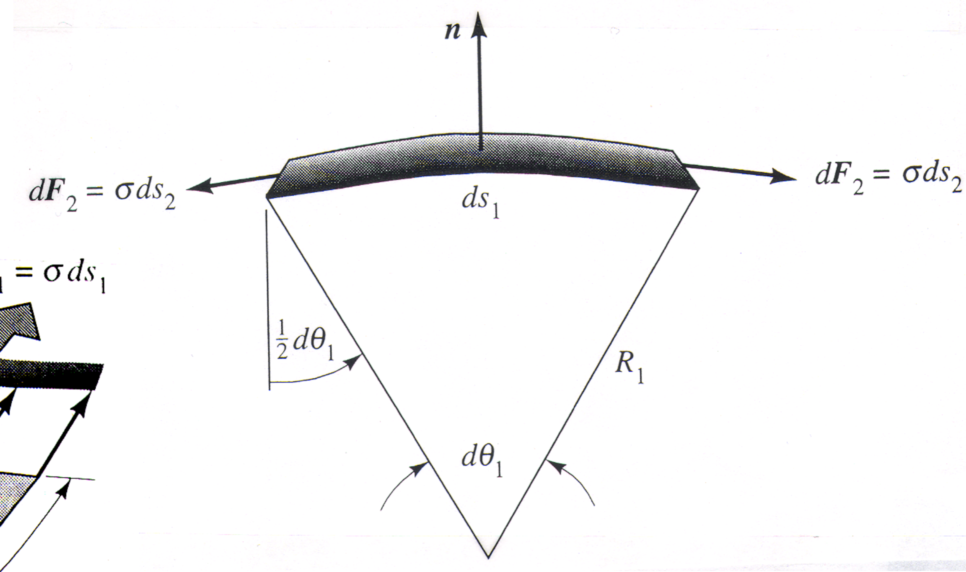
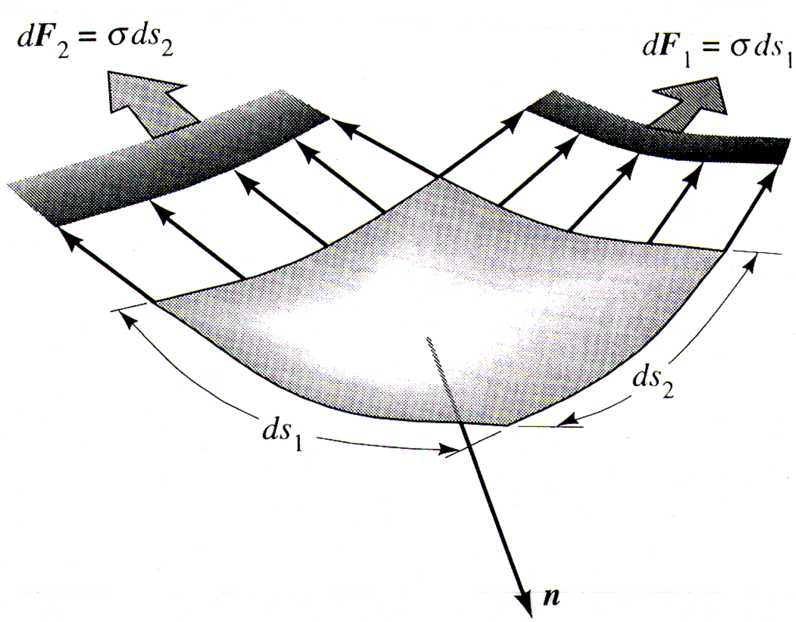


The following relationships are well known in trigonometry:

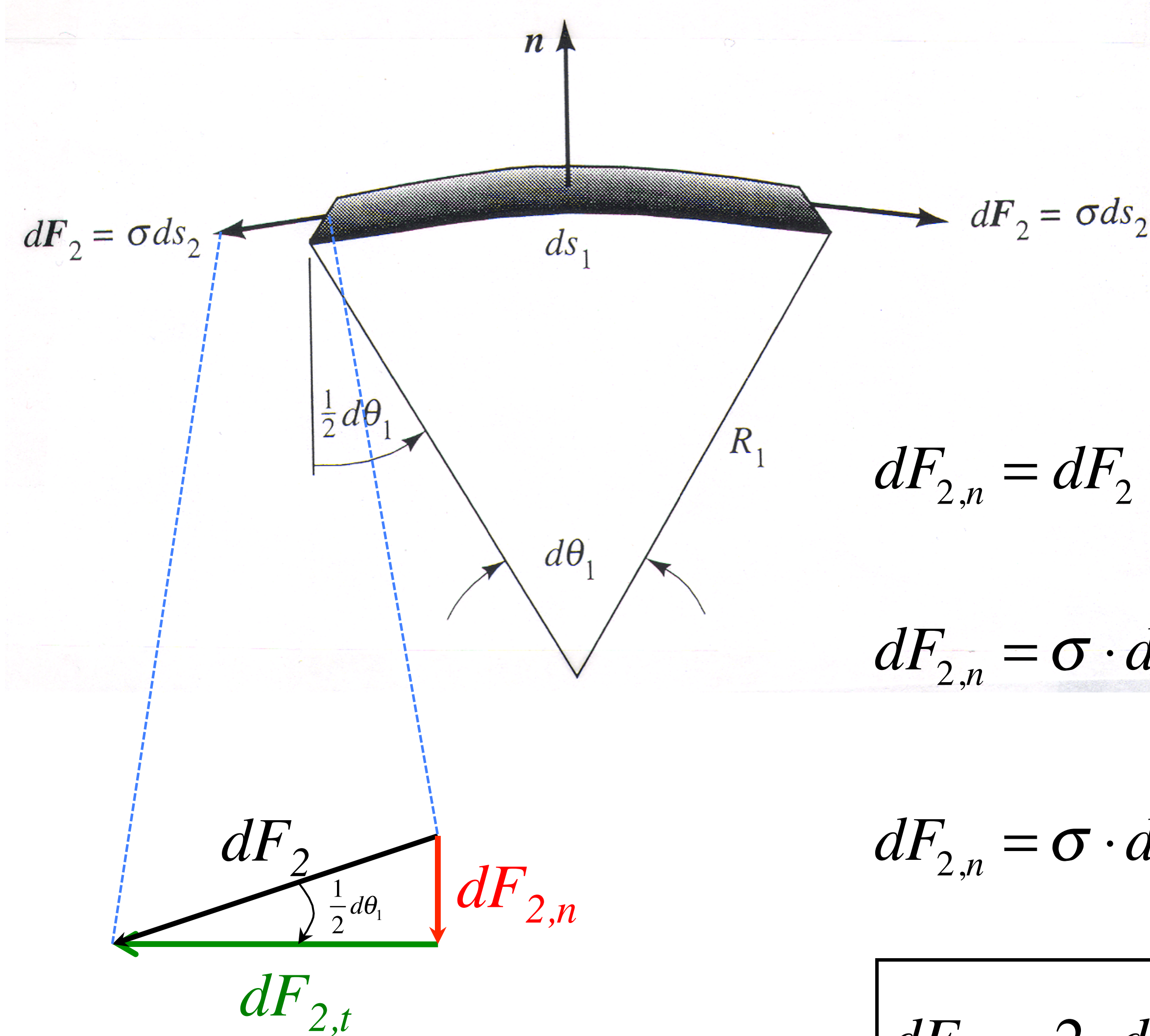
$$ds_1 = R_1 d\theta_1 \quad and \quad ds_2 = R_2 d\theta_2$$

$$\sin\left(\frac{1}{2}d\theta_1\right) \cong \frac{1}{2}d\theta_1 \quad and \quad \sin\left(\frac{1}{2}d\theta_2\right) \cong \frac{1}{2}d\theta_2$$





Cohesion Forces and Surface Tension



$$dF_{2,n} = dF_2 \cdot \sin\left(\frac{1}{2} d\theta_1\right)$$

$$dF_{2,n} = \sigma \cdot ds_2 \cdot \left(\frac{1}{2} d\theta_1\right)$$

$$dF_{2,n} = \sigma \cdot ds_2 \cdot \left(\frac{1}{2}\frac{ds_1}{R_1}\right)$$

$$\boxed{dF_{n_2} = 2 \cdot dF_{2,n} = \frac{\sigma \cdot ds_2 ds_1}{R_1}}$$

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Cohesion Forces and Surface Tension

Similarly for the force pulling in the other directions we may write:

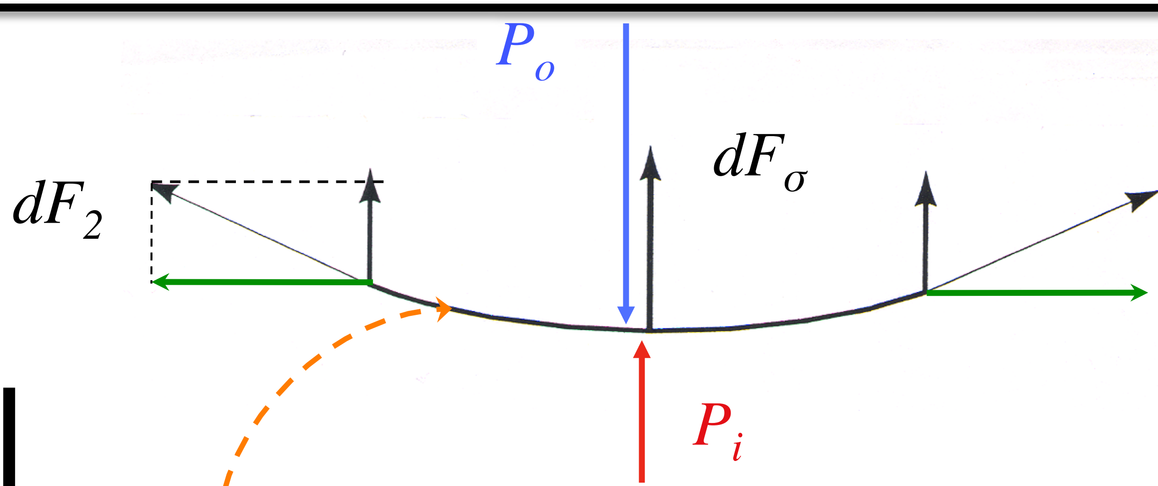
$$\boxed{dF_{n_1} = 2 \cdot dF_{1,n} = \frac{\sigma \cdot ds_1 ds_2}{R_2}}$$

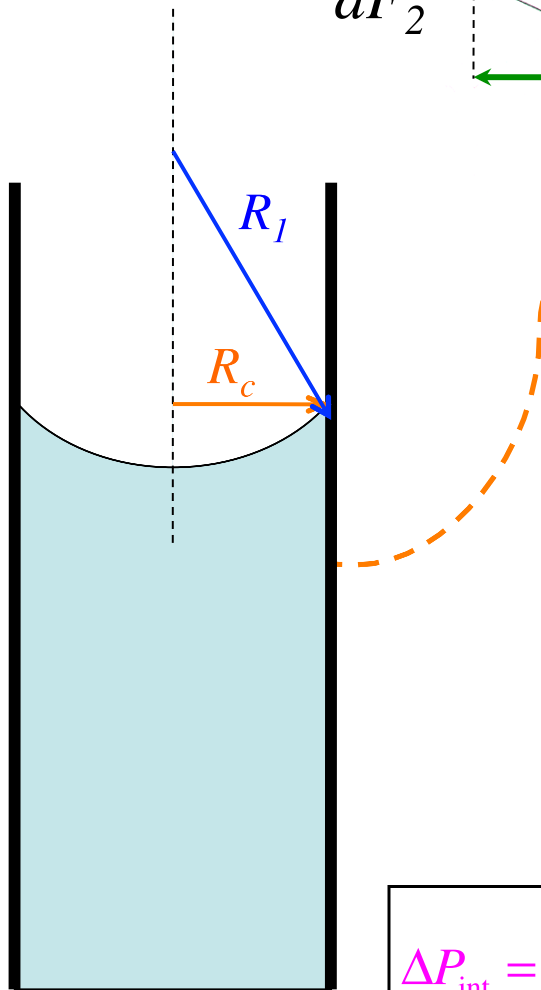
Finally, the total surface tension force acting on the differential surface area in the direction of the surface vector "*n*":

$$dF_\sigma = dF_{n_1} + dF_{n_2} = \frac{\sigma \cdot ds_1 ds_2}{R_2} + \frac{\sigma \cdot ds_2 ds_1}{R_1}$$

$$\boxed{dF_\sigma = \sigma \cdot ds_1 ds_2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right)}$$

Liquid Rise in a Capillary





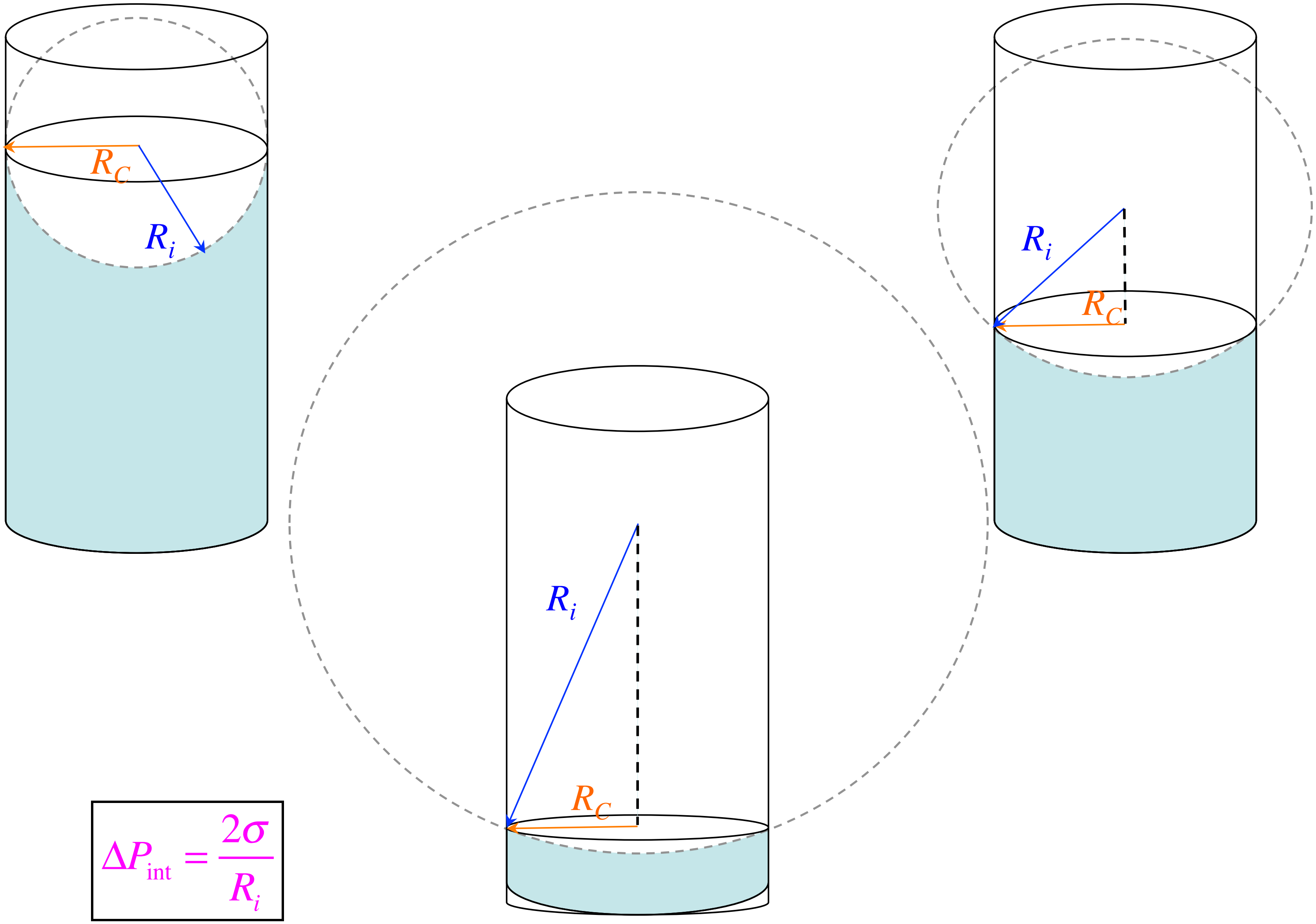
$$-P_o \cdot ds_1 ds_2 + dF_\sigma + P_i \cdot ds_1 ds_2 = 0$$

$$\left(P_o - P_i\right) ds_1 ds_2 = dF_\sigma$$

$$\left(P_o - P_i\right) \cancel{ds_1} \cancel{ds_2} = \sigma \left[\frac{1}{R_1} + \frac{1}{R_2}\right] \cancel{ds_1} \cancel{ds_2}$$

$$\boxed{\Delta P_{\text{int}} = \sigma \left[\frac{1}{R_1} + \frac{1}{R_2}\right]} \qquad if \quad R_1 = R_2 \quad \Rightarrow \quad \Delta P_{\text{int}} = \frac{2\sigma}{R_i}$$

Liquid Rise in a Capillary



$$\Delta P_{\text{int}} = \frac{2\sigma}{R_i}$$

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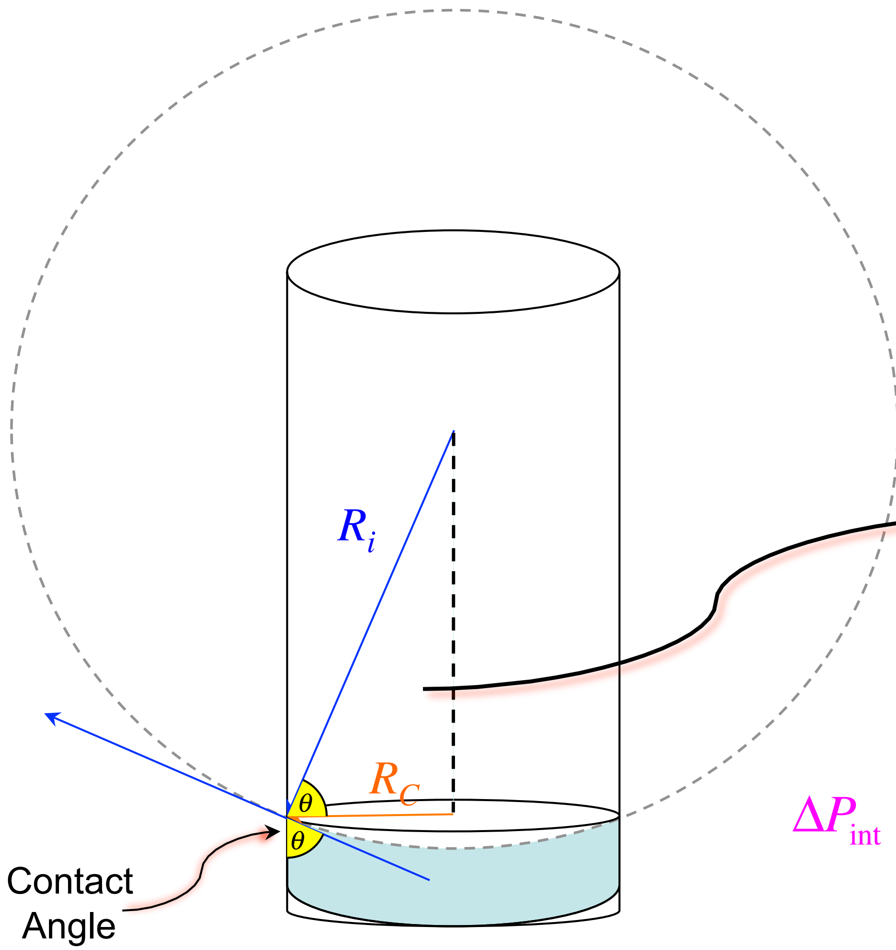
Liquid Rise in a Capillary



$$\frac{R_C}{R_i} = \cos(\theta)$$

$$R_i = \frac{R_C}{\cos(\theta)}$$

$$\Delta P_{\text{int}} = \frac{2\sigma}{R_i}$$

$$\Delta P_{\text{int}} = \frac{2\sigma}{R_C}\cos(\theta) = \frac{4\sigma}{D_C}\cos(\theta)$$

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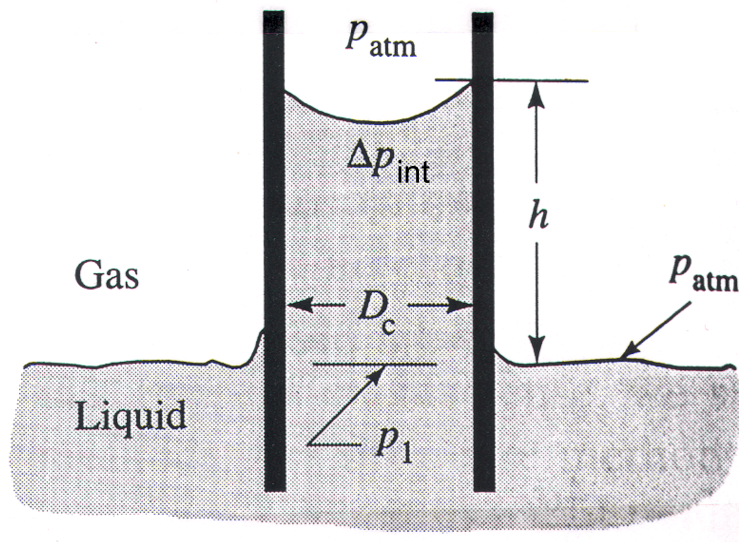
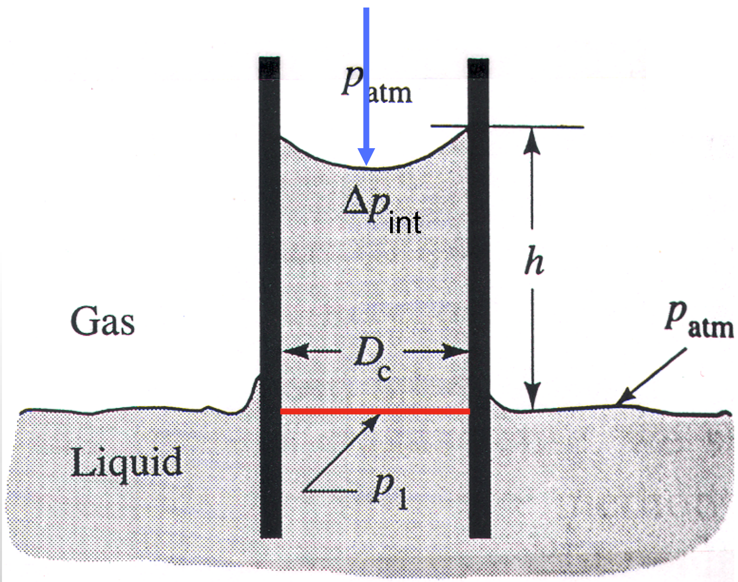
Liquid Rise in a Capillary

A narrow capillary tube is dipped into a liquid that wets the tube. We observe that the liquid rises in the tube, above the level of the free surface. We want to derive a mathematical model that relates the height of capillary rise 'h' to the surface tension 'σ' and the inner diameter of the capillary D_c. This model will suggest the method for measuring the surface tension.



Liquid Rise in a Capillary



$$P_{atm} - \Delta P_{int} + \rho g h = P_1 = P_{atm} \quad \Rightarrow \quad \Delta P_{int} = \rho g h$$

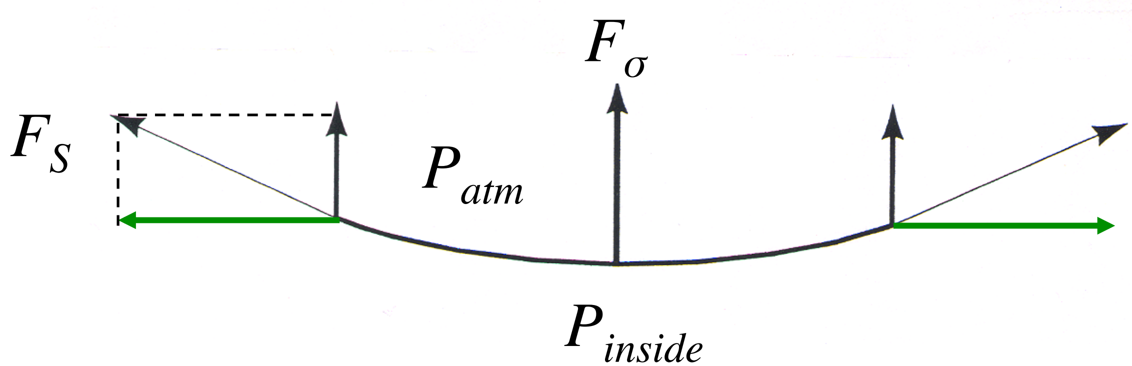
$$\sigma\left[\frac{1}{R_1}+\frac{1}{R_2}\right] = \rho g h \quad \text{and if} \quad R_1 = R_2$$

$$\frac{2\sigma}{R_1} = \frac{4\sigma}{D_c}\cos(\theta) = \rho g h \quad \Rightarrow \quad \boxed{h = \frac{4\sigma}{\rho g D_c}\cos(\theta)}$$

$$Also \quad \boxed{\sigma = \frac{h\rho g D_c}{4}\cos(\theta)}$$

If fluid wets capilary wall prefectly, ie., $\theta = 0$

$$\sigma = \frac{h\rho g D_c}{4}\cos(\theta) \quad \Rightarrow \quad \boxed{\sigma = \frac{h\rho g D_c}{4}}_{\theta=0}$$



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Thank you for your attention!