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# OREGON STATE UNIVERSITY CBEE
# DEPARTMENT OF CHEMICAL ENGINEERING

## CHE 331
## Transport Phenomena I


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# Viscosity

(pp 97-104 in Middleman Text)

**Please turn-off cell phones**

# Viscosity - Definition

$$\boxed{\mu (=) \left[\frac{kg}{m \cdot s}\right] \text{ or } [\mathrm{Pa \cdot s}]}$$

*Viscosity is a material property, related to the resistance a fluid exhibits to being deformed by the imposition of <u>stresses</u>.*

*Viscosity is a measure of stresses exerted by a fluid on a surrounding media when the fluid is undergoing <u>deformation</u>.*

*Viscosity is a material property that characterizes fluid ability to <u>transfer momentum</u> (mv).*

## Viscosity - Definition

There is a great similarity between mass, heat, and momentum transfer. These transport phenomena are all mathematically described in a similar way:

$$\Phi_{mass} = -D\frac{\partial C}{\partial x} \quad \left[\frac{mol}{m^2 s}\right] \qquad \text{Fick's Law}$$

$$\Phi_{heat} = -k\frac{\partial T}{\partial x} \quad \left[\frac{J}{m^2 s}\right] \qquad \text{Fourier's Law}$$

$$\Phi_{mom} = -\mu\frac{\partial u}{\partial x} \quad \left[\frac{\left(kg \times \frac{m}{s}\right)}{m^2 s}\right] \qquad \text{Newton's Law}$$

# Viscosity - Definition

The role of the coefficients in the three *Laws of Transport* is somewhat similar too:

$$D \; (=) \; \left[\frac{m^2}{s}\right]$$

molecular mass diffusivity

$$\alpha = \left(\frac{k}{\rho C_p}\right) \; (=) \; \left[\frac{m^2}{s}\right]$$

molecular thermal diffusivity

$$\nu = \left(\frac{\mu}{\rho}\right) \; (=) \; \left[\frac{m^2}{s}\right]$$

molecular momentum diffusivity

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# Viscosity - Physical Phenomena

If the viscosity is a material property that relates to the ability of materials to transfer momentum when exposed to deformation, we could pose a question;

***"What is a real physical phenomenon that 'creates/causes' viscosity"***

Predominant mechanisms of momentum transfer in liquid and gasses are somewhat different.

In liquids molecules collide, and hook-up to each other, thus transferring momentum. (Velcro effect)

In (ideal) gasses molecules collide into each other thus transferring momentum.

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# Viscosity – Of Liquids

Viscosity of liquids is strong function of temperature:

$$\boxed{\mu = A \times e^{\frac{B}{T}}} \quad where\ \ T\ \ is\ in\ \left[K\right].$$

It is challenging to predict the viscosity of liquids from characteristic molecular properties of materials. However, several models attempt to do precisely that: predict the viscosity of liquids. One of them is the method of Morris [The Properties of Gasses and Liquids, Reid, Prausnitz and Poling (McGraw-Hill 1987)]

$$\boxed{log\frac{\mu}{\mu^+} = J\left(\frac{1}{T_r} - 1\right)} \Leftrightarrow \ \mu = \mu^+ \times 10^{J\left(\frac{1}{T_r}-1\right)} \qquad T_r = \frac{T}{T_c}$$

*where*

$\mu^+ = pseudocritical\ \ viscosity;\quad J = structure\ \ function$

$T_r = reduced\ \ temperature;\quad T_c = critical\ \ temperature$

# Viscosity – Of Liquids Prediction

lculate the viscosity of liquid propane at -150[°C]; $T_c$=369.8[K]

## Pseudocritical Viscosity $\mu^+$ for Some Common Liquids

| Type of liquid | $\mu^+$ (Pa · s × 10$^{-5}$) |
|---|---|
| Hydrocarbons | 8.75 |
| Halogenated hydrocarbons | 14.8 |
| ~~Benzene derivatives~~ | 8.95 |
| Halogenated benzene derivatives | 12.3 |
| Alcohols | 8.19 |
| Organic acids | 11.7 |
| Ethers, ketones, aldehydes, and acetates | 9.6 |
| Phenols | 1.26 |

[a]Each table entry must be multiplied by 10$^{-5}$.

*Source:* Reid et al., 1987, Table 9.8.

$$\boxed{\mu = \mu^+ \times 10^{J\left(\frac{1}{T_r}-1\right)}}$$

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**Table** Group Contributions to $J$

| Group | $b_i$ | Group | $b_i$ |
|---|---|---|---|
| $CH_3$, $CH_2$, CH | 0.0825 | Additional H in ring | 0.1446 |
| Halogen-substituted $CH_3$ | 0 | $CH_3$, $CH_2$, CH adjoining ring | 0.0520 |
| Halogen-substituted $CH_2$ | 0.0893 | $NH_2$ adjoining ring | 0.7645 |
| Halogen-substituted CH | 0.0667 | F, Cl adjoining ring | 0 |
| Halogen-substituted C | 0 | OH for alcohols | 2.0446 |
| Br | 0.2058 | COOH for acids | 0.8896 |
| Cl | 0.147 | C=O for ketones | 0.3217 |
| F | 0.1344 | O=C—O for acetates | 0.4369 |
| I | 0.1908 | OH for phenols | 3.4420 |
| Double bond | −0.0742 | —O— for ethers | 0.1090 |
| $C_6H_4$ benzene ring | 0.3558 | | |

*Source:* Reid et al., 1987, Table 9.10.

$$J = \left[0.0577 + \sum_i b_i n_i\right]^{1/2} = \sqrt{0.0577 + 3 \times 0.0825} = 0.552$$

$$\mu = \mu^+ \times 10^{J\left(\frac{1}{T_r} - 1\right)} = 8.75 \times 10^{-5} \times 10^{0.552\left(\frac{1}{0.333} - 1\right)}$$

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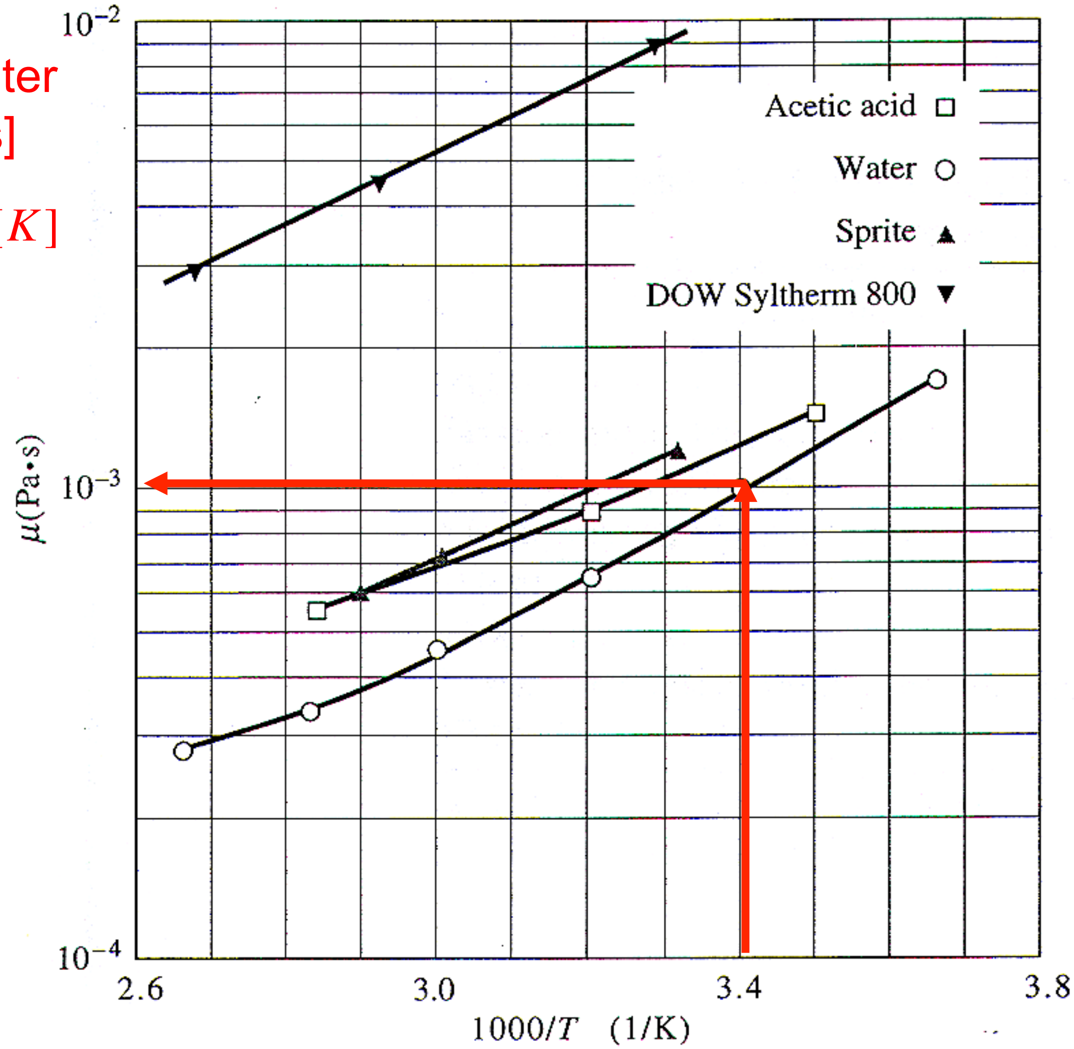
$$\mu_{propane} = 1.11 \times 10^{-3} [Pa \cdot s]; \quad also \quad T_r = \frac{T}{T_c} = \frac{273 - 150}{369.8} = 0.333$$

Calculate viscosity of water at 20[°C] μ=0.001 [Pas]

$$t = 20\ [^oC] \quad \Leftrightarrow \quad T = 293\ [K]$$

$$\frac{1000}{T} = \frac{1000}{293} = 3.41$$



(a)

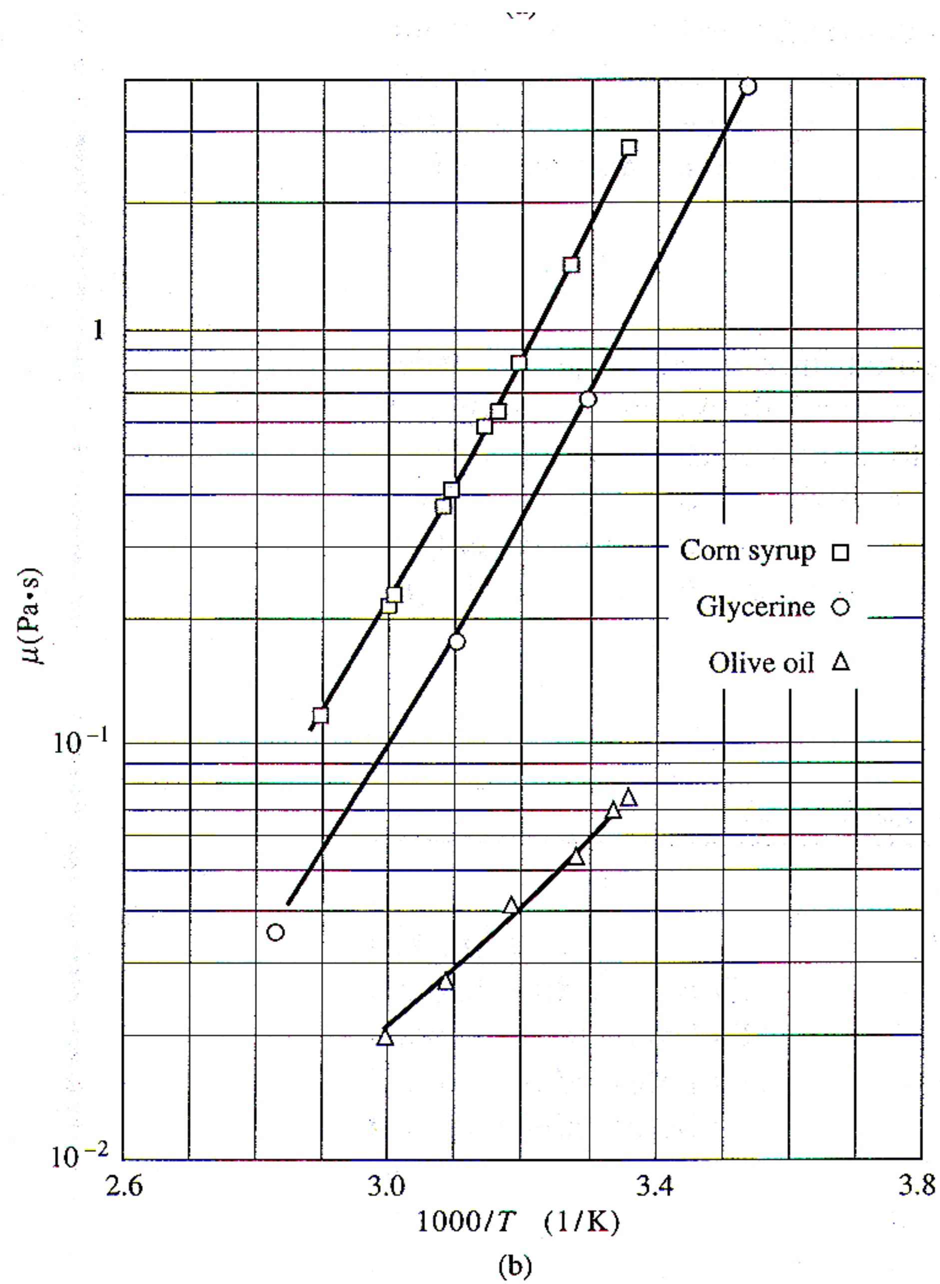



**Figure** Data for viscosity of several liquids at $P = 1$ atm as a function of temperature: (a) low viscosity liquids and (b) high viscosity liquids.

# Viscosity - Of Gases

For pure gasses away from critical pressure the *Kinetic Theory of Gasses* provides accurate method for predicting viscosities. One of the most popular method for predicting viscosities of pure gasses is based on the *Chapman-Enskog Equation:*

$$\boxed{\mu = 2.67\times10^{-6}\,\frac{\sqrt{(MW)\times T}}{\sigma^2\Omega_V}}\quad [Pa\cdot s]$$

where MW= molecular weight

$T$ = absolute temperature

$\sigma$ = collision diamter

$\Omega_V$ = collision integral

$\varepsilon$ = characteristic energy

$k$ = Boltzman const. $\left(1.38\cdot10^{-23}\right)\left[\frac{N\cdot m}{K}\right]$

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**Table** Parameters for the Chapman–Enskog Equation and $T_c$ Values

| Compound | | $\sigma$ (Å) | $\varepsilon/k$ (K) | $M$ | $T_c$(K) |
|---|---|---|---|---|---|
| Air | | 3.71 | 78.6 | 29 | 132 |
| Carbon tetrachloride | $CCl_4$ | 5.95 | 323 | 154 | 556 |
| Chloroform | $CHCl_3$ | 5.39 | 340 | 119 | 536 |
| Methanol | $CH_3OH$ | 3.63 | 482 | 32 | 512.6 |
| Methane | $CH_4$ | 3.76 | 149 | 16 | 191 |
| Carbon monoxide | $CO$ | 3.69 | 91.7 | 28 | 133 |
| Carbon dioxide | $CO_2$ | 3.94 | 195 | 44 | 304 |
| Ethane | $C_2H_6$ | 4.44 | 216 | 30 | 305 |
| Propane | $C_3H_8$ | 5.12 | 237 | 44 | 370 |
| *n*-Butane | $C_4H_{10}$ | 4.69 | 531 | 58 | 408 |
| Acetone | $CH_3COCH_3$ | 4.6 | 560 | 58 | 508 |
| Benzene | $C_6H_6$ | 5.35 | 412 | 78 | 562 |
| Hydrogen | $H_2$ | 2.83 | 59.7 | 2 | 33.2 |
| Water | $H_2O$ | 2.64 | 809 | 18 | 647 |
| Ammonia | $NH_3$ | 2.9 | 558 | 17 | 405.6 |
| Nitrogen | $N_2$ | 3.8 | 71.4 | 28 | 126.2 |
| Oxygen | $O_2$ | 3.47 | 107 | 32 | 154.6 |
| Silane | $SiH_4$ | 4.08 | 208 | 18 | 269.5 |
| Helium | $He$ | 2.58 | 10.2 | 4 | 5.2 |

$$T^* = \frac{k \cdot T}{\varepsilon} = \frac{T}{(\varepsilon / k)} = \frac{(220+273)}{558} = 0.884$$

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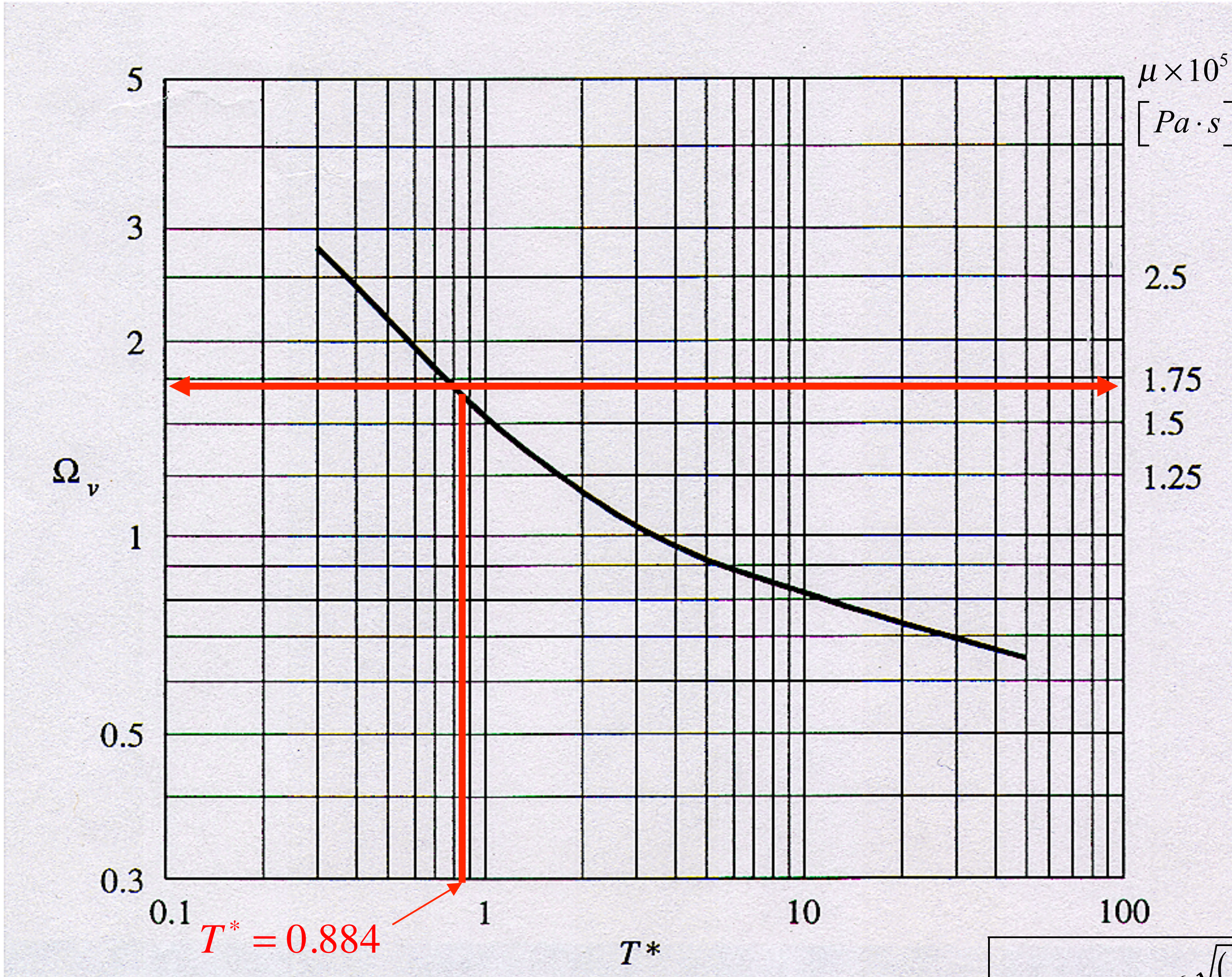


$$\mu = 2.67 \times 10^{-6} \frac{\sqrt{(MW) \times T}}{\sigma^2 \Omega_V}$$

$$\mu = 2.67 \times 10^{-6} \frac{\sqrt{(17) \times 493}}{2.9^2 \times 1.7} = 1.71 \times 10^{-5} \, [Pa \cdot s]$$

# Viscosity – Of Gases

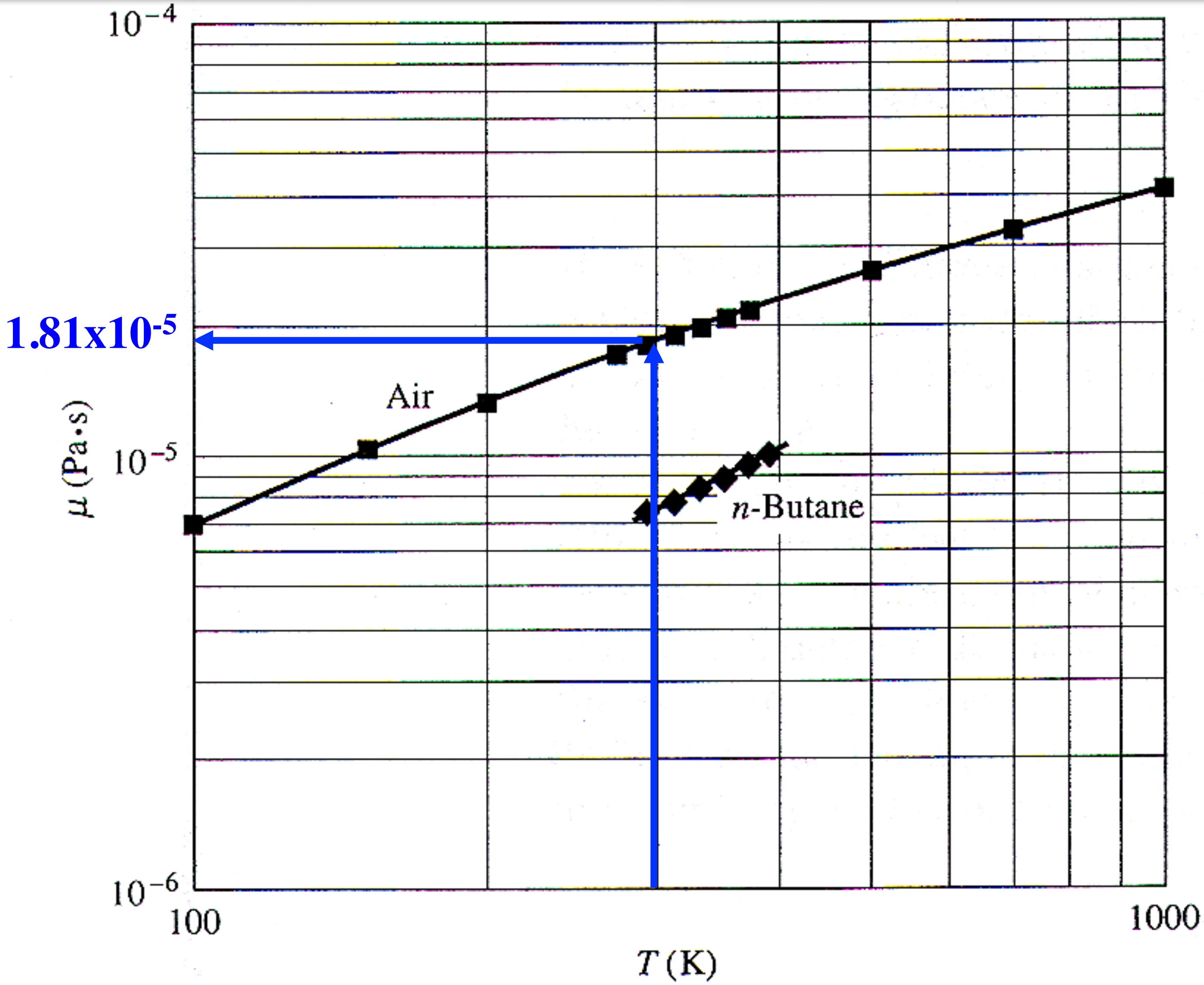


**Figure** Gas viscosity as a function of temperature, at $P$ = 1 atm.

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*People. Ideas. Innovation.*

*Thank you for your attention!*