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OREGON STATE UNIVERSITY
School of Chemical, Biological, and Environmental Engineering

CHE 331
Transport Phenomena I

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Elements of Turbulent Flow

Please turn-off cell phones

CHE 331
Fall 2022

Introduction

Turbulent fluid motion is an irregular condition of flow in which the various quantities show a random variation with time and space coordinates so that statistically distinct average values can be discerned.

A thorough understanding of turbulence has not been achieved to date and may never be attained in the foreseeable future.

As nice as solutions for laminar flow are, turbulence is the more normal state of affairs in fluid flow.

“I am an old man now, and when I die and go to heaven, there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of a fluid. About the former, I am rather optimistic.” Horace Lamb 1932

Introduction - What is Turbulence

In experiments in fluid systems at flow conditions below the so-called critical Reynolds number Re_{crit} the flow is smooth and adjacent layers of fluid slide past each other in an orderly fashion. This regime is called *laminar flow*.

At values of the Reynolds number above a Re_{crit} radical flow change occurs through a complicated chain of events. The motion of fluid becomes *intrinsically unsteady* even with constant, imposed boundary conditions. The flow state is chaotic, i.e., the velocity and all other flow properties vary randomly. This regime is called *turbulent flow*.

These changes occur "in line" with the universal principle of *minimization of potential energy* and the ever-growing magnitude of *flow instabilities*.

Where Re number comes from?

$$\rho\left[\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}\right]=-\frac{\partial P}{\partial x}+\mu\left[\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right]+\rho g_x$$

One could introduce dimensionless variables; i.e., normalize the above equation;

$$\tilde{u}_x=\frac{u_x}{U};\quad \tilde{u}_y=\frac{u_y}{U};\quad \tilde{u}_z=\frac{u_z}{U}\quad and\quad \tilde{x}=\frac{x}{D};\quad \tilde{y}=\frac{y}{D};\quad \tilde{z}=\frac{z}{D};$$

$$\tilde{t}=t\left(\frac{U}{D}\right)\quad\Rightarrow\quad \partial t=\partial\tilde{t}\left(\frac{D}{U}\right);\quad say\quad g_x=0$$

$$\boxed{\mathrm{Re}=\frac{UD}{(\mu/\rho)}}$$

and obtain:

$$\frac{\rho UD}{\mu}\left[\frac{\partial\tilde{u}_x}{\partial\tilde{t}}+\tilde{u}_x\frac{\partial\tilde{u}_x}{\partial\tilde{x}}+\tilde{u}_y\frac{\partial\tilde{u}_x}{\partial\tilde{y}}+\tilde{u}_z\frac{\partial\tilde{u}_x}{\partial\tilde{z}}\right]=-\frac{\partial P}{\partial\tilde{x}}\left(\frac{D}{\mu U}\right)+\left[\frac{\partial^2\tilde{u}_x}{\partial\tilde{x}^2}+\frac{\partial^2\tilde{u}_x}{\partial\tilde{y}^2}+\frac{\partial^2\tilde{u}_x}{\partial\tilde{z}^2}\right]$$

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Introduction

At low Reynolds numbers, viscous effects dominate, and will *dampen* out any small perturbations in the flow; i.e.,

$$\frac{\mu}{\rho} >> UD$$

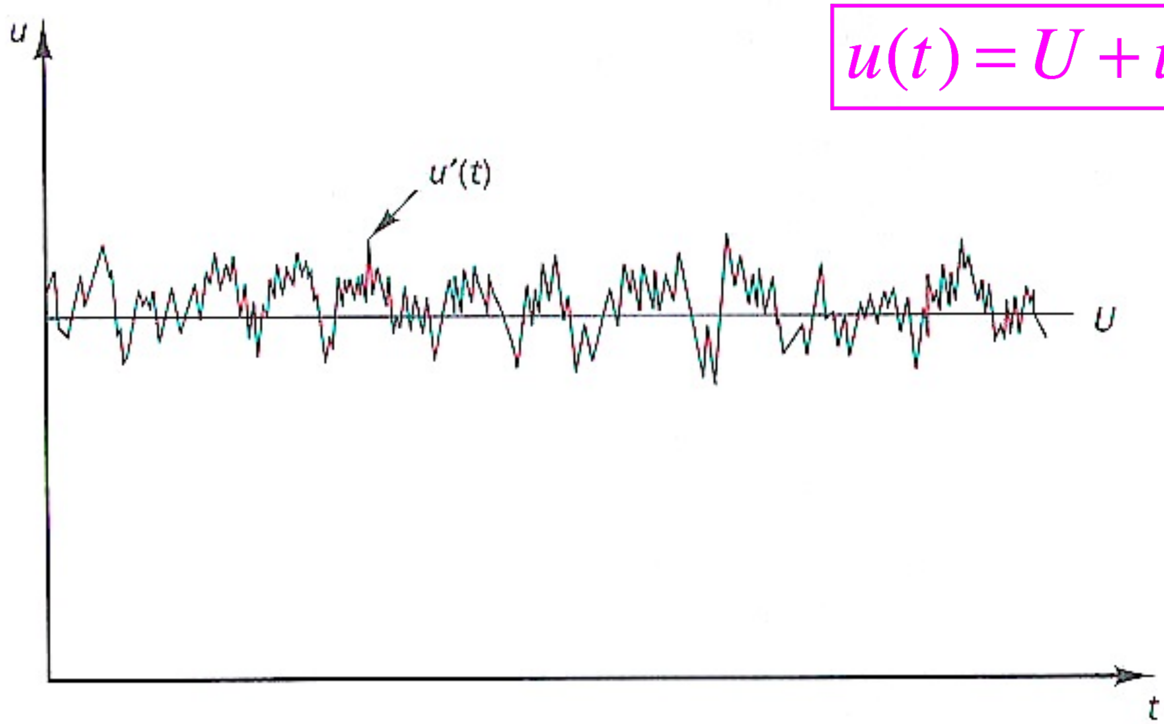
But at higher Reynolds numbers this is no longer the case, and the predominance of the nonlinear inertial terms in the Navier-Stokes equations will lead to *amplification* of small perturbations.

$$\frac{\mu}{\rho} << UD$$

The consequence is that in addition to to the general trend of flow there will be superimposed *eddies*. Eddies are regions that are typically rotating, and thus have vorticity, and behaving as little whirlpools.

Introduction - What is Turbulence

A typical point velocity measurement (of velocity component ‘u’) exhibits the form shown below:



$$\boxed{u(t) = U + u'(t)}$$

A random nature of turbulent flow makes it practically impossible to represent deterministically the motion of all fluid particles. Instead the velocity shown above is ***decomposed*** into a steady mean value ***U*** with fluctuating component ***u’(t)***, superimposed on it:

Introduction - What is Turbulence

The key component in this consideration is that the time average value of the fluctuating component is defined as:

$$\overline{u'} = \frac{1}{\Delta t}\int_0^{\Delta t} u'(t) \equiv 0$$

Thus one can write::

$$\overline{u(t)} = \overline{[U + u'(t)]} = \frac{1}{\Delta t}\int_0^{\Delta t} u(t)dt = \frac{1}{\Delta t}\int_0^{\Delta t} \left(U + u'(t)\right)dt =$$

$$= \frac{1}{\Delta t}\int_0^{\Delta t} U\,dt + \underbrace{\frac{1}{\Delta t}\int_0^{\Delta t} u'(t)\,dt}_{0} = U$$

Reynolds-averaged Navier-Stokes Eq. For incompressible flow.

Consider Continuity and Navier-Stokes equations in Cartesian coordinate system;

$$div\ \mathbf{u} = 0$$

$$\frac{\partial u}{\partial t} + div(u\mathbf{u}) = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho} div(grad(u))$$

$$\frac{\partial v}{\partial t} + div(v\mathbf{u}) = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho} div(grad(v))$$

$$\frac{\partial w}{\partial t} + div(w\mathbf{u}) = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho} div(grad(w))$$

Reynolds-averaged Navier-Stokes Eq. For incompressible flow.

Consider Continuity and Navier-Stokes equations in Cartesian coordinate system;

$$div\left(\mathbf{U}+\mathbf{u}'\right)=0$$

$$\frac{\partial(U+u')}{\partial t}+div\left((U+u')\mathbf{u}\right)=-\frac{1}{\rho}\frac{\partial(P+p')}{\partial x}+\frac{\mu}{\rho}div\left(grad\left(U+u'\right)\right)$$

$$\frac{\partial(V+v')}{\partial t}+div\left((V+v')\mathbf{u}\right)=-\frac{1}{\rho}\frac{\partial(P+p')}{\partial y}+\frac{\mu}{\rho}div\left(grad\left(V+v'\right)\right)$$

$$\frac{\partial(W+w')}{\partial t}+div\left((W+w')\mathbf{u}\right)=-\frac{1}{\rho}\frac{\partial(P+p')}{\partial z}+\frac{\mu}{\rho}div\left(grad\left(W+w'\right)\right)$$

Reynolds-averaged Navier-Stokes Eq. For incompressible flow.

Take the average value of each term:

$$div\left(\overline{\mathbf{U}+\mathbf{u}'}\right)=0$$

$$\frac{\partial\left(\overline{U+u'}\right)}{\partial t}+div\left(\overline{\left(U+u'\right)\mathbf{u}}\right)=-\frac{1}{\rho}\frac{\partial\left(\overline{P+p'}\right)}{\partial x}+\frac{\mu}{\rho}div\left(grad\left(\overline{U+u'}\right)\right)$$

$$\frac{\partial\left(\overline{V+v'}\right)}{\partial t}+div\left(\overline{\left(V+v'\right)\mathbf{u}}\right)=-\frac{1}{\rho}\frac{\partial\left(\overline{P+p'}\right)}{\partial y}+\frac{\mu}{\rho}div\left(grad\left(\overline{V+v'}\right)\right)$$

$$\frac{\partial\left(\overline{W+w'}\right)}{\partial t}+div\left(\overline{\left(W+w'\right)\mathbf{u}}\right)=-\frac{1}{\rho}\frac{\partial\left(\overline{P+p'}\right)}{\partial z}+\frac{\mu}{\rho}div\left(grad\left(\overline{W+w'}\right)\right)$$

Reynolds-averaged Navier-Stokes Eq. For incompressible flow.

And obtain:

$$div\left(\mathbf{U}\right)=0$$

$$\frac{\partial(U)}{\partial t}+div(U\mathbf{U})+div\left(\overline{u'\mathbf{u}'}\right)=-\frac{1}{\rho}\frac{\partial(P)}{\partial x}+\frac{\mu}{\rho}div\left(grad(U)\right)$$

$$\frac{\partial(V)}{\partial t}+div(V\mathbf{U})+div\left(\overline{v'\mathbf{u}'}\right)=-\frac{1}{\rho}\frac{\partial(P)}{\partial y}+\frac{\mu}{\rho}div\left(grad(V)\right)$$

$$\frac{\partial(W)}{\partial t}+div(W\mathbf{U})+div\left(\overline{w'\mathbf{u}'}\right)=-\frac{1}{\rho}\frac{\partial(P)}{\partial z}+\frac{\mu}{\rho}div\left(grad(W)\right)$$

where $\mathbf{U}=\mathbf{U}(U,V,W)$

Reynolds-averaged Navier-Stokes Eq. For incompressible flow.

We can rewrite the above equations into a more familiar form:

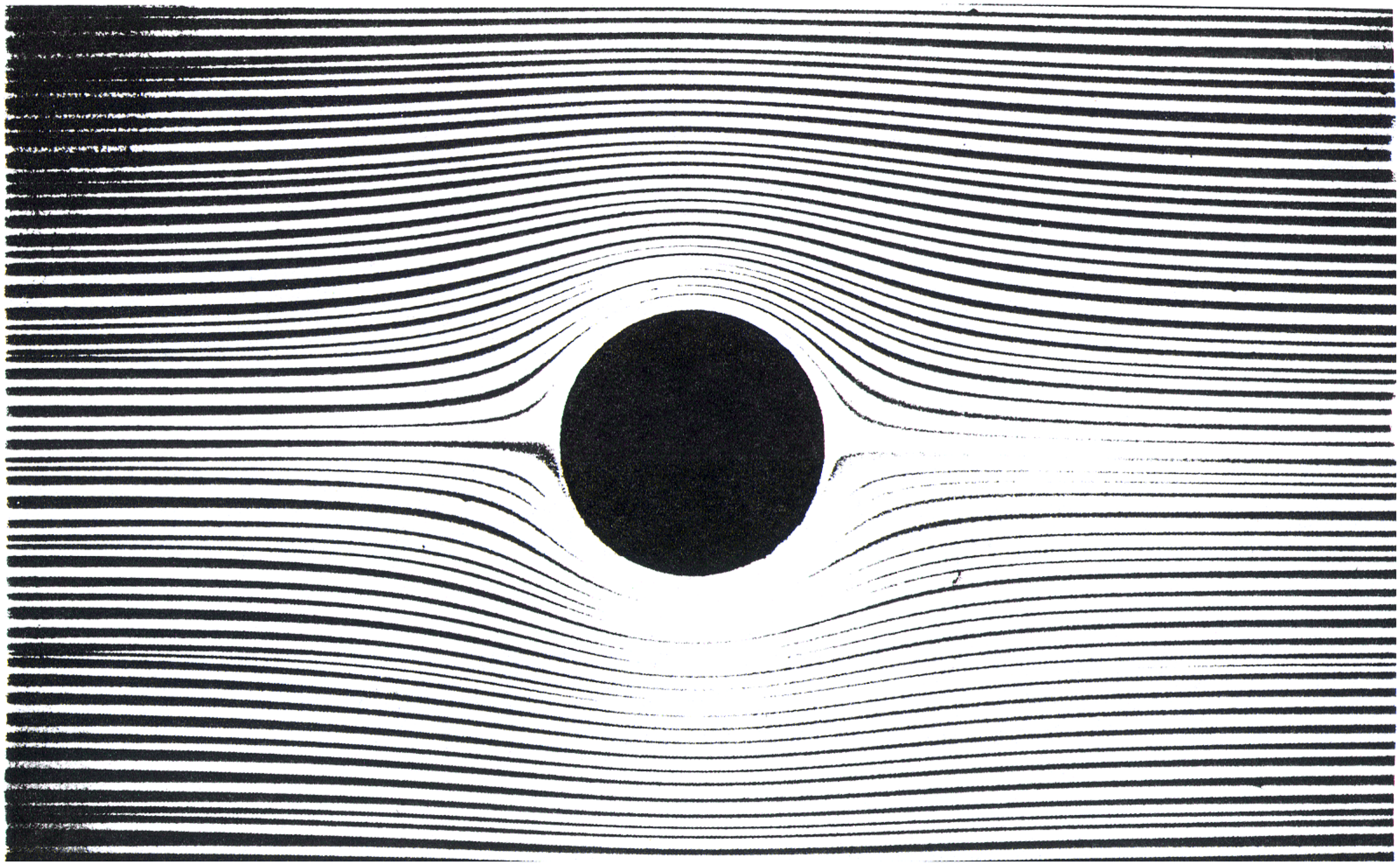
$$\frac{\partial(U)}{\partial t}+div(U\mathbf{U})=-\frac{1}{\rho}\frac{\partial(P)}{\partial x}+\frac{\mu}{\rho}div(grad(U))-\frac{1}{\rho}\left[\frac{\partial\left(\rho\overline{u'^2}\right)}{\partial x}+\frac{\partial\left(\rho\overline{u'v'}\right)}{\partial y}+\frac{\partial\left(\rho\overline{u'w'}\right)}{\partial z}\right]$$

$$\frac{\partial(V)}{\partial t}+div(V\mathbf{U})=-\frac{1}{\rho}\frac{\partial(P)}{\partial y}+\frac{\mu}{\rho}div(grad(V))-\frac{1}{\rho}\left[\frac{\partial\left(\rho\overline{v'u'}\right)}{\partial x}+\frac{\partial\left(\rho\overline{v'^2}\right)}{\partial y}+\frac{\partial\left(\rho\overline{v'w'}\right)}{\partial z}\right]$$

$$\frac{\partial(W)}{\partial t}+div(W\mathbf{U})=-\frac{1}{\rho}\frac{\partial(P)}{\partial z}+\frac{\mu}{\rho}div(grad(W))-\frac{1}{\rho}\left[\frac{\partial\left(\rho\overline{w'u'}\right)}{\partial x}+\frac{\partial\left(\rho\overline{w'v'}\right)}{\partial y}+\frac{\partial\left(\rho\overline{w'^2}\right)}{\partial z}\right]$$

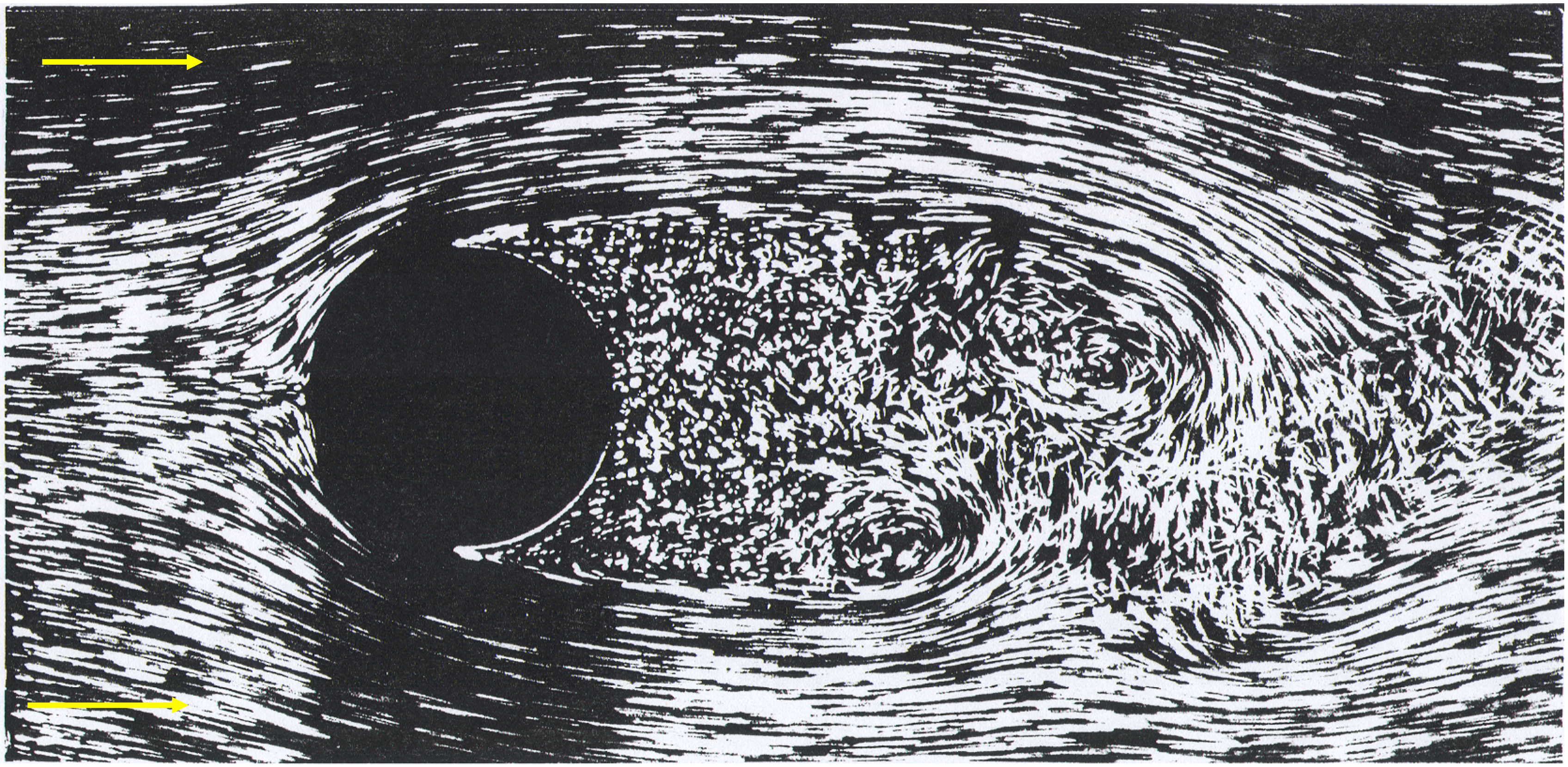
The extra stress terms have been written out in a longhand for clarity.

Creeping Laminar Flow Pass Cylinder



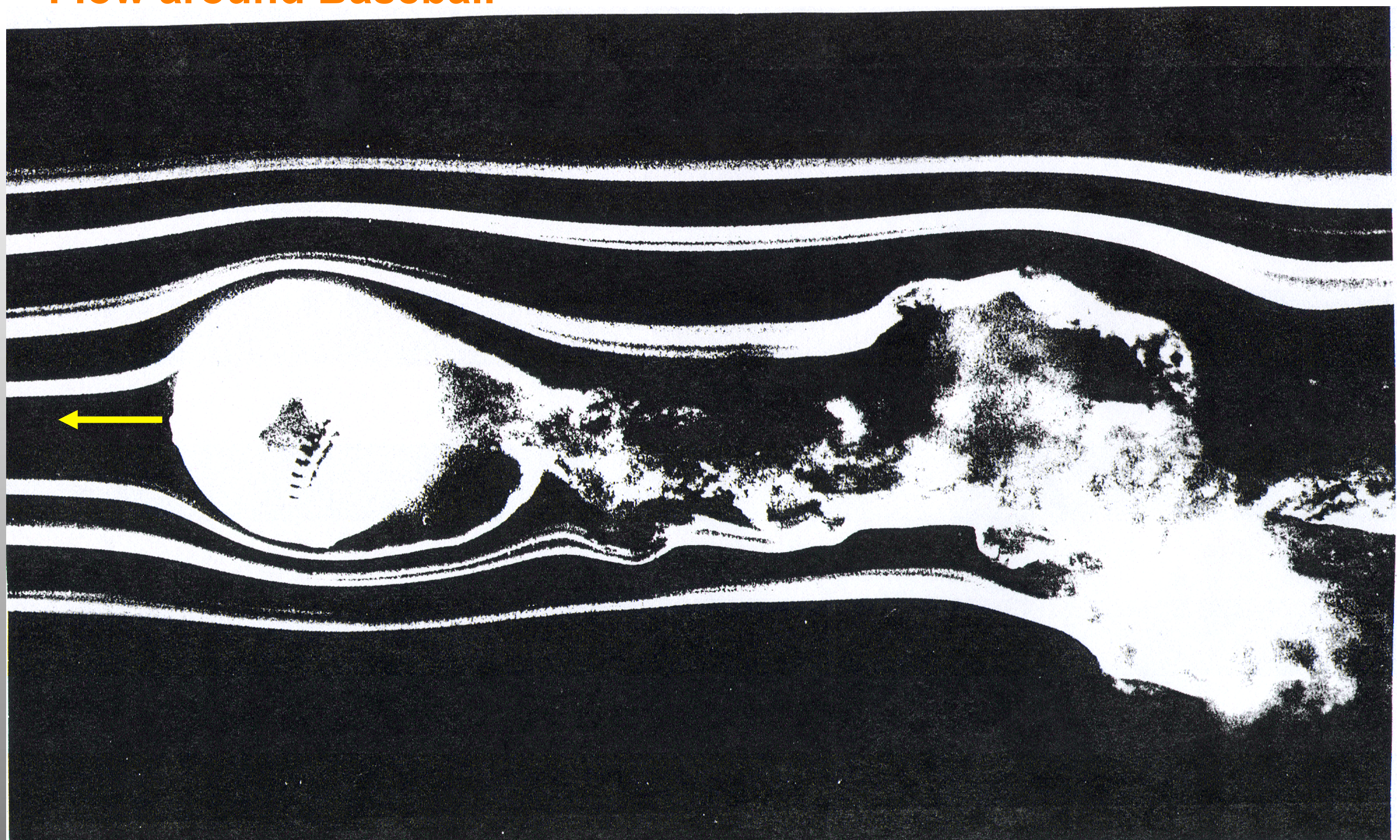
Turbulent Flow Around Cylinder

Flow around Cylinder



Turbulent Flow

Flow around Baseball



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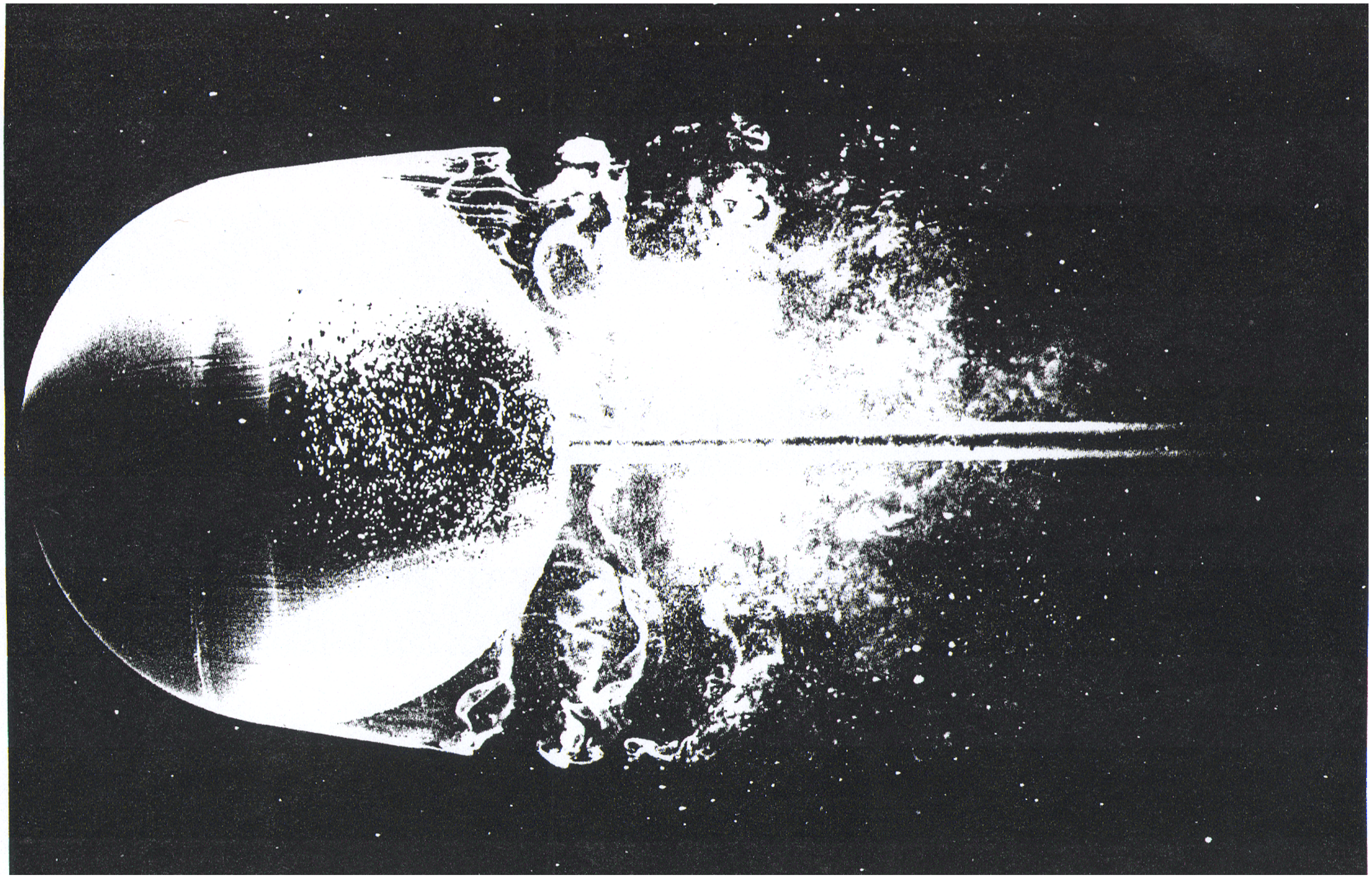
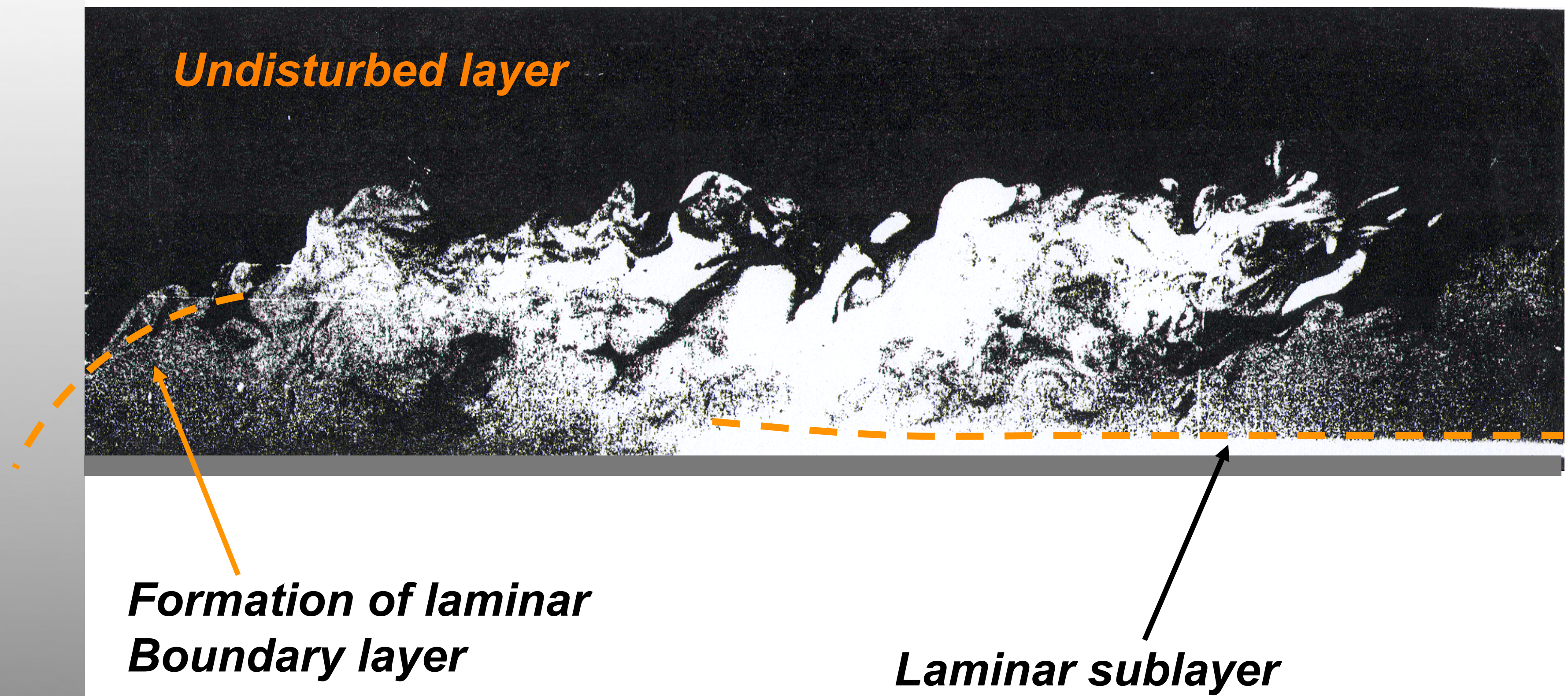
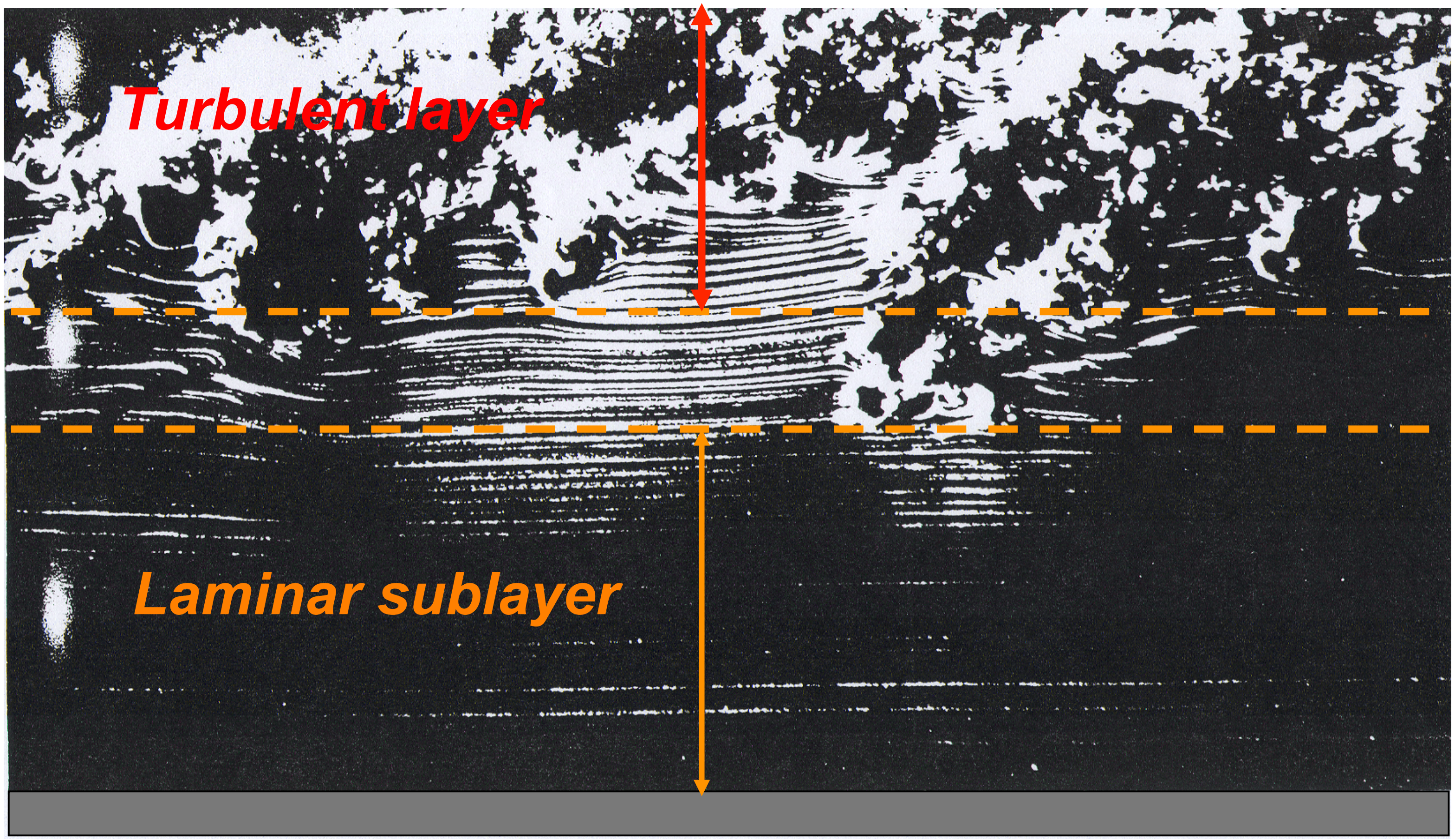
Turbulent Flow Pass Sphere



Turbulent Flow Over Flat Plate-Boundary Layer

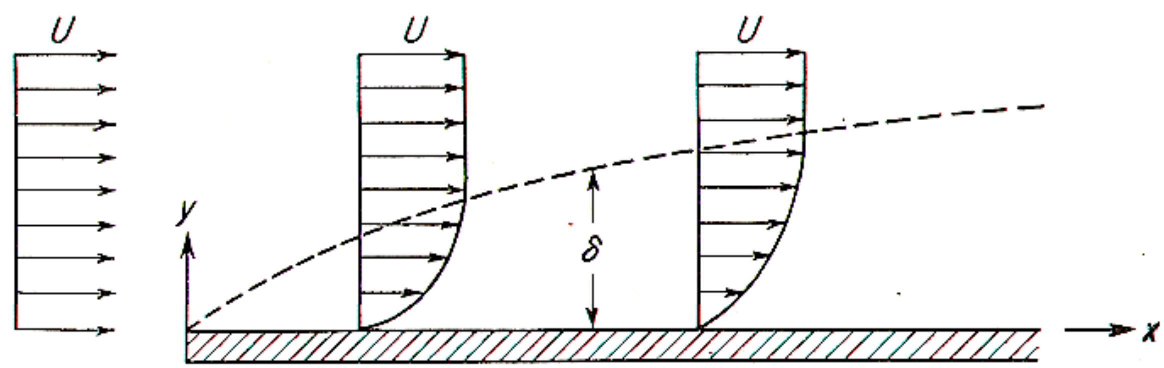


Turbulent Flow Over Flat Plate - Boundary Layer

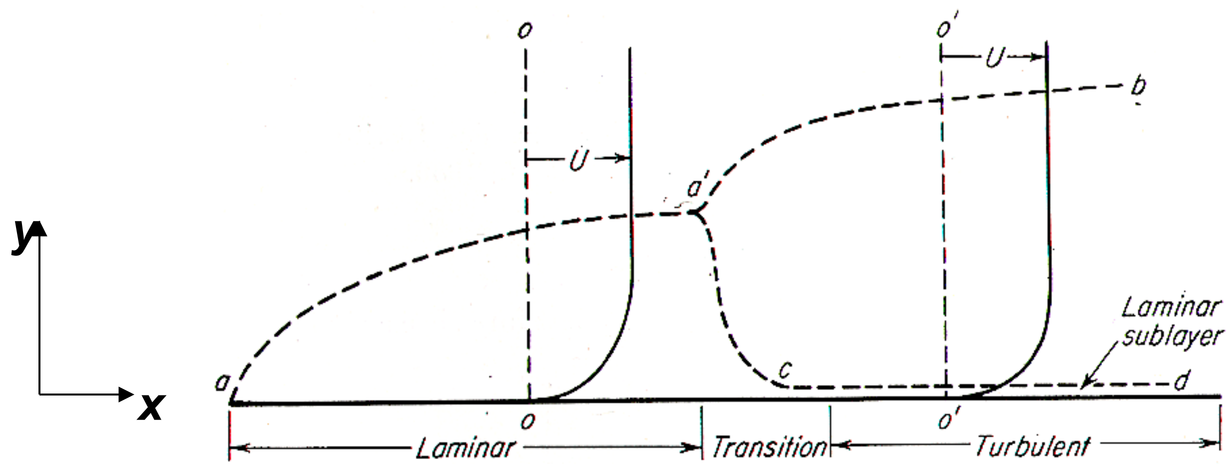


Boundary Layer on Immersed Bodies

In laminar flow the boundary layer over the flat plate has relatively simple "architecture":



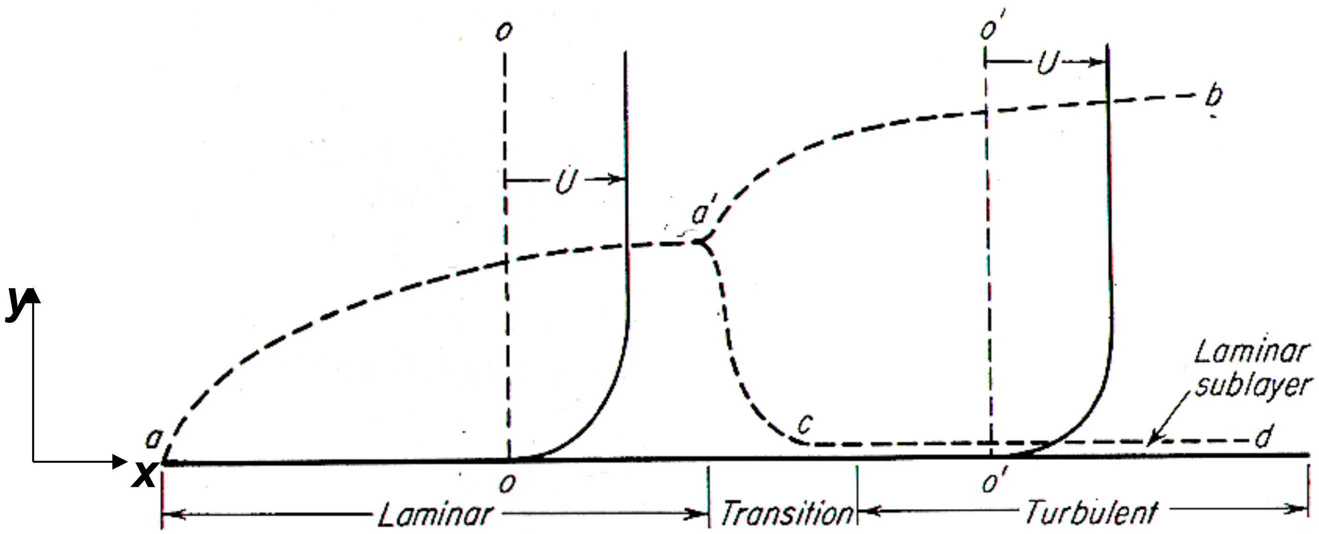
In turbulent flow the "architecture" of the boundary layer is quite different, and perhaps more complex.



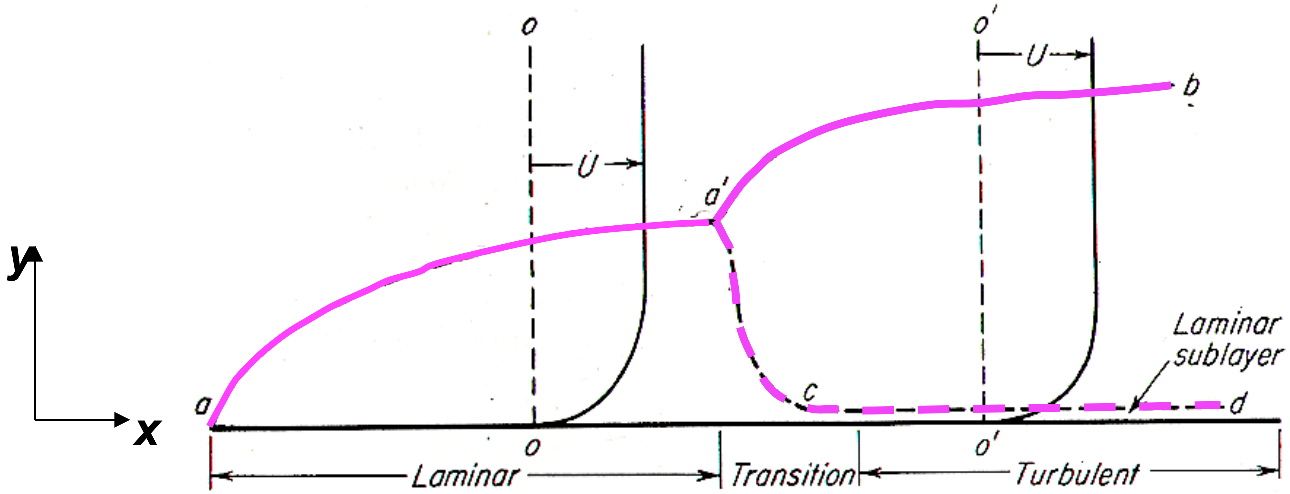
Boundary Layer on Immersed Bodies

The undisturbed bulk flow has a velocity *U in x direction only (we are considering two dimensional flow only)*, although the nature of the turbulence is three dimensional.

The fluid next to the plate surface has zero velocity. Initially, closer to the leading edge, the boundary layer is laminar. As the boundary layer increases in thickness, it becomes unstable, and flow becomes turbulent. At a certain distance along the plate the laminar zone collapses into a boundary layer that has a ***laminar sub-layer*** and a turbulent part above the laminar sub-layer.



Turbulent Boundary Layer on Flat Plate



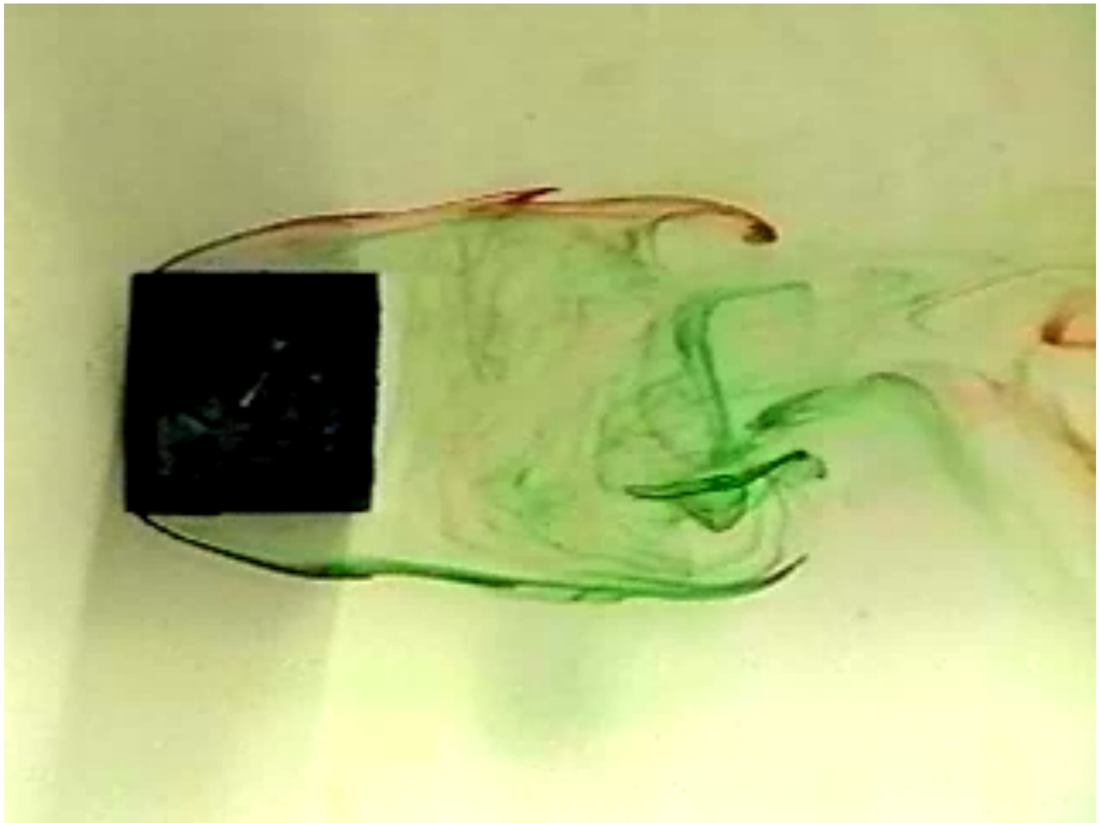
Line ***a-a'*** represents the outer edge of the laminar boundary layer;

Line ***a'-b*** represents the outer edge of the turbulent boundary layer;

The laminar sub-layer exist adjacent to the wall for the turbulent boundary layer, so laminar flow always exists between line ***a-a'-c-d*** and the surface of the plate.

Turbulence Introduction

To illustrate the concept of the scale of turbulence one can look at experiments in hydraulic channel:



Turbulence Introduction

Size of Eddy	Eddy Energy	Energy Source	Fate of Eddy
Large	High	From Pressure; Energy of Flow	Decomposes Into Medium Eddies
Medium	Medium	From Larger Eddies	Decomposes Into Small Eddies
Small	Low	From Medium Eddies	Is Annihilated by Viscous Action

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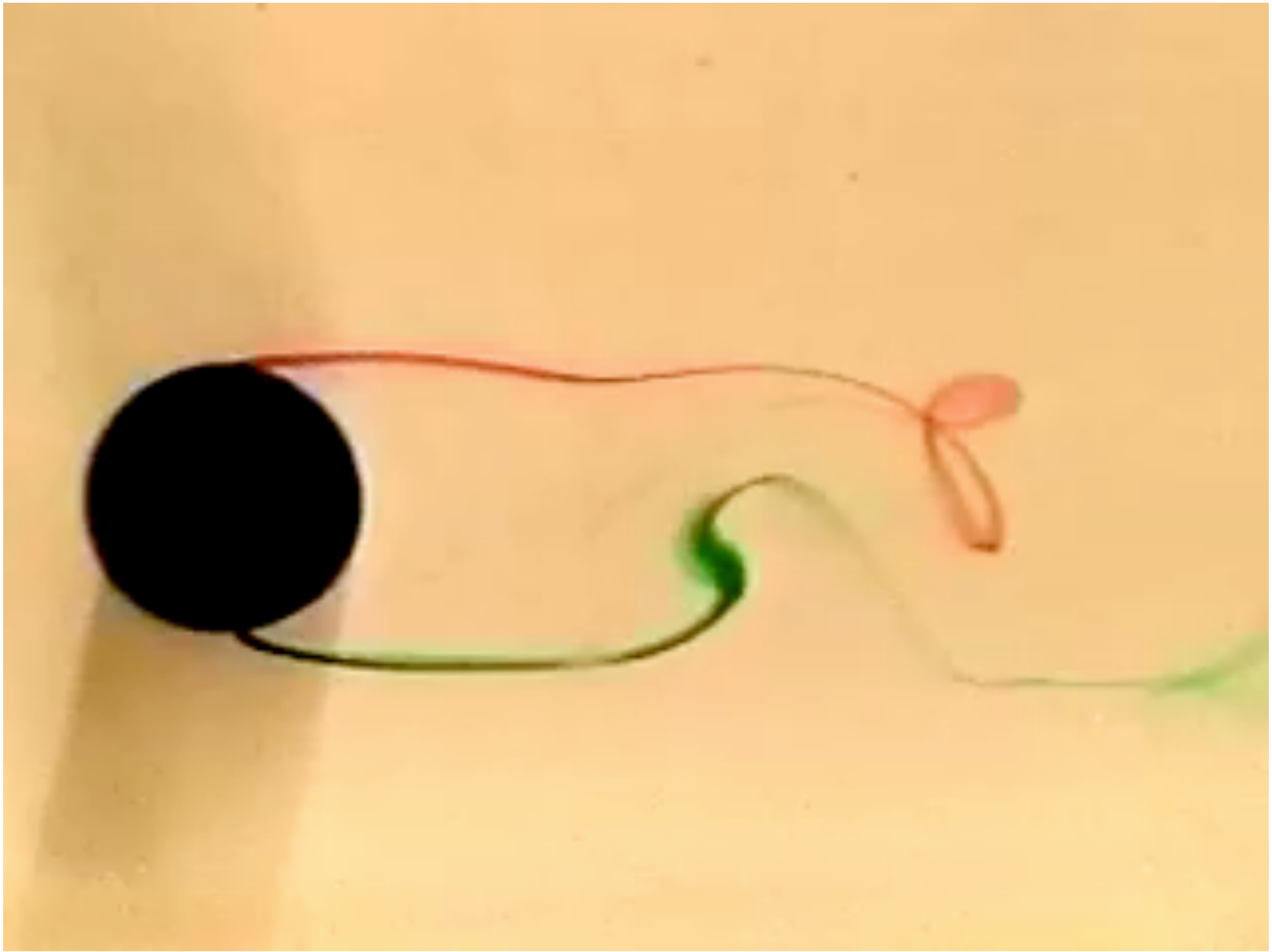

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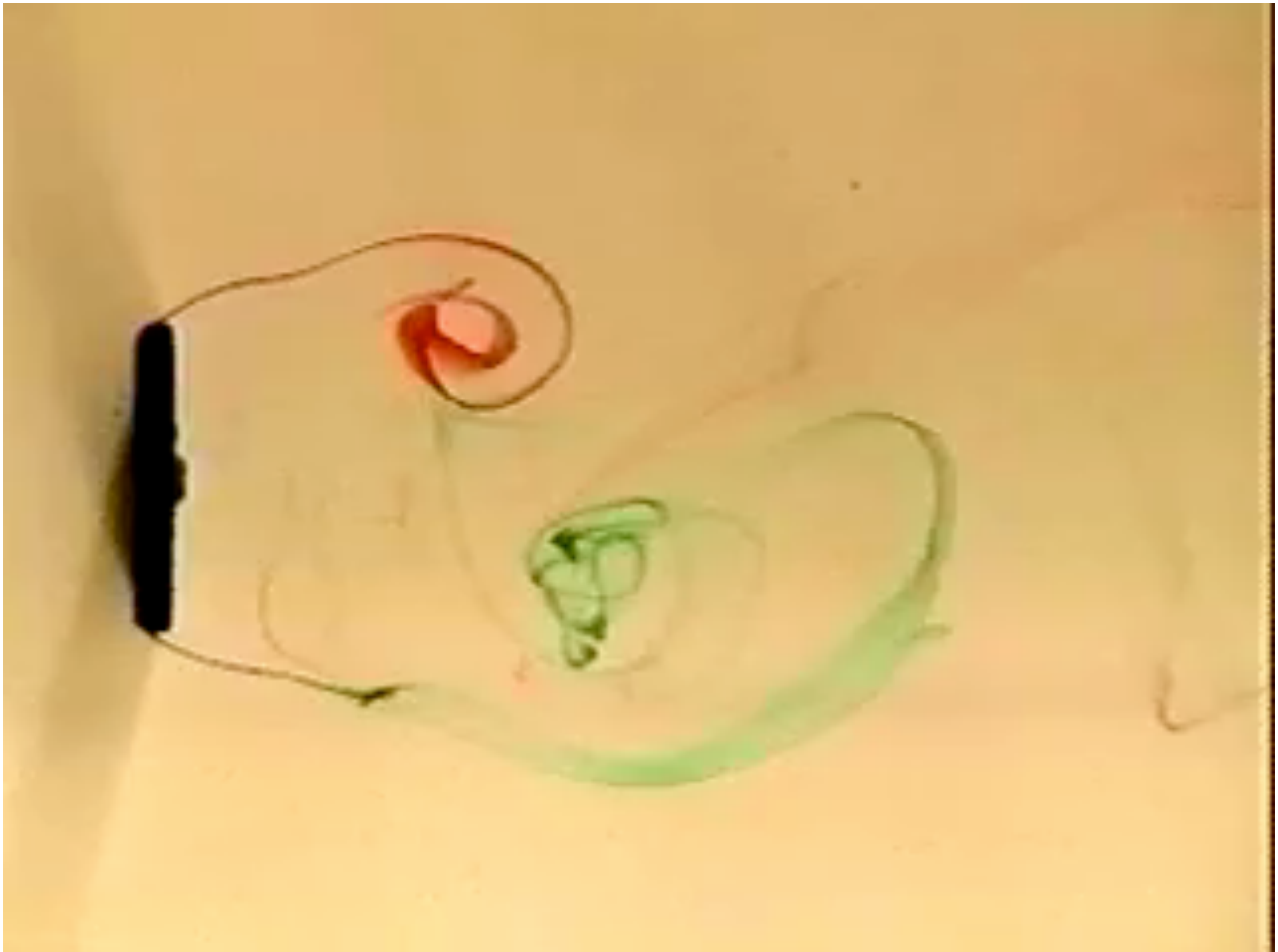

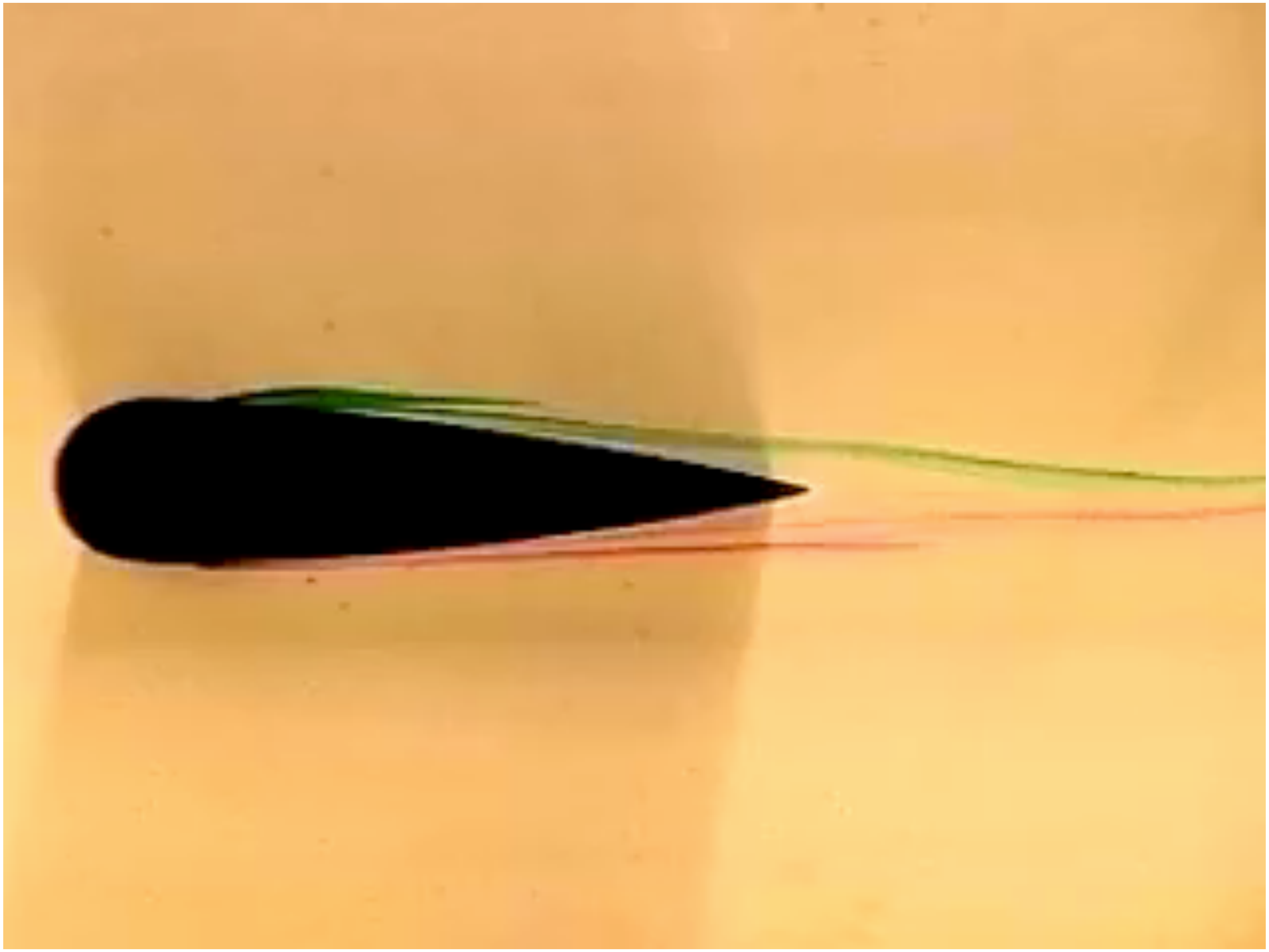

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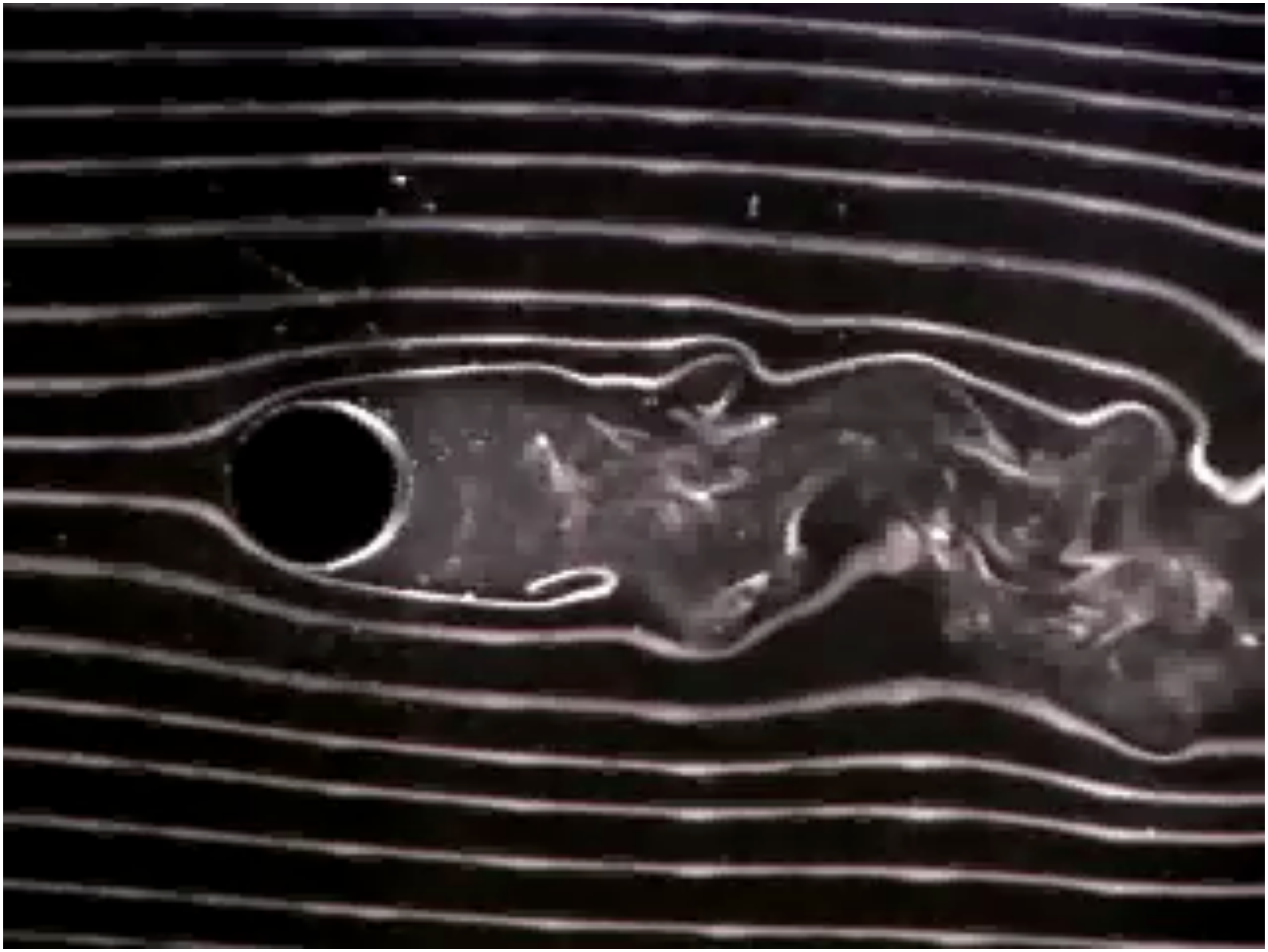

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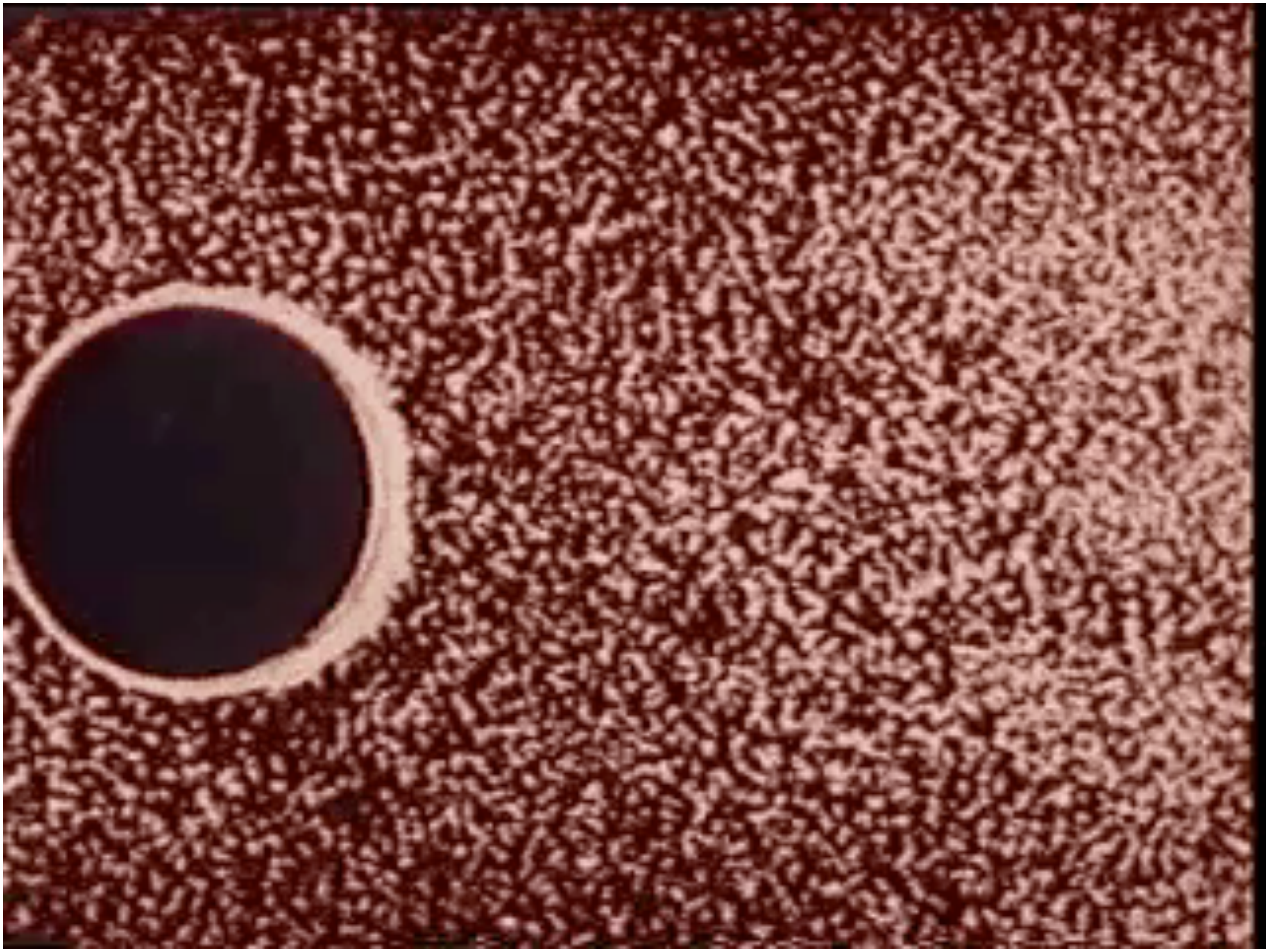

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Thank you for your attention!