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OREGON STATE UNIVERSITY
CBEE Department of Chemical Engineering

CHE 331
Transport Phenomena1

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Mechanical Energy Balance Equation for flow through Packed Beds

Please turn-off cell phones

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Fall 2022

Flow Trough Packed Beds

There are two types of porous media

Packed Beds

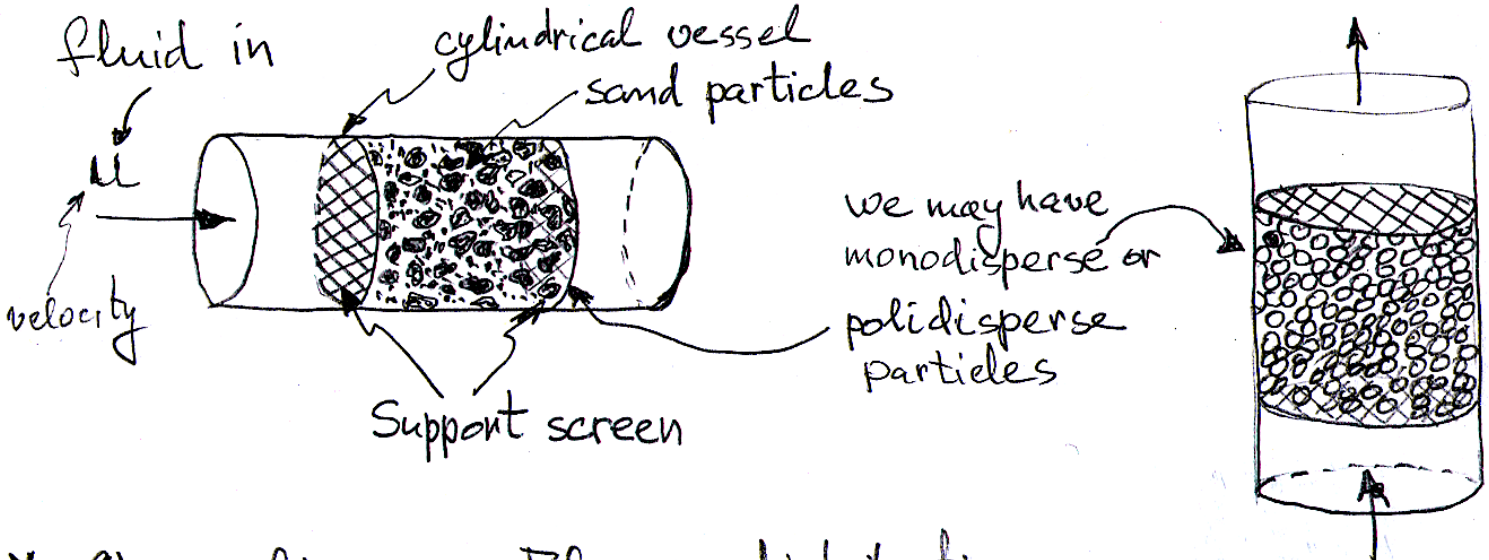
1. Rock piles
2. Sand filters
3. Cigarettes
4. Packed bed react.
5. . . . . .

Porous Solids

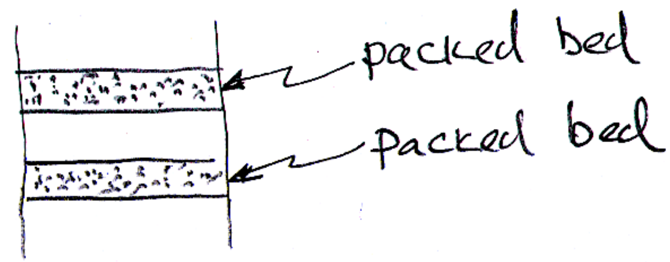
1. Oil shale
2. Sponges
3. Foams
4. Sintered metals
5. . . . . .

We will consider flow of fluids through packed beds only. There are several parameters/variables of the flow through packed beds that we have to consider, define, or evaluate. For example we need to define *Voidage* and *Interstitial Fluid Velocity*.

MECHANICAL ENERGY BALANCE EQUATION



✱ Channeling → Flow redistribution



$$V_{Total} = V_{particles} + V_{fluid} \quad \Rightarrow \quad 1 = \frac{V_{particles}}{V_{Total}} + \frac{V_{fluid}}{V_{Total}}$$

MECHANICAL ENERGY BALANCE EQUATION

$$1 = \frac{V_{particles}}{V_{Total}} + \frac{V_{fluid}}{V_{Total}} \quad \Rightarrow \quad 1 = \delta + \varepsilon = (1 - \varepsilon) + \varepsilon$$

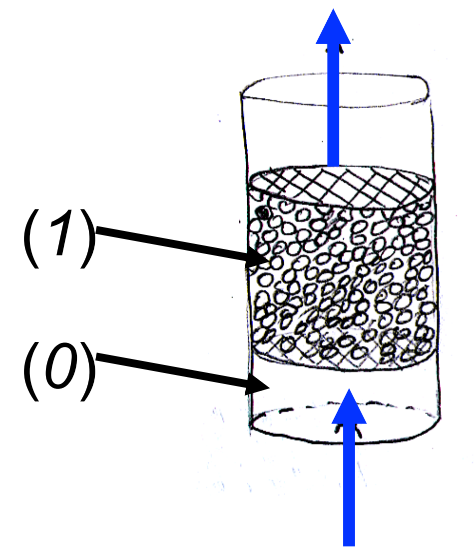
Voidage

$$Voidage = \frac{V_{fluid}}{V_{Total}} = \varepsilon$$

It is also true: $Voidage = \dfrac{A_{fluid}}{A_{Total}} = \varepsilon$

MECHANICAL ENERGY BALANCE EQUATION

Consider continuity of mass flow at two different bed cross sections *1* and *0*;



Mass flow at (0) = Mass flow at (1)

$$\underbrace{A_0 \cdot \bar{u}_0 \cdot \rho}_{\left[m^2\right]\left[\frac{m}{s}\right]\left[\frac{kg}{m^3}\right]} = \underbrace{A_1 \cdot \bar{u}_1 \cdot \rho}_{\left[\frac{kg}{s}\right]}$$

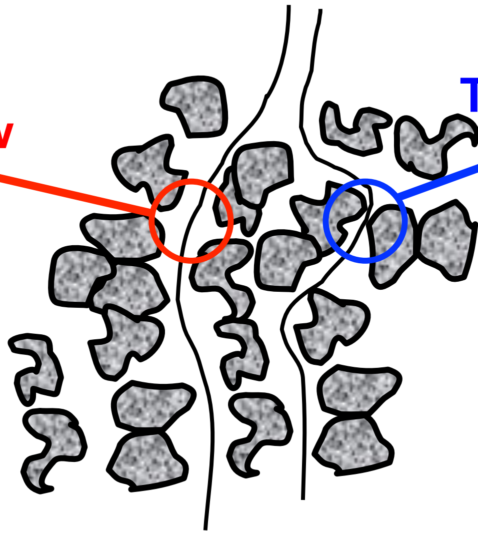
$$\varepsilon = \frac{A_1}{A_o}$$

$$\Rightarrow \quad A_0 \bar{u}_0 = A_0 \varepsilon \bar{u}_1 \quad \Rightarrow \quad \frac{\bar{u}_0}{\varepsilon} = \bar{u}_1$$

$$Interstitial\ \ velocity = \bar{u}_{\text{int}} = \bar{u}_1 = \frac{\bar{u}_0}{\varepsilon}\left[\frac{m}{s}\right]$$

$$Superficial\ \ velocity = \bar{u}_0$$

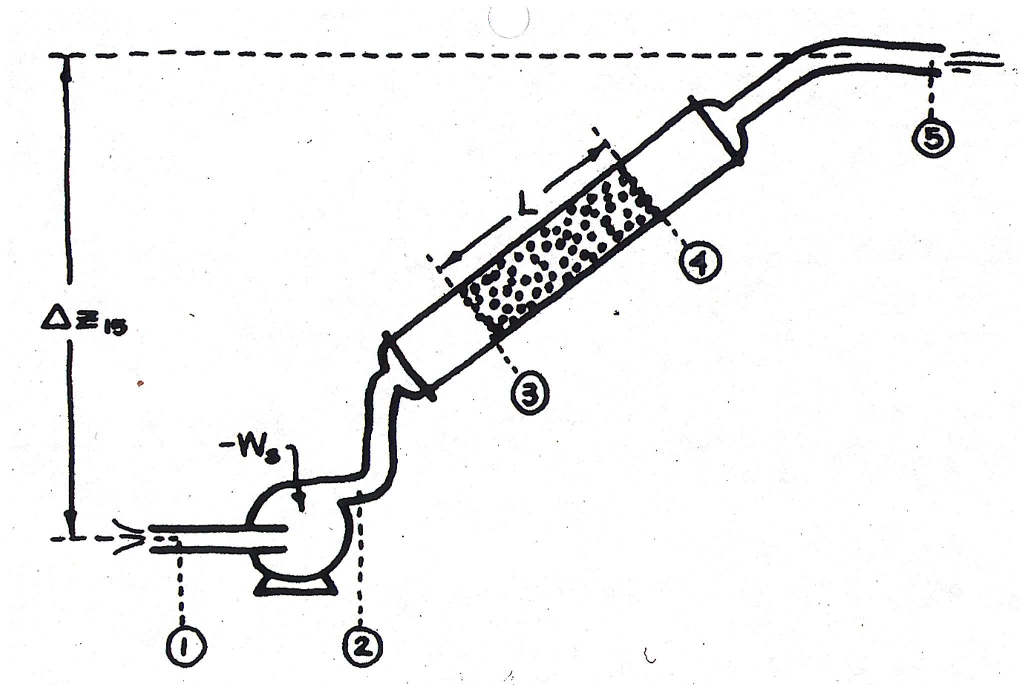
MECHANICAL ENERGY BALANCE EQUATION



The local flow condition can be turbulent and laminar simultaneously. It is obvious that laminar flow could be predominant if fluid mass flow rate is not large, or it could be predominantly turbulent at higher mass flow rates.

Therefore both flow regimes can exists simultaneously at different locations.

MECHANICAL ENERGY BALANCE EQUATION – Packed Bed



$$g\Delta Z+\frac{\Delta\bar{u}^2}{2}+\int\frac{dP}{\rho}+W_{S\,out}+\sum F=0$$

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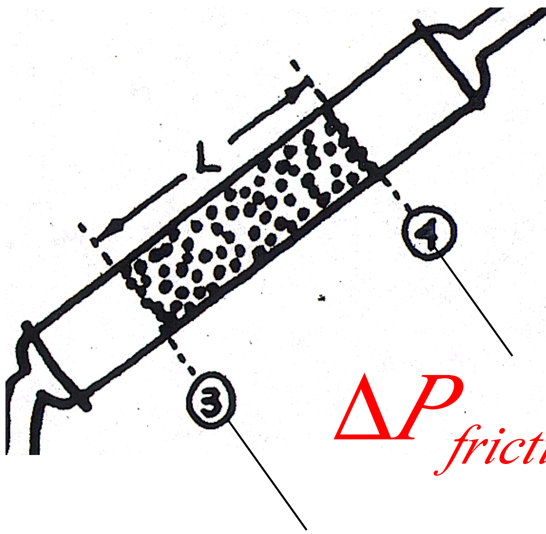
MECHANICAL ENERGY BALANCE EQUATION

Frictional Losses in Packed Bed



$\Delta P_{friction}$

Turbulent Losses

$$\int_3^4 \frac{dP}{\rho} = \frac{\Delta P_{friction}}{\rho} = \sum F = \frac{150(1-\varepsilon)^2 \mu \bar{u}_0 L}{\varepsilon^3 d_{part}^2 \rho} + \frac{1.75(1-\varepsilon)\bar{u}_0^2 L}{\varepsilon^3 d_{part}}$$

Viscous Losses

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MECHANICAL ENERGY BALANCE EQUATION

Earlier we derived an expression for frictional losses in a tube:

$$\sum F = \frac{2 f_F L \bar{u}^2}{d_{tube}}$$

For frictional losses in a packed bed we could have, similarly:

$$\sum F = \frac{(1-\varepsilon)}{\varepsilon^3} * \frac{f_F L \bar{u}_0^2}{d_{part}}$$

where:

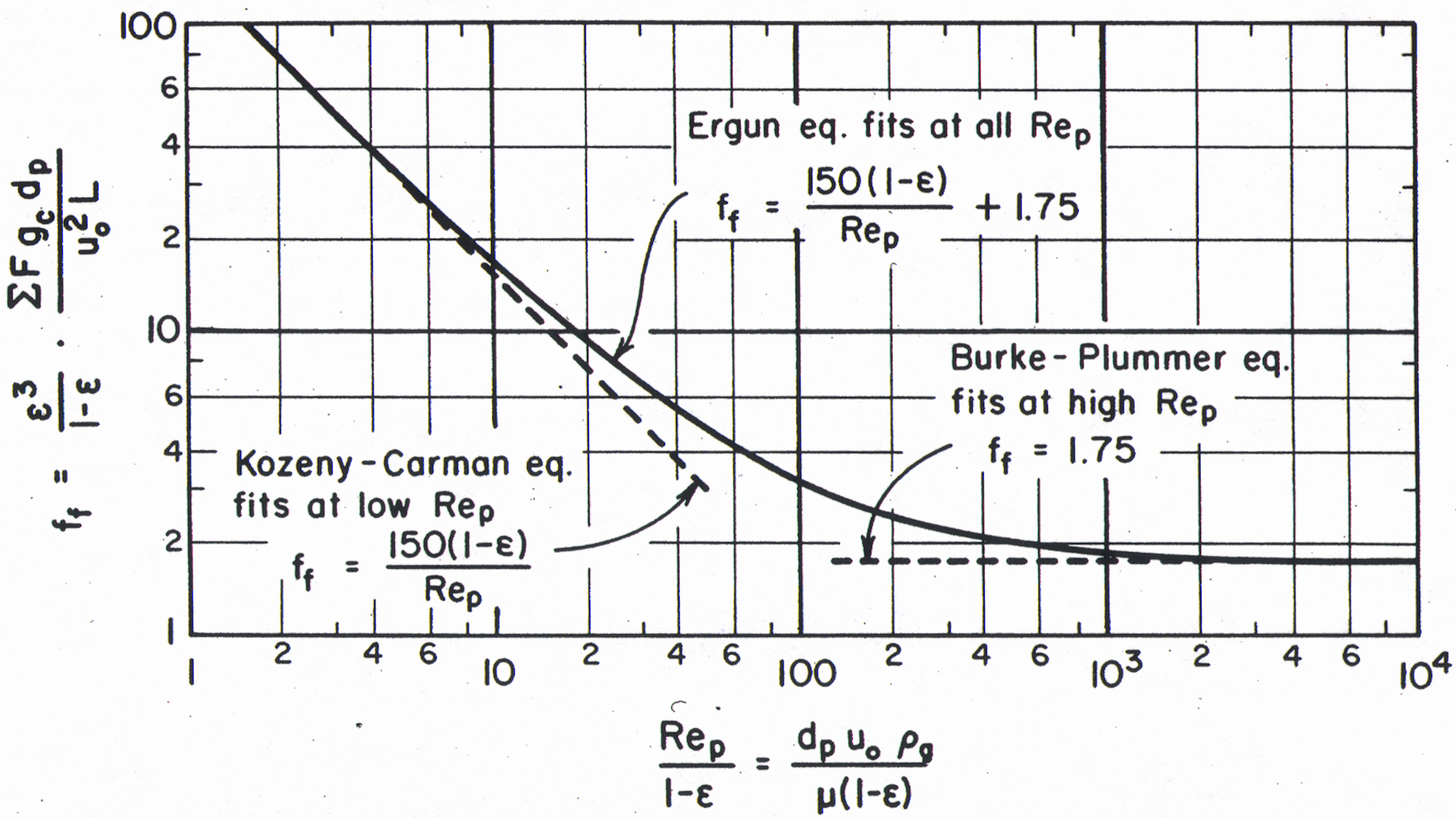
$$f_F = \frac{150(1-\varepsilon)}{\mathrm{Re}_{part}} + 1.75$$

and

$$\boxed{\mathrm{Re}_{part} = \frac{\rho \bar{u}_0 d_{part}}{\mu}}$$

MECHANICAL ENERGY BALANCE EQUATION

Friction Factor vs. Reynolds number for flow through **Packed Beds**



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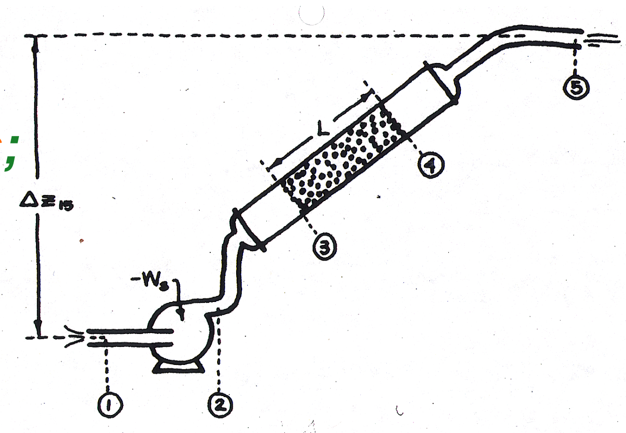
MECHANICAL ENERGY BALANCE EQUATION

Potential Energy Term $[g\Delta Z]$

This term is always negligible for gasses; however, it could be important for liquid fluids.



Kinetic Energy Term $\left[\frac{\Delta \bar{u}^2}{2}\right]$

This term is usually negligible for both gasses and liquids. However if this term has to be incorporated into the mechanical energy balance equation than one has to use an appropriate equation; perhaps one that reflects compressibility of gasses:

$$gdZ + udu + \frac{dP}{\rho} + d\left(\sum F\right) + dW_{Sout} = 0$$

$$2\ln\left[\frac{P_2}{P_1}\right] - \frac{(MW)}{G^2 RT}\left(P_1^2 - P_2^2\right) + \frac{4 f_F L}{d_p} = 0$$

where

$$G = \rho\bar{u}\left[\frac{kg}{m^2 s}\right]$$

For compressible gasses.

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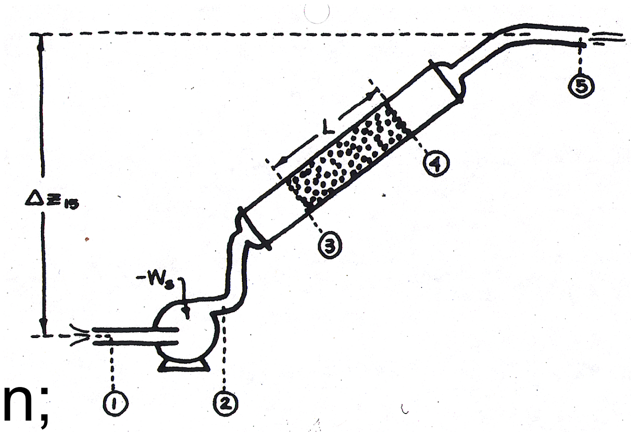
MECHANICAL ENERGY BALANCE EQUATION

The Work Term $\left[W_{Sout}\right]$



One can set the energy balance between points 1 and 5, and obtain the shaft work from the MEB equation;

$$g\left(Z_5 - Z_1\right) + \frac{\bar{u}_5^2 - \bar{u}_1^2}{2} + \frac{P_5 - P_1}{\rho} + \sum F_{1-5} = -\left(\dot{W}_{Sout}\right)$$

Or, one could make a similar energy balance between points 1 and 2;

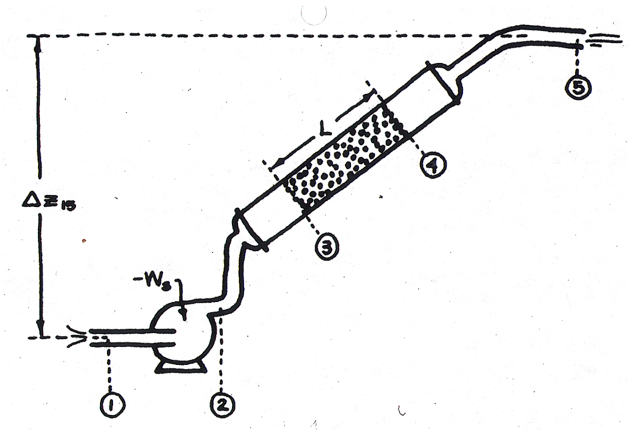
$$-W_{Sout} = \int_1^2 \frac{dP}{\rho} = \frac{\Delta P_{1-2}}{\bar{\rho}}$$

where $\bar{\rho}$ is the density calculated at average pressure $\bar{P}$.

This could be done as long as $\Delta P \leq 0.1\bar{P}$

MECHANICAL ENERGY BALANCE EQUATION

The Work Term $\left[W_{S\,out}\right]$



One can set the energy balance between points 1 and 5, and obtain the shaft work from the MEB equation;

$$g\left(Z_5 - Z_1\right) + \frac{\bar{u}_5^2 - \bar{u}_1^2}{2} + \frac{P_5 - P_1}{\rho} + \sum F_{1-5} + \Delta \dot{W}_S = 0$$

$$g\left(Z_5 - Z_1\right) + \frac{\bar{u}_5^2 - \bar{u}_1^2}{2} + \frac{P_5 - P_1}{\rho} + \sum F_{1-5} = -\Delta \dot{W}_S = -\left(\cancelto{0}{\dot{W}_{Sout}} - \dot{W}_{Sin}\right)$$

$$g\left(Z_5 - Z_1\right) + \frac{\bar{u}_5^2 - \bar{u}_1^2}{2} + \frac{P_5 - P_1}{\rho} + \sum F_{1-5} = \left(\dot{W}_{Sin}\right)$$

Or if we mandate, by definition, that W_{Sin} *does not exists:*

$$g\left(Z_5 - Z_1\right) + \frac{\bar{u}_5^2 - \bar{u}_1^2}{2} + \frac{P_5 - P_1}{\rho} + \sum F_{1-5} = -\dot{W}_{Sout}$$

MECHANICAL ENERGY BALANCE EQUATION

The Flow Work Term $\left[\int_a^b \frac{dP}{\rho}\right]$

For small changes of fluid density use density that is calculated at average pressure.

$$\overline{P} = \frac{P_a + P_b}{2} \quad \text{also} \quad \Delta P \le 0.1\overline{P}$$

For large changes in density - compressible gasses, i.e. $\Delta P > 0.1\overline{P}$ one could have the following case:

$$\cancel{g\Delta Z} + \cancel{\frac{\Delta\overline{u}^2}{2}} + \int \frac{dP}{\rho} + \cancel{\dot{W}}_{Sout} + \sum F = 0 \Bigg| * \frac{1}{\overline{u}_0^2}$$

$$\sum F = \frac{150(1-\varepsilon)^2 \mu \overline{u}_0 L}{\varepsilon^3 d_p^2 \rho} + \frac{1.75(1-\varepsilon)\overline{u}_0^2 L}{\varepsilon^3 d_p}$$

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MECHANICAL ENERGY BALANCE EQUATION

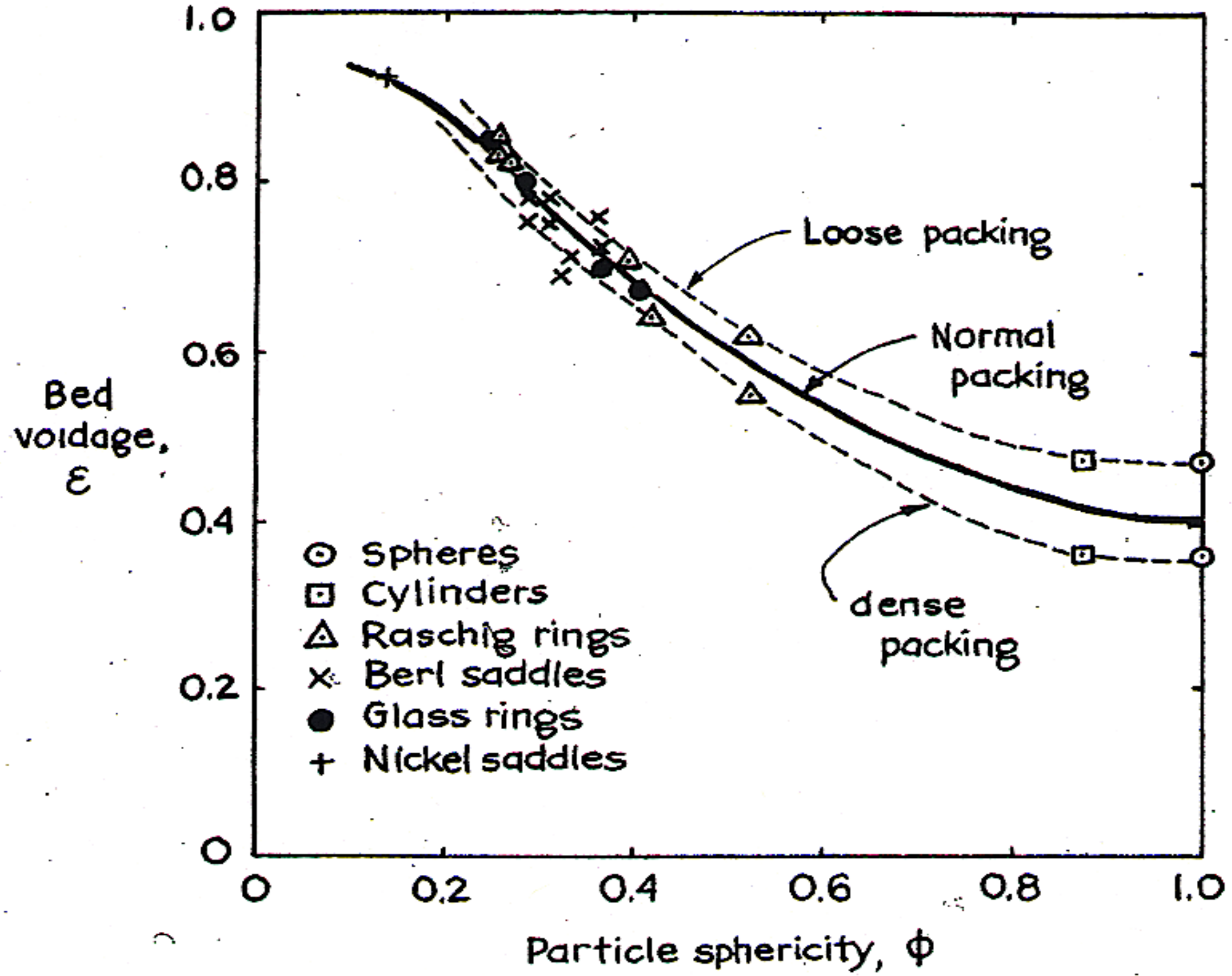
$$\int_3^4 \frac{\rho dP}{\underbrace{\rho\rho u_0^2}_{G_0^2}} + \frac{150(1-\varepsilon)^2 \mu L}{\varepsilon^3 d_p^2 G_0} + \frac{1.75(1-\varepsilon)L}{\varepsilon^3 d_p} = 0 \quad \text{where } G_0 = \rho u_0$$

For isothermal non-reversible process one can write:

$$\frac{(MW)}{2G_0^2 RT}\left(P_4^2 - P_3^2\right) + \frac{150(1-\varepsilon)^2 \mu L}{\varepsilon^3 d_p^2 G_0} + \frac{1.75(1-\varepsilon)L}{\varepsilon^3 d_p} = 0$$

where $\rho = \dfrac{P(MW)}{RT}$

MECHANICAL ENERGY BALANCE EQUATION



The voidage increases as the sphericity decreases for randomly packed beds of uniform particles.

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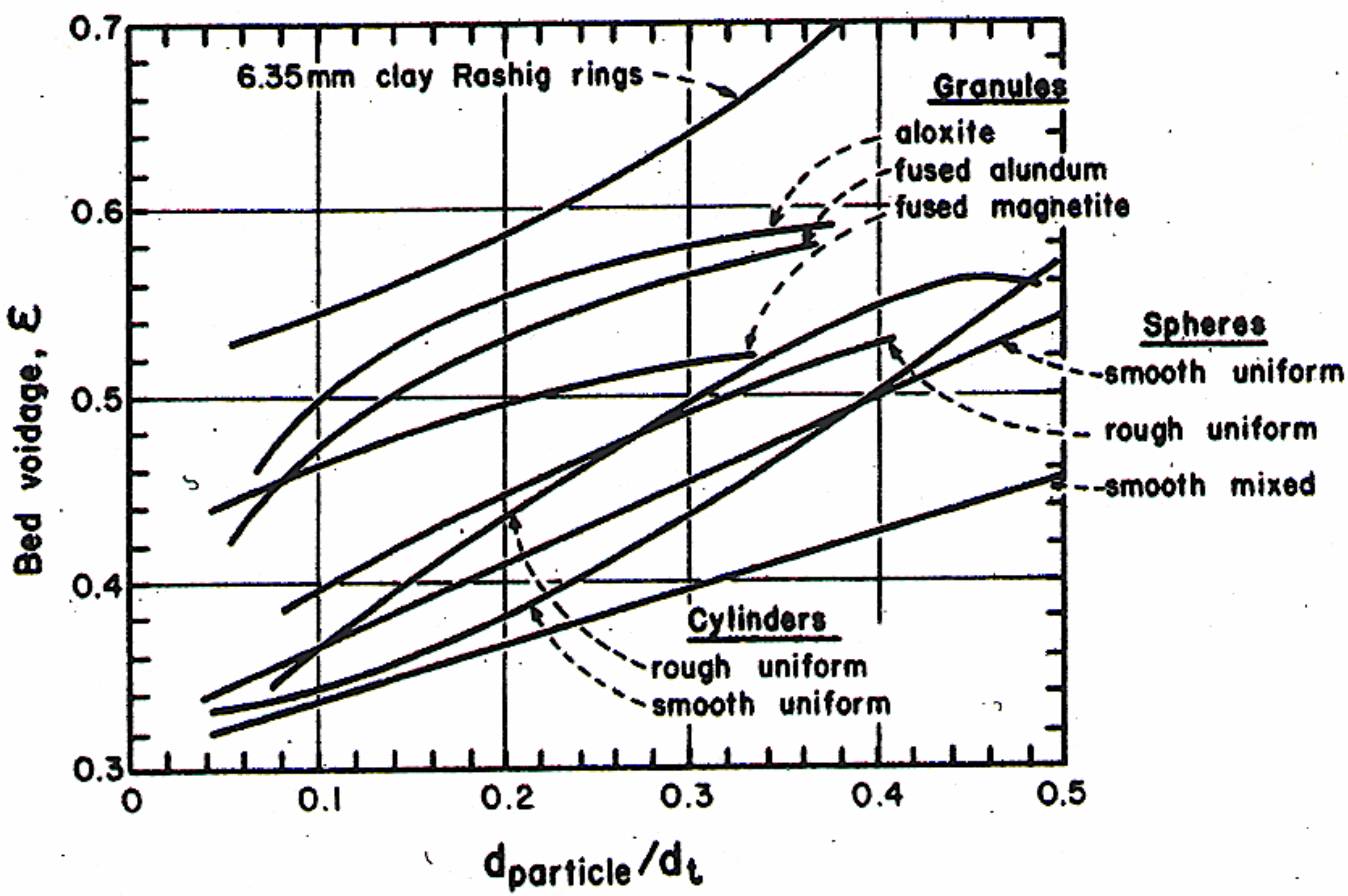
People. Ideas. Innovation.

Thank you for your attention!

MECHANICAL ENERGY BALANCE EQUATION

Bed Voidage vs. Particle Diameter



MECHANICAL ENERGY BALANCE EQUATION

Tyler Standard Screen Sizes

Mesh number (number of wires/in)	Aperture, μm (opening between adjacent wires)	Mesh number (number of wires/in)	Aperture, μm (opening between adjacent wires)
3	6680	35	417
4	4699	48	295
6	3327	65	208
8	2362	100	147
10	1651	150	104
14	1168	200	74
20	833	325	53
28	589	400	38

Unfortunately there is no general relationship between d_{scr} and d_p. The best we can say for pressure drop considerations in packed beds is

- for irregular particles with no seeming long or shorter dimension take

$$d_p \cong \phi d_{scr} = \phi d_{sph}$$

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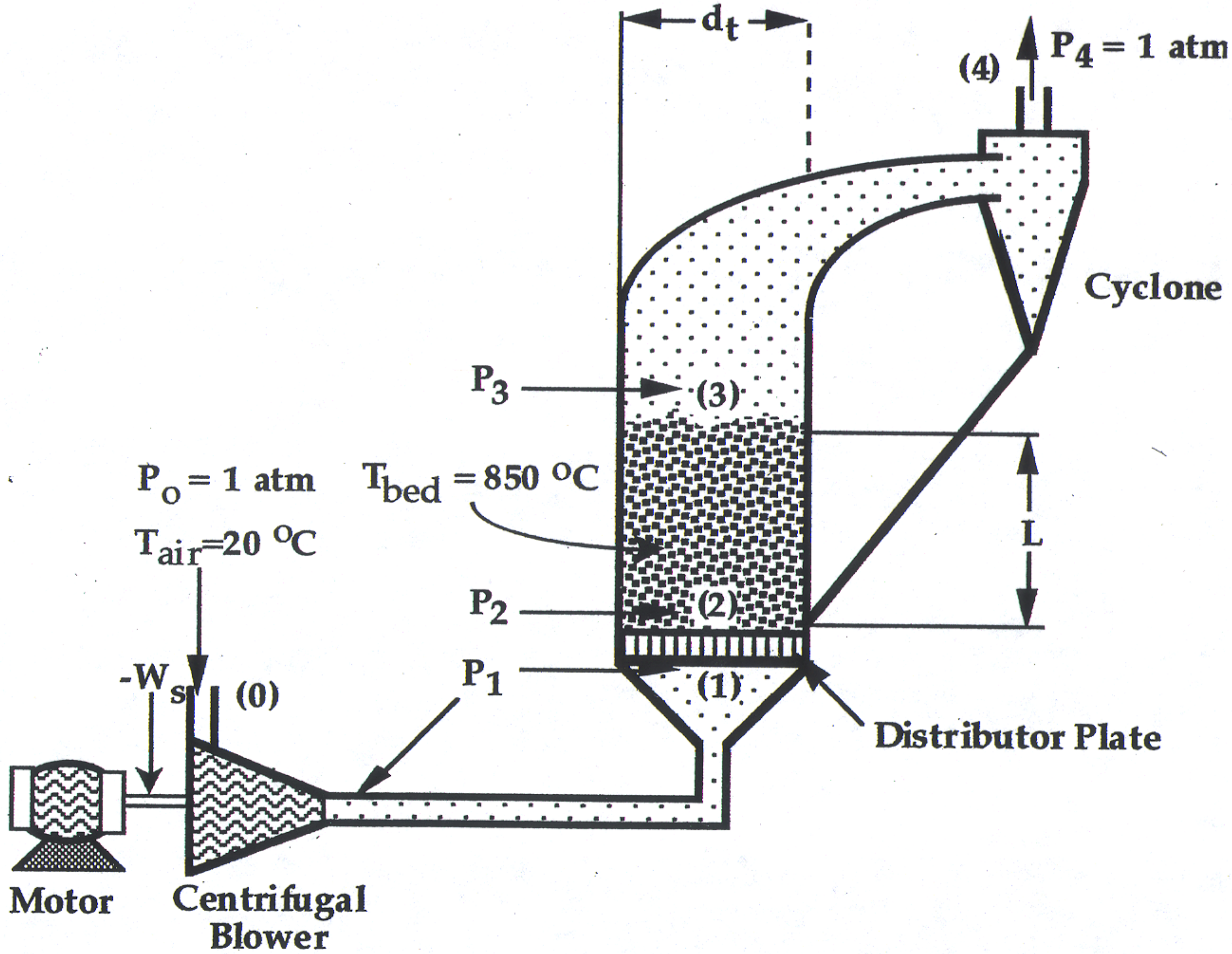

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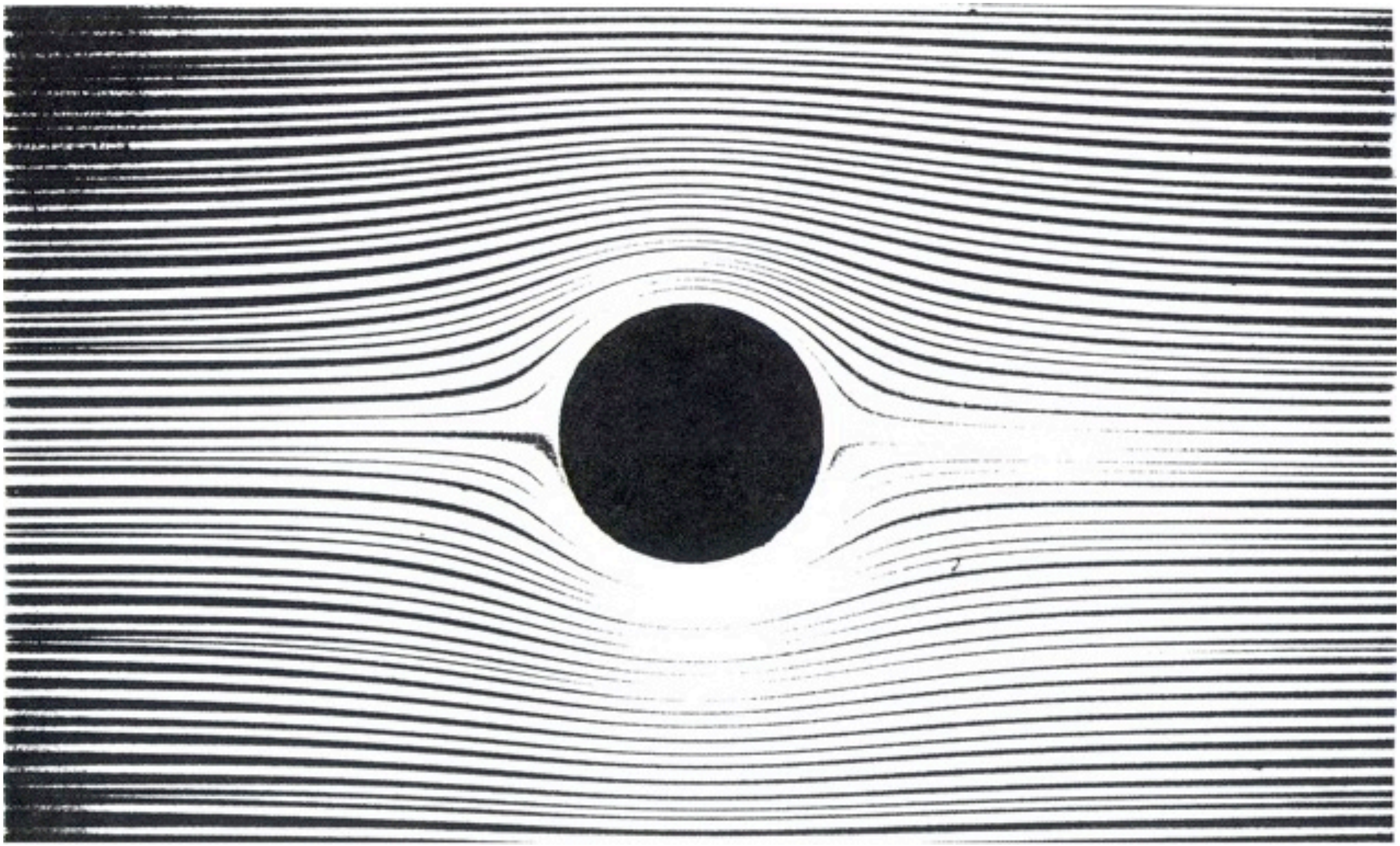

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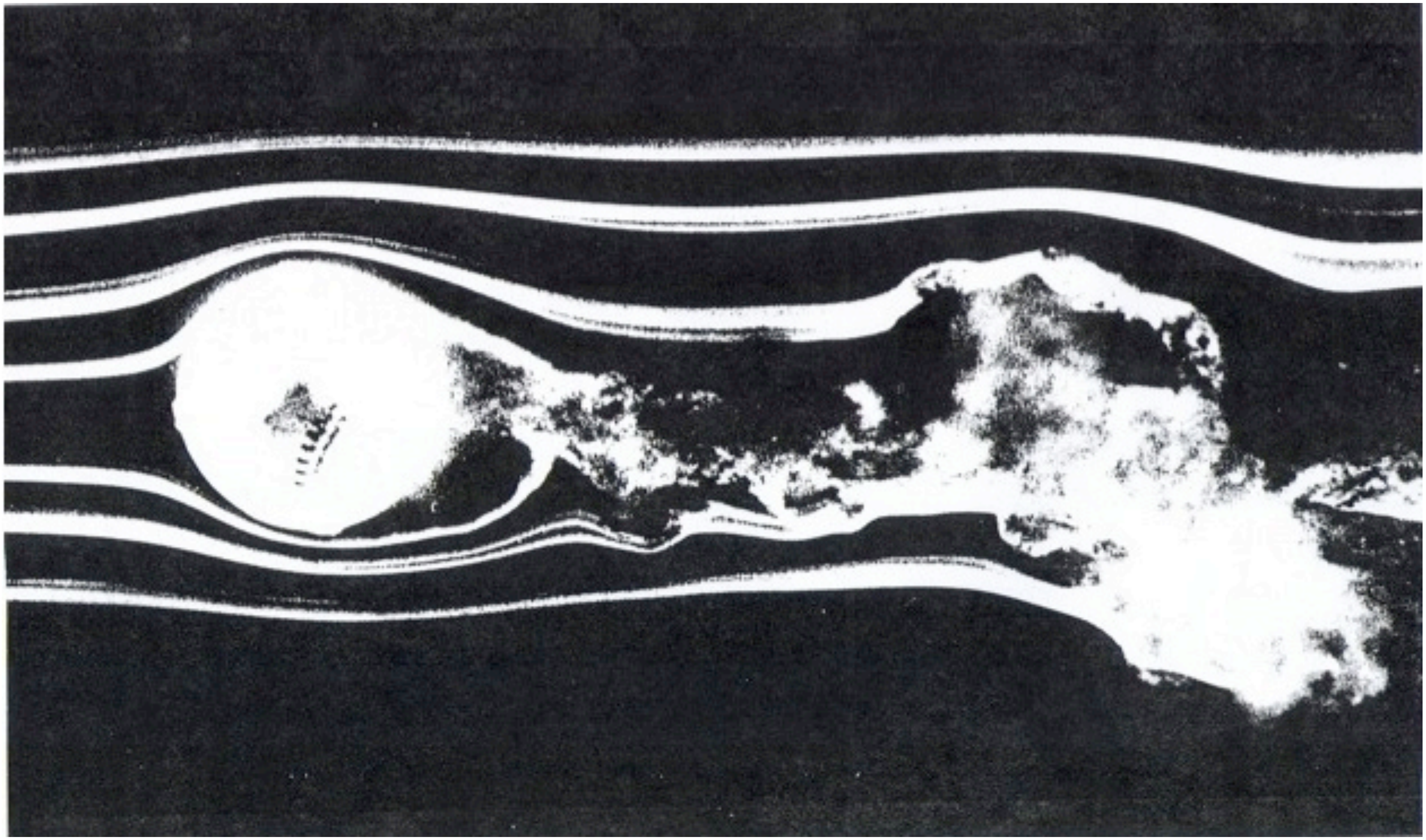

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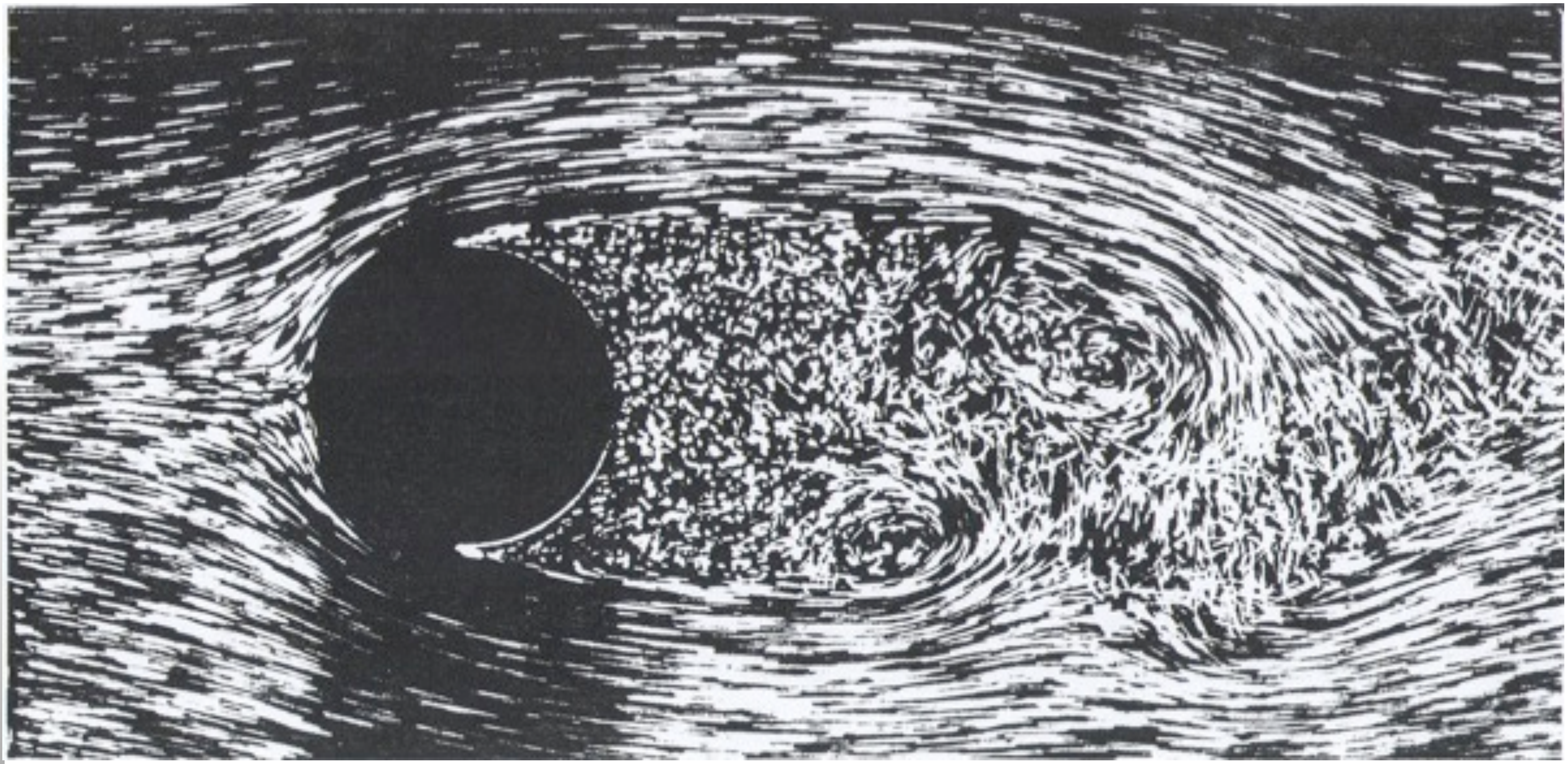

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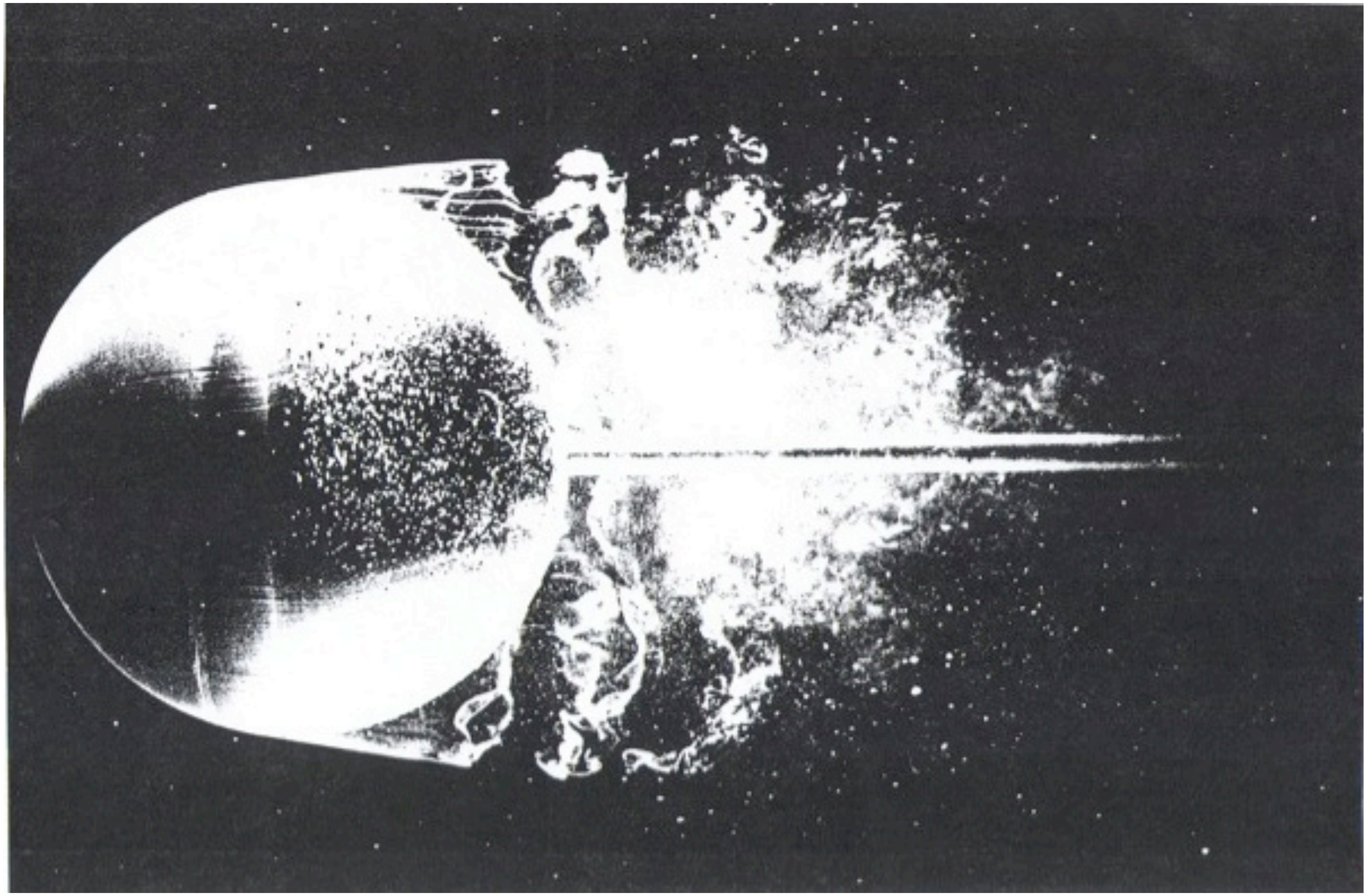

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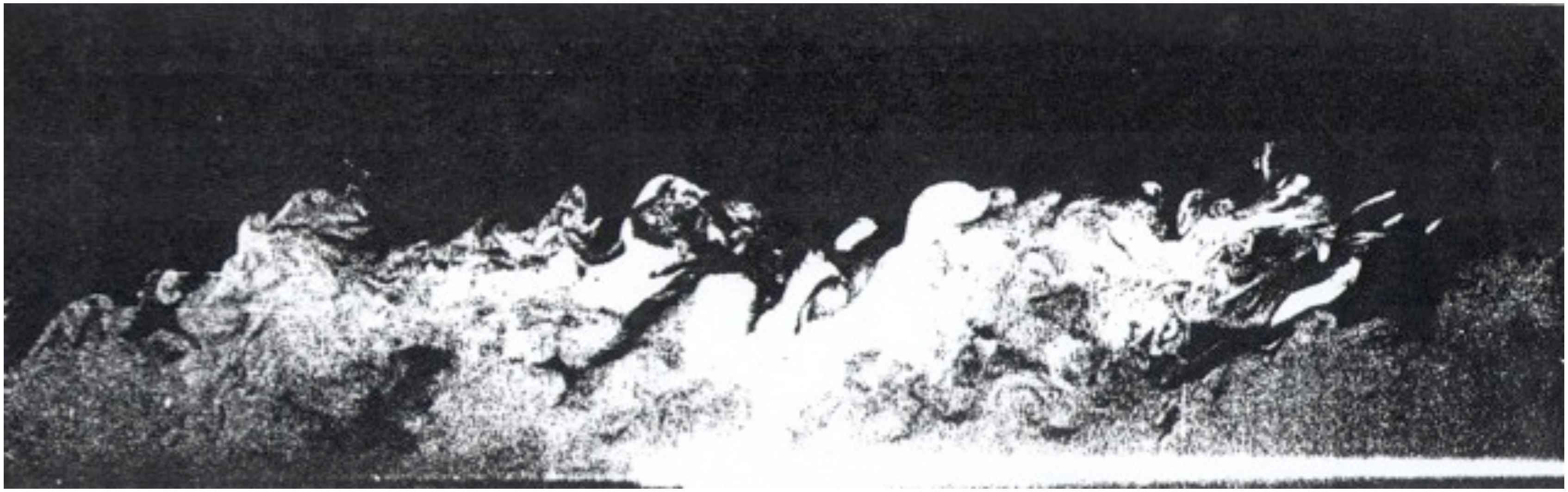

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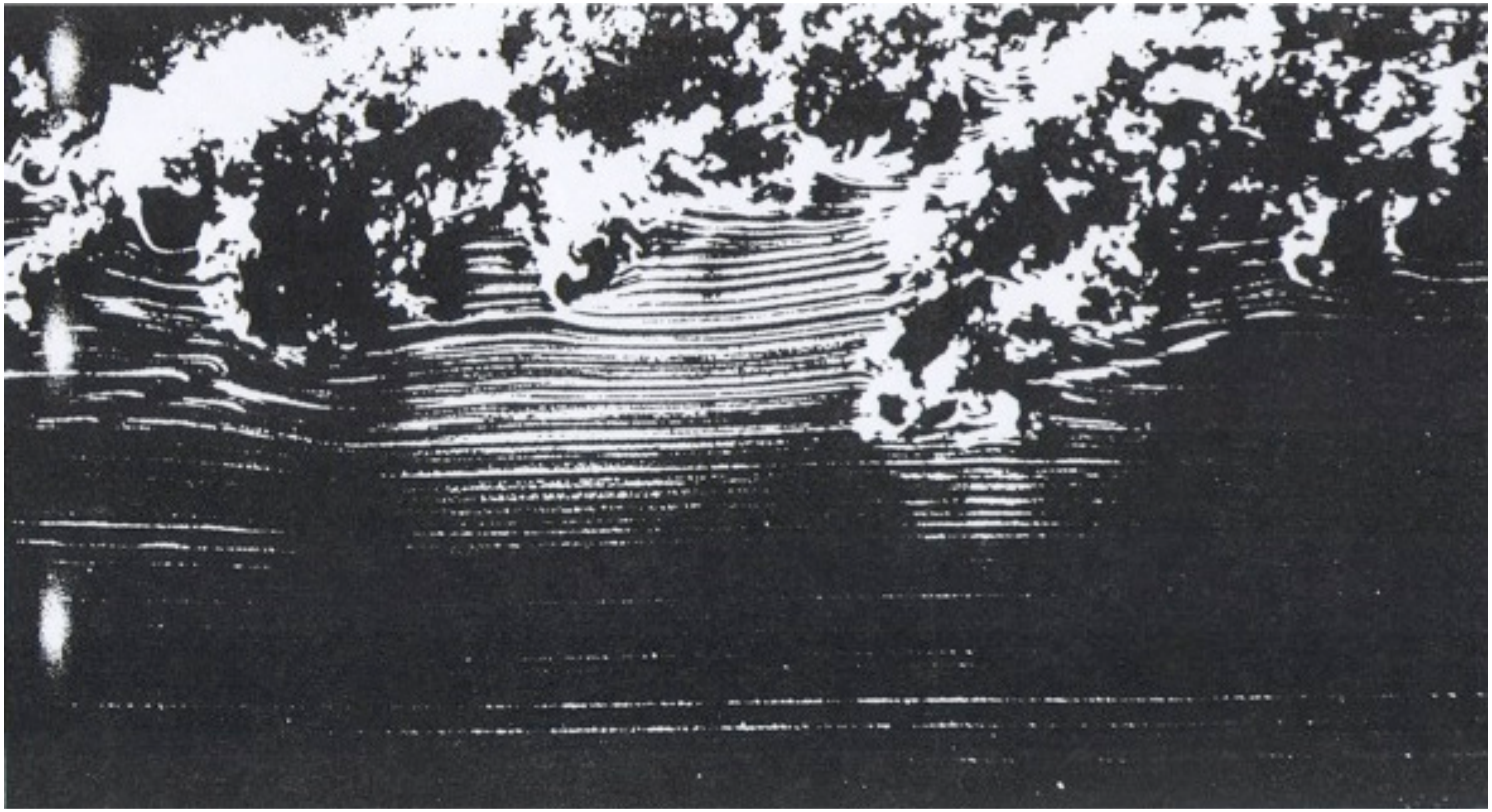

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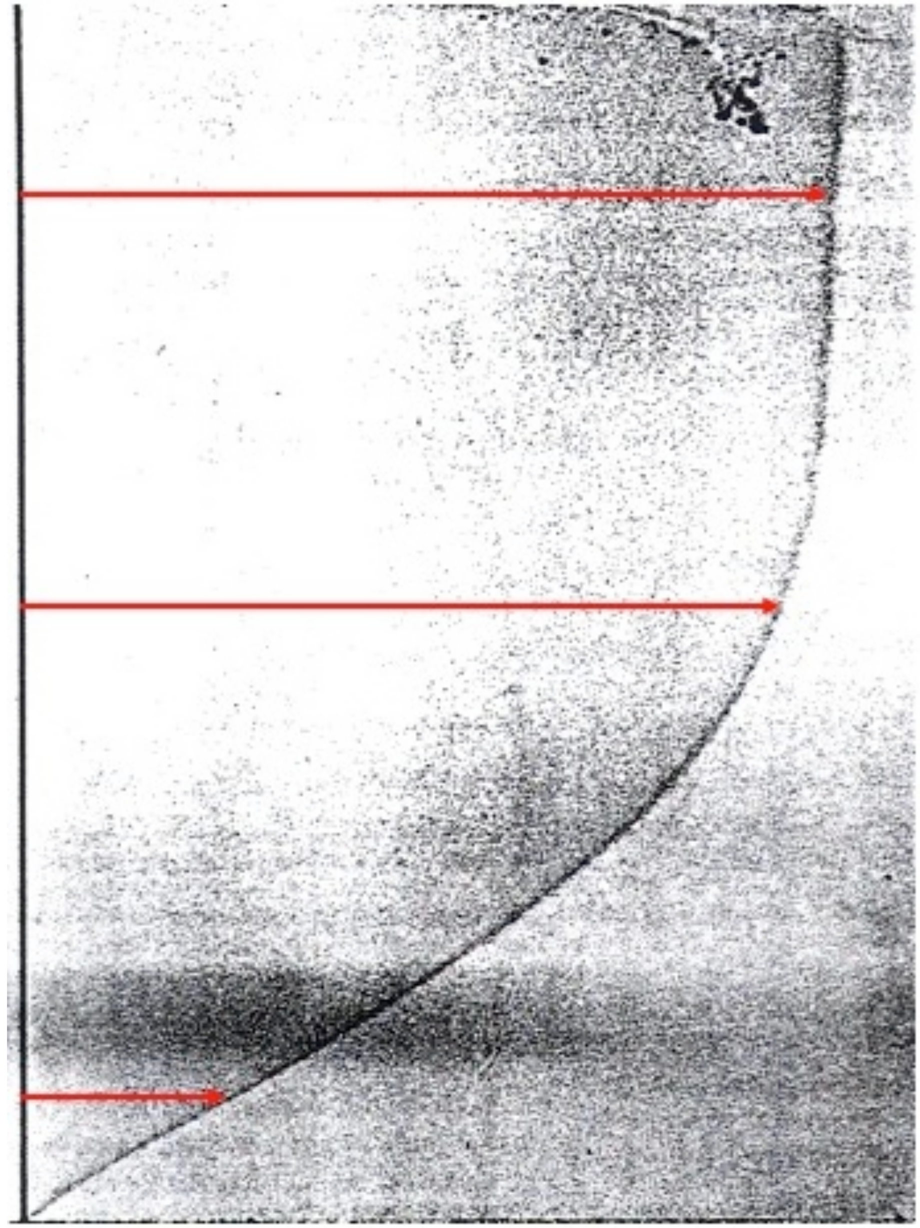

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