Consider an isothermal, steady state, unidirectional, laminar flow of an incompressible ***non-Newtonian*** liquid streaming along a vertical plate as shown in the illustration below. The liquid film of thickness $\boldsymbol{\delta}$ is flowing due to gravity only. The liquid is a ***power-law fluid***, which could be represented by the following *Shear Stress – Shear Rate* relationship:

$$\tau_{xy} = -\eta\left(\frac{\partial u_x}{\partial y}\right)^n$$

a) Develop a mathematical model [differential equation(s) + boundary conditions] that will represent the flow of the liquid film. List the variables and your assumptions. Start from the conservation of momentum equations.

b) Solve the mathematical model developed in (***a***) and obtain an algebraic expression that will represent the velocity profile $u_x(y)$ of this power-law fluid.

c) If the exponent '*n*' in the solution obtained in part (***b***) is set to *n* =1, (and $\eta = \mu$) does your solution reduce itself to a velocity profile that could be obtained for a ***Newtonian*** fluid?

d) Develop an expression for the volumetric flow rate of the ***non-Newtonian*** fluid per unit width of the film, (Q/W) where *W* is the width of the plate (normal to the plane of the paper).

e) If the exponent '*n*' in the solution obtained in part (***d***) is set to *n* =1, (and $\eta = \mu$) does your solution reduce itself to the volumetric flow rate that could be obtained for a ***Newtonian*** fluid?

Assumptions: The flow is assumed to be fully developed, which implies that the velocity profile does not change along the direction of flow. Momentum transfer between liquid and air is negligible. Also, ignore effects at each end of the plate. *State any additional assumption.*