



**Oregon State**  
University

# **OREGON STATE UNIVERSITY SCHOOL of CBEE**

## **DEPARTMENT OF CHEMICAL ENGINEERING**

### **CHE 331**

## **Transport Phenomena I**

### **Non-Newtonian Fluids**

## **Dr. Goran Jovanovic**

Reading Assignment:

- 1) Liquid Viscosity Correlations for Flowmeter Calculations;  
Roger Gilmont
- 2) Non-Newtonian Flow I - Characterization of Fluid Behavior;  
Boger D.V and Halmos A.

**Please turn-off cell phones**



## CLASSIFICATION OF FLUIDS

Fluids give and flow under stress. Newtonians are the simplest of fluids, and they are characterized by the following property:

*The velocity gradient (Shear Rate) at a point within the fluid flow field is proportional to the Shear Stress at that point.*

*Shear rate*  $\propto$  *Shear stress*

$$\frac{du}{dy} \propto \tau$$

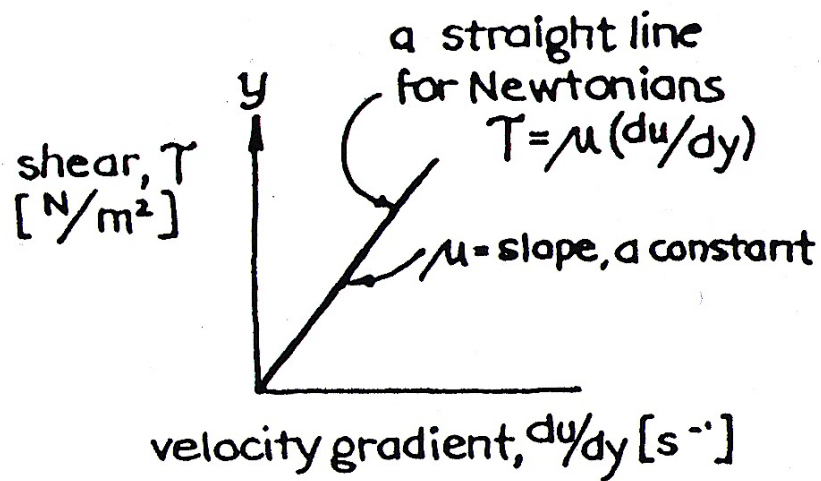
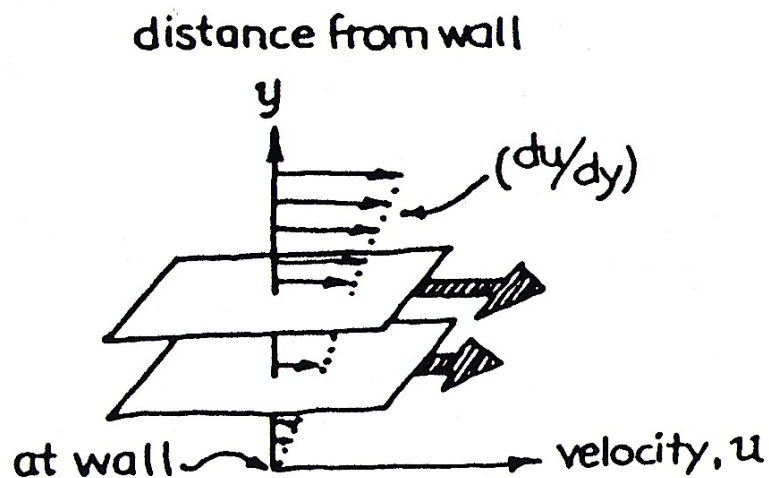
Newton's Law

$$\tau_{xy} = -\mu \left( \frac{d\vec{u}_x}{dy} \right)$$
$$\tau_{xy} = \mu \left( \left| \frac{du_x}{dy} \right| \right)$$

*All other fluids whose Shear rate – Shear stress relationship cannot be described by the above equation are non-Newtonian Fluids!!*



# CLASSIFICATION OF FLUIDS





# CLASSIFICATION OF FLUIDS

## Newtonian Fluids

Air  
Water  
Steam  
Almost all Gases  
Fluids of simple molecules

## Non-Newtonian Fluids

Suspensions  
Slurries  
Emulsions  
Polymer Solutions  
Paints  
Biological Fluids  
Concrete Mix

.....

### Time Independent

$$\left( \begin{matrix} \text{Shear} \\ \text{Rate} \end{matrix} \right) = f \left( \begin{matrix} \text{Shear} \\ \text{Stress} \\ \text{alone} \end{matrix} \right)$$

### Time Dependent

(non elastic)

$$\left( \begin{matrix} \text{Shear} \\ \text{Rate} \end{matrix} \right) = f \left( \begin{matrix} \text{Shear Stress} + \\ \text{History of Stress} \end{matrix} \right)$$

### Viscoelastic



# CLASSIFICATION OF FLUIDS

How about this fluid:

Is it time dependent?

Is it elastic?

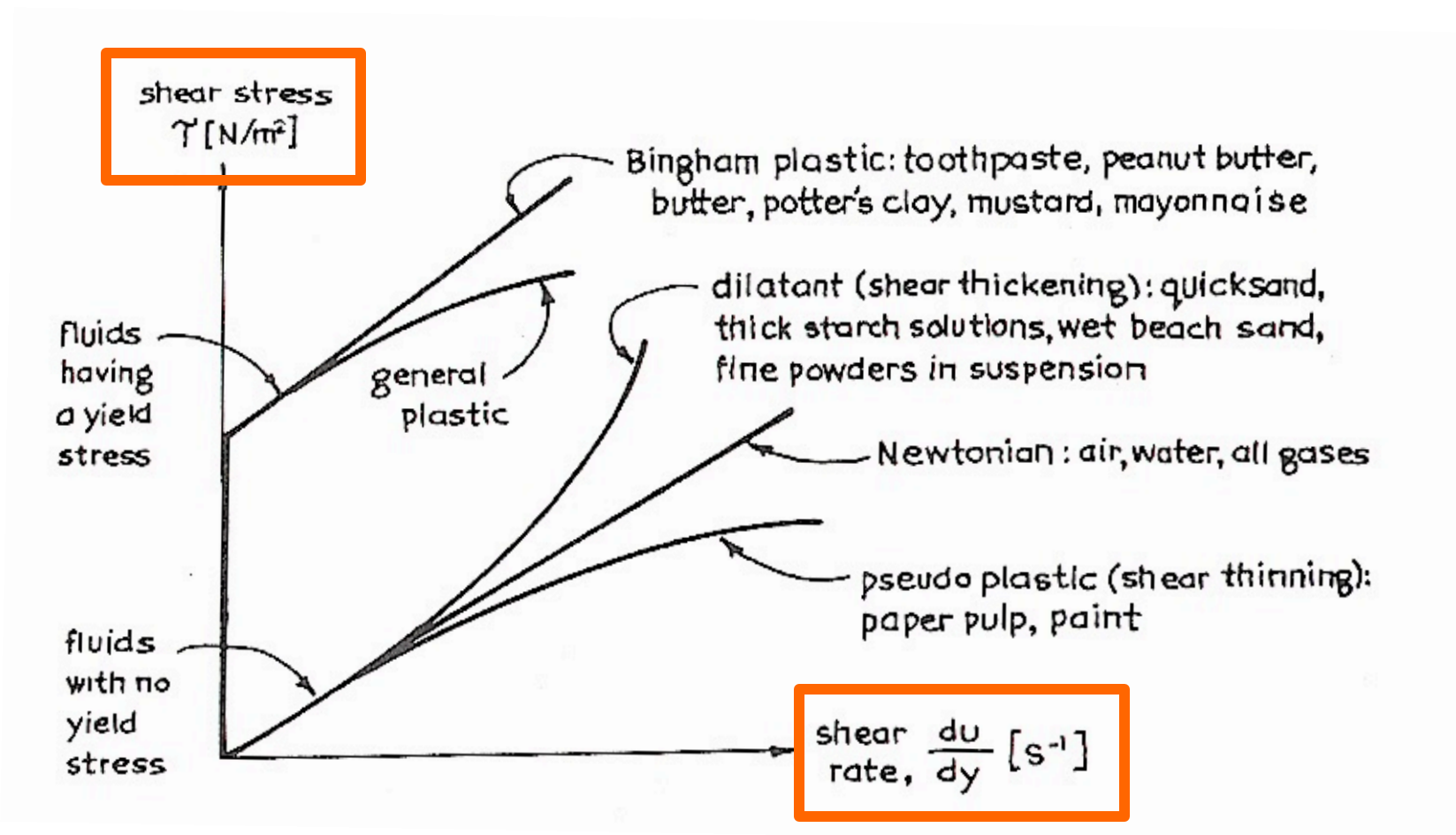
Does it have memory?

So, what kind of fluid this is?



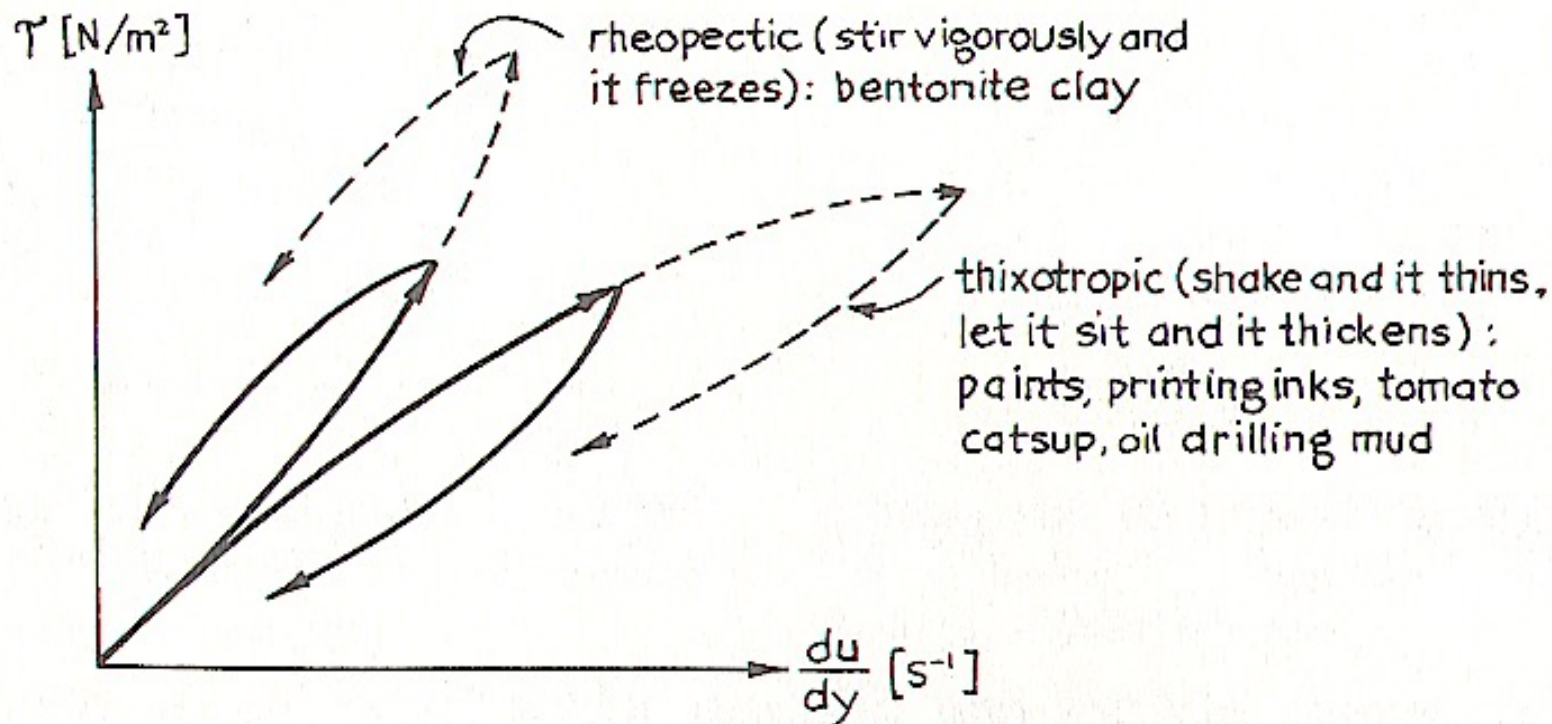


# CLASSIFICATION OF FLUIDS (time independent)





# CLASSIFICATION OF FLUIDS (time dependent viscoelastic)





# CLASSIFICATION OF FLUIDS

**Newtonian Fluids** - The velocity gradient is proportional to the imposed shear stress on fluid.

$$\Rightarrow \tau = \mu \left( \left| \frac{du}{dy} \right| \right)$$

$\Downarrow$

**Bingham Plastic Fluids** - The velocity gradient is proportional to the shear stress but these fluids have Initial Yield Stress  $\tau_o$

$$\Leftarrow \tau = \tau_o + \eta \left( \left| \frac{du}{dy} \right| \right)$$

$\Downarrow$

**Power Law Fluids** - Shear stress Shear rate relationship is not linear. K is not viscosity (shear thickening and shear thinning)

$$\Leftarrow \tau = K \left( \left| \frac{du}{dy} \right| \right)^n$$

$\Downarrow$

**General Plastic Fluids**

$$\Leftarrow \tau = \tau_o + K \left( \left| \frac{du}{dy} \right| \right)^n$$





## CLASSIFICATION OF FLUIDS - *Average Velocity in Pipe*

### Newtonian Fluids

$$\tau = \mu \left( \left| \frac{du}{dy} \right| \right)$$

### *Average velocity in Pipe*

$$u_{avg} = \frac{d_p^2 \rho \sum F}{32 \mu L}$$

### Bingham Plastic Fluids

$$\tau = \tau_o + \eta \left( \left| \frac{du}{dy} \right| \right)$$

$$u_{avg} = \frac{d_p^2 \rho \sum F}{32 \eta L} \left( 1 - \frac{4}{3} m + \frac{1}{3} m^4 \right)$$

$$\text{where } m = \frac{4 \tau_o L}{\rho d_p \sum F} = \frac{\tau_o}{\tau_w} \quad \text{and} \quad m \leq 1$$

$$\text{if } \tau_o \geq \tau_w \quad \text{or} \quad m \geq 1 \quad \text{or} \quad \frac{4 \tau_o L}{d_p} \geq \rho \sum F \quad \text{the flow is frozen}$$



## Average velocity in Pipe

### Power Law Fluids

$$\tau = K \left( \left| \frac{du}{dy} \right| \right)^n$$

$$u_{avg} = \frac{d_p^2 \rho \sum F}{32 K \cdot L} \frac{4n}{1+3n} \left( \frac{d_p \rho \sum F}{4 K \cdot L} \right)^{\frac{(1-n)}{n}}$$

### General Plastic Fluids

$$\tau = \tau_o + K \left( \left| \frac{du}{dy} \right| \right)^n$$

$$u_{avg} = \frac{d_p^2 \rho \sum F}{32 K \cdot L} \left( \frac{d_p \rho \sum F}{4 K \cdot L} \right)^{\frac{(1-n)}{n}} \times$$

$$\times 4n(1-m)^{\frac{(n+1)}{n}} \left( \frac{(1-m)^2}{1+3n} + \frac{2m(1-m)}{1+2n} + \frac{m^2}{1+n} \right)$$

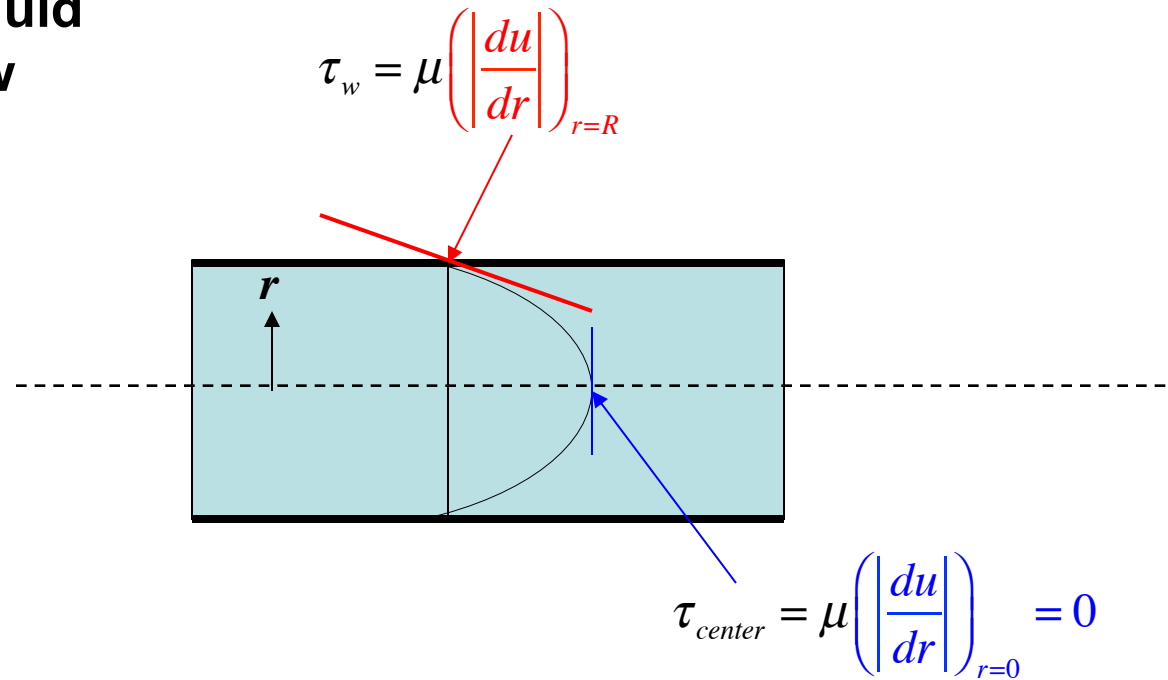
where  $m = \frac{4\tau_o L}{\rho d_p \sum F} = \frac{\tau_o}{\tau_w}$  and  $m \leq 1$

if  $\tau_o \geq \tau_w$  or  $m \geq 1$  or  $\frac{4\tau_o L}{d_p} \geq \rho \sum F$  the flow is frozen

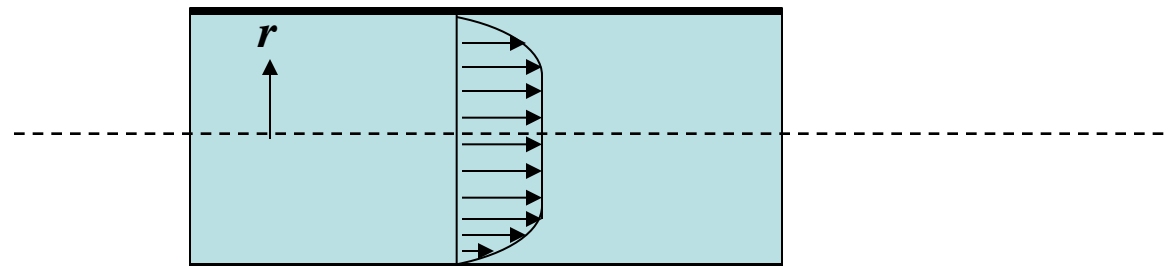


# CLASSIFICATION OF FLUIDS - *Velocity Distribution in Pipe*

## Newtonian Fluid Laminar Flow



## Bingham Plastic Fluid Laminar Flow

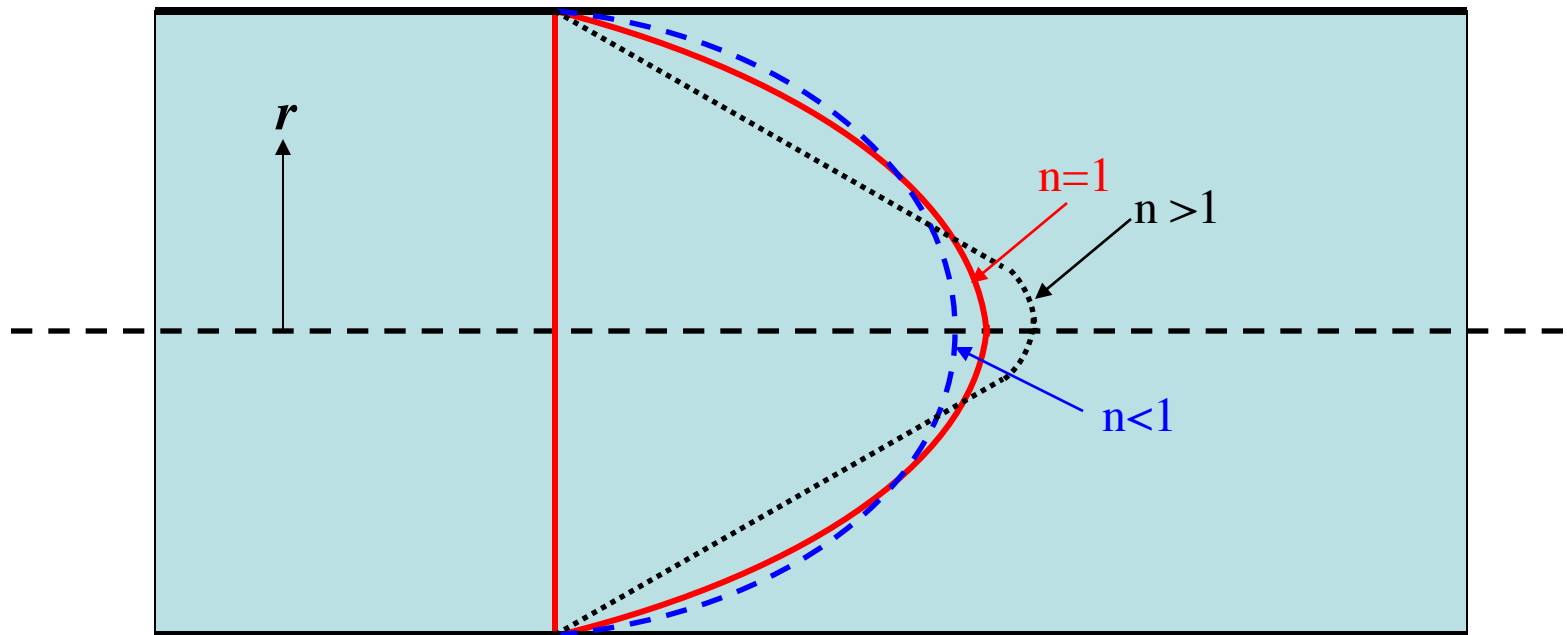




# CLASSIFICATION OF FLUIDS - *Velocity Distribution in Pipe*

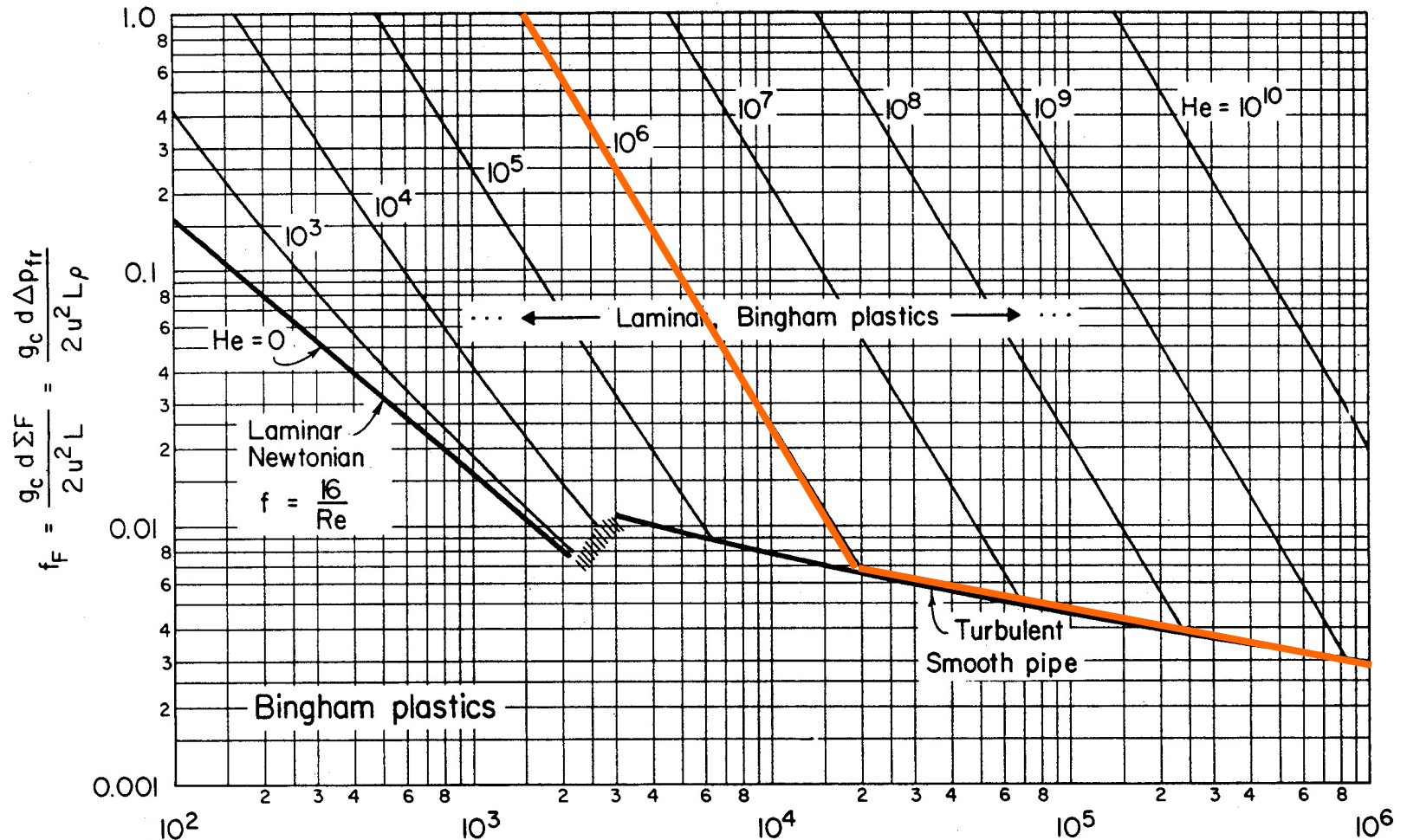
## Power Law Fluid

$$\tau = K \left( \frac{du}{dy} \right)^n$$





# CLASSIFICATION OF FLUIDS – Bingham Plastic - Friction Factor



$$He = \frac{\tau_o d^2 \rho}{\eta^2}$$

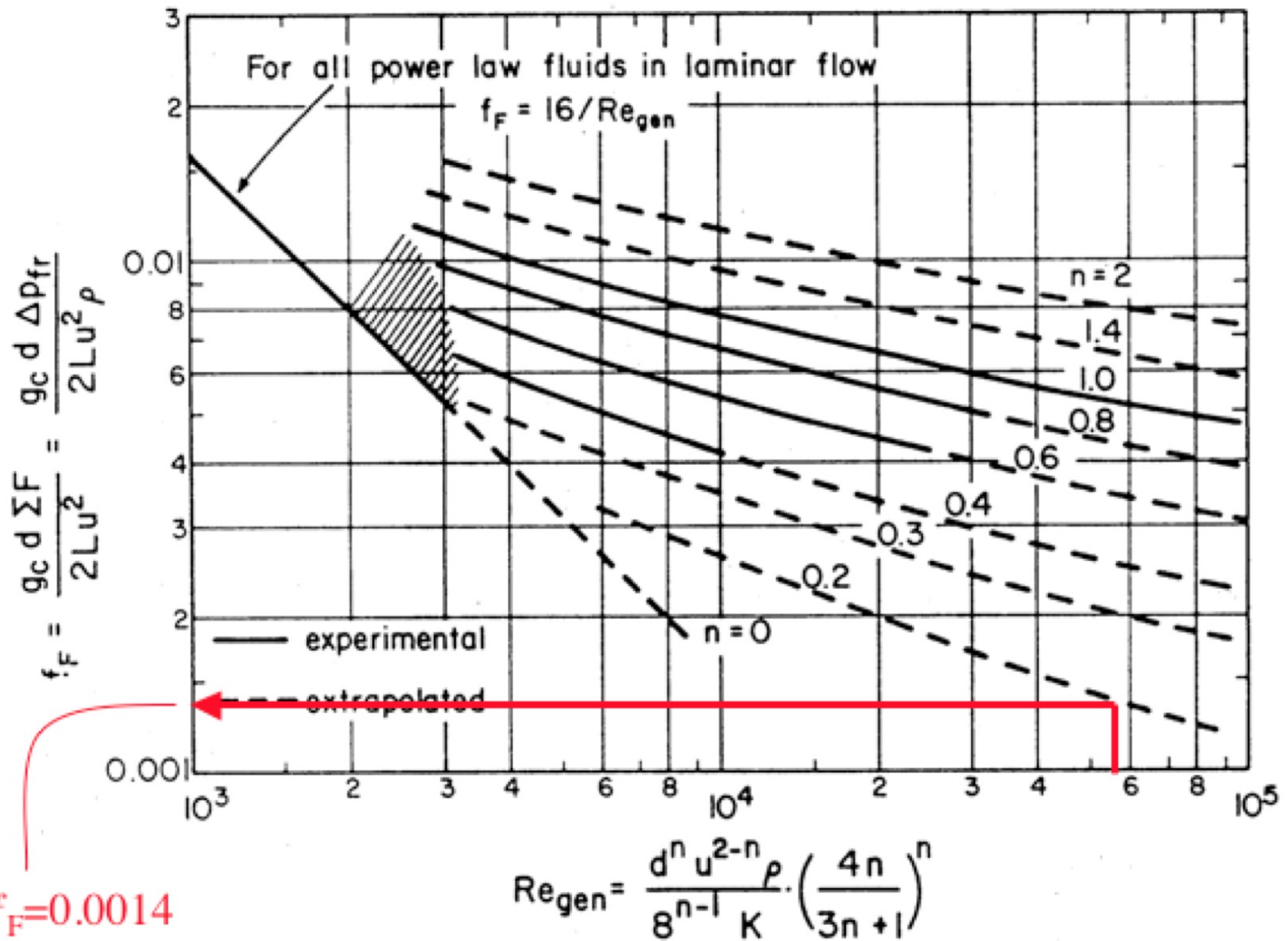
$$\tau = \tau_o + \eta \left( \left| \frac{du}{dy} \right| \right)$$

$$Re = \frac{du \rho}{\eta}$$

Example  $He = 1,000,000$ .



# CLASSIFICATION OF FLUIDS – Power law Fluids Friction Factor





*People. Ideas. Innovation.*

*Thank you for your attention!*