## Practice Problem 1

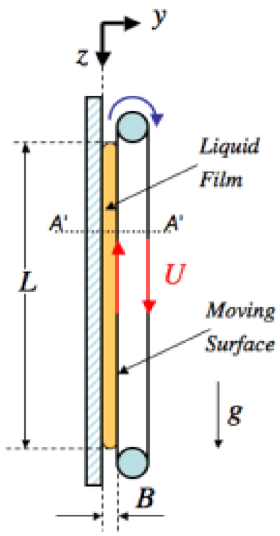
A Newtonian viscous liquid is flowing between two wide planer surfaces as shown in the illustration below. The surface along ***y*** = *0* is fixed in space, and the surface along ***y*** = ***B*** has a steady speed ***U*** parallel to the fixed plane such that the liquid neither falls nor rises. (If ***U*** were zero the liquid film would flow downward under the action of gravity.)

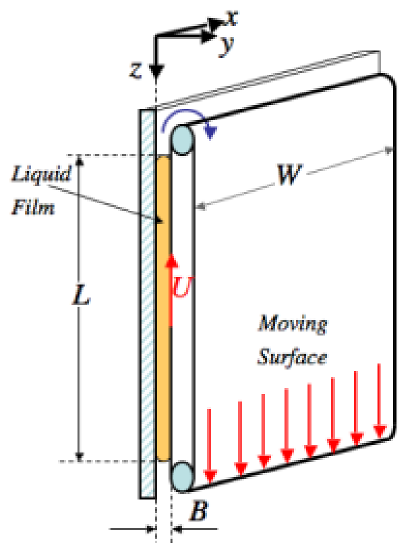
***a)*** Derive a differential equation that will reflect the steady state flow of the fluid such that the liquid neither falls nor rises. Start from appropriate Navier-Stocks equations.

***b)*** Derive appropriate boundary conditions.

***c)*** Develop an algebraic expression for the velocity profile of fluid between the plates from the mathematical model obtained in (a) & (b) above.

***d)*** Sketch a graph of the velocity $u_z(y)$.




### Navier Stokes Equations in Cartesian Coordinates

$$\rho\left[\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}\right]=-\frac{\partial P}{\partial x}+\mu\left[\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right]+\rho g_x$$

$$\rho\left[\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_y}{\partial z}\right]=-\frac{\partial P}{\partial y}+\mu\left[\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right]+\rho g_y$$

$$\rho\left[\frac{\partial u_z}{\partial t}+u_x\frac{\partial u_z}{\partial x}+u_y\frac{\partial u_z}{\partial y}+u_z\frac{\partial u_z}{\partial z}\right]=-\frac{\partial P}{\partial z}+\mu\left[\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial y^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+\rho g_z$$