Non-Newtonian Fluids – Power-Law Fluid

A Seattle based company, Starbucks, is planning* to pump its world famous drink, *Café-Laté*, in a slurry pipeline from their new coffee roasting plant in Washington all away to the East coast. This world's first coffee pipeline, 5000 [km] long, will go to the city of New York, (initially) so that citizens of the can finally taste this civilized drink. The coffee roasting plant is located 1400 [m] above the see on the slopes of Mt. Rainier, where fresh and clean water is available in unlimited quantities. Starbucks scientist recently discovered that one can make excellent coffee in a cold water (snow melt from Mt. Rainier), providing that a 3 : 1 mixture (by volume) of water and ground coffee is stirred continuously for 15 days. A group of chemical engineering students from Oregon State University was engaged in this project one fogy, rainy, and perfectly gray morning. They provided basic data for the coffee-water mixture (slurry). They claim that this particular mixture behaves like a power low-fluid; $n = 0.2$, $K = 0.65$ [kg/m s]$^{1.8}$ all in SI units. It is estimated that the daily consumption of *Café-Laté* on this new market is around 1200 [m^3/day]. What will be the pumping cost per one cup (0.0002 [m^3]) of the *Café-Laté* transported this way, if energy cost 0.08 [$/kWh], and if the whole pumping system is 75% efficient? Assume $\rho_{\text{ground coffee}} = 1500$ [kg/m^3].

* According to: "All Things Considered"-- PBS News Program, April 22, 1996

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Point-1 Mt.Renier	Z_1 =	1400	[m]
Point-2 New York City	Z_2 =	0.0	[m]
Mt. Renier to New York Distance	L =	5000	[km]
Coffee - Water Slurry			
Daily consumption of Cafe -Late	$\boxed{\dot{V} =}$	1200	[m^3/day]
Volumetric Flow Ratio	1 part coffee 3 parts water		
Power Law Fluid	$\boxed{\tau = K\left(\frac{\partial u_x}{\partial r}\right)^n}$	$\boxed{\tau = 0.65\left(\frac{\partial u_x}{\partial r}\right)^{0.2}}$	

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SOLUTION : Set Mechanical Energy Balance Equation between New York (2) and Mt. Rainier (1):

$$g\Delta Z + \frac{\Delta \cancel{u}^2}{2} + \frac{\Delta \cancel{P}}{\rho} + W_{Sout} + \Sigma F = 0 \quad \Rightarrow \quad g\left(Z_1 - \cancel{Z_2}\right) = W_{Sout} + \frac{2 f_F L u_2^2}{d}$$

$$W_{Sout} + \frac{2 f_F L u_2^2}{d} = 9.81 \times 1400 = 13734 \left(\frac{J}{kg}\right)$$

Now we have to find ***d***, ***u***, and ***f***$_F$. We know that daily consumption of *Café-Laté* has to be 1200[m^3/day].

$$\dot{V}_{cafe\ \ late} = 1200\left[\frac{m^3}{day}\right] = \frac{1200}{24 \times 3600} = 0.01388\left[\frac{m^3}{s}\right]$$

Café-Laté

$$\dot{V}_{solid\ \ coffe} = \frac{1200}{3} = 400\left[\frac{m^3}{day}\right] = \frac{400}{24 \times 3600} = 0.00463\left[\frac{m^3}{s}\right]$$

Solid Ground Coffee

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$$\dot{V}_{total} = \dot{V}_{solid\ \ coffe} + \dot{V}_{cafe\ \ late} = 400 + 1200 = 1600\left[\frac{m^3}{day}\right] = \frac{1600}{24\times 3600}$$

$$\dot{V}_{total} = 0.0185\left[\frac{m^3}{s}\right]$$ ← Coffee Brew

To calculate the pipe diameter one has to use the recommended mean residence time of the coffee brew in the pipeline, $\bar{t} = 15[days]$

$$\bar{t} = \frac{V_{pipe-line}}{\dot{V}_{total}} = 15[day] \Rightarrow V_{pipe\ \ line} = \bar{t}\times\dot{V}_{total} = 15\times 1600 = 24000\left[m^3\right]$$

$$V_{pipe\ \ line} = \frac{\pi d^2}{4}L \Rightarrow d = \sqrt{\frac{4\cdot 24000}{\pi\cdot 5\cdot 10^6}} = 0.0782[m] \quad \boxed{d = 0.0782[m]}$$

$$\dot{V}_{total} = \frac{\pi d^2}{4}\bar{u} \Rightarrow \bar{u} = \frac{4\dot{V}_{total}}{\pi d^2} = \frac{4\cdot 0.0185}{\pi(0.0782)^2} = 3.85\left[\frac{m}{s}\right] \quad \boxed{\bar{u} = 3.85\left[\frac{m}{s}\right]}$$

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Now one can calculate the friction factor for this power-law fluid.

$$\tau_{xy} = K\left(\frac{\partial u_x}{\partial y}\right)^n = 0.65 \times \left(\frac{\partial u_x}{\partial y}\right)^{0.2}$$

First we calculate generalized Re_{gen} number for which, among other properties need ρ_{mix}, i.e. the density of the coffee brew. We can obtain the density of the coffee brew from the following consideration:

$$\dot{m}_{total} = \dot{m}_{cafe-late} + \dot{m}_{solid\ \ coffe} \quad \Rightarrow \quad \rho_{mixture}\dot{V}_{total} = \rho_{cafe\ \ late}\dot{V}_{cafe\ \ late} + \rho_{solid\ \ coffe}\dot{V}_{solid\ \ coffe}$$

$$\rho_{mixture} = \rho_{cafe\ \ late}\frac{\dot{V}_{cafe\ \ late}}{\dot{V}_{total}} + \rho_{solid\ \ coffe}\frac{\dot{V}_{solid\ \ coffe}}{\dot{V}_{total}} = \rho_{cafe\ \ late} \times 0.75 + \rho_{solid\ \ coffe} \times 0.25$$

$$\rho_{mixture} = 1000 \times 0.75 + 1500 \times 0.25 = 1125\left[\frac{kg}{m^3}\right]$$

$$\boxed{\rho_{mixture} = 1125\left[\frac{kg}{m^3}\right]}$$

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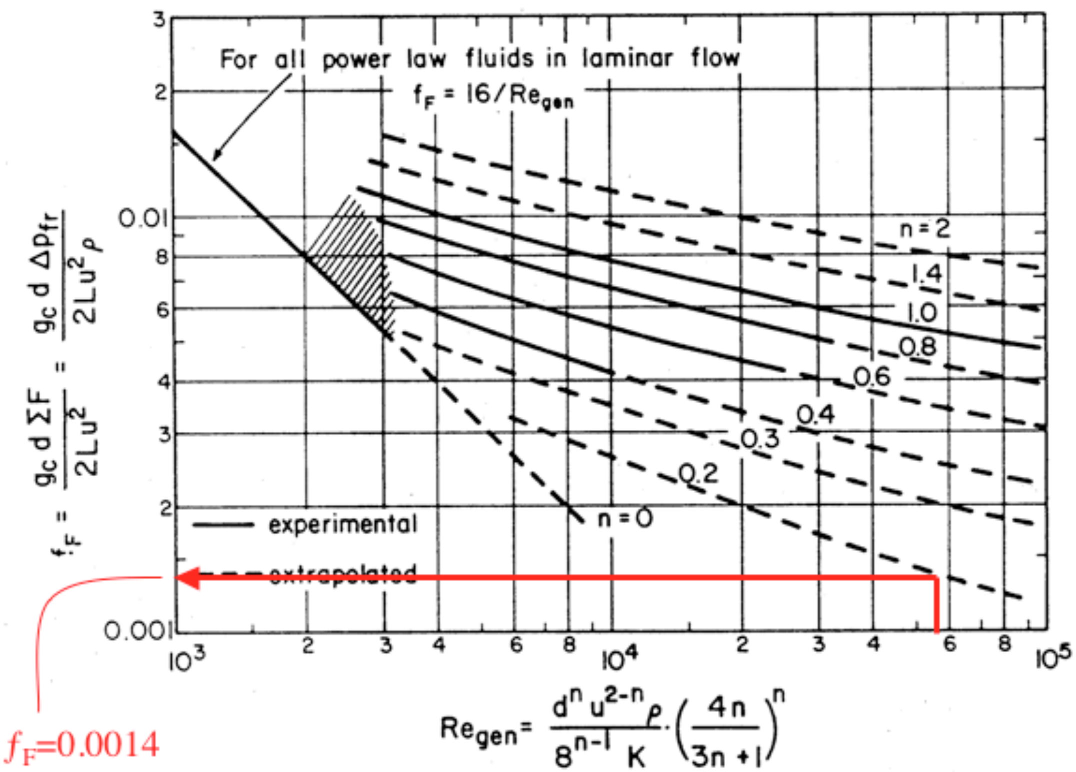
Then, we can calculate Re_{gen}:

$$Re_{gen} = \frac{d^n \ \bar{u}^{2-n} \rho_{mix}}{8^{n-1} K}\left(\frac{4n}{1+3n}\right)^n = \frac{(0.0782)^{0.2} \times 3.85^{2-0.2} \times 1125}{8^{0.2-1} \times 0.65}\left(\frac{4\times 0.2}{1+3\times 0.2}\right)^{0.2}$$

$$\boxed{Re_{gen} = 54272}$$

Now we can read, from the Chart, the value for the f_F:



f_F=0.0014

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Now we can calculate friction losses:

$$\Sigma F = \frac{2 f_F \bar{u}^2 L}{d} = \frac{2 \times 0.0014 \times (3.43)^2 \times 5{,}000{,}000.}{0.0782} = 2{,}665{,}521.$$

$$\boxed{\boxed{\Sigma F = 2{,}665{,}521.\left[\frac{J}{kg}\right]}}$$

And needed shaft work:

$$W_{Sout} = -g\Delta Z - \Sigma F = 13{,}734. - 2{,}655{,}521. = -2{,}651{,}787.\left(\frac{J}{kg}\right)$$

$$\boxed{\boxed{W_{Sout} = -2{,}651.8\left(\frac{kJ}{kg}\right)}}$$

Negative sign on W_{Sout} means that we have to provide this energy through pumping.

$$\cancelto{0}{W_{Sout}} - W_{S-in} = -g\Delta Z - \Sigma F = -2{,}651.8 \quad \Rightarrow \quad \boxed{W_{S-in} = 2{,}651.8\left(\frac{kJ}{kg}\right)}$$

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Power Requirement:

$$P = \frac{\left|W\right|_{Sout} \times \dot{V}_{total} \times \rho_{mixter}}{\eta} = \frac{2651787 \times 0.0185 \times 1125}{0.75} = 73{,}587{,}089.\left[\frac{J}{s}\right]$$

$$\boxed{\boxed{P = 73{,}587.\left[kW\right]}}$$

Energy Cost Per Cup of Café-Laté :

$$E_{energy} = P \times time = 73{,}587.\left[kW\right] \times 24\left[\frac{hours}{day}\right] = 1{,}766{,}090.\left[\frac{kWh}{day}\right]$$

$$C_{\$} = 1{,}766{,}090.\left[\frac{kWh}{day}\right]0.08\left[\frac{\$}{kWh}\right] = 141{,}287.\left[\frac{\$}{day}\right] \qquad \boxed{\boxed{C_{\$} = 141{,}287.\left[\frac{\$}{day}\right]}}$$

$$CC_{\$\$} = \frac{141{,}287.\left[\frac{\$}{day}\right]}{1200\left[\frac{\cancel{m}^3 cafe\ late}{day}\right]1000\left[\frac{\cancel{l}}{\cancel{m}^3}\right]5\left[\frac{cups}{\cancel{l}}\right]} \Rightarrow \boxed{\boxed{CC_{\$\$} = 0.024\left[\frac{\$}{cup}\right]}}$$

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Thank you for your attention!