

**OREGON STATE UNIVERSITY**  
**CBEE Department of Chemical Engineering**

**CHE 331**  
**Transport Phenomena1**

**Dr. Goran Jovanovic**

**Mechanical Energy Balance Equation**  
**Flow Through Fluidized Beds**

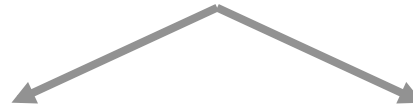
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## MECHANICAL ENERGY BALANCE EQUATION

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### Fluidized Bed Operations

There are two types of Fluidized Bed applications



#### Fluidized Bed Reactors

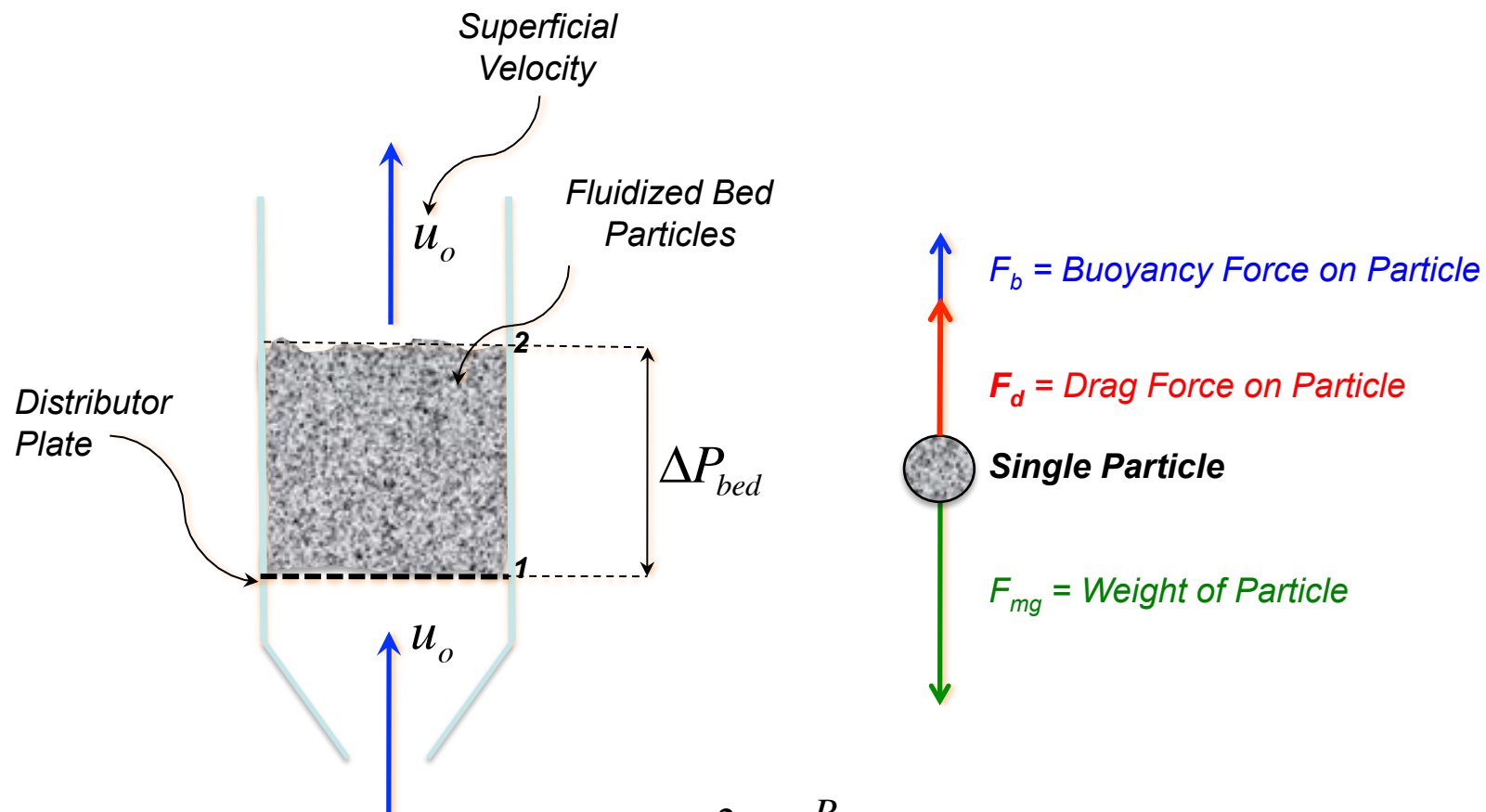
1. Oil Cracking
2. Coal Burning
3. Pyrolysis of FeS
4. Fuel Synthesis
5. Penicillin Production
6. ....
7. ....

#### Fluidized Bed Operations

1. Drying of solids
2. Heat recuperation
3. Mixing of Particles
4. Separation of Particles
5. Coating of Particles
6. Adsorption
7. Cooling of fluids
8. ....

# MECHANICAL ENERGY BALANCE EQUATION

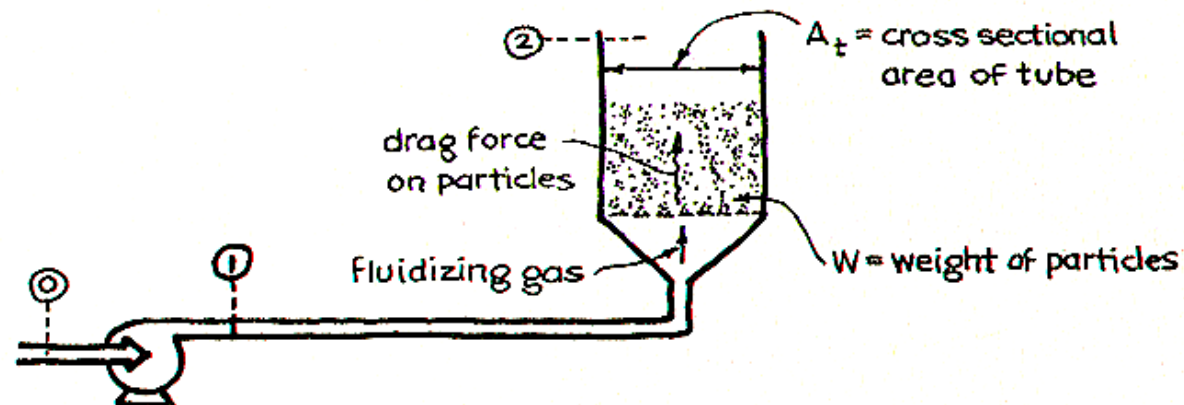
## Fluidized Bed Operations



$$g\Delta Z + \frac{\Delta \bar{u}^2}{2} + \int_{P_1}^{P_2} \frac{dP}{\rho} + W_{Sout} + \sum F = 0$$

# MECHANICAL ENERGY BALANCE EQUATION

## Fluidized Bed Operations



**Frictional  
Pressure  
Drop**

x

**Cross  
Sectional  
Area of Bed**

=

**Volume  
of Bed**

x

**Fraction of Bed  
Consisting of  
Solids**

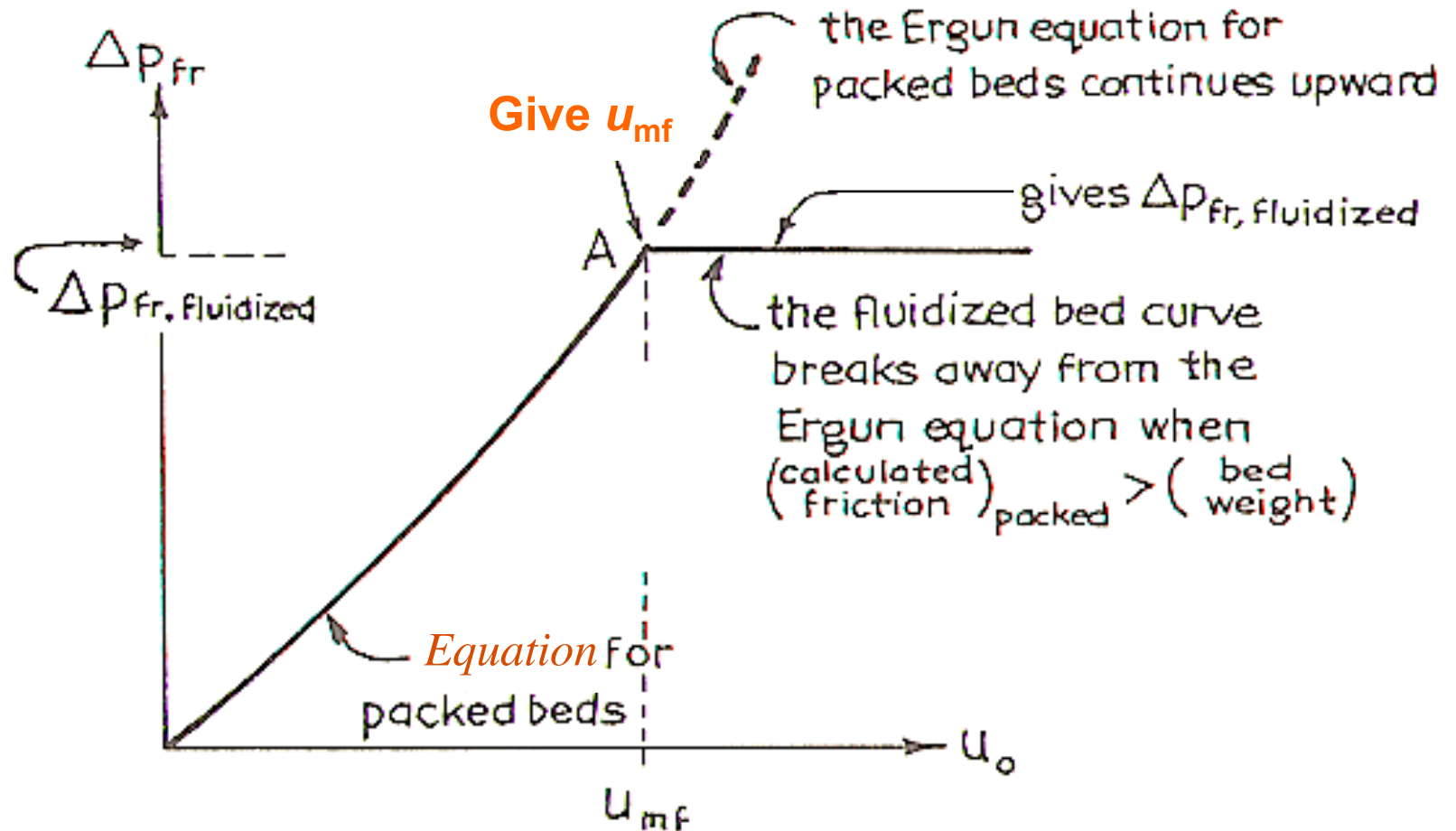
x

**Specific  
Buoyant Weight  
of Solids**

$$\Delta P_{Bed} \times \left( \frac{\pi D_{Bed}^2}{4} \right) = \left( \frac{\pi D_{Bed}^2}{4} \right) \cdot L_{Bed} \times (1 - \epsilon) \times [(\rho_{par} - \rho_{flu}) \cdot g]$$

# MECHANICAL ENERGY BALANCE EQUATION

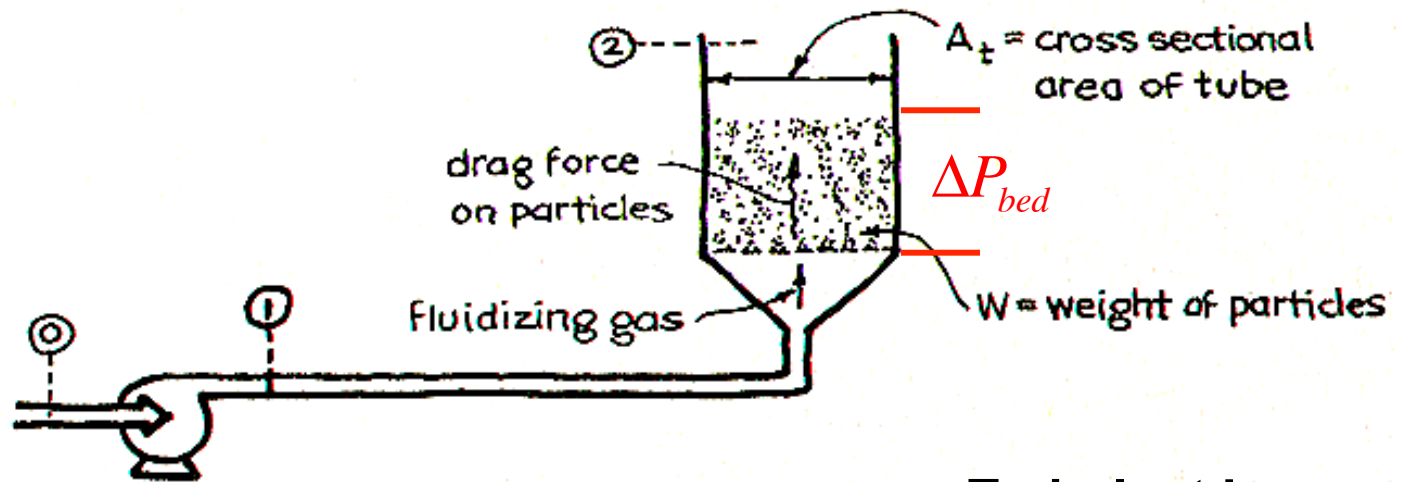
## Fluidized Bed Operations



## MECHANICAL ENERGY BALANCE EQUATION

### Fluidized Bed Operations

Consider Ergun Equation at the moment just before the fluidization occurs; i.e. while the bed is still packed bed.



Turbulent Losses

$$\sum F = \frac{150(1 - \epsilon_{mf})^2 \mu u_{mf} L_{mf}}{\epsilon_{mf}^3 d_{part}^2 \rho} + \frac{1.75(1 - \epsilon_{mf}) u_{mf}^2 L_{mf}}{\epsilon_{mf}^3 d_{part}}$$

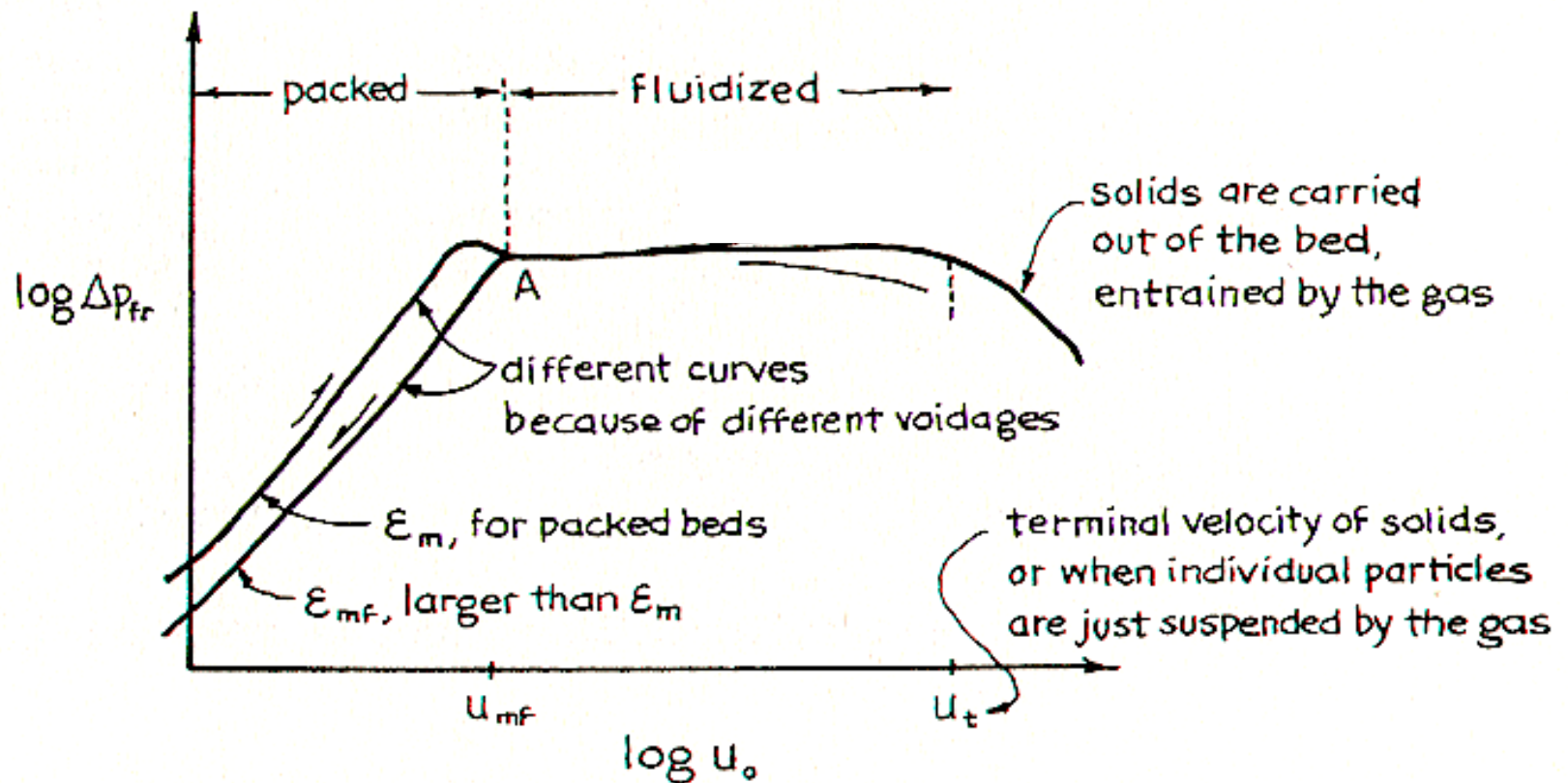
*Eq.1*



Viscous Losses

# MECHANICAL ENERGY BALANCE EQUATION

## Fluidized Bed Operations



## MECHANICAL ENERGY BALANCE EQUATION

### Fluidized Bed Operations

$$\left\{ \begin{array}{l} \text{Total Drag Force between} \\ \text{fluid on particles} \end{array} \right\} = \left\{ \begin{array}{l} \text{Weight of Solids} \\ \text{in the bed} \end{array} \right\}$$

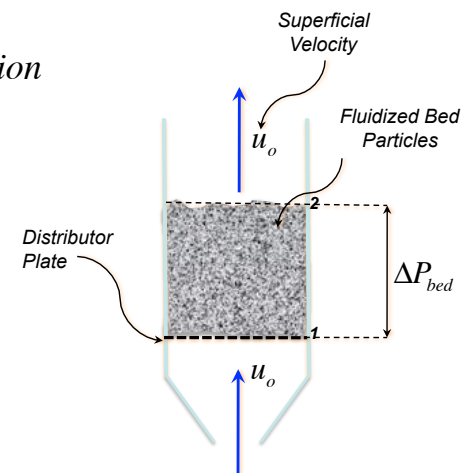
$$\left\{ \Delta P_{friction} A_t \right\} = \left\{ A_t L_{mf} (1 - \varepsilon_{mf}) (\rho_{part} - \rho_{fluid}) g \right\} \quad \text{Eq.2}$$

From the MEB equation we can estimate:  $\Delta P_{friction}$

$$g\Delta Z + \frac{\Delta \bar{u}^2}{2} + \int_{P_1}^{P_2} \frac{dP}{\rho} + \cancel{W_{Sout}} + \sum F = 0$$

If we assume that:  $\Delta P_{bed} < 0.1 P_{average}$

$$g\Delta Z + \frac{\Delta P}{\rho} + \sum F = 0 \Rightarrow \sum F = -g\Delta Z - \frac{\Delta P}{\rho}$$





## MECHANICAL ENERGY BALANCE EQUATION

### Fluidized Bed Operations

For gas-solid system: 
$$\sum F = -g\Delta Z - \frac{\Delta P}{\rho}$$

Now we can substitute **Eq.1** and **Eq.2** into above equation and obtain:

$$\frac{150(1-\epsilon_{mf})^2 \mu u_{mf} L_{mf}}{\epsilon_{mf}^3 d_{part}^2} + \frac{1.75 \rho (1-\epsilon_{mf}) u_{mf}^2 L_{mf}}{\epsilon_{mf}^3 d_{part}} =$$

$$= \left\{ L_{mf} (1-\epsilon_{mf}) (\rho_{part} - \rho_{fluid}) g \right\}$$

$$\frac{150(1-\epsilon_{mf})}{\epsilon_{mf}^3} \text{Re}_{p,mf} + \frac{1.75}{\epsilon_{mf}^3} \text{Re}_{p,mf}^2 = \frac{d_p^3 \rho_f (\rho_p - \rho_f) g}{\mu^2}$$

## MECHANICAL ENERGY BALANCE EQUATION

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For predominantly viscous/laminar flow:

$$u_{mf} = \frac{d_p^2 (\rho_p - \rho_f)}{150\mu} \frac{g\varepsilon_{mf}^3}{1 - \varepsilon_{mf}} \quad \text{for } Re_{p,mf} < 20$$

For predominantly turbulent flow:

$$u_{mf}^2 = \frac{d_p (\rho_p - \rho_f)}{1.75\rho_f} g\varepsilon_{mf}^3 \quad \text{for } Re_{p,mf} > 1000$$

And for any Re:

$$\frac{d_p u_{mf} \rho_f}{\mu} = \sqrt{28.7^2 + \frac{0.0494 d_p^3 \rho_f (\rho_p - \rho_f) g}{\mu^2}} - 28.7$$

## MECHANICAL ENERGY BALANCE EQUATION

### Fluidized Bed Operations

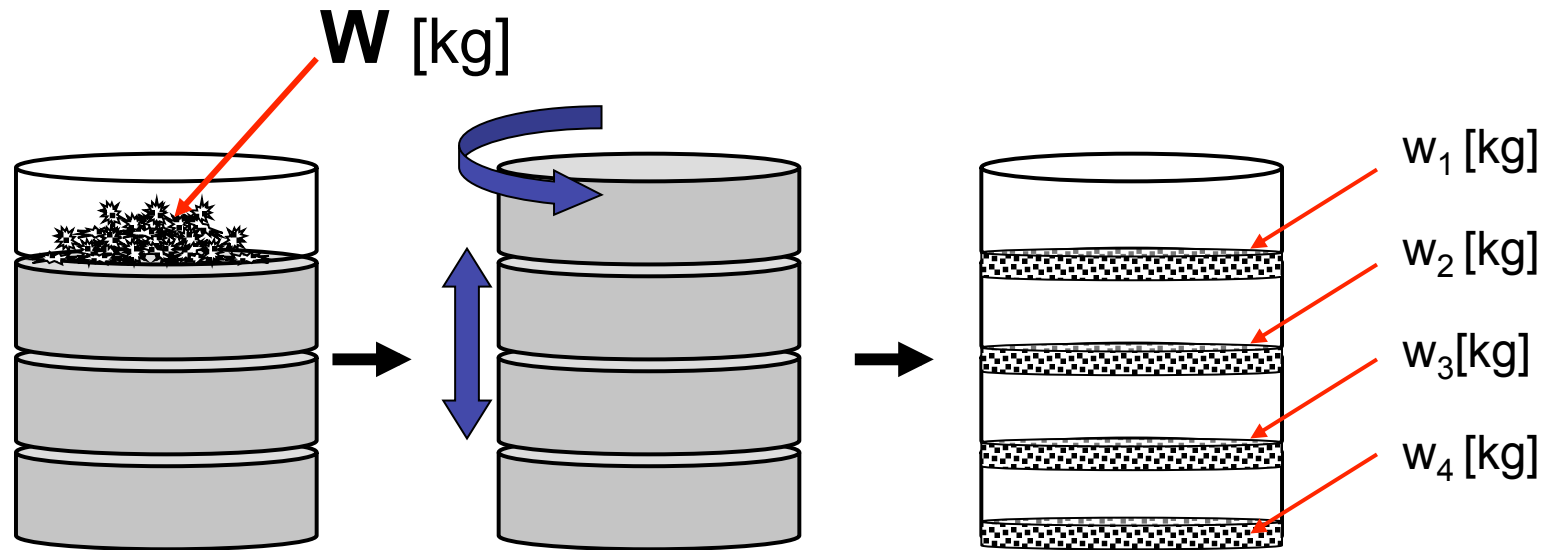
$$1 = \frac{V_{particles}}{V_{Total}} + \frac{V_{fluid}}{V_{Total}} \Rightarrow 1 = \delta + \varepsilon = (1 - \varepsilon) + \varepsilon$$

$$Voidage = \frac{V_{fluid}}{V_{Total}} = \varepsilon$$

Voidage

It is also true:  $Voidage = \frac{A_{fluid}}{A_{Total}} = \varepsilon$

## MECHANICAL ENERGY BALANCE EQUATION – Particle Size



$$\left. \begin{aligned} x_1 &= \frac{w_1}{W}; \quad x_2 = \frac{w_2}{W}; \quad x_3 = \frac{w_3}{W}; \quad x_4 = \frac{w_4}{W} \\ d_{scr.i} &= \frac{(\text{upper screen size} + \text{lower screen size})}{2} \\ d_{pi} &= d_{scr.i} \times \Phi \end{aligned} \right\} \bar{d}_p = \frac{1}{\sum_{\text{all size cuts}} \left( \frac{x_i}{d_{pi}} \right)}$$

## MECHANICAL ENERGY BALANCE EQUATION – Particle Size

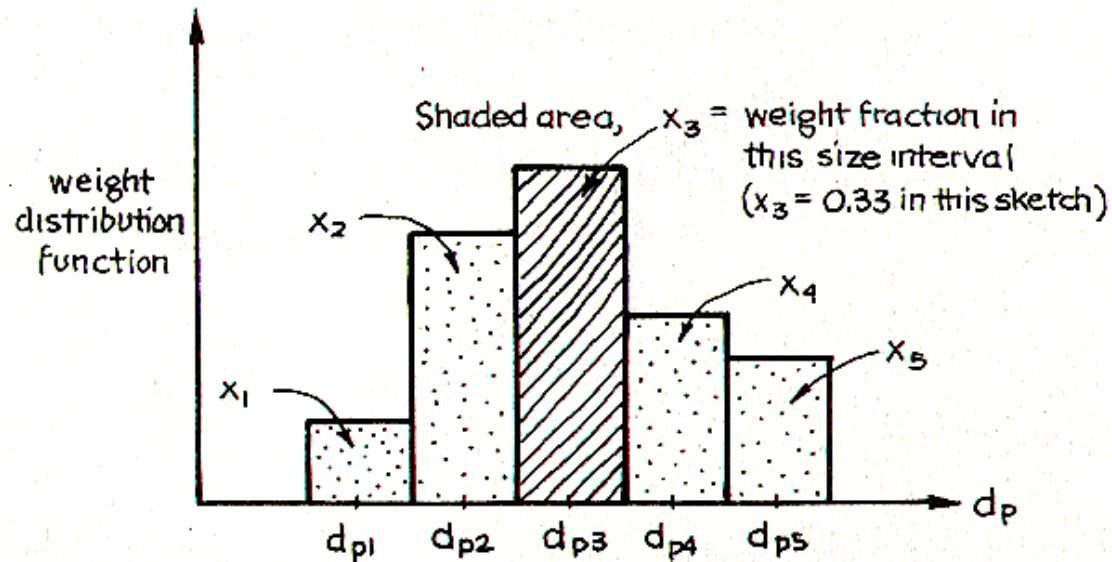


Fig. 6.2. Histogram representing the size distribution of particles in a packed bed.

$$\bar{d}_p = \left( \begin{array}{l} \text{the nominal size of particles which would have the same total} \\ \text{surface area as the size mixture in question - same total bed} \\ \text{volume and same bed voidage in both cases} \end{array} \right)$$

$$\bar{d}_p = \frac{1}{\sum_{\text{all size cuts}} \left( \frac{x_i}{d_{pi}} \right)}$$

## MECHANICAL ENERGY BALANCE EQUATION

### Tyler Standard Screen Sizes

Mesh number (number of wires/in)	Aperture, $\mu\text{m}$ (opening between adjacent wires)	Mesh number (number of wires/in)	Aperture, $\mu\text{m}$ (opening between adjacent wires)
3	6680	35	417
4	4699	48	295
6	3327	65	208
8	2362	100	147
10	1651	150	104
14	1168	200	74
20	833	325	53
28	589	400	38

Unfortunately there is no general relationship between  $d_{\text{scr}}$  and  $d_p$ . The best we can say for pressure drop considerations in packed beds is

- for irregular particles with no seeming long or shorter dimension take

$$d_p \cong \phi d_{\text{scr}} = \phi d_{\text{sph}}$$



## MECHANICAL ENERGY BALANCE EQUATION – Particle Size

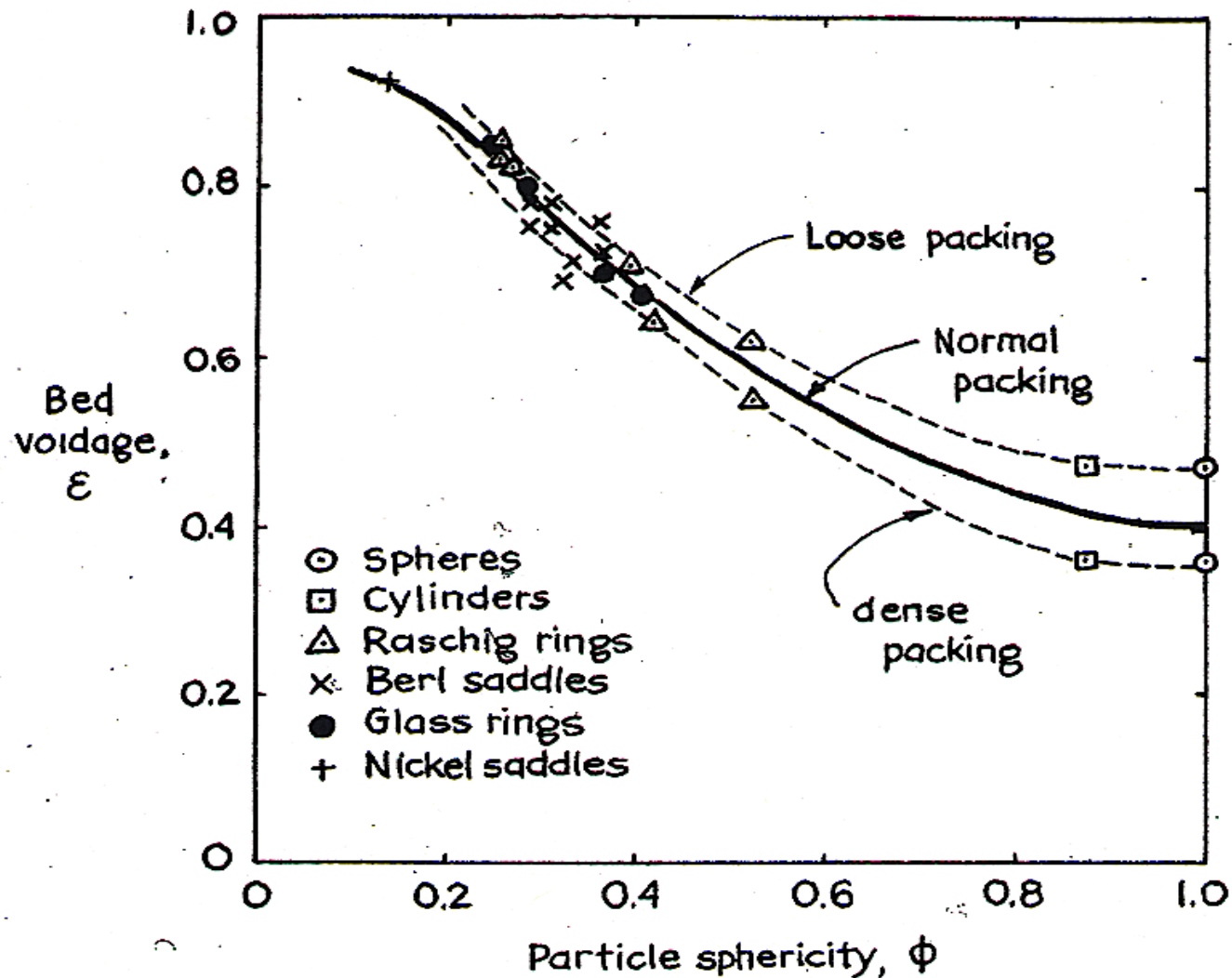
Particle shape	Sphericity $\phi$
Sphere	1.00
Cube	0.81
Cylinder	
$h = d$	0.87
$h = 5d$	0.70
$h = 10d$	0.58
Disks	
$h = d/3$	0.76
$h = d/6$	0.60
$h = d/10$	0.47
Old beach sand	As high as 0.86
Young river sand	As low as 0.53
Average for various types of sand	0.75
Crushed solids	0.5–0.7
Granular particles	0.7–0.8
Wheat	0.85
Raschig rings	0.26–0.53
Berl saddles	0.30–0.37
Nickel saddles	0.14

$$\Phi = \left( \frac{\text{Surface of Sphere}}{\text{Surface of Particle}} \right)_{\text{SAME VOLUME}}$$
$$\Phi \leq 1$$

<sup>a</sup>Data from Brown (1950), and from geometrical considerations.



## MECHANICAL ENERGY BALANCE EQUATION



*The voidage increases as the sphericity decreases for randomly packed beds of uniform particles.*





*People. Ideas. Innovation.*

*Thank you for your attention!*