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OREGON STATE UNIVERSITY
CBEE
DEPARTMENT OF CHEMICAL ENGINEERING

CHE 331
Transport Phenomena I

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Mechanical Energy Balance Equation I

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Fall 2022

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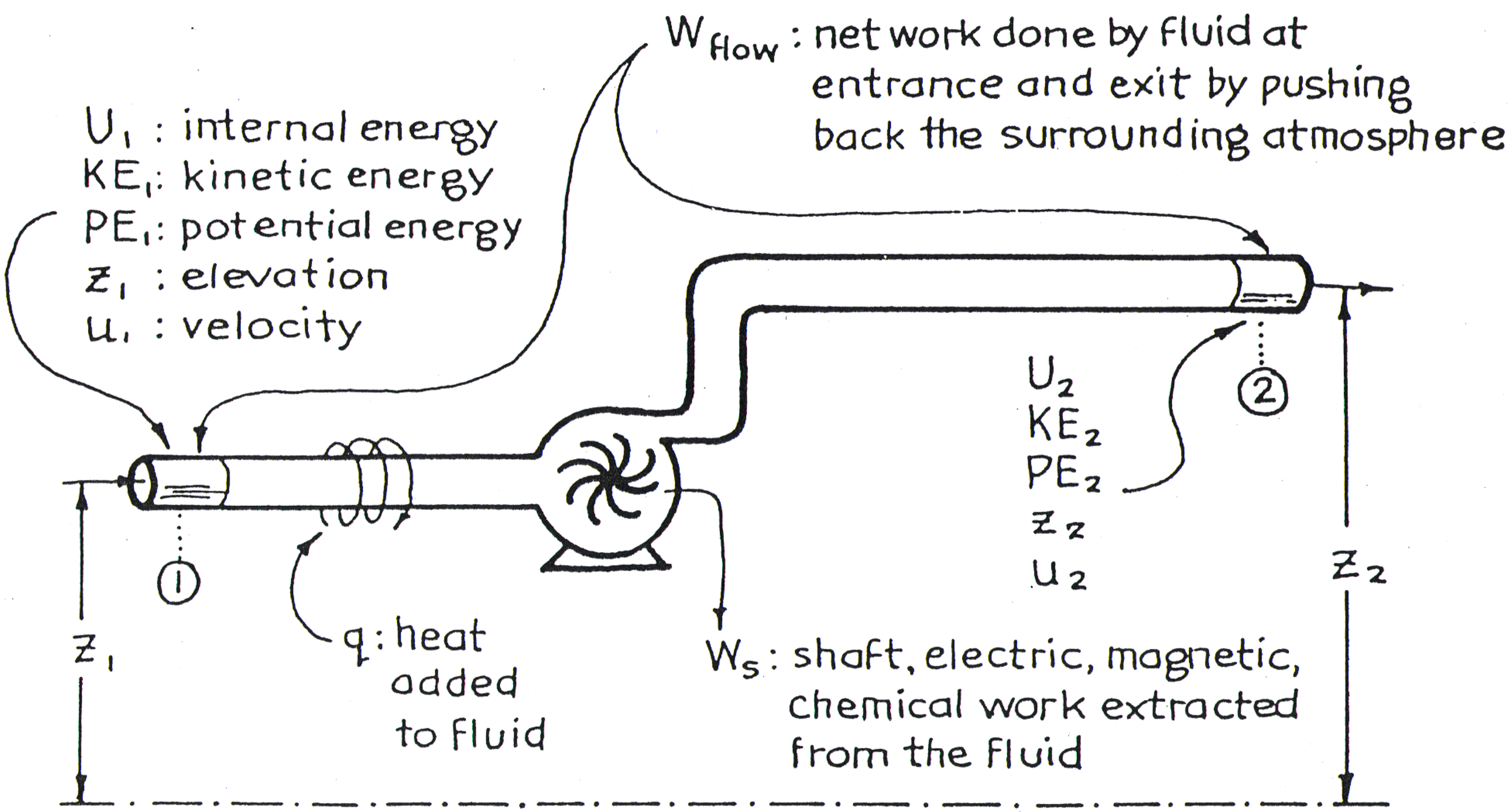
MECHANICAL ENERGY BALANCE EQUATION

INTERNAL ENERGY(*U*) is the energy of a substance associated with the motions, interactions and bonding of its constituent molecules rather than,

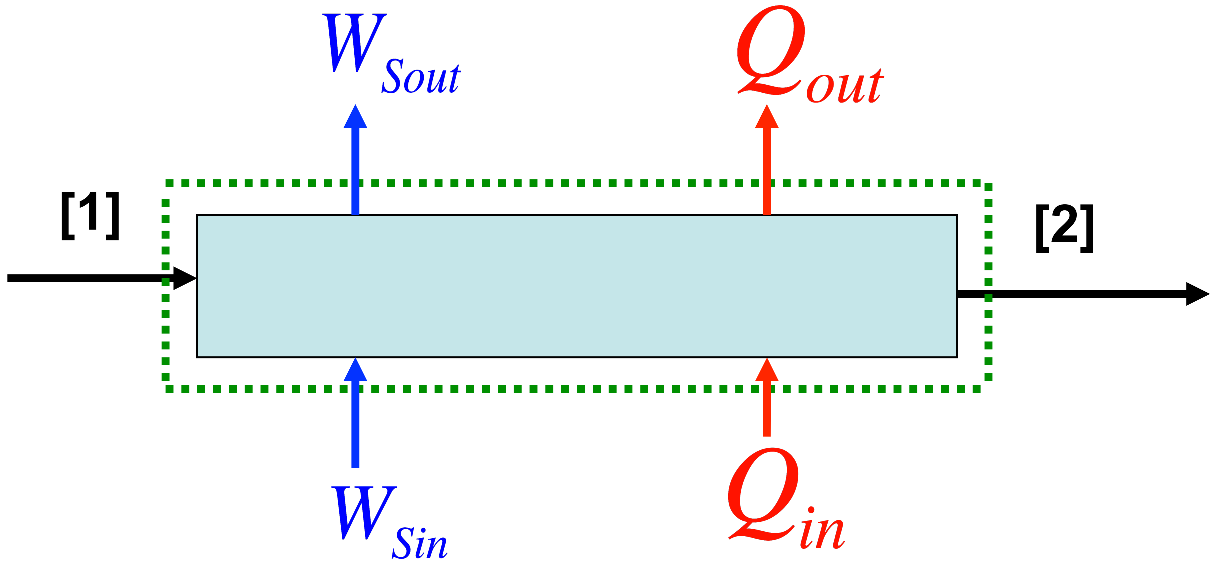
EXTERNAL ENERGY, which is associated with the velocity and location of its center of mass which is of primary interest in mechanics.

Flow System



O. Levenspiel, *Engineering Flow and Heat Exchange,* Plenum Press1988

Mechanical Energy Balance [MEB] Equation



Energy Balance – Conservation of Energy [J]
(at steady state):

$$\text{Input} - \text{Output} = \cancel{\text{Accumulation}}^{\,0}$$

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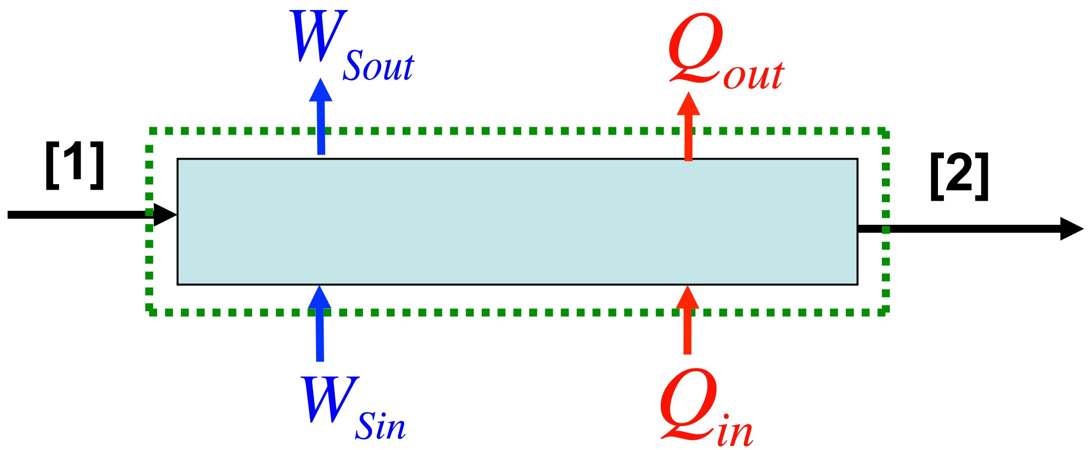
Mechanical Energy Balance [MEB] Equation



Energy Balance: Input – Output = 0

$$\overbrace{mU_1 + mgZ_1 + \frac{mu_1^2}{2} + (PV)_1 + W_{Sin} + Q_{in}}^{Input} - \underbrace{\left(mU_2 + mgZ_2 + \frac{mu_2^2}{2} + (PV)_2 + Q_{out} + W_{Sout} \right)}_{Output} = 0 \quad [J]$$

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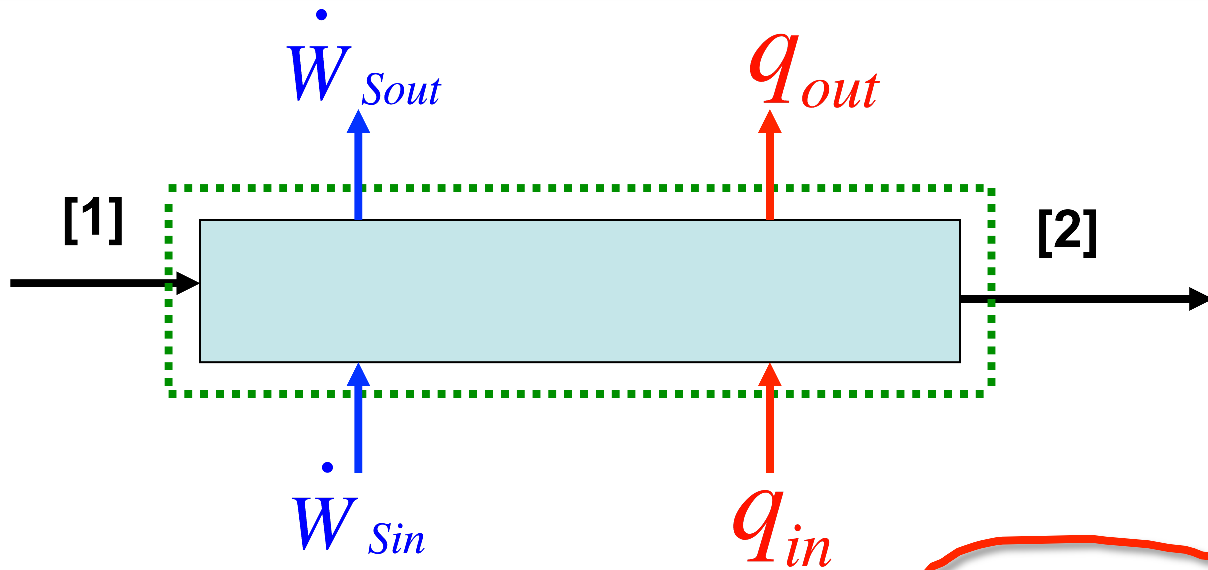
Mechanical Energy Balance [MEB] Equation



Energy Balance [J/kg] (steady state):

$$\frac{PV}{m}=\frac{P}{\rho}=P\vartheta$$

$$\overbrace{\frac{U_1}{m}+\frac{mgZ_1}{m}+\frac{mu_1^2}{2m}+\left(\frac{PV}{m}\right)_1+\frac{W_{Sin}}{m}+\frac{Q_{in}}{m}}^{Input}-\underbrace{\left(\frac{U_2}{m}+\frac{mgZ_2}{m}+\frac{mu_2^2}{2m}+\left(\frac{PV}{m}\right)_2+\frac{Q_{out}}{m}+\frac{W_{Sout}}{m}\right)}_{Output}=0\ \left[\frac{J}{kg}\right]$$

$$mU_1+mgZ_1+\frac{mu_1^2}{2}+(PV)_1+W_{Sin}+Q_{in}-\left(mU_2+mgZ_2+\frac{mu_2^2}{2}+(PV)_2+Q_{out}+W_{Sout}\right)=0\ [J]$$

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Mechanical Energy Balance [MEB] Equation

$$-(U_2 - U_1) - g(Z_2 - Z_1) - \frac{1}{2}(u_2^2 - u_1^2) - [(P\vartheta)_2 - (P\vartheta)_1] -$$

$$(q_{out} - q_{in}) - (\dot{W}_{Sout} - \dot{W}_{Sin}) = 0$$

Let Δ be (=) (2 - 1) or (out - in)

Shaft Work

Heat

$$\Delta U + g\Delta Z + \frac{1}{2}\Delta u^2 + \Delta(P\vartheta) + \Delta q + \Delta \dot{W} = 0$$

Flow Work

Kinetic Energy

Potential Energy

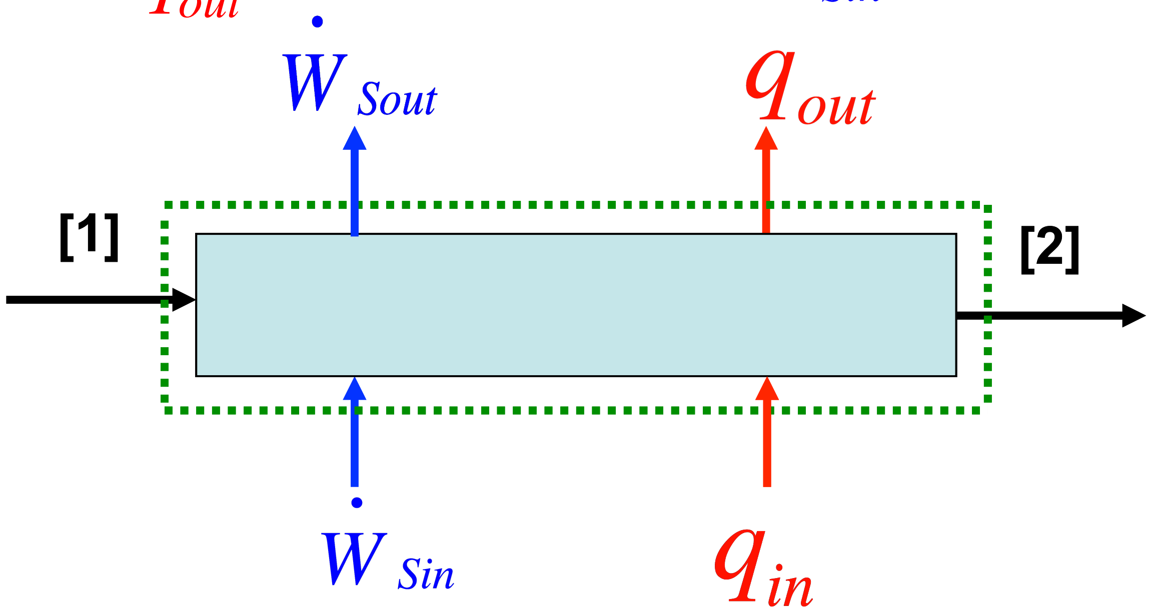
Internal Energy

Remember: $\Delta q = q_{out} - q_{in}$ *and* $\Delta \dot{W} = \dot{W}_{Sout} - \dot{W}_{Sin}$

$$\Delta U + g\Delta Z + \frac{1}{2}\Delta u^2 + \Delta(P\vartheta) = -\Delta q - \Delta \dot{W}$$

Often people consider (in the spirit of an old thermodynamic tradition) that:

$$q_{out} = 0 \quad \textit{and} \quad \dot{W}_{Sin} = 0$$



Mechanical Energy Balance [MEB] Equation

Which converts the above equation into:

$$\Delta U + g\Delta Z + \frac{1}{2}\Delta u^2 + \Delta(P\vartheta) = q_{in} - \dot{W}_{Sout}$$

We could rewrite the last equation:

$$[\Delta U + \Delta(P\vartheta)] + g\Delta Z + \frac{1}{2}\Delta u^2 = q_{in} - \dot{W}_{Sout}$$

$$\Delta H + g\Delta Z + \frac{1}{2}\Delta u^2 = q_{in} - \dot{W}_{Sout}$$

Because: $\Delta H = \Delta U + \Delta(P\vartheta)$

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Second Law of Thermodynamics

From the Second Law of Thermodynamics we know

$$\Delta U = \int T\,dS - \int P\,d\vartheta + [all\ \ other\ \ forms\ \ of\ \ energy]$$

The term $\int T\,dS$ accounts for heat exchange with surrounding and heat generated by friction losses.

$$\int T\,dS = q_{in} + \Sigma F$$

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$$\Delta U + g\Delta Z + \frac{1}{2}\Delta u^2 + \Delta(P\vartheta) = q_{in} - \dot{W}_{Sout}$$

$$\left[\int T\,dS - \int P\,d\vartheta\right] + g\Delta Z + \frac{1}{2}\Delta u^2 + \Delta(P\vartheta) = q_{in} - \dot{W}_{Sout}$$

$$\left[\cancel{q_{in}} + \sum F - \int P\,d\vartheta\right] + g\Delta Z + \frac{1}{2}\Delta u^2 + \Delta(P\vartheta) = \cancel{q_{in}} - \dot{W}_{Sout}$$

We can differentiate the above expression:

$$\delta\left(\sum F\right) - \cancel{P\,d\vartheta} + g\,dZ + u\,du + \cancel{P\,d\vartheta} + \vartheta\,dP = -\delta\left(\dot{W}_{Sout}\right)$$

Mechanical Energy Balance [MEB] Equation

And we can obtain the ***differential form*** of the Mechanical Energy Balance Equation:

$$gdZ + udu + \vartheta dP + \delta\left(\sum F\right) + \delta\left(\dot{W}_{Sout}\right) = 0$$

$$gdZ + udu + \frac{dP}{\rho} + \delta\left(\sum F\right) + \delta\left(\dot{W}_{Sout}\right) = 0$$

Or in the ***difference form*** and for constant density:

$$g\Delta Z + \frac{\Delta u^2}{2} + \frac{\Delta P}{\rho} + \sum F + \dot{W}_{Sout} = 0$$

Mechanical Energy Balance [MEB] Equation

$$g\Delta Z + \frac{\Delta u^2}{2} + \frac{\Delta P}{\rho} + \sum F + \dot{W}_{Sout} = 0$$

$$\updownarrow \updownarrow \updownarrow$$

$$\boxed{g(Z_2 - Z_1) + \frac{(u_2^2 - u_1^2)}{2} + \frac{(P_2 - P_1)}{\rho} + \sum F + \dot{W}_{Sout} = 0}$$

Mechanical Energy balance Equation

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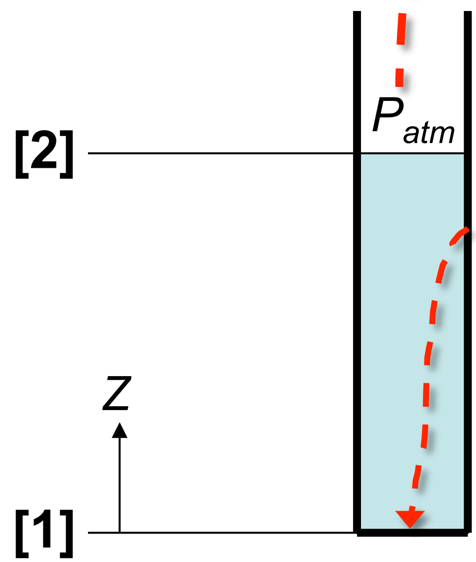
Consider the following Static Fluid Situation:



$$P_1 = P_{atm} + \rho g Z_2$$

$$P_1 = P_2 + \rho g Z_2$$

$$g\left(Z_2 - \cancel{Z_1}\right) + \cancel{\frac{\left(u_2^2 - u_1^2\right)}{2}} + \frac{\left(P_2 - P_1\right)}{\rho} + \cancel{\sum F} + \cancel{\dot{W}_{s\,out}} = 0$$

$$g\left(Z_2\right) + \frac{\left(P_2 - P_1\right)}{\rho} = 0$$

$$\rho g\left(Z_2\right) + \left(P_2 - P_1\right) = 0$$

(Dr. Levenspiel Book contains all equations for MEB)

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Thank you for your attention!