Oregon State University

OREGON STATE UNIVERSITY

School of Chemical, Biological, and Environmental Engineering

CHE 331
Transport Phenomena I

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Flow Through Porous Media
Darcy and Brinkman Equations

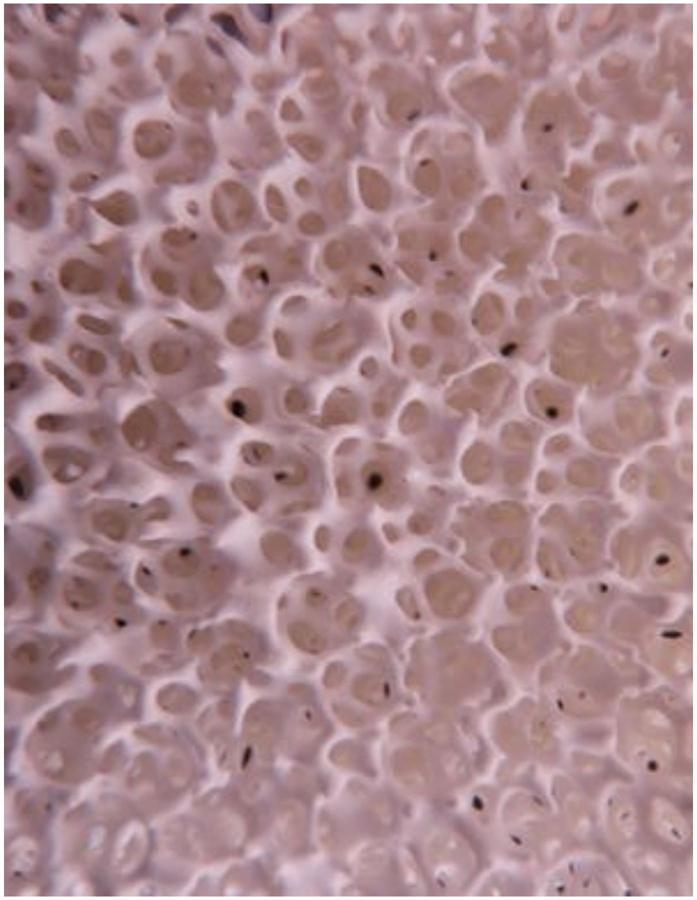
Please turn-off cell phones

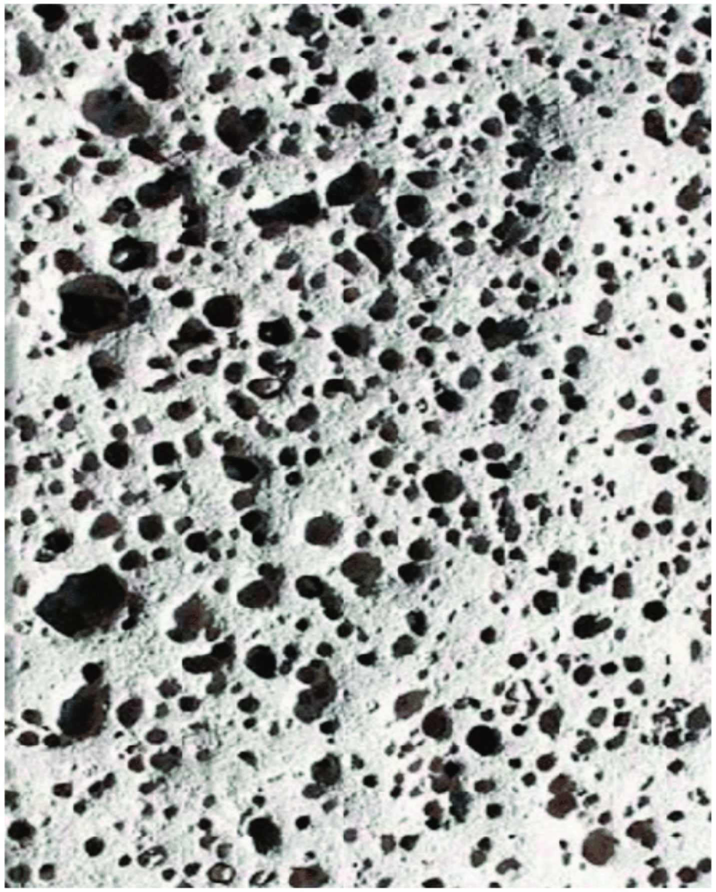
CHE 331
Fall 2022

Flow Through Consolidated Media

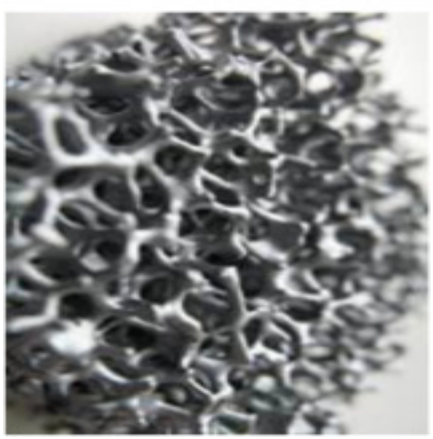
Not all porous media are made of particles; such as flow of groundwater in aquifers, flow of gas in petroleum reservoirs, etc. We refer to these cemented solids as consolidated media. We use basic principals to model flow through consolidated materials.

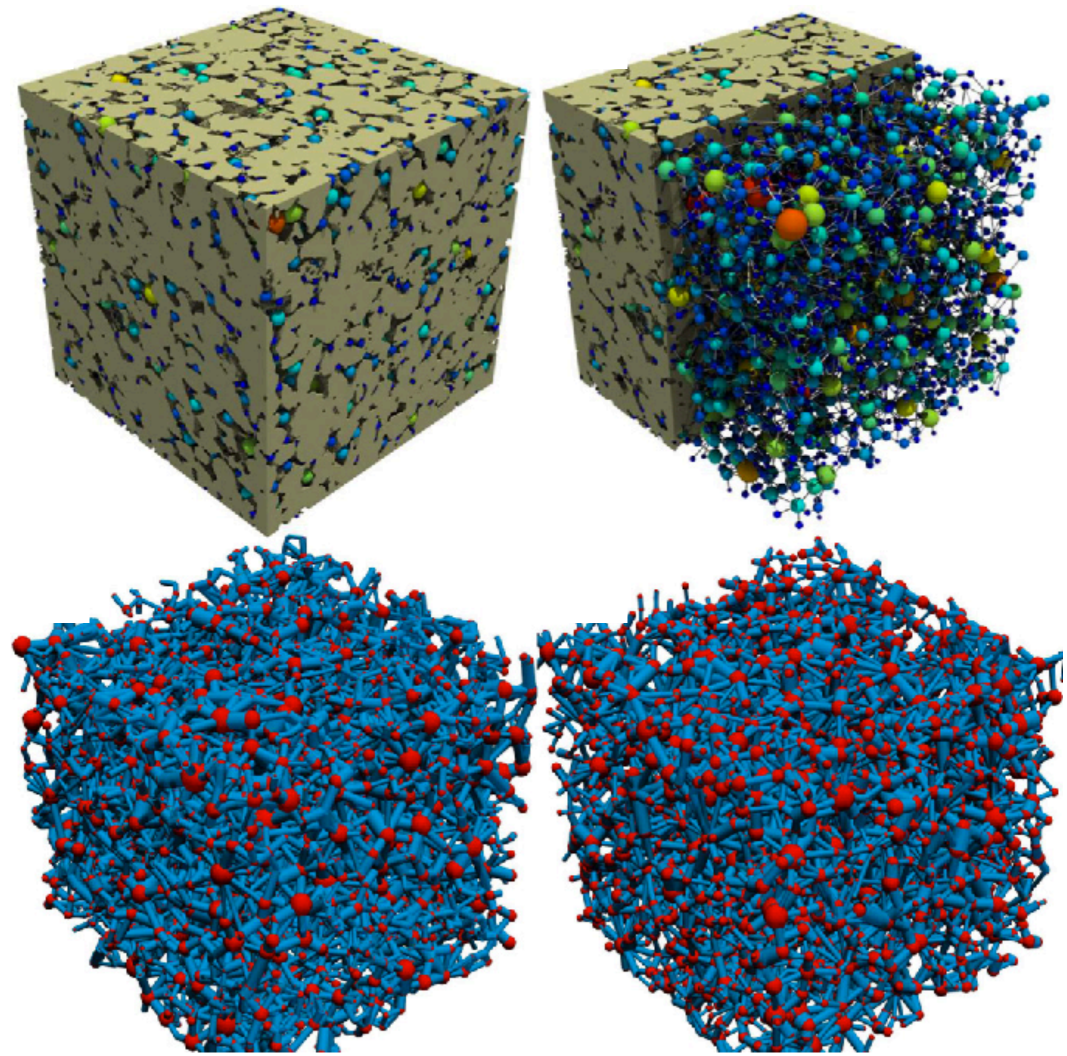
Consolidated Media





Consolidated Media





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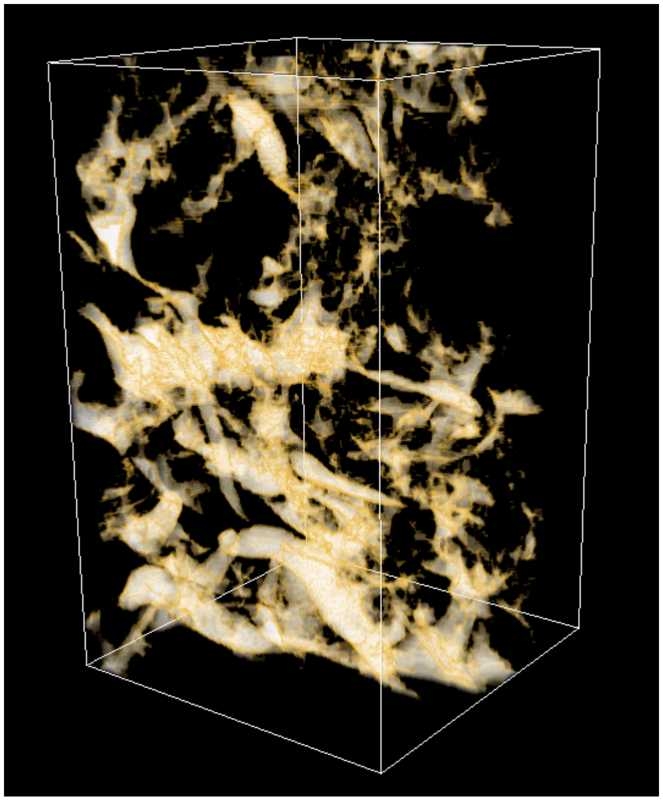

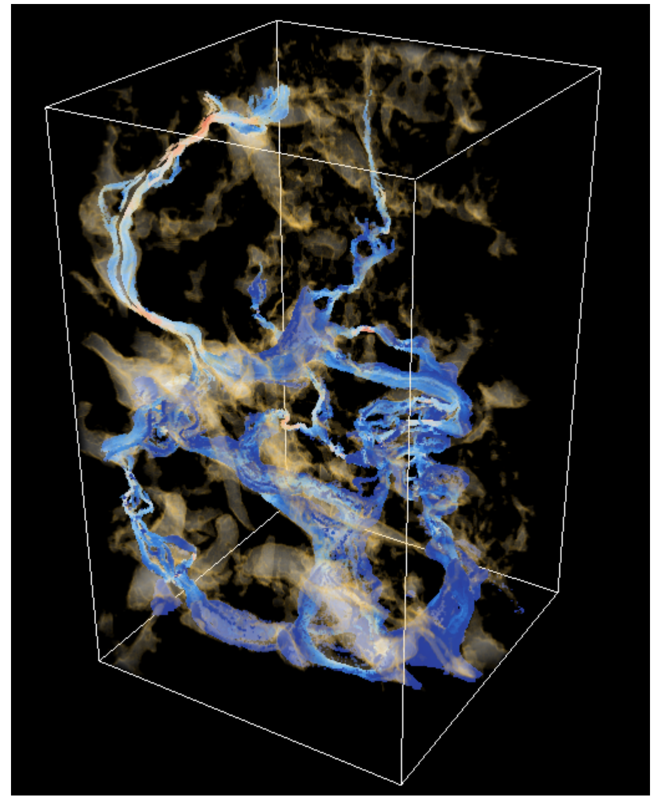

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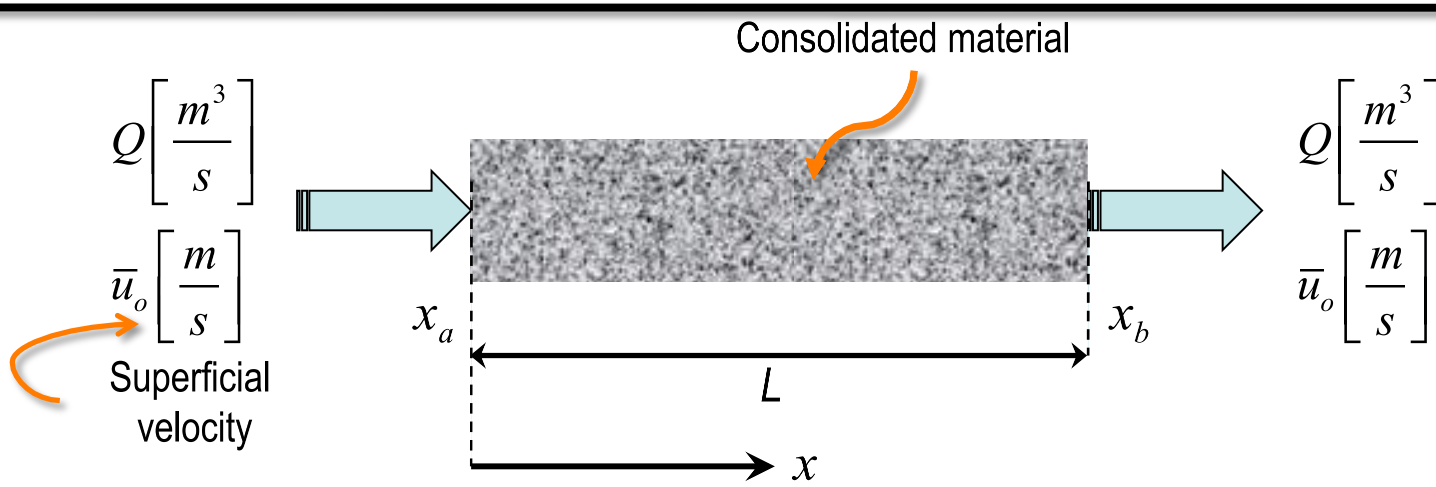
Darcy's Law for Laminar Flow through Porous Media



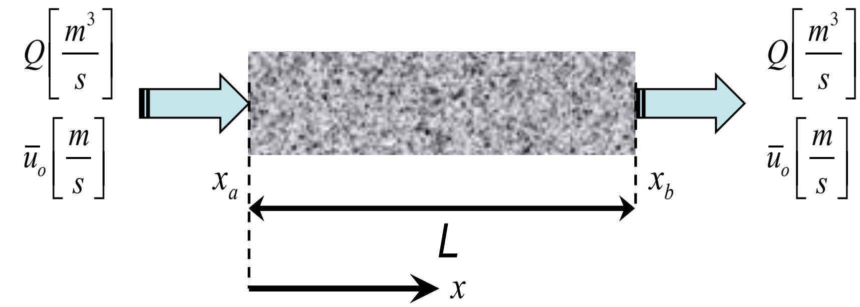
Darcy's Law is (actually not a Law) but an equation that describes the flow of a fluid through a porous medium. Henry Darcy derived the equation from the experimental results (*Q vs.* ΔP) of water flow through sand beds. This equation is used in hydrogeology, civil engineering, and other applications.

$$\vec{u}_o = -\frac{k}{\mu}\nabla P \quad \Rightarrow \quad \vec{u}_{ox} = -\frac{k}{\mu}\frac{dP}{dx} \qquad \text{(one dimensional case)}$$

$$\left(\frac{m}{s}\right) \sim \frac{(m^2)}{(Pa \cdot s)}\frac{(Pa)}{(m)}$$

Darcy's Law for Laminar Flow through Porous Media

$$\vec{u}_{ox} = -\frac{\kappa}{\mu}\frac{dP}{dx} \Rightarrow \vec{u}_{ox}\int_{x=0}^{x=L} dx = -\frac{\kappa}{\mu}\int_{P_o}^{P_L} dP \Rightarrow \vec{u}_{ox} = -\frac{\kappa}{\mu}\frac{\Delta P}{L}$$



κ = permeability of porous media $\left[m^2\right]$

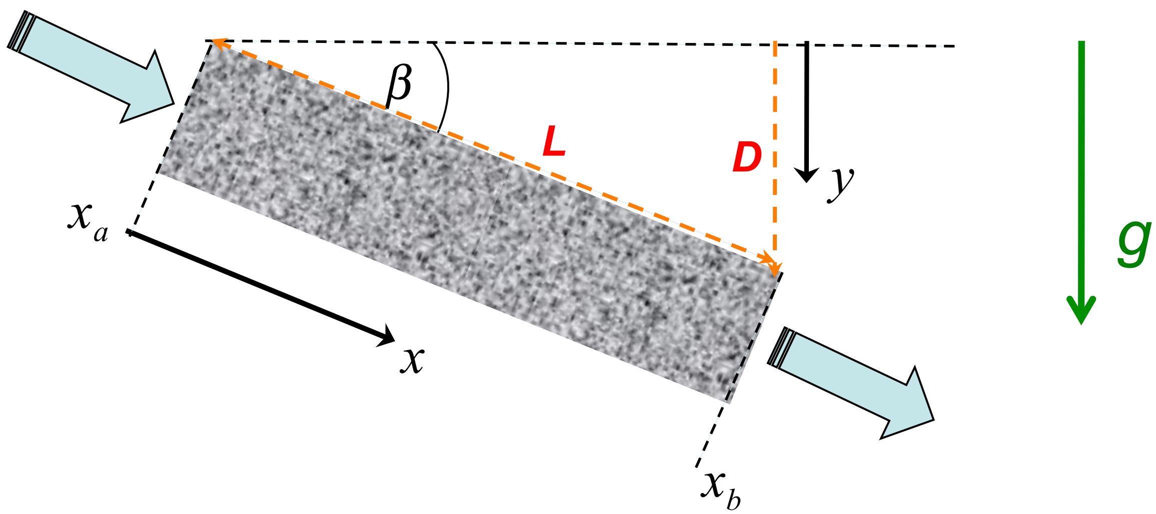
Permeability of porous media is function of porous structure, voidage, size distributions of pores, tortuosity, nature of porous media (particles, consolidated media, fibers, . . .)

$$\kappa = C \cdot \bar{d}_p^2 \qquad \kappa = \bar{d}_f^2 \cdot C_1\left(\sqrt{\frac{1-\varepsilon_c}{1-\varepsilon}} - 1\right)^{C_2}$$

where d_p is the average pore diameter, and d_f is the average fiber diameter.

Darcy's Law for Laminar Flow through Porous Media

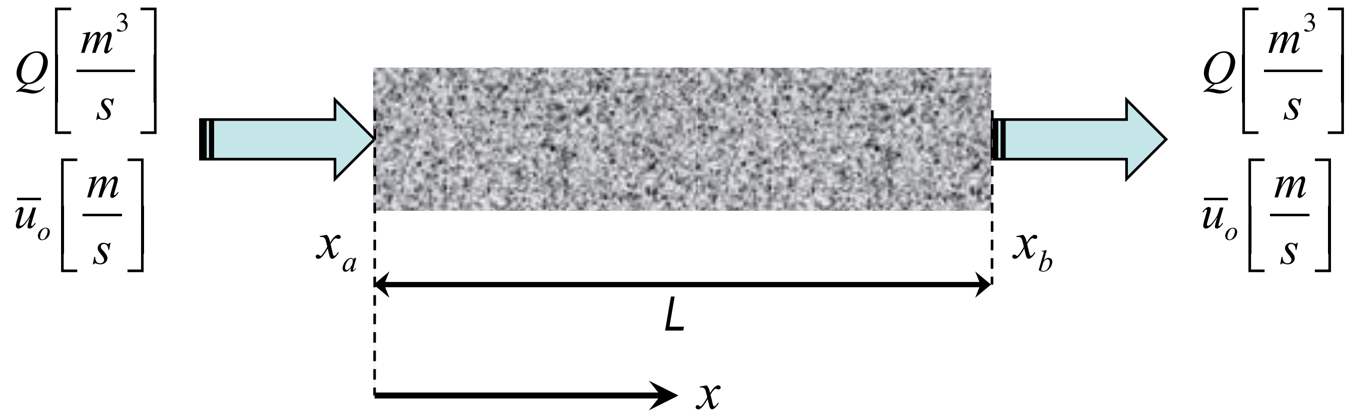
For flow in inclined media where hydrostatic pressure may be important, the Darcy equation becomes:



$$\vec{u}_{ox} = -\frac{\kappa}{\mu}\left(\frac{dP}{dx} - \rho g \frac{dy}{dx}\right) \Rightarrow \vec{u}_{ox}\int_{x_a}^{x_b} dx = \frac{\kappa}{\mu}\left(\int_{P_{x_b}}^{P_{x_a}} dP + \rho g \int_{y=0}^{D} dy\right)$$

$$\vec{u}_{ox} = \frac{\kappa}{\mu}\left(\frac{\Delta P}{L} + \rho g \frac{D}{L}\right) \Leftrightarrow \vec{u}_{ox} = \frac{\kappa}{\mu}\left(\frac{\Delta P}{L} + \rho g \sin(\beta)\right)$$

Darcy's Law for Laminar Flow through Porous Media



Now, if we write the energy balance equation for this system:

$$g\Delta Z + \cancel{\frac{\Delta u_o^2}{2}} + \frac{\Delta P}{\rho} + \sum F + \cancel{W}_{Sout} = 0$$

$$-\rho g \Delta Z - \Delta P = \rho \sum F \quad \Rightarrow \quad \vec{u}_{ox} = \frac{\kappa}{\mu}\left(\frac{\Delta P + \rho g D}{L}\right)$$

$$\Downarrow$$

$$\vec{u}_{ox}\frac{\mu \cdot L}{\kappa} = \rho \sum F \qquad \left(\Delta P + \rho g D\right) = \vec{u}_{ox}\frac{\mu \cdot L}{\kappa}$$

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Brinkman Equation

One could use Ergun's equation to define permeability of flow in packed beds:

$$\vec{u}_{ox}\frac{\mu\cdot L}{\kappa}=\rho\sum F \quad\Rightarrow\quad \kappa=\vec{u}_{ox}\frac{\mu\cdot L}{\rho\sum F}$$

Use Ergun Equation

The *Brinkman* Equation for laminar flow through porous and non-porous media is:

Pressure drop in laminar flow through pipes/channels

$$-\left(\frac{dP}{dx}-\rho g\sin(\beta)\right)=\vec{u}_{ox}\frac{\mu}{\kappa}-\mu\frac{d^2\vec{u}_{ox}^{\,2}}{dx^2}$$

Pressure drop in porous media

Darcy Equation in Multidimensional Flow

In operator form, *Darcy* equation can be written:

$$\vec{u}_o = -\frac{\kappa}{\mu}\left(\nabla P - \rho g \nabla D\right)$$

Darcy equation in Cartesian coordinate system:

$$\vec{u}_{ox} = -\frac{\kappa_x}{\mu}\left(\frac{dP}{dx} - \rho g \frac{dD}{dx}\right)$$

$$\vec{u}_{oy} = -\frac{\kappa_y}{\mu}\left(\frac{dP}{dy} - \rho g \frac{dD}{dy}\right)$$

$$\vec{u}_{oz} = -\frac{\kappa_z}{\mu}\left(\frac{dP}{dz} - \rho g \frac{dD}{dz}\right)$$