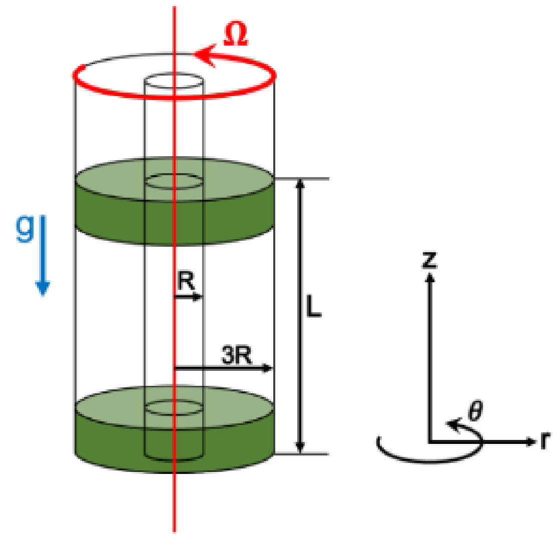
Practice Problem 2

Consider a fully developed, isothermal, steady state unidirectional (in θ direction), laminar flow of an incompressible **non-Newtonian** fluid between two concentric cylinders. The outer cylinder rotates steadily about its axis at an **angular velocity of Ω [rad/s]** and the inner cylinder is stationary. The cylinders are long compared to the radii (i.e. L >>> 3R). The fluid flow is driven only by the rotation of the outer cylinder



a) State all pertinent assumptions; also, use the continuity equation.

b) Develop a mathematical model [differential equation(s) + boundary conditions] that will represent the flow of this fluid between the two concentric cylinders. Start with the momentum equations shown above. Solve the model if you wish.

Momentum equation in r direction:

$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{{u_\theta}^2}{r}\right)=-\frac{\partial P}{\partial r}-\left[\frac{1}{r}\frac{\partial(r\tau_{rr})}{\partial r}+\frac{1}{r}\frac{\partial(\tau_{\theta r})}{\partial \theta}+\frac{\partial(\tau_{zr})}{\partial z}-\frac{\tau_{\theta\theta}}{r}\right]+\rho g_r$$

Momentum equation in θ direction:

$$\rho\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}-\frac{u_r u_\theta}{r}\right)=-\frac{1}{r}\frac{\partial P}{\partial \theta}-\left[\frac{1}{r^2}\frac{\partial(r^2\tau_{r\theta})}{\partial r}+\frac{1}{r}\frac{\partial(\tau_{\theta\theta})}{\partial \theta}+\frac{\partial(\tau_{z\theta})}{\partial z}-\frac{\tau_{\theta r}-\tau_{r\theta}}{r}\right]+\rho g_\theta$$

Momentum equation in z direction:

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial P}{\partial z}-\left[\frac{1}{r}\frac{\partial(r\tau_{rz})}{\partial r}+\frac{1}{r}\frac{\partial(\tau_{\theta z})}{\partial \theta}+\frac{\partial(\tau_{zz})}{\partial z}\right]+\rho g_z$$

Also, the law of viscosity for the Power Law fluid with constant density may be expressed as:

$$\tau_{rr}=-K\left[2\left(\frac{\partial u_r}{\partial r}\right)^n\right];\quad \tau_{\theta\theta}=-K\left[2\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right)^n+\left(\frac{u_r}{r}\right)^n\right];\quad \tau_{zz}=-K\left[2\left(\frac{\partial u_z}{\partial z}\right)^n\right]$$

$$\tau_{r\theta}=\tau_{\theta r}=-K\left[\left(r\frac{\partial(u_\theta/r)}{\partial r}\right)^n+\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta}\right)^n\right];\quad \tau_{\theta z}=\tau_{z\theta}=-K\left[\left(\frac{1}{r}\frac{\partial u_z}{\partial \theta}\right)^n+\left(\frac{\partial u_\theta}{\partial z}\right)^n\right];$$

$$\tau_{zr}=\tau_{rz}=-K\left[\left(\frac{\partial u_r}{\partial z}\right)^n+\left(\frac{\partial u_z}{\partial r}\right)^n\right]$$