CHAPTER 30: CONVECTIVE MASS TRANSFER CORRELATIONS

Convective Mass Correlations are based on

- Theoretical Analysis
- Experimental Data
- Momentum, Heat and Mass Transfer Analogies

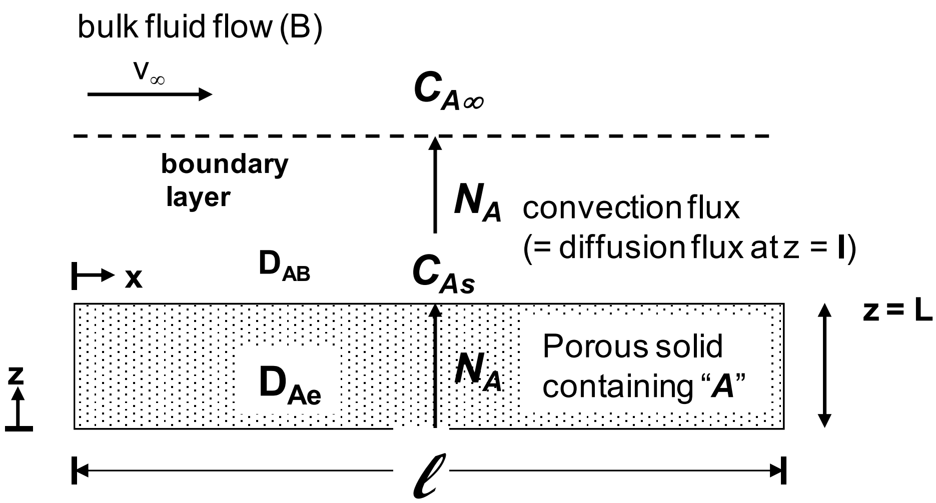
or any combination of the three.

30.1 Mass Transfer to Plates, Spheres, and Cylinders (External Flow)

Flat Plate *(length* ***L)***

Please refer to section 28.5

The Biot Number (Bi) for External Flow over a Surface



$$\mathrm{Bi} = \frac{k_c}{\dfrac{D_{Ae}}{L}} = \frac{k_c \cdot L}{D_{Ae}} = \frac{\text{convection over surface}}{\text{diffusion through medium that defines the surface}}$$

L = *thickness* of slab

ℓ = *length* of slab

The Biot Number (Bi) for External Flow (cont.)

At ***z = L*** by continuity $$N_A(L,t) = -D_{Ae}\frac{\partial C_A(L,t)}{\partial x} = k_c\left(C_{As} - C_{A\infty}\right)$$

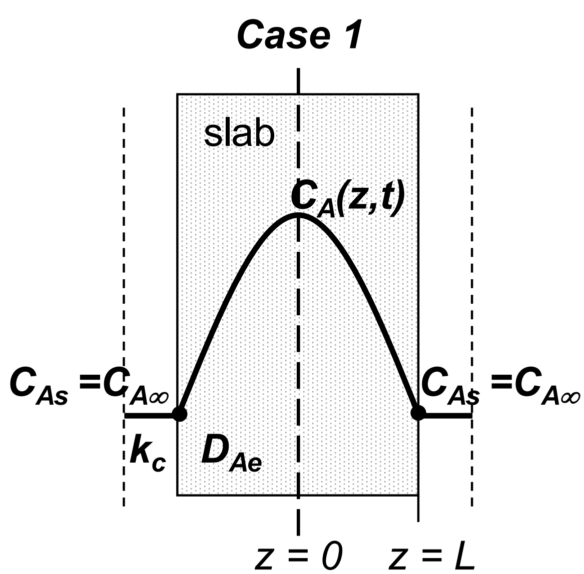
let $z^{/} = \dfrac{z}{L}$ $$-\frac{\partial C_A(L,t)}{\partial z^{/}} = \frac{k_c\,L}{D_{Ae}}\left(C_{As} - C_{A\infty}\right)$$

or $$-\frac{\partial C_A(L,t)}{\partial z^{/}} = Bi\left(C_{As} - C_{A\infty}\right)$$

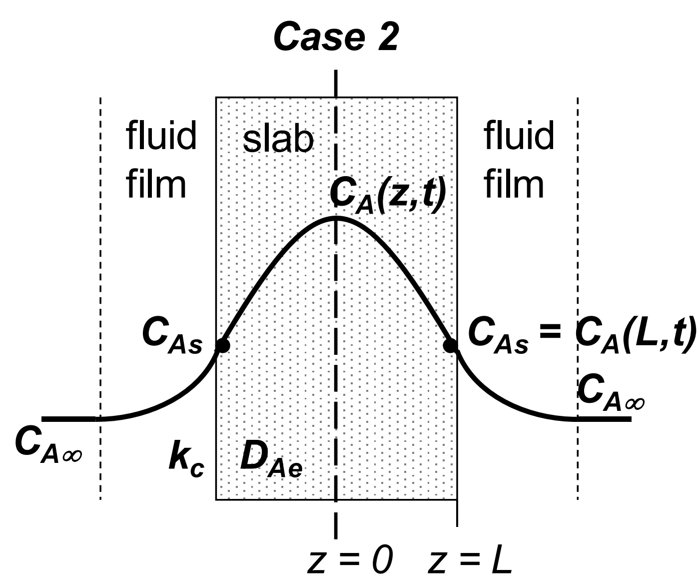
Case 1: Slab Diffusion dominates, no convective boundary layer resistance, ***Bi > 20***

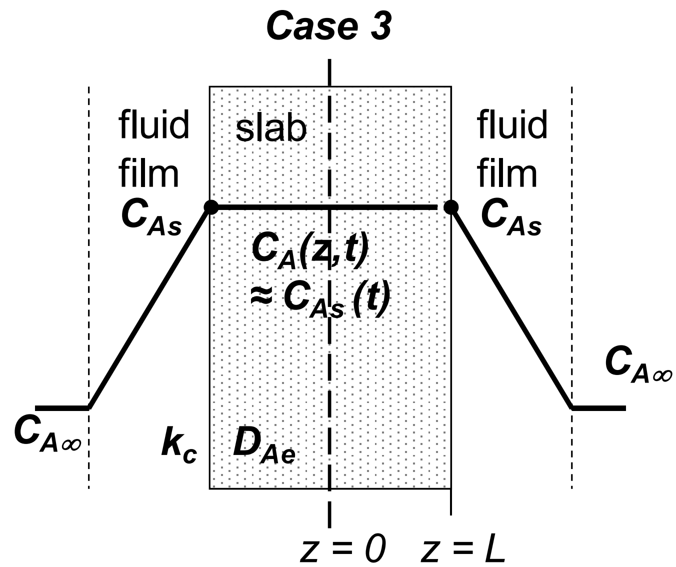
Case 2: Both diffusion and convection resistances to mass transfer

Case 3: Convective mass transfer dominates, no slab diffusion resistance with uniform concentration of A in slab, ***Bi < 0.10***

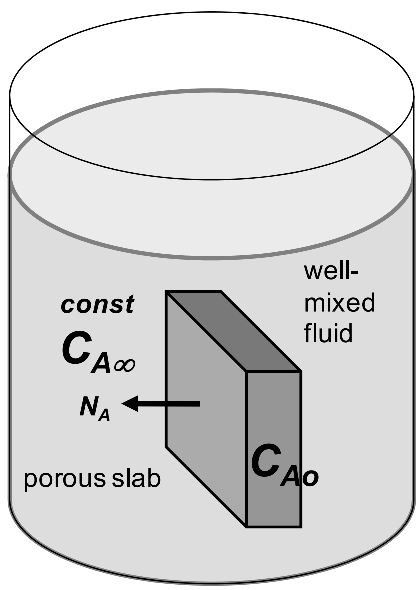


high Bi (> 20)
low "D_{Ae}", high "k_c"
thick slab



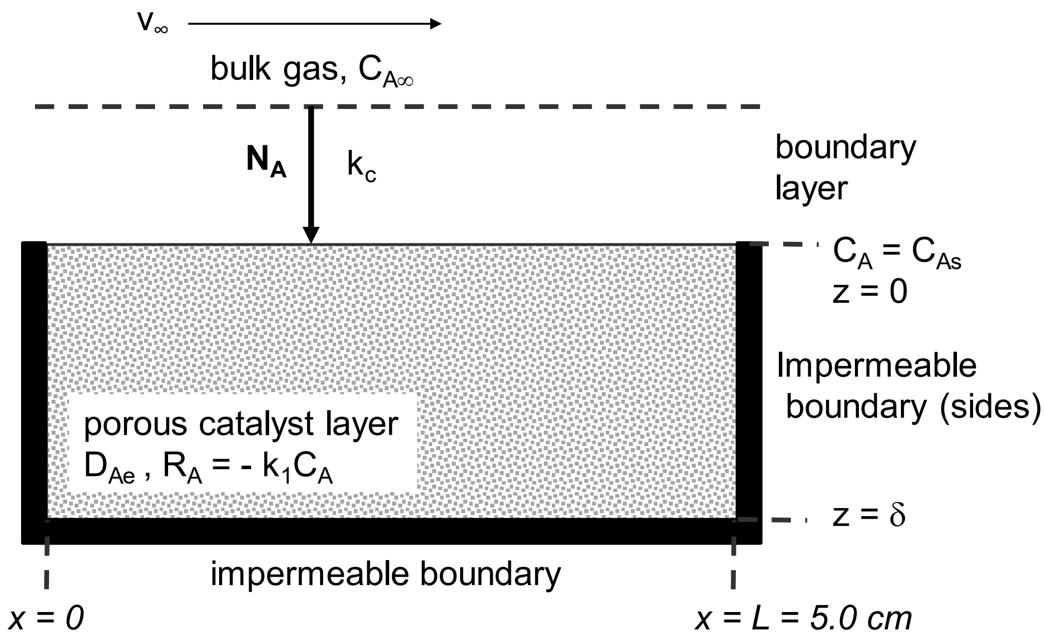


low Bi (< 0.10)
high "D_{Ae}", low "k_c"
thin slab



Lumped Parameter Analysis: Sometimes it is not so important to know $C_A(z,t)$. At low ***Bi*** numbers (< 0.10), the slab can be considered to have a uniform concentration profile of at a given time ***t***, i.e. $C_A(z,t) = C_{As}(t)$. Consider a single slab of initial uniform concentration of A, C_{Ao}, placed in a well-mixed fluid maintained at a uniform lower concentration of A, $C_{A\infty}$. An unsteady-state material balance can be performed on this process which will not require solution of the partial differential equations for mass transfer.

Convective mass transfer over a porous, reactive slab



Recall first-order homogeneous reaction within a slab with flux in z-direction and impermeable boundary at *z = 0* (section 26.2)

Differential Model

$$\frac{d^2C_A}{dz^2} - \frac{k_1}{D_{Ae}} C_A = 0$$

with B.C.

$$z = 0, C_A = C_{As} \quad z = \delta, \frac{dC_A}{dz} = 0$$

Analytical Solution for concentration profile

$$C_A(z) = \frac{C_{As} \cosh\left((\delta - z)\sqrt{k_1 / D_{Ae}}\right)}{\cosh\left(\delta\sqrt{k_1 / D_{Ae}}\right)}$$

and flux into the slab

$$N_A\big|_{z=0} = -D_{Ae} \left.\frac{dC_A}{dz}\right|_{z=0} = +\frac{D_{Ae} c_{As}}{\delta}\left(\delta\sqrt{k_1 / D_{Ae}}\right) \tanh\left(\delta\sqrt{k_1 / D_{Ae}}\right)$$

At the surface, the flux through the boundary layer must equal the flux into the reacting slab

$$N_A = k_c\left(C_{A\infty} - C_{As}\right) = N_A\Big|_{z=0} = -D_{Ae}\frac{dC_A}{dz}\bigg|_{z=0}$$

Use the above relationship to the get the concentration at the surface, C_{As}

$$k_c\left(C_{A\infty} - C_{As}\right) = \frac{D_{Ae}c_{As}}{\delta}\left(\delta\sqrt{k_1/D_{Ae}}\right)\ \tanh\left(\delta\sqrt{k_1/D_{Ae}}\ \right)$$

$$C_{As} = \frac{C_{A\infty}}{1+\dfrac{D_{Ae}}{k_c\delta}\left(\delta\sqrt{k_1/D_{Ae}}\right)\ \tanh\left(\delta\sqrt{k_1/D_{Ae}}\ \right)}$$

Recall $\phi = \delta\sqrt{k_1/D_{Ae}}$ Thiele modulus

And see also that $Bi = k_c\delta/D_{Ae}$ (thickness of slab δ is characteristic length)

$$C_{As} = \frac{C_{A\infty}}{1+\dfrac{\phi\ \tanh(\phi)}{Bi}}$$

As Bi → ∞, $C_{As} \to C_{A\infty}$. What physical situation does this refer to?

As Bi → 0, $C_{As} \to 0$. What physical situation does this refer to?

30.1 cont. Forced Convection – External Flow around Spheres and Cylinders

There is no theoretical basis for estimation of the convective heat transfer coefficient for external fluid flow around spheres or cylinders. Instead, the heat convective heat transfer coefficients are measured by experiment at different fluid velocities, and the data to fit to equations of the form

$$Nu = A + B \cdot \mathrm{Re}^n \, Sc^m$$

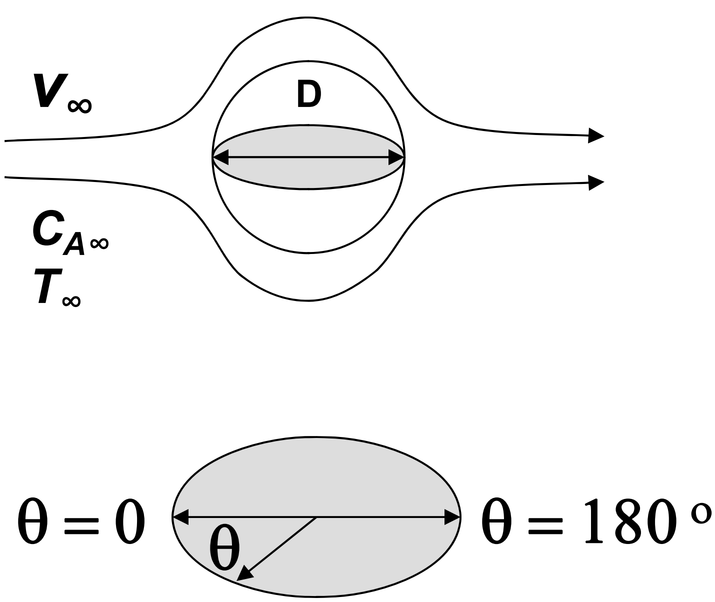
where ***A, B, m** and **n*** are adjustable parameters, and

$$\mathrm{Re}_D = \frac{v_\infty D}{\nu} \qquad \mathrm{Sh} = \frac{k_c D}{D_{AB}} \text{ (gases)} \qquad \mathrm{Sh} = \frac{k_L D}{D_{AB}} \text{ (liquids)}$$

D is the outer diameter of the sphere or cylinder

Fluid Flow around a Sphere (D = outer diameter of sphere)

When fluid flows around the surface of a sphere, the local heat and mass transfer coefficients will change as function of position around the surface of the sphere. Mass transfer correlations consider the *averaged* mass transfer coefficient over the whole external surface.



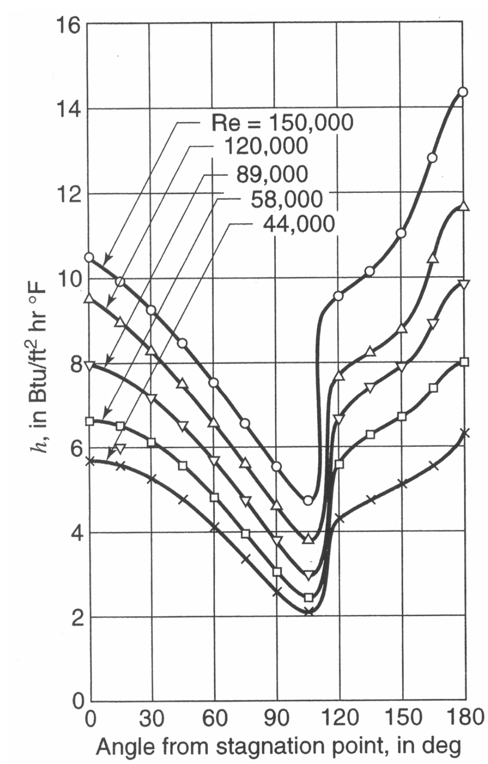


Fig. 20-11. W³-R, 5th

Fluid Flow around a Single Sphere (D = outer diameter of sphere)
Common Convective Mass Transfer Correlations

Brian and Hales Correlation (*Liquids*, $Pe_{AB} \leq 10{,}000$, where $Pe_{AB} = Re_D Sc$)

$$Sh = (4.0 + 1.21\,Pe_{AB}^{2/3})^{1/2}$$

Levich Correlation (*Liquids*, $Pe_{AB} \geq 10{,}000$, where $Pe_{AB} = Re_D Sc$)

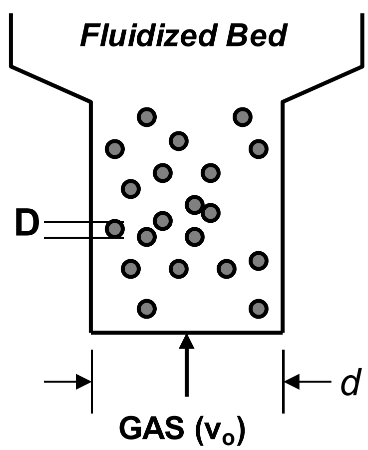
$$Sh = 1.01\,Pe_{AB}^{1/3}$$

Fröessling Correlation (validated for *Gases* at $2 \leq Re_D \leq 800$, $0.6 \leq Sc \leq 2.7$)

$$Sh = 2.0 + 0.552\,Re_D^{1/2}\,Sc^{1/3}$$

Aside: why does $\boldsymbol{Sh = 2.0}$ for a still fluid with $\boldsymbol{Re_D = 0}$?

The *Fröessling Correlation* is also approximately true for flow of gas around individual spheres (particles) suspended in a fluidized bed:



D = *average* diameter of particle equivalent to a sphere

Superficial gas velocity $v_\infty = \dfrac{v_o}{\pi d^2/4}$

Fluid Flow around a Cylinder (D = outer diameter of cylinder)

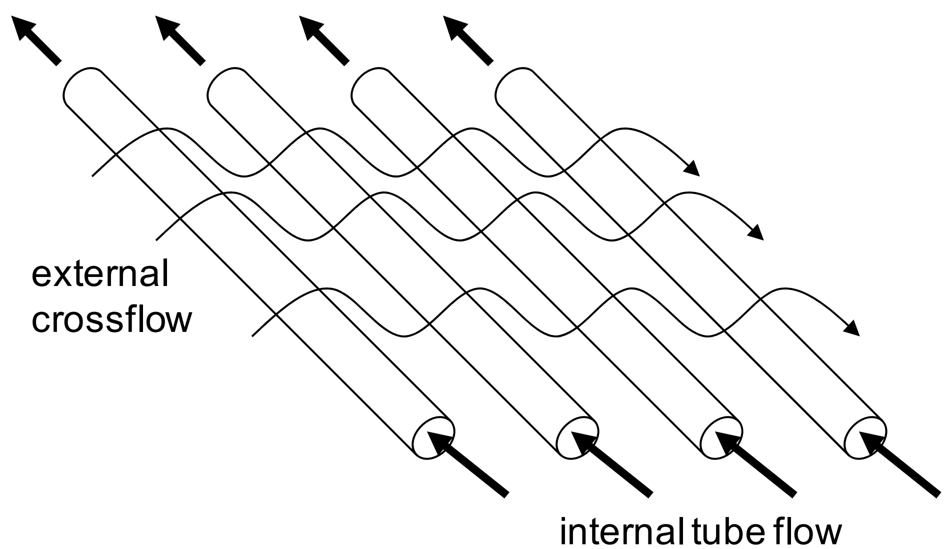
Forced Convection, External Gas Flow *Normal* to Cylinder Axis (cross flow)

$$Re = \frac{v_\infty D}{\nu}$$

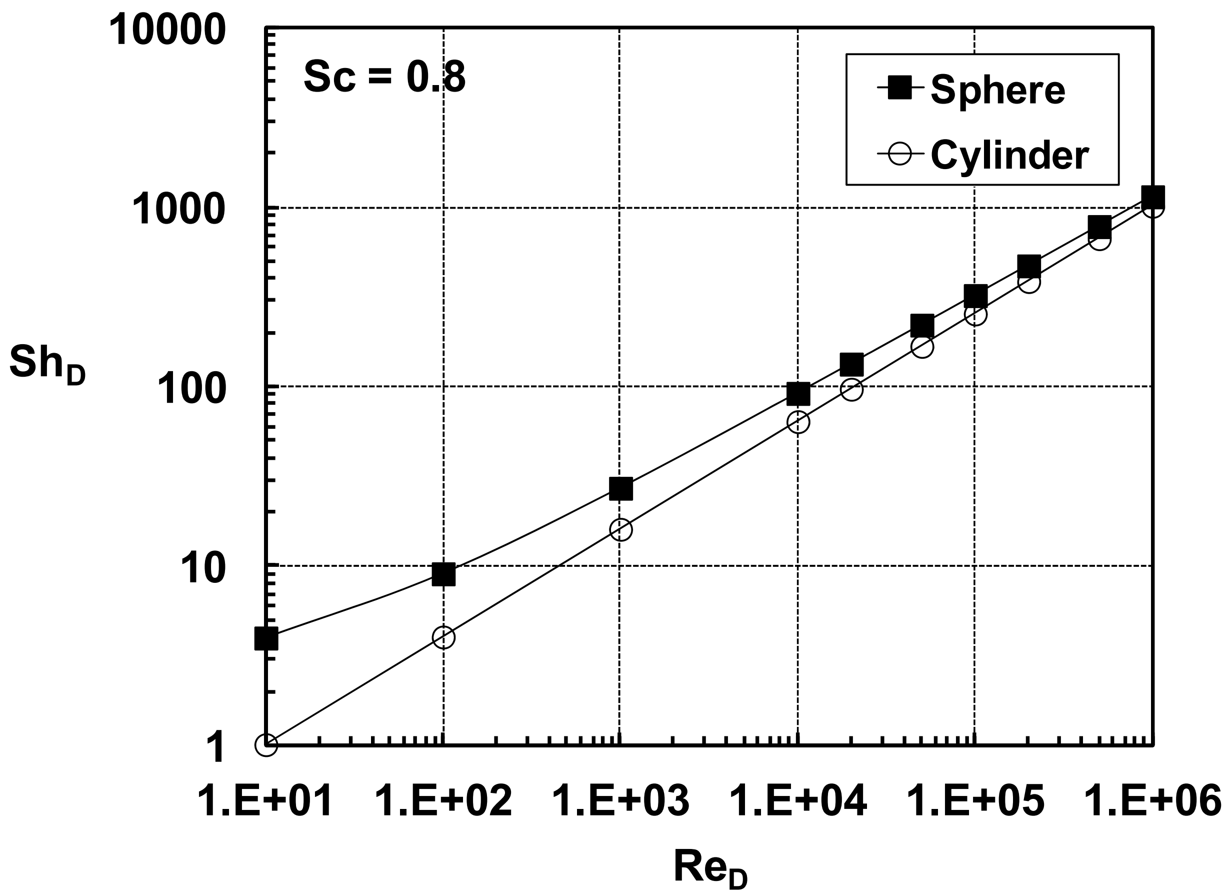
D = outer diameter of cylinder

$$Sh_D = \frac{k_c D}{D_{AB}} = 0.281\,(Re_D)^{0.6} Sc^{0.44}$$

Experimentally validated for $400 < Re_D < 25{,}000$ and $0.6 < Sc < 2.6$ (i.e. gases), but can also be used for liquids



Sample comparison of convection mass transfer for flow around a sphere and cylinder for gases with $Sc = 0.8$



10000
1000
100
10
1
Sc = 0.8
Sphere
Cylinder
Sh$_D$
1.E+01
1.E+02
1.E+03
1.E+04
1.E+05
1.E+06
Re$_D$

30.2 Mass Transfer Involving Flow through Pipes (Internal Flow)

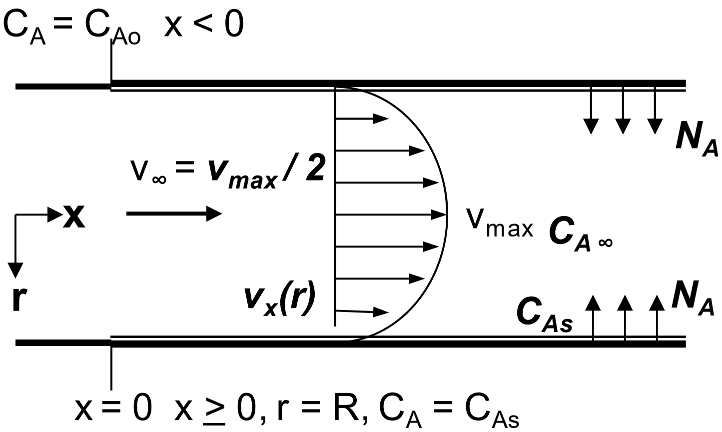
Fluid Flow Inside a Tube, Laminar Flow (Re < 2000)

For laminar fluid flow inside a tube, the convective mass coefficient k_c can also be determined from first principles.

Velocity distribution for fluid flowing inside a tube (***R*** = radius of tube)

$$v_x(r) = v_{\max}\left[1-\left(\frac{r}{R}\right)^2\right]$$

$$= 2\cdot v_\infty \cdot \left[1-\left(\frac{r}{R}\right)^2\right]$$



Graetz-Nusselt Local Mass Transfer Coefficient

$\boldsymbol{k_c}$ is function of position $\boldsymbol{x}$ within tube of length $\boldsymbol{L}$. The theoretical result is (from the Graetz-Nusselt Problem)

$$k_{c,x} = \frac{1}{\Gamma(4/3)}\left(\frac{8}{9}\frac{v_\infty D_{AB}^2}{D\cdot x}\right)^{1/3} \quad \text{with } \Gamma(4/3) = 0.89 \text{ (Gamma function)}$$

From integration of this theoretical result over x (from $x = 0$ to $x = L$) (show in your notes), the average average transfer coefficient is

$$\frac{k_c \cdot D}{D_{AB}} = 1.62\left(\frac{\mathrm{v}_\infty \mathrm{D}^2}{\mathrm{L\,D_{AB}}}\right)^{1/3} = 1.62\left(\frac{\rho\cdot v_\infty \cdot D}{\mu}\right)^{1/3}\left(\frac{\mu}{\rho D_{AB}}\right)^{1/3}\left(\frac{D}{L}\right)^{1/3} = 1.62\left(Pe\cdot\frac{D}{L}\right)^{1/3}$$

Sieder-Tate Equation for Mass Transfer (Pe (D/L) > 10)

(validated for gases or liquids, *Pe (D/L) > 10* and *Re < 2000*; the 1.62 term from theory is adjusted to 1.86)

$$\mathrm{Sh} = 1.86\left(\frac{v_\infty D^2}{L\, D_{AB}}\right)^{1/3} = 1.86\left(\frac{D}{L}\frac{v_\infty D}{\nu}\cdot\frac{\nu}{D_{AB}}\right)^{1/3} = 1.86\left(\frac{D}{L}\mathrm{Re\,Sc}\right)^{1/3} = 1.86\left(\frac{D}{L}\mathrm{Pe}\right)^{1/3}$$

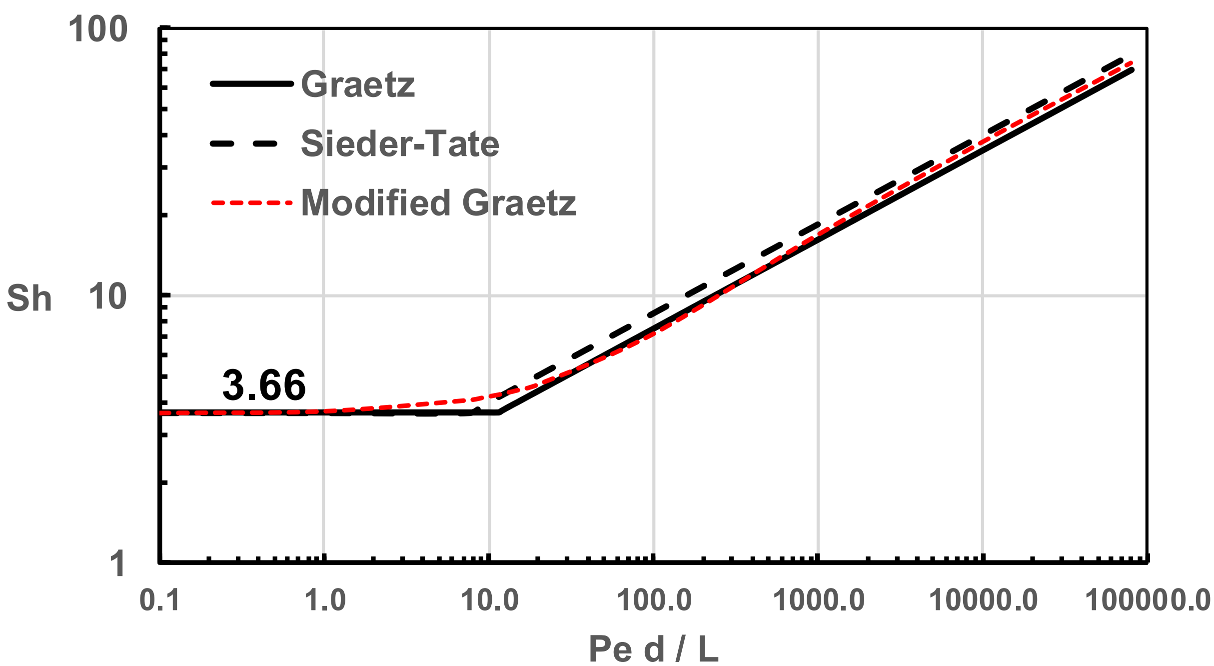
$$\mathrm{Re} = \frac{v_\infty D}{\nu} \quad \mathrm{Sh} = \frac{k_c D}{D_{AB}} \quad \mathrm{Pe} = \mathrm{Sc}\cdot\mathrm{Re} = \frac{v_\infty D}{D_{AB}} \qquad \text{Note: } \frac{v_\infty D^2}{L\, D_{AB}} > 20$$

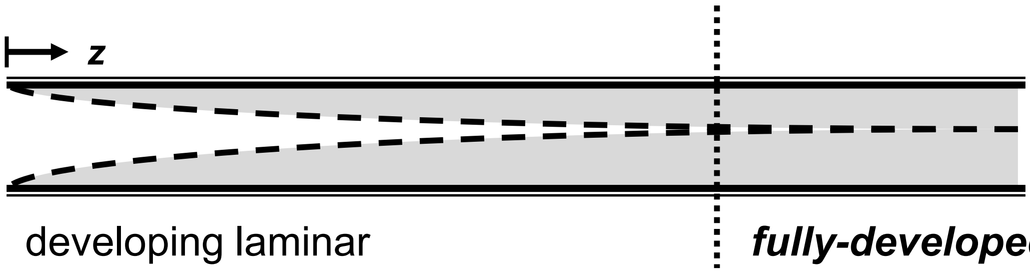
L = the *length* of the pipe (accounts for development of the laminar boundary layer at the tube wall)

D = diameter of the pipe

As ***D / L*** becomes small (e.g. a long pipe), the condition of *fully-developed laminar flow* occurs. If C_{As} is constant along the tube length, then it can be shown that

$$\mathrm{Sh} = \frac{k_c D}{D_{AB}} = 3.66$$ (*Pe (D/L)< 10*) *Fully Developed Laminar F Boundary Layer*





developing laminar boundary layer
Pe D/L $\geq$ 10
Sh > 3.66

fully-developed laminar boundary layer
Pe D/L < 10
Sh = 3.66

To account for fully-developed laminar flow, the following correlation is valid (modified Graetz model for Re < 2000)

$$Sh = 3.66 + \frac{0.0668\left(Pe \cdot \dfrac{D}{L}\right)}{1 + 0.04\left(Pe \cdot \dfrac{D}{L}\right)^{2/3}}$$

Fuid Flow Inside a Tube, Turbulent Flow
(*Re* $\geq$ *2000, inner diameter* ***D***)

Gilliland and Sherwood Correlation for Gases

$$\frac{k_c D}{D_{AB}} y_{B,lm} = 0.023\ Re^{0.83}\ Sc^{0.44} \qquad Re = \frac{v_\infty D}{\nu} \qquad Sh = \frac{k_c D}{D_{AB}}$$

$$y_{B,lm} = \frac{y_{B,s} - y_{B,bulk}}{\ln\left[\dfrac{y_{B,s}}{y_{B,bulk}}\right]} = \frac{(1-y_{A,s}) - (1-y_{A,bulk})}{\ln\left[\dfrac{1-y_{A,s}}{1-y_{A,bulk}}\right]}$$ If dilute w.r.t. solute A, note $y_A \to 0, \quad \therefore y_{B,lm} \to 1$

Linton and Sherwood Correlation for Liquids

$$Sh = 0.023\, Re^{0.83} Sc^{1/3} \qquad Re = \frac{v_\infty D}{\nu} \qquad Sh = \frac{k_L D}{D_{AB}}$$

Note similarity to the Dittus-Boelter equation for convective heat transfer

To neglect entrance effects ***L / D*** > 60

Forced Convection for Fluid Flow Inside non-Circular Conduits

Equivalent diameter

$$D_e = 4 \cdot r_H = 4 \frac{\text{cross sectional area}}{\text{perimeter}} = \frac{4A}{P}$$

r_H = "hydraulic radius" (m)

The basic idea is to estimate D_e and then use D_e as the "equivalent diameter" in existing mass transfer correlations for fluid flow inside a tube

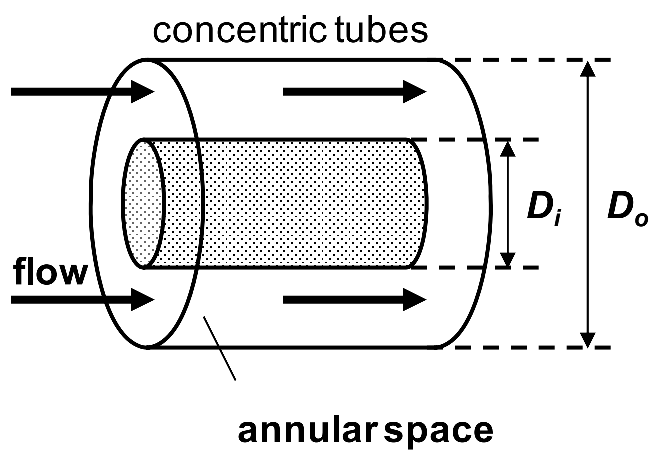
Annular Flow

$$D_e = 4 \cdot r_H = 4 \frac{\left(\frac{\pi D_o^2}{4} - \frac{\pi D_i^2}{4} \right)}{\pi D_o + \pi D_i}$$

$$D_e = D_o - D_i$$

$$\mathrm{Re}_{D,e} = \frac{\rho \cdot v_\infty D_e}{\mu}$$

$$v_\infty = \frac{v_o}{\frac{\pi D_o^2}{4} - \frac{\pi D_i^2}{4}}$$



v_o = volumetric flowrate (m^3/sec)

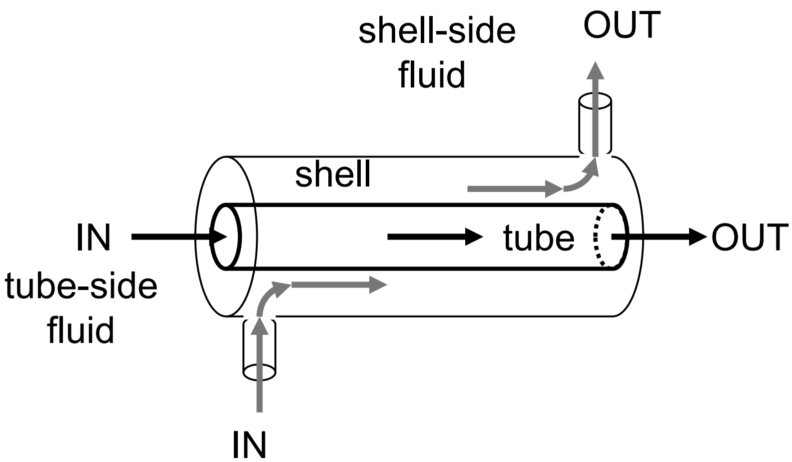
For example, for turbulent flow in the annular space *($Re_{D,e}$ > 10,000)*
Simply use *Linton and Sherwood Correlation* with $D = D_e$

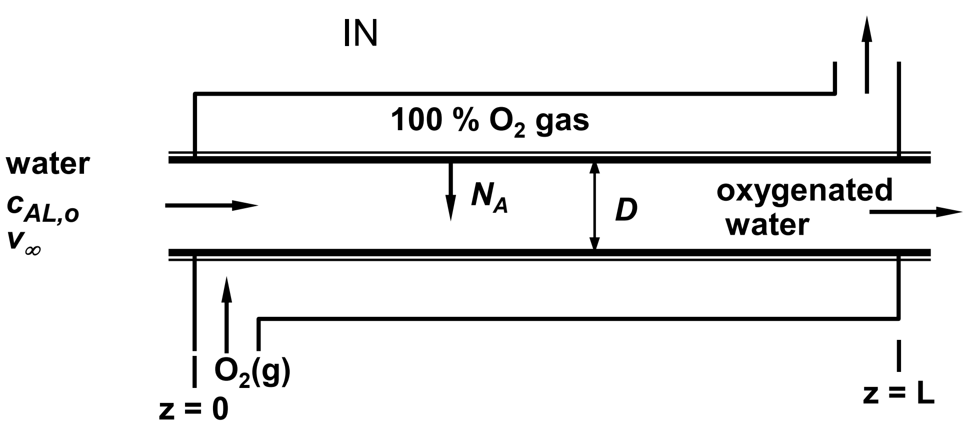
$$\frac{k_c \cdot D_e}{D_{AB}} = 0.023 \left(\frac{\rho \cdot v_\infty \cdot D_e}{\mu} \right)^{0.8} (Sc)^{1/3}$$

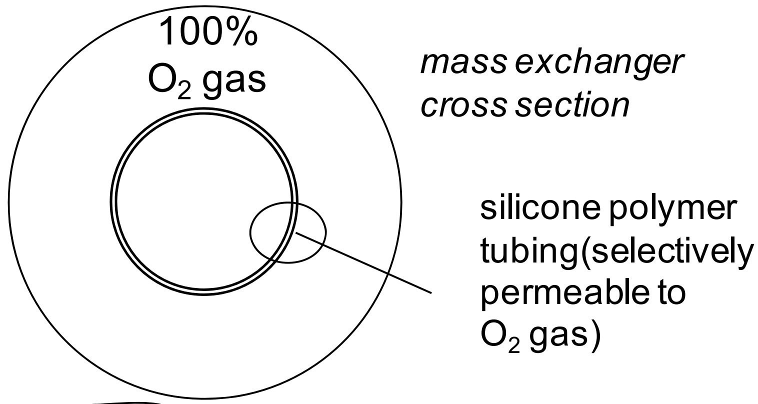
What is D_e of a *rectangular duct* with height d_1 and width d_2? Draw a picture.

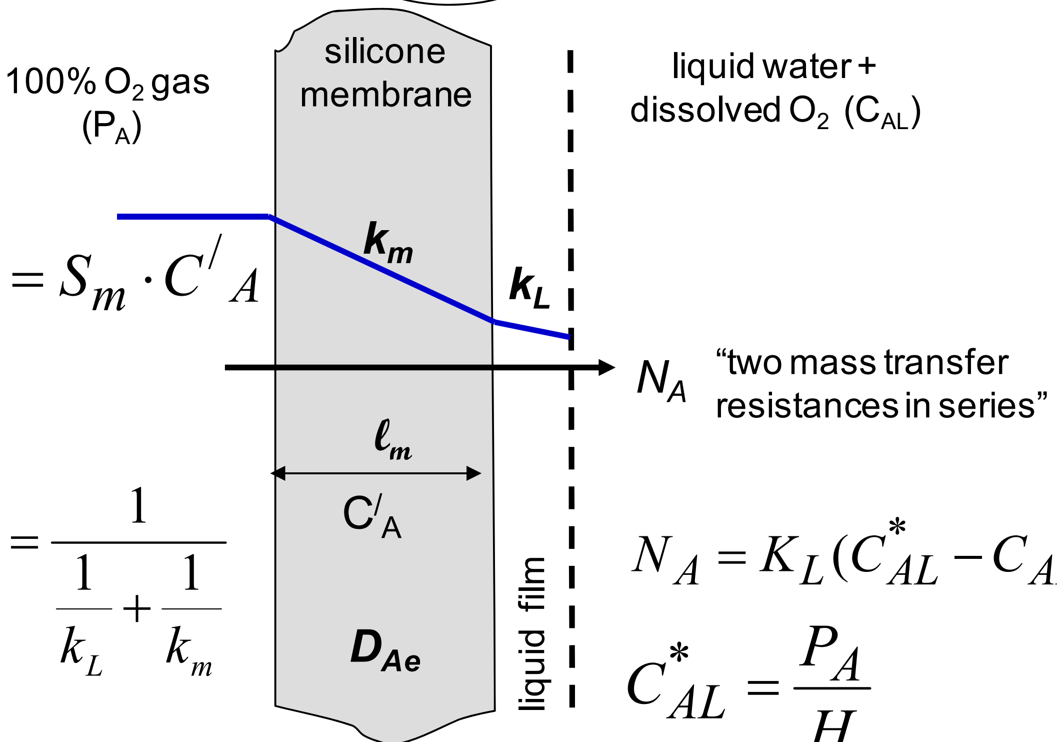
30.2 Aside: membrane-based mass exchanger

Shell-and-Tube Arrangement









$$P_A = S_m \cdot C'_A$$

$$K_L = \frac{1}{\dfrac{1}{k_L} + \dfrac{1}{k_m}}$$

$$N_A = K_L(C^*_{AL} - C_{AL})$$

$$C^*_{AL} = \frac{P_A}{H}$$

$$k_m = \frac{H \cdot D_{Ae}}{S_m \cdot l_m}$$

If ℓ_m is small (thin membrane)
S_m is small (highly soluble membrane)
D_{Ae} is large (high membrane diffusivity)

then $K_L \sim k_L$

Steps for Modeling Mass Transfer Processes Involving Convection

In real process systems, the flux N_A is coupled to a material balance on the control volume of physical system. Processes of this type are modeled similarly to the modeling procedures described earlier in Section 25.4.

Step 1. Draw a picture of the physical system. Label the important features, including the boundary surface where convective mass transfer occurs. Decide where the **SOURCE** and the **SINK** for mass transfer are located.

Step 2. Make a "list of assumptions" based on your consideration of the physical system.

Step 3. Formulate material balance(s) on the transferring species (solute A) with respect to one phase of the process, and then incorporate the appropriate relationship for N_A into the material balance.

Processes dominated by convective mass transfer fall into two types:

- Well-mixed control volume of uniform concentration of the transferring species, i.e. absorption of a solute (A) into a stirred tank of liquid (B); process can be steady state (open flow tank) or unsteady state (batch tank)

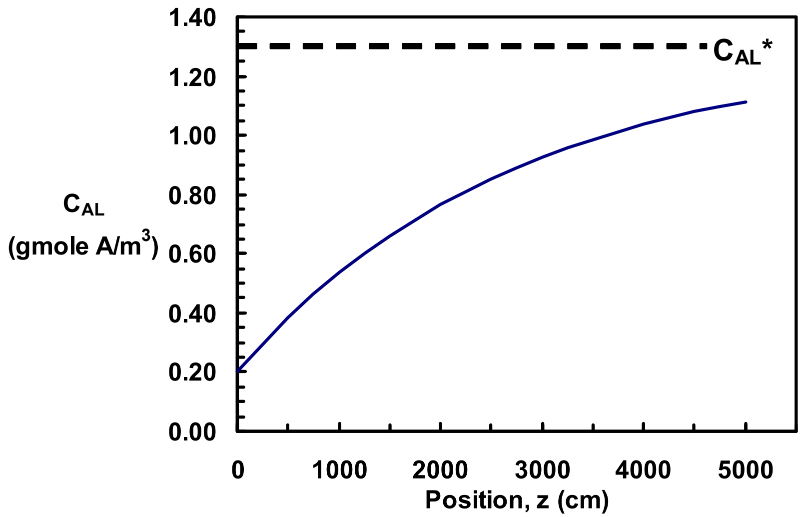
- Differential control volume with a one-dimensional variation in concentration of the transferring species down the length of a open or closed condiut, i.e. flow through through a tube, where mass transfer flux originates from the wall of the tube (steady state process)

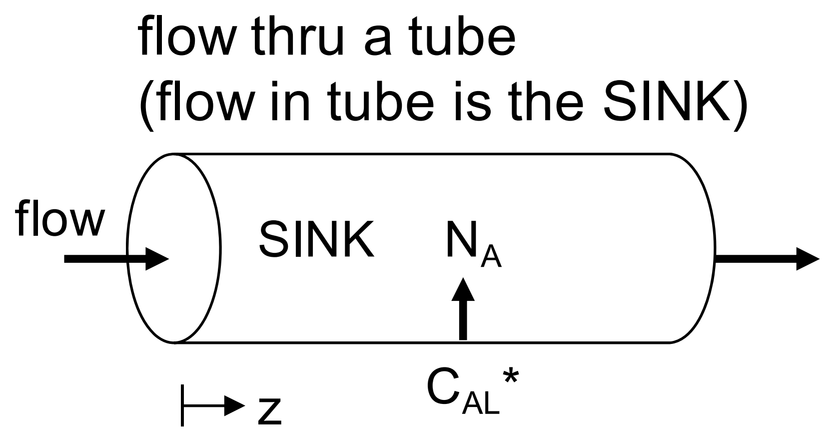
Step 4. Recognize and specify the process boundary and initial conditions associated with the process unit.

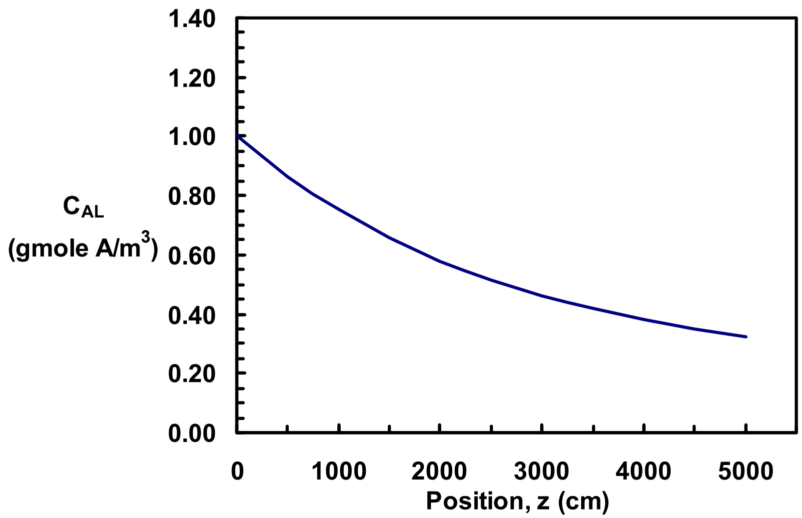
Step 5. Solve the algebraic or differential equation(s) resulting from the material balance(s) to obtain the concentration profile, flux, or other parameters of engineering interest.

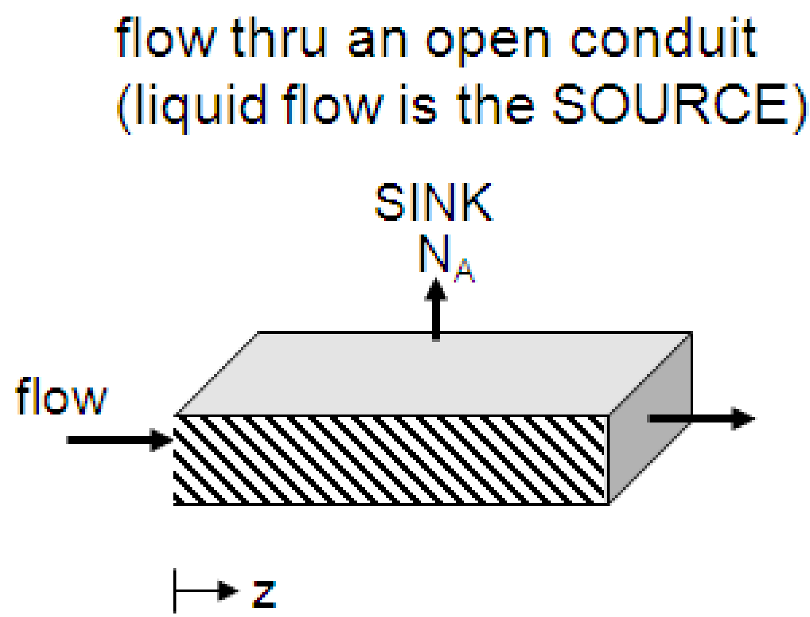
Step 6. Plug in the numbers to the model. In many cases, k_c can be upfront. If so, get *Re*, *Sc*, and *Sh* appropriate for the physical system.

Steady State, Differential Control Volume Examples (no reaction of transferring solute A)

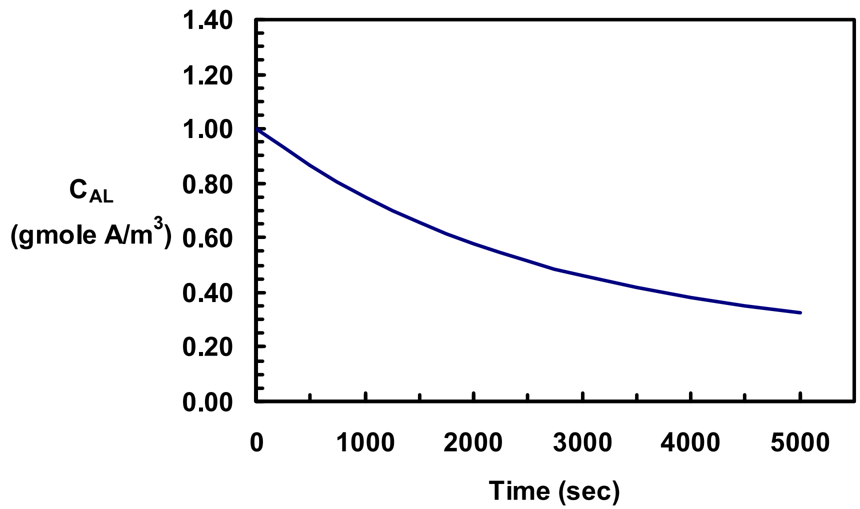


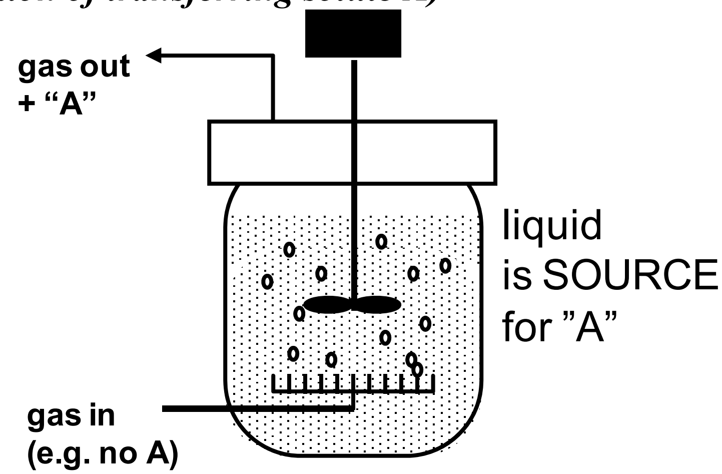


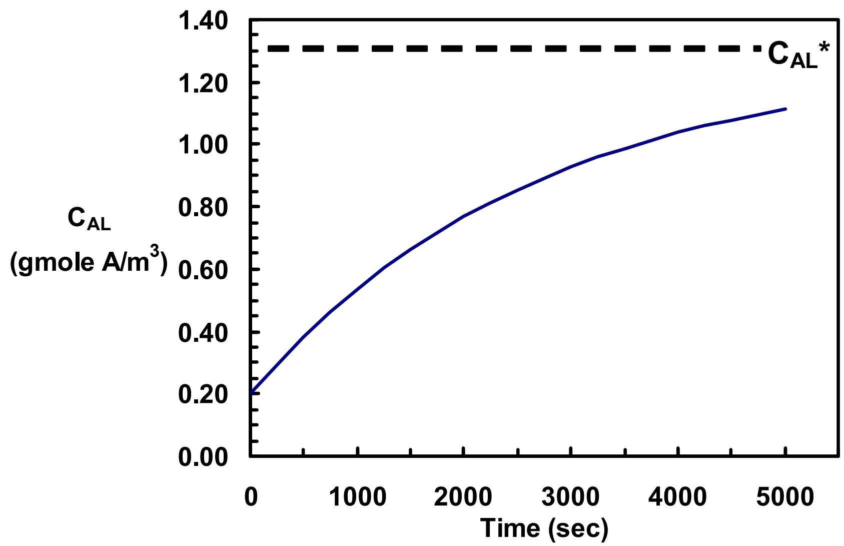


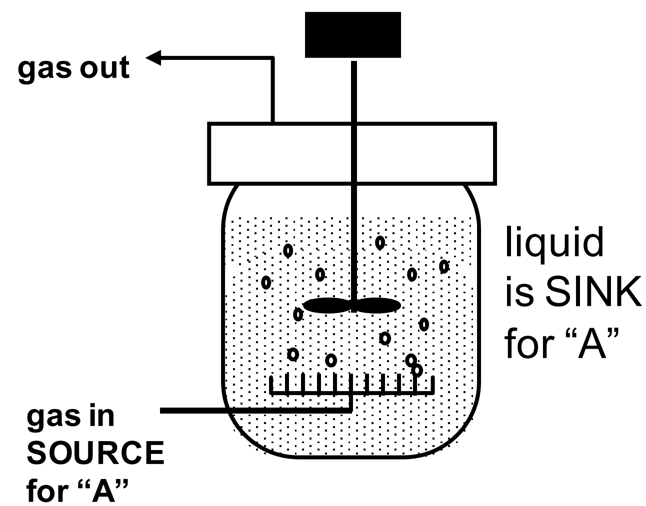


Well Mixed, Unsteady State Control Volume Examples (no reaction of transferring solute A)

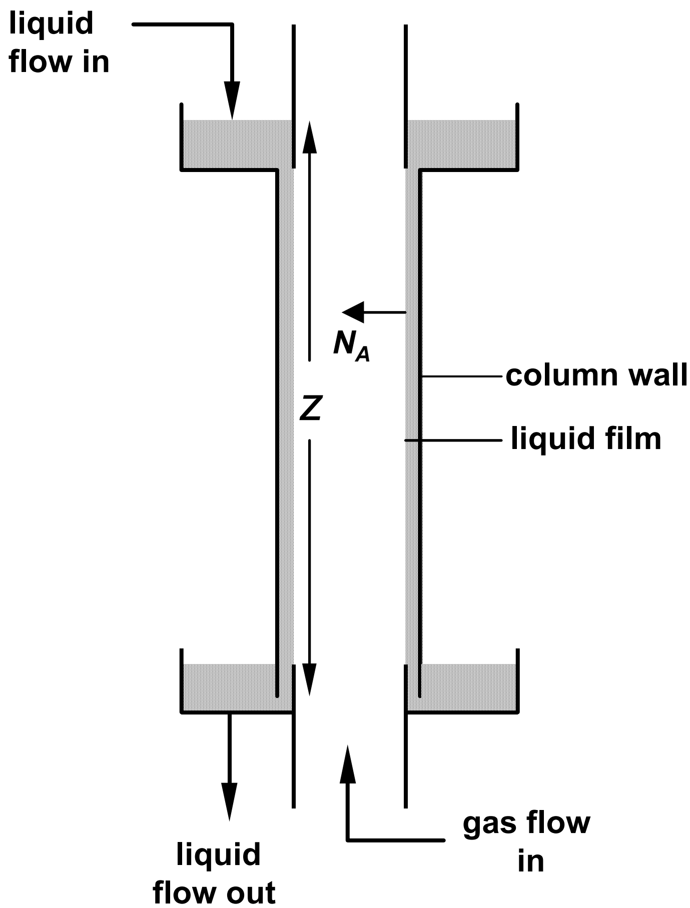








30.3 Mass Transfer in Wetted Wall Columns



Liquid phase mass transfer coefficient, $\boldsymbol{k_L}$

$$\frac{k_L z}{D_{AB}} = 0.433\,(Sc)^{1/2}\left(\frac{\rho_L{}^2 g z^3}{\mu_L{}^2}\right)^{1/6}(Re_L)^{0.4}$$

z	length of contact down the falling film (m)
D_{AB}	the mass diffusivity of the diffusing component A into liquid solvent B (m^2/sec)
ρ_L	the density of the liquid (kg/m^3)
μ_L	the viscosity of liquid (kg/m-sec)
g	the acceleration due to gravity (9.8 m/sec^2)
Sc	Schmidt number for the solute dissolved in the liquid evaluated at the liquid film temperature

Reynolds number of the liquid flowing down the inner surface of the tube

$$Re_L = \frac{4\,\Gamma}{\mu_L} = \frac{4w}{\pi D \mu_L}$$

w the mass flowrate of liquid (kg/sec)
D inner diameter of the cylindrical column (m)
Γ mass flow rate of liquid *per unit wetted perimeter* of the column (kg/m-sec)

Note: The Reynolds number of the gas flowing through the tube is still

$Re = \dfrac{\rho v_\infty D}{\mu}$ where v_∞ and ρ/μ refer to the gas phase

30.4 Mass Transfer in Packed Beds

Packed Bed (simplest correlation does not account for bed porosity)

$$j_D = \frac{k_c}{u_{ave}}(Sc)^{2/3} = 1.17 \cdot Re^{-0.415} \qquad \text{with} \quad Re = \frac{d_p u_{ave} \rho}{\mu}$$

d_p average particle diameter equivalent to a sphere (m)

u_{ave} superficial gas velocity of the empty bed (m/sec)

$$u_{ave} = \frac{v_o}{A}$$

v_o = volumetric flowrate of fluid into the bed (m^3/sec)

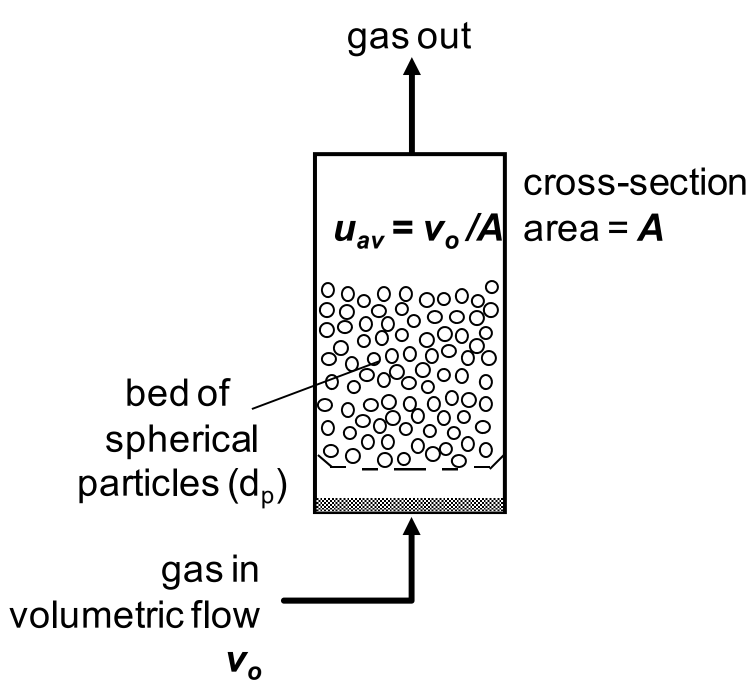
A = cross-sectional area of the empty bed (m^2)

The "surface area" for flux to/from the particles in the packed bed

$$A_i = V \frac{A_{sphere}}{V_e} = V \frac{\pi}{d_p}$$

V = volume of the empty bed (m^3)

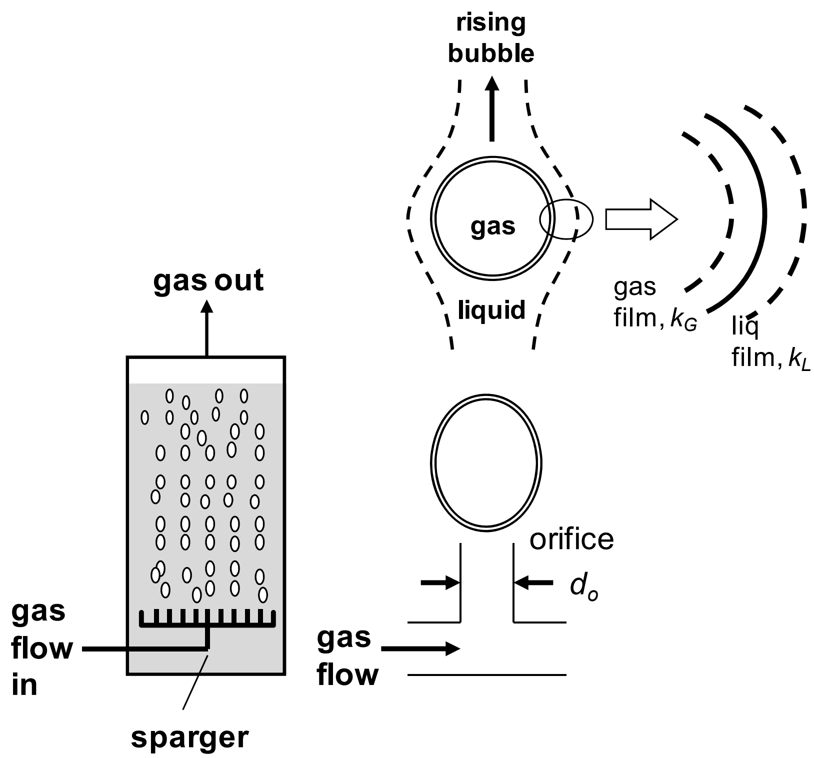
$V_e = d_p^3$ = volume of a "box" around a single sphere (m^3)



30.5 Gas-Liquid Mass Transfer in Bubble Columns and Stirred Tanks

Bubble Column

A vessel filled with liquid is "sparged" with gas bubbles. The rising bubbles mix the liquid.



Gas is forced through an orifice (sparger) into the liquid to form the gas bubble.

Mass Transfer Correlations for Spherical Bubble Swarms

Gas bubbles rise up in the liquid by *natural convection* process where a hydrodynamic boundary layer is formed in the *liquid* surrounding the outer surface of the gas bubble. The mass transfer coefficient for the *liquid film* (***k_L***) is determined by the following correlations based on bubble size:

d_b = gas bubble diameter

$d_b < 2.5$ mm $\qquad \mathrm{Sh} = \dfrac{k_L\, d_b}{D_{AB}} = 0.31\,\mathrm{Gr}^{1/3}\,\mathrm{Sc}^{1/3}$

$d_b \geq 2.5$ mm $\qquad \mathrm{Sh} = \dfrac{k_L\, d_b}{D_{AB}} = 0.42\,\mathrm{Gr}^{1/3}\,\mathrm{Sc}^{1/2}$

Grashof number based on d_b $\quad \mathrm{Gr} = \dfrac{d_b^{\,3}\ \rho_L\ g\,\Delta\rho}{\mu_L^{\,2}}$ "Natural Convection"

$\Delta\rho = \rho_L - \rho_G$ difference of the density of the liquid and the density of the gas inside the bubble

ρ_L bulk liquid mass density, kg/m^3

ρ_G bulk gas bubble mass, kg/m^3

μ_L bulk liquid viscosity (kg/m-sec)

D_{AB} diffusion coefficient of dissolved gaseous solute *A* in solvent *B* (m^2/sec)

g gravitational constant (9.8 m/sec^2)

If the gaseous solute A is spargingly soluble in the liquid solvent, then commonly $\boldsymbol{k_L \cong K_L}$ for the interphase mass transfer process (liquid film controlling)

Gas Bubble Diameter

For low gas flowrates, where the rising gas bubbles are separated, by equating the buoyant force with the force to due surface tension at the orifice, the bubble diameter is

$$d_b = \left(\frac{6\, d_o \sigma_L}{g\,(\rho_L - \rho_G)} \right)^{1/3}$$

where $\boldsymbol{d_o}$ is the diameter of the orifice, and $\boldsymbol{\sigma_L}$ is the surface tension of the liquid

The stable gas bubble diameter in a bubble column without external agitation is approximated by the Sauter-mean diameter

$$\frac{d_b}{D} = 26 \left(\frac{g\, D^2\, \rho_L}{\sigma_L} \right)^{-0.50} \left(\frac{g\, D^3}{\nu_L^2} \right)^{-0.12} \left(\frac{u_{gs}}{\sqrt{g\, D}} \right)^{-0.12}$$

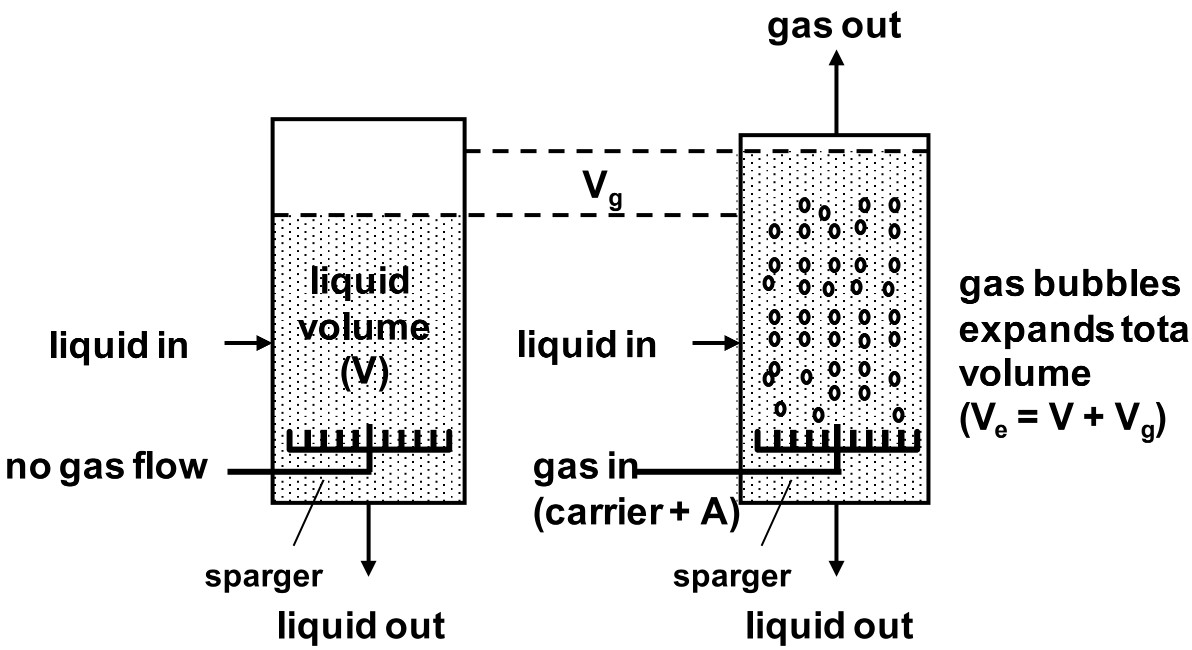
where $\boldsymbol{D}$ is the diameter of the column, and $\boldsymbol{u_{gs}}$ is the superficial gas velocity through the empty column.

Interphase Mass Transfer Area in Bubbled Liquids

How do we relate flux N_A to transfer rate W_A in cases where the interphase mass transfer area ($\boldsymbol{A_i}$) is not readily determined?

Gas Holdup Ratio in Liquid **(ϕ_g)**

$$\phi_g = \frac{\text{volume of gas bubbles}}{\text{volume of aerated liquid}} = \frac{V_g}{V_e}$$



(ϕ_g) can be estimated by correlation or by experiment. For example, for a bubble column using water-like liquids without external agitation, ϕ_g is estimated by

$$\frac{\phi_g}{(1-\phi_g)^4} = 0.20\left(\frac{g\,D^2\,\rho_L}{\sigma_L}\right)^{1/8}\left(\frac{g\,D^3}{\nu_L^2}\right)^{1/12}\left(\frac{u_{gs}}{\sqrt{g\,D}}\right)$$

where σ_L is the surface tension of the liquid, and $\boldsymbol{D}$ is the diameter of the column, and $\boldsymbol{u_{gs}}$ is the superficial gas velocity through the empty column.

Interphase mass transfer area per unit liquid volume (***A_i /V***) for bubbles of average diameter ***d_b***

$$\frac{A_i}{V} = \frac{V_g}{V} \cdot \frac{\text{single bubble area}}{\text{single bubble volume}} = \frac{6\phi_g}{d_b}$$

The gas holdup is typically less than 0.2 for most gas-liquid sparging operations

Volumetric Mass Transfer Coefficients (Capacity Coefficients)

$$a = \frac{A_i}{V} = \frac{\text{area available for interphase mass transfer (m}^2\text{)}}{\text{liquid volume (m}^3\text{)}}$$

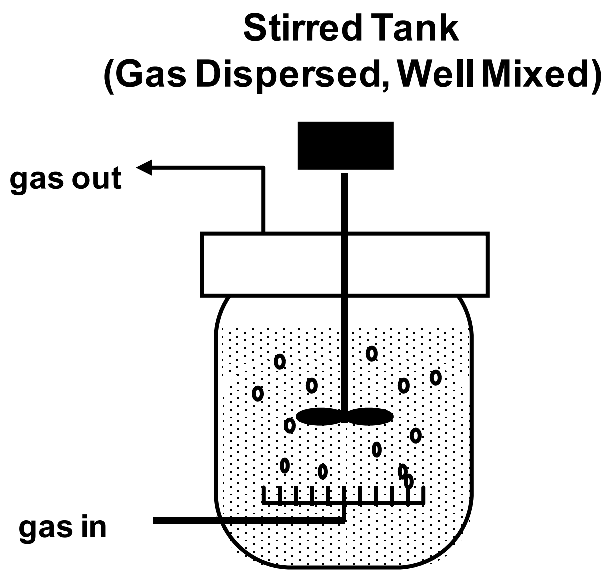
"***a***" is often "lumped" with the convective mass transfer coefficient to yield the "volumetric capacity coefficient", e.g. k_L for a gas-sparged liquid can become

$$k_L a = k_L \frac{A_i}{V} = \left(\frac{m}{s}\right)\left(\frac{m^2}{m^3}\right) = s^{-1}$$

Capacity coefficients cannot be used to calculate flux N_A directly. Instead, they are used to compute the total rate of interphase transfer W_A , for example

$$W_A = N_A \cdot \frac{A_i}{V} \cdot V = K_L a \cdot V\left(C_{AL}^* - C_{AL}\right)$$

Stirred Tank



Oxygen mass transfer to water, coalescing air bubbles

$$(k_L a)_{O_2} = 2.6 \cdot 10^{-2} \left(\frac{P_g}{V} \right)^{0.4} (u_{gs})^{0.5}$$

valid for $V < 2.6$ m^3 of liquid and $500 < P_g / V < 10{,}000$ W/m^3

Oxygen mass transfer to water, non-coalescing air bubbles

$$(k_L a)_{O_2} = 2.0 \cdot 10^{-3} \left(\frac{P_g}{V} \right)^{0.7} (u_{gs})^{0.2}$$

valid for $V < 4.4$ m^3 of liquid and $500 < P_g / V < 10{,}000$ W/m^3

In both correlations, the following units must be strictly followed

$(k_L a)_{O_2}$ liquid phase film capacity coefficient for O_2 in water in units of s^{-1}

P_g / V the power consumption of the bubble-aerated vessel per unit *liquid* volume in units of W/m^3

u_{gs} superficial velocity of the gas flowing through the *empty* vessel in units of m/s (divide the volumetric flowrate of the gas by the cross-sectional area of the empty vessel)

The above correlations work for any design of impeller (e.g. paddle, marine or flat-blade disk turbine impeller).

Power Input per Liquid Volume (P / V)

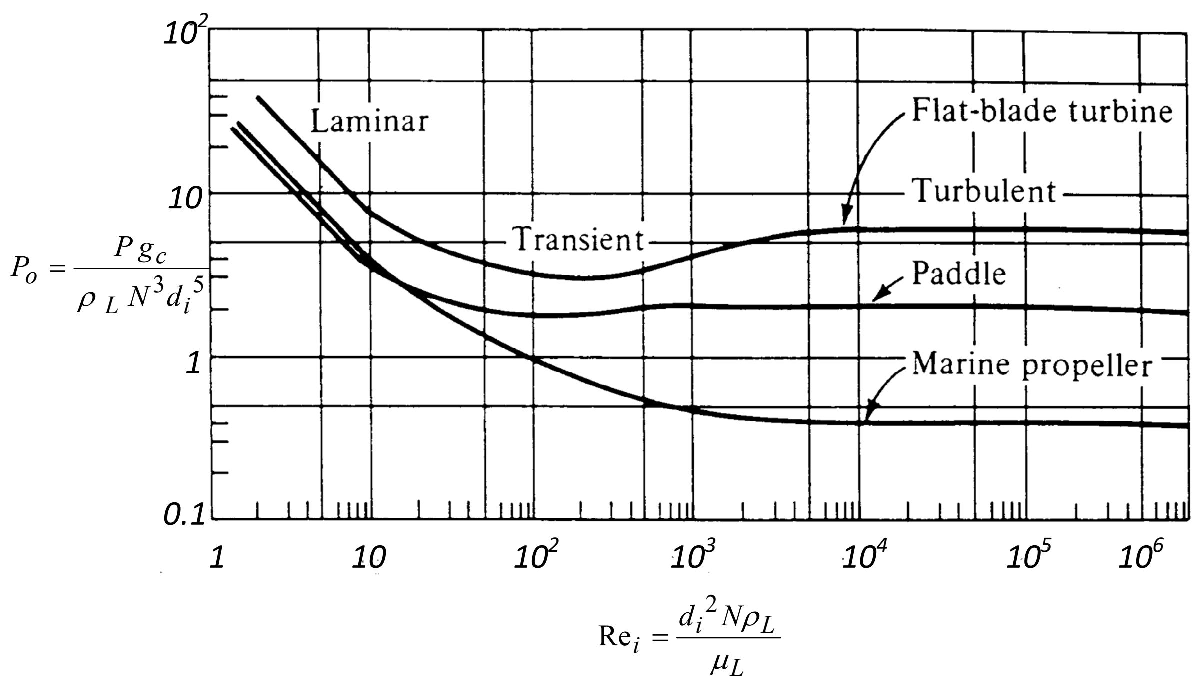
Define impeller Reynolds number $\mathrm{Re}_i = \dfrac{d_i^2 N \rho_L}{\mu_L}$

d_i = impeller diameter (e.g. m)

N = impeller rotation rate (e.g rev/sec)

Define Power number $P_o = \dfrac{P\, g_c}{\rho\ N^3 d_i^5}$

Use the following correlation to get " P ", the *ungasssed* power input (no bubbles) into a stirred tank of liquid (Fig. 30.5, W^3-R 5^{th})



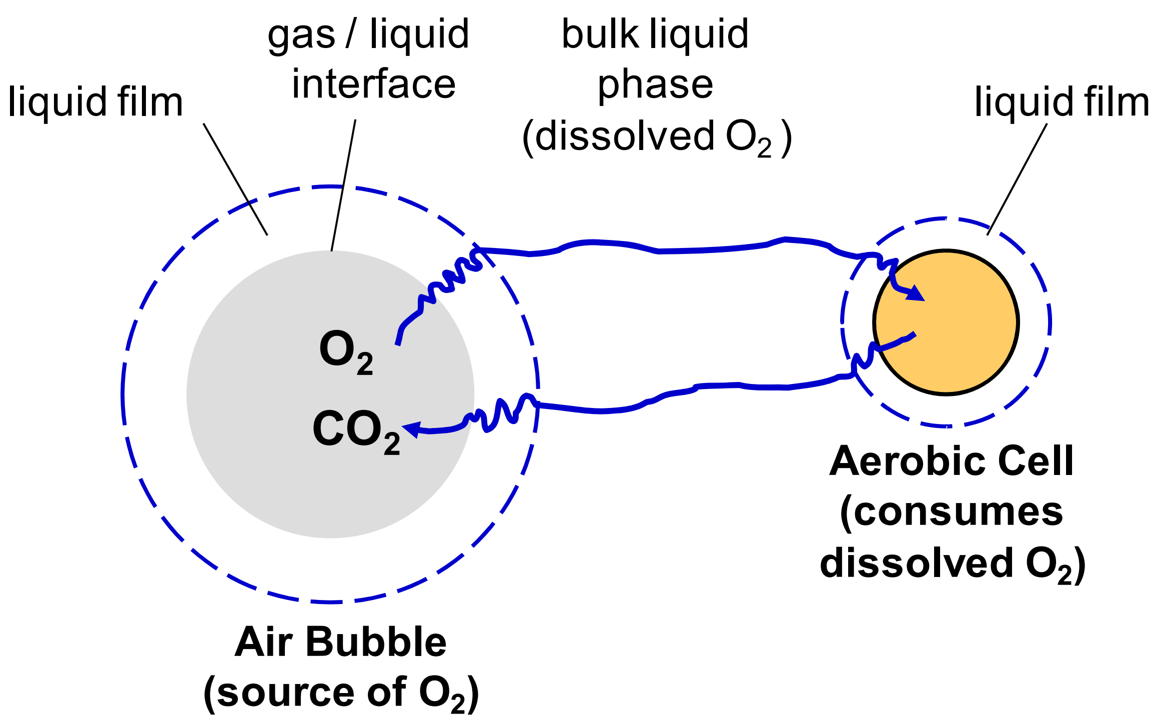
Aerating a stirred tank of liquid lowers the impeller power input. Get " P_g ", the *gassed* power input, by correcting ***P*** for the input volumetric gas flowrate:

$$\log_{10}\left(\frac{P_g}{P}\right) = -192\left(\frac{d_i}{d_T}\right)^{4.38}\left(\frac{d_i^{\,2} N \rho_L}{\mu_L}\right)^{0.115}\left(\frac{d_i N^2}{g}\right)^{1.96\left(\frac{d_i}{d_T}\right)}\left(\frac{Q_g}{d_i^3 N}\right)$$

d_T = diameter of the vessel
d_i = impeller diameter (correlation validated with flat-blade turbine)
Q_g = gas volumetric flowrate (e.g. m^3/sec)

O_2 Transfer Rate (OTR) in Liquid Cell Suspension Culture

If culture becomes O_2 transfer limited, dissolved O_2 conc. in bulk liquid goes to zero ($C_{AL} \approx 0$)



$$OTR = C_{AL}^{*} \cdot k_L a = \frac{P_{O_2}}{H_{O_2}} k_L a \quad \left(\frac{mmol\ O_2}{L - hr}\right)$$

30.6 Capacity Coefficients for Gas-Liquid Packed Towers

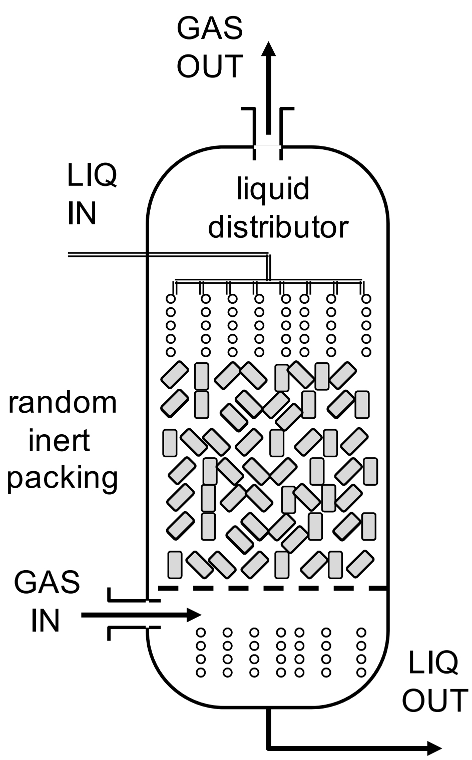
In a gas-liquid packed tower, gas flows upward from the bottom to the top of the tower, and the liquid flows from the top to the bottom of the tower through a bed of randomly packed pieces of material called "packing". The packing material is inert, and the size and shape of the packing is designed to increase the surface area for interphase mass transfer and to promote the flow of both gas and liquid through the packed bed.

Sherwood And Holloway Correlation for the Liquid Film Capacity Coefficient

$$\frac{k_L a}{D_{AB}} = \alpha \left(\frac{L}{\mu} \right)^{1-n} \left(\frac{\mu}{\rho D_{AB}} \right)^{0.5} = \alpha \left(\frac{L}{\mu} \right)^{1-n} Sc^{0.5}$$

α and n are the packing coefficients for a specific type of packing (see Table below), which require the following units be followed:

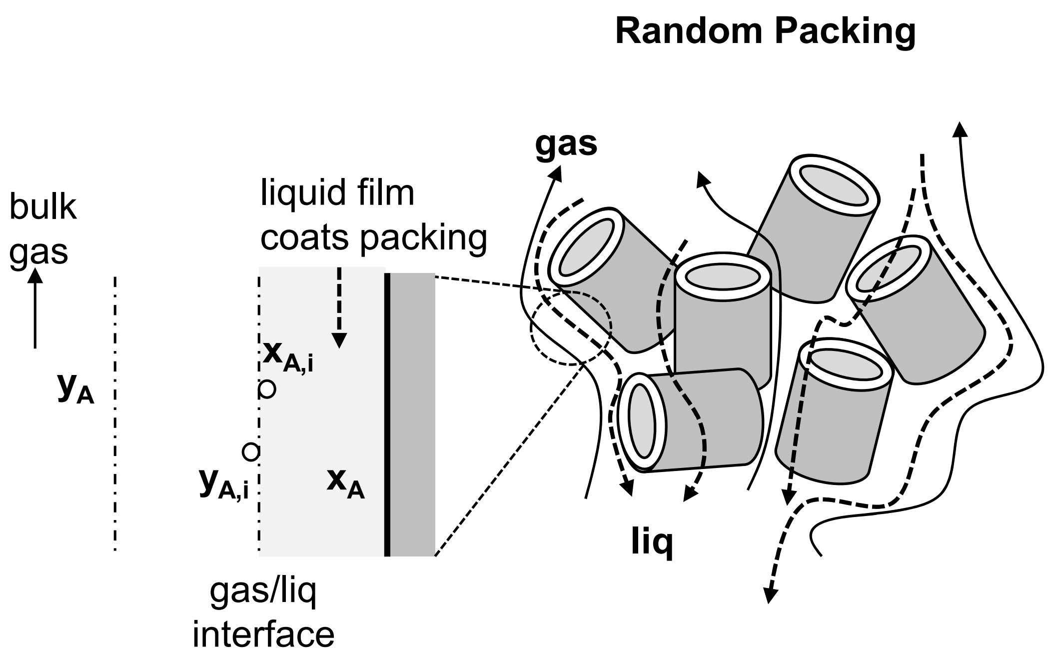
$k_L a$	mass-transfer capacity coefficient, in hr^{-1}
L	liquid mass flow rate per cross-sectional area of the empty tower, $lb_m/ft^2 \cdot hr$
μ	viscosity of the liquid, $lb_m/ft \cdot hr$
ρ	density of the liquid, lb_m/ft^3
D_{AB}	liquid mass diffusivity of transferring component A into inert liquid B, ft^2/hr

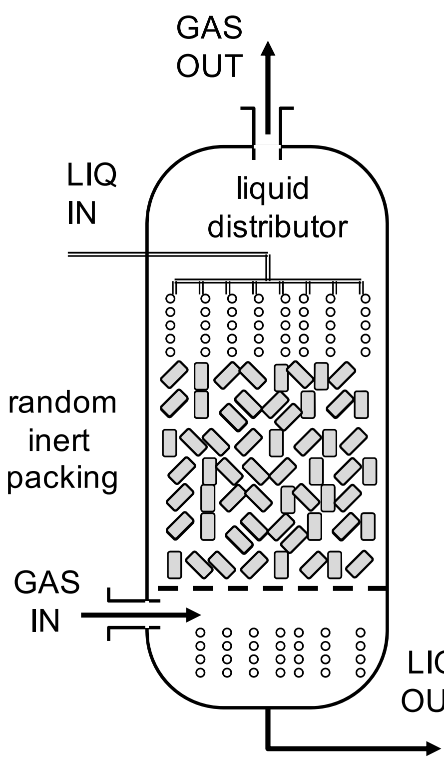


Counter-current flow gas-liquid packed tower

Packing	α	n
2.0 inch rings	80	0.22
1.5-inch rings	90	0.22
1.0 inch rings	100	0.22
0.5-inch rings	280	0.35
0.375 inch rings	550	0.46
1.5 inch saddles	160	0.28
1.0 inch saddles	170	0.28
0.375 inch saddles	150	0.28
3.0 inch spiral tiles	110	0.28

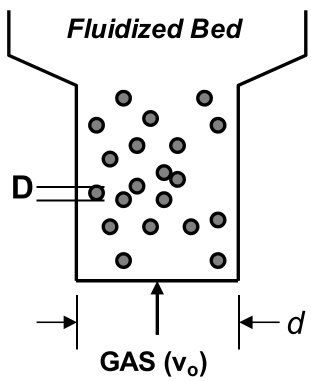
Packing coefficients (α and n) of various packings for use in the Sherwood and Holloway correlation


Random Packing
gas
bulk
gas
liquid film
coats packing
y_A
$x_{A,i}$
$y_{A,i}$
x_A
gas/liq
interface
liq

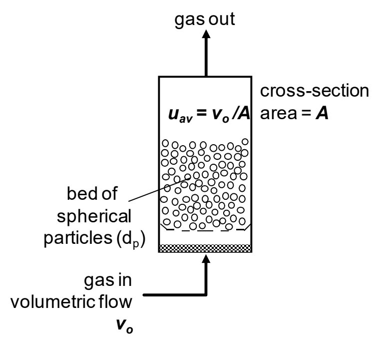

GAS
OUT
LIQ
IN
liquid
distributor
random
inert
packing
GAS
IN
LIQ
OUT

30.2-20.6 Summary of process systems for promoting convective mass transfer

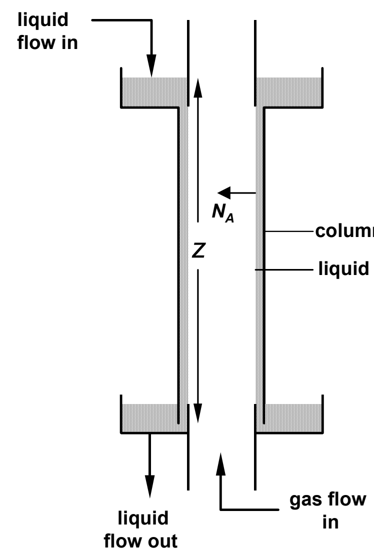
30.2 Fluidized Bed



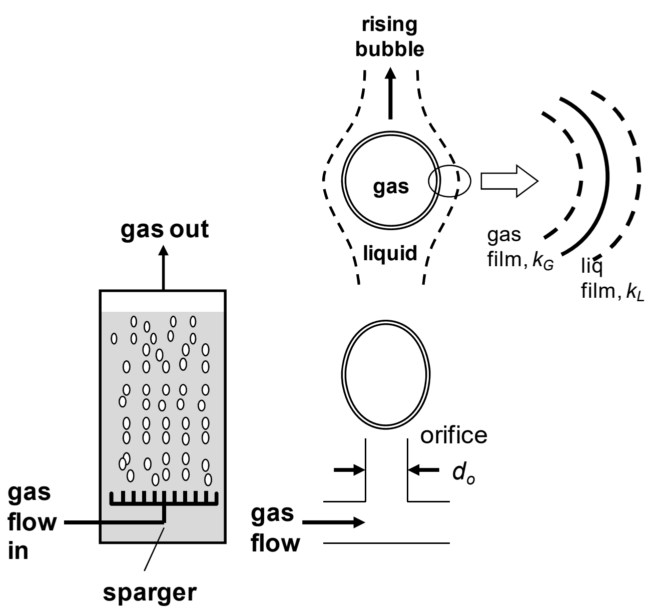
30.4 Packed Bed



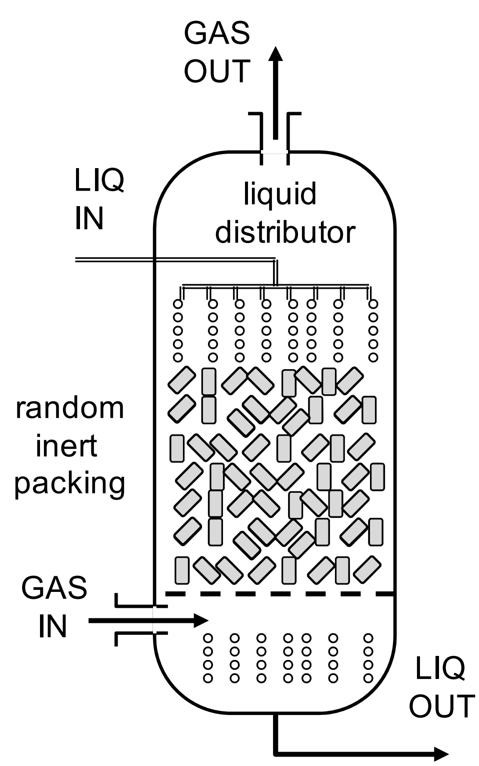
30.3 Wetted Wall Column



30.5 Bubble Column



30.6 Gas-Liquid Packed Column

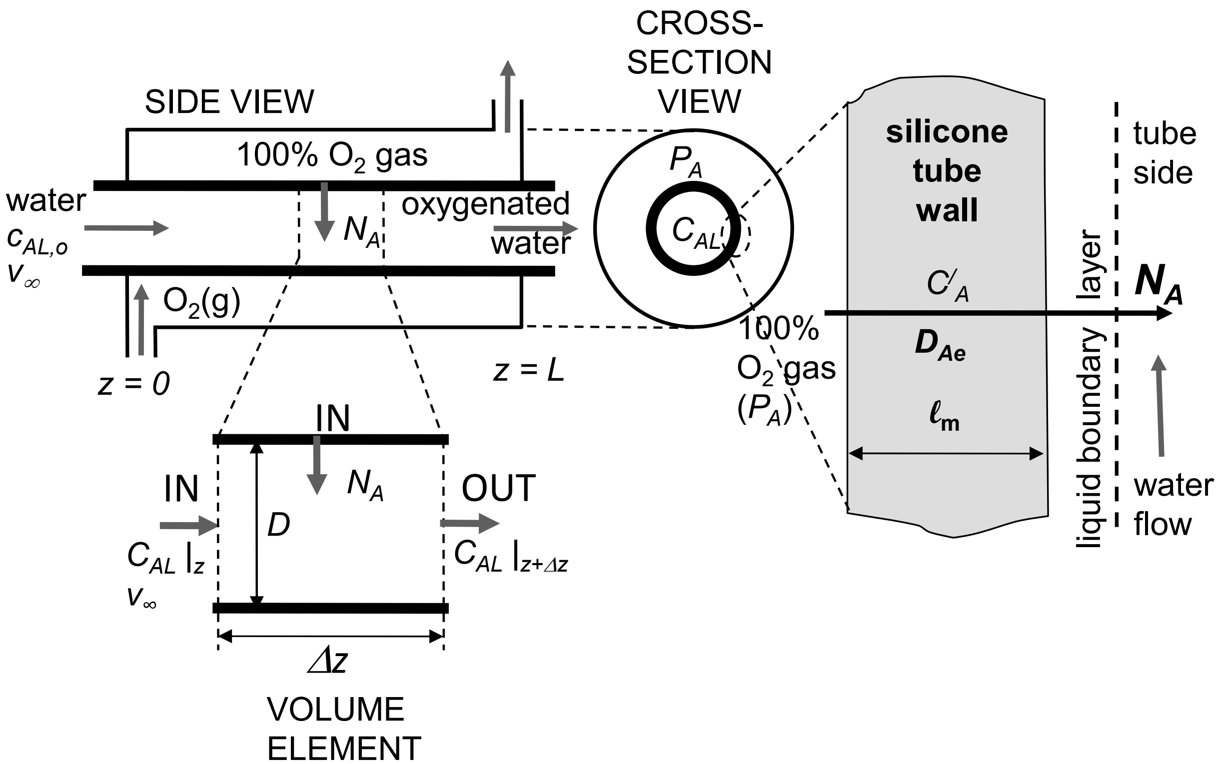


Bubbleless Shell and Tube Membrane Aerator (Example 6 WRF $6^{th}/7^{th}$)

The "bubbleless" shell-and-tube membrane aeration system shown below is used to transfer oxygen gas (O_2) to liquid water. The tube wall is made of silicone, a polymer that is highly permeable to O_2 gas but not to water vapor. Pure oxygen gas (100% O_2) is maintained at a constant pressure in the annular space surrounding the tube. The O_2 gas partitions into silicone polymer, and then diffuses through the tube wall to reach the flowing water. As the fluid flows down the length of the tube, the absorption of oxygen will increase the concentration of the dissolved oxygen.

(a) Develop a model to predict the concentration of dissolved oxygen down the length the tube, $\boldsymbol{C_{AL}(z)}$, evaluated at $\boldsymbol{z = L}$.

(b) Then determine required length z = L for $\boldsymbol{C_{AL}}$ to be at 30% of dissolved oxygen saturation concentration for the parameters given.



Differential material balance (all terms moles O_2/time)

$$\begin{pmatrix}\text{rate of } O_2 \text{ carried by fluid}\\ \text{into volume element (IN)}\end{pmatrix} + \begin{pmatrix}\text{mass transfer of } O_2\\ \text{into volume elenent (IN)}\end{pmatrix} - \begin{pmatrix}\text{rate of } O_2 \text{ carried by fluid}\\ \text{into volume element (OUT)}\end{pmatrix} +$$

$$\begin{pmatrix}\text{rate of } O_2 \text{ generation within}\\ \text{volume element (GEN)}\end{pmatrix} = \begin{pmatrix}\text{rate of } O_2 \text{ accumulation within}\\ \text{volume element (ACC)}\end{pmatrix}$$

Source for O_2 mass transfer: O_2 gas

Sink for O_2 mass transfer: flowing water

Assumptions:
(1) Process is at steady state
(2) Concentration profile of interest is along the *z*-direction only and represents the local bulk concentration and fluid properties
(3) No reaction of O_2 in the water
(4) Process is dilute with respect to dissolved O_2.

IN – OUT = 0

$$\frac{\pi D^2}{4} \mathrm{v}_\infty \, c_{AL}\Big|_z + N_A \, \pi \, D \, \Delta z - \frac{\pi D^2}{4} \mathrm{v}_\infty \, c_{AL}\Big|_{z+\Delta z} + 0 = 0$$

$$-\frac{\pi D^2}{4} \mathrm{v}_\infty \left(\frac{c_{AL}\big|_{z+\Delta z} - c_{AL}\big|_z}{\Delta z} \right) + K_L \, \pi \, D \left(c_{AL}^* - c_{AL} \right) = 0$$

$\Delta z \to 0$,

$$-\frac{dc_{AL}}{dz} + \frac{4K_L}{\mathrm{v}_\infty D} \left(c_{AL}^* - c_{AL} \right) = 0$$

Integration:

$$\int_{c_{AL,o}}^{c_{AL}} \frac{-dc_{AL}}{c_{AL}^* - c_{AL}} = -\frac{4K_L}{\mathrm{v}_\infty D} \int_0^L dz$$

or

$$\ln \left(\frac{c_{AL}^* - c_{AL,o}}{c_{AL}^* - c_{AL}} \right) = \frac{4K_L L}{\mathrm{v}_\infty D}$$

Concentration Profile $c_{AL}(z)$

$$c_{AL}(\mathrm{z}) = c_{AL}^* - \left(c_{AL}^* - c_{AL,o} \right) \exp\left(-\frac{4K_L z}{\mathrm{v}_\infty D} \right)$$

(b) Required tube length, ***L*** for parameters given below

Tube side convective liquid flow: $\mathbf{v}_\infty$ = 50 cm/sec, ***d*** = 1.0 cm,
$\boldsymbol{\nu}_L$ = 9.12 x 10^{-3} cm^2/sec, $\boldsymbol{D_{AB}}$ = 2.1 x 10^{-5} cm^2/sec (A = O_2, B = liquid H_2O)
Membrane parameters: $\boldsymbol{D_{Ae}}$ = 5.0 x 10^{-6} cm^2/sec, $\boldsymbol{S_m}$ = 0.029 atm m^3/gmole,
$\boldsymbol{l_m}$ = 0.1 cm
Thermodynamic: ***H*** = 0.78 m^{3}atm/gmole for O_2 gas in H_2O liquid, $\boldsymbol{p_A}$ = 1.5 atm
100% O_2 on shell side

Determine K_L

Tube side liquid k_L

$$\mathrm{Re} = \frac{\mathrm{v}_\infty D}{\nu_L} = \frac{(50\ \text{cm/s})(1.0\ \text{cm})}{(9.12\ \text{x}\ 10^{-3}\text{cm}^2/\text{s})} = 5482\ \text{(turbulent)}$$

$$Sc = \frac{\nu_L}{D_{AB}} = \frac{9.12\ \text{x}\ 10^{-3}\ \text{cm}^2/\text{s}}{2.1\ \text{x}\ 10^{-5}\ \text{cm}^2/\text{s}} = 434$$

$$Sh = 0.023\,\mathrm{Re}^{0.83}\,Sc^{1/3} = 0.023(5482)^{0.83}(434)^{1/3} = 221$$

$$k_L = \frac{Sh}{D}D_{AB} = \frac{221}{1.0\ \text{cm}}2.1\ \text{x}\ 10^{-5}\text{cm}^2/\text{s} = 4.64\ \text{x}\ 10^{-3}\ \text{cm/s}$$

Membrane constant k_m

$$k_m = \frac{H\,D_{Ae}}{S_m l_m} = \frac{(0.78\ \text{atm}\cdot\text{m}^3/\text{gmole})(5.0\ \text{x}\ 10^{-6}\ \text{cm}^2/\text{s})}{(0.029\ \text{atm}\cdot\text{m}^3/\text{gmole})(0.1\ \text{cm})} = 1.35\ \text{x}\ 10^{-3}\ \text{cm/s}$$

Overall mass transfer coefficient K_L

$$K_L = \frac{k_L k_m}{k_L + k_m} = \frac{(4.64\ \text{x}\ 10^{-3}\ \text{cm/s})(1.35\ \text{x}\ 10^{-3}\ \text{cm/s})}{(4.64\ \text{x}\ 10^{-3}\ \text{cm/s})+(1.35\ \text{x}\ 10^{-3}\ \text{cm/s})} = 1.04\ \text{x}\ 10^{-3}\ \text{cm/s}$$

$$c_{AL}^* = \frac{p_A}{H} = \frac{1.5\ \text{atm}}{0.78\ \text{atm}\cdot\text{m}^3/\text{gmole}} = 1.92\ \text{gmole O}_2/\text{m}^3$$

Required tube length L

At 30% of saturation c_{AL} at $z = L$ is $0.3c_{AL}^*$ = 0.577 gmole O_2/m^3

Therefore, the required length L is

$$L = \frac{\mathrm{v}_\infty D}{4K_L}\ln\left(\frac{c_{AL}^* - c_{AL,o}}{c_{AL}^* - c_{AL}}\right) = \frac{(50\ \text{cm/s})(1.0\ \text{cm})}{4\cdot(1.04\ \text{x}\ 10^{-3}\ \text{cm/s})}\ln\left(\frac{(1.92\text{-}0)\,\text{gmole/m}^3}{(1.92\text{-}0.577)\,\text{gmole/m}^3}\right) = 4287\ \text{cm}$$

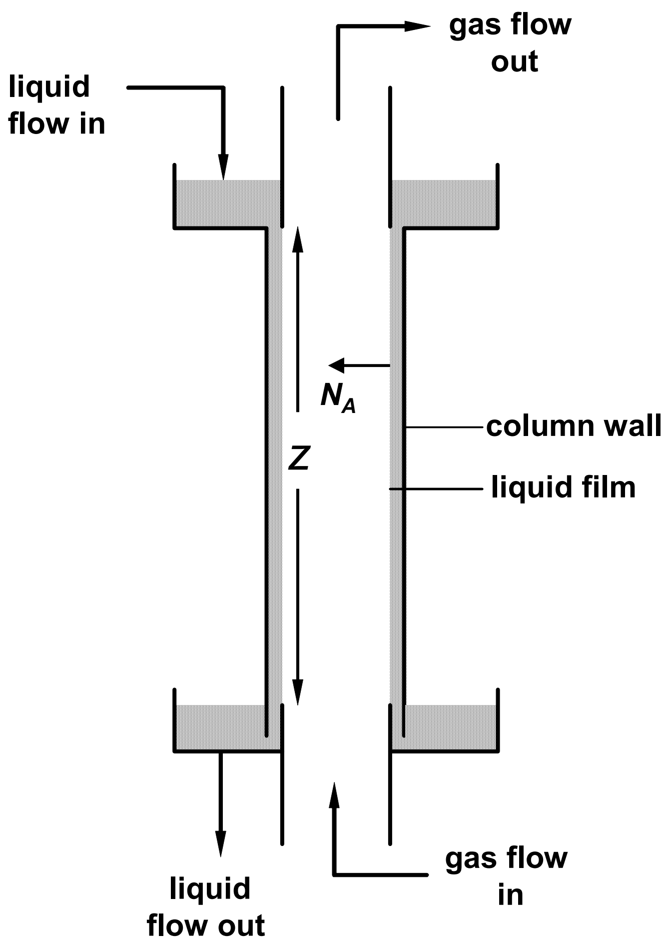
Problem Example, Topic 30.3 and 29.2

Problem statement. A wetted-wall column is used to study the stripping of TCE from water to air at a constant temperature of 293 K and total system pressure of 1.0 atm. The column inner diameter is 4.0 cm and the height is 2.0 m. The volumetric air flowrate into the column is 2000 cm^3/s ($2.0 \cdot 10^{-3}$ m^3/s) and the volumetric flowrate of water is 50 cm^3/s ($5.0 \cdot 10^{-4}$ m^3/s). *Estimate K_L, the overall liquid phase mass transfer coefficient for TCE across the liquid and gas film.*

Assume that water loss by evaporation is negligible. The process is very dilute so that the bulk gas has the properties of air and bulk liquid has the properties of water. The equilibrium solubility of TCE in water is described by Henry's law:

$$p_A = H \cdot x_A$$

where H is 550 atm at 293 K. The binary gas phase diffusivity of TCE in air is $8.08 \cdot 10^{-6}$ m^2/s at 1.0 atm and 293 K, as determined by the Fuller-Shettler-Giddings correlation. The binary liquid phase diffusivity of TCE in water at 293 K is $8.9 \cdot 10^{-10}$ m^2/s, as determined by the Hayduk-Laudie correlation.



Gas Film Coefficient k_G

Re for air flow through inside of wetted wall column

$$v_\infty = \frac{4Q_g}{\pi D^2} = \frac{4\cdot\left(2.0\cdot10^{-3}\,\frac{m^3}{s}\right)}{\pi\,(0.04\,m)^2} = 1.59\,\frac{m}{s}$$

$$\mathrm{Re} = \frac{\rho_{air}\,v_\infty D}{\mu_{air}} = \frac{\left(1.19\,\frac{kg}{m^3}\right)\left(1.59\,\frac{m}{s}\right)(0.04\ m)}{1.84\cdot10^{-5}\,\frac{kg}{m\cdot s}} = 4113$$

$$Sc = \frac{\mu_{air}}{\rho_{air}\,D_{TCE-air}} = \frac{1.84\cdot10^{-5}\,\frac{kg}{m\cdot s}}{\left(1.19\,\frac{kg}{m^3}\right)\left(8.08\cdot10^{-6}\,\frac{m^2}{s}\right)} = 1.91$$

Re > 2000. Therefore

$$k_c = \frac{D_{AB}}{D}0.023\,Re^{0.83}\,Sc^{0.44} = \left(\frac{8.08\cdot10^{-6}\ m^2/s}{0.04\,m}\right)(0.023)(4113)^{0.83}(1.91)^{0.44} = 6.17\cdot10^{-3}\,\frac{m}{s}$$

$$k_G = \frac{k_c}{RT} = \frac{6.17\cdot10^{-3}\,\frac{m}{s}}{\left(0.08206\,\frac{m^3\cdot atm}{kgmole\cdot K}\right)(293\,K)} = 2.57\cdot10^{-4}\,\frac{kgmole}{m^2\cdot s\cdot atm}$$

Liquid film coefficient k_L

Re for falling liquid film

$$\mathrm{Re}_L = \frac{4\rho_L v_L}{\pi D\mu_L} = \frac{4\cdot\left(998.2\,\frac{kg}{m^3}\right)\left(5\cdot10^{-5}\,\frac{m^3}{s}\right)}{\pi\cdot(0.04\,m)\left(9.93\cdot10^{-4}\,\frac{kg}{m\cdot s}\right)} = 1600$$

$$Sc = \frac{\mu_L}{\rho_L\,D_{TCE-H_2O}} = \frac{9.93\cdot10^{-4}\,\frac{kg}{m\cdot s}}{\left(998.2\,\frac{kg}{m^3}\right)\left(8.90\cdot10^{-10}\,\frac{m^2}{s}\right)} = 1118$$

$$k_L = \frac{D_{AB}}{z} 0.433 (Sc)^{1/2} \left(\frac{\rho_L{}^2 \, g \, z^3}{\mu_L{}^2} \right)^{1/6} (Re_L)^{0.4}$$

$$= \frac{8.9 \cdot 10^{-10} \frac{m^2}{s}}{2.0\, m} 0.433 \cdot (1118)^{1/2} \left(\frac{\left(998.2 \frac{kg}{m^3}\right)^2 \left(\frac{9.8\, m}{s^2}\right) (2.0\, m)^3}{\left(9.93 \cdot 10^{-4} \frac{kg}{m \cdot s}\right)^2} \right)^{1/6} (1600)^{0.4} = 2.55 \cdot 10^{-5} \frac{m}{s}$$

Overall liquid mass transfer coefficient (K_L)

$$H = 550\, atm \frac{M_{H_2O}}{\rho_{H_2O}} = (550\, atm) \frac{\left(\frac{18\, kg}{kgmole}\right)}{\left(993.2 \frac{kg}{m^3}\right)} = 9.97 \frac{atm \cdot m^3}{kgmole}$$

$$\frac{1}{K_L} = \frac{1}{k_L} + \frac{1}{H\, k_G} = \frac{1}{2.55 \cdot 10^{-5} \frac{m}{s}} + \frac{1}{9.97 \frac{atm \cdot m^3}{kgmole} 2.57 \cdot 10^{-4} \frac{kgmole}{m^2 \cdot s \cdot atm}}$$

$K_L = 2.52 \cdot 10^{-5}$ m / s

Note $K_L \cong k_L$ (why?)