

# Compact and Provably Secure Lattice-Based Signatures in Hardware

**James Howe, Ciara Rafferty, Ayesha Khalid, Máire O'Neill.**  
May 2017



@CSIT\_QUB  
@SAFECRYPTO

DYNAMIC

REPEAT

48

0

0

0

0

0

0

0

0

IEEE ISCAS 2017

- Jointly collaborative work between:
  - James Howe, Ciara Rafferty, Ayesha Khalid, and Máire O'Neill,
  - Centre of Secure Information Technologies (CSIT), Queen's University Belfast, UK.



European Union H2020 SAFEcrypto Project:  
Advancing lattice-based cryptography in theory  
and practice (2015-2018).



@SAFEcrypto



[www.SAFEcrypto.eu](http://www.SAFEcrypto.eu)

- Motivation
- Introduction to (ideal) lattice-based cryptography
- Hardware design and optimisations
- Results and performance comparison
- Future research directions

What happens when quantum computers become a reality 10/15 years from now?

Commonly used public-key cryptographic algorithms  
(based on integer factorisation and discrete log problem) such as:

**RSA, DSA, Diffie-Hellman Key Exchange, ECC, ECDSA**

will be vulnerable to Shor's algorithm and **will no longer be secure.**

What happens when quantum computers become a reality 10/15 years from now?

Commonly used public-key cryptographic algorithms  
(based on integer factorisation and discrete log problem) such as:

**RSA, DSA, Diffie-Hellman Key Exchange, ECC, ECDSA**

will be vulnerable to Shor's algorithm and **will no longer be secure.**

**IBM's quantum cloud computer  
goes commercial -March 2017** nature

What happens when quantum computers become a reality 10/15 years from now?

Commonly used public-key cryptographic algorithms  
(based on integer factorisation and discrete log problem) such as:

**RSA, DSA, Diffie-Hellman Key Exchange, ECC, ECDSA**

will be vulnerable to Shor's algorithm and **will no longer be secure**.

**IBM's quantum cloud computer  
goes commercial** -March 2017 **nature**

**Scientists are close to building a quantum  
computer that can beat a conventional one**  
-December 2016

**Science**

What happens when quantum computers become a reality 10/15 years from now?

Commonly used public-key cryptographic algorithms  
(based on integer factorisation and discrete log problem) such as:

**RSA, DSA, Diffie-Hellman Key Exchange, ECC, ECDSA**

will be vulnerable to Shor's algorithm and **will no longer be secure**.

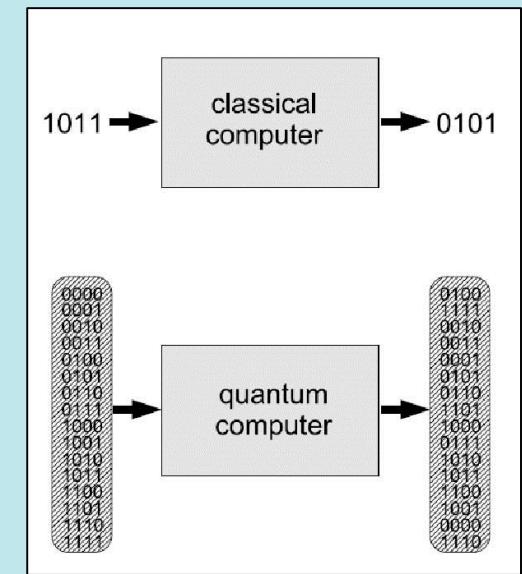
**IBM's quantum cloud computer  
goes commercial** -March 2017 **nature**

**Google moves closer to a universal  
quantum computer** -June 2016 **nature**

**Scientists are close to building a quantum  
computer that can beat a conventional one**  
-December 2016

**Science**

- Why do we need post-quantum cryptography?
  - Quantum computers break ECC and RSA.
  - Classically hard computational problems are now trivial.
  - Governments and companies in preparation.
- Quantum computers exploit the power of quantum parallelism:
- Shor's Algorithm (1994)
  - Used to quickly factorise large numbers (exponential speedup).
  - Significant implications for current cryptographic techniques.
- Grover's Algorithm(1996)
  - Can be used to search an unsorted database faster than a conventional computer, effects security of AES, so AES-128 now 64-bit secure.



- Quantum computers will cause complete insecurities in all currently implemented cryptography on the Internet, such as RSA, DSA, ECC, and ECDSA.
- Post-quantum cryptography currently protects against classical and quantum attacks.

- Quantum computers will cause complete insecurities in all currently implemented cryptography on the Internet, such as RSA, DSA, ECC, and ECDSA.
- Post-quantum cryptography currently protects against classical and quantum attacks.

**Revealed: Google's plan for  
quantum computer supremacy**

-August 2016

**New  
Scientist**

- Quantum computers will cause complete insecurities in all currently implemented cryptography on the Internet, such as RSA, DSA, ECC, and ECDSA.
- Post-quantum cryptography currently protects against classical and quantum attacks.

**Revealed: Google's plan for  
quantum computer supremacy**

-August 2016

**New  
Scientist**

**The quantum clock is ticking on  
encryption – and your data is under  
threat -October 2016**

**WIRED**

- Quantum computers will cause complete insecurities in all currently implemented cryptography on the Internet, such as RSA, DSA, ECC, and ECDSA.
- Post-quantum cryptography currently protects against classical and quantum attacks.

**Revealed: Google's plan for quantum computer supremacy**

-August 2016

New  
Scientist

**Hacking, cryptography, and the countdown to quantum computing -September 2016**

THE  
NEW YORKER

**The quantum clock is ticking on encryption – and your data is under threat -October 2016**

WIRED

- Quantum computers will cause complete insecurities in all currently implemented cryptography on the Internet, such as RSA, DSA, ECC, and ECDSA.
- Post-quantum cryptography currently protects against classical and quantum attacks.

**Revealed: Google's plan for quantum computer supremacy**

-August 2016

New  
Scientist

**Hacking, cryptography, and the countdown to quantum computing** -September 2016

THE  
NEW YORKER

**Google tests new crypto in Chrome to fend off quantum attacks**

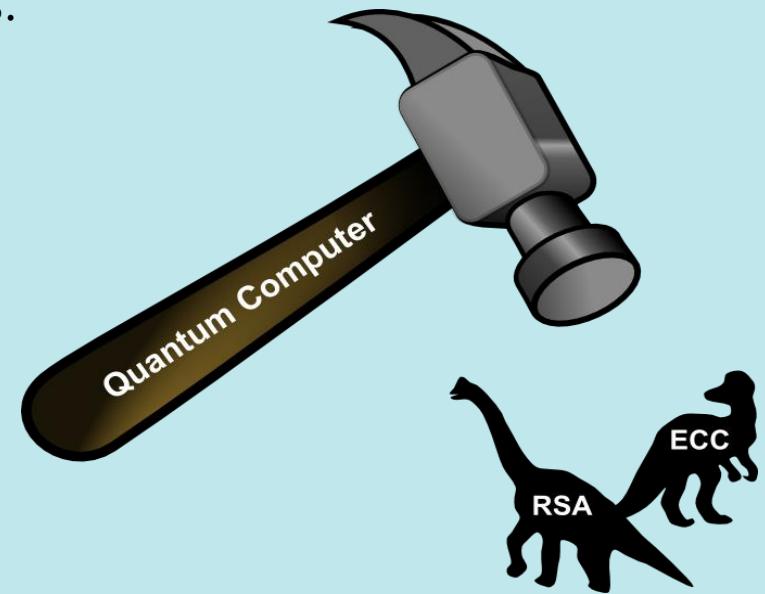
-July 2016

WIRED

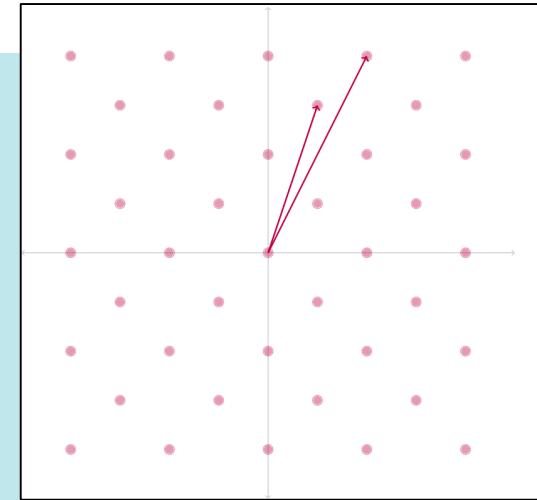
**The quantum clock is ticking on encryption – and your data is under threat** -October 2016

WIRED

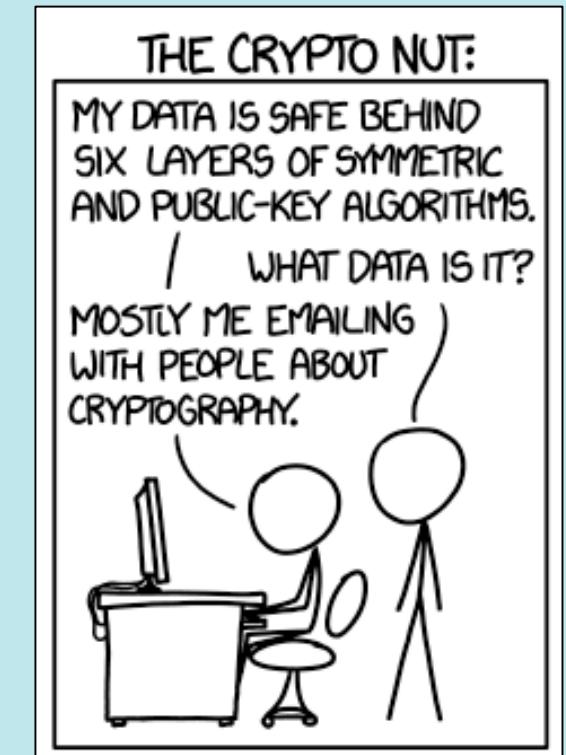
- ETSI researching industrial requirements for quantum-safe real-world deployments.
  - ETSI Quantum-Safe Cryptography (QSC) Industry Specification Group (ISG).
- NIST plan to start post-quantum standardisation 30 Nov 2017
  - Similar to previous AES and SHA-3 standardisations.
- Why focus on lattice-based cryptography?
  - More versatile than code-based, MQ, and hash-based schemes.
  - Theoretical foundations are well-studied.
  - Uses in encryption, signatures, FHE, IBE, etc...



- Lattice-based cryptography is important in its own right.
- Research in lattice-based cryptography is flourishing:
  - “New Hope” key exchange created.
  - “LPR” encryption outperforms RSA and ECC in s/w and h/w.
  - “BLISS” signatures outperform RSA and ECDSA in s/w and h/w.
- Lattice-based cryptography is already being considered:
  - VPN strongSwan supports signature and encryption within post-quantum mode.
  - New Hope awarded Internet Defense Prize Winner 2016.
  - Google experimenting with “New Hope” key exchange.
  - Horizon 2020 SAFECrypto Project.
    - Advancing lattice-based cryptography in theory and practice.



- Digital signatures are very important.
  - Authenticates message source.
  - Validates that data sent is unaltered/trusted.
  - Identifies person, legally, like written signature.
- Used everyday within bank transfers, smart cards, SSL etc...
  - Currently uses RSA or ECDSA.
  - These will be obsolete with quantum computers.
- Hardware-based signatures are becoming more prominent.
  - Will become prominent within IoT & the cloud.
  - Required for V2X communications.



## Ring-TESLA vs BLISS

- For lattice-based digital signatures, current state-of-the-art is BLISS.
- Recently announced was Ring-TESLA, an efficient lattice-based signature scheme.
- Shown to compete with state-of-the-art in software.

BLISS	vs.	Ring-TESLA
<b>Patented</b> NTRU assumptions.		<b>Strong</b> security assumptions (Ring-LWE).
<b>No</b> worst-case to average-case hardness.		<b>Includes</b> worst-case to average-case hardness.
<b>Costly</b> discrete Gaussian sampling.		<b>No</b> on-device discrete Gaussian sampling.
<b>Large</b> polynomial multiplier used.		<b>Evaluate</b> generic low-area poly. multiplier.
<b>Parameters</b> not chosen via security reduction.		<b>Simpler</b> parameter selection, tightly secure.

- Attacks found in the BLISS algorithm.
- Cache attack (software) targets the discrete Gaussian sampler component.
- Discrete Gaussian samplers are known to be a side-channel target in software and hardware.
- Not known yet how to fix this issue in software.
- Ring-TESLA uses discrete Gaussian samplers independent of secret computations.

### Flush, Gauss, and Reload – A Cache Attack on the BLISS Lattice-Based Signature Scheme

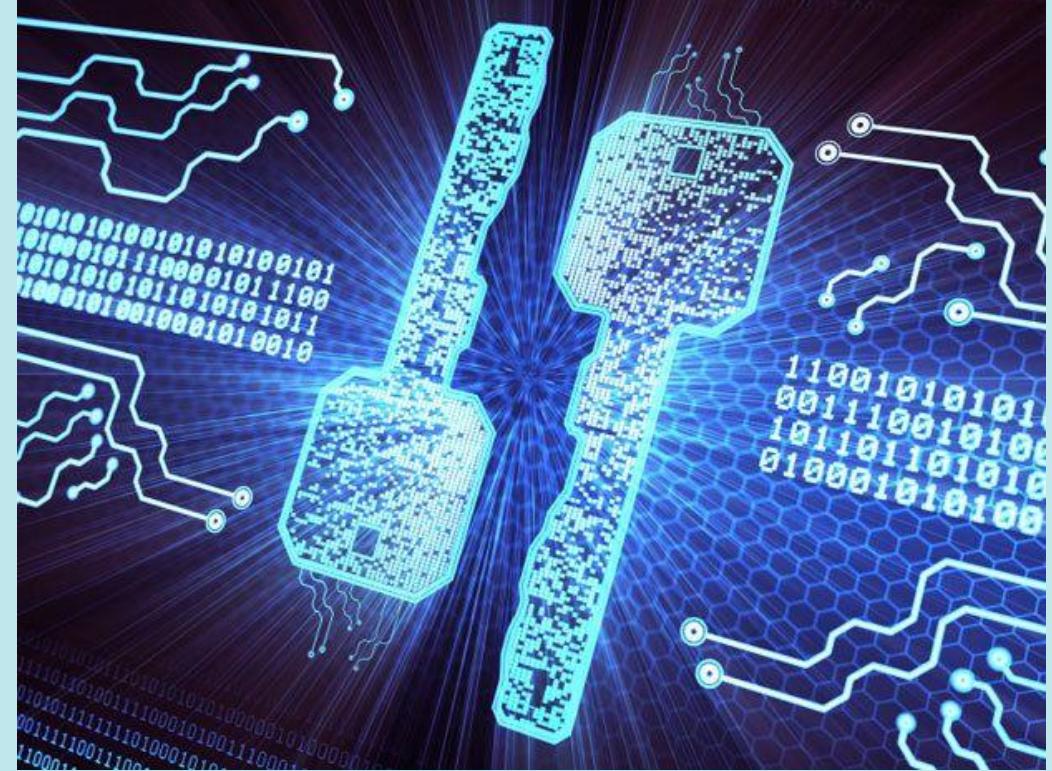
Leon Groot Bruinderink<sup>1</sup>, Andreas Hülsing<sup>1</sup>, Tanja Lange<sup>1</sup>, and Yuval Yarom<sup>2</sup>

<sup>1</sup> Department of Mathematics and Computer Science  
Technische Universiteit Eindhoven, P.O. Box 513, 5600 MB Eindhoven, NL  
[1.groot.bruinderink@tue.nl](mailto:1.groot.bruinderink@tue.nl), [andreas@huelsing.net](mailto:andreas@huelsing.net), [tanja@hyperelliptic.org](mailto:tanja@hyperelliptic.org)

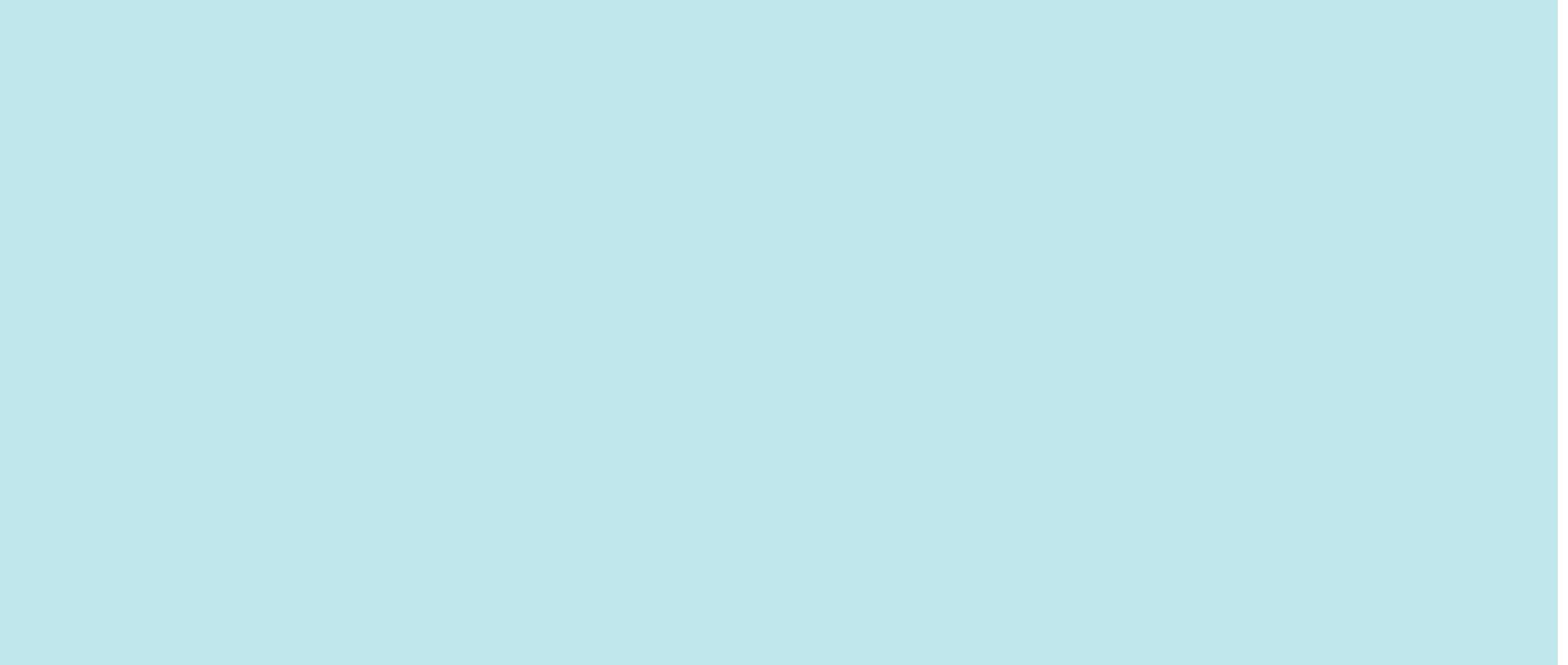
<sup>2</sup> The University of Adelaide and NICTA  
[yval@cs.adelaide.edu.au](mailto:yval@cs.adelaide.edu.au)

## Hardware Design Approach

- Generic hardware architecture.
- Evaluate low-area hardware design.
- Offers better security, slower throughput.
- Spartan-6 FPGA targeted for comparisons.



## Ring-TESLA signature scheme<sup>1</sup> (Akleylek et al. '16)



<sup>1</sup>) Sedat Akleylek, Nina Bindel, Johannes A. Buchmann, Julianne Krämer, and Giorgia Azzurra Marson. An efficient lattice-based signature scheme with provably secure instantiation. In AFRICACRYPT, pages 44–60, 2016.

**KeyGen( $a_1, a_2$ ):**

Discrete Gaussian polynomials:  $\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2 \leftarrow D_\sigma^n$ ,  $\mathbf{t}_1 \equiv \mathbf{a}_1 \mathbf{s} + \mathbf{e}_1 \pmod{q}$ ,  $\mathbf{t}_2 \equiv \mathbf{a}_2 + \mathbf{e}_2 \pmod{q}$   
Secret-Key:  $(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2)$  // Public-Key:  $(\mathbf{t}_1, \mathbf{t}_2)$ .

<sup>1</sup>) Sedat Akleylek, Nina Bindel, Johannes A. Buchmann, Julianne Krämer, and Giorgia Azzurra Marson. An efficient lattice-based signature scheme with provably secure instantiation. In AFRICACRYPT, pages 44–60, 2016.

**KeyGen( $a_1, a_2$ ):**

Discrete Gaussian polynomials:  $\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2 \leftarrow D_\sigma^n$ ,  $\mathbf{t}_1 \equiv \mathbf{a}_1 \mathbf{s} + \mathbf{e}_1 \pmod{q}$ ,  $\mathbf{t}_2 \equiv \mathbf{a}_2 + \mathbf{e}_2 \pmod{q}$   
 Secret-Key:  $(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2)$  // Public-Key:  $(\mathbf{t}_1, \mathbf{t}_2)$ .

**Sign( $\mu; a_1, a_2, s, e_1, e_2$ ):**

Uniform polynomial:  $\mathbf{y} \leftarrow \mathbb{Z}_q[x]/(x^n + 1)$

- $\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$ ,  $\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$

Compute the hash function:

- $\mathbf{c} = H(\mathbf{v}_1 || \mathbf{v}_2, \mu)$

Compute signature/rejections:

- $\mathbf{z} \equiv \mathbf{y} + \mathbf{s}\mathbf{c} \quad \leftarrow \text{signature}$
- $\mathbf{w}_1 \equiv \mathbf{v}_1 + \mathbf{e}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}_2 \equiv \mathbf{v}_2 + \mathbf{e}_2 \mathbf{c} \pmod{q}$

<sup>1</sup>) Sedat Akleylek, Nina Bindel, Johannes A. Buchmann, Juliane Krämer, and Giorgia Azzurra Marson. An efficient lattice-based signature scheme with provably secure instantiation. In AFRICACRYPT, pages 44–60, 2016.

### KeyGen( $\mathbf{a}_1, \mathbf{a}_2$ ):

Discrete Gaussian polynomials:  $\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2 \leftarrow D_\sigma^n$ ,  $\mathbf{t}_1 \equiv \mathbf{a}_1 \mathbf{s} + \mathbf{e}_1 \pmod{q}$ ,  $\mathbf{t}_2 \equiv \mathbf{a}_2 + \mathbf{e}_2 \pmod{q}$   
 Secret-Key:  $(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2)$  // Public-Key:  $(\mathbf{t}_1, \mathbf{t}_2)$ .

### Sign( $\mu; \mathbf{a}_1, \mathbf{a}_2, \mathbf{s}, \mathbf{e}_1, \mathbf{e}_2$ ):

Uniform polynomial:  $\mathbf{y} \leftarrow \mathbb{Z}_q[x]/(x^n + 1)$

- $\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$ ,  $\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$

Compute the hash function:

- $\mathbf{c} = H(\mathbf{v}_1 || \mathbf{v}_2, \mu)$

Compute signature/rejections:

- $\mathbf{z} \equiv \mathbf{y} + \mathbf{s}\mathbf{c} \quad \leftarrow \text{signature}$
- $\mathbf{w}_1 \equiv \mathbf{v}_1 + \mathbf{e}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}_2 \equiv \mathbf{v}_2 + \mathbf{e}_2 \mathbf{c} \pmod{q}$

### Verify( $\mu; \mathbf{z}, \mathbf{c}; \mathbf{a}_1, \mathbf{a}_2, \mathbf{t}_1, \mathbf{t}_2$ ):

Compute hash inputs:

- $\mathbf{w}'_1 \equiv \mathbf{a}_1 \mathbf{z} + \mathbf{t}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}'_2 \equiv \mathbf{a}_2 \mathbf{z} + \mathbf{t}_2 \mathbf{c} \pmod{q}$

Compute the hash function:

- $\mathbf{c}' = H(\mathbf{w}'_1 || \mathbf{w}'_2, \mu)$

Accept/reject signature:

- If  $\mathbf{c} = \mathbf{c}'$

<sup>1</sup>) Sedat Akleylek, Nina Bindel, Johannes A. Buchmann, Juliane Krämer, and Giorgia Azzurra Marson. An efficient lattice-based signature scheme with provably secure instantiation. In AFRICACRYPT, pages 44–60, 2016.

### KeyGen( $a_1, a_2$ ):

Discrete Gaussian polynomials:  $\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2 \leftarrow D_\sigma^n$ ,  $\mathbf{t}_1 \equiv \mathbf{a}_1 \mathbf{s} + \mathbf{e}_1 \pmod{q}$ ,  $\mathbf{t}_2 \equiv \mathbf{a}_2 + \mathbf{e}_2 \pmod{q}$   
 Secret-Key:  $(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2)$  // Public-Key:  $(\mathbf{t}_1, \mathbf{t}_2)$ .

### Sign( $\mu; a_1, a_2, s, e_1, e_2$ ):

Uniform polynomial:  $y \leftarrow \mathbb{Z}_q[x]/(x^n + 1)$

- $v_1 \equiv a_1 y \pmod{q}$ ,  $v_2 \equiv a_2 y \pmod{q}$

Compute the hash function:

- $c = H(v_1 || v_2, \mu)$

Compute signature/rejections:

- $\mathbf{z} \equiv \mathbf{y} + \mathbf{s}\mathbf{c} \quad \leftarrow \text{signature}$
- $\mathbf{w}_1 \equiv v_1 + \mathbf{e}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}_2 \equiv v_2 + \mathbf{e}_2 \mathbf{c} \pmod{q}$

### Verify( $\mu; z, c; a_1, a_2, t_1, t_2$ ):

Compute hash inputs:

- $\mathbf{w}'_1 \equiv \mathbf{a}_1 \mathbf{z} + \mathbf{t}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}'_2 \equiv \mathbf{a}_2 \mathbf{z} + \mathbf{t}_2 \mathbf{c} \pmod{q}$

Compute the hash function:

- $c' = H(\mathbf{w}'_1 || \mathbf{w}'_2, \mu)$

Accept/reject signature:

- If  $c = c'$

<sup>1</sup>) Sedat Akleylek, Nina Bindel, Johannes A. Buchmann, Juliane Krämer, and Giorgia Azzurra Marson. An efficient lattice-based signature scheme with provably secure instantiation. In AFRICACRYPT, pages 44–60, 2016.

### KeyGen( $a_1, a_2$ ):

Discrete Gaussian polynomials:  $\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2 \leftarrow D_\sigma^n$ ,  $\mathbf{t}_1 \equiv \mathbf{a}_1 \mathbf{s} + \mathbf{e}_1 \pmod{q}$ ,  $\mathbf{t}_2 \equiv \mathbf{a}_2 + \mathbf{e}_2 \pmod{q}$   
 Secret-Key:  $(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2)$  // Public-Key:  $(\mathbf{t}_1, \mathbf{t}_2)$ .

### Sign( $\mu; a_1, a_2, s, e_1, e_2$ ):

Uniform polynomial:  $y \leftarrow \mathbb{Z}_q[x]/(x^n + 1)$

- $v_1 \equiv a_1 y \pmod{q}$ ,  $v_2 \equiv a_2 y \pmod{q}$

Compute the hash function:

- $c = H(v_1 || v_2, \mu)$

Compute signature/rejections:

- $\mathbf{z} \equiv \mathbf{y} + \mathbf{s}c \quad \leftarrow \text{signature}$
- $\mathbf{w}_1 \equiv v_1 + \mathbf{e}_1 c \pmod{q}$
- $\mathbf{w}_2 \equiv v_2 + \mathbf{e}_2 c \pmod{q}$

### Verify( $\mu; z, c; a_1, a_2, t_1, t_2$ ):

Compute hash inputs:

- $\mathbf{w}'_1 \equiv \mathbf{a}_1 \mathbf{z} + \mathbf{t}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}'_2 \equiv \mathbf{a}_2 \mathbf{z} + \mathbf{t}_2 \mathbf{c} \pmod{q}$

Compute the hash function:

- $c' = H(\mathbf{w}'_1 || \mathbf{w}'_2, \mu)$

Accept/reject signature:

- If  $c = c'$

<sup>1</sup>) Sedat Akleylek, Nina Bindel, Johannes A. Buchmann, Juliane Krämer, and Giorgia Azzurra Marson. An efficient lattice-based signature scheme with provably secure instantiation. In AFRICACRYPT, pages 44–60, 2016.

### KeyGen( $a_1, a_2$ ):

Discrete Gaussian polynomials:  $\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2 \leftarrow D_\sigma^n$ ,  $\mathbf{t}_1 \equiv \mathbf{a}_1 \mathbf{s} + \mathbf{e}_1 \pmod{q}$ ,  $\mathbf{t}_2 \equiv \mathbf{a}_2 + \mathbf{e}_2 \pmod{q}$   
 Secret-Key:  $(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2)$  // Public-Key:  $(\mathbf{t}_1, \mathbf{t}_2)$ .

### Sign( $\mu; a_1, a_2, s, e_1, e_2$ ):

Uniform polynomial:  $y \leftarrow \mathbb{Z}_q[x]/(x^n + 1)$

- $v_1 \equiv a_1 y \pmod{q}$ ,  $v_2 \equiv a_2 y \pmod{q}$

Compute the hash function:

- $c = H(v_1 || v_2, \mu)$

Compute signature/rejections:

- $\mathbf{z} \equiv \mathbf{y} + \mathbf{s}c \quad \leftarrow \text{signature}$
- $w_1 \equiv v_1 + e_1 c \pmod{q}$
- $w_2 \equiv v_2 + e_2 c \pmod{q}$

### Verify( $\mu; z, c; a_1, a_2, t_1, t_2$ ):

Compute hash inputs:

- $w'_1 \equiv a_1 z + t_1 c \pmod{q}$
- $w'_2 \equiv a_2 z + t_2 c \pmod{q}$

Compute the hash function:

- $c' = H(w'_1 || w'_2, \mu)$

Accept/reject signature:

- If  $c = c'$

<sup>1</sup>) Sedat Akleylek, Nina Bindel, Johannes A. Buchmann, Juliane Krämer, and Giorgia Azzurra Marson. An efficient lattice-based signature scheme with provably secure instantiation. In AFRICACRYPT, pages 44–60, 2016.

### KeyGen( $a_1, a_2$ ):

Discrete Gaussian polynomials:  $\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2 \leftarrow D_\sigma^n$ ,  $\mathbf{t}_1 \equiv \mathbf{a}_1 \mathbf{s} + \mathbf{e}_1 \pmod{q}$ ,  $\mathbf{t}_2 \equiv \mathbf{a}_2 + \mathbf{e}_2 \pmod{q}$   
 Secret-Key:  $(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2)$  // Public-Key:  $(\mathbf{t}_1, \mathbf{t}_2)$ .

### Sign( $\mu; a_1, a_2, s, e_1, e_2$ ):

Uniform polynomial:  $y \leftarrow \mathbb{Z}_q[x]/(x^n + 1)$

- $v_1 \equiv a_1 y \pmod{q}$ ,  $v_2 \equiv a_2 y \pmod{q}$

Compute the hash function:

- $c = H(v_1 || v_2, \mu)$

Compute signature/rejections:

- $\mathbf{z} \equiv \mathbf{y} + \mathbf{s}\mathbf{c} \quad \leftarrow \text{signature}$
- $\mathbf{w}_1 \equiv v_1 + \mathbf{e}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}_2 \equiv v_2 + \mathbf{e}_2 \mathbf{c} \pmod{q}$

### Verify( $\mu; z, c; a_1, a_2, t_1, t_2$ ):

Compute hash inputs:

- $w'_1 \equiv a_1 z + t_1 c \pmod{q}$
- $w'_2 \equiv a_2 z + t_2 c \pmod{q}$

Compute the hash function:

- $c' = H(w'_1 || w'_2, \mu)$

Accept/reject signature:

- If  $c = c'$

<sup>1</sup>) Sedat Akleylek, Nina Bindel, Johannes A. Buchmann, Juliane Krämer, and Giorgia Azzurra Marson. An efficient lattice-based signature scheme with provably secure instantiation. In AFRICACRYPT, pages 44–60, 2016.

### KeyGen( $a_1, a_2$ ):

Discrete Gaussian polynomials:  $\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2 \leftarrow D_\sigma^n$ ,  $\mathbf{t}_1 \equiv \mathbf{a}_1 \mathbf{s} + \mathbf{e}_1 \pmod{q}$ ,  $\mathbf{t}_2 \equiv \mathbf{a}_2 + \mathbf{e}_2 \pmod{q}$   
 Secret-Key:  $(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2)$  // Public-Key:  $(\mathbf{t}_1, \mathbf{t}_2)$ .

### Sign( $\mu; a_1, a_2, s, e_1, e_2$ ):

Uniform polynomial:  $\mathbf{y} \leftarrow \mathbb{Z}_q[x]/(x^n + 1)$

- $\mathbf{v}_1 \equiv a_1 \mathbf{y} \pmod{q}$ ,  $\mathbf{v}_2 \equiv a_2 \mathbf{y} \pmod{q}$

Compute the hash function:

- $\mathbf{c} = H(\mathbf{v}_1 \parallel \mathbf{v}_2, \mu)$

Compute signature/rejections:

- $\mathbf{z} \equiv \mathbf{y} + \mathbf{s}\mathbf{c} \quad \leftarrow \text{signature}$
- $\mathbf{w}_1 \equiv \mathbf{v}_1 + \mathbf{e}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}_2 \equiv \mathbf{v}_2 + \mathbf{e}_2 \mathbf{c} \pmod{q}$

### Verify( $\mu; z, c; a_1, a_2, t_1, t_2$ ):

Compute hash inputs:

- $\mathbf{w}'_1 \equiv a_1 \mathbf{z} + t_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}'_2 \equiv a_2 \mathbf{z} + t_2 \mathbf{c} \pmod{q}$

Compute the hash function:

- $\mathbf{c}' = H(\mathbf{w}'_1 \parallel \mathbf{w}'_2, \mu)$

Accept/reject signature:

- If  $\mathbf{c} = \mathbf{c}'$

<sup>1</sup>) Sedat Akleylek, Nina Bindel, Johannes A. Buchmann, Juliane Krämer, and Giorgia Azzurra Marson. An efficient lattice-based signature scheme with provably secure instantiation. In AFRICACRYPT, pages 44–60, 2016.

### KeyGen( $a_1, a_2$ ):

Discrete Gaussian polynomials:  $\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2 \leftarrow D_\sigma^n$ ,  $\mathbf{t}_1 \equiv \mathbf{a}_1 \mathbf{s} + \mathbf{e}_1 \pmod{q}$ ,  $\mathbf{t}_2 \equiv \mathbf{a}_2 + \mathbf{e}_2 \pmod{q}$   
 Secret-Key:  $(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2)$  // Public-Key:  $(\mathbf{t}_1, \mathbf{t}_2)$ .

### Sign( $\mu; a_1, a_2, s, e_1, e_2$ ):

Uniform polynomial:  $\mathbf{y} \leftarrow \mathbb{Z}_q[x]/(x^n + 1)$

- $\mathbf{v}_1 \equiv a_1 \mathbf{y} \pmod{q}$ ,  $\mathbf{v}_2 \equiv a_2 \mathbf{y} \pmod{q}$

Compute the hash function:

- $\mathbf{c} = H(\mathbf{v}_1 \parallel \mathbf{v}_2, \mu)$

Compute signature/rejections:

- $\mathbf{z} \equiv \mathbf{y} + \mathbf{s}\mathbf{c} \quad \leftarrow \text{signature}$
- $\mathbf{w}_1 \equiv \mathbf{v}_1 + \mathbf{e}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}_2 \equiv \mathbf{v}_2 + \mathbf{e}_2 \mathbf{c} \pmod{q}$

### Verify( $\mu; z, c; a_1, a_2, t_1, t_2$ ):

Compute hash inputs:

- $\mathbf{w}'_1 \equiv a_1 \mathbf{z} + t_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}'_2 \equiv a_2 \mathbf{z} + t_2 \mathbf{c} \pmod{q}$

Compute the hash function:

- $\mathbf{c}' = H(\mathbf{w}'_1 \parallel \mathbf{w}'_2, \mu)$

Accept/reject signature:

- If  $\mathbf{c} = \mathbf{c}'$

<sup>1</sup>) Sedat Akleylek, Nina Bindel, Johannes A. Buchmann, Juliane Krämer, and Giorgia Azzurra Marson. An efficient lattice-based signature scheme with provably secure instantiation. In AFRICACRYPT, pages 44–60, 2016.

### KeyGen( $a_1, a_2$ ):

Discrete Gaussian polynomials:  $\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2 \leftarrow D_\sigma^n$ ,  $\mathbf{t}_1 \equiv \mathbf{a}_1 \mathbf{s} + \mathbf{e}_1 \pmod{q}$ ,  $\mathbf{t}_2 \equiv \mathbf{a}_2 + \mathbf{e}_2 \pmod{q}$   
 Secret-Key:  $(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2)$  // Public-Key:  $(\mathbf{t}_1, \mathbf{t}_2)$ .

### Sign( $\mu; a_1, a_2, s, e_1, e_2$ ):

Uniform polynomial:  $\mathbf{y} \leftarrow \mathbb{Z}_q[x]/(x^n + 1)$

- $\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$ ,  $\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$

Compute the hash function:

- $\mathbf{c} = H(\mathbf{v}_1 \parallel \mathbf{v}_2, \mu)$

Compute signature/rejections:

- $\mathbf{z} \equiv \mathbf{y} + \mathbf{s}\mathbf{c} \quad \leftarrow \text{signature}$
- $\mathbf{w}_1 \equiv \mathbf{v}_1 + \mathbf{e}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}_2 \equiv \mathbf{v}_2 + \mathbf{e}_2 \mathbf{c} \pmod{q}$

### Verify( $\mu; z, c; a_1, a_2, t_1, t_2$ ):

Compute hash inputs:

- $\mathbf{w}'_1 \equiv \mathbf{a}_1 \mathbf{z} + \mathbf{t}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}'_2 \equiv \mathbf{a}_2 \mathbf{z} + \mathbf{t}_2 \mathbf{c} \pmod{q}$

Compute the hash function:

- $\mathbf{c}' = H(\mathbf{w}'_1 \parallel \mathbf{w}'_2, \mu)$

Accept/reject signature:

- If  $\mathbf{c} = \mathbf{c}'$

<sup>1</sup>) Sedat Akleylek, Nina Bindel, Johannes A. Buchmann, Juliane Krämer, and Giorgia Azzurra Marson. An efficient lattice-based signature scheme with provably secure instantiation. In AFRICACRYPT, pages 44–60, 2016.

### KeyGen( $a_1, a_2$ ):

Discrete Gaussian polynomials:  $\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2 \leftarrow D_\sigma^n$ ,  $\mathbf{t}_1 \equiv \mathbf{a}_1 \mathbf{s} + \mathbf{e}_1 \pmod{q}$ ,  $\mathbf{t}_2 \equiv \mathbf{a}_2 + \mathbf{e}_2 \pmod{q}$   
 Secret-Key:  $(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2)$  // Public-Key:  $(\mathbf{t}_1, \mathbf{t}_2)$ .

### Sign( $\mu; a_1, a_2, s, e_1, e_2$ ):

Uniform polynomial:  $\mathbf{y} \leftarrow \mathbb{Z}_q[x]/(x^n + 1)$

- $\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$ ,  $\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$

Compute the hash function:

- $\mathbf{c} = H(\mathbf{v}_1 \parallel \mathbf{v}_2, \mu)$

Compute signature/rejections:

- $\mathbf{z} \equiv \mathbf{y} + \mathbf{s}\mathbf{c} \quad \leftarrow \text{signature}$
- $\mathbf{w}_1 \equiv \mathbf{v}_1 + \mathbf{e}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}_2 \equiv \mathbf{v}_2 + \mathbf{e}_2 \mathbf{c} \pmod{q}$

### Verify( $\mu; z, c; a_1, a_2, t_1, t_2$ ):

Compute hash inputs:

- $\mathbf{w}'_1 \equiv \mathbf{a}_1 \mathbf{z} + \mathbf{t}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}'_2 \equiv \mathbf{a}_2 \mathbf{z} + \mathbf{t}_2 \mathbf{c} \pmod{q}$

Compute the hash function:

- $\mathbf{c}' = H(\mathbf{w}'_1 \parallel \mathbf{w}'_2, \mu)$

Accept/reject signature:

- If  $\mathbf{c} = \mathbf{c}'$

<sup>1</sup>) Sedat Akleylek, Nina Bindel, Johannes A. Buchmann, Juliane Krämer, and Giorgia Azzurra Marson. An efficient lattice-based signature scheme with provably secure instantiation. In AFRICACRYPT, pages 44–60, 2016.

### KeyGen( $a_1, a_2$ ):

Discrete Gaussian polynomials:  $\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2 \leftarrow D_\sigma^n$ ,  $\mathbf{t}_1 \equiv \mathbf{a}_1 \mathbf{s} + \mathbf{e}_1 \pmod{q}$ ,  $\mathbf{t}_2 \equiv \mathbf{a}_2 + \mathbf{e}_2 \pmod{q}$   
 Secret-Key:  $(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2)$  // Public-Key:  $(\mathbf{t}_1, \mathbf{t}_2)$ .

### Sign( $\mu; a_1, a_2, s, e_1, e_2$ ):

Uniform polynomial:  $y \leftarrow \mathbb{Z}_q[x]/(x^n + 1)$

- $\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$ ,  $\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$

Compute the hash function:

- $\mathbf{c} = H(\mathbf{v}_1 \parallel \mathbf{v}_2, \mu)$

Compute signature/rejections:

- $\mathbf{z} \equiv \mathbf{y} + \mathbf{sc} \quad \leftarrow \text{signature}$
- $\mathbf{w}_1 \equiv \mathbf{v}_1 + \mathbf{e}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}_2 \equiv \mathbf{v}_2 + \mathbf{e}_2 \mathbf{c} \pmod{q}$

### Verify( $\mu; z, c; a_1, a_2, t_1, t_2$ ):

Compute hash inputs:

- $\mathbf{w}'_1 \equiv \mathbf{a}_1 \mathbf{z} + \mathbf{t}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}'_2 \equiv \mathbf{a}_2 \mathbf{z} + \mathbf{t}_2 \mathbf{c} \pmod{q}$

Compute the hash function:

- $\mathbf{c}' = H(\mathbf{w}'_1 \parallel \mathbf{w}'_2, \mu)$

Accept/reject signature:

- If  $\mathbf{c} = \mathbf{c}'$

<sup>1</sup>) Sedat Akleylek, Nina Bindel, Johannes A. Buchmann, Juliane Krämer, and Giorgia Azzurra Marson. An efficient lattice-based signature scheme with provably secure instantiation. In AFRICACRYPT, pages 44–60, 2016.

### KeyGen( $a_1, a_2$ ):

Discrete Gaussian polynomials:  $\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2 \leftarrow D_\sigma^n$ ,  $\mathbf{t}_1 \equiv a_1 \mathbf{s} + \mathbf{e}_1 \pmod{q}$ ,  $\mathbf{t}_2 \equiv a_2 + \mathbf{e}_2 \pmod{q}$   
 Secret-Key:  $(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2)$  // Public-Key:  $(\mathbf{t}_1, \mathbf{t}_2)$ .

### Sign( $\mu; a_1, a_2, s, e_1, e_2$ ):

Uniform polynomial:  $y \leftarrow \mathbb{Z}_q[x]/(x^n + 1)$

- $v_1 \equiv a_1 y \pmod{q}$ ,  $v_2 \equiv a_2 y \pmod{q}$

Compute the hash function:

- $c = H(v_1 || v_2, \mu)$

Compute signature/rejections:

- $\mathbf{z} \equiv y + sc \quad \leftarrow \text{signature}$
- $w_1 \equiv v_1 + e_1 c \pmod{q}$
- $w_2 \equiv v_2 + e_2 c \pmod{q}$

### Verify( $\mu; z, c; a_1, a_2, t_1, t_2$ ):

Compute hash inputs:

- $w'_1 \equiv a_1 z + t_1 c \pmod{q}$
- $w'_2 \equiv a_2 z + t_2 c \pmod{q}$

Compute the hash function:

- $c' = H(w'_1 || w'_2, \mu)$

Accept/reject signature:

- If  $c = c'$

<sup>1</sup>) Sedat Akleylek, Nina Bindel, Johannes A. Buchmann, Juliane Krämer, and Giorgia Azzurra Marson. An efficient lattice-based signature scheme with provably secure instantiation. In AFRICACRYPT, pages 44–60, 2016.

### KeyGen( $a_1, a_2$ ):

Discrete Gaussian polynomials:  $\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2 \leftarrow D_\sigma^n$ ,  $\mathbf{t}_1 \equiv a_1 \mathbf{s} + \mathbf{e}_1 \pmod{q}$ ,  $\mathbf{t}_2 \equiv a_2 + \mathbf{e}_2 \pmod{q}$   
 Secret-Key:  $(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2)$  // Public-Key:  $(\mathbf{t}_1, \mathbf{t}_2)$ .

### Sign( $\mu; a_1, a_2, s, e_1, e_2$ ):

Uniform polynomial:  $y \leftarrow \mathbb{Z}_q[x]/(x^n + 1)$

- $\mathbf{v}_1 \equiv a_1 y \pmod{q}$ ,  $\mathbf{v}_2 \equiv a_2 y \pmod{q}$

Compute the hash function:

- $\mathbf{c} = H(\mathbf{v}_1 || \mathbf{v}_2, \mu)$

Compute signature/rejections:

- $\mathbf{z} \equiv \mathbf{y} + \mathbf{s}\mathbf{c} \quad \leftarrow \text{signature}$
- $\mathbf{w}_1 \equiv \mathbf{v}_1 + \mathbf{e}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}_2 \equiv \mathbf{v}_2 + \mathbf{e}_2 \mathbf{c} \pmod{q}$

### Verify( $\mu; z, c; a_1, a_2, t_1, t_2$ ):

Compute hash inputs:

- $\mathbf{w}'_1 \equiv a_1 \mathbf{z} + \mathbf{t}_1 \mathbf{c} \pmod{q}$
- $\mathbf{w}'_2 \equiv a_2 \mathbf{z} + \mathbf{t}_2 \mathbf{c} \pmod{q}$

Compute the hash function:

- $\mathbf{c}' = H(\mathbf{w}'_1 || \mathbf{w}'_2, \mu)$

Accept/reject signature:

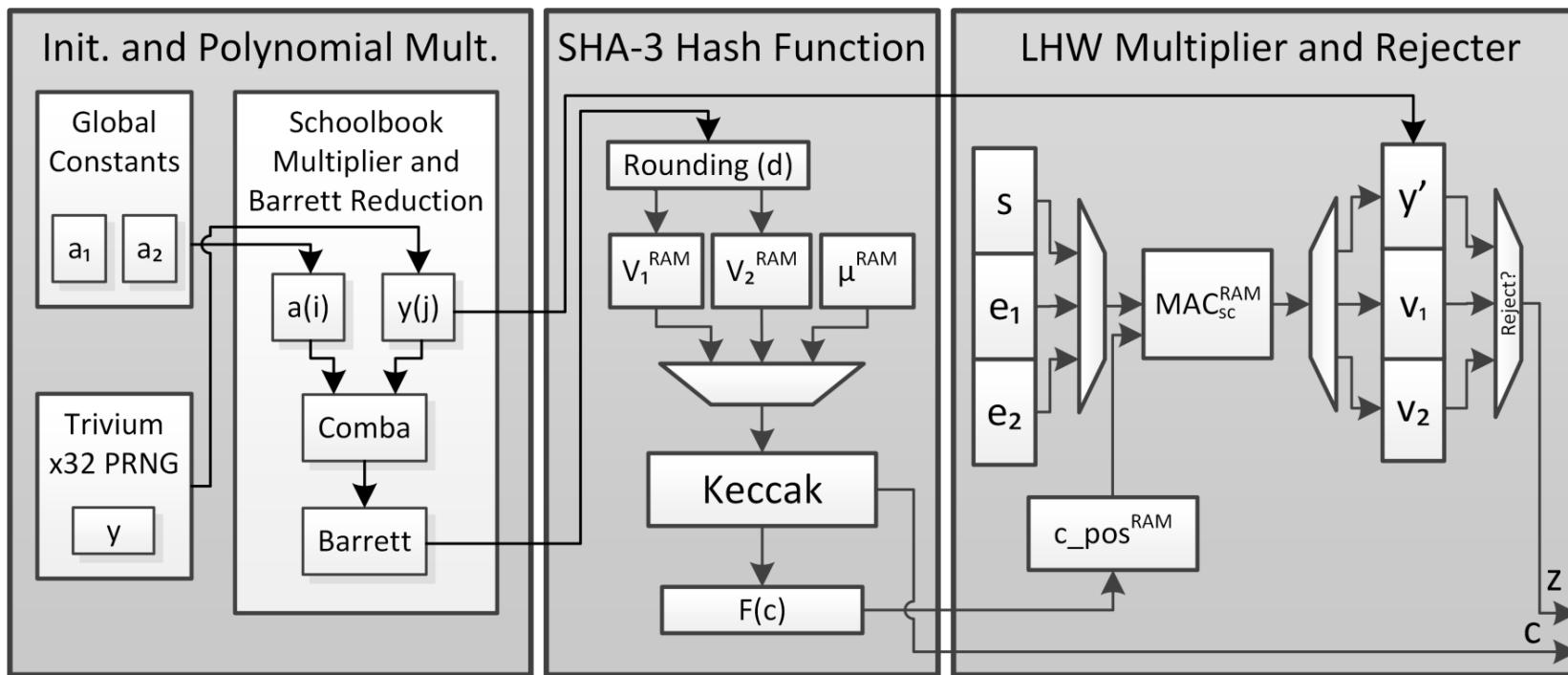
- If  $\mathbf{c} = \mathbf{c}'$

128-bit security parameters:  
 $n = 512$ ,  
 $q = 51750913$ ,  
 $\sigma = 52$ .

Signature is 11.9 kb,  
 public-key is 26 kb,  
 and secret-key is 13.7 kb.

<sup>1</sup>) Sedat Akleylek, Nina Bindel, Johannes A. Buchmann, Juliane Krämer, and Giorgia Azzurra Marson. An efficient lattice-based signature scheme with provably secure instantiation. In AFRICACRYPT, pages 44–60, 2016.

## Architecture of Ring-TESLA Sign



High-level architecture of the Ring-LWE signature scheme, Ring-TESLA.

- The first hardware design of a Ring-LWE signature scheme.
- First low-area signature scheme in lattice-based cryptography.
- Generic hardware designs for sign and verify.
- Numerous parallel multipliers used for a variety of results.

---

## Algorithm 1 Ring-TESLA Sign

---

**procedure** SIGN( $\mu, \mathbf{a}_1, \mathbf{a}_2, \mathbf{s}, \mathbf{e}_1, \mathbf{e}_2$ )

$\mathbf{y} \leftarrow \mathcal{R}_{q,[B]}$

$\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$

$\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$

$c = H([\mathbf{v}_1]_{d,q}, [\mathbf{v}_2]_{d,q}, \mu)$

$\mathbf{c} = F(c)$

$\mathbf{z} \leftarrow \mathbf{y} + \mathbf{s}\mathbf{c}$

$\mathbf{w}_1 \equiv \mathbf{v}_1 - \mathbf{e}_1 \mathbf{c} \pmod{q}$

$\mathbf{w}_2 \equiv \mathbf{v}_2 - \mathbf{e}_2 \mathbf{c} \pmod{q}$

**if**  $[\mathbf{w}_1]_{2^d}, [\mathbf{w}_2]_{2^d} \notin \mathcal{R}_{2^d-L}$   
or  $\mathbf{z} \notin \mathcal{R}_{B-U}$  **then**

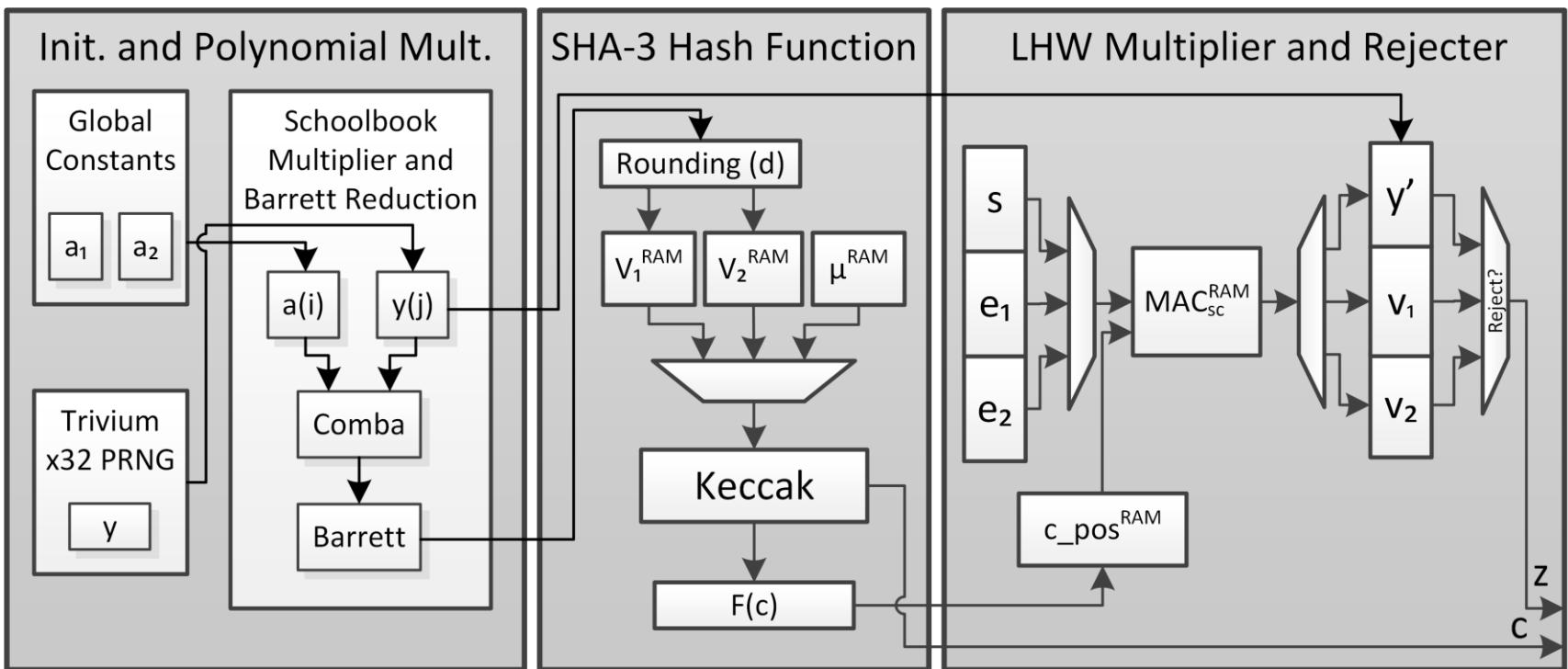
    Restart

**end if**

**return**  $(\mathbf{z}, c)$

**end procedure**

---



# Architecture of the Ring-TESLA Signature Scheme

---

## Algorithm 1 Ring-TESLA Sign

---

**procedure** SIGN( $\mu, \mathbf{a}_1, \mathbf{a}_2, \mathbf{s}, \mathbf{e}_1, \mathbf{e}_2$ )

$\mathbf{y} \leftarrow \mathcal{R}_{q,[B]}$

$\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$

$\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$

$c = H([\mathbf{v}_1]_{d,q}, [\mathbf{v}_2]_{d,q}, \mu)$

$\mathbf{c} = F(c)$

$\mathbf{z} \leftarrow \mathbf{y} + \mathbf{s}\mathbf{c}$

$\mathbf{w}_1 \equiv \mathbf{v}_1 - \mathbf{e}_1 \mathbf{c} \pmod{q}$

$\mathbf{w}_2 \equiv \mathbf{v}_2 - \mathbf{e}_2 \mathbf{c} \pmod{q}$

**if**  $[\mathbf{w}_1]_{2^d}, [\mathbf{w}_2]_{2^d} \notin \mathcal{R}_{2^d-L}$   
or  $\mathbf{z} \notin \mathcal{R}_{B-U}$  **then**

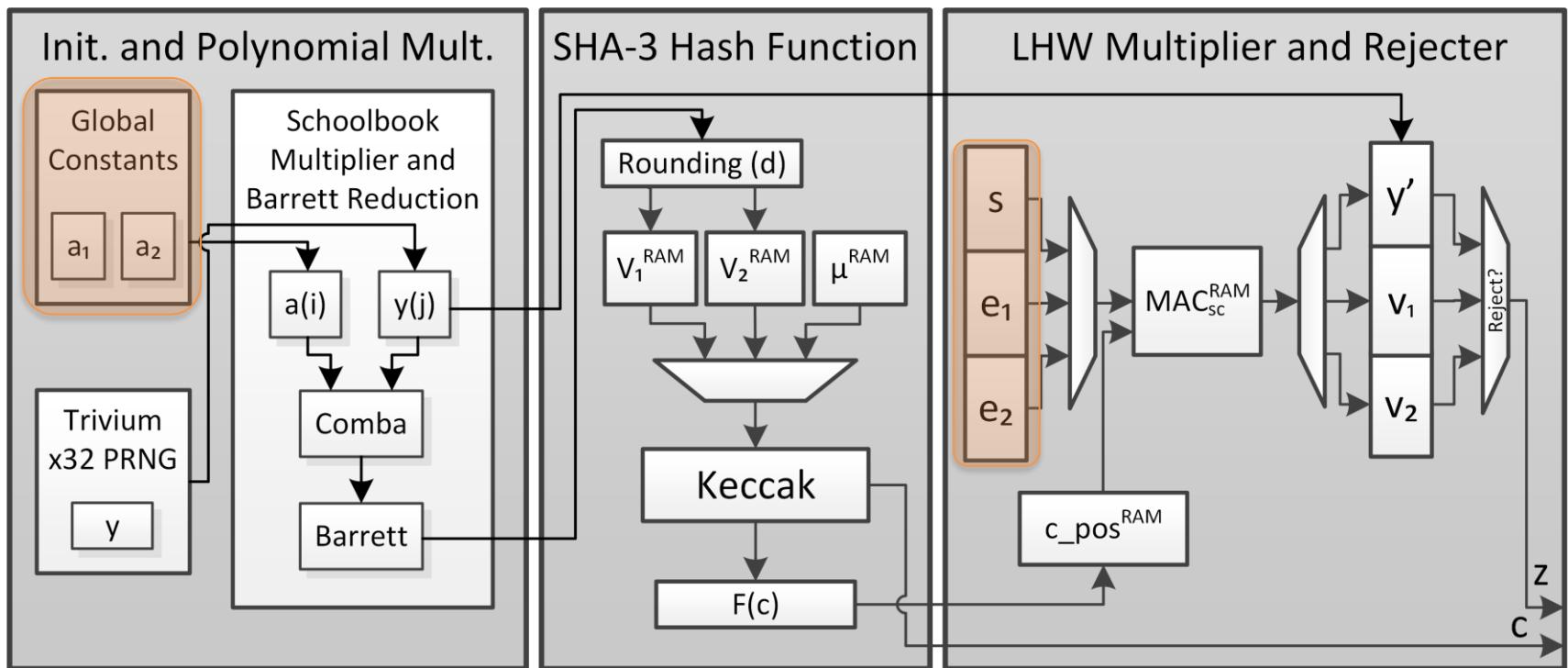
    Restart

**end if**

**return**  $(\mathbf{z}, c)$

**end procedure**

---



# Architecture of the Ring-TESLA Signature Scheme

---

## Algorithm 1 Ring-TESLA Sign

---

**procedure** SIGN( $\mu, \mathbf{a}_1, \mathbf{a}_2, \mathbf{s}, \mathbf{e}_1, \mathbf{e}_2$ )

$\mathbf{y} \leftarrow \mathcal{R}_{q,[B]}$

$\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$

$\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$

$c = H([\mathbf{v}_1]_{d,q}, [\mathbf{v}_2]_{d,q}, \mu)$

$\mathbf{c} = F(c)$

$\mathbf{z} \leftarrow \mathbf{y} + \mathbf{s}\mathbf{c}$

$\mathbf{w}_1 \equiv \mathbf{v}_1 - \mathbf{e}_1 \mathbf{c} \pmod{q}$

$\mathbf{w}_2 \equiv \mathbf{v}_2 - \mathbf{e}_2 \mathbf{c} \pmod{q}$

**if**  $[\mathbf{w}_1]_{2^d}, [\mathbf{w}_2]_{2^d} \notin \mathcal{R}_{2^d-L}$   
or  $\mathbf{z} \notin \mathcal{R}_{B-U}$  **then**

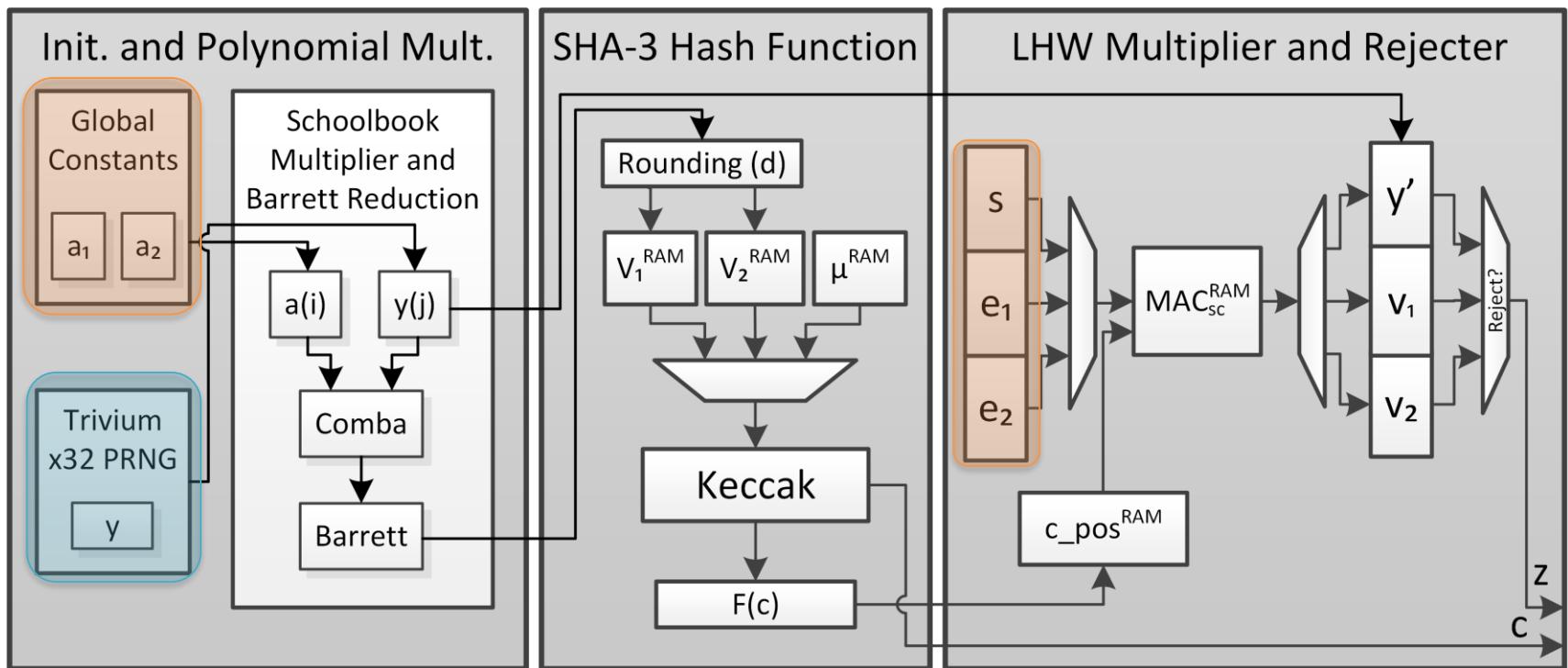
    Restart

**end if**

**return**  $(\mathbf{z}, c)$

**end procedure**

---



# Architecture of the Ring-TESLA Signature Scheme

---

## Algorithm 1 Ring-TESLA Sign

---

**procedure** SIGN( $\mu, \mathbf{a}_1, \mathbf{a}_2, \mathbf{s}, \mathbf{e}_1, \mathbf{e}_2$ )

$\mathbf{y} \leftarrow \mathcal{R}_{q,[B]}$

$\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$

$\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$

$c = H([\mathbf{v}_1]_{d,q}, [\mathbf{v}_2]_{d,q}, \mu)$

$\mathbf{c} = F(c)$

$\mathbf{z} \leftarrow \mathbf{y} + \mathbf{s}\mathbf{c}$

$\mathbf{w}_1 \equiv \mathbf{v}_1 - \mathbf{e}_1 \mathbf{c} \pmod{q}$

$\mathbf{w}_2 \equiv \mathbf{v}_2 - \mathbf{e}_2 \mathbf{c} \pmod{q}$

**if**  $[\mathbf{w}_1]_{2^d}, [\mathbf{w}_2]_{2^d} \notin \mathcal{R}_{2^d-L}$   
or  $\mathbf{z} \notin \mathcal{R}_{B-U}$  **then**

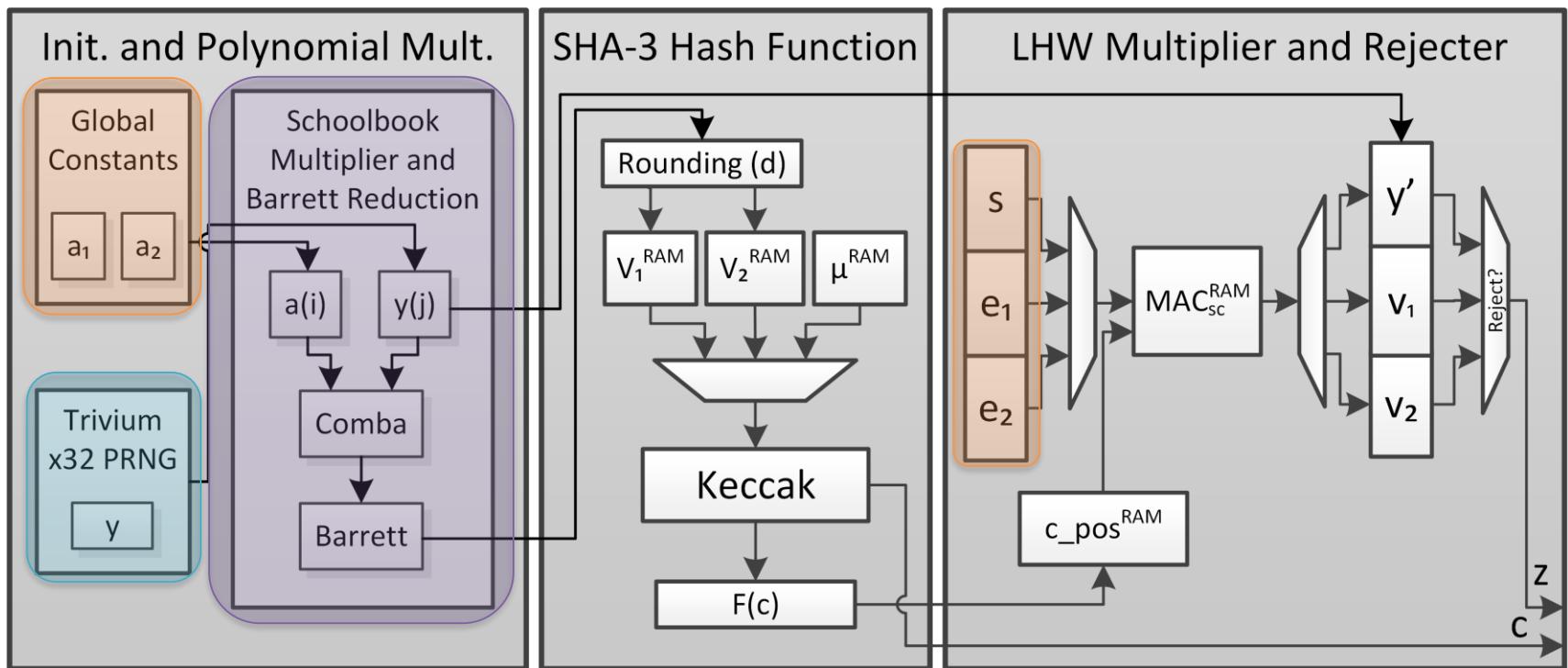
    Restart

**end if**

**return**  $(\mathbf{z}, c)$

**end procedure**

---



# Architecture of the Ring-TESLA Signature Scheme

---

## Algorithm 1 Ring-TESLA Sign

---

**procedure** SIGN( $\mu, \mathbf{a}_1, \mathbf{a}_2, \mathbf{s}, \mathbf{e}_1, \mathbf{e}_2$ )

$\mathbf{y} \leftarrow \mathcal{R}_{q,[B]}$

$\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$

$\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$

$c = H([\mathbf{v}_1]_{d,q}, [\mathbf{v}_2]_{d,q}, \mu)$

$\mathbf{c} = F(c)$

$\mathbf{z} \leftarrow \mathbf{y} + \mathbf{s}\mathbf{c}$

$\mathbf{w}_1 \equiv \mathbf{v}_1 - \mathbf{e}_1 \mathbf{c} \pmod{q}$

$\mathbf{w}_2 \equiv \mathbf{v}_2 - \mathbf{e}_2 \mathbf{c} \pmod{q}$

**if**  $[\mathbf{w}_1]_{2^d}, [\mathbf{w}_2]_{2^d} \notin \mathcal{R}_{2^d-L}$

or  $\mathbf{z} \notin \mathcal{R}_{B-U}$  **then**

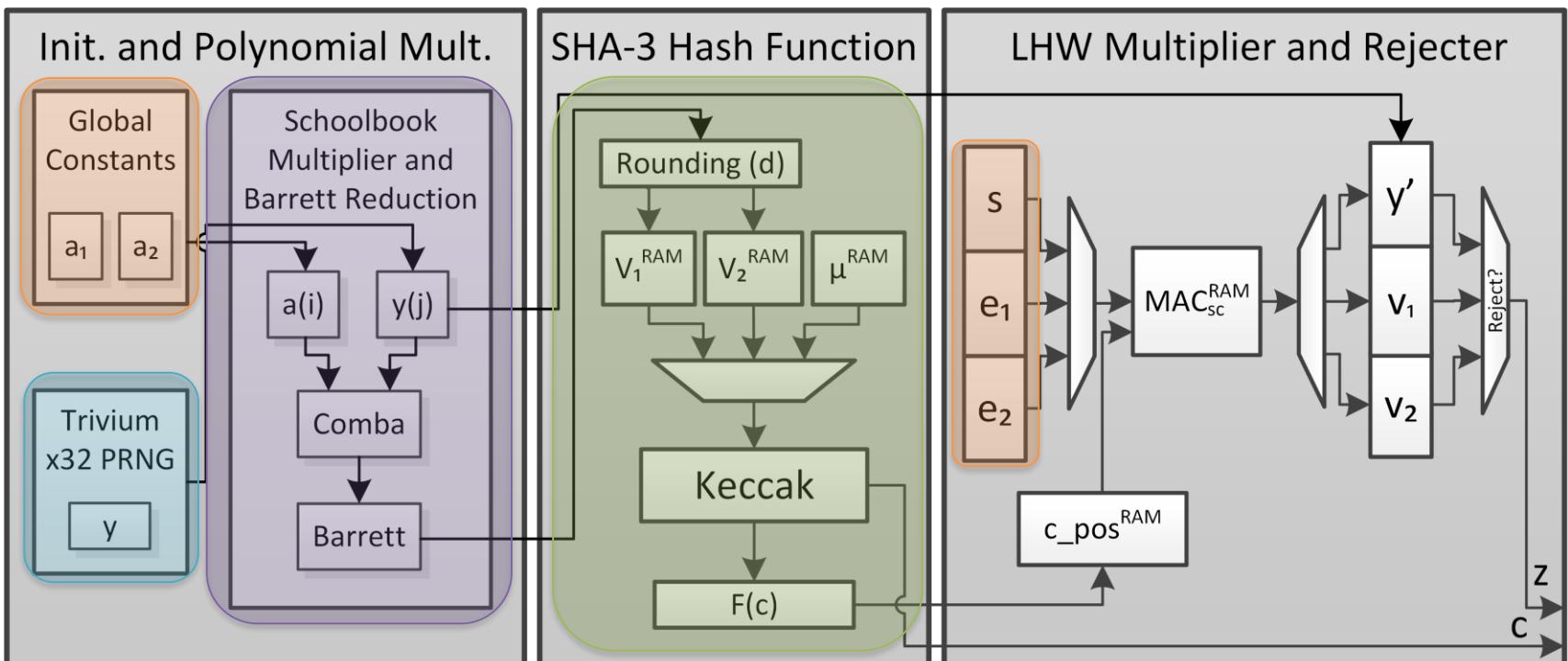
Restart

**end if**

**return**  $(\mathbf{z}, c)$

**end procedure**

---



# Architecture of the Ring-TESLA Signature Scheme

## Algorithm 1 Ring-TESLA Sign

```
procedure SIGN( $\mu, \mathbf{a}_1, \mathbf{a}_2, \mathbf{s}, \mathbf{e}_1, \mathbf{e}_2$ )
```

$\mathbf{y} \leftarrow \mathcal{R}_{q,[B]}$

$\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$

$\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$

$c = H([\mathbf{v}_1]_{d,q}, [\mathbf{v}_2]_{d,q}, \mu)$

$\mathbf{c} = F(c)$

$\mathbf{z} \leftarrow \mathbf{y} + \mathbf{s}\mathbf{c}$

$\mathbf{w}_1 \equiv \mathbf{v}_1 - \mathbf{e}_1 \mathbf{c} \pmod{q}$

$\mathbf{w}_2 \equiv \mathbf{v}_2 - \mathbf{e}_2 \mathbf{c} \pmod{q}$

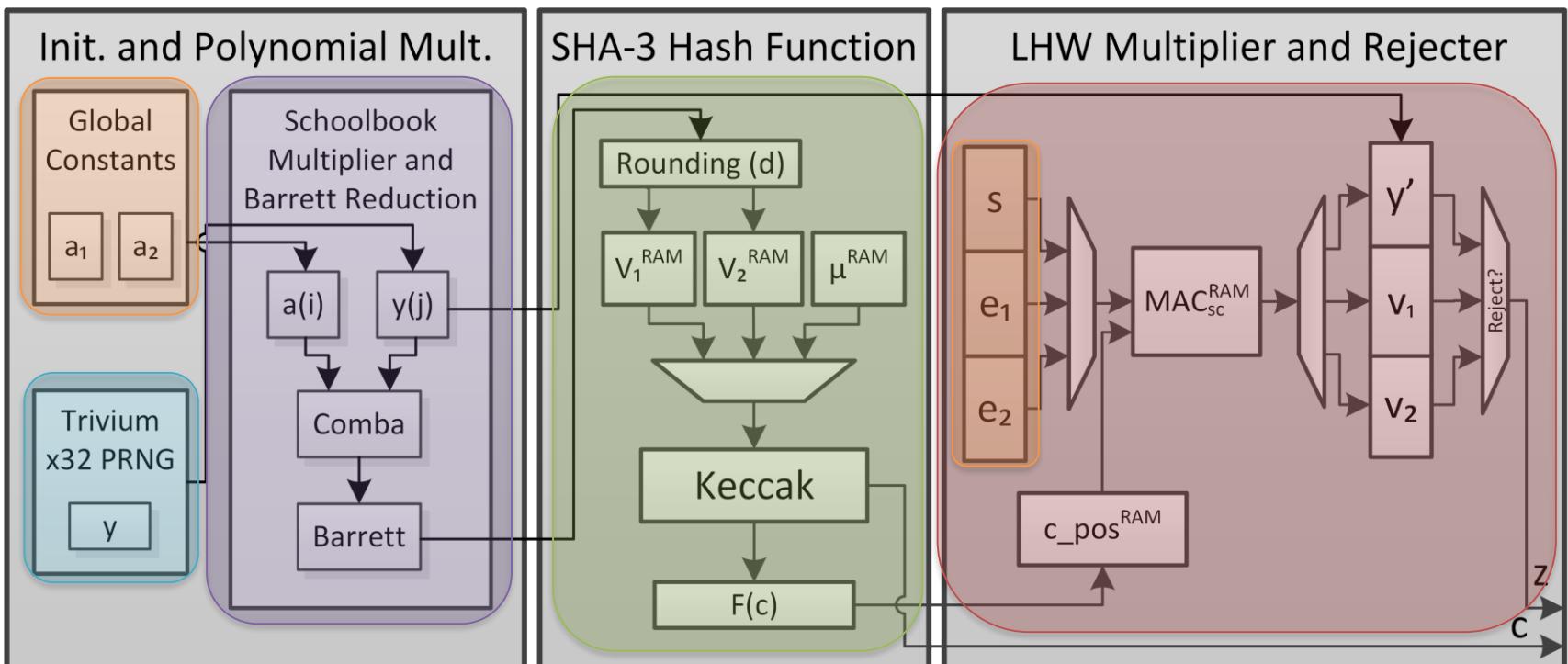
if  $[\mathbf{w}_1]_{2^d}, [\mathbf{w}_2]_{2^d} \notin \mathcal{R}_{2^d-L}$   
or  $\mathbf{z} \notin \mathcal{R}_{B-U}$  then

Restart

end if

return  $(\mathbf{z}, c)$

end procedure



---

## Algorithm 1 Ring-TESLA Sign

---

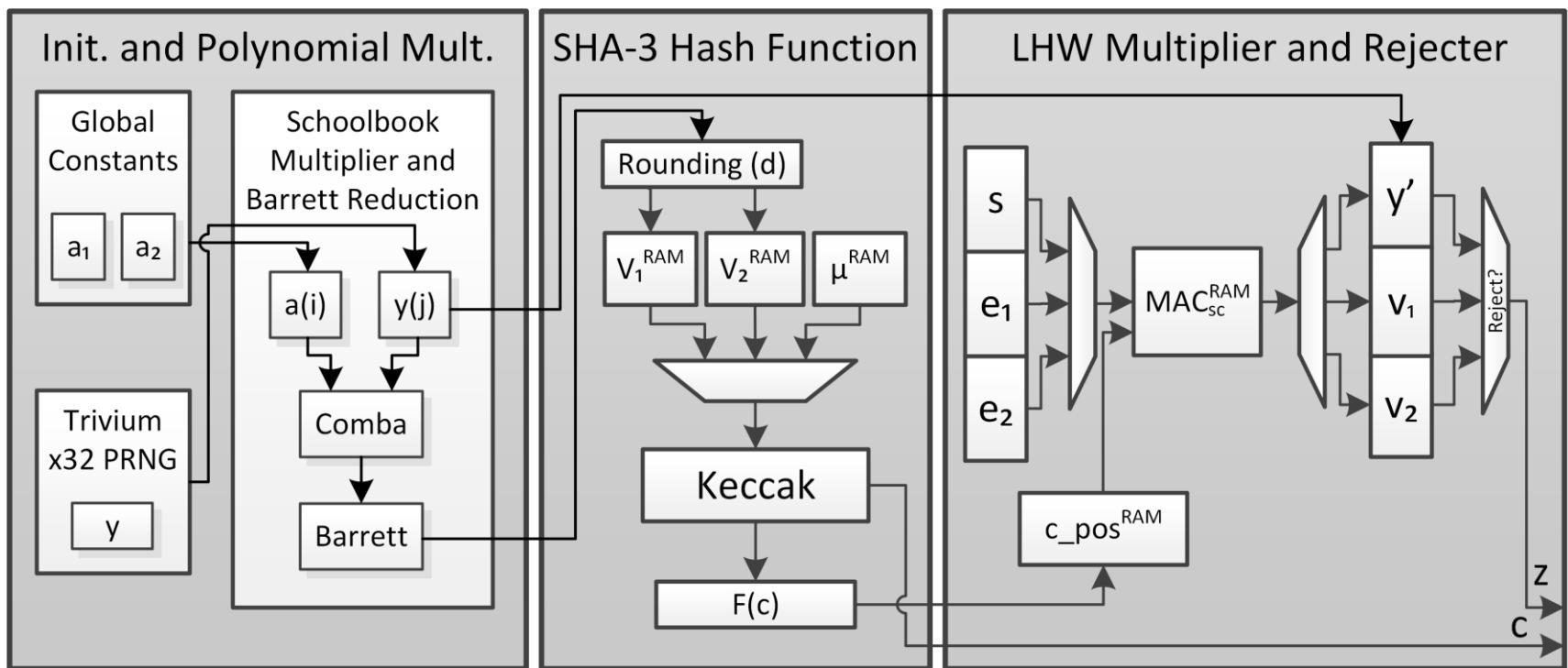
**procedure** SIGN( $\mu, \mathbf{a}_1, \mathbf{a}_2, \mathbf{s}, \mathbf{e}_1, \mathbf{e}_2$ )

```

 $\mathbf{y} \leftarrow \mathcal{R}_{q,[B]}$ 
 $\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$ 
 $\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$ 
 $c = H([\mathbf{v}_1]_{d,q}, [\mathbf{v}_2]_{d,q}, \mu)$ 
 $\mathbf{c} = F(c)$ 
 $\mathbf{z} \leftarrow \mathbf{y} + \mathbf{s}\mathbf{c}$ 
 $\mathbf{w}_1 \equiv \mathbf{v}_1 - \mathbf{e}_1\mathbf{c} \pmod{q}$ 
 $\mathbf{w}_2 \equiv \mathbf{v}_2 - \mathbf{e}_2\mathbf{c} \pmod{q}$ 
if  $[\mathbf{w}_1]_{2^d}, [\mathbf{w}_2]_{2^d} \notin \mathcal{R}_{2^d-L}$ 
    or  $\mathbf{z} \notin \mathcal{R}_{B-U}$  then
        Restart
    end if
    return ( $\mathbf{z}, c$ )
end procedure

```

---



---

## Algorithm 1 Ring-TESLA Sign

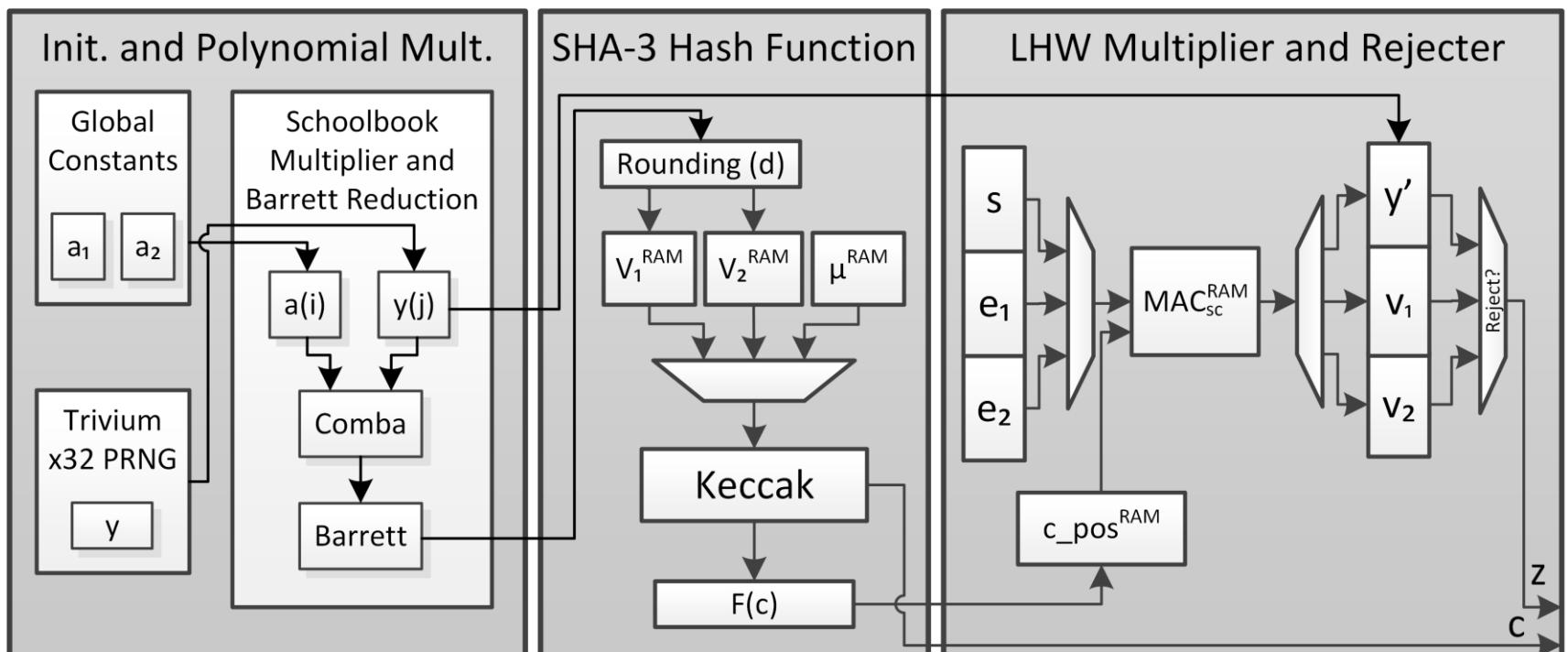
---

```
procedure SIGN( $\mu, \mathbf{a}_1, \mathbf{a}_2, \mathbf{s}, \mathbf{e}_1, \mathbf{e}_2$ )
```

```

 $\mathbf{y} \leftarrow \mathcal{R}_{q,[B]}$ 
 $\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$ 
 $\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$ 
 $c = H([\mathbf{v}_1]_{d,q}, [\mathbf{v}_2]_{d,q}, \mu)$ 
 $\mathbf{c} = F(c)$ 
 $\mathbf{z} \leftarrow \mathbf{y} + \mathbf{s}\mathbf{c}$ 
 $\mathbf{w}_1 \equiv \mathbf{v}_1 - \mathbf{e}_1\mathbf{c} \pmod{q}$ 
 $\mathbf{w}_2 \equiv \mathbf{v}_2 - \mathbf{e}_2\mathbf{c} \pmod{q}$ 
if  $[\mathbf{w}_1]_{2^d}, [\mathbf{w}_2]_{2^d} \notin \mathcal{R}_{2^d-L}$ 
    or  $\mathbf{z} \notin \mathcal{R}_{B-U}$  then
        Restart
    end if
    return  $(\mathbf{z}, c)$ 
end procedure

```



Pre-Hash

---

## Algorithm 1 Ring-TESLA Sign

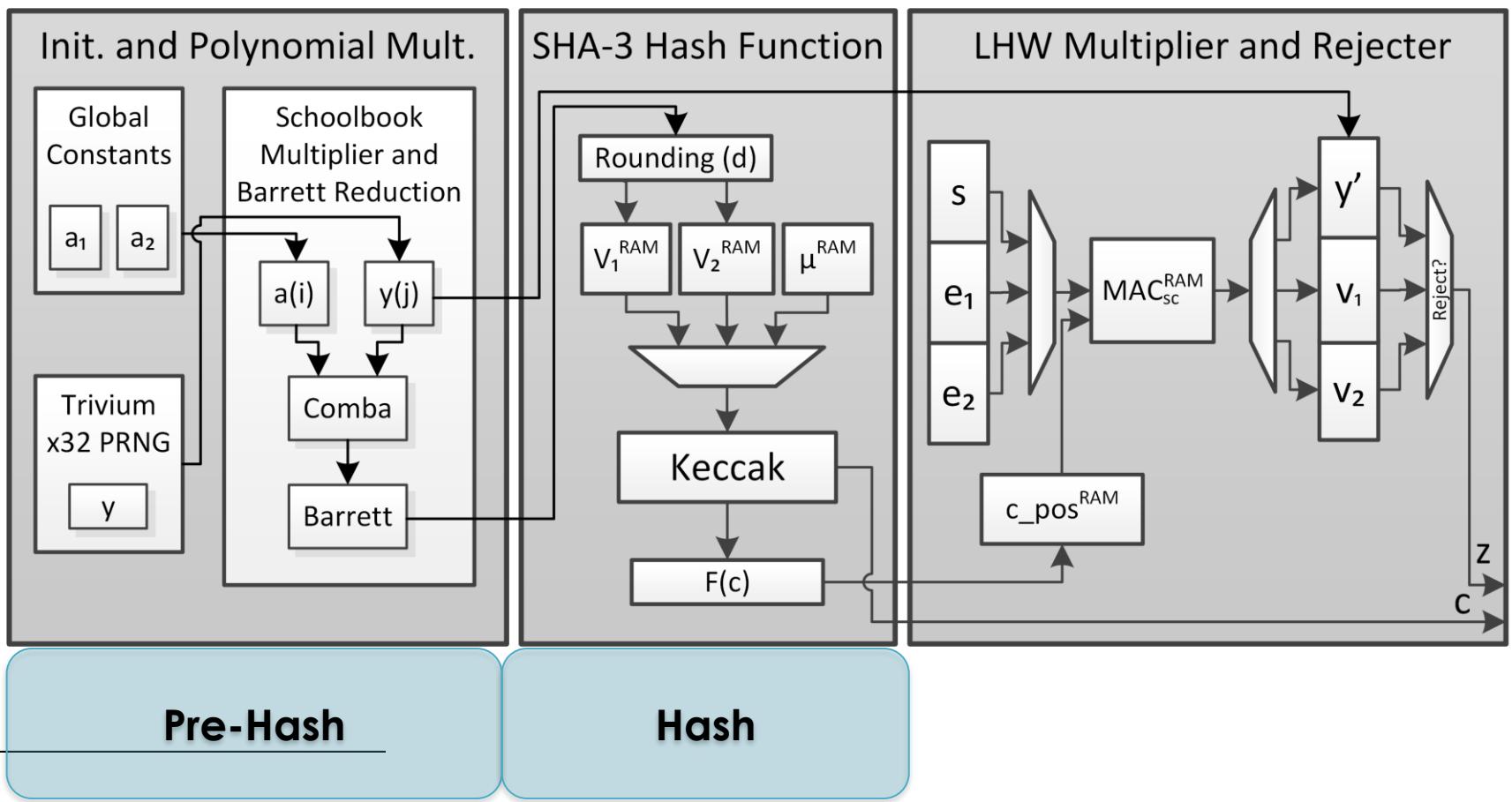
---

```
procedure SIGN( $\mu, \mathbf{a}_1, \mathbf{a}_2, \mathbf{s}, \mathbf{e}_1, \mathbf{e}_2$ )
```

```

 $\mathbf{y} \leftarrow \mathcal{R}_{q,[B]}$ 
 $\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$ 
 $\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$ 
 $c = H([\mathbf{v}_1]_{d,q}, [\mathbf{v}_2]_{d,q}, \mu)$ 
 $\mathbf{c} = F(c)$ 
 $\mathbf{z} \leftarrow \mathbf{y} + \mathbf{s}\mathbf{c}$ 
 $\mathbf{w}_1 \equiv \mathbf{v}_1 - \mathbf{e}_1\mathbf{c} \pmod{q}$ 
 $\mathbf{w}_2 \equiv \mathbf{v}_2 - \mathbf{e}_2\mathbf{c} \pmod{q}$ 
if  $[\mathbf{w}_1]_{2^d}, [\mathbf{w}_2]_{2^d} \notin \mathcal{R}_{2^d-L}$ 
    or  $\mathbf{z} \notin \mathcal{R}_{B-U}$  then
        Restart
    end if
    return  $(\mathbf{z}, c)$ 
end procedure

```



---

## Algorithm 1 Ring-TESLA Sign

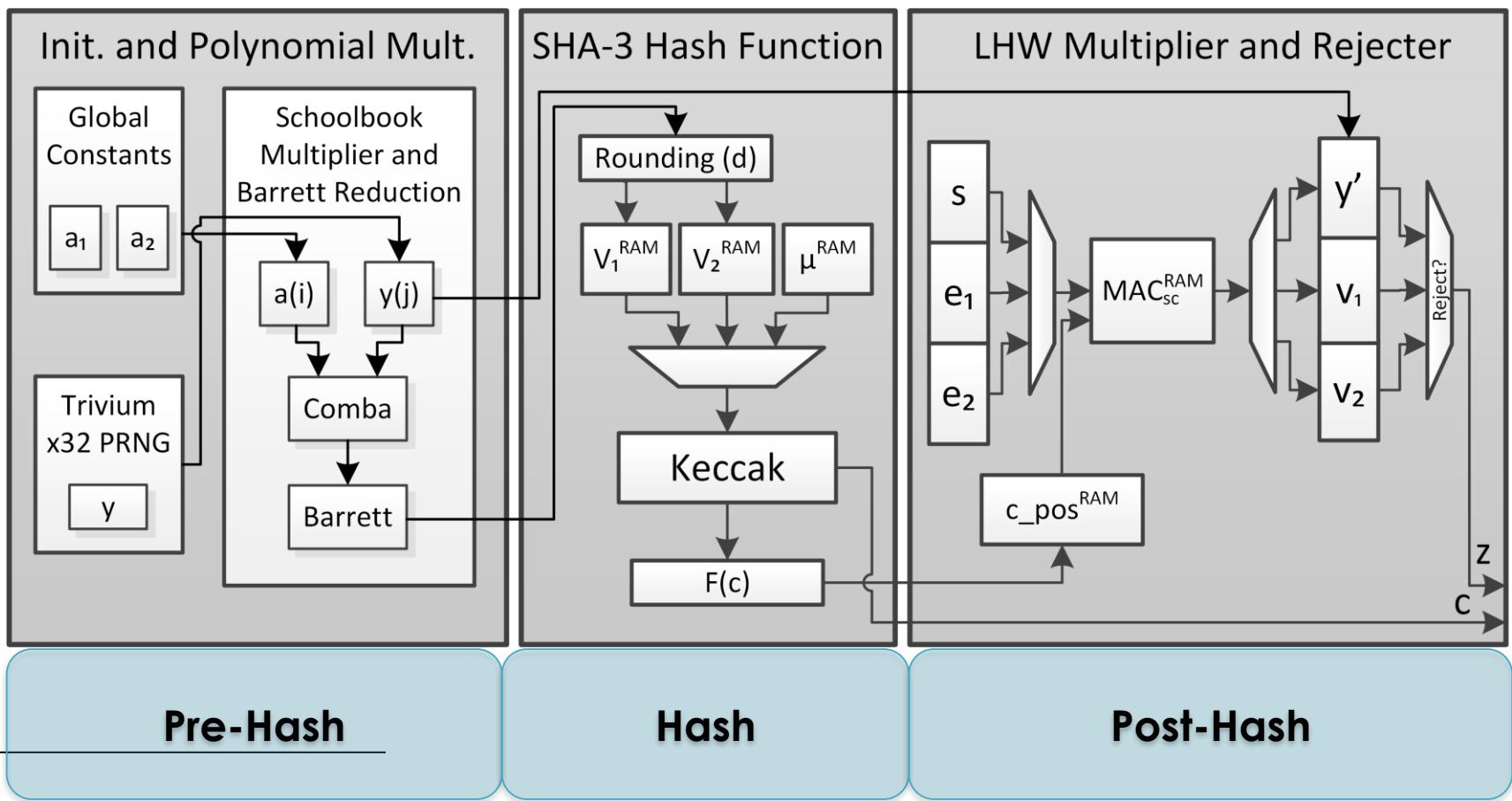
---

```
procedure SIGN( $\mu, \mathbf{a}_1, \mathbf{a}_2, \mathbf{s}, \mathbf{e}_1, \mathbf{e}_2$ )
```

```

 $\mathbf{y} \leftarrow \mathcal{R}_{q,[B]}$ 
 $\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$ 
 $\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$ 
 $c = H([\mathbf{v}_1]_{d,q}, [\mathbf{v}_2]_{d,q}, \mu)$ 
 $\mathbf{c} = F(c)$ 
 $\mathbf{z} \leftarrow \mathbf{y} + \mathbf{s}\mathbf{c}$ 
 $\mathbf{w}_1 \equiv \mathbf{v}_1 - \mathbf{e}_1\mathbf{c} \pmod{q}$ 
 $\mathbf{w}_2 \equiv \mathbf{v}_2 - \mathbf{e}_2\mathbf{c} \pmod{q}$ 
if  $[\mathbf{w}_1]_{2^d}, [\mathbf{w}_2]_{2^d} \notin \mathcal{R}_{2^d-L}$ 
    or  $\mathbf{z} \notin \mathcal{R}_{B-U}$  then
        Restart
    end if
    return  $(\mathbf{z}, c)$ 
end procedure

```



**Algorithm 1** Ring-TESLA Sign

```

procedure SIGN( $\mu, \mathbf{a}_1, \mathbf{a}_2, \mathbf{s}, \mathbf{e}_1, \mathbf{e}_2$ )
     $\mathbf{y} \leftarrow \mathcal{R}_{q,[B]}$ 
     $\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$ 
     $\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$ 
     $c = H([\mathbf{v}_1]_{d,q}, [\mathbf{v}_2]_{d,q}, \mu)$ 
     $\mathbf{c} = F(c)$ 
     $\mathbf{z} \leftarrow \mathbf{y} + \mathbf{s}\mathbf{c}$ 
     $\mathbf{w}_1 \equiv \mathbf{v}_1 - \mathbf{e}_1 \mathbf{c} \pmod{q}$ 
     $\mathbf{w}_2 \equiv \mathbf{v}_2 - \mathbf{e}_2 \mathbf{c} \pmod{q}$ 
    if  $[\mathbf{w}_1]_{2^d}, [\mathbf{w}_2]_{2^d} \notin \mathcal{R}_{2^d-L}$ 
        or  $\mathbf{z} \notin \mathcal{R}_{B-U}$  then
            Restart
    end if

```

- Pipeline created for pre-hash computations.
- After pre-hash polynomial multiplication;
  - $\mathbf{y}$  is copied to another register for  $\mathbf{z}$ .
  - $\mathbf{y}$  is generated for next signature in parallel.
- Hash, LHW calculations of  $\mathbf{z}$ ,  $\mathbf{w}_1$ , and  $\mathbf{w}_2$ , and rejections then outside the critical path.
- Sign/Verify critical path thus pre-hash phase.

**Algorithm 1** Ring-TESLA Sign

```

procedure SIGN( $\mu, \mathbf{a}_1, \mathbf{a}_2, \mathbf{s}, \mathbf{e}_1, \mathbf{e}_2$ )
     $\mathbf{y} \leftarrow \mathcal{R}_{q,[B]}$ 
     $\mathbf{v}_1 \equiv \mathbf{a}_1 \mathbf{y} \pmod{q}$ 
     $\mathbf{v}_2 \equiv \mathbf{a}_2 \mathbf{y} \pmod{q}$ 
     $c = H([\mathbf{v}_1]_{d,q}, [\mathbf{v}_2]_{d,q}, \mu)$ 
     $\mathbf{c} = F(c)$ 
     $\mathbf{z} \leftarrow \mathbf{y} + \mathbf{s}\mathbf{c}$ 
     $\mathbf{w}_1 \equiv \mathbf{v}_1 - \mathbf{e}_1 \mathbf{c} \pmod{q}$ 
     $\mathbf{w}_2 \equiv \mathbf{v}_2 - \mathbf{e}_2 \mathbf{c} \pmod{q}$ 
    if  $[\mathbf{w}_1]_{2^d}, [\mathbf{w}_2]_{2^d} \notin \mathcal{R}_{2^d-L}$ 
        or  $\mathbf{z} \notin \mathcal{R}_{B-U}$  then
            Restart
    end if

```

- Pipeline created for pre-hash computations.
- After pre-hash polynomial multiplication;
  - $\mathbf{y}$  is copied to another register for  $\mathbf{z}$ .
  - $\mathbf{y}$  is generated for next signature in parallel.
- Hash, LHW calculations of  $\mathbf{z}$ ,  $\mathbf{w}_1$ , and  $\mathbf{w}_2$ , and rejections then outside the critical path.
- Sign/Verify critical path thus pre-hash phase.

.

Signature #1

Poly. Mult.  $\Rightarrow$ Hash  $\Rightarrow$ 

LHW

Signature #2

Poly. Mult.  $\Rightarrow$ Hash  $\Rightarrow$ 

LHW

:

Signature #n

..

Poly. Mult.  $\Rightarrow$ Hash  $\Rightarrow$ 

LHW

## Ring-TESLA Hardware Results

- Ring-TESLA, ideal lattice-based signatures on a Spartan 6 – LX25 FPGA.
- Significantly smaller than other lattice-based signature designs, suffers in throughput.
- Significantly smaller and faster in comparison to RSA and ECDSA.

Operation, Configuration	Security	Device	LUT/FF/SLICE	BRAM/DSP	MHz	Cycles	Ops/sec
Ring-TESLA (Sign, SB-I)	128-bits	S6 LX25	4447/3345/1257	3/6	190	1835540	104
Ring-TESLA (Sign, SB-II)	128-bits	S6 LX25	4828/3790/1513	4/8	196	917771	214
Ring-TESLA (Sign, SB-IV)	128-bits	S6 LX25	5071/3851/1503	4/12	187	458891	408
Ring-TESLA-(Sign, SB-VIII)	128-bits	S6 LX25	6848/5457/2254	4/20	180	229446	785
Ring-TESLA (Verify, SB-I)	128-bits	S6 LX25	3714/3023/1172	3/6	188	1835540	102
Ring-TESLA (Verify, SB-II)	128-bits	S6 LX25	3917/3253/1238	3/8	194	917771	212
Ring-TESLA (Verify, SB-IV)	128-bits	S6 LX25	4793/3939/1551	3/12	186	458891	406
Ring-TESLA (Verify, SB-VIII)	128-bits	S6 LX25	6473/5582/2103	3/20	178	229446	776
GLP (Sign, Schoolbook x3)	80-bits	S6 LX16	7465/8993/2273	30/28	162	-	931
GLP (Verify, Schoolbook x3)	80-bits	S6 LX16	6225/6663/2263	15/8	158	-	998
BLISS (Sign, NTT)	128-bits	S6 LX25	7193/6420/2291	6/5	139	15864	8761
BLISS (Verify NTT)	128-bits	S6 LX25	5065/4312/1687	4/3	166	16346	17101
RSA (Sign)	103-bits	V5 LX30	3237 slices	7/17	200	-	89
ECDSA (Sign)	128-bits	V5 LX110	32299 LUT/FF pairs	10/37	139	-	-
ECDSA (Verify)	128-bits	V5 LX110	32299 LUT/FF pairs	10/37	110	-	-

## Ring-TESLA Hardware Results

- Ring-TESLA, ideal lattice-based signatures on a Spartan 6 – LX25 FPGA.
- Significantly smaller than other lattice-based signature designs, suffers in throughput.
- Significantly smaller and faster in comparison to RSA and ECDSA.

↓

Operation, Configuration	Security	Device	LUT/FF/SLICE	BRAM/DSP	MHz	Cycles	Ops/sec
Ring-TESLA (Sign, SB-I)	128-bits	S6 LX25	4447/3345/1257	3/6	190	1835540	104
Ring-TESLA (Sign, SB-II)	128-bits	S6 LX25	4828/3790/1513	4/8	196	917771	214
Ring-TESLA (Sign, SB-IV)	128-bits	S6 LX25	5071/3851/1503	4/12	187	458891	408
Ring-TESLA-(Sign, SB-VIII)	128-bits	S6 LX25	6848/5457/2254	4/20	180	229446	785
Ring-TESLA (Verify, SB-I)	128-bits	S6 LX25	3714/3023/1172	3/6	188	1835540	102
Ring-TESLA (Verify, SB-II)	128-bits	S6 LX25	3917/3253/1238	3/8	194	917771	212
Ring-TESLA (Verify, SB-IV)	128-bits	S6 LX25	4793/3939/1551	3/12	186	458891	406
Ring-TESLA (Verify, SB-VIII)	128-bits	S6 LX25	6473/5582/2103	3/20	178	229446	776
GLP (Sign, Schoolbook x3)	80-bits	S6 LX16	7465/8993/2273	30/28	162	-	931
GLP (Verify, Schoolbook x3)	80-bits	S6 LX16	6225/6663/2263	15/8	158	-	998
BLISS (Sign, NTT)	128-bits	S6 LX25	7193/6420/2291	6/5	139	15864	8761
BLISS (Verify NTT)	128-bits	S6 LX25	5065/4312/1687	4/3	166	16346	17101
RSA (Sign)	103-bits	V5 LX30	3237 slices	7/17	200	-	89
ECDSA (Sign)	128-bits	V5 LX110	32299 LUT/FF pairs	10/37	139	-	-
ECDSA (Verify)	128-bits	V5 LX110	32299 LUT/FF pairs	10/37	110	-	-

## Ring-TESLA Hardware Results

- Ring-TESLA, ideal lattice-based signatures on a Spartan 6 – LX25 FPGA.
- Significantly smaller than other lattice-based signature designs, suffers in throughput.
- Significantly smaller and faster in comparison to RSA and ECDSA.

Operation, Configuration	Security	Device	LUT/FF/SLICE	BRAM/DSP	MHz	Cycles	Ops/sec
Ring-TESLA (Sign, SB-I)	128-bits	S6 LX25	4447/3345/1257	3/6	190	1835540	104
Ring-TESLA (Sign, SB-II)	128-bits	S6 LX25	4828/3790/1513	4/8	196	917771	214
Ring-TESLA (Sign, SB-IV)	128-bits	S6 LX25	5071/3851/1503	4/12	187	458891	408
Ring-TESLA-(Sign, SB-VIII)	128-bits	S6 LX25	6848/5457/2254	4/20	180	229446	785
Ring-TESLA (Verify, SB-I)	128-bits	S6 LX25	3714/3023/1172	3/6	188	1835540	102
Ring-TESLA (Verify, SB-II)	128-bits	S6 LX25	3917/3253/1238	3/8	194	917771	212
Ring-TESLA (Verify, SB-IV)	128-bits	S6 LX25	4793/3939/1551	3/12	186	458891	406
Ring-TESLA (Verify, SB-VIII)	128-bits	S6 LX25	6473/5582/2103	3/20	178	229446	776
GLP (Sign, Schoolbook x3)	80-bits	S6 LX16	7465/8993/2273	30/28	162	-	931
GLP (Verify, Schoolbook x3)	80-bits	S6 LX16	6225/6663/2263	15/8	158	-	998
BLISS (Sign, NTT)	128-bits	S6 LX25	7193/6420/2291	6/5	139	15864	8761
BLISS (Verify NTT)	128-bits	S6 LX25	5065/4312/1687	4/3	166	16346	17101
RSA (Sign)	103-bits	V5 LX30	3237 slices	7/17	200	-	89
ECDSA (Sign)	128-bits	V5 LX110	32299 LUT/FF pairs	10/37	139	-	-
ECDSA (Verify)	128-bits	V5 LX110	32299 LUT/FF pairs	10/37	110	-	-

## Ring-TESLA Hardware Results

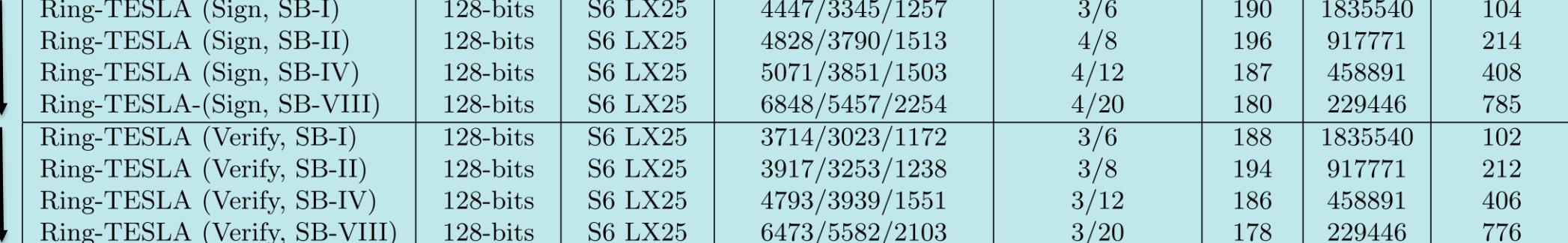
- Ring-TESLA, ideal lattice-based signatures on a Spartan 6 – LX25 FPGA.
- Significantly smaller than other lattice-based signature designs, suffers in throughput.
- Significantly smaller and faster in comparison to RSA and ECDSA.

↓ ↓ ↓

Operation, Configuration	Security	Device	LUT/FF/SLICE	BRAM/DSP	MHz	Cycles	Ops/sec
Ring-TESLA (Sign, SB-I)	128-bits	S6 LX25	4447/3345/1257	3/6	190	1835540	104
Ring-TESLA (Sign, SB-II)	128-bits	S6 LX25	4828/3790/1513	4/8	196	917771	214
Ring-TESLA (Sign, SB-IV)	128-bits	S6 LX25	5071/3851/1503	4/12	187	458891	408
Ring-TESLA-(Sign, SB-VIII)	128-bits	S6 LX25	6848/5457/2254	4/20	180	229446	785
Ring-TESLA (Verify, SB-I)	128-bits	S6 LX25	3714/3023/1172	3/6	188	1835540	102
Ring-TESLA (Verify, SB-II)	128-bits	S6 LX25	3917/3253/1238	3/8	194	917771	212
Ring-TESLA (Verify, SB-IV)	128-bits	S6 LX25	4793/3939/1551	3/12	186	458891	406
Ring-TESLA (Verify, SB-VIII)	128-bits	S6 LX25	6473/5582/2103	3/20	178	229446	776
GLP (Sign, Schoolbook x3)	80-bits	S6 LX16	7465/8993/2273	30/28	162	-	931
GLP (Verify, Schoolbook x3)	80-bits	S6 LX16	6225/6663/2263	15/8	158	-	998
BLISS (Sign, NTT)	128-bits	S6 LX25	7193/6420/2291	6/5	139	15864	8761
BLISS (Verify NTT)	128-bits	S6 LX25	5065/4312/1687	4/3	166	16346	17101
RSA (Sign)	103-bits	V5 LX30	3237 slices	7/17	200	-	89
ECDSA (Sign)	128-bits	V5 LX110	32299 LUT/FF pairs	10/37	139	-	-
ECDSA (Verify)	128-bits	V5 LX110	32299 LUT/FF pairs	10/37	110	-	-

## Ring-TESLA Hardware Results

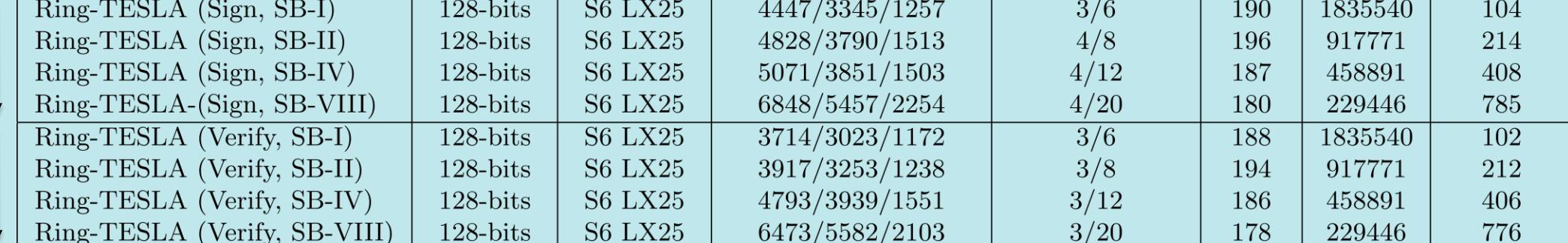
- Ring-TESLA, ideal lattice-based signatures on a Spartan 6 – LX25 FPGA.
- Significantly smaller than other lattice-based signature designs, suffers in throughput.
- Significantly smaller and faster in comparison to RSA and ECDSA.



Operation, Configuration	Security	Device	LUT/FF/SLICE	BRAM/DSP	MHz	Cycles	Ops/sec
Ring-TESLA (Sign, SB-I)	128-bits	S6 LX25	4447/3345/1257	3/6	190	1835540	104
Ring-TESLA (Sign, SB-II)	128-bits	S6 LX25	4828/3790/1513	4/8	196	917771	214
Ring-TESLA (Sign, SB-IV)	128-bits	S6 LX25	5071/3851/1503	4/12	187	458891	408
Ring-TESLA-(Sign, SB-VIII)	128-bits	S6 LX25	6848/5457/2254	4/20	180	229446	785
Ring-TESLA (Verify, SB-I)	128-bits	S6 LX25	3714/3023/1172	3/6	188	1835540	102
Ring-TESLA (Verify, SB-II)	128-bits	S6 LX25	3917/3253/1238	3/8	194	917771	212
Ring-TESLA (Verify, SB-IV)	128-bits	S6 LX25	4793/3939/1551	3/12	186	458891	406
Ring-TESLA (Verify, SB-VIII)	128-bits	S6 LX25	6473/5582/2103	3/20	178	229446	776
GLP (Sign, Schoolbook x3)	80-bits	S6 LX16	7465/8993/2273	30/28	162	-	931
GLP (Verify, Schoolbook x3)	80-bits	S6 LX16	6225/6663/2263	15/8	158	-	998
BLISS (Sign, NTT)	128-bits	S6 LX25	7193/6420/2291	6/5	139	15864	8761
BLISS (Verify NTT)	128-bits	S6 LX25	5065/4312/1687	4/3	166	16346	17101
RSA (Sign)	103-bits	V5 LX30	3237 slices	7/17	200	-	89
ECDSA (Sign)	128-bits	V5 LX110	32299 LUT/FF pairs	10/37	139	-	-
ECDSA (Verify)	128-bits	V5 LX110	32299 LUT/FF pairs	10/37	110	-	-

## Ring-TESLA Hardware Results

- Ring-TESLA, ideal lattice-based signatures on a Spartan 6 – LX25 FPGA.
- Significantly smaller than other lattice-based signature designs, suffers in throughput.
- Significantly smaller and faster in comparison to RSA and ECDSA.



Operation, Configuration	Security	Device	LUT/FF/SLICE	BRAM/DSP	MHz	Cycles	Ops/sec
Ring-TESLA (Sign, SB-I)	128-bits	S6 LX25	4447/3345/1257	3/6	190	1835540	104
Ring-TESLA (Sign, SB-II)	128-bits	S6 LX25	4828/3790/1513	4/8	196	917771	214
Ring-TESLA (Sign, SB-IV)	128-bits	S6 LX25	5071/3851/1503	4/12	187	458891	408
Ring-TESLA-(Sign, SB-VIII)	128-bits	S6 LX25	6848/5457/2254	4/20	180	229446	785
Ring-TESLA (Verify, SB-I)	128-bits	S6 LX25	3714/3023/1172	3/6	188	1835540	102
Ring-TESLA (Verify, SB-II)	128-bits	S6 LX25	3917/3253/1238	3/8	194	917771	212
Ring-TESLA (Verify, SB-IV)	128-bits	S6 LX25	4793/3939/1551	3/12	186	458891	406
Ring-TESLA (Verify, SB-VIII)	128-bits	S6 LX25	6473/5582/2103	3/20	178	229446	776
GLP (Sign, Schoolbook x3)	80-bits	S6 LX16	7465/8993/2273	30/28	162	-	931
GLP (Verify, Schoolbook x3)	80-bits	S6 LX16	6225/6663/2263	15/8	158	-	998
BLISS (Sign, NTT)	128-bits	S6 LX25	7193/6420/2291	6/5	139	15864	8761
BLISS (Verify NTT)	128-bits	S6 LX25	5065/4312/1687	4/3	166	16346	17101
RSA (Sign)	103-bits	V5 LX30	3237 slices	7/17	200	-	89
ECDSA (Sign)	128-bits	V5 LX110	32299 LUT/FF pairs	10/37	139	-	-
ECDSA (Verify)	128-bits	V5 LX110	32299 LUT/FF pairs	10/37	110	-	-

- The first hardware design of a Ring-LWE signature scheme.
- First low-area signature scheme in lattice-based cryptography.
- Generic hardware designs for sign and verify, important for parameters changes.
- Numerous parallel multipliers used for a variety of results.
- Consider hardware-friendly parameters in the future.
- Consider high-throughput, large polynomial multiplier in the future.

# Thank you!

Any questions?