Approximate Gumbel Last Passage Percolation

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0.1 Auxiliary Functions

Definition 1. For a real number N, we define the function $h_N : \mathbb{R} \to \mathbb{R}$ by

$$h_N(x) = -\log\left(N\left(1 - \exp\left(-\frac{e^{-x}}{N}\right)\right)\right) - x.$$

Lemma 2. *Let $N \geq 1$. The function h_N satisfies the following properties:*

1. *h_N is convex on $\mathbb{R}$;*
2. *For all $x \in \mathbb{R}$, $0 < h_N(x) \leq \frac{e^{-x}}{N}$;*
3. *For all $x > 0$, $\frac{e^{-x}}{3N} \leq h_N(x)$.*

Proof. The proof is by calculation of the first and second derivatives and application of the mean value theorem. □