

# Approximate Gumbel Last Passage Percolation

James Stephens

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## 0.1 Auxiliary Functions

**Definition 1.** For a real number  $N$ , we define the function  $h_N : \mathbb{R} \rightarrow \mathbb{R}$  by

$$h_N(x) = -\log \left( N \left( 1 - \exp \left( -\frac{e^{-x}}{N} \right) \right) \right) - x.$$

**Lemma 2.** Let  $N \geq 1$ . The function  $h_N$  satisfies the following properties:

1.  $h_N$  is convex on  $\mathbb{R}$ ;
2. For all  $x \in \mathbb{R}$ ,  $0 < h_N(x) \leq \frac{e^{-x}}{N}$ ;
3. For all  $x > 0$ ,  $\frac{e^{-x}}{3N} \leq h_N(x)$ .

*Proof.* The proof is by calculation of the first and second derivatives and application of the mean value theorem.  $\square$