James Cooper
CSC 555
Final Project

Loss Modeling in the Industrial Sector – Monte Carlo Simulation

1. Introduction

Managing risk is an essential function applied in virtually every industry, effecting businesses large and small, from the largest bank to the smallest local business. Businesses must take into account unfortunate events and seek to provide assurances that objectives can still be met in the face of uncertainties. Examples are commonly seen in finance, [1] and [2], where financial institutions must project and account for losses by holding enough required capital to offset both market as well as operational risk. Retail stores must take into account theft, while technology companies may need to account for technological failures.

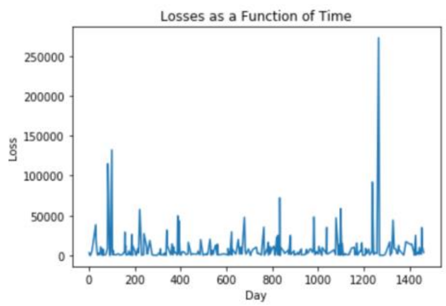
The industrial sector is not immune to the necessity to analyze risk, specifically in our case, to analyze accidents and the losses that can occur from these events. Plant accidents can involve dangerous possibilities where accidents can account for not only monetary losses, but also dangers to the population. Thus, it is extremely important to take into account as pointed out in [4], in considering risk the probability of an accident is just as important as the consequences of an accident. In order to properly model risk, solid historical data must be examined to determine why accidents happen, where and when they happen, and how much they are costing. In this study, we focus on the latter categories – both the time frequency of accident events and the severity of losses. A thorough Monte-Carlo simulation of time intervals of losses for one year will provide a powerful tool to examine these risk aspects and predict with confidence how much money a company may budget in 90% of simulated scenarios to account for these losses.

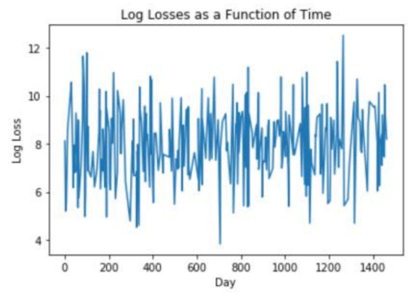
This study examines two plants, Plant A and Plant B, and focuses on accidents over a four-year period, the frequency in which they happen, and the incurred severity of loss for each of these accidents. Two separate distributions are taken into account, the frequency, or time between events as well as the severity of losses over the four-year period. Objectives include determining any trend in days of an event and severity of loss, and a simulation to model the average yearly loss with a relative precision of 10%. This will give a confident threshold as to how much the company should budget to cover losses in 90% of simulated scenarios, representing a 10% Value at Risk (VaR). Finally, the simulation is repeated using the bootstrap method to resample and define the confidence intervals with which we can predict average yearly loss.

1.1 Data

Loss data is represented in the form of time t_i (day of event) and severity of loss s_i represented in dollars (t_i, s_i). Times of losses are assumed to be independent. Multiple events do happen on the same day in some cases, but occur at different times during the day. The data is sorted and divided into Plant A and

Plant B. Time is represented as starting day 1, starting at 4 years in the past, up to day 1,462. This represents four years of accident recordings. A column was added to take the natural logarithm of losses, which will be explained futher below. Seen here is a plot of loss values as a function of time, as well as taking the natural log of the data which seems to make it stationary:

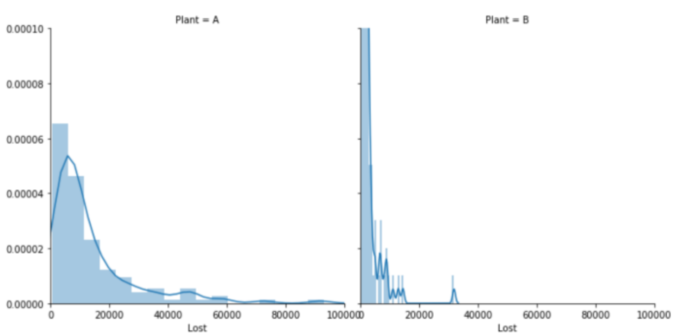




From this data we will extract and study different distributions here, the study of the time between losses and the study of the severity of the loss. Using the Python programming language, the data was read into a data frame using Jupyter Notebook. Upon initial examination of the data, there existed 291 non-null rows for the Plant, Day, and Loss columns. The data contains losses incurred ranging from $46.00 to $272,851.00 from day 1 to day 1,462. The data was manipulated for exploratory analysis to examine vectors and their distributions in yearly intervals from year 1 to year 4.

2. Distribution of Loss Values

The severity of loss is examined by breaking the data into plant A and B separately. In examining the distribution of the loss data, it can be seen that the majority of individual losses are less than $20,000, but there exists a much fatter tail in Plant A where the maximum individual loss is $272,851. The maximum loss in Plant B is $31,769. The range in the severity of loss in Plant B trends a bit lower than in Plant A where losses are generally higher.

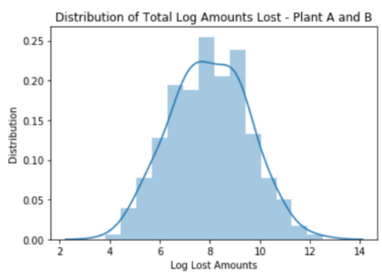


Distribution of losses over a 4-year period

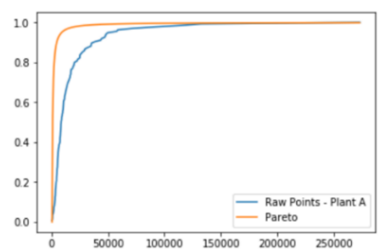
	Plant A	Plant B
Average No. Accidents per Year	33.75	39
Average Loss per Accident	$17,470.16	$2,022.96
Average Loss in Total per Year	$589,617.75	$78,895.00

As can be seen, the average individual loss in Plant A is higher at $17,470.16 compared with Plant B at $2,022.96. This distribution was consistent throughout each year in the data where over 50% of losses in each plant were in the 0-$20,000 range. In total, Plant A has a much higher average loss per year than Plant B, while actually having fewer accidents. The distribution of the loss seems to follow a lognormal distribution. Upon taking the natural logarithm of the loss column, we can see that the distribution displays a much more normal distribution with a *mu* value of Plant A at 9.15 and the *mu* value of Plant B at 6.96. To confirm the normality of the log data, two different tests were performed to test the null hypothesis that the sample comes from a normal distribution using the normal test from Python's scipy. This uses a function to test the null hypothesis that the sample comes from a normal distribtution. It gives a p value combining skew and kurtosis. We get a value of 0.44 and conclude that we cannot reject the null and that it does in fact come from the normal distribution. This will further be bolstered running the simulations.

Using the lognormal data will be useful to model the data in simulation using a Gaussian/lognormal simulation. In order to describe quantities that are distributed in a lognormal fashion, the Pareto distribution can examined as well upon the un-logged data. Upon sorting the un-logged loss data and plotting, it was clear that the data followed what is typical of the Pareto distribution as can be seen in the graph below – we can see that almost 90% of the data represents losses below $50,000. The Pareto distribution can be analyzed as a separate study, for our purposes we will focus on Gaussian.



Natural log of the losses in Plant A and Plant B



Sorted Loss in Plant A – Pareto Distribution of non-logged data

2.1 Loss Data - Normal/Gaussian Distribution

Because we have lognormal data for losses, it was appropriate to use a simulation using a Gaussian function in Python. The probability mass function, as defined in (DiPerro, 275) is:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where $E[X] = \mu$ and $E[(x-\mu)^2] = \sigma^2$.

Looking at our lognormal data, we can see that in Plant A we have $\mu = 9.146$ and $\sigma = 6.194$ and in Plant B we have $\mu = 6.959$ and $\sigma = 3.829$. We will leverage the random.gauss (mu, sigma) function in Python to simulate the loss data using the non-logged data. This function in Python maps Gaussian random numbers with $\mu = 0$ and $\sigma = 1$ into our data with $\mu = 17{,}470.16$ and $\sigma = 490$ for Plant A and $\mu = 2{,}022.96$ and $\sigma = 46$ for Plant B. We can say that:

$$y' = \mu + y\sigma$$

where y' is a Gaussian number equaling the mean of our data added to y times the standard deviation of our data. We can say y is a Gaussian number with mean 0 and standard deviation 1. (DiPerro, 277). Once we have the parameters needed for simulating the loss data, we turn to the distribution of time intervals between accident events. In order to run this we also need to look at the time distribution so that we can run these distributions simultaneously. The time distribution will be a vital factor to simulate the data.

3. Distribution of Time Between Events – Exponential Distribution

For the time data, upon initial observation appear to be exponentially distributed, examining the plot below. The time intervals between events seem to follow an exponential distribution. More specifically, events can be said to be occurring with a constant probability per time unit, being represented by a parameter lambda as specified in (DiPerro, 275) so that “the time at which the state actually changes is described by an exponential random variable with parameter λ”. The probability distribution function of the exponential distribution is defined as:

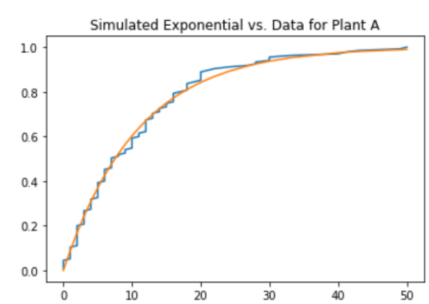
$$p(x) = \lambda e^{-\lambda x}$$

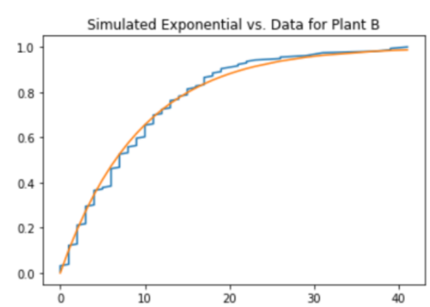
In addition, the cumulative distribution function can be defined, using the inversion method as:

$$1 - e^{-\lambda x}$$

So that the exponential distribution implemented in Python takes a random variable x and the lambda in our distribution and returns P(X > x). Passing our data through, we get the CDF function. The exponential distribution is commonly used in scenarios such as time between events. In our specific case we can use it to model time intervals between accidents. Looking at our data, we can difference the data so that we have the actual time between events as a vector. We take the *lambda* value as being the length of the data, or the number of days in the distribution, divided by four years in days (1,462 days). Using our *lambda* values for Plants A and B with the Exponential function defined below in Python, we create a loop giving us the distribution of our data as well as a simulated distribution using the F_exponential function. The simulated exponential distribution closely matches our actual data, which can be seen in the plot below. The orange line represents the simulated data and the blue line the differenced data from Plants A and B.

```
def F_exponential(x, lamb):
    return 1.0 - exp(-lamb*x)
```





Now that we see the time distribution is exponential, we have the parameters we need to simulate the full set of data using both distributions to simulate one year of losses in both plants.

4. Simulation Model

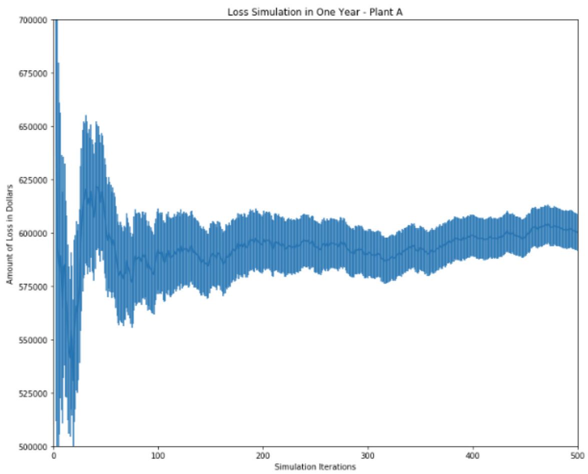
The ultimate goal of the study is to model the average yearly loss for both plants using the parameters and distributions we have identified. Once this is accomplished, we can make a recommendation on how much the company should budget in order to cover losses in 90% of simulated scenarios. In order to simulate one year of losses, a model is implemented using parameters *lambda, mu and sigma* to run a 'simulate_once' within a 'simulate many' function that will run a simulation as many times as specified, as defined in (DiPerro, 287). The program is created by making a class called Simulator and running to run the functions. The function utilizes two Python 'random' functions to simulate the distributions that we have specified – exponential (random.expovariate) and Gaussian (random.gauss). The random.expovariate function takes as its parameters our *lambda* values defined above for both plants. The random.gauss function takes in as parameters the *mu* and *sigma* values as defined above. When running the simulation with a precision of 10% we return k (number of iterations), *y_bar* (the output of simulate many divided by the number of iterations, k), and *dy_bar* (the standard error).

After running 10,000 iterations, printing the y_bar parameter gives us an average yearly loss value. In Plant A, the value returned is $605,406.97 and in Plant B we get an average yearly loss of $80,740.95.

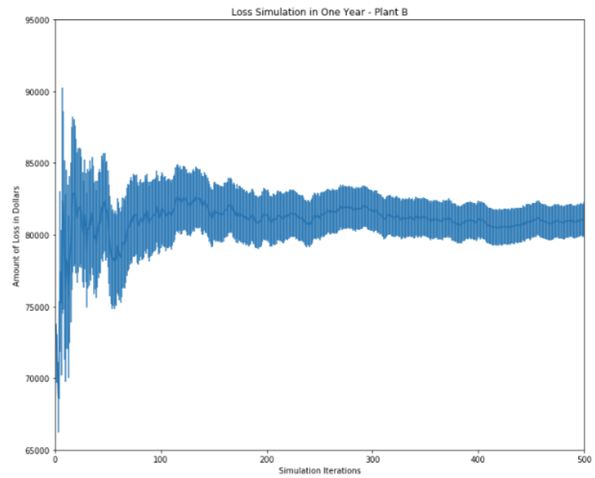
	Average Yearly Loss After 10,000 iterations
Plant A	$605,406.97
Plant B	$80,740.95

Upon initial examination, the numbers seem to be within a reasonable amount above the actual mean of our data per year. When running the bootstrap algorithm in our data, we print confidence intervals of 50%, 68%, 80%, and 90%. When run 1,000 times, the data gives intervals at the 90% level of between $605,126.65 for the lower bound and $605,129.29 for the upper bound in Plant A. In Plant B the range at 90% is between $80,931.60 for the lower bound and $80,933.66 for the upper bound.

Providing a visualization of the error over the iterations we can clearly see that the simulations in both plants converge to the mean as the error gets smaller and smaller. Plotting the iterations on the x axis and loss in the y axis, with error bars, we get the plots below:

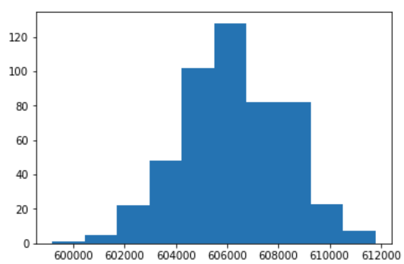


Simulation converging to mean in Plant A

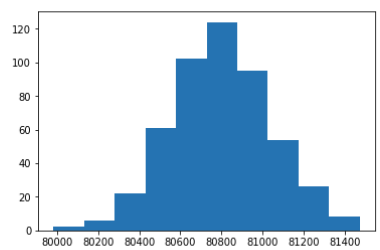


Simulation converging to mean in Plant B

With the simulation, we can also see that the distribution appears normal as expected and the histograms below show the mean values of the simulations:



Loss Distribution Plant A



Loss Distribution Plant B

Finally, we can sort the values of the simulation from smallest to largest, and give the values for each plant that correspond to 90th percentile of the distribution. This gives us the value that will cover losses in 90% of simulated scenarios or 10% Value at Risk. After running the simulations, we can say the recommended value to budget in both plants comes to $608,901.87 for Plant A and $81,116.06 in Plant B. In examining the actual means for losses per year, we can say with 90% confidence that these amounts will give a comfortable cushion in budget amounts for each year.

5. Conclusion

This study attempted to give some sound advice based on empirical evidence and Monte-Carlo analysis how much the company should budget to cover losses. As stated, accidents are a large risk in the industrial sector and it is critical to have the capital to cover them. This analysis shows that the simulation fit the data very well and gives a very good approach to simulating losses in these plants. The distributions used in the simulation were shown to fit and the final analysis gave a 10% VaR with bootstrap errors at 90%. In the future, this model can be extended to simulate new data in Plants A and B in order to shape a profitable business.

Appendix

Code from Python Jupyter Notebook:

```
import os
import csv
import math
import random
import numpy as np
import pandas as pd
from math import exp
import matplotlib.pyplot as plt
import matplotlib.mlab as mlab
%matplotlib inline
import seaborn as sns
from termcolor import colored
from itertools import combinations
```

In [4]:

os.chdir("C:\\Users\\James Cooper\\Desktop\\DePaul\\Monte Carlo Algorithms")
Dataset read into pandas dataframe¶
In [5]:
dat = pd.read_csv("accidents.csv", sep=',', header=0)
A logged losses column is added¶
As shown in graphs below, we can see that the losses display a lognormal distribution
In [6]:
dat['Log_Loss'] = np.log(dat['Lost'])
dat.head()
Out[6]:

	Plant	Day	Lost	Log_Loss
0	A	1	3348	8.116118
1	B	4	181	5.198497
2	B	7	250	5.521461
3	A	13	5446	8.602637
4	A	30	38549	10.559685

In [7]:

```
dat.info()
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 291 entries, 0 to 290
Data columns (total 4 columns):
Plant      291 non-null object
Day        291 non-null int64
Lost       291 non-null int64
Log_Loss   291 non-null float64
dtypes: float64(1), int64(2), object(1)
memory usage: 9.2+ KB
In [8]:
dat.describe()
Out[8]:
```

	Day	Lost	Log_Loss
count	291.000000	291.000000	291.000000
mean	697.202749	9189.185567	7.973683
std	427.485991	21853.188417	1.552151
min	1.000000	46.000000	3.828641
25%	327.000000	987.000000	6.894670
50%	706.000000	2886.000000	7.967627
75%	1041.500000	8717.000000	9.072894
max	1462.000000	272851.000000	12.516681

Losses over 4 years broken down by Plant A and B¶

```
In [7]:
g = sns.FacetGrid(dat, col="Plant", size=5, aspect=1)
g.map(sns.distplot, "Lost")

plt.xlim(0, 100000)
plt.ylim(0, 0.00010)
Out[7]:
(0, 0.0001)
```

Log value of total losses over 4 years¶

```
In [43]:
fig2 = sns.distplot(dat['Log_Loss'])
plt.xlabel("Log Lost Amounts")
plt.ylabel("Distribution")
plt.title("Distribution of Total Log Amounts Lost - Plant A and B")
plt.show()
```

CDF log distribution¶

```
In [44]:
pointsA = F(np.array(LogA))
plt.plot(pointsA[:,0], pointsA[:,1], label = 'Raw Points - Plant A')
plt.legend()
plt.show()
```

Data broken down into 4 years in days¶

In [9]:

```
Year_1 = dat[dat.Day <= 365]
Year_2 = dat[(dat['Day'] > 365) & (dat['Day'] <= 730)]
Year_3 = dat[(dat['Day'] > 730) & (dat['Day'] <= 1095)]
Year_4 = dat[(dat['Day'] > 1095)]
```

In [10]:

```
A_y1 = Year_1['Lost'][Year_1['Plant'] == 'A']
A_y2 = Year_2['Lost'][Year_2['Plant'] == 'A']
A_y3 = Year_3['Lost'][Year_3['Plant'] == 'A']
A_y4 = Year_4['Lost'][Year_4['Plant'] == 'A']
B_y1 = Year_1['Lost'][Year_1['Plant'] == 'B']
B_y2 = Year_2['Lost'][Year_2['Plant'] == 'B']
B_y3 = Year_3['Lost'][Year_3['Plant'] == 'B']
B_y4 = Year_4['Lost'][Year_4['Plant'] == 'B']
```

Question 1: Average Number of accidents per year - broken down by Plant A and Plant B¶

In [16]:

```
a = colored('Average number of accidents per year in Plant A: %.2f' % ((A_y1.count() + A_y2.count() +
A_y3.count() + A_y4.count())/4),"blue", attrs = ['bold'])

print(a)
b = colored('Average number of accidents per year in Plant B: %.2f' % ((B_y1.count() + B_y2.count() +
B_y3.count() + B_y4.count())/4), 'red', attrs = ['bold'])
print(b)
```

Average number of accidents per year in Plant A: 33.75

Average number of accidents per year in Plant B: 39.00

Question 2: Average Loss per accident - broken down by Plant A and B¶

In [11]:

```
data_A = dat['Lost'][dat['Plant'] == 'A']
c = colored('The Average loss per accident in Plant A: %.3f' % (sum(data_A)/len(data_A)), "blue", attrs =
['bold'])
print(c)
data_B = dat['Lost'][dat['Plant'] == 'B']
d = colored('The Average loss per accident in Plant B: %.3f' % (sum(data_B)/len(data_B)), "red", attrs =
['bold'])
print(d)
```

The Average loss per accident in Plant A: 17470.156

The Average loss per accident in Plant B: 2022.962

Question 3: Average Loss in total per year - broken down by Plant A and Plant B¶

In [12]:

```
av_lossA = sum(data_A)/4
av_loss_printA = colored('The average loss in total per year for Plant A: %.3f' % (av_lossA), "blue", attrs =
['bold'])
print(av_loss_printA)

av_lossB = sum(data_B)/4
av_loss_printB = colored('The average loss in total per year for Plant B: %.3f' % (av_lossB), "red", attrs =
['bold'])
```

```
print(av_loss_printB)
The average loss in total per year for Plant A: 589617.750
The average loss in total per year for Plant B: 78895.500
```

Distribution and Average for each year Plant A¶

In [24]:

```
def mu(X, plant, year):
    for i in X:
        #u = np.array(X)
        means = np.mean(X)
    answer = colored(('Mean Lost for year %d - Plant %s: ' % (year, plant)), "blue", attrs = ['bold'])
    print(answer, means)

    fig_1 = sns.distplot(X)
    #X.hist(figsize=[8,6])
    plt.title('Distribution of Losses')
    plt.show()


#analysis = colored(('We can see here a mean across all four years are fairly similar, the distributions also show that the losses display an exponential distrubution, taking the log of the losses produces a much more normal distribution'),'blue')
#print(analysis'\n')


print(mu(A_y1, 'A', 1))
print(mu(A_y2, 'A', 2))
print(mu(A_y3, 'A', 3))
print(mu(A_y4, 'A', 4))

Mean Lost for year 1 - Plant A:  18206.405405405407
Mean Lost for year 2 - Plant A:  13648.466666666667
Mean Lost for year 3 - Plant A:  14588.194444444445
Mean Lost for year 4 - Plant A:  23443.90625
```

Distribution and Average for each year Plant B¶

In [26]:

```
def mu(X, plant, year):
    for i in X:
        #u = np.array(X)
        means = np.mean(X)
    answer = colored(('Mean Lost in year %d - Plant %s: ' % (year, plant)), "blue", attrs = ['bold'])
    print(answer, means)

    fig_1 = sns.distplot(X)
    #X.hist(figsize=[8,6])
    plt.title('Distribution of Losses')
    plt.show()
```

#analysis = colored(('We can see here a mean across all four years are fairly similar, the distributions also show that the losses display an exponential distrubution, taking the log of the losses produces a much more normal distribution'),'blue')
#print(analysis'\n')

```
print(mu(B_y1, 'B', 1))
print(mu(B_y2, 'B', 2))
print(mu(B_y3, 'B', 3))
print(mu(B_y4, 'B', 4))
```

Mean Lost in year 1 - Plant B: 1908.2553191489362
Mean Lost in year 2 - Plant B: 1778.264705882353
Mean Lost in year 3 - Plant B: 2238.175
Mean Lost in year 4 - Plant B: 2168.7428571428572

None

Plotting Distributions¶

In [13]:

```
#Function takes in a vector, sorts from smallest to largest, makes another column which is the proportion
#that each value is of the total - or the percentile

def F(v):
    v.sort()
    n = len(v)
    data = []
    for k in range(n):
        point = (v[k], float(k+1)/n)
        data.append(point)
    return np.array(data)
```

Parameters to use in simulations¶

In [14]:

```
mu_A = (sum(data_A)/len(data_A))
mu_B = (sum(data_B)/len(data_B))
A_min = min(data_A)
B_min = min(data_B)
alphaA = mu_A/(mu_A - A_min)
alphaB = mu_B/(mu_B - B_min)
sigmaA = np.std(data_A)
sigmaB = np.std(data_B)

i = colored(' mu of A = %f \n mu of B = %f \n sigma A = %f \n sigmaB = %f \n min of A = %f \n min of B = %f \n alpha A = %f \n alpha B = %f ' %
      (mu_A, mu_B, A_min, B_min, alphaA, alphaB, sigmaA, sigmaB), 'blue', attrs = ['bold'])
'\t'
print(i)
```

```
 mu of A = 17470.155556
 mu of B = 2022.961538
 sigma A = 490.000000
 sigmaB = 46.000000
 min of A = 1.028857
 min of B = 1.023268
 alpha A = 29751.865078
 alpha B = 3325.907709
```
In [15]:
```
LogA = dat['Log_Loss'][dat['Plant'] == 'A']
LogB = dat['Log_Loss'][dat['Plant'] == 'B']
```
In [16]:
```
log_mu_A = (sum(LogA)/len(LogA))
log_mu_B = (sum(LogB)/len(LogB))
logA_min = min(LogA)
logB_min = min(LogB)
log_alphaA = log_mu_A/(log_mu_A - logA_min)
log_alphaB = log_mu_B/(log_mu_B - logB_min)
log_sigmaA = np.std(LogA)
log_sigmaB = np.std(LogB)

print('\n')
i = colored(' mu of A = %f \n mu of B = %f \n log sigma A = %f \n log sigma B = %f \n min of A = %f \n min of B = %f \n alpha A = %f \n alpha B = %f' %
      (log_mu_A, log_mu_B, logA_min, logB_min, log_alphaA, log_alphaB, log_sigmaA, log_sigmaB), 'blue', attrs = ['bold'])
'\t'
print(i)
```
```
 mu of A = 9.146100
 mu of B = 6.959092
 log sigma A = 6.194405
 log sigma B = 3.828641
 min of A = 3.098593
 min of B = 2.223032
 alpha A = 1.061511
 alpha B = 1.133381
```
Plotting the Pareto distribution with the unlogged data¶

In [40]:
```
def pareto_distA(xvalues, xMb = A_min, alphaa = alphaA):
   return 1.0 - (xMb/xvalues)**alphaa

pointsA = F(np.array(data_A))
paretoA = [(xvalues, pareto_distA(xvalues)) for (xvalues,y) in pointsA]
pareto_pointsA = np.array(paretoA)


plt.plot(pointsA[:,0], pointsA[:,1], label = 'Raw Points - Plant A')
```

```
plt.plot(pareto_pointsA[:,0], pareto_pointsA[:,1], label = 'Pareto')
#plt.plot(pareto_points)
plt.legend()
plt.show()
```

In [41]:

```
def pareto_distB(xvalues, xMb = B_min, alphab = alphaB):
   return 1.0 - (xMb/xvalues)**alphab

pointsB = F(np.array(data_B))
paretoB = [(xvalues, pareto_distB(xvalues)) for (xvalues,y) in pointsB]
pareto_pointsB = np.array(paretoB)


plt.plot(pointsB[:,0], pointsB[:,1], label = 'Raw Points - Plant B')
plt.plot(pareto_pointsB[:,0], pareto_pointsB[:,1], label = 'Pareto')
#plt.plot(pareto_points)
plt.legend()
plt.show()
```

Exponential Distribution for the time between events¶

In [ ]:

Lambda value of Time disri

The lambda value is the length of the data for each plant divided by 1462 - which is 4 years in days and the number of days in the data, this provides for a good match to what the actual distribution looks like when simulating the exponential distribution seen in the plot below

In [17]:

```
TimeA = dat['Day'][dat['Plant'] == 'A']
lambA = float(len(TimeA))/1462

TimeB = dat['Day'][dat['Plant'] == 'B']
lambB = float(len(TimeB))/1462
```

Differenced data - in order to get the time intervals between events, we need to take the difference of our data

In [49]:

```
diffA = np.diff(TimeA)
diff_tA = np.concatenate([[1], diffA])
diff_timeA = diff_tA.astype(float)
print(diff_timeA)
[ 1. 12. 17.  1. 20. 10. 20.  2.  5. 12.  2.  4. 50.  1.  0.  4. 12.  1.
  8.  6.  0.  2. 10.  2. 15.  5. 18. 12.  6.  8. 43.  3.  0. 30.  6. 12.
  3.  7.  2.  3. 13.  5.  2.  0. 12.  7. 20. 41.  1. 12. 18. 14.  9.  7.
 16.  4.  5. 28. 18. 16.  2. 27. 11. 16. 18. 10.  9. 15. 33. 19. 24.  2.
 11.  5.  4.  1.  3.  4. 14. 20.  9.  1.  0.  3. 12. 21.  6.  5. 22.  3.
  6.  5.  6.  5.  5.  2. 10.  5.  7. 15. 20. 40. 13.  7.  7. 30.  4.  0.
  6. 13. 10. 16.  3.  2.  2. 16. 28.  1.  6.  4. 18. 49. 14. 13. 11.  2.
 30. 20. 16.  7.  3. 10.  3. 10.  2.]
```

In [53]:

```
diffB = np.diff(TimeB)
diff_tB = np.concatenate([[1], diffB])
diff_timeB = diff_tB.astype(float)
print(diff_timeB)
[ 1.  3. 29.  2.  1.  4.  4.  2.  9.  1.  1.  4. 10. 17.  3. 11.  3. 11.
  9.  7. 11. 16.  7. 15.  6.  0. 14.  3.  6. 17.  9. 14.  4. 19. 21. 10.
  7. 11.  0.  2.  0.  2.  1.  1.  6.  3. 17. 10.  0. 19.  2. 10.  4. 23.
  8. 13.  2. 37. 13.  9.  6.  2.  2. 17.  4.  4. 14.  4.  1.  7. 39.  3.
  6.  9. 16.  2. 15. 10. 15.  3. 20. 30.  2. 15. 13. 11.  1.  6.  1.  6.
  0. 10. 12. 11.  8.  3.  1. 18. 18.  1.  8.  1. 15.  1.  5.  6.  2. 17.
 17. 41.  7.  7.  2. 13.  2. 21.  1.  5. 26.  7. 13.  7.  7.  6.  3.  6.
  3. 39.  6.  6. 22. 10.  2.  6.  9.  4. 19. 13. 11. 18.  3.  4. 22.  7.
 10. 12.  4. 26.  3. 31. 12.  8. 12.  8.  3. 10.]
In [69]:
def F_exponential(x, lamb):
   return 1.0 - exp(-lamb*x)
```

Exponential plot for Plant A¶

```
In [73]:
points_tA = F(np.array(diff_timeA))
points_exp = [(x, F_exponential(x, lambA)) for (x,y) in points_tA]
points_expA = np.array(points_exp)
plt.plot(points_tA[:,0], points_tA[:,1])
plt.plot(points_expA[:,0], points_expA[:,1])
Out[73]:
[<matplotlib.lines.Line2D at 0x2203525b2b0>]
```

Exponential plot for Plant B¶

```
In [74]:
points_tB = F(np.array(diff_timeB))
points_exp = [(x, F_exponential(x, lambB)) for (x,y) in points_tB]
points_expB = np.array(points_exp)
plt.plot(points_tB[:,0], points_tB[:,1])
plt.plot(points_expB[:,0], points_expB[:,1])
Out[74]:
[<matplotlib.lines.Line2D at 0x220354d30b8>]
```

Question 4 - Simulation using simulate_once function and then simulate_many¶

Plant A Simulation¶

```
In [217]:
class Simulator(object):

   def __init__(self):
      pass

   def simulate_once(self):
      t = 0
      total_loss = 0
      while t < 365:
```

```
            t = t + random.expovariate(lambA)
            amount = random.gauss(mu_A, sigmaA)
            #amount = random.paretovariate(log_alphaA)*logA_min
            total_loss = total_loss + amount
        return total_loss

    def simulate_many(self, ap=0.1, rp=0.1, ns=10000):
        sy = 0.0
        sy2 = 0.0
        self.history = []
        for k in range(1, ns+1):
            y = self.simulate_once()
            sy = sy + y
            sy2 = sy2 + y*y #sum of y^2 to y^2
            y_bar = sy/k #average
            y2_bar = sy2/k
            sigma = (y2_bar - y_bar**2)**0.5
            dy_bar = sigma/k**0.5 #error on the average
            #self.history.append((k, y_bar, dy_bar))
            self.history.append((y_bar))
            if dy_bar<ap and k>100:
                return y_bar
        return y_bar


sim = Simulator()
print('mu=',mu_A, 'lambda value=',lambA, 'alpha=',alphaA)
c = colored('The average yearly loss for Plant A with 10% precision = ')
print(c)
print('loss =',sim.simulate_many(ap=0.1, ns=100000))
historya = sim.history[:100000]
mu= 17470.155555555557 lambda value= 0.09233926128590972 alpha= 1.0288572150235504
The average yearly loss for Plant A with 10% precision =
loss = 605406.9731260132
Bootstrap Error in Results for Plant A¶
In [219]:
y_bars = np.array(historya)
y_bar = y_bars.tolist()

def resample(v):
    #return [random.choice(v)]
    #v = y_bar
    n = len(v)
    u = [random.choice(v) for k in range(n)]
    return u

def bootstrap(scenarios, confidence=90):
    samples = []
```

```
    for x in range(1000):
        samples.append(np.mean(resample(scenarios)))
        samples.sort()
    i = int((1000-confidence)/2)
    j = 99-i
    mu_plus = samples[j]
    mu_minus = samples[i]
    return mu_minus, mu_plus

for confidence in (50, 68, 80, 90):
    mu = np.mean(y_bar)
    mu_minus, mu_plus = bootstrap(y_bar)
    print (confidence, mu_minus, mu, mu_plus)
```

```
50 605126.5891187396 605127.2336195105 605129.0952236425
68 605126.6931001974 605127.2336195105 605129.4540232203
80 605126.2775814689 605127.2336195105 605128.904740706
90 605126.6497171395 605127.2336195105 605129.290323567
```

In [132]:

```
h_A = np.array(historya)
```

Plot of Error bars for Plant A¶

We can see here that the errorbars get smaller as the losses converge

In [200]:

```
#from matplotlib import rcParams
rcParams['figure.figsize'] = 8,6
plt.errorbar(h_A[:,0], h_A[:,1], h_A[:,2])
#plt.xlim(0, 500)
#plt.ylim(65000, 95000)
plt.xlabel("Simulation Iterations")
plt.ylabel("Amount of Loss in Dollars")
plt.title("Loss Simulation in One Year - Plant A")
plt.show()
```

In [201]:

```
from matplotlib import rcParams
rcParams['figure.figsize'] = 12,10
plt.errorbar(h_A[:,0], h_A[:,1], h_A[:,2])
plt.xlim(0, 500)
plt.ylim(500000, 700000)
plt.xlabel("Simulation Iterations")
plt.ylabel("Amount of Loss in Dollars")
plt.title("Loss Simulation in One Year - Plant A")
plt.show()
```

In [18]:

```
class Simulator(object):

    def __init__(self):
        pass
```

```
    def simulate_once(self):
        t = 0
        total_loss = 0
        while t < 365:
            t = t + random.expovariate(lambA)
            amount = random.gauss(mu_A, sigmaA)
            #amount = random.paretovariate(log_alphaA)*logA_min
            total_loss = total_loss + amount
        return total_loss

    def simulate_many(self, ap=0.1, rp=0.1, ns=10000):
        sy = 0.0
        sy2 = 0.0
        self.history = []
        for k in range(1, ns+1):
            y = self.simulate_once()
            sy = sy + y
            sy2 = sy2 + y*y #sum of y^2 to y^2
            y_bar = sy/k #average
            y2_bar = sy2/k
            sigma = (y2_bar - y_bar**2)**0.5
            dy_bar = sigma/k**0.5 #error on the average
            self.history.append((k, y_bar, dy_bar))
            #self.history.append((y_bar))
            if dy_bar<ap and k>100:
                return y_bar
        return y_bar


sim = Simulator()
print('mu=',mu_A, 'lambda value=',lambA, 'alpha=',alphaA)
c = colored('The average yearly loss for Plant A with 10% precision = ')
print(c)
print('loss =',sim.simulate_many(ap=0.1, ns=100000))
historya = sim.history[:100000]
mu= 17470.155555555557 lambda value= 0.09233926128590972 alpha= 1.0288572150235504
The average yearly loss for Plant A with 10% precision =
loss = 606163.3261370979
In [24]:
from matplotlib import rcParams
scenarios = []
for k in range(500):
    y = sim.simulate_many()
    scenarios.append(y)
    scenarios.sort
#print(scenarios)
```

```
rcParams['figure.figsize'] = 6,4
plt.hist(scenarios)
Out[24]:
(array([  1.,   5.,  22.,  48., 102., 128.,  82.,  82.,  23.,   7.]),
 array([599192.13178073, 600451.65959373, 601711.18740673, 602970.71521973,
        604230.24303273, 605489.77084573, 606749.29865873, 608008.82647172,
        609268.35428472, 610527.88209772, 611787.40991072]),
 <a list of 10 Patch objects>)
```

How much the company should budget to ensure it can cover losses in 90% of simulated scenarios in Plant A¶

```
In [33]:
scenarios.sort()
print('10% VaR=', scenarios[450])
10% VaR= 608901.872850586
```

Plant B Simulation¶

```
In [202]:
class Simulator(object):

    def __init__(self):
        pass

    def simulate_once(self):
        t = 0.0
        total_loss = 0.0
        while t < 365:
            t = t + random.expovariate(lambB)
            amount = random.gauss(mu_B, sigmaB)
            #amount = random.paretovariate(log_alphaA)*logA_min
            total_loss = total_loss + amount
        return total_loss

    def simulate_manyb(self, ap=0.1, rp=0.1, ns=10000):
        sy = 0.0
        sy2 = 0.0
        self.history = []
        for k in range(1, ns+1):
            y = self.simulate_once()
            sy = sy + y
            sy2 = sy2 + y*y #sum of y^2 to y^2
            y_bar = sy/k #average
            y2_bar = sy2/k
            sigma = (y2_bar - y_bar**2)**0.5
            dy_bar = sigma/k**0.5 #error on the average
            self.history.append((k, y_bar, dy_bar))
            #self.history.append((y_bar))
            if dy_bar<ap and k>100:
                return y_bar
```

```
    return y_bar



sim = Simulator()
print('mu=',mu_B, 'lambda value=',lambB, 'alpha=',alphaB)
c = colored('The average yearly loss for Plant B with 10% precision = ')
print(c)
print(sim.simulate_manyb(ap=0.1, ns=100000))
historyb = sim.history[:100000]
```

mu= 2022.9615384615386 lambda value= 0.106703146374829 alpha= 1.0232680298048675
The average yearly loss for Plant B with 10% precision =
80740.95289179828

Bootstrap Error in Results for Plant B¶

In [91]:

```
y_barsb = np.array(historyb)
y_bar = y_barsb.tolist()

def resample(v):
  #return [random.choice(v)]
  #v = y_bar
  n = len(v)
  u = [random.choice(v) for k in range(n)]
  return u

def bootstrap(scenarios, confidence=90):
  samples = []
  for x in range(100):
    samples.append(np.mean(resample(scenarios)))
    samples.sort()
  i = int((100-confidence)/2)
  j = 99-i
  mu_plus = samples[j]
  mu_minus = samples[i]
  return mu_minus, mu_plus

for confidence in (50, 68, 80, 90):
  mu = np.mean(y_bar)
  mu_minus, mu_plus = bootstrap(y_bar)
  print (confidence, mu_minus, mu, mu_plus)
```

50 80931.60123242464 80932.58619637636 80933.792963159
68 80931.44707596392 80932.58619637636 80933.69555461928
80 80931.67628188511 80932.58619637636 80933.57489558832
90 80931.61026559037 80932.58619637636 80933.65674364725

In [133]:

```
h = np.array(historyb)
```

Plot of Error bars for Plant B¶

We can see that the errorbars get smaller as the number of iterations rises and the total loss converges

```
In [104]:
plt.plot(h[:,0], h[:,1])
plt.xlim(0, 3000)
plt.ylim(76000, 84000)
Out[104]:
(76000, 84000)

In [210]:
from matplotlib import rcParams
rcParams['figure.figsize'] = 12,10
plt.errorbar(h[:,0], h[:,1], h[:,2])
plt.xlim(0, 500)
plt.ylim(65000, 95000)
plt.xlabel("Simulation Iterations")
plt.ylabel("Amount of Loss in Dollars")
plt.title("Loss Simulation in One Year - Plant B")
plt.show()

In [39]:
#Here we are using simulate many, but only with return y_bar, not with dy_bar or k
scenariosb = []
for k in range(500):
   yb = sim.simulate_manyb()
   scenariosb.append(yb)
#print(scenarios)

rcParams['figure.figsize'] = 6,4
plt.hist(scenariosb)
Out[39]:
(array([  2.,   6.,  22.,  61., 102., 124.,  95.,  54.,  26.,   8.]),
 array([79982.24621653, 80131.45237551, 80280.65853448, 80429.86469345,
       80579.07085243, 80728.2770114 , 80877.48317037, 81026.68932934,
       81175.89548832, 81325.10164729, 81474.30780626]),
 <a list of 10 Patch objects>)
```

How much the company should budget to ensure it can cover losses in 90% of simulated scenarios in Plant B¶

```
In [43]:
scenariosb.sort()
print('10% VaR=', scenariosb[450])
10% VaR= 81116.05651844747
```

References

[1] Amraja Mohamed, Mohamed, et al. "Approximation of Aggregate Losses Using Simulation." *Journal of Mathematics and Statistics*. 2010, pp 233-239. DOI: http://thescipub.com/pdf/10.3844/jmssp.2010.233.239

[2] DiPerro, Massimo and Nandy, Ananda. "Comprehensive Modeling of Operational Risk Economic Capital An Empirical Appraoch. *DePaul University, School of CTI.*

[3] DiPerro, Massimo." Annotated Algorithms in Python With Applications in Physics, Biology, and Finance (2nd ED.)" Chicago, IL. Experts4Solutions. 2013.

[4] Lochbaum, David. "Nuclear Plant Risk Studies Failing the Grade." Cambridge, MA. Union of Concerned Sciences. 2000.