Cooling Computer Components
An Examination of Heat Dissipation Effectiveness

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1 Introduction

This paper will examine the mechanisms behind various methodologies of cooling computer components. Specifically, the basis behind a passive-cooling system, an air-cooling system, and a liquid-cooling system will be explored. The focus will be on the cooling of a central processing unit, or CPU, though other components will likely adhere to the same basic principles that we will discover in our analysis.

2 Preliminary Formulations

By Fourier's Law, the rate of heat transfer through a body by conduction is

$$\frac{dQ}{dt}_{\text{conduction}} = kA\frac{dT}{dx},$$

where Q is the thermal energy, A is the cross-sectional area, $\frac{dT}{dx}$ is the rate of change of temperature across the length of the body, and k is the thermal conductivity of the material. This formula is limited in that it does not describe the rate of heat flow across a boundary – in this case, it does not describe the interface between the CPU and whatever cooling system is being used. For the purpose of constructing a more useful formulation, this equation can be re-written as

$$\frac{\Delta Q}{\Delta t}_{\text{conduction}} = kA\frac{\Delta T}{\Delta x}.$$

If the substitution $R = \frac{\Delta x}{kA}$ is made (deemed the *thermal resistivity*), then, for a system of two bodies in series,

$$R_{\text{total}} = R_1 + R_2.$$

Hence, we get

$$\frac{dQ}{dt}_{\text{conduction}} = \frac{T_{\text{CPU}}(t) - T_{\text{cooling}}(t)}{\left(\frac{\Delta x_{\text{CPU}}}{k_{\text{CPU}}A} + \frac{\Delta x_{\text{cooling}}}{k_{\text{cooling}}A}\right)} = \frac{T_{\text{CPU}}(t) - T_{\text{cooling}}(t)}{B},$$

where the Δx's are the thicknesses of the CPU and the base of the cooling system and A is the cross-sectional area of the interface between the CPU and the cooling system. This equation expresses the rate at which heat is being transferred from the CPU to the cooling system via conduction. We use $T(t)$ to remind ourselves that temperature is a function of time, and replace the constant expression $\left(\frac{\Delta x_{\text{CPU}}}{k_{\text{CPU}}A} + \frac{\Delta x_{\text{cooling}}}{k_{\text{cooling}}A}\right)$ with B for clarity.

An object is cooled via *convection* to the surroundings at a rate given by Newton's Law of Cooling,

$$\frac{dQ}{dt}_{\text{convection}} = hS\big(T(t) - T_0\big),$$

where h is the heat transfer coefficient of the fluid, S is the heat transfer surface area (that is, the area exposed to the surroundings), and T_0 is the temperature of the surroundings, which will be assumed to be constant.

The object can *radiate* heat to the surroundings as well, at a rate governed by the Stefan-Boltzmann Law,

$$\frac{dQ}{dt}_{\text{radiation}} = \epsilon\sigma S\big(T^4(t) - T_0^4\big),$$

where ϵ is the emissivity coefficient and σ is the Stefan-Boltzmann constant.

The specific heat capacity c of a particular material is given by

$$c = \frac{Q}{m\Delta T},$$

where m is the mass used of the material. Taking Q and T as functions of time and differentiating both sides yields

$$\frac{dQ}{dt} = mc\frac{dT}{dt}.$$

3 Passive Cooling

Consider first, for the sake of simplicity, a cooling system relying solely on passive heat expenditure. In particular, suppose a CPU is being cooled purely by a large heatsink, made of some material, attached to itself. In this scenario, we will assume that no convective currents are present or created. Thus, convection cooling is zero and the radiation of heat from the heatsink will be the *only* form of cooling. Using this result, the net rate of heat flow to the heatsink can be written as

$$\frac{dQ}{dt}_{\text{heatsink}} = \frac{T_{\text{CPU}}(t) - T_{\text{heatsink}}(t)}{B} - \epsilon\sigma S\big(T^4_{\text{heatsink}}(t) - T_0^4\big).$$

The net flow of heat to the CPU can be written as the heat generated by the CPU – which will be approximately constant under load and directly proportional to the CPU's power consumption – minus the amount of heat conducted to the heatsink. In particular,

$$\frac{dQ}{dt}_{\text{CPU}} = P - \frac{T_{\text{CPU}}(t) - T_{\text{heatsink}}(t)}{B},$$

where P is the heat generated under load.

Hence, we can characterize the temperature of the CPU and heatsink in a passively-cooled mechanism over time as a system of differential equations,

$$\frac{dT_{\text{CPU}}}{dt} = \frac{1}{m_{\text{CPU}}c_{\text{CPU}}}\left(P - \frac{T_{\text{CPU}} - T_{\text{heatsink}}}{B}\right)$$

$$\frac{dT_{\text{heatsink}}}{dt} = \frac{1}{m_{\text{heatsink}}c_{\text{heatsink}}}\left(\frac{T_{\text{CPU}} - T_{\text{heatsink}}}{B} - \epsilon\sigma S\left(T_{\text{heatsink}}^4 - T_0^4\right)\right).$$

It is not necessary to solve this system (indeed, it is quite difficult to do so) in order to understand the underlying mechanism. We want to minimize $\frac{dT_{\text{CPU}}}{dt}$, and we can do so by ideally adjusting certain parameters. Since $\frac{dT_{\text{CPU}}}{dt}$ depends on T_{heatsink}, it is nearly sufficient to minimize $\frac{dT_{\text{heatsink}}}{dt}$, so that the temperature disparity allows for more conduction of heat from the CPU to the heatsink. However, since the heatsink must take heat *away* from the CPU, the term B must be minimized as well.

Analyzing the equation for the rate of change of the temperature of the heatsink, the ideal cooling solution would:

- use a material for the heatsink with a large thermal conductivity, large specific heat capacity, and large thermal emissivity coefficient
- maximize the mass of the heatsink
- maximize the surface area of the heatsink.

Other parameters, such as the material of the CPU, cannot be altered.

To optimize these parameters, two different materials can be chosen. One material, with a large thermal conductivity, is based at the bottom of the heatsink and is solely used to conduct heat from the CPU to the heatsink. The other material is the primary component of the heatsink and radiates heat to the surroundings, so it must have a relatively large emissivity coefficient. This material must also have a large specific heat capacity, so that any temperature gradient that develops between the heatsink and the surroundings is minimally translated to an increase in the temperature of the heatsink.

For the first material, the only consideration is thermal conductivity. Silver and copper have thermal conductivities of 429 $\frac{\mathrm{W}}{\mathrm{m \cdot K}}$ and 401 $\frac{\mathrm{W}}{\mathrm{m \cdot K}}$, respectively [1]. Silver is thus a better objective choice than copper. However, due to cost considerations (silver is far more expensive per kilogram than copper), the latter is often used in place of silver; the difference in thermal conductivities does not justify the cost increase.

For the second material, the primary material of the heatsink, a number of factors must be attended to. First, both the specific heat capacity and emissivity coefficient must be maximized. In addition, the heat transfer surface area must be maximized, so the material must allow itself to be machined – in other words, the material cannot be brittle. Lastly, the material must maintain its shape; that is, it cannot melt or deform under high temperatures. Keeping these considerations in mind, it is clear that some metal would be the most suitable choice.

The metal in question must have both a high specific heat capacity and a high emissivity coefficient. While magnesium has a quite high specific heat capacity of 1050 $\frac{\mathrm{J}}{\mathrm{kg \cdot K}}$ [2], it has an extremely low emissivity coefficient of 0.13 [3]. Similarly, while steel has a rather high emissivity coefficient of 0.88 [3], it has a relatively low specific heat capacity of 490 $\frac{\mathrm{J}}{\mathrm{kg \cdot K}}$ [2]. The optimal balance between the two is achieved with aluminum, which has a specific heat capacity of 897 $\frac{\mathrm{J}}{\mathrm{kg \cdot K}}$ [2] and a thermal emissivity coefficient of 0.77.

Both the surface area and the mass of the second material, aluminum, must be as large as possible for an ideal cooling system. To maximize the surface area, one can simply machine the metal in such a way as to form numerous "fins" at which heat can be most effectively dissipated. In theory, there is no limit to the amount of the material that can be used; thus, the mass, and subsequently the surface area, can be as large as necessary. In practice, the amount that can be used is hindered by cost limitations.

We have seen that the most effective passive-cooling heatsink is one that employs a copper base, that conducts heat from the CPU, and numerous aluminum fins, that emit heat to the surroundings.

4 Air Cooling

Consider now a case where the CPU is being air-cooled. In particular, we use the same metal heatsink that was used for passive cooling, but append to the system some mechanism that causes forced convection, such as a fan (or several fans). In this way, the heatsink is now not only radiating heat, but is being cooled by convection as well. The net rate at which heat is being transferred to the heatsink is then

$$\left.\frac{dQ}{dt}\right._{\text{heatsink}} = \frac{T_{\text{CPU}}(t) - T_{\text{heatsink}}(t)}{B} - hS\big(T_{\text{heatsink}}(t) - T_0\big) - \epsilon\sigma S\big(T_{\text{heatsink}}^4(t) - T_0^4\big).$$

The heat flow to the CPU remains the same as in the passive cooling system. The resulting system of differential equations governing the temperature of the CPU and heatsink, proceeding in a similar manner as previously, is

$$\frac{dT_{\text{CPU}}}{dt} = \frac{1}{m_{\text{CPU}}c_{\text{CPU}}}\Big(P - \frac{T_{\text{CPU}} - T_{\text{heatsink}}}{B}\Big)$$

$$\frac{dT_{\text{heatsink}}}{dt} = \frac{1}{m_{\text{heatsink}}c_{\text{heatsink}}}\Big(\frac{T_{\text{CPU}} - T_{\text{heatsink}}}{B} - hS\big(T_{\text{heatsink}} - T_0\big) - \epsilon\sigma S\big(T_{\text{heatsink}}^4 - T_0^4\big)\Big).$$

Again, this system is quite difficult to solve, so we will try to qualitatively understand the basic mechanism. These equations are quite similar to the ones found for the passive cooling scenario, so much of the findings from the previous analysis is still applicable. However, there is a new term in the equation for the temperature of the heatsink, h, the heat transfer coefficient, that must be considered.

The heat transfer coefficient of convection depends primarily on the fluid. In this case, since the CPU is being air-cooled, the fluid *must* be air; that is, we cannot freely change which fluid to use as was done for the heatsink material. Air has a heat transfer coefficient of around 10 $\frac{\text{W}}{\text{m}^2\cdot\text{K}}$ in low-speed forced convection and around 100 $\frac{\text{W}}{\text{m}^2\cdot\text{K}}$ in medium-speed forced convection [4] (more precise values must take into consideration the heat transfer material and fin density, among other factors, which may affect the resistance to convection). Evidently, a higher fluid speed leads to an increase in the heat transfer coefficient, which is more favourable for cooling. Thus, for optimal cooling, the fastest air speed allowable should be achieved, being limited by considerations such as fan noise and fan power draw.

The surface area, S, in $hS\big(T_{\text{heatsink}} - T_0\big)$, the expression added for convection, is the same surface area that was examined for passive cooling. In particular, this surface area must remain quite large to allow for adequate heat transfer by convection and radiation.

Lastly, since the majority of the factors governing the heatsink itself did not change, it is still ideal to use a copper-based, aluminum-finned design.

5 Liquid Cooling

Now, consider a liquid cooling system. Instead of a large metal heatsink, the CPU will be cooled by a "block" attached to it; liquid is pumped through this block and absorbs some of the generated heat. From there, the liquid goes through a radiator, is cooled by several fans, then goes back to the block and repeats the loop. The temperature disparity between any two points in the loop is no larger than a few degrees, since the liquid is being pumped relatively fast (we will assume the difference is zero). The heat flow to the CPU is

$$\frac{dQ}{dt}_{\text{CPU}} = P - \frac{T_{\text{CPU}}(t) - T_{\text{block}}(t)}{B}.$$

Instead of considering the heat flow and temperature of a heatsink, we examine the temperature of the block and the liquid. The block is being cooled by convection by the liquid *only*, so the heat flow to the block is

$$\frac{dQ}{dt}_{\text{block}} = \frac{T_{\text{CPU}}(t) - T_{\text{block}}(t)}{B} - h_{\text{liquid}} A\big(T_{\text{block}}(t) - T_{\text{liquid}}(t)\big).$$

Here, A, the cross-sectional area of the interface between the CPU and the block, is being used in place of S, since the block is not the main mode of cooling and thus has no need to have a large surface area. The surface area by which it is cooled by the liquid is usually the same as the area used to conduct heat from the CPU as a result. Additionally, h_{liquid} is specified to distinguish it from h_{air}.

The net rate of heat flow to the liquid is the rate of heat input minus the rate of heat output, or

$$\frac{dQ}{dt}_{\text{liquid}} = h_{\text{liquid}} A\big(T_{\text{block}}(t) - T_{\text{liquid}}(t)\big) - h_{\text{air}} S\big(T_{\text{liquid}}(t) - T_0\big),$$

where S is the surface area of the radiator.

Hence, the rates of temperature change of the CPU, block, and liquid as a system of differential equations is

$$\frac{dT_{\text{CPU}}}{dt} = \frac{1}{m_{\text{CPU}} c_{\text{CPU}}}\left(P - \frac{T_{\text{CPU}} - T_{\text{block}}}{B}\right)$$

$$\frac{dT_{\text{block}}}{dt} = \frac{1}{m_{\text{block}} c_{\text{block}}}\left(\frac{T_{\text{CPU}} - T_{\text{block}}}{B} - h_{\text{liquid}} A\big(T_{\text{block}} - T_{\text{liquid}}\big)\right)$$

$$\frac{dT_{\text{liquid}}}{dt} = \frac{1}{m_{\text{liquid}} c_{\text{liquid}}}\Big(h_{\text{liquid}} A\big(T_{\text{block}} - T_{\text{liquid}}\big) - h_{\text{air}} S\big(T_{\text{liquid}} - T_0\big)\Big).$$

In spite of this system being linear, unlike the respective systems found for passive and air cooling, it is still rather difficult to solve. Accordingly, like for the previous analyses, we will examine it qualitatively and attempt to understand it holistically.

Again, it is necessary to maximize the conduction of heat from the CPU to the block. To do so, we must minimize B, or, equivalently, maximize the thermal conductivity of the block. Since the rate of temperature change of the CPU depends on the temperature disparity between itself and the block, it is also necessary to minimize $\frac{dt_{\text{block}}}{dt}$ to maximize this disparity. We can do this by choosing a material for the block with a large specific heat capacity and a liquid for the loop with a large heat transfer coefficient. Increasing the mass of the block and increasing the speed of the liquid (thus increasing its heat transfer coefficient) will also slow down the rate of temperature increase.

Additionally, we can minimize $\frac{dt_{\text{block}}}{dt}$ by maximizing the temperature disparity between the block and the liquid. In other words, minimizing $\frac{dt_{\text{water}}}{dt}$ is also required. To do so, we can choose a liquid with a large specific heat capacity, maximize the surface area of the radiator, and increase the heat transfer coefficient of the cooling air (i.e. by increasing the air speed). Using *more* liquid will also help in this case, though as more liquid is added, our assumption that the temperature difference between any two points in the loop is zero will become less and less applicable.

In summary, it is sufficient to:

- use a material for the block with a large thermal conductivity and large specific heat capacity
- make the block as massive as required
- use a liquid with a large heat transfer coefficient and large specific heat capacity
- maximize the speed of the liquid and amount of liquid used (this is equivalent to creating a longer loop)
- maximize the surface area of the radiator
- maximize the air flow through the radiator

Let us begin with the material of the block. Since we are trying to maximize the thermal conductivity, the best choices are still copper and silver. However, copper is now likely the better choice – while the two have very similar thermal conductivities (401 $\frac{\text{W}}{\text{m}\cdot\text{K}}$ for copper and 429 $\frac{\text{W}}{\text{m}\cdot\text{K}}$ for silver [1]), copper has a significantly higher specific heat capacity than silver (385 $\frac{\text{J}}{\text{kg}\cdot\text{K}}$ as opposed to 235 $\frac{\text{J}}{\text{kg}\cdot\text{K}}$ [2]). Note that it is not generally necessary to use *two* metals for the block, like was done for heatsink, as the block is not meant to radiate heat. Note also that increasing the mass of the block eventually results in diminishing returns in cooling, and, since the block is not meant to be the primary method of cooling anyway, it is customary and more cost-effective to use a rather small block (which is more than can be said about a heatsink).

Let us turn our attention to the liquid. The liquid must have both a large heat transfer coefficient and a large specific heat capacity. Very roughly (since the heat transfer coefficient is dependent on numerous factors), the heat transfer coefficients of water, organic solvents, and oils are 6000 $\frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}}$, 1500 $\frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}}$, and 400 $\frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}}$, respectively [5]. Water is the obvious choice here, and this is further justified by the fact that its specific heat capacity, 4182 $\frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}$ [2], is far higher than the specific heat capacities of its competitors. Benzene, an organic solvent, for example, only has a specific heat capacity of 1744 $\frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}$ [6].

Furthermore, we can make the loop larger, thus increasing the mass of water used, or pump the water through the loop faster, thus increasing the heat transfer coefficient, in order to increase the cooling performance of the system.

Consider now the performance of the radiator. Even though it is not mentioned in the equations, the radiator must be made of a material that has a large emissivity coefficient *and* a large thermal conductivity so that the water is allowed to be cooled. Aluminum and copper both satisfy these criteria, with respective emissivity coefficients of 0.77 and 0.78 [3] and respective thermal conductivities of 205 $\frac{\mathrm{W}}{\mathrm{m} \cdot \mathrm{K}}$ and 401 $\frac{\mathrm{W}}{\mathrm{m} \cdot \mathrm{K}}$ [1]. It is clear that copper is the better choice.

The radiator must also have a large surface area, so again the material, copper, must be machined into fins. In addition, several fans should be used to increase the airflow through the radiator, increasing the heat transfer coefficient of the air and thereby maximizing the cooling performance of the system.

Thus, the most effective liquid-cooling system employs a copper block, a large water loop with a powerful-enough pump to move the water with sufficient speed, a copper radiator with a large surface area, and several fans moving air across the radiator.

6 Conclusion

We have constructed and examined systems of differential equations that govern various cooling mechanisms for a CPU. Namely, the principles behind passive cooling, air cooling, and liquid cooling were dissected. It was demonstrated that the most effective design for a passive-cooling system is a metal heatsink comprised of a copper base and aluminum fins. Moreover, it was discovered that the ideal air-cooling system simply incurred forced convection upon that heatsink to as large of an extent as possible. The most effective design for a liquid-cooling system utilized a copper block and a large copper radiator employing several fans, as well as a large water loop.

6.1 Future considerations

It may be interesting to apply these basic principles to more exotic forms of computer cooling. In particular, phase-change cooling – in which the processor is cooled by the absorption of heat by a gas compressed into a liquid – and liquid nitrogen (LN2) cooling – in which liquid nitrogen is "poured" onto the processor and creates an extremely large temperature gradient – may be compelling to analyze.

References

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