REVIEW
FOR Midterm!

HIGH SCHOOL & Beyond

providing a lifetime of insight

HS&B began in 1980 as a nationally representative sample of 30,030 sophomores and 28,240 seniors in 1,015 public and private high schools in the United States. From the initial sample of 58,270 public and private high school students, 14,825 sophomores and 11,995 seniors were selected to be re-interviewed over their early adult years. Each school contained a representative sample of 36 sophomores and 36 seniors, making possible inferences about each school and its student body. The student questionnaires in 1980 gathered important information about educational experiences, cognitive skills

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HSB2 Data Set:

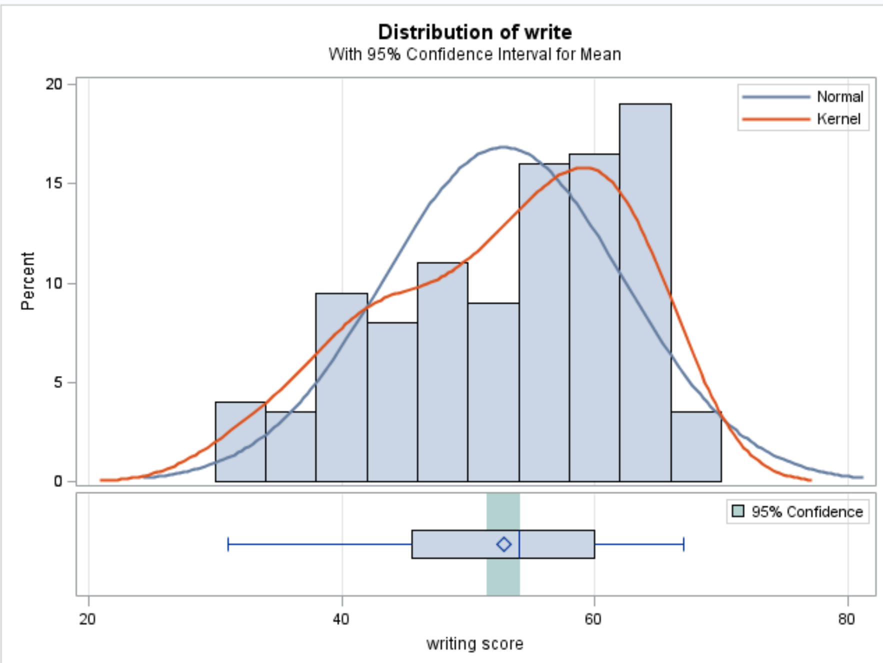
This data file contains 200 observations from a sample of high school students with demographic information about the students, such as their gender (**female**), socio-economic status (**ses**) and ethnic background (**race**). It also contains a number of scores on standardized tests, including tests of reading (**read**), writing (**write**), mathematics (**math**) and social studies (**socst**).

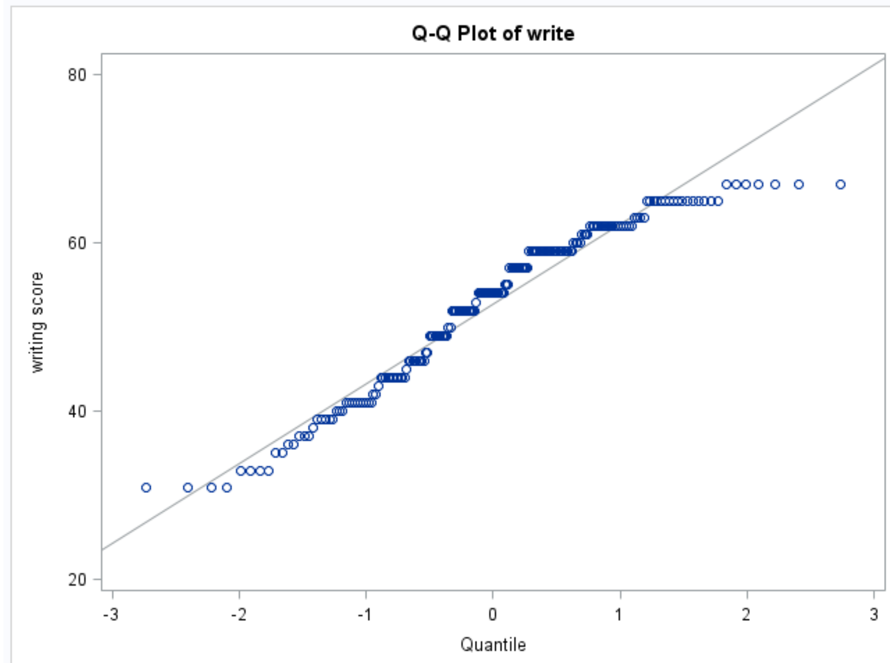
id	female	race	ses	schtyp	prog	read	write	math	science	socst
70	0	4	1	1	1	57	52	41	47	57
121	1	4	2	1	3	68	59	53	63	61
86	0	4	3	1	1	44	33	54	58	31
141	0	4	3	1	3	63	44	47	53	56
172	0	4	2	1	2	47	52	57	53	61
113	0	4	2	1	2	44	52	51	63	61
50	0	3	2	1	1	50	59	42	53	61
11	0	1	2	1	2	34	46	45	39	36
84	0	4	2	1	1	63	57	54	58	51
48	0	3	2	1	2	57	55	52	50	51
75	0	4	2	1	3	60	46	51	53	61
60	0	4	2	1	2	57	65	51	63	61
95	0	4	3	1	2	73	60	71	61	71
104	0	4	3	1	2	54	63	57	55	46
38	0	3	1	1	2	45	57	50	31	56
115	0	4	1	1	1	42	49	43	50	56
76	0	4	3	1	2	47	52	51	50	56
195	0	4	2	2	1	57	57	60	58	56
114	0	4	3	1	2	68	65	62	55	61

Review!!!!

1. Download the HSB2 Dataset and import into a SAS session.
2. Test the claim that the mean writing score is equal to 50. Write all 6 steps.
3. Test the claim that the mean writing score is different for males and females. Include all 6 steps.
4. There are 4 unique race categories. Test to see if the 1st race has a different mean writing score than the 4th race.
5. Use proc power to find the power of a test to detect a difference of 3 in the means if the two groups have standard deviations of 5 and 8 and the first group has a sample size of 20 and the second has a sample size of 40. Assume we want to use a satherwaite approximation (Wilcoxon test of difference of means / different standard deviations.)

2. Test the claim that the mean writing score is equal to 50. Write all 6 steps.





The histogram and qq plot provide strong evidence of a left skew although the sample size of 200 should ensure that the means will be normally distributed (central limit theorem). We will assume the scores are independent of one another and proceed with a t test.

2. Test the claim that the mean writing score is equal to 50. Write all 6 steps.

DF	t Value	Pr > |t|
199	4.14	<.0001

1. $H_0: \mu_{writing} = 50$
   $H_A: \mu_{writing} \neq 50$

2. *Critical Value*: $t_{.975,199} \cong 1.96$

3. *Test Statistic*: 4.14

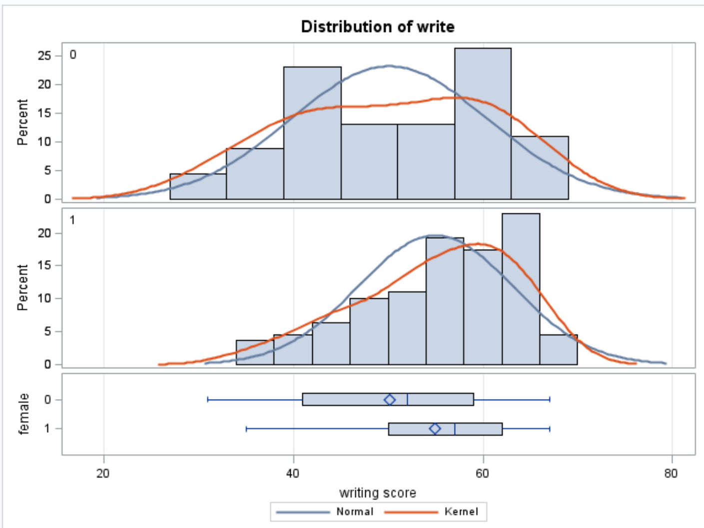
4. *Pvalue* $< .0001$

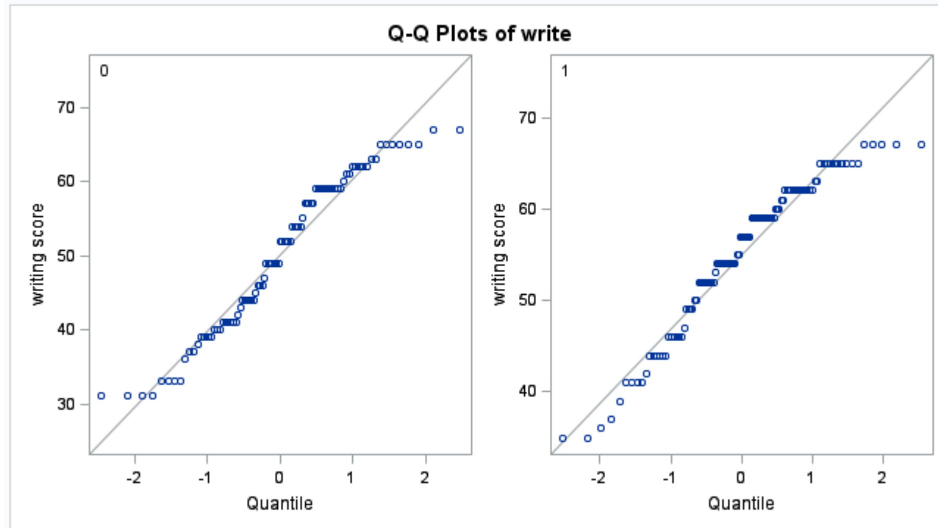
5. *Reject* H_0

6. There is strong evidence to suggest at the alpha = .05 level of significance (pvalue = .0001) that the mean writing score is different than 50 points. A 95% confidence interval for the true mean writing score is: (51.5 points, 54.1 points).

```
proc ttest data = TMP1.hsb2 H0 = 50;
var write;
run;
```

3. Test the claim that the mean writing score is different for males and females. Include all 6 steps.





While there is not significant evidence that the male writing scores (female = 0) are not normally distributed, the histograms and qq plots provide evidence of a left skewed distribution of writing scores for females (female = 1). However, since there are 91 in the male group and 109 in the female group, the central limit theorem will ensure that means from these distributions are normally distributed thus making the t test robust to the normality assumption.

3. Test the claim that the mean writing score is different for males and females. Include all 6 steps.

Brown and Forsythe's Test for Homogeneity of write Variance ANOVA of Absolute Deviations from Group Medians					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
female	1	262.6	262.6	9.62	0.0022
Error	198	5404.9	27.2976		

The histograms on the last slide were somewhat inconclusive on the question of equality of variances. For this reason we seek secondary evidence in the form of a formal hypothesis test. Since there is evidence that the writing scores are not normally distributed, the Brown-Forsythe test of equality of variance should be used instead of the F-Test. There is significant evidence at the alpha = .05 level of significance (pvalue = .0022) to suggest that the variance of the male writing scores is different from that of the females.

There is some evidence that the standard deviations are different therefore, since the Welche's test is nearly as powerful as the t test even when the standard deviations are the same, we will proceed with the Welch's test of the difference of means.

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	198	-3.73	0.0002
Satterthwaite	Unequal	169.71	-3.66	0.0003

3. Test the claim that the mean writing score is different for males and females. Include all 6 steps.

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	198	-3.73	0.0002
Satterthwaite	Unequal	169.71	-3.66	0.0003

1. $H_0: \mu_{female} = \mu_{male}$
   $H_A: \mu_{female} \neq \mu_{male}$

2. *Critical Value*: $t_{.975,169.71} \cong 1.96$

3. *Test Statistic*: -3.66

4. *Pvalue* $= .0003$

5. *Reject* H_0

6. There is strong evidence to suggest at the alpha = .05 level of significance (pvalue = .0003) that the mean writing score of female high school students in the US is different than the mean writing score of males . This was an observational study and thus no causal inference can be deduced. A 95% confidence interval for this difference is: (2.2 points, 7.5 points).

3. Test the claim that the mean writing score is different for males and females. Include all 6 steps.

ALTERNATIVE SOLUTION: NON PARAMETRIC

Variable: write (writing score)

female	N	Mean	Std Dev	Std Err	Minimum	Maximum
0	91	50.1209	10.3052	1.0803	31.0000	67.0000
1	109	54.9908	8.1337	0.7791	35.0000	67.0000
Diff (1-2)		-4.8699	9.1846	1.3042		

female	Method	Mean	95% CL Mean		Std Dev	95% CL Std Dev	
0		50.1209	47.9747	52.2670	10.3052	8.9947	12.0662
1		54.9908	53.4466	56.5351	8.1337	7.1786	9.3843
Diff (1-2)	Pooled	-4.8699	-7.4418	-2.2981	9.1846	8.3622	10.1878
Diff (1-2)	Satterthwaite	-4.8699	-7.4992	-2.2407			

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	198	-3.73	0.0002
Satterthwaite	Unequal	169.71	-3.66	0.0003

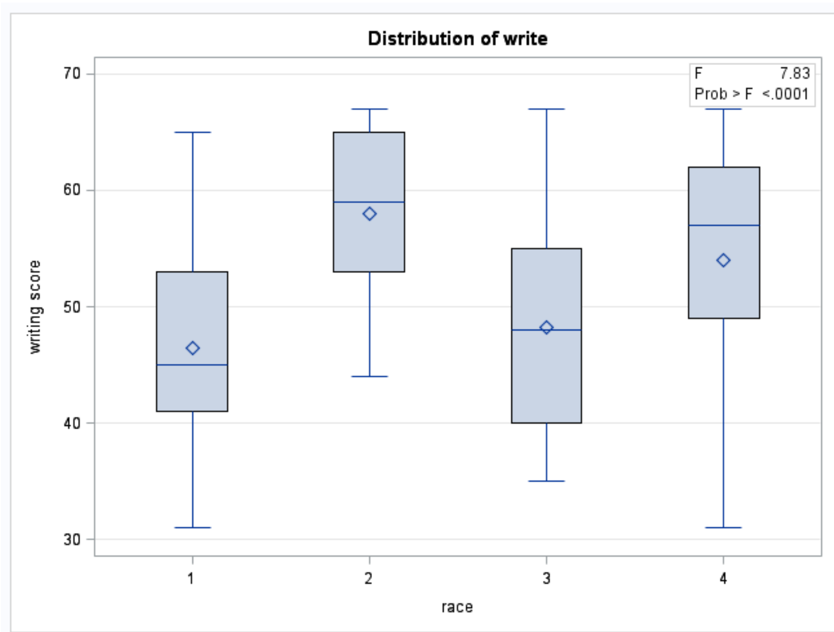
Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	90	108	1.61	0.0187

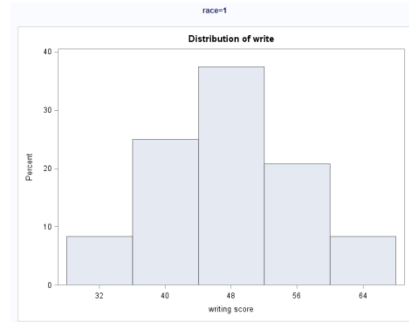
Wilcoxon Scores (Rank Sums) for Variable write Classified by Variable female					
female	N	Sum of Scores	Expected Under H0	Std Dev Under H0	Mean Score
0	91	7792.0	9145.50	406.559086	85.626374
1	109	12308.0	10954.50	406.559086	112.917431
Average scores were used for ties.					

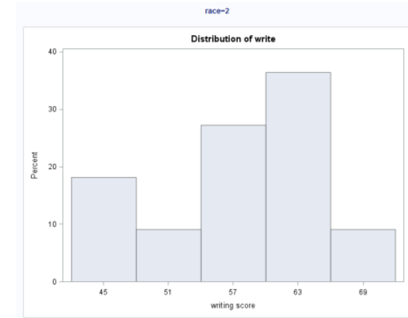
Wilcoxon Two-Sample Test	
Statistic	7792.0000
Normal Approximation	
Z	-3.3279
One-Sided Pr < Z	0.0004
Two-Sided Pr > |Z|	0.0009
t Approximation	
One-Sided Pr < Z	0.0005
Two-Sided Pr > |Z|	0.0010
Z includes a continuity correction of 0.5.	

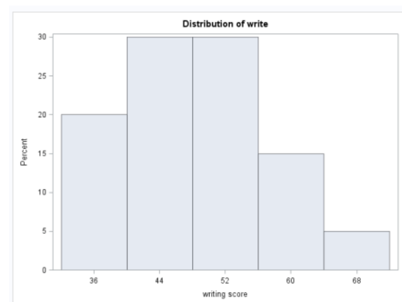
Kruskal-Wallis Test	
Chi-Square	11.0833
DF	1
Pr > Chi-Square	0.0009

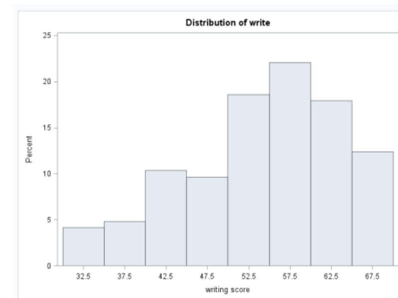
4. There are 4 unique race categories. Test to see if the 1st race has a different mean writing score than the 4th race.







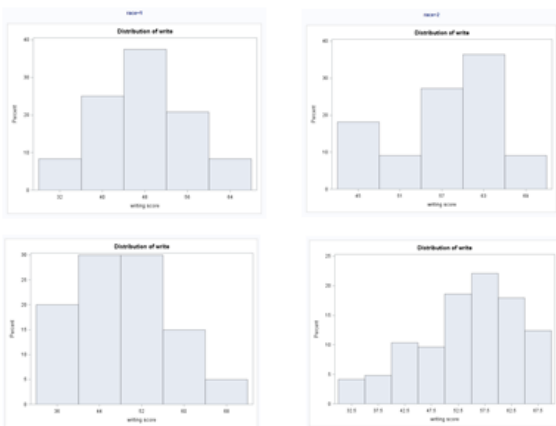




Level of race	N	write Mean	write Std Dev
1	24	46.4583333	8.27242232
2	11	58.0000000	7.89936706
3	20	48.2000000	9.32229924
4	145	54.0551724	9.17255819

While the only histogram that provides strong evidence (n = 145 and a considerable left skew) against normality is race = 4, the histograms of the small sample sizes of the other groups does not provide strong evidence that their distribution are also not normal. Since the sample sizes are small, we will proceed with the Kruskal Wallace nonparametric test.

4. There are 4 unique race categories. Test to see if the 1st race has a different mean writing score than the 4th race.



Again, given that the other groups have small sample sizes and there is no reason to believe the shape of the other races should be any different than the shape of the distribution of writing scores of race = 4, we will assume that the shapes (although maybe not the locations) of the distributions are the same. For this reason we will conduct a Kruskal-Wallis Test and make our inference about the Median.

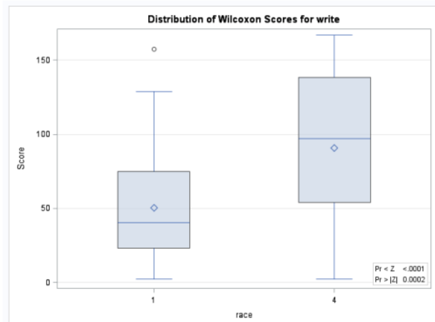
Kruskal-Wallis Test	
Chi-Square	22.1942
DF	3
Pr > Chi-Square	<.0001

There is sufficient evidence at the alpha = .05 level of significance (pvalue < .0001) that at least one of medians is different between the race groups.

Now we shall test to see if the 1st race has a different mean than the fourth race. A multiple comparison adjustment is not needed here since our hypothesis was formulated before we looked at the data.

4. There are 4 unique race categories. Test to see if the 1st race has a different mean writing score than the 4th race.

A Wilcoxon Rank Sum test will be performed between the two groups.



1. $H_0: Median_1 = Median_4$
   $H_A: Median_1 \neq Median_4$

2. $Critical\ Value: z \cong \pm 1.96$

3. $Test\ Statistic: z = -3.72$

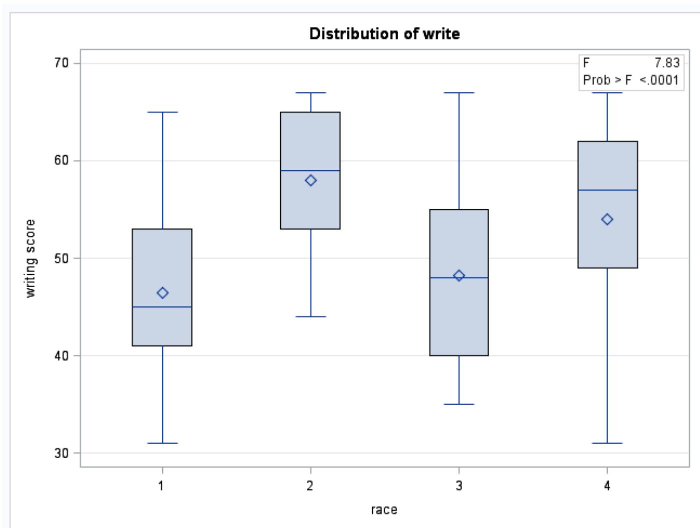
4. $Pvalue = .0002$

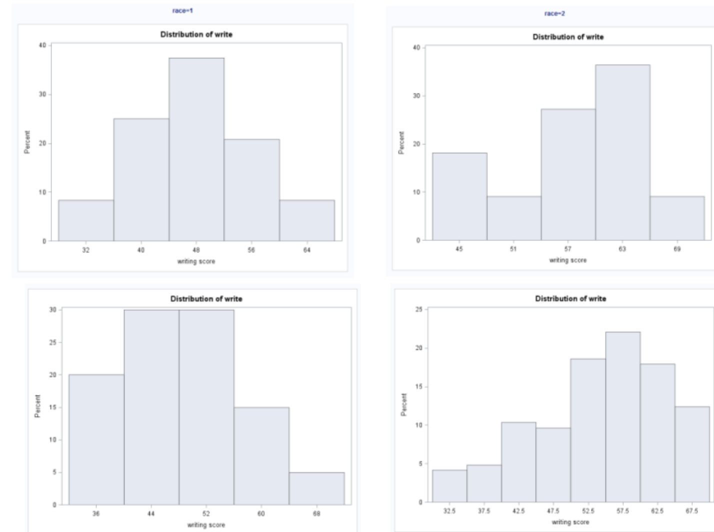
5. $Reject\ H_0$

6. There is strong evidence to suggest at the alpha = .05 level of significance (pvalue = .0002 from the Rank Sum Test) that the median writing score of US high school students with race = 1 is different than the median writing score of those of race = 4. This was an observational study and thus no causal inference can be deduced. The best estimate of the difference is the difference in sample medians: 45 – 57 = -12

4. There are 4 unique race categories. Test to see if the 1st race has a different mean writing score than the 4th race.

ALTERNATIVE SOLUITON: CONTRASTS





Level of race	N	write	
		Mean	Std Dev
1	24	46.4583333	8.27242232
2	11	58.0000000	7.89936706
3	20	48.2000000	9.32229924
4	145	54.0551724	9.17255819

The race = 4 histogram provides strong evidence (n = 145 and a considerable left skew) against normality. While the other histograms do not provide much evidence against normality, they are based on small sample sizes and may indeed be left skewed as the evidence suggests race = 4 is. However, it can be argued that even the smallest group n = 11 has a sufficient sample size to ensure the central limit theorem will provide normally distributed means. For this reason we will proceed with an ANOVA and t-tests for the planned analysis. In addition we will assume the standard deviations are equal although this is a risky assumption given the data.

4. There are 4 unique race categories. Test to see if the 1st race has a different mean writing score than the 4th race. ALTERNATIVE SOLUITON: CONTRASTS

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
Race 1 v. Race 4	1	1188.388371	1188.388371	14.59	0.0002

There is strong evidence at the alpha = .05 level of significance (pvalue = .0002 from a two sample t-test) to suggest that the mean writing score of race = 1 US high school students is different than that of those who are race = 4.

Compare the contrast to building your own F-table.

$\mu_1 \ \mu_2 \ \mu_3 \ \mu_4$

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	1914.15805	638.05268	7.83	<.0001
Error	196	15964.71695	81.45264		
Corrected Total	199	17878.87500			

$\mu_o \ \mu_2 \ \mu_3 \ \mu_o$

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	725.76967	362.88484	4.17	0.0169
Error	197	17153.10533	87.07160		
Corrected Total	199	17878.87500			

Source	DJ	SS	MS	F	Pr > F
Model	1	1188.388	1188.388	14.59	.0002
Error	196	15964.71695	81.45264		
Total	197	17153.10533			

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
Race 1 v. Race 4	1	1188.388371	1188.388371	14.59	0.0002

5. Use proc power to find the power of a test to detect a difference of 3 in the means if the two groups have standard deviations of 5 and 8 and the first group has a sample size of 20 and the second has a sample size of 40. Assume we want to use a Satterthwaite approximation (Wilcoxon test of difference of means / different standard deviations.)

```
proc power;
      twosamplemeans test=diff_satt
         meandiff = 3
         groupstddevs = 5 | 8
         groupweights = (1 2)
         ntotal = 60
         power = .;
   run;
```

Fixed Scenario Elements	
Distribution	Normal
Method	Exact
Mean Difference	3
Group 1 Standard Deviation	5
Group 2 Standard Deviation	8
Group 1 Weight	1
Group 2 Weight	2
Total Sample Size	60
Number of Sides	2
Null Difference	0
Nominal Alpha	0.05

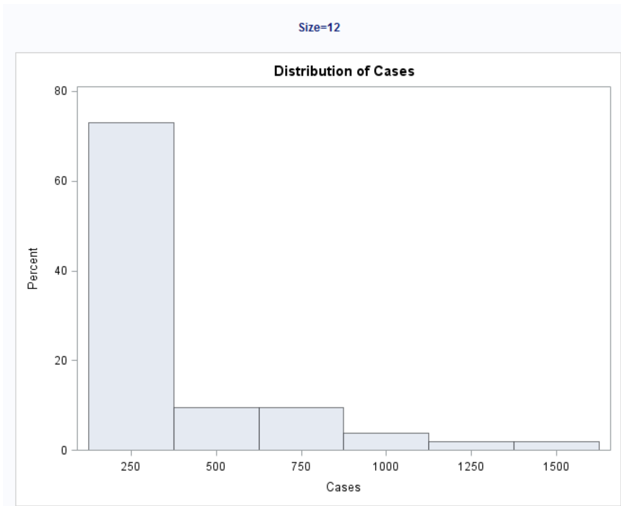
Computed Power	
Actual Alpha	Power
0.0498	0.415

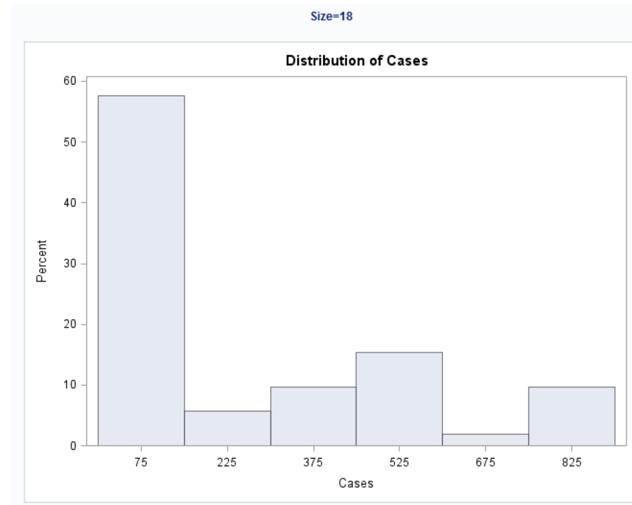
Beer Prices!!!

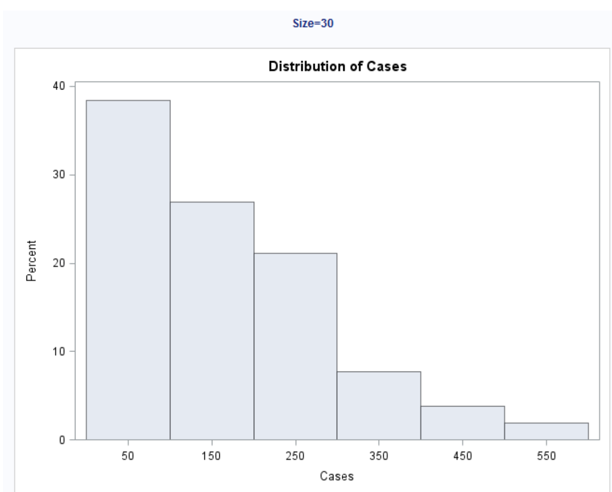
PRICE 18PK	CASES 18PK
14.10	439
18.65	98
18.65	70
18.65	52
18.65	64
18.65	72
18.65	47
18.73	85
18.75	59
18.75	63
18.75	57
18.75	54
13.87	404
14.27	380
18.76	65
18.77	40
13.87	456
14.14	176
18.76	61
18.72	91
18.76	59
18.76	83
18.74	41
18.75	47
18.75	84
18.75	85
18.75	116
13.79	544

The data file contains 52 weeks of average-price and total-sales records for three different carton sizes: 12-packs, 18-packs, and 30-packs. (This is real data, apart from some very minor adjustments for the 30-packs.) The data can be found in the HW folder on blackboard. We would like to perform an analysis to detect any differences in the mean sales between the 3 sizes of cases.

Beer Prices!!!



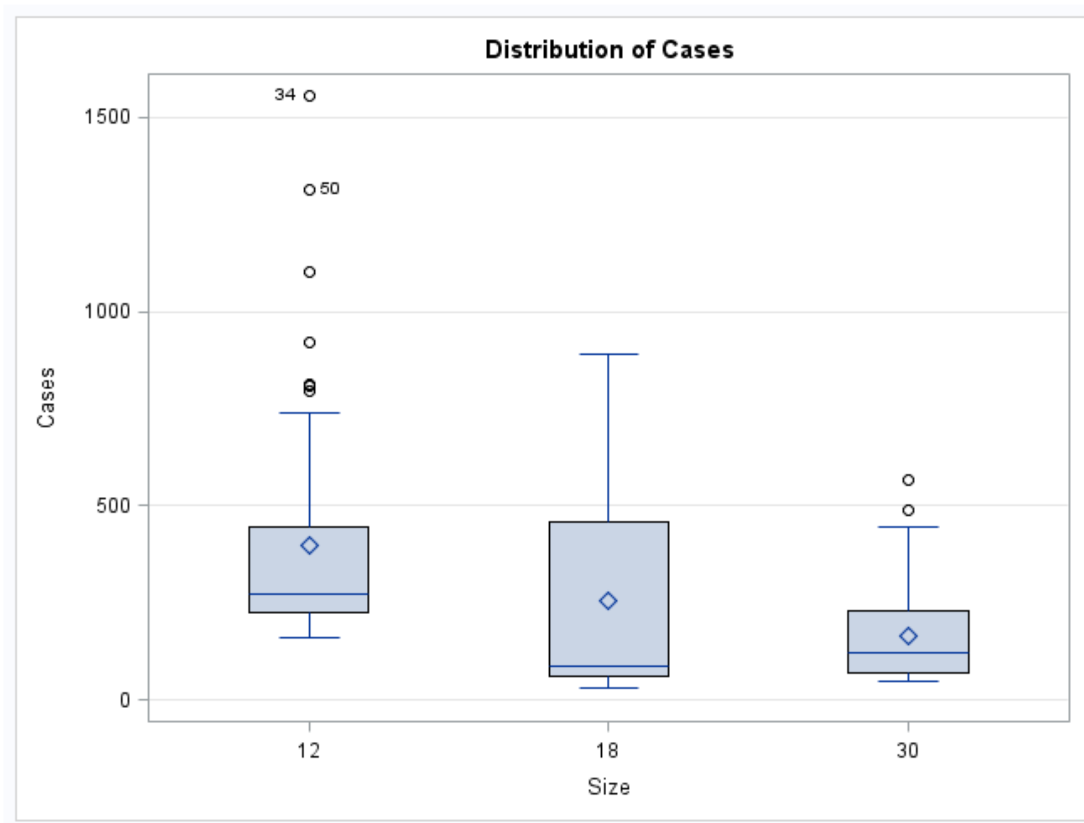




Clearly, there is strong visual evidence that the distribution of the amount of each size case sold is heavily right skewed. Looking closely at the units of the x-axis we can see that there is strong visual evidence that the standard deviations are quite different.

Beer Prices!!!!

Visual evidence of the right skew and heteroskedacity is also shown in the box plot:



Beer Prices!!!

We know that equality among variances is a crucial assumption of ANOVA thus we will seek a non-parametric solution: Kruskal-Wallis. Given the difference in standard deviations, the shapes cannot be said to be the same, so we will have to limit our tests to differences of distribution.

The NPAR1WAY Procedure

Wilcoxon Scores (Rank Sums) for Variable Cases Classified by Variable Size

Size	N	Sum of Scores	Expected Under H0	Std Dev Under H0	Mean Score
12	52	5530.50	4082.0	265.994971	106.355769
18	52	3579.50	4082.0	265.994971	68.836538
30	52	3136.00	4082.0	265.994971	60.307692

Average scores were used for ties.

Kruskal-Wallis Test	
Chi-Square	30.5811
DF	2
Pr > Chi-Square	<.0001

1. $H_0 : F_{twelve} = F_{eighteen}$
   $H_A : F_{twelve} \neq F_{eighteen}$

There is strong evidence at the alpha = .05 level of significance (pvalue < .0001 from the Kruskal -Wallis test) that at least of the distributions is different than the other two. Three separate rank sum tests will be conducted with a Bonferroni multiple comparison adjustment in order to test for these differences.

Beer Prices!!!

Wilcoxon Scores (Rank Sums) for Variable Cases Classified by Variable Size					
Size	N	Sum of Scores	Expected Under H0	Std Dev Under H0	Mean Score
18	52	2765.0	2730.0	153.814382	53.173077
30	52	2695.0	2730.0	153.814382	51.826923
Average scores were used for ties.					

Wilcoxon Two-Sample Test	
Statistic	2765.0000
Normal Approximation	
Z	0.2243
One-Sided Pr > Z	0.4113
Two-Sided Pr > |Z|	0.8225
t Approximation	
One-Sided Pr > Z	0.4115
Two-Sided Pr > |Z|	0.8230
Z includes a continuity correction of 0.5.	

Wilcoxon Scores (Rank Sums) for Variable Cases Classified by Variable Size					
Size	N	Sum of Scores	Expected Under H0	Std Dev Under H0	Mean Score
12	52	3267.50	2730.0	153.814792	62.836538
18	52	2192.50	2730.0	153.814792	42.163462
Average scores were used for ties.					

Wilcoxon Two-Sample Test	
Statistic	3267.5000
Normal Approximation	
Z	3.4912
One-Sided Pr > Z	0.0002
Two-Sided Pr > |Z|	0.0005
t Approximation	
One-Sided Pr > Z	0.0004
Two-Sided Pr > |Z|	0.0007
Z includes a continuity correction of 0.5.	

Wilcoxon Scores (Rank Sums) for Variable Cases Classified by Variable Size					
Size	N	Sum of Scores	Expected Under H0	Std Dev Under H0	Mean Score
12	52	3641.0	2730.0	153.816844	70.019231
30	52	1819.0	2730.0	153.816844	34.980769
Average scores were used for ties.					

Wilcoxon Two-Sample Test	
Statistic	3641.0000
Normal Approximation	
Z	5.9194
One-Sided Pr > Z	<.0001
Two-Sided Pr > |Z|	<.0001
t Approximation	
One-Sided Pr > Z	<.0001
Two-Sided Pr > |Z|	<.0001
Z includes a continuity correction of 0.5.	

Since there are three simultaneous tests, we will adjust the alpha level down to .05/2(3) = .05/6 = .008. Therefore at the alpha = .05 family wise significance level, there is strong evidence that the distribution of 12 pack cases sold is different that the distribution of both 18 and 30 packs sold (pvalue = .0005 and < .0001 respectively.)