Problem Solving and Computer Science Education: Why the Process of Deduction is important for U.S. High Schools

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February 17, 2014

Introduction

If you've ever read any of the pieces in my blog, I warn you that this piece is nothing like the existential babble I usually generate in the late hours of the night. Today, I want to talk about education, mathematics, and why so many people hated math in high school.

I know that it's probably a little hypocritical of me to be claiming that everyone hates math when I'm typesetting this on LaTeX, but bear with me for the moment.

"I hate math"

Think back to high school, or even middle school. Did you enjoy your high school mathematics courses? Chances are, you probably didn't enjoy every minute of Honors Precalculus or Geometry. In fact, when I ask most math or computer science majors why they chose math, almost always, their response is that the things they learned in university, or math competitions, or computer camp, was fascinating. There are seldom few who began ranting on about their love of memorizing integration by parts.

Most people, when asked why they dislike mathematics, will say, "I'm never going to need to know calculus in real life." And the truth is, for the vast majority of students who do not wish to be engineers or scientists, this is entirely true.

"But James", you argue. "Math isn't all about applications. Math is supposed to teach us how to think! No one cares about whether you know what a partial derivative or a one-to-one function is; they just want you to learn to reason quantitatively."

To that I respond with: I completely agree. Math is *supposed* to teach us how to think and reason. Math is *supposed* to allow us to reason quantitatively, or reason at all. Unfortunately, the systems we've ingrained into our society for what every high school graduate should learn isn't exactly doing that.

But math is the most applicative and concepts based class there is in high school! We can leave vocabulary memorization and biology terms and come to the awesome world of conceptual analysis. How can math, of all subjects, not teach us conceptual based learning?

The truth of the matter is, throughout the next few pages, I hope to convince you that the math they teach you in high school is actually one big set of flashcards in disguise. Yes, it's definitely more conceptualized than biology or spanish class. But for a subject that's supposed to be the crucial basis of teaching students to think critically, it actually falls quite short.

Not convinced? I'll quantify it for you.

Steps of Logic

Consider this:

$$7 * 5 =?$$

Any reasonable person would probably expect someone who has graduated high school to know how to do this problem. In fact, I will assume that things like this, times tables, as a somewhat inherent piece of knowledge, an axiom or fact that most people graduating high school should know. Disagree with me here if you wish. I do believe that this sort of basic arithmetic is a fair starting point.

Then consider the following problem:

Solve the following equation for x:

$$7 * 5 - x = 0$$

How long did it take you? An average high school student could probably deduce that the solution is $x = 35$. I will define this type of problem as taking 1 *step of logic*. It takes us about one cycle in our brain computers to do this.

In our heads, we think, well, I learned that with equations, I just move the x over and simplify. Since this is an equation, I should do that. And then we do that.

What about these ones?

Solve the following equations for x:

$$7 * 5 - x^2 = 0$$

$$7x - x^2 + 8 < 0$$

$$\int_0^1 cos(x)\, dx = 7 * 5$$

I will argue with you that these problems ***also take only one step of logic***.

This will take some convincing. How can something as complicated as an integral be considered only one step of logic?

Well, although we assume a higher base of knowledge, consider whether the instructions "move x to one side and then simplify" still work on the third equation. We may have to know what the weird symbol on the left means, or what the product rule is, or how to integrate by parts, but these are just $facts$. Overall, you see an equation, you will move x to one side and simplify the other side. As long as someone teaches you how to simplify that side, you need only make one deduction: that you must simplify it.

Yes, one could argue that the third one requires an acute knowledge of integration and fundamental theorem of calculus and whatever else, but ask yourself, are you making any logical *deductions* by doing this?

No, it's just pure computation. And unfortunately, that's what most classes in mathematics are in high school. Yes, word problems kind of improve this, give us that little "basis" or "application" we're looking for. But again, that increases the steps of logic at most about 1 level. We learn to read mathy word problems, and then convert them to equations, and solve.

They teach you how to simplify something a little bit more complicated every year. We learn to solve one bigger equation and one "bigger" problem. But our real deductive reasoning skills really haven't improved beyond a better sense of pattern matching. High school students of all levels every year are trained to be computers. And we already have *computers* for that. It's very understandable why students probably don't like this type of math. If my calculator can do something better than me, why should I learn it?

Instead of focusing on giving us a large breadth of material to apply the same concept of solving an equation toward, we should increase the depth at which we give problems. Teaching deductive skills, or, strengthening one's ability to take more logical steps at a time, is imperative to this approach.

Deduce, don't reduce

Let's take an example of a problem given to college students in a discrete math course:

James and Joe are playing a game involving nine coins in the center of a table, each worth a different amount of money between 1 and 9 dollars(the first coin is worth 1 dollar, second worth 2, up until 9). Every turn, a player may choose one coin to take from the center. Whoever obtains exactly three coins that add up to exactly 15 dollars wins. James goes first. Does someone always win? If so, who?

Consider how a student may try to solve this problem. He may start listing all of the possible ways to win the game. $9 + 5 + 1$? $7 + 2 + 6$?

Then, he may play the games in his head, and try to figure out whether there's a good way to always win.

Notice that this involves a whole lot more mental exercise than the previous problems we've seen. Also, note that it seems to require much less inherent background knowledge than previous problems.

On the surface, it looks like this problem has no real intrinsic value. Who would be playing such a game? What applications would this have in the *real* world?

The truth is, most students will learn more from this question than any calculus question you can possibly throw at them. It's true that many engineers would probably appreciate understanding what a hyperbolic sine function does, but not every high school student needs that kind of tool. Instead, this problem teaches students to think logically. It's what most of us call a puzzle. The reason it feels more interesting and more difficult is because the number of logical steps necessary to solve such a problem is much higher than the problems we've seen before.

It teaches the ability *deduction* rather than raw knowledge.

This immediately presents two benefits over the existing, memorization-based system:

1. First, it increases our ability to think critically. By increasing the number of logical steps necessary to solve a problem, and therefore increasing the *depth* rather than *breadth* of a problem, we are teaching students analytical skills in logical deduction that they can apply to almost every career choice.

2. Second, it makes doing problems more *interesting*. For most mathematically inclined students, problems like these are fascinating, exciting, and mysterious.

Obviously, it has some clear weaknesses as well. For one, the less applicative nature of the material may make students who are struggling lose interest quickly. “When are we ever going to *use* this?” they'll say. The answer, of course, is that they'll use the problem solving and analytical skills they gain from this every day. But it's harder to convince them of this.

Moreover, the material is considerably more challenging to process than calculus. The ability to argue and prove is a lot more difficult to master than the ability to compute. When a problem requires more logical steps to get there, a person doing the problem is more likely to step in the wrong direction. This causes a lot of effort to be sometimes wasted on hard problems, when a student goes on a misconception. On the other hand, the relative difficulty of the problems encourage collaboration and cooperation in solving problems, another skill seldom promoted in today's cutthroat high schools.

The magic solution

Let's skip ahead a bit to the solution to the first problem, which I bet at least one person reading this may be pondering. If we put the numbers 1 through 9 into a ”magic” square, a 3 by 3 table where every row or column adds to the same number(in this case 15), we see something interesting:

6	1	8
7	5	3
2	9	4

We represent ”taking a coin” as taking control of one of these squares. Notice that since every single row, column or diagonal adds to fifteen, controlling any row, column or diagonal wins the game.

Thus, this becomes something most of us are familiar with: tic tac toe. Since James and Joe are essentially playing tic tac toe, we know that since no one actually can force a win in the game, there's no automatically winning strategy.

The solution to this problem may seem to have come right out of the blue. That's what it's like being told an answer to the problems: arbitrary. But when a student really sits down and discovers the solution on his own, he will attempt to relate a foreign problem to something he already understands. In this case, the proper technique was tic tac toe.

Yes, getting to the answer is hard, but that's the point. In going from not knowing the solution to figuring out the solution, the student has actually *learned.* This difficulty, this logical leap, is what we should be training students to accomplish.

Unlike equations, solving these problems can be very rewarding: the process of struggling and reasoning and taking logical steps is difficult, and reaching the end is quite satisfying.

An alternative

I understand that the benefits of teaching calculus are way more than I probably play them out to be. But the point still stands: students who learn to deduce logic rather than reduce equations gain a very different, yet extraordinarily useful form of insight. Teaching deductive reasoning in math classes in high schools will lead to more students enjoying math and ready to learn more once they get to college. Furthermore, the problem solving skills they learn in these courses are useful in an innumerable set of environments. Ultimately, these benefits can not and should not be overlooked.