MAST10006 Calculus 2, Semester 2, 2020
Assignment 4

School of Mathematics and Statistics, The University of Melbourne

- Submit your assignment solutions online in Canvas before **6pm, Monday 14 September 2020**
- Submit your solutions as a single PDF file with the pages in the right order and correct orientation. You may be penalised a mark if you do not.
- This assignment is worth 2.22% of your final MAST10006 mark.
- Answer all questions below. Of these questions, one will be chosen for marking.
- Marks may be awarded for:
  - Correct use of appropriate mathematical techniques
  - Accuracy and validity of any calculations or algebraic manipulations
  - Clear justification or explanation of techniques and rules used
  - Use of correct mathematical notation and terminology
- You must use methods taught in MAST10006 Calculus 2 to solve the assignment questions.

1. Mathematical models are an important tool in understanding and managing infectious disease epidemics. According to the SIR model (first studied by Kermack & McKendrick in 1927), the expected incidence of a disease in the early stages of an epidemic is approximated by

$$f(t) = k\,\mathrm{sech}^2\left(\alpha t - \phi\right), \quad t \geq 0$$

where k, α and ϕ are positive constants and t is the time in days since the start of the disease outbreak.

The *incidence* of a disease is the rate of new cases of the disease per unit time, typically measured as the number of new cases in a single day. $f(t)$ thus represents the expected number of new cases at day t.

For simplicity, assume the unrealistic but simple values $k = 20000, \alpha = 2, \phi = 30$, which gives

$$f(t) = 20000\,\mathrm{sech}^2\left(2t - 30\right), \quad t \geq 0.$$

(a) Sketch by hand the graph of $y = f(t)$. The graph should have the correct shape, exact value labels for any intercepts, labels for asymptotes and stationary points.

(b) Suppose that 1% of the new disease cases on any given day require hospitalisation, and that the hospital system's capacity can handle at most 10 new hospitalisations per day. Find the exact value of t for which the hospital system is first expected to be over capacity. Then use any device invented after the 19th century to find the day for which this occurs.

(c) The expected total reported cases at time t days is given by

$$T(t) = \int_0^t f(u)\,du.$$

Find an expression for $T(t)$ in terms of t.

(d) Suppose that the population consists of 1 million people. According to this model, how many people are expected to remain uninfected by the disease in the long term?

2. Evaluate the following integrals. State which method you use, document each step carefully and keep track of any assumptions that you make.

   (a) $\displaystyle\int 6x \sin(x^2)\, \mathrm{d}x.$

   (b) $\displaystyle\int \frac{x^2}{\sqrt{9x^2 - 1}}\, \mathrm{d}x.$

   (c) $\displaystyle\int e^{-x} \cos(x)\, \mathrm{d}x.$ Use integration by parts.

   (d) $\displaystyle\int e^{-x} \cos(x)\, \mathrm{d}x.$ Use the complex exponential.

   (e) Comment briefly on which of (c) and (d) you found simpler, giving a brief reason.

End of assignment