Calculus 2 Written Assignment 3

1. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$

a) $S_0 = \frac{(-1)^0}{0!} = 1$ $\qquad S_1 = 0$

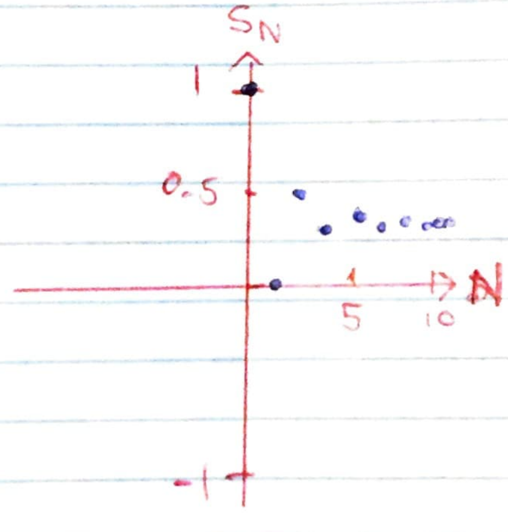
$S_2 = 0.5$ $\qquad S_3 \approx 0.333$

$S_4 = 0.375$ $\qquad S_5 \approx 0.367$

$S_6 \approx 0.368$ $\qquad S_7 \approx 0.368$

$S_8 \approx 0.368$ $\qquad S_9 \approx 0.368$

$S_{10} \approx 0.368$



The series appears to converge to approximately 0.368

b) I would like to apply the ratio test but the series does not satisfy the condition that it must be a positive term series.

c) Ratio test from Wikipedia:

$$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(-1)^{n+1}}{n+1!} \cdot \frac{n!}{(-1)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{-1}{(n+1)n!} \cdot n! \right|$$

$$= \lim_{n\to\infty} \left| \frac{-1}{n+1} \right|$$

$$= \lim_{n\to\infty} \left| \frac{-\frac{1}{n}}{1+\frac{1}{n}} \right| \quad \left(\times \frac{1}{n}\right)$$

$$= \left| \frac{0}{1+0} \right| \quad , \text{ standard limits and limit laws}$$

$$= 0$$

$\therefore L<1$ so the series is convergent by the ratio test.

2.

a) Y-intercept

$x = 0$

$$f(x) = \left(\frac{e^0 - e^0}{e^0 + e^0}\right)^2 - \frac{5}{\frac{1}{2}(e^0 + e^0)}$$

$$= 0 - \frac{5}{\frac{1}{2} \cdot 2} = -5$$

x-intercept

$$f(x) = \frac{\sinh^2(x)}{\cosh^2(x)} - \frac{5}{\cosh(x)} = 0$$

$$\sinh^2(x) - 5\cosh(x) = 0$$
$$\cosh^2(x) - 1 - 5\cosh(x) = 0$$

let $\cosh(x) = z \Rightarrow z^2 - 5z - 1 = 0$

$$z = \frac{5 \pm \sqrt{25 - 4 \cdot 1 \cdot -1}}{2} = \frac{5 \pm \sqrt{29}}{2}$$

$$\therefore x = \pm \operatorname{arccosh}\left(\frac{5 + \sqrt{29}}{2}\right)$$ as $\cosh(x) > 1$

and $\tanh^2(x) = \tanh^2(-x)$
and $\operatorname{sech}(x) = \operatorname{sech}(-x)$

b) stationary point(s) occur when $f'(x)=0$

$$\frac{d}{dx}(\tanh^2(x)) - \frac{d}{dx}(5\operatorname{sech}(x))$$

$$= \frac{d}{dx}(\cosh^2(x)) - 5\frac{d}{dx}(\cosh(x)) - \frac{d}{dx}(1)$$

$$= 2\cosh x \sinh x - 5\sinh x$$

$$\cancel{\Rightarrow \sinh(2x) - 5\sinh x = 0}$$

$$\cancel{\sinh(2x) = 5\sinh(x)}$$

$$\Rightarrow \sinh(x)(2\cosh x - 5) = 0$$

$$\sinh(x) = 0$$

$$x = 0$$

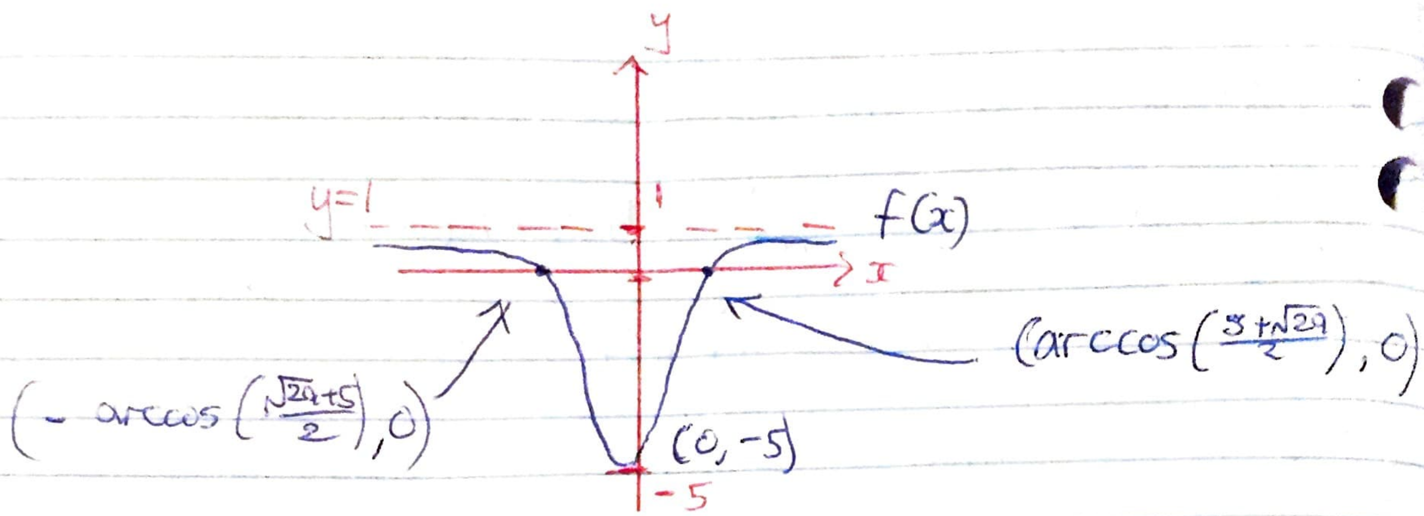
stationary point at $(0, -5)$

c) $f(x)$ is an even function as

$$\tanh^2(x) - 5\operatorname{sech}(x) = \tanh^2(-x) - 5\operatorname{sech}(-x)$$

d) The function is continuous for all $x \in \mathbb{R}$ according to continuity theorems 1 and 3 which state that the addition of two continuous functions will give a continuous function and that hyperbolic functions including $\tanh^2(x)$ (preserves continuity by theorem 1 $f\circ g$) and $\operatorname{sech}(x)$ are continuous.

e)



7.

$$e^{-t+1}\cos t = e^{-t}e^{1}\,\mathrm{Re}\left(e^{it}\right)$$

$$= e^{1}\,\mathrm{Re}\left(e^{(-1+i)t}\right), \quad \text{since } e^{-t} \in \mathbb{R}$$

$$\frac{d^{61}}{dt^{61}}\left(\cancel{e}\ \mathrm{Re}\left(e^{(-1+i)t}\right)\right)$$

$$= e\,\mathrm{Re}\left[(-1+i)^{61}e^{(-1+i)t}\right], \quad \frac{d}{dt}\left(e^{kt}\right) = ke^{kt}$$

$$(-1+i)^{61} = \left(\sqrt{2}\,e^{3\pi i/4}\right)^{61}$$

$$= 2^{30}\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$$

$$= \frac{2^{30}\sqrt{2}}{2} - \frac{2^{30}\sqrt{2}\,i}{2} = 2^{30} - 2^{30}i$$

Hence $\frac{d^{61}}{dt^{61}}\left(e^{-t+1}\cos t\right) = e\,Re\left[(2^{30}-2^{30}i)\left(e^{(-1+i)t}\right)\right]$

$= e\,Re\left[(2^{30}-2^{30}i)\,e^{-t}(\cos t + i\sin t)\right]$

$= e\,Re\left[(2^{30}e^{-t}-2^{30}ie^{-t})(\cos t+i\sin t)\right]$

$= e\left(2^{30}e^{-t}\cos t+2^{30}e^{-t}\sin t\right)$

$= 2^{30}e^{-t+1}\cos t + 2^{30}e^{-t+1}\sin t$