Assignment 6 Due: 6:00PM, Friday 15 May.

Name: James La Fontaine

Student ID: 1079860

Explainer: Question 1 should be completed in **WebWork** by 6:00PM, Friday 15 May. WebWork should be accessed via Assignment 6 WebWork in the Assignments panel of the MAST10005 LMS Site.

You should upload a scan of neatly presented solutions to Question 2 in **Gradescope**, which should be accessed via Assignment 6 Written Part in the Assignments section of the MAST10005 LMS Site. Please do not include your answers to Question 1 in your solutions to the written part.

1. You should complete this question in WebWork by 6:00PM, Friday 15 May. It will test your ability to calculate implicit derivatives. Completing Question 1 *before* you attempt Question 2 will make Question 2 easier because you will have already checked that your formula for the implicit derivative of the curve $C = \{(x, y) \in \mathbb{R}^2 \mid x^4 + y^2 = 4x^2 + 3y\}$ correct.

2. Consider the curve

$$C = \{(x, y) \in \mathbb{R}^2 \mid x^4 + y^2 = 4x^2 + 3y\}$$

(a) Find all intercepts of C.

$x^4 + y^2 = 4x^2 + 3y$ $\qquad \frac{dy}{dx} = \frac{4x^3 - 8x}{3 - 2y}$

x-intercepts where $y = 0$:

$\Leftrightarrow x^4 = 4x^2$

$x^4 - 4x^2 = 0$

$x^2(x^2 - 4) = 0$

$x^2 = 0 \qquad x^2 - 4 = 0$

$x = 0 \qquad x^2 = 4$

$(0,0) \qquad x = \pm 2$

$(\pm 2, 0)$

y-intercepts where $x = 0$:

$\Leftrightarrow y^2 = 3y$

$y^2 - 3y = 0$

$y(y-3) = 0$

$y = 0 \qquad y - 3 = 0$

$(0,0) \qquad y = 3$

$(0,3)$

(b) Find the points in C where the tangent line is *horizontal*. Be careful to check the conditions given in Theorem 3.34. [Hint: there are 6 points in total.]

$\Rightarrow 4x^3 - 8x = 0$ (a) and $3 - 2y \neq 0$ (b)

$\Leftrightarrow 4x(x^2 - 2) = 0$ $\Leftrightarrow y \neq \frac{3}{2}$

$4x = 0$ $x^2 - 2 = 0$

$x = 0$ $x^2 = 2$

$x = \pm\sqrt{2}$

$x^4 + y^2 = 4x^2 + 3y$ (c)

(sub $x=0$ into (c))

from (a)

$y = 0, 3 \neq \frac{3}{2}$

(sub $x = \sqrt{2}$ into (c))

$(\sqrt{2})^4 + y^2 = 4(\sqrt{2})^2 + 3y$

$4 + y^2 = 8 + 3y$

$y^2 - 3y = 4$

$y^2 - 3y - 4 = 0$

$(y-4)(y+1) = 0$

$y = 4, -1 \neq \frac{3}{2}$

(sub $x = -\sqrt{2}$ into (c))

$(-\sqrt{2})^4 + y^2 = 4(-\sqrt{2})^2 + 3y$

$4 + y^2 = 8 + 3y$

$\therefore y = 4, -1$

$\therefore$ There are horizontal tangent lines at the points $(0,3), (0,0), (-\sqrt{2}, -1), (-\sqrt{2}, 4), (\sqrt{2}, 4)$, $(\sqrt{2}, -1)$

(c) Find the points in C where the tangent line is *vertical*. Be careful to check the conditions given in Theorem 3.34.

$\Rightarrow 4x^3 - 8x \neq 0$ (a) and $3 - 2y = 0$ (b)

$\Leftrightarrow x \neq 0, \pm\sqrt{2}$ $\quad\Leftrightarrow y = \frac{3}{2}$

$x^4 + y^2 = 4x^2 + 3y$ (c)

(sub $y = \frac{3}{2}$ into (c))

$x^4 + \left(\frac{3}{2}\right)^2 = 4x^2 + 3\left(\frac{3}{2}\right)$

$x^4 + \frac{9}{4} = 4x^2 + \frac{9}{2}$

$x^4 - 4x^2 = \frac{9}{4}$

$x^4 - 4x^2 - \frac{9}{4} = 0$

$\frac{1}{4}(4x^4 - 16x - 9) = 0$

$\frac{1}{4}(2x^2+1)(2x^2-9) = 0$

$2x^2 = -1$ $\qquad 2x^2 = 9$

$x^2 = -\frac{1}{2}$ $\qquad x^2 = \frac{9}{2}$

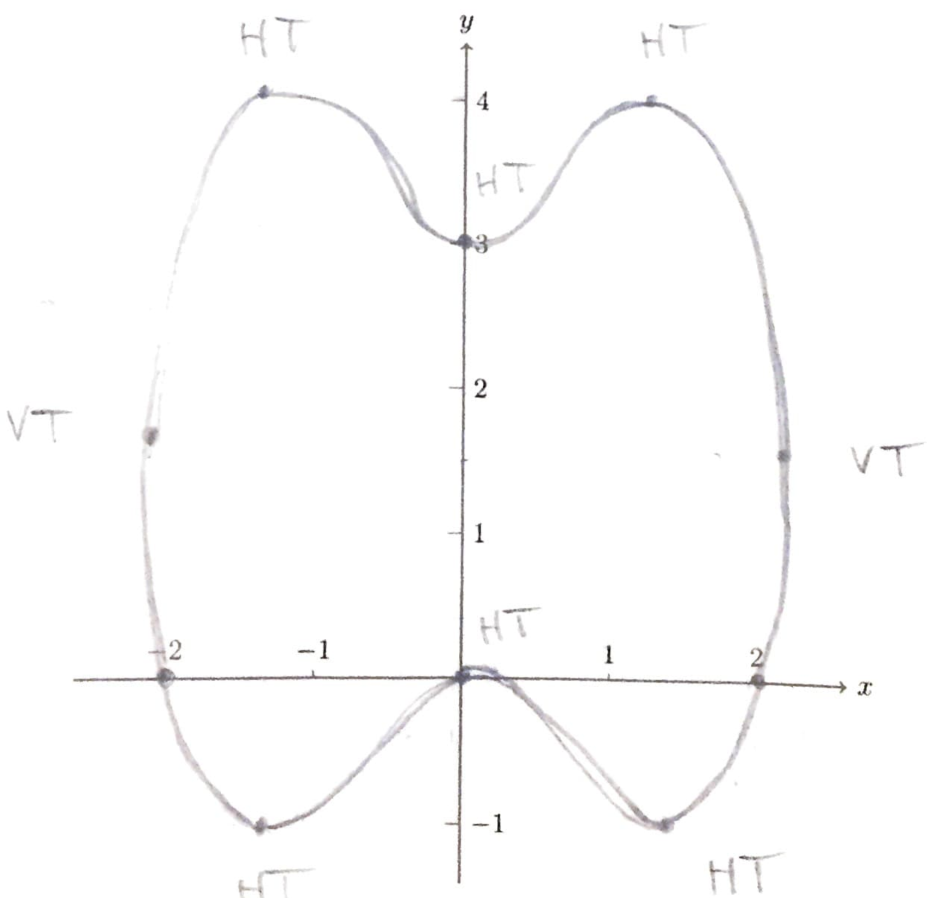
$x = \pm\frac{i}{\sqrt{2}}$ $\qquad x = \pm\frac{3}{\sqrt{2}}$

$x = \pm\frac{3}{\sqrt{2}}$ as $x \in \mathbb{R}$

$\neq 0, \pm\sqrt{2}$

$\therefore$ There are vertical tangent lines at $\left(-\frac{3}{\sqrt{2}}, \frac{3}{2}\right), \left(\frac{3}{\sqrt{2}}, \frac{3}{2}\right)$

(d) Sketch C on the axes. Label all the points you found in (b) and (c). Abbreviations HT for Horizontal tangent and VT for Vertical tangent will save space in your labelling.



Extra challenge problem [No marks]:
Find $a > 0$ and formulas for functions $f_1 : [-a, a] \longrightarrow \mathbb{R}$ and $f_2 : [-a, a] \longrightarrow \mathbb{R}$ such that C is the union of the graphs of f_1 and f_2.

Important Note: If you decide not to do the challenge problem, you should still include this page in your solutions. If you don't do this, Gradescope will not allow you to submit.

$a = \frac{3}{\sqrt{2}}$ as this is where the vertical tangents are and therefore where the curve must be split into functions.

$y = y(x)$

$$y^2 - 3y + x^4 - 4x^2 = 0$$

$$y = \frac{3 \pm \sqrt{9 - 4(1)(x^4 - 4x^2)}}{2}$$

$$y = \frac{3 \pm \sqrt{9 - 4(x^4 - 4x^2)}}{2}$$

$$y = \frac{3 \pm \sqrt{9 - 4x^4 + 16x^2}}{2} \quad \therefore \quad f_1(x) = \frac{3 + \sqrt{-4x^4 + 16x^2 + 9}}{2}$$

$$f_2(x) = \frac{3 - \sqrt{-4x^4 + 16x^2 + 9}}{2}$$

Assignment Information

This assignment is worth $\frac{20}{9}$% of your final MAST10005 mark.

Full working should be shown in your solutions to Question 2. There will be 1 mark overall for correct mathematical notation.

Full solutions to the assignment will be uploaded to the LMS site approximately 3 days after the assignment is due.