2

(a) $\theta = \frac{\pi}{2}$

(b) $\overrightarrow{OP} = \vec{v}_{11} = \left(-\frac{4}{3}, \frac{4}{3}, \frac{8}{3}\right) = -\frac{4}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{8}{3}\hat{k}$

$r = \|\overrightarrow{PC}\| = \sqrt{\left(\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = \sqrt{\frac{3}{9}} = \sqrt{\frac{1}{3}} = \frac{\sqrt{3}}{3}$

3 $h: \mathbb{C} \to \mathbb{R}$

$h(z) = Re(z) + Im(z)$ $\qquad f(z) = \log(h(z))$

$g(x) = \log(x)$

(a) $h: \mathbb{C} \to \mathbb{R}$ is surjective if for every $b \in \mathbb{R}$ there exists at least one $z \in \mathbb{C}$ such that $h(z) = b$

Take any $b \in \mathbb{R}$.

$h(z) = b$

$Re(z) + Im(z) = b$

For any $z = x + iy \in \mathbb{C}$:

$Re(z) = x, \quad x \in \mathbb{R}$ $\qquad Im(z) = y, \quad y \in \mathbb{R}$

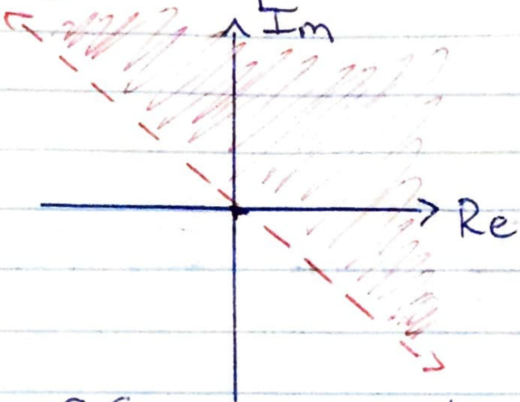
Therefore $b = Re(z) + Im(z) \in \mathbb{R}$

This completes the proof that h is surjective. $\square$

(b) For f to be defined, $h(z) > 0$

$$\therefore \operatorname{Re}(z) + \operatorname{Im}(z) > 0$$

$$\therefore \operatorname{dom}(f) = \{ z \in \mathbb{C} \mid \operatorname{Re}(z) + \operatorname{Im}(z) > 0 \}$$



(c) range f = $f(\text{range } h \cap \text{dom } g)$

$$= f(\mathbb{R} \cap (0, \infty))$$

$$= f(0, \infty) = \mathbb{R}$$

(4) $r_1(t) = t\cos(t)\hat{i} + t\sin(t)\hat{j}$

$r_2(t) = \frac{t}{\sqrt{2}}\hat{i} + \frac{t}{\sqrt{2}}\hat{j}$

(a) $t\cos(t) = \frac{t}{\sqrt{2}}$ ① $\Rightarrow \cos(t) = \frac{1}{\sqrt{2}} \Rightarrow t = \frac{\pi}{4}, \frac{7\pi}{4}$ $\quad t \in [0, 2\pi]$

$t\sin(t) = \frac{t}{\sqrt{2}}$ ② $\Rightarrow \sin(t) = \frac{1}{\sqrt{2}}$

$\sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$ ✓ from ①

$\sin(\frac{7\pi}{4}) = -\frac{1}{\sqrt{2}}$ ✗

The particles collide at $t = 2k\pi + \frac{\pi}{4}$, $k \in \mathbb{Z} \cap k \geq 0$ as $t \geq 0$

$$\therefore A = \{ 2k\pi + \frac{\pi}{4} \mid k \in \mathbb{Z} \cap k \geq 0 \}$$

(b) from $r_2(t)$

$x = \frac{t}{\sqrt{2}} \Rightarrow t = \sqrt{2}x$ $\quad$ $t = 2k\pi + \frac{\pi}{4}, k \in \mathbb{Z} \cap k \geq 0$

$y = \frac{t}{\sqrt{2}} \Rightarrow t = \sqrt{2}y$

$y = x$

$\therefore B = \left\{ \left(\frac{2k\pi + \frac{\pi}{4}}{\sqrt{2}}, \frac{2k\pi + \frac{\pi}{4}}{\sqrt{2}} \right) \middle| k \in \mathbb{Z} \cap k \geq 0 \right\}$