Assignment 1 (Challenges 5 & 6)

Graded

Student

James La Fontaine

Total Points

2 / 4 pts

Question 1

Challenge 5A

0 / 0.5 pts

Marks

✔ **+ 0 pts** Little understanding shown; or significant issues with interpretation; or no interpretation given at all

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Issues identified (for feedback only)

Tutors, feel free to add more if you find any

✔ **+ 0 pts** Interpretation does not make F false
NB: No need to show two interpretations (one making F true, one false), it's not an error if given however.

1 What's this here? We can't just pick a value for x

2 Great to see this level of detail, but the initial description would suffice

3 Cannot pull quantifier in like this. $\exists$ is over whole $\implies$, flips to $\forall$ if you do pull it in

Question 2

Challenge 5B

1.5 / 1.5 pts

Marks

✔ **+ 1.5 pts** Correct resolution proof

Question 3

Challenge 6A

0 / 1 pt

Marks

✔ **+ 0 pts** Little understanding shown; or significant issues with proof

4 Good

5 Incomplete

6 Not equivalence if you're removing quantifiers.

Question 4

Challenge 6B

0.5 / 1 pt

Marks

+ 0.5 pts Some non-trivial issues but demonstrates understanding
1-2 major issues, or >1 minor issue

Major Issues identified (for feedback only)

Tutors, feel free to add more if you find any

+ 0 pts Substitution for x not identified

7 What is n? There is a unifier for it.

8 Not a bad conclusion from the proof you've given, but you missed the crucial unifier {n -> a}.

9 Not strictly true. Pay attention to the implication in 6.6.

10 Not an MGU

TASK 5A

$I_D = \{1, 2, 3\}$ $\quad F: (\forall x\,(Q(x))) \vee \exists x((\forall y\, R(y,x) \vee Q(x)) \Rightarrow \exists z \forall y\; P(z,y))$

Predicates:

$Q(x)$: x is an odd number

$Q(x)$	
1	t
2	f
3	t

$P(z,y): z = y$

$P(z,y)$	1	2	3
1	t	f	f
2	f	t	f
3	f	f	t

$R(y,x): y > x$

$R(y,x)$	1	2	3
1	f	f	f
2	t	f	f
3	t	t	f

Valuation: $\sigma(x) = 1$, $\sigma(z) = 2$

$Q(2) = f$ so $\forall x\; Q(x)$ evaluates to f

$\sigma(x) = 1$ so $\exists x((\forall y\; R(y,x) \vee Q(x))$ evaluates to t as $Q(1) = t$

$\exists z\, \forall y\, (P(z,y))$ evaluates to f as there is no number z in I_D which is equal to all other numbers

$f \vee (t \Rightarrow f)$ evaluates to f so F is non-valid.

TASK 5B

Show that $F \vee \neg G$ is valid $\equiv$ show that $\neg F \wedge G$ is unsatisfiable

$$\neg((\forall x\, Q(x)) \vee \exists x((\forall y\, R(y,x) \vee Q(x)) \Rightarrow \exists z \forall y\, P(z,y)))$$
$$\wedge (\exists x \forall y (P(x,y) \vee (\exists z\, R(y,z) \Rightarrow \forall w\, Q(w))))$$

① Remove $\Rightarrow$

$$\neg((\forall x\, Q(x)) \vee \exists x(\neg(\forall y\, R(y,x) \vee Q(x)) \vee \exists z \forall y\, P(z,y)))$$
$$\wedge (\exists x \forall y (P(x,y) \vee (\neg(\exists z\, R(y,z)) \vee \forall w\, Q(w))))$$

② Push negations in

$$((\exists x\, \neg Q(x)) \wedge \forall x((\forall y\, R(y,x) \vee Q(x)) \wedge \forall z \exists y\, \neg P(z,y)))$$
$$\wedge (\exists x \forall y (P(x,y) \vee (\forall z\, \neg R(y,z)) \vee \forall w\, Q(w)))$$

③ Standardize bound variables apart

$$((\exists x\, \neg Q(x)) \wedge \forall u((\forall y\, R(y,u) \vee Q(u)) \wedge \forall z \exists v\, \neg P(z,v)))$$
$$\wedge (\exists r \forall t (P(r,t) \vee (\forall q\, \neg R(t,q)) \vee \forall w\, Q(w)))$$

④ Remove $\exists$ $\quad x \mapsto a,\ v \mapsto f(z,u),\ r \mapsto b$

$$((\neg Q(a)) \wedge \forall u((\forall y\, R(y,u) \vee Q(u)) \wedge \forall z\, \neg P(z, f(z,u)))$$
$$\wedge (\forall t (P(b,t) \vee (\forall q\, \neg R(t,q)) \vee \forall w\, Q(w)))$$

⑤ + ⑥ Remove $\forall$ and convert to CNF

$$\neg Q(a) \wedge (R(y,u) \vee Q(u)) \wedge \neg P(z, f(z,u))$$
$$\wedge (P(b,t) \vee \neg R(t,q) \vee Q(w))$$

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Question assigned to the following page: 2

$\{\neg Q(a)\}, \{R(y,u), Q(u)\}, \{\neg P(z, f(z,u))\},$
$\{P(b,t), \neg R(t,q), Q(w)\}$

$\neg Q(a) \qquad P(b,t), \neg R(t,q), Q(w)$

$w \mapsto a$

$P(b,t), \neg R(t,q) \qquad R(y,u), Q(u)$

$t \mapsto y, q \mapsto u$

$P(b,y), Q(u) \qquad \neg P(z, f(z,u))$

$z \mapsto b,\ y \mapsto f(b,u)$

$Q(u)$

$u \mapsto a$

$\perp$

Therefore $\neg(F \vee \neg G)$ is unsatisfiable and so $F \vee \neg G$ is valid

Question assigned to the following page: 3

TASK 6A

Show $(\forall x\, P(a,x,x)) \wedge (\forall x \forall y \forall z(\neg P(x,y,z) \vee P(s(x),y,s(z))))$
$\wedge\ (\forall x \forall y \forall z(\neg P(x,y,z) \vee P(y,x,z)))$
$\wedge\ (\forall x \exists y(\neg E(x) \vee P(y,y,x)))$
$\wedge\ (\forall x \forall y(\neg P(y,y,x) \vee E(x)))$
$\wedge\ \neg(\forall x\ (\neg E(x) \vee E(s(s(x)))))$ is unsatisfiable

$\forall x\ P(a,x,x) = \{P(a,x,x)\}$ (6.1)

$\forall x \forall y \forall z(\neg P(x,y,z) \vee P(s(x),y,s(z)))$

$= \{\neg P(w,y,z), P(s(w),y,s(z))\}$ (6.2)

$\forall x \forall y \forall z(\neg P(x,y,z) \vee P(y,x,z))$

$= \{\neg P(q,r,t), P(r,q,t)\}$ (6.3)

$\forall x \exists y(\neg E(x) \vee P(y,y,x))$ $\quad y \mapsto f(u)$

$= \{\neg E(u), P(f(u),f(u),u)\}$ (6.4)

$\forall x \forall y(\neg P(y,y,x) \vee E(x))$

$= \{\neg P(v,v,p), E(p)\}$ (6.5)

$\neg(\forall x(\neg E(x) \vee E(s(s(x)))))$ $\quad x \mapsto b$

$= \{E(b)\}, \{\neg E(s(s(b)))\}$ ¬(6.6)

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Question assigned to the following page: [3]

$E(b)$ $\qquad$ $\neg E(u), P(fu, fu, u)$

$u \mapsto b$

$P(fb, fb, b)$

?

$\neg E(s(s(b)))$ $\qquad$ $\neg P(v, v, p), E(p)$

$p \mapsto s(s(b))$

$\neg P(v, v, s(s(b)))$

?

TASK 6B

clauses from 6A

$\{P(a,x,x)\}$ (6.1)

$\{\neg P(w,y,z),\ P(s(w),y,s(z))\}$ (6.2)

$\{\neg P(q,r,t),\ P(r,q,t)\}$ (6.3)

$\{\neg E(u),\ P(f(u),f(u),u)\}$ (6.4)

$\{\neg P(v,v,p),\ E(p)\}$ (6.5)

$\{\neg E(0),\ E(s(s(0)))\}$ (6.6)

$\{\neg E(s(s(s(n))))\}$ $\neg$ (6.7)

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Question assigned to the following page: 4

$\neg E(o), E(s(s(o)))$ $\quad\quad$ $\neg P(v, v, p), E(p)$

$p \mapsto o$

$E(s(s(o))), \neg P(v, v, o)$

$\neg E(s(s(s(n))))$

$o \mapsto s(n)$

$\neg P(v, v, s(n))$ $\quad\quad$ $\neg P(w, y, z), P(s(w), y, s(z))$

$z \mapsto n,$
$v \mapsto s(w),$
$y \mapsto s(w),$

$P(a, x, x)$ $\quad\quad$ $\neg P(w, \cancel{y}\, s(w), \cancel{z}\, n)$

$\cancel{w \mapsto a}, \cancel{y \mapsto x}, \cancel{z \mapsto x}$

$w \mapsto a, x \mapsto s(w), n \mapsto s(w)$

$\perp$

Therefore 6.7 is a logical consequence of axioms (6.1)–(6.5) and theorem (6.6)

$p \mapsto o \mapsto s($)

p is any natural number such that it is the addition of 2 equal natural numbers and is therefore an even number under (6.5). p can be mapped to o where o is any natural even number implying $s(s(o))$ (or $o+2$) is also an even number under (6.6). o can be mapped to $s(n)$ where n is any number which satisfies $E(s(s(s(n))))$ (i.e. any odd number). (6.7) is satisfied as $s(s(s(n)))$ can be rewritten as $s(s(o))$ which we know to be an even number from (6.6). It can be concluded that all successors of odd numbers are even numbers in $\mathbb{N}$