MAST10006 Calculus 2, Semester 2, 2020

Assignment 3

School of Mathematics and Statistics, The University of Melbourne

- Submit your assignment solutions online in Canvas before **6pm, Monday 7 September 2020**
- This assignment is worth 2.22% of your final MAST10006 mark.
- Answer all questions below. Of these questions, one will be chosen for marking.
- Marks may be awarded for:
  - Correct use of appropriate mathematical techniques
  - Accuracy and validity of any calculations or algebraic manipulations
  - Clear justification or explanation of techniques and rules used
  - Clear communication of mathematical ideas through diagrams
  - Use of correct mathematical notation and terminology
- You must use methods taught in MAST10006 Calculus 2 to solve the assignment questions.

1. Consider the series
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$
   (a) Calculate the first 11 partial sums, S_N for $N \in \{0, 1 \ldots, 10\}$. You can write the values correct to 3 decimal places.
   Sketch the partials sums on a graph with N on the horizontal axis and S_N on the vertical axis. Hand draw on grid paper or use an app of your choice such as Desmos or Mathematica. Make an educated guess about its convergence (or divergence) behaviour. You don't need to justify your guess.

   (b) Which test from our class would you like to apply to test the convergence of this series, and what condition gets in the way?

   (c) Find that test on Wikipedia. The test on Wikipedia will have exactly the same name, but there is a crucial difference compared to the way it is stated in class. State a version of the test from Wikipedia, and use it to test convergence of the series.

2. Consider the function
$$f(x) = \tanh^2(x) - 5\,\mathrm{sech}(x)$$
   (a) Find the axis intercepts of the graph $y = f(x)$.

   (b) Find the stationary points of $y = f(x)$.

   (c) Determine if f is odd, even or neither.

   (d) For which value(s) of $x \in \mathbb{R}$ is the function f continuous? Justify your answer with reference to continuity theorems from lectures.

   (e) Hence sketch the graph of $y = f(x)$.

   Give numerical answers as exact values, in terms of inverse hyperbolic functions if necessary. In your graphs, label all curves, axis intercepts and asymptotes (if any).

3. Use the complex exponential to evaluate the derivative
$$\frac{d^{61}}{dt^{61}}\left(e^{-t+1}\cos(t)\right)$$

End of assignment