Assignment 5 Due: 6:00PM, Friday 8 May.

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Explainer: Question 1 should be completed in **WebWork** by 6:00PM, Friday 8 May. WebWork should be accessed via Assignment 5 WebWork in the Assignments panel of the MAST10005 LMS Site.

You should upload a scan of neatly presented solutions to Question 2 in **Gradescope**, which should be accessed via Assignment 5 Written Part in the Assignments section of the MAST10005 LMS Site. Please do not include your answers to Question 1 in your solutions to the written part.

1. You should complete this question in WebWork by 6:00PM, Friday 8 May. It will test your ability to calculate first and second derivatives. Completing Question 1 *before* you attempt Question 2 will make Question 2 easier because you will have already checked that your calculations of the first and second derivatives of $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = \arctan(\sin(x))$ are both correct.

2. Here we use the answers from Question 1 to understand the graph of $f : \mathbb{R} \longrightarrow \mathbb{R}$ where

$$f(x) = \arctan(\sin(x)).$$

Simplifying the formulas for $f'(x)$ and $f''(x)$ as far as possible will make all of the calculations in this question much easier.

(a) Find all x and y intercepts of f. Explain your reasoning.

y-intercepts occur when $x=0$:

$f(0) = \arctan(\sin(0))$

$= \arctan(0) = 0$

$\boxed{y=0}$

$(0,0)$

x-intercepts occur when $f(x)=0$:

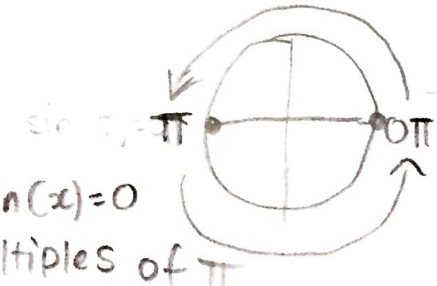
$\arctan(\sin(x)) = 0$

$\sin(x) = \tan(0)$

$\sin(x) = 0$

$\boxed{x = k\pi \text{ for } k \in \mathbb{Z}}$

$(k\pi, 0),\ k \in \mathbb{Z}$



$\sin(x)=0$ at multiples of π

(b) Use your answer to WebWork Problem 3 to find the set of all stationary points of f. Be sure to give the y values at the stationary points. A simple way to do this is by expressing your answers in the form $(x, f(x))$.

For $(x, f(x))$ to be a stationary point, $f'(x) = 0$

$$\Rightarrow \frac{\cos(x)}{1+\sin^2(x)} = 0$$

$$\cos(x) = 0$$

$$x = k\pi + \frac{\pi}{2} \text{ for } k \in \mathbb{Z}$$

Let S be the set of all stationary points of f

$$S = \left\{ \left(k\pi + \frac{\pi}{2}, \arctan\left(\sin\left(k\pi + \frac{\pi}{2}\right)\right)\right) \middle| k \in \mathbb{Z} \right\}$$

(c) Use your answer to WebWork Problem 3 to find the intervals on which f is concave up. Show full reasoning.

f is concave up on an interval I if and only if $f''(x) > 0$ for all x in I

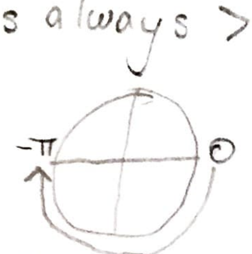
$$\Rightarrow \frac{-\sin(x)(\cos^2(x)+2)}{(\sin^2(x)+1)^2} > 0$$

① $(\cos^2(x)+2)$ is always >0

② $(\sin^2(x)+1)^2$ is always >0

③ $-\sin(x) > 0$

$\sin(x) < 0$



over the interval $(-\pi, 0)$ for $x \in [-2\pi, 0]$

$\therefore f$ is concave up over the intervals $(2k\pi - \pi, 2k\pi)$ for any $k \in \mathbb{Z}$

(d) State the intervals on which f is concave down. You may use your answer to (c).

from (c):
f must be concave down over the intervals
$\boxed{(2k\pi, 2k\pi+\pi) \text{ for any } k \in \mathbb{Z}}$

(e) Find the set of inflection points of f. Explain your answer. Be sure to include the y values of the inflection points in your answer.

A point of inflection is a point in dom(f) where changes between being concave up and concave down.
From (c) and (d): f changes between being concave up and concave down every $k\pi$ for any $k \in \mathbb{Z}$
Let P be the set of inflection points of f.

$$\boxed{P = \{(k\pi, \arctan(\sin(k\pi))) \mid k \in \mathbb{Z}\}}$$

(f) Use your answers to (d) and (e) to decide which of the stationary points you found in (b) are local maxima and which are local minima.

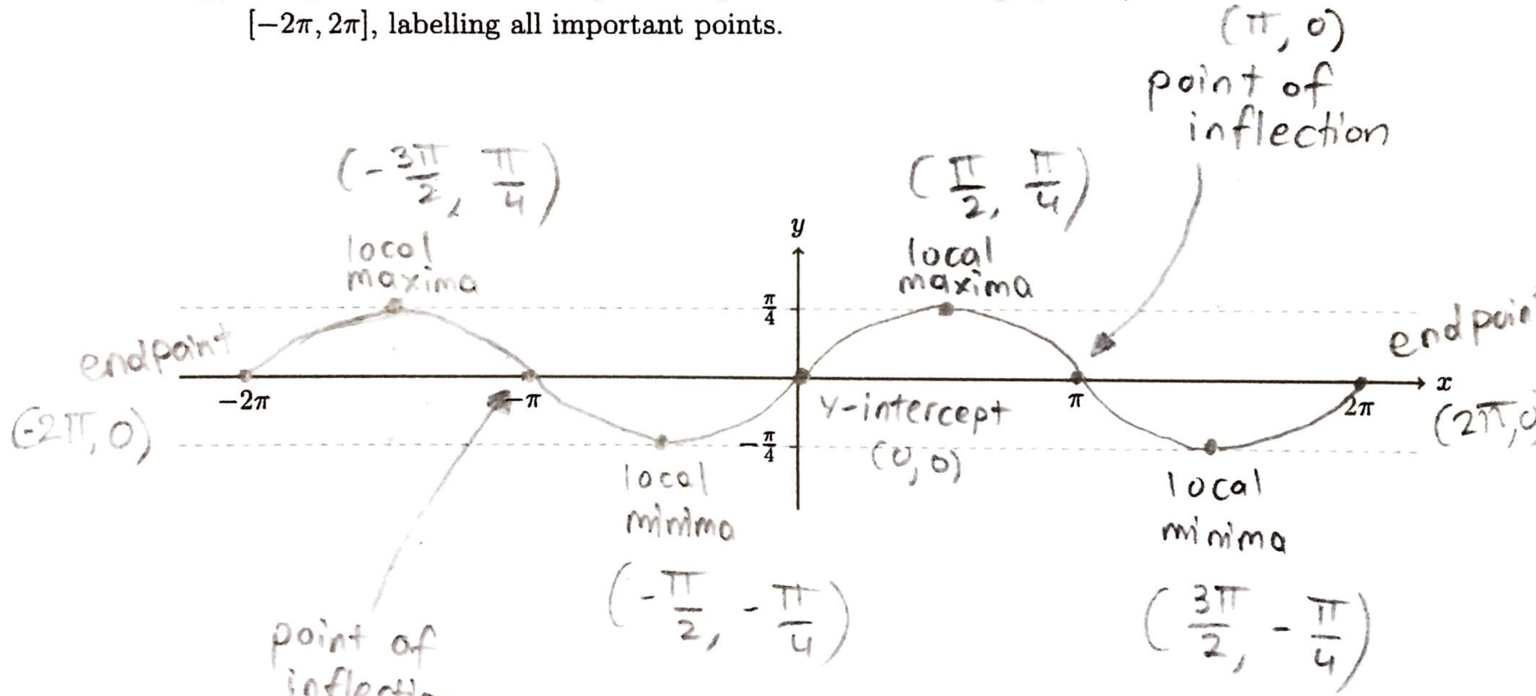
All stationary points contained in the intervals $(2k\pi, 2k\pi+\pi)$, $k \in \mathbb{Z}$ will be local maxima as $f'(x)$ is decreasing over this interval

$\therefore$ $\boxed{\text{Local maxima: } x = 2k\pi + \frac{\pi}{2},\ k \in \mathbb{Z}}$

$\therefore$ Local minima must be all the stationary points contained in the intervals $(2k\pi-\pi, 2k\pi)$, $k \in \mathbb{Z}$

$\boxed{\text{Local minima: } x = 2k\pi - \frac{\pi}{2},\ k \in \mathbb{Z}}$

(g) Use your answers to the previous parts to sketch the graph of f on the interval $[-2\pi, 2\pi]$, labelling all important points.



$(-\pi, 0)$

Assignment Information

This assignment is worth $\frac{20}{9}$% of your final MAST10005 mark.

Full working should be shown in your solutions to Question 2. There will be 1 mark overall for correct mathematical notation.

Full solutions to the assignment will be uploaded to the LMS site approximately 3 days after the assignment is due.