Lat Stress var ①

Pressure gradient ? $-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} = \frac{\hat{\tau}_0}{\rho h}$

$\bar{\cdot}$: x-t avg $\Rightarrow f(y,z)$

$\hat{\cdot}$: x-y-t avg. $\Rightarrow f(z)$

$\langle\ \rangle$: z avg $\Rightarrow f(x,y,t)$

$$\frac{\partial \overline{uv}}{\partial y} + \frac{\partial \overline{uw}}{\partial z} = \nu\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\bar{u} + \bar{F}$$

$$\Rightarrow 0 = \frac{\partial}{\partial y}\left(\nu\frac{\partial \bar{u}}{\partial y} - \overline{uv}\right) + \frac{\partial}{\partial z}\left(\nu\frac{\partial \bar{u}}{\partial z} - \overline{uw}\right) + \bar{F}$$

$$= \frac{\partial}{\partial y}\frac{\bar{\tau}_{12}}{\rho} + \frac{\partial}{\partial z}\frac{\bar{\tau}_{13}}{\rho} + \frac{1}{h}\frac{\hat{\tau}_0}{\rho}$$

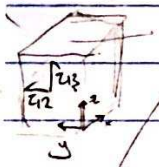
Bad:
Too much rough working leads to no logical flow of working.
Not all handwriting is legible.
Unreferenced diagrams are distracting and ambiguous.
Phone camera scanning yield poor scan quality:
...Pages are distorted and not exactly rectangular A4;
...small writing can become unreadable;
...poor lighting may lead to shadow artefacts on the page;

Avg in y:

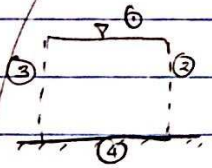
$$\Rightarrow 0 = 0 + \frac{\partial}{\partial z}\hat{\tau}_{13} + \frac{1}{h}\hat{\tau}_0$$

int. in z. $\Rightarrow \hat{\tau}_{13}(z) = -\frac{z}{h}\hat{\tau}_0$ | rearrange: $\frac{1}{h}\hat{\tau}_0 = -\frac{\partial}{\partial z}$

$$\Rightarrow 0 = \frac{\partial}{\partial y}\bar{\tau}_{12} + \frac{\partial}{\partial z}\underbrace{\left(\bar{\tau}_{13} - \hat{\tau}_{13}\right)}_{\Delta\tau}$$



$\tau_{12} -$



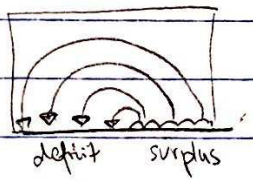
①: free surface: $\frac{\partial u}{\partial z} = 0 \Rightarrow \tau_{13} = 0$

flux: $\int_0^w \frac{\partial}{\partial y}\bar{\tau}_{12}\,dy = \bar{\tau}_{12}\big|_0^w = 0$ since periodic.

②+③ = 0 since periodic

④: $u = 0 \Rightarrow \frac{\partial u}{\partial y} = 0 \Rightarrow \tau_{12} = 0$.

$$\Rightarrow \int_0^w \left.\frac{\partial \Delta\tau}{\partial z}\right|_{z=0} dy = 0$$



i.e. plot distribution of $\Delta\tau$ & $\bar{\tau}_{12}$

(maybe stations later)

Integrate in z:

$$\frac{\partial}{\partial y}\langle\bar{\tau}_{12}\rangle = \left.\bar{\tau}_{13} - \hat{\tau}_{13}\right|_{z=0} = (\bar{\tau}_0 - \hat{\tau}_0)\frac{1}{h}$$

idealised model? $\bar{\tau}_0 = \hat{\tau}_0 \pm \frac{1}{2}\Delta\tau_0$ @ rough/smooth