Assignment 2 Due: **6:00PM, Friday 3 April**.

Penalties will be automatically applied to late assignments by Canvas.

Explainer: Question 1 will be completed in WebWork by 6:00PM, Friday 3 April. You can access this part of the assignment via Assignment 2 WebWork in the Assignments section of the MAST10005 LMS Site. You should upload a scan of your solutions to Question 2 via Assignment 2 Written Part in the Assignments section of the MAST10005 LMS Site. You should present your work written neatly.

1. [5 marks] This question will assess your ability to perform calculations involving complex numbers. Completing Question 1 *before* you attempt Question 2 will make Question 2 easier because you will have already done some of the necessary calculations. In particular, Problem 2(b) in WebWork asks you to express all complex solutions of the equation:

$$w^3 = \frac{\sqrt{3}+i}{2}$$

in polar form. You should make sure that you know the correct solution to this problem before attempting Question 2.

2. [3 method marks, 6 answer marks]

   (a) In the WebWork part of this assignment, you found the 3 elements of

$$A = \left\{ z \in \mathbb{C} \;\middle|\; z^3 = \frac{\sqrt{3}+i}{2} \right\}$$

   in exponential polar form. Use a similar method to find the 3 elements of

$$B = \left\{ z \in \mathbb{C} \;\middle|\; z^3 = \frac{\sqrt{3}-i}{2} \right\}$$

   in exponential polar form.

   (b) Sketch $A \cup B$ in the complex plane. Try to plot the positions of the 6 elements accurately. This will help you to complete part (d).

   (c) Use your previous solutions and the quadratic formula to find all complex solutions of the equation

$$z^6 - \sqrt{3}z^3 + 1 = 0$$

   in *exponential polar form*. Reviewing Example 1.83 in the Lecture Slides may be useful if you can't get started.

   (d) Do the solutions in (c) come in *conjugate pairs* as predicted by Theorem 1.81? If so, list the pairs.

   (e) Use your answer to (c) to express the polynomial $P(z) = z^6 - \sqrt{3}z^3 + 1$ as a product of linear factors.

   (f) Let $\theta \in \mathbb{R}$. Use Examplex 1.52, Theorem 1.53 and Theorem 1.39 to express

$$(z - e^{i\theta})(z - e^{-i\theta})$$

as a quadratic function of z *with all coefficients real*. You should justify the steps in your argument by stating where you use Example 1.52, Theorems 1.53 and Theorem 1.39.

[Hint: One of the coefficients can be expressed in terms of $\cos(\theta) = \mathrm{Re}(e^{i\theta})$.]

(g) Use your previous answers to express $P(z)$ as a product of three quadratics with real coefficients.

Assignment Instructions

This assignment is worth $\frac{20}{9}\%$ of your final MAST10005 mark.

Full working should be shown in your solutions to Question 2. There will be 1 mark overall for correct mathematical notation.

There is some advice on suitable apps for scanning your assignment work on Canvas page "Guidelines for online assignment submission (written parts)" in the Assignments panel.

Full solutions to the assignment will be uploaded to the LMS site approximately 3 days after the assignment is due.