

①

a) In a CD, sound is stored digitally in binary format (as a pattern of ones and zeros). After sound is recorded it is converted into numbers by a process called sampling; where a piece of electronic equipment measures the sound nearly 50,000 times a second and turns the measurement into a number stored in binary format. Pits (bumps) are etched into a CD and represent 0s while flat, unburned areas represent 1s. A CD player reads these pits and flat areas with a laser and converts the binary data back into the sound we hear.

b) Grooves are etched into a vinyl record with smooth transitions between them and are essentially a type of fingerprint of the analog sound wave being recorded as opposed to the sudden pits and flat spots of a digital CD. A vinyl record player has a needle that moves over the wavy grooves as the record rotates. The record player generates electrical signals based on the grooves that are then fed to the amplifier to create sound through the speakers.

c) I think that digital CDs are superior to analog vinyl records as they don't degrade as easily as vinyl, they are more portable, and are easier to use. Digital CD audio quality is also superior as there is less interference and no variation in playback speed as opposed to vinyl where imperfections in the grooves can affect the playback quality severely and the speed can even be inconsistent.

② $5x^2 - 50x + 125 = 0$, $x = 5, x = 8$

(sub in 8) $b = \text{base}$

$$(5 \times b^0) \times (8 \times b^0)^2 - (5 \times b^1) \times (8 \times b^0) + ((1 \times b^2) + (2 \times b^1) + (5 \times b^0)) = 0$$

$$(5 \times 64) - 40b + b^2 + 2b + 5 = 0$$

$$b^2 - 38b + 325 = 0$$

$$(b-25)(b-13) = 0$$

$$b = 13, b = 25$$

check $x = 5, b = 13$

$$(5 \times (5)^2) - (5 \times 13) \times 5 + ((1 \times 169) + (2 \times 13) + (5))$$

$$125 - 325 + 169 + 26 + 5 = 0 \quad \checkmark$$

\therefore base 13

\therefore These beings have 13 fingers.

(3)

a) Z_1 is 1 if both A_1 and B_1 are 1
 BUT A_2 and B_2 are not both 1

Z_2 is 1 if both A_2 and B_2 are 1
 BUT A_1 and B_1 are not both 1

A_1	B_1	A_2	B_2	Z_1	Z_2
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	1
0	1	0	0	0	0
0	1	0	1	0	0
0	1	1	0	0	0
0	1	1	1	0	1
1	0	0	0	0	0
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	0	1
1	0	0	0	1	0
1	0	0	1	1	0
1	0	1	0	1	0
1	0	1	1	1	0
1	1	0	0	1	0
1	1	0	1	1	0
1	1	1	0	1	0
1	1	1	1	1	0

b) Z_1

	$\bar{A}2\bar{B}2$	$\bar{A}2B2$	$A2B2$	$A2\bar{B}2$
$\bar{A}1B1$	0	0	0	0
$\bar{A}1B1$	0	0	0	0
$A1B1$	1	1	0	1
$A1\bar{B}1$	0	0	0	0

$$Z_1 = A1B1\bar{B}2 + A1B1\bar{A}2$$

Z_2

	$\bar{A}2\bar{B}2$	$\bar{A}2B2$	$A2B2$	$A2\bar{B}2$
$\bar{A}1B1$	0	0	1	0
$\bar{A}1B1$	0	0	1	0
$A1B1$	0	0	0	0
$A1\bar{B}1$	0	0	1	0

$$Z_2 = \bar{A}1A2B2 + \bar{B}1A2B2$$

(4)

	A	B	C	D	F	G
	0	0	0	0	1	0
	0	0	0	1	1	0
	0	0	1	0	1	0
	0	0	1	1	1	0
	0	1	0	0	0	0
	0	1	0	1	1	1
	0	1	1	0	0	0
	1	0	0	0	0	0
	1	0	0	1	0	0
	1	0	1	0	0	0
	1	0	1	1	0	0
	1	1	0	0	0	1
	1	1	0	1	0	1
	1	1	1	0	0	1
	1	1	1	1	0	1

a) K-map for F

	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD	
$\bar{A}\bar{B}$	1	1	1	1	$F = \bar{A}\bar{B} + \bar{A}D$
$\bar{A}B$	0	1	1	0	
$A\bar{B}$	0	0	0	0	
AB	0	0	0	0	

b) K-map for G

	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	1	1	0
$A\bar{B}$	1	1	1	1
AB	0	0	0	0

$$G = AB + BD$$

c) $F = \bar{A}\bar{B} + \bar{A}D$

$$= \bar{A}(\bar{B} + D)$$
 1 product vs 2

$$G = AB + BD$$

$$= B(A + D)$$
 1 product vs 2

5)

a) $C_7 C_6 C_5 C_4 C_3 C_2 C_1 = b_4 b_3 b_2 P_3 b_1 P_2 P_1$

P_3 group: $b_4 b_3 b_2 P_3 = C_7 C_6 C_5 C_4$

P_2 group: $b_4 b_3 b_1 P_2 = C_7 C_6 C_3 C_2$

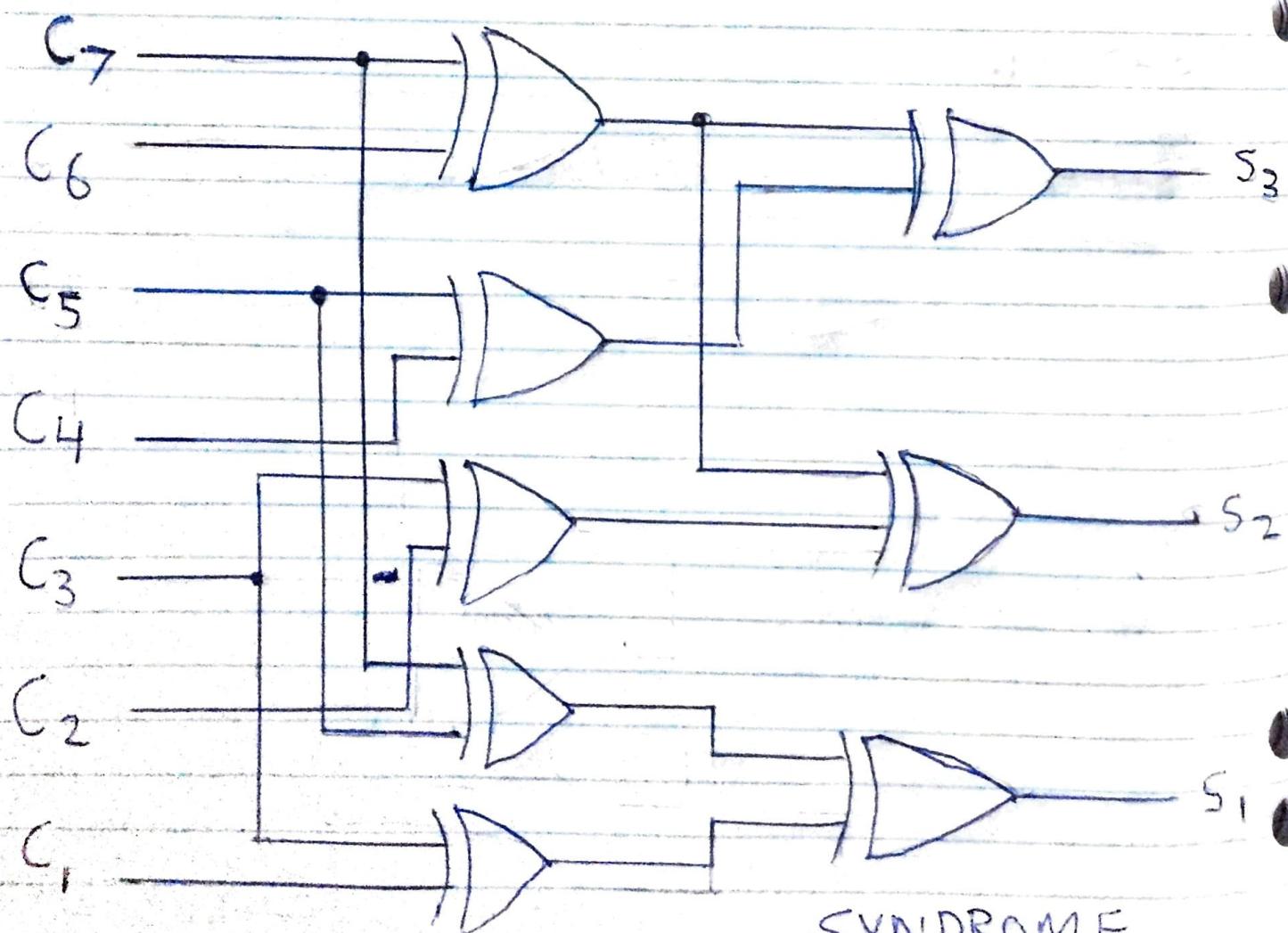
P_1 group: $b_4 b_2 b_1 P_1 = C_7 C_5 C_3 C_1$

$$S_3 = b_4 \oplus b_3 \oplus b_2 \oplus P_3$$

check if parity
of each group
is correct

$$S_2 = b_4 \oplus b_3 \oplus b_1 \oplus P_2$$

$$S_1 = b_4 \oplus b_2 \oplus b_1 \oplus P_1$$



SYNDROME
GENERATOR

b) $d_4 = s_3 s_2 s_1 \oplus b_4$

→ If all 3 parity bits were incorrect then b_4 must be wrong

$$d_3 = s_3 s_2 \overline{s}_1 \oplus b_3$$

→ If p_3 and p_2 were incorrect then b_3 must be wrong

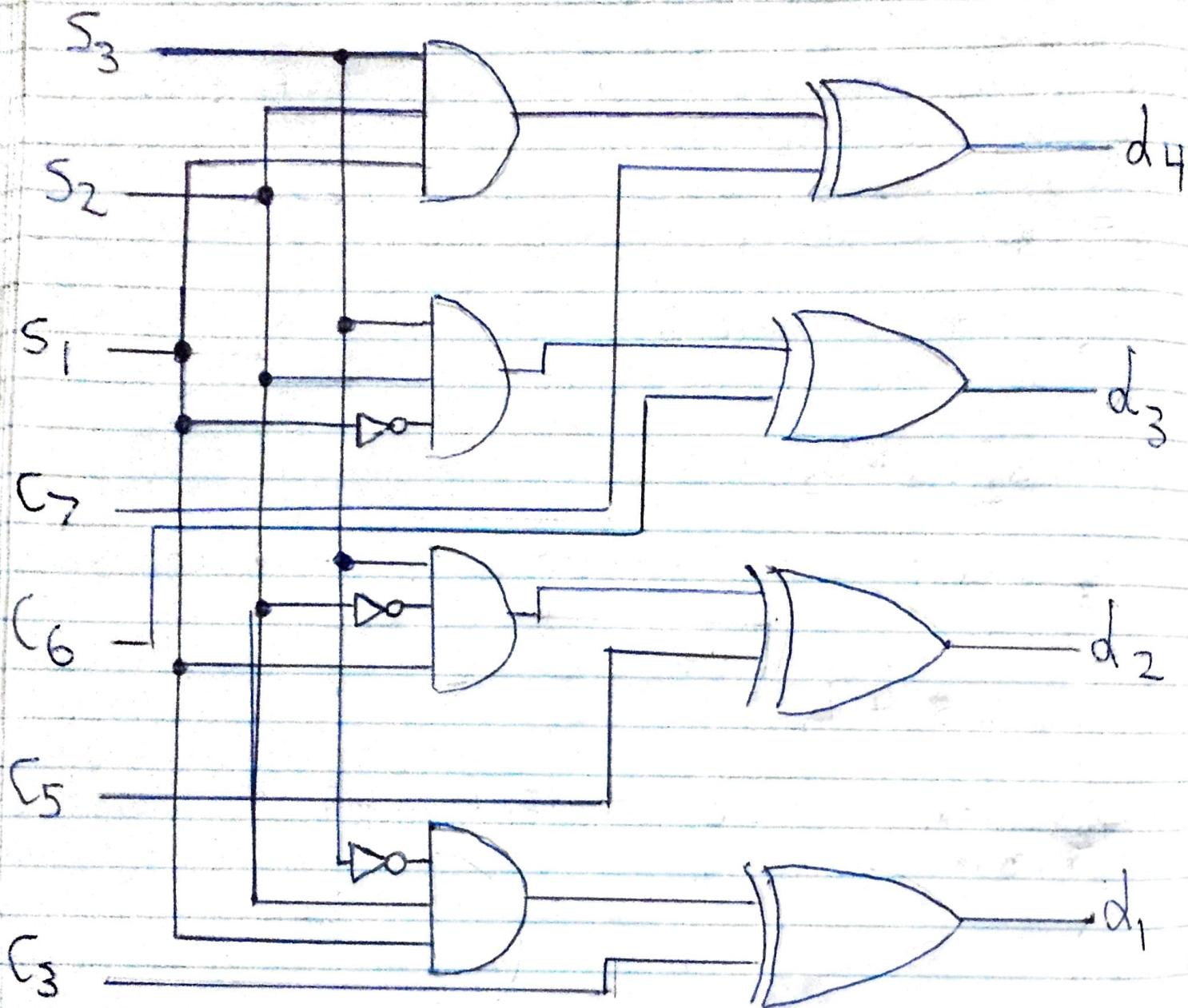
d₂ = $s_3 \overline{s}_2 s_1 \oplus b_2$

→ If p_3 and p_1 were incorrect then b_2 must be wrong

d₁ = $\overline{s}_3 s_2 s_1 \oplus b_1$

→ If p_2 and p_1 were incorrect then b_1 must be wrong

ERROR CORRECTOR



The error corrector block does not have C_4 , C_2 or S_1 as inputs because these are the parity bits and they have already been used to check for errors in the received codeword with the syndrome generator.

⑥

a)

i) $1111111 = 1111 \times 2^3 + 111 = 1111 \times 2^{11} + 111$

ii) $1111000 = 1111 \times 2^3 + 0 = 1111 \times 2^{11} + 0$

iii) $0000000 = 0000 \times 2^0 + 0$

iv) $00001100 = 1100 \times 2^0 + 0$

v) $1010101 = 1010 \times 2^3 + 101 = 1010 \times 2^{11} + 101$

b) The exponent correlates with the number of significant bits

- $E = 00$ when B_6 AND B_5 AND $B_4 = 0$
- $E = 01$ when B_6 AND $B_5 = 0$ AND $B_4 = 1$
- $E = 10$ when $B_6 = 0$ AND $B_5 = 1$
- $E = 11$ when $B_6 = 1$

$E_0 = 1$ when B_6 AND $B_5 = 0$ AND $B_4 = 1$ OR
when $B_6 = 1$

$\therefore E_0 = \overline{B_6} \overline{B_5} B_4 + B_6 = \overline{B_5} B_4 + B_6$

$E_1 = 1$ when $B_6 = 0$ AND $B_5 = 1$ OR $B_6 = 1$

$\therefore E_1 = \overline{B_6} B_5 + B_6 = B_5 + B_6$

The mantissa digits correspond with one of the original bits depending on the exponent (or total number of significant bits).

$M_0 = 1$ when $B_0 = 1$ AND $E = 00$ OR $B_1 = 1$ AND $E = 01$ OR $B_2 = 1$ AND $E = 10$ OR $B_3 = 1$ AND $E = 11$

$M_1 = 1$ when $B_1 = 1$ AND $E = 00$ OR "

$M_2 = 1$ when $B_2 = 1$ AND $E = 00$ OR "

$M_3 = 1$ when $B_3 = 1$ AND $E = 00$ OR "

$$\therefore M_0 = B_0 \overline{B}_6 \overline{B}_5 \overline{B}_4 + B_1 \overline{B}_6 \overline{B}_5 B_4 + B_2 \overline{B}_6 B_5 + B_3 B_6$$

$$M_1 = B_1 \overline{B}_6 \overline{B}_5 \overline{B}_4 + B_2 \overline{B}_6 \overline{B}_5 B_4 + B_3 \overline{B}_6 B_5 + B_4 B_6$$

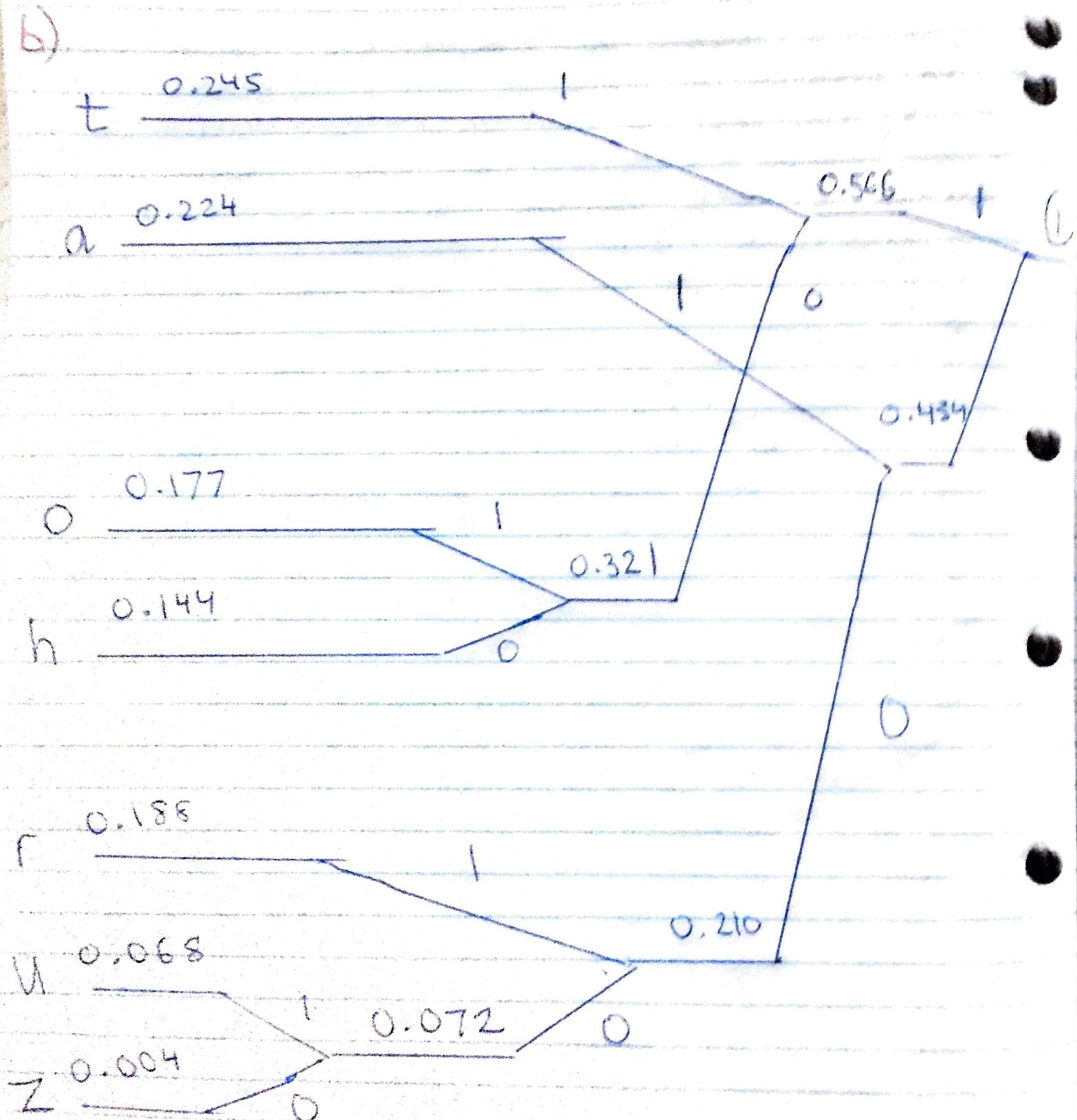
$$M_2 = B_2 \overline{B}_6 \overline{B}_5 \overline{B}_4 + B_3 \overline{B}_6 \overline{B}_5 B_4 + B_4 \overline{B}_6 B_5 + B_5 B_6$$

$$M_3 = B_3 \overline{B}_6 \overline{B}_5 \overline{B}_4 + B_4 \overline{B}_6 \overline{B}_5 + B_5 \overline{B}_6 + B_6$$

7

a)

letter	fixed length
a	000
h	001
o	010
r	011
t	100
u	101
z	110



$$\cdot t = 11$$

$$\cdot r = 0.01$$

$$\cdot a = 0.1$$

$$\cdot u = 0.001$$

$$\cdot o = 101$$

$$\cdot z = 0.0000$$

$$\cdot h = 100$$

c)

The average number of bits per symbol for the fixed length code:

$$\bar{L} = \frac{4 \times 7}{7} = 4$$

The average number of bits per symbol for the Huffman code:

$$\begin{aligned}\bar{L} = & (0.224 \times 2) + (0.245 \times 2) + (0.144 \times 3) + (0.127 \times 3) \\ & + (0.138 \times 3) + (0.0068 \times 4) + (0.004 \times 4)\end{aligned}$$

$$\bar{L} = 2.603$$

Comparing the average number of bits in both codes shows the efficiency of Huffman code as it has a smaller average.

d) Huffman code uses the probability of a certain letter to occur to decode it. The higher the probability of the letter is, fewer bits will be decoded, which reduces the total number of bits used at the end. That is what makes it so efficient and special.