Assignment 1 Due: **6:00PM, Friday 20 March**.

Penalties will be automatically applied to late assignments by Canvas.

Explainer: Question 1 is a *peer assessment exercise*. A scan or clear photo of your solutions should be submitted via "Assignment 1 Peer Review" in the Assignments section of the MAST10005 Canvas Site. Three of your fellow students will be asked to comment on your work, paying particular attention to the way you justify steps in your argument. Writing your solutions in the boxes below and then scanning this page may make it easier for the students reviewing your answers to read your work, but if you need further space, feel free to write your solutions up on blank pages. You should **not** include details on your solution page that identify you.

1. [2 marks for submitting, 3 marks for peer reviews] In this question, you should explain your reasoning using *only* the properties discussed on slides 57 to 72 of the lecture slides. Although the marks for this question are purely for submitting a solution and reviewing the work of other students, taking both of these tasks seriously should put you in a better position to answer Question 3, where marks will be awarded for giving a correct argument.

   (a) Express the set $A = \{x \in \mathbb{R} \mid x^2 - x - 6 > 0\}$ as a union of intervals.

   (b) Use your answer to (a) to explain briefly why $\{x \in \mathbb{R} \mid x^2 - x - 6 < 0\} = (-2, 3)$.

Explainer: You should upload scan or clear photo of your solutions your answers to questions 2 and 3 via the Assignment 1 Written Part in the Assignments section of the MAST10005 LMS Site. You should present your work written neatly.

2. [2 answer marks, 3 method marks] Recall that a *Pythagorean triple* is a triple (a, b, c) of natural numbers such that $a^2 + b^2 = c^2$. These triples give the side lengths of right angled triangles for which the side lengths all have whole number values. Question 1 (c) of the Week 1 Workshop uses the best known Pythagorean triple $(3, 4, 5)$. Reviewing that Question should help you with this one. In this question, you may assume that $\sqrt{3},\ \frac{\sqrt{3}}{2} \notin \mathbb{Q}$. We define:

$$P = \left\{ \frac{a}{c} \;\middle|\; a, c \in \mathbb{N} \text{ and } a^2 + b^2 = c^2 \text{ for some } b \in \mathbb{N} \right\}$$
$$S = \{x \in \mathbb{R} \mid \sin(x) \in \mathbb{Q}\}$$
$$T = \{x \in \mathbb{R} \mid \sin(x) \in P\}$$
$$C = \{x \in \mathbb{R} \mid \cos(x) \in \mathbb{Q}\}$$

   (a) List at least one element of each of the following sets:

$$P,\ S \setminus C,\ C \setminus S \text{ and } C \cap S.$$

   (b) Prove that $S \not\subseteq T$. Don't forget to *prove* that your counter example is not an element of T. A proof by contradiction will probably be easiest.

   (c) Prove carefully that $T \subseteq C$. You will need the *Pythagorean Identity* (see the Formula sheet in the Subject Booklet).

3. [1 answer mark, 3 method marks] Using your answers to parts (a) and (b) of Question 1, express the following set as a union of intervals:

$$D = \left\{ x \in \mathbb{R} \;\middle|\; \frac{2x^2}{x^2 - x - 6} > 2 \right\}.$$

   Explain your reasoning carefully.

Assignment Instructions

This assignment is worth $\frac{20}{9}\%$ of your final MAST10005 mark.

Full working should be shown in your solutions to Questions 2 and 3. There will be 1 mark overall for correct mathematical notation.

There is some advice on suitable apps for scanning your assignment work on Canvas page "Guidelines for online assignment submission (written parts)" in the Assignments panel.

Full solutions to the assignment will be uploaded to the LMS site approximately 3 days after the assignment is due.