② $f_1(x,y) = (-y, x)$
- Reflection across the line $y=x$
- Reflection across the y-axis

$f_3(x,y) = (y, -x)$
- Reflection across the y-axis
- Reflection across the line $y=x$

For f_3 to be the inverse of f_1,

$f_3 \circ f_1(x,y) = (x,y)$
- Reflection across $y=x$
- Reflection across y-axis

$f_3(-y, x) = (x, y)$

$(x,y) = (x,y)$ ✓
- Reflection across y-axis
- Reflection across $y=x$

AND

$f_1 \circ f_3(x,y) = (x,y)$ = " "

$f_1(y, -x) = (x,y)$ " "

$(x,y) = (x,y)$ ✓ " "

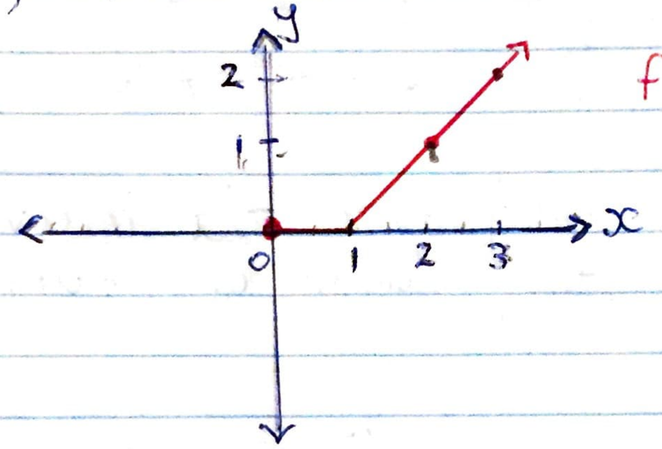
Therefore $f_3 : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is the inverse function of $f_1 : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$. $\square$

③ $f:[0,\infty)\to[0,\infty)$ $\quad g:[0,\infty)\to[0,\infty)$

$f(x)=\max(x-1,0)$ $\quad g(x)=x+1$

$$=\begin{cases} x-1, & \text{if } x-1\geq 0 \\ 0, & \text{if } x-1<0 \end{cases}$$

(a)



$f:[0,\infty)\to[0,\infty)$

(b) $g\circ f(x)$

$$=g\left(\begin{cases} x-1, & \text{if } x\geq 1 \\ 0, & \text{if } x<1 \end{cases}\right)$$

$$\boxed{=\begin{cases} x, & \text{if } x\geq 1 \\ 1, & \text{if } x\in[0,1) \end{cases} \quad \text{OR} \quad \max(x,1),\ x\in[0,\infty)}$$

(c) $f\circ g(x)$

$=f(x+1)=\max(x+1-1,0)$

$=\max(x,0)$ OR $\begin{cases} x, & \text{if } x\geq 0 \\ 0, & \text{if } x<0 \end{cases}$

$$\boxed{=x,\quad x\in[0,\infty)}$$

(d) For g to be the inverse of f,

$g(f(x)) = x$ AND $f(g(x)) = x$

Suppose that $g(f(x)) = x$

$$\Rightarrow g(f(x)) = \begin{cases} x, & \text{if } x \geq 1 \quad = x \\ 1, & \text{if } x < 1 \quad \neq x \end{cases}$$

$g(f(x)) = x$ only over the interval $[1, \infty)$ and not the domain of function f which is $[0, \infty)$.

Therefore g is not the inverse of f. $\square$

4

(a) OA

$\cos\theta = \dfrac{OA}{1}$

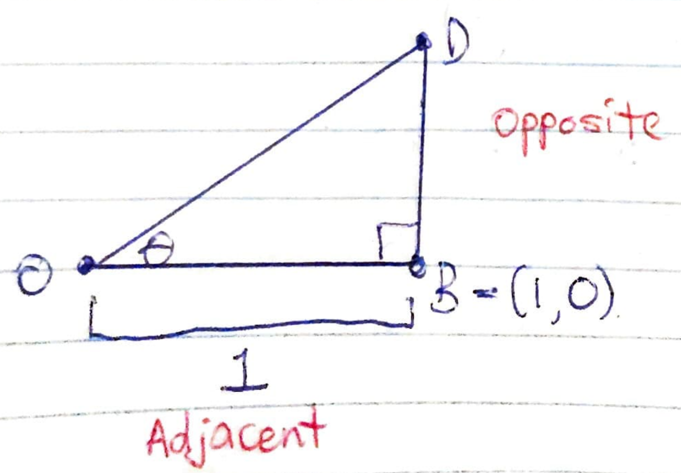
$\therefore OA = \cos\theta$

(b) AC

$\sin\theta = \dfrac{AC}{1}$

$\therefore AC = \sin\theta$

(c)

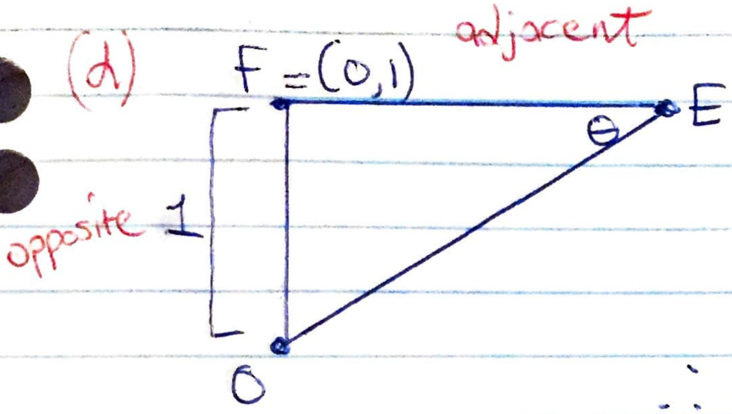


$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

$\therefore \tan\theta = \dfrac{BD}{1}$

$\therefore BD = \tan\theta$

(d)



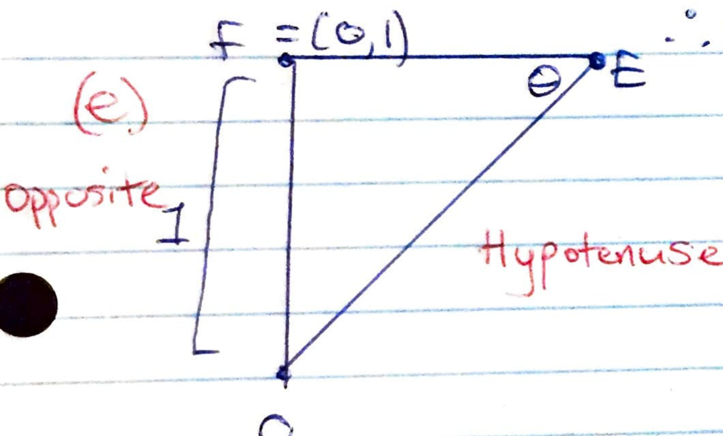
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\therefore \tan\theta = \frac{1}{EF}$$

$$\therefore EF = \frac{1}{\tan\theta} \qquad \left(\frac{1}{\tan\theta} = \cot\theta\right)$$

$$\therefore EF = \cot\theta$$

(e)



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\therefore \sin\theta = \frac{1}{OE}$$

$$\therefore OE = \frac{1}{\sin\theta} \qquad \left(\frac{1}{\sin\theta} = \operatorname{cosec}\theta\right)$$

$$\therefore OE = \operatorname{cosec}\theta$$