Assignment 5 Due: 6:00PM, Friday 8 May.

Name:

Student ID:

Explainer: Question 1 should be completed in **WebWork** by 6:00PM, Friday 8 May. WebWork should be accessed via Assignment 5 WebWork in the Assignments panel of the MAST10005 LMS Site.

You should upload a scan of neatly presented solutions to Question 2 in **Gradescope**, which should be accessed via Assignment 5 Written Part in the Assignments section of the MAST10005 LMS Site. Please do not include your answers to Question 1 in your solutions to the written part.

1. You should complete this question in WebWork by 6:00PM, Friday 8 May. It will test your ability to calculate first and second derivatives. Completing Question 1 *before* you attempt Question 2 will make Question 2 easier because you will have already checked that your calculations of the first and second derivatives of $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = \arctan(\sin(x))$ are both correct.

2. Here we use the answers from Question 1 to understand the graph of $f : \mathbb{R} \longrightarrow \mathbb{R}$ where

$$f(x) = \arctan(\sin(x)).$$

Simplifying the formulas for $f'(x)$ and $f''(x)$ as far as possible will make all of the calculations in this question much easier.

(a) Find all x and y intercepts of f. Explain your reasoning.

Solution:

y intercept is $f(0) = 0$. x intercepts are

$$\arctan(\sin(x)) = 0 \Rightarrow \sin(x) = 0$$
$$\Rightarrow x \in \{k\pi \mid k \in \mathbb{Z}\}$$

1A

For x intercepts. Need not use set notation.

(b) Use your answer to WebWork Problem 3 to find the set of all stationary points of f. Be sure to give the y values at the stationary points. A simple way to do this is by expressing your answers in the form $(x, f(x))$.

Solution:

From WebWork Problem 3 $f'(x) = \dfrac{\cos(x)}{\sin^2(x)+1}$.

Hence $f'(x) = 0 \Rightarrow \cos(x) = 0 \Rightarrow x \in \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\}$

1M

–

so stationary points are

$$\{(\tfrac{\pi}{2} + 2k\pi, \tfrac{\pi}{4}) \mid k \in \mathbb{Z}\} \cup \{(\tfrac{3\pi}{2} + 2k\pi, -\tfrac{\pi}{4}) \mid k \in \mathbb{Z}\}$$

2A

1 for correct x values, 1 for y values

(c) Use your answer to WebWork Problem 3 to find the intervals on which f is concave up. Show full reasoning.

Solution:

From WebWork Problem 3 $f''(x) = \dfrac{-\sin(x)(2+\cos^2(x))}{(\sin^2(x)+1)^2}$.

The factors $2+\cos^2(x)$ and $(\sin^2(x)+1)^2$ in this expression are positive, so the sign of $f''(x)$ is determined by the remaining factor $-\sin(x)$:

$f''(x) > 0 \Rightarrow -\sin(x) > 0 \Rightarrow \sin(x) < 0$

1M

–

Hence f is concave up on intervals of the form

$$((2k-1)\pi, 2k\pi)$$

1A

Or equivalent expression

where $k \in \mathbb{Z}$.

(d) State the intervals on which f is concave down. You may use your answer to (c).

Solution:

Using (c) and the fact that $\sin(x) = 0$ when $x = k\pi$ for some $k \in \mathbb{Z}$, we see that f is concave down on intervals of the form

$$(2k\pi, (2k+1)\pi)$$

where $k \in \mathbb{Z}$.

0A

Not Marked

(e) Find the set of inflection points of f. Explain your answer. Be sure to include the y values of the inflection points in your answer.

Solution:

From (d) the concavity changes at every point $x = k\pi$ for $k \in \mathbb{Z}$.

Hence set of inflection points is

$$\{(k\pi, 0) \mid k \in \mathbb{Z}\}.$$

1A

Correct x values.

(f) Use your answers to (d) and (e) to decide which of the stationary points you found in (b) are local maxima and which are local minima.

Solution:

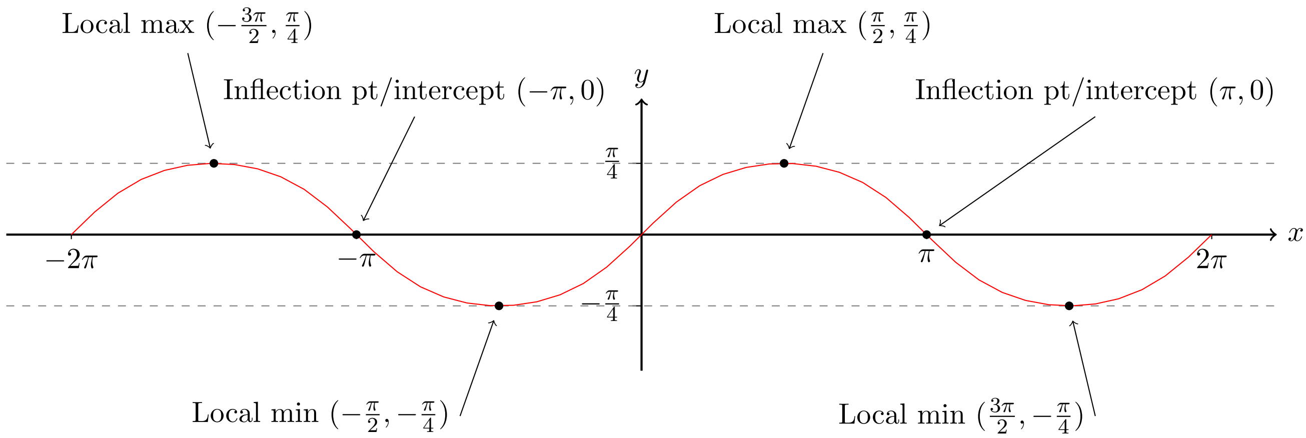
Since f is concave down on intervals $(2k\pi, (2k+1)\pi)$ for $k \in \mathbb{Z}$ the stationary points in $\{(\frac{\pi}{2} + 2k\pi, \frac{\pi}{4}) \mid k \in \mathbb{Z}\}$ are local maxima.

Similarly the stationary points in $\{(\frac{3\pi}{2} + 2k\pi, \frac{\pi}{4}) \mid k \in \mathbb{Z}\}$ are local minima.

1M

Explanation based on concavity at each stationary point.

(g) Use your answers to the previous parts to sketch the graph of f on the interval $[-2\pi, 2\pi]$, labelling all important points.



1M

Label at least one of each type of point.

1L

Whole written part: clear structure, and ALL mathematical notation is correct.

Assignment Information

This assignment is worth $\frac{20}{9}\%$ *of your final MAST10005 mark.*

Full working should be shown in your solutions to Question 2. There will be 1 mark overall for correct mathematical notation.

Full solutions to the assignment will be uploaded to the LMS site approximately 3 days after the assignment is due.