Question 1

1a) $n=9, \quad \sigma=0.6, \quad \bar{x}=8$

$$\Pr\left(-c < \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} < c\right) = 0.95 \quad \text{where } c = \Phi^{-1}(0.975)$$

$$\Pr\left(-1.96 < \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} < 1.96\right) = 0.95$$

$$\Pr\left(\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}\right) = 0.95$$

A 95% CI for μ is $\left(8 - 1.96\frac{0.6}{3},\ 8 + 1.96\frac{0.6}{3}\right)$

$$= (7.608,\ 8.392)$$

1b) $\frac{\text{width}}{2} = c\frac{\sigma}{\sqrt{n}}$

$$\frac{0.2}{2} = 1.96\frac{0.6}{\sqrt{n}}$$

$$n = \left(\frac{1.96 \times 0.6}{0.1}\right)^2 = 138.298 \approx 139 \text{ samples required}$$

1c)

$$\Pr\left(-c < \frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}} < c\right) = 0.95 \qquad \frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}} \sim t_{n-1}$$

$$\Pr\left(\bar{X} - c\frac{S}{\sqrt{n}} < \mu < \bar{X} + c\frac{S}{\sqrt{n}}\right) = 0.95$$

$c = 2.306 \qquad s^2 = 0.425 \qquad s = 0.652$

A 95% CI for μ is $\left(8 - 2.306\frac{0.652}{3}, 8 + 2.306\frac{0.652}{3}\right)$

$= (7.499, 8.501)$

This confidence interval is slightly wider than the confidence interval from part (a) due to the sample standard deviation being higher than the assumed σ.

Question 2

2a)

$$n = \frac{c^2 \hat{p}(1-\hat{p})}{\epsilon^2}$$

assume $\hat{p} = 0.8$ as the lowest sample proportion we expect and use it as our estimate to cover the case with maximum uncertainty

$$\Rightarrow n = \frac{1.96^2 \times 0.8 \times 0.2}{0.05^2} \approx 246 \text{ samples required}$$

2b) $n = \frac{1.96^2 \times 0.8 \times 0.2}{0.02^2} \approx 1537$ samples required

Question 4

$n=8$ $m=12$ $\bar{x}=8.21$ $\bar{y}=7.36$ $S_x=1.610$
$S_y=0.956$

Let enriched air plant growth $= X$
normal air plant growth $= Y$

$$W = \frac{\bar{X}-\bar{Y}-(\mu_x-\mu_y)}{\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}} \approx t_{\frac{\left(\frac{S_x^2}{n}+\frac{S_y^2}{m}\right)^2}{\frac{S_x^4}{n^2(n-1)}+\frac{S_y^4}{m^2(m-1)}}}$$

$$\approx t_{10.31}$$

$$Pr\left(-c < \frac{\bar{X}-\bar{Y}-(\mu_x-\mu_y)}{\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}} < c\right)=0.95$$

$$Pr\left(\bar{X}-\bar{Y}-c\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}} < \mu_x-\mu_y < \bar{X}-\bar{Y}+c\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}\right)$$
$$=0.95$$

A 95% CI for $\mu_x-\mu_y$ is

$$\left(8.21-7.36-2.219\sqrt{\frac{1.61^2}{8}+\frac{0.956^2}{12}},\ 8.21-7.36+2.219\sqrt{\frac{1.61^2}{8}+\frac{0.956^2}{12}}\right)$$

$$=(0.501,\ 1.199)$$

Therefore there is strong evidence that a CO_2-enriched atmosphere increases plant growth.