1.

a) $f(n) = n^3$

$g(n) = (n+2)^3 = n^3 +$ lower order terms

$\Rightarrow f(n) \in \Theta(g(n))$

b) $f(n) = 3^{(2n)} \approx 3^n \quad (c^n)$

$g(n) = 3^{n+2} \approx 3^n \quad (c^n)$

$\Rightarrow f(n) \in \Theta(g(n))$

c) $f(n) = (3n)^2 = 9n^2$

$g(n) = 3^{n^2} = c^n$

$n^c \prec c^n \Rightarrow n^2 \prec 3^{n^2}$

$\Rightarrow f(n) \in O(g(n))$

d) $f(n) = (\log_2 n)^{\log_2 n}$ (n^n growth)

$g(n) = \sqrt{n} = n^{1/2}$

$n^{\varepsilon} \prec n^n \Rightarrow n^{1/2} \prec \log_2 n^{\log_2 n}$

$\Rightarrow f(n) \in \Omega(g(n))$

2.

a) the basic operation is the distance comparison within the nested loop

Let the cost of this comparison = 1

$g(n) :=$ the number of times this is performed

The worst case runtime is the same as the best case runtime as all elements are checked every time (i.e. the algorithm is input-insensitive).

Therefore,

$T(n) = 1 \cdot g(n)$

$$T(n) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} n-1-i+1+1 = \sum_{i=0}^{n-2} n-i+1$$

$$= (n+1)(n-2-0+1) - \sum_{i=0}^{n-2} i$$ (as $n+1$ is unaffected by the sum)

$$= (n+1)(n-1) - \frac{(n-1)n}{2} = \frac{(n-1)(n+2)}{2} = \Theta(n^2)$$

b)

3.

a) A, C, D, F, G, E, B

b) A, C, D, B, F, E, G

c) Using one of the previous solutions would not provide a minimal cost solution as BFS is more suited to finding a shortest path between nodes and DFS is better suited for topological sorting and finding cycles. Prims algorithm would be more appropriate to find a minimal cost network, or a minimum spanning tree

d) for v in V
  BFS from each existing node to X and Y
  if path from v to X cost < cost min X
    min X ← path from v to X, cost min X ← cost p
  if path from v to Y cost < cost min Y
    min Y ← path from v to Y, cost min Y ← cost p

 return min Y and min X

(p = path)

4.

a) The worst case complexity would involve the case in which the $^2/_3$ size array is searched every single time and so

$T(n) \in O(\log_3(2n))$ worst case

b) $T(1) = 0$

$T(n) = T(n/5) + n$

$= T(n/5^2) + n/5 + n$

$\vdots$

$= T\left(\frac{n}{5^k}\right) + \sum_{i=1}^{k} \frac{n}{5^k}$

$\vdots$

$k = \log_5 n \quad \Rightarrow T\left(\frac{n}{5^{\log_5 n}}\right) + \sum_{i=1}^{\log_5 n} \frac{n}{5^{\log_5 n}}$

$= T(1) + \sum_{i=1}^{\log_5 n} 1$

$= \log_5 n$

$\Rightarrow R(n) \in \Theta(\log_5 n) \quad \Rightarrow R(n) \in \Theta(\log n)$