COMP20007 Mid-Semester Test

Graded

Student

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Total Points

7 / 10 pts

Question 1

Question 1

1 / 2 pts

(a)

- ✔ **− 0 pts** Correct with full justification
- **− 0.25 pts** Correct with insufficient justification, or minor error in justification
- **− 0.5 pts** Correct with no, or incorrect, justification
- **− 0.5 pts** Incorrect or ununaswered

(b)

- **− 0 pts** Correct with full justification
- **− 0.25 pts** Correct with insufficient justification, or minor error in justification
- **− 0.5 pts** Correct with no, or incorrect, justification
- ✔ **− 0.5 pts** Incorrect or unanswered

(c)

- **− 0 pts** Correct with full justification
- ✔ **− 0.25 pts** Correct with insufficient justification, or minor error in justification
- **− 0.5 pts** Correct with no, or incorrect, justification
- **− 0.5 pts** Incorrect or unanswered

(d)

- **− 0 pts** Correct with full justification
- ✔ **− 0.25 pts** Correct with insufficient justification, or minor error in justification
- **− 0.5 pts** Correct with no, or incorrect, justification
- **− 0.5 pts** Incorrect or unanswered

1 working needed

Question 2

Question 2

1 / 2 pts

✔ **– 0 pts** Part (a) correct answer and justification

– 0.5 pts Part (a) correct answer, incorrect/missing justification

– 1 pt Part (a) incorrect answer and justification

– 0 pts Part (b) correct

– 0.5 pts Part (b) correct idea, incomplete or inaccurate implementation

✔ **– 1 pt** Part (b) unanswered or incorrect algorithms (-1.0)

Question 3

Question 3

3 / 4 pts

Q 3a: max mark = 1

✔ **+ 1 pt** Correct, or just with a minor problem.

+ 0.5 pts Incorrect, but is a valid DFS.

+ 0.5 pts seems correct, but the answer not in a right format and could be understood in different ways.

+ 0 pts Not attempted, or not a valid DFS

Q 3b: max mark= 1

✔ **+ 1 pt** Correct, or with a minor mistake.

+ 0.5 pts Incorrect, but is a valid BFS, or a MST (which reflects an incorrect interpretation of choosing lowest weight when dequeuing rather than enqueuing)

+ 0 pts Incorrect, or not attempted

+ 0.5 pts seems correct, but the answer is not in a right format and could be understood in different ways.

Q 3c: max mark = 1

✔ **+ 1 pt** Correct, and the justification is reasonable or just has a minor problem.

+ 0.5 pts Correct answer, but the justification is flawed or missing.

+ 0.5 pts Incorrect answer, but the discussion is mostly correct and shows good understanding.

+ 0 pts Not attempted, or mostly wrong answer and justifications.

Q 3d: max mark= 1

+ 1 pt Correct, or just with a minor mistake.

+ 0.5 pts Correct or mostly correct algorithm, but with no or incorrect complexity.

+ 0.5 pts Not a right algorithm, but the complexity analysis seems OK.

✔ **+ 0 pts** Not attempted, or largely incorrect.

+ 0.5 pts Incorrect, but the discussion shows some good understanding.

Question 4

Question 4

2 / 2 pts

- ✔ **− 0 pts** Correct
- **− 0.5 pts** Missing/unclear justification for 4a
- **− 0.5 pts** Incorrect complexity for 4a
- **− 1 pt** Missing 4a
- **− 0.5 pts** Missing/Incorrect Recurrence Relation for 4b
- **− 0.5 pts** Recurrence Relation for 4b substantially incorrectly reduced to closed form
- **− 1 pt** Missing 4b

1.

a) $f(n) = n^3$

$g(n) = (n+2)^3 = n^3 +$ lower order terms

$\Rightarrow f(n) \in \Theta(g(n))$

b) $f(n) = 3^{(2n)} \approx 3^n \quad (c^n)$

$g(n) = 3^{n+2} \approx 3^n \quad (c^n)$

$\Rightarrow f(n) \in \Theta(g(n))$

c) $f(n) = (3n)^2 = 9n^2$

$g(n) = 3^{n^2} = c^n$

$n^c \prec c^n \Rightarrow n^2 \prec 3^{n^2}$

$\Rightarrow f(n) \in O(g(n))$

d) $f(n) = (\log_2 n)^{\log_2 n} \quad$ (n^n growth)

$g(n) = \sqrt{n} = n^{1/2}$

$n^\varepsilon \prec n^n \Rightarrow n^{1/2} \prec \log_2 n^{\log_2 n}$

$\Rightarrow f(n) \in \Omega(g(n))$

Question assigned to the following page: 2

2.

a) the basic operation is the distance comparison within the nested loop

Let the cost of this comparison $= 1$

$g(n) ::=$ the number of times this is performed

The worst case runtime is the same as the best case runtime as all elements are checked every time (i.e. the algorithm is input-insensitive).

Therefore,

$T(n) = 1 \cdot g(n)$

$T(n) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} n-1-i+1+1 = \sum_{i=0}^{n-2} n-i+1$

$= (n+1)(n-2-0+1) - \sum_{i=0}^{n-2} i$ (as $n+1$ is unaffected by the sum)

$= (n+1)(n-1) - \frac{(n-1)n}{2} = \frac{(n-1)(n+2)}{2} = \Theta(n^2)$

b)

Question assigned to the following page: 3

3.

a) A, C, D, F, G, E, B

b) A, C, D, B, F, E, G

c) Using one of the previous solutions would not provide a minimal cost solution as BFS is more suited to finding a shortest path between nodes and DFS is better suited for topological sorting and finding cycles. Prims algorithm would be more appropriate to find a minimal cost network, or a minimum spanning tree

d) for v in V
    BFS from each existing node to X and Y
  if path from v to X cost < cost min X
    min X ← path from v to X, cost min X ← cost p
  if path from v to Y cost < cost min Y
    min Y ← path from v to Y, cost min Y ← cost p

  return min Y and min X   (p = path)

Question assigned to the following page: 4

4.

a) The worst case complexity would involve the case in which the 2/3 size array is searched every single time and so

$T(n) \in O(\log_3(2n))$ worst case

b) $T(1) = 0$

$T(n) = T(n/5) + n$

$= T(n/5^2) + n/5 + n$

$\vdots$

$= T\left(\frac{n}{5^k}\right) + \sum_{i=1}^{k} \frac{n}{5^k}$

$\vdots$

$k = \log_5 n \quad \Rightarrow T\left(\frac{n}{5^{\log_5 n}}\right) + \sum_{i=1}^{\log_5 n} \frac{n}{5^{\log_5 n}}$

$= T(1) + \sum_{i=1}^{\log_5 n} 1$

$= \log_5 n$

$\Rightarrow R(n) \in \Theta(\log_5 n) \Rightarrow R(n) \in \Theta(\log n)$