Assignment 4: Solutions and marking scheme

1. As this question was completed online in WebWork, no marks are shown.

   Problem 1. As this part varies individually for students, no solutions are provided.

   Problem 2. As this part varies individually for students, no solutions are provided.

   Problem 3. As this part varies individually for students, no solutions are provided.

   Problem 4. a. $k = \frac{4}{9}$.

   b. $\mathbf{v}_{\parallel} = (-\frac{4}{3}, \frac{4}{3}, \frac{8}{3})$.

   c. $\mathbf{v}_{\perp} = (\frac{1}{3}, -\frac{1}{3}, \frac{1}{3})$.

2. (a) $\dfrac{\pi}{2}$. $\boxed{1A}$

   (b) P is given by $\mathbf{v}_{\parallel}$, so $P = (-\frac{4}{3}, \frac{4}{3}, \frac{8}{3})$. $\boxed{1A}$

   The radius r is the length of $\mathbf{v}_{\perp} = \frac{1}{3}(1, -1, 1)$, so

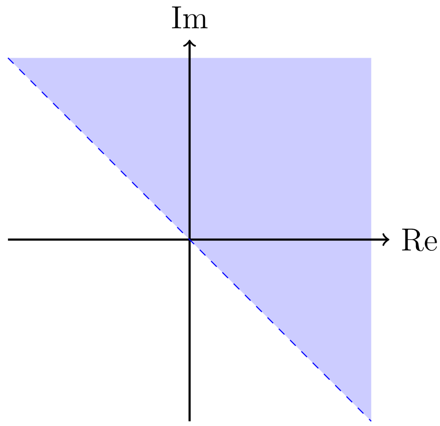
$$r = \|\frac{1}{3}(1, -1, 1)\| = \frac{1}{3}\|(1, -1, 1)\| = \frac{\sqrt{3}}{3}. \qquad \boxed{1A}$$

3. (a) Let $x \in \mathbb{R}$. Then $x \in \mathbb{C}$ and $h(x) = \mathrm{Re}(x) = x$. Hence h is surjective.

   (b) $\boxed{1M}$ Writing a general element of $z \in \mathbb{C}$ in Cartesian form $x + iy$ gives:

$$\begin{aligned} \mathrm{dom}(\log \circ h) &= \{x + iy \in \mathrm{dom}(h) \mid h(x) \in \mathrm{dom}(\log)\} \\ &= \{x + iy \in \mathbb{C} \mid x + y \in (0, \infty)\} \\ &= \{x + iy \in \mathbb{C} \mid y > -x\} \qquad \boxed{1A} \end{aligned}$$

   [Sketch not marked]



   (c) By (a) we have $\mathrm{range}(h) = \mathbb{R}$, so:

$$\begin{aligned} \mathrm{range}(\log \circ h) &= \log(\mathrm{range}(h) \cap \mathrm{dom}(\log)) \\ &= \log(\mathbb{R} \cap (0, \infty)) \\ &= \log((0, \infty)) = \mathbb{R} \qquad \boxed{1A} \end{aligned}$$

4. (a) $\boxed{\text{1M}}$

$$\begin{aligned}\mathbf{r}_1(t) = \mathbf{r}_2(t) &\Rightarrow t\cos(t)\mathbf{i} + t\sin(t)\mathbf{j} = \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j} \\ &\Rightarrow t = 0 \text{ or } \cos(t) = \frac{1}{\sqrt{2}} = \sin(t) \\ &\Rightarrow t = 0 \text{ or } t = \frac{\pi}{4} + 2k\pi \text{ for some } k \in \mathbb{Z}\end{aligned}$$

so $A = \{0\} \cup \{\frac{\pi}{4} + 2k\pi \mid k \in \mathbb{Z}\}$ $\boxed{\text{1A}}$

(b)

$$\begin{aligned}B = \mathbf{r}_2(A) &= \{0\} \cup \left\{\mathbf{r}_2(\tfrac{\pi}{4} + 2k\pi) \mid k \in \mathbb{Z}\right\} \\ &= \{0\} \cup \left\{ \frac{\frac{\pi}{4} + 2k\pi}{\sqrt{2}}\mathbf{i} + \frac{\frac{\pi}{4} + 2k\pi}{\sqrt{2}}\mathbf{j} \;\middle|\; k \in \mathbb{Z} \right\}\end{aligned}$$ $\boxed{\text{1A}}$

You might like to run an animation of this (very interesting) example in our interactive applet:

https://www.geogebra.org/m/BeRSE00O.

$\boxed{\text{1L}}$ *Clear structure, and ALL mathematical notation is correct.*