Assignment 8 Due: 6:00PM, Friday 29 May.

Name:

Student ID:

Explainer: Question 1 should be completed in **WebWork** by 6:00PM, Friday 29 May. WebWork should be accessed via Assignment 8 WebWork in the Assignments panel of the MAST10005 LMS Site.

You should upload a scan of neatly presented solutions to Questions 2, 3 and 4 in **Gradescope**, which should be accessed via Assignment 8 Written Part in the Assignments section of the MAST10005 LMS Site. Please do not include your answers to Question 1 in this part.

1. You should complete this question in WebWork by 6:00PM, Friday 29 May. Completing this question first will make the the written part easier because you will have already checked your partial fractions decomposition of $\dfrac{x-x^2}{x^3+x^2+x+1}$ needed in Question 2.

2. Use your answer to Problem 3 in WebWork to find the antiderivative $\int f(x)\ \mathrm{d}x$ where
$$f(x) = \frac{x^4 - x^2 + x - 1}{x^3 + x^2 + x + 1}.$$
[Hint: As the degree of the numerator is greater than the degree of the denominator, you must first perform polynomial long division.]

Solution: Polynomial division gives:

$$f(x) = x - 1 + \frac{x - x^2}{x^3 + x^2 + x + 1} = x - 1 + \frac{x - x^2}{(x^2+1)(x+1)}.$$

Using our calculations from WebWork:

$$f(x) = x - 1 + \frac{1}{x^2+1} - \frac{1}{x+1}.$$

1M Break up expression

Hence using an antiderivative from our table:

$$\int f(x)\ \mathrm{d}x = \int x - 1 + \frac{1}{x^2+1} - \frac{1}{x+1}\ \mathrm{d}x$$
$$= \frac{1}{2}x^2 - x + \arctan(x) - \log(|x+1|) + C.$$

1A –

3. (a) By completing the square in the denominator, find the antiderivative

$$\int \frac{1}{x^2 + \sqrt{2}x + 1}\, dx.$$

Solution: Completing the square gives

$$x^2 + \sqrt{2}x + 1 = (x + \tfrac{1}{\sqrt{2}}) + \tfrac{1}{2} = u^2 + a^2$$

where $u = x + \frac{1}{\sqrt{2}}$ and $a = \frac{1}{\sqrt{2}}$, so $\dfrac{du}{dx} = 1$.

1M Completing the square

Hence

$$\int \frac{1}{x^2 + \sqrt{2}x + 1}\, dx = \int \frac{1}{u^2 + a^2} \frac{du}{dx}\, dx$$

1M Integration

$$= \int \frac{1}{u^2 + a^2}\, du \qquad \text{[Substitution]}$$

$$= \tfrac{1}{a} \arctan(\tfrac{u}{a}) + C$$

$$= \sqrt{2} \arctan(\sqrt{2}(x + \frac{1}{\sqrt{2}})) + C$$

$$= \sqrt{2} \arctan(\sqrt{2}x + 1) + C$$

1A –

(b) Using your work from (a), find the antiderivative $\int \frac{\sqrt{2}x+2}{x^2+\sqrt{2}x+1}\,\mathrm{d}x$.

Solution: Our notation from (a) gives $x = u - \frac{1}{\sqrt{2}}$, so

$$\int \frac{\sqrt{2}x+2}{x^2+\sqrt{2}x+1}\,\mathrm{d}x = \int \frac{\sqrt{2}(u-\frac{1}{\sqrt{2}})+1}{u^2+a^2}\frac{du}{dx}\,\mathrm{d}x$$

1M Integration

$$= \int \frac{\sqrt{2}u+1}{u^2+a^2}\,\mathrm{d}u$$

$$= \tfrac{1}{\sqrt{2}}\int \frac{2u}{u^2+a^2}\frac{du}{dx}\,\mathrm{d}x + \int \frac{1}{u^2+a^2}\,\mathrm{d}u$$

$$= \tfrac{1}{\sqrt{2}}\int \frac{1}{v}\frac{dv}{du}\,\mathrm{d}u + \sqrt{2}\arctan(\sqrt{2}x+1)+C \quad [v = u^2+a^2, \text{part}(a)]$$

$$= \tfrac{1}{\sqrt{2}}\int \frac{1}{v}\,\mathrm{d}v + \sqrt{2}\arctan(\sqrt{2}x+1)+C \quad [\text{substitution}]$$

$$= \frac{1}{\sqrt{2}}\log(|v|)\,\mathrm{d}v + \sqrt{2}\arctan(\sqrt{2}x+1)+C$$

$$= \tfrac{1}{\sqrt{2}}\log(x^2+\sqrt{2}x+1) + \sqrt{2}\arctan(\sqrt{2}x+1)+C$$

1A $|\cdot|$ OK

4. (a) Use *integration by substitution* to convert the definite integral

$$\int_0^{\frac{\pi}{2}} \cos(x)e^{-\sin^2(x)}\,\mathrm{d}x$$

into a definite integral with a *simpler* integrand (on a different interval of integration).

Solution:

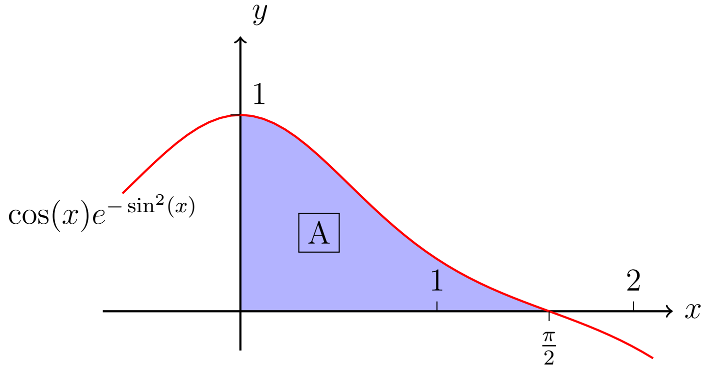
$$\int_0^{\frac{\pi}{2}} \cos(x)e^{-\sin^2(x)}\,dx = \int_0^{\frac{\pi}{2}} \sin'(x)e^{-\sin^2(x)}\,\mathrm{d}x$$

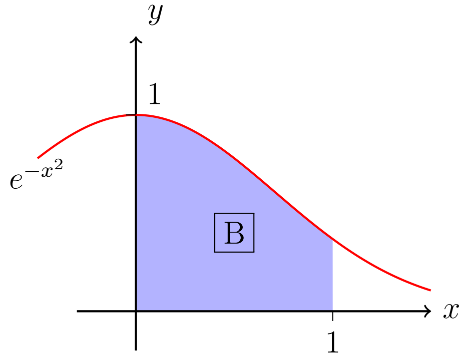
1M –

$$= \int_{\sin(0)}^{\sin(\frac{\pi}{2})} e^{-x^2}\,\mathrm{d}x \qquad [\text{Substitution}]$$

$$= \int_0^1 e^{-x^2}\,\mathrm{d}x$$

(b) Illustrate the results of your calculation in (a) by shading two equal areas in the following diagrams (as in Example 4.7 in the lecture slides) on the diagrams below.





Areas $\boxed{A}$ and $\boxed{B}$ are equal.

1A

correct areas shaded.

1L

Whole written part: clear structure, and ALL mathematical notation is correct.

Assignment Information

This assignment is worth $\frac{20}{9}$% of your final MAST10005 mark.

Full working should be shown in your solutions to Questions 2, 3 and 4. There will be 1 mark overall for correct mathematical notation.
Full solutions to the assignment will be uploaded to the LMS site approximately 3 days after the assignment is due.