NS vorticity transport ①

Acceptable:
Handwriting is legible.
Equations are neatly aligned and easy to follow.
University scanners provide high quality scanning, prevent page and text distortion.
No extraneous rough working to distract.

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial}{\partial x_j}\frac{\partial u_i}{\partial x_j}$$

let $\underline{\omega} = \nabla \times \underline{u}$

- note that $\nabla \times \underline{\omega} = \nabla \times (\nabla \times \underline{u})$

$$\equiv \varepsilon_{ijk} \frac{\partial}{\partial x_j} \varepsilon_{kmn} \frac{\partial}{\partial x_m} u_n$$

$$= \varepsilon_{ijk}\, \varepsilon_{kmn} \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_m} u_n$$

$$= \varepsilon_{ijk}\, \varepsilon_{mnk}$$

$$= (\delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}) \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_m} u_n$$

$$= \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_i} u_j - \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_j} u_i$$

$$\nabla \times \underline{\omega} = \nabla(\cancel{\nabla \cdot \underline{u}}) - \nabla^2 u$$

- Also note that $\underline{u} \times \underline{\omega} = \underline{u} \times (\nabla \times \underline{u})$

$$= \varepsilon_{ijk} u_j\, \varepsilon_{kmn} \frac{\partial}{\partial x_m} u_n$$

$$= \varepsilon_{ijk}\, \varepsilon_{mnk} u_j \frac{\partial}{\partial x_m} u_n$$

$$= (\delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}) u_j \frac{\partial}{\partial x_m} u_n$$

$$= u_j \frac{\partial}{\partial x_i} u_j - u_j \frac{\partial}{\partial x_j} u_i$$

$$= \frac{\partial}{\partial x_i}\left(\tfrac{1}{2} u_j^2\right) - u_j \frac{\partial}{\partial x_j} u_i$$

$$\equiv \nabla\left(\tfrac{1}{2} u \cdot u\right) - u \cdot \nabla u$$