Assignment 8 Written Part

● Graded

Student

James La Fontaine

Total Points

9 / 10 pts

Question 1

Q2

2 / 2 pts

✔ **+ 1 pt** Method

✔ **+ 1 pt** Correct answer

+ 0 pts No marks could be given

Question 2

Q3a

3 / 3 pts

✔ **+ 1 pt** Complete the square

✔ **+ 1 pt** Integration

✔ **+ 1 pt** Correct answer

+ 0 pts No marks can be given

Question 3

Q3b

1 / 2 pts

✔ **+ 1 pt** Method

+ 1 pt Correct answer

+ 0 pts No marks can be given

+arctan NOT - arctan

Question 4

Q4a

1 / 1 pt

✔ **+ 1 pt** Method

+ 0 pts No mark could be given

Question 5

Q4b

1 / 1 pt

✔ **+ 1 pt** Correct areas shaded

+ 0 pts No mark could be given

Question 6

Notation

1 / 1 pt

- ✔ **+ 1 pt** No significant lapses
- **+ 0 pts** Significant lapses

Assignment 8 Due: 6:00PM, Friday 29 May.

Name: James La Fontaine

Student ID: 1079860

Explainer: Question 1 should be completed in **WebWork** by 6:00PM, Friday 29 May. WebWork should be accessed via Assignment 8 WebWork in the Assignments panel of the MAST10005 LMS Site.

You should upload a scan of neatly presented solutions to Questions 2, 3 and 4 in **Gradescope**, which should be accessed via Assignment 8 Written Part in the Assignments section of the MAST10005 LMS Site. Please do not include your answers to Question 1 in this part.

1. You should complete this question in WebWork by 6:00PM, Friday 29 May. Completing this question first will make the the written part easier because you will have already checked your partial fractions decomposition of $\frac{x-x^2}{x^3+x^2+x+1}$ needed in Question 2.
2. Use your answer to Problem 3 in WebWork to find the antiderivative $\int f(x)\,dx$ where
$$f(x) = \frac{x^4 - x^2 + x - 1}{x^3 + x^2 + x + 1}.$$
[Hint: As the degree of the numerator is greater than the degree of the denominator, you must first perform polynomial long division.]

$$\begin{array}{r} x - 1 \\ x^3+x^2+x+1 \overline{\big)\, x^4 + 0x^3 - x^2 + x - 1} \\ -(x^4 + x^3 + x^2 + x) \\ \hline -x^3 - 2x^2 - 1 \\ -(-x^3 - x^2 - x - 1) \\ \hline -x^2 + x \end{array}$$

$$\therefore \int f(x) = \int x - 1\,dx + \int \frac{x - x^2}{x^3+x^2+x+1}\,dx$$

$$= \frac{1}{1+1}x^{1+1} - x + \int \frac{1}{x^2+1}\,dx - \int \frac{1}{x+1}\,dx$$

(from Webwork)

$$= \frac{1}{2}x^2 - x + \arctan(x) - \log|x+1| + C$$

3. (a) By completing the square of the denominator, find the antiderivative

$$\int \frac{1}{x^2+\sqrt{2}x+1}\,dx.$$

$$x^2+\sqrt{2}x+1=\left(x+\frac{\sqrt{2}}{2}\right)^2+\left(1-\frac{2}{4}\right)$$
$$=\left(x+\frac{\sqrt{2}}{2}\right)^2+\frac{1}{2}$$

$$\therefore \int \frac{1}{x^2+\sqrt{2}x+1}\,dx = \int \frac{1}{\left(x+\frac{\sqrt{2}}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2}\,dx$$

$u=x+\frac{\sqrt{2}}{2}$

$\frac{du}{dx}=1$

$$=\int \frac{1}{u^2+\left(\frac{1}{\sqrt{2}}\right)^2}\frac{du}{\cancel{dx}}\,\cancel{dx}$$

$$=\frac{1}{\frac{1}{\sqrt{2}}}\arctan\left(\frac{u}{\frac{1}{\sqrt{2}}}\right)+C$$

$$=\sqrt{2}\arctan\left(\frac{x+\frac{\sqrt{2}}{2}}{\frac{1}{\sqrt{2}}}\right)+C$$

$$=\sqrt{2}\arctan\left(\sqrt{2}x+1\right)+C$$

(b) Using your work from (a), find the antiderivative $\int \frac{\sqrt{2}x+2}{x^2+\sqrt{2}x+1}\,dx$.

$$= \int \frac{\sqrt{2}x+2}{\left(x+\frac{\sqrt{2}}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2}\,dx$$

$u = x+\frac{\sqrt{2}}{2}$ $\therefore x = u-\frac{\sqrt{2}}{2}$

$\frac{du}{dx} = 1$

$$= \int \frac{\sqrt{2}\left(u-\frac{\sqrt{2}}{2}\right)+2}{u^2+\left(\frac{1}{\sqrt{2}}\right)^2}\frac{du}{\cancel{dx}}\,\cancel{dx}$$

$$= \int \frac{\sqrt{2}u-1}{u^2+\left(\frac{1}{\sqrt{2}}\right)^2}\,du$$

$v = u^2+\left(\frac{1}{\sqrt{2}}\right)^2$

$\frac{dv}{du} = 2u$

$\frac{1}{2}\frac{dv}{du} = u$

$$= \sqrt{2}\int \frac{u}{u^2+\left(\frac{1}{\sqrt{2}}\right)^2}\,du - \int \frac{1}{u^2+\left(\frac{1}{\sqrt{2}}\right)^2}\,du$$

$$= \sqrt{2}\int \frac{1}{v}\frac{1}{2}\frac{dv}{\cancel{du}}\,\cancel{du} - \sqrt{2}\arctan(\sqrt{2}x+1)+C$$

$$= \frac{\sqrt{2}}{2}\log|v| - \sqrt{2}\arctan(\sqrt{2}x+1)+C$$

(as $v>0$)

$$= \frac{\sqrt{2}}{2}\log\left(\left(x+\frac{\sqrt{2}}{2}\right)^2+\frac{1}{2}\right) - \sqrt{2}\arctan(\sqrt{2}x+1)+C$$

4. (a) Use *integration by substitution* to convert the definite integral

$$\int_0^{\frac{\pi}{2}} \cos(x)e^{-\sin^2(x)}\,dx$$

into a definite integral with a *simpler* integrand (on a different interval of integration).

$u = \sin(x)$

$\frac{du}{dx} = \cos(x)$

when $x=0$, $u=0$

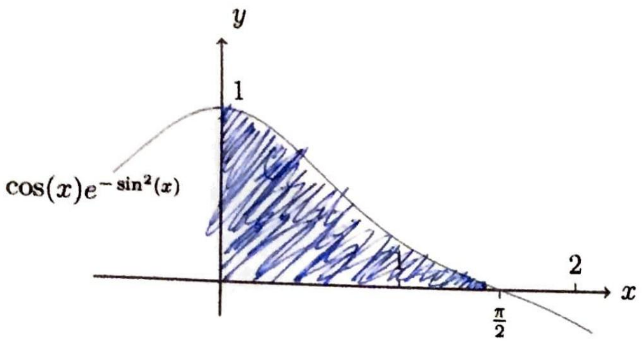
when $x=\frac{\pi}{2}$, $u=1$

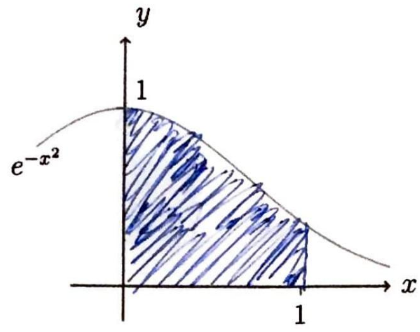
$$\therefore \int_0^{\pi/2} \cos(x)e^{-\sin^2(x)}\,dx$$

$$= \int_0^{\pi/2} e^{-u^2}\frac{du}{dx}\,dx$$

$$= \int_0^1 e^{-u^2}\,du$$

(b) Illustrate the results of your calculation in (a) by shading two equal areas in the following diagrams (as in Example 4.7 in the lecture slides) on the diagrams below.





Assignment Information

This assignment is worth $\frac{20}{9}$% of your final MAST10005 mark.

Full working should be shown in your solutions to Questions 2, 3 and 4. There will be 1 mark overall for correct mathematical notation.

Full solutions to the assignment will be uploaded to the LMS site approximately 3 days after the assignment is due.