Assignment 4 Due: **6:00PM, Friday 1 May**.

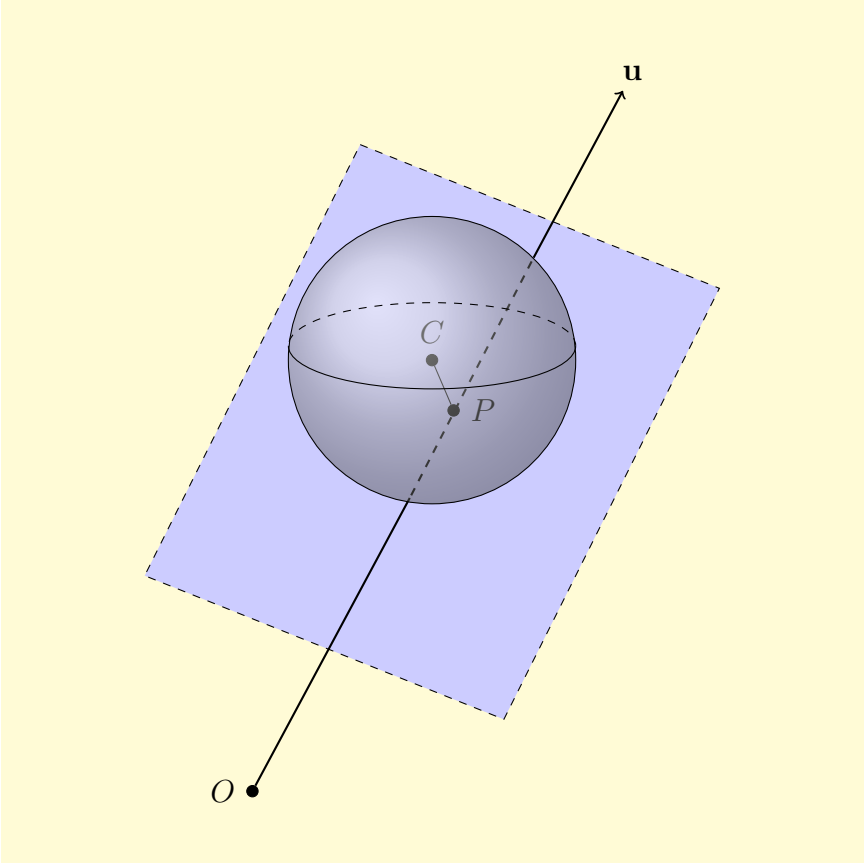
Penalties will be automatically applied to late assignments by Canvas.

Explainer: Question 1 should be completed in WebWork by 6:00PM, Friday 1 May, accessed via Assignment 4 WebWork in the Assignments section of the MAST10005 LMS Site. You should upload a scan of neatly presented solutions to Questions 2 and 3 via Assignment 4 Written Part in the Assignments section of the MAST10005 LMS Site. There is no need to include your answers to Question 1.

1. You should complete this question in WebWork by 6:00PM, Friday 1 May. It will test your ability to do calculations involving vectors. Completing Question 1 *before* you attempt Question 2 will make Question 2 easier because you will have already checked that your calculations of the parallel and perpendicular projections of $\mathbf{v} = (-1, 1, 3)$ onto $\mathbf{u} = (-3, 3, 6)$ are both correct.

2. [3 answer marks] As illustrated, a spherical ball with centre C and radius r is rolling down a sloping plane in $\mathbb{R}^3$ that passes through the origin O and contains the vector $\mathbf{u} = -3\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$. As it rolls, the point of contact P with the plane rolls along the line defined by $\mathbf{u}$. Suppose that at some particular moment in time, the vector

$$\mathbf{v} = \overrightarrow{OC} = -\mathbf{i} + \mathbf{j} + 3\mathbf{k}$$



   (a) What angle does $\overrightarrow{PC}$ make with $\mathbf{u}$?

   (b) Use the vector projections you calculated in Question 1, to find P and the radius r of the ball.

3. [1 method mark, 2 answer marks] Define $h : \mathbb{C} \longrightarrow \mathbb{R}$ by:

$$h(z) = \mathrm{Re}(z) + \mathrm{Im}(z)$$

and define f by the formula $f(z) = \log(h(z))$.

   (a) Prove that h is surjective. Easy!

   (b) Find the implied domain of f and sketch it in a complex plane diagram.

   (c) Find the implied range of f.

4. [1 method mark, 2 answer marks] Suppose the motion of two particles is given by the two parametric curves $\mathbf{r}_1 : \mathbb{R} \longrightarrow \mathbb{R}^2$ and $\mathbf{r}_2 : \mathbb{R} \longrightarrow \mathbb{R}^2$ defined as follows:

$$\mathbf{r}_1(t) = t\cos(t)\mathbf{i} + t\sin(t)\mathbf{j}, \qquad \mathbf{r}_2(t) = \frac{t}{\sqrt{2}}\mathbf{i} + \frac{t}{\sqrt{2}}\mathbf{j}.$$

   (a) Find the set A of t-values in $\mathbb{R}$ for which the particles *collide* (there are infinitely many).

   (b) Find the set B of points in $\mathbb{R}^2$ at which these collisions occur.

Assignment Instructions

This assignment is worth $\frac{20}{9}\%$ of your final MAST10005 mark.

Full working should be shown in your solutions to Question 2 and 3. There will be 1 mark overall for correct mathematical notation.

There is some advice on suitable apps for scanning your assignment work on Canvas page "Guidelines for online assignment submission (written parts)" in the Assignments panel.

Full solutions to the assignment will be uploaded to the LMS site approximately 3 days after the assignment is due.