Assignment 9 Due: 6:00PM, Friday 5 June.

Name: James La Fontaine

Student ID: 1079860

Explainer: Question 1 should be completed in **WebWork** by 6:00PM, Friday 5 June. WebWork should be accessed via Assignment 9 WebWork in the Assignments panel of the MAST10005 LMS Site.

You should upload a scan of neatly presented solutions to Questions 2 and 3 in **Gradescope**, which should be accessed via Assignment 9 Written Part in the Assignments section of the MAST10005 LMS Site. Please do not include your answers to Question 1 in this part.

1. You should complete this question in WebWork by 6:00PM, Friday 5 June. Completing this question first will make the the written part easier because you will have already found and checked the equivalent expression for $\cos^2(x)\sin^4(x)$ needed in Question 2.

2. Use your answer to Problem 3 in WebWork to find the antiderivative

$$\int \cos^2(x)\sin^4(x)\ dx.$$

3. Consider the following separable differential equation:

$$\frac{dy}{dt} = \cos(t)\cos^2(y).$$

(a) Find all constant solutions of this differential equation (there are infinitely many).

Here $G(y) = \cos^2(y)$

$G(c) = 0 \Leftrightarrow \cos^2(c) = 0$

$\Leftrightarrow \cos(c) = 0$

$\Leftrightarrow c = \frac{\pi}{2} + k\pi, \; k \in \mathbb{Z}$

So $y \equiv \frac{\pi}{2} + k\pi, \; k \in \mathbb{Z}$

(b) Use your answer to (a) and Theorem 4.44 to explain why range$(y) \subseteq (-\frac{\pi}{2}, \frac{\pi}{2})$ for the solution of this equation satisfying the initial condition $y = 0$ when $t = 0$.

Since $\frac{\pi}{2} + k\pi$ is a constant solution, by Theorem 4.44 no other solution can take the value of multiples of $\frac{\pi}{2}$ by $k\pi$ and furthermore the curves of other solutions cannot cross the lines $y = \frac{\pi}{2} + k\pi$ so for the IVP in which $y = 0$, y must be contained in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ to not cross these lines.

(c) Solve the initial value problem with initial condition $y = 0$ when $t = 0$. You should explain how you use the result from (b) when solving this problem. [Hint: Review lecture slide 237.]

$\frac{dy}{dt} = \cos(t)\cos^2(y)$ $\quad G(y) = \cos^2(y),\ f(t) = \cos(t)$

$\frac{1}{\cos^2(y)} \frac{dy}{dt} = \cos(t)$ ← assuming that $\cos^2(y)$ is never zero, which is justified as $\cos^2(y) = 0$ only when $y = \frac{\pi}{2} + k\pi$, which is a constant solution (from (b))

$\int \frac{1}{\cos^2(y)}\, dy = \int \cos(t)\, dt$

$= \sin(t) + C$

$u = \tan(y) = \frac{\sin(y)}{\cos(y)}$ ← assuming that tan is defined (y is $\subset (-\frac{\pi}{2}, \frac{\pi}{2})$) which we know from (b)

$\frac{du}{dy} = \frac{\cos(y)\cdot\cos(y) - \sin(y)\cdot(-\sin(y))}{\cos^2(y)}$

$= \frac{1}{\cos^2(y)}$

$\Rightarrow \int du = u = \tan(y) + C$

$y = \arctan(\sin(0)) + C$ ← assuming that arctan is defined, which it is as range$(y) \subset (-\frac{\pi}{2}, \frac{\pi}{2})$

$y = 0$ ✓

$\therefore y = \arctan(\sin(t))$ is the solution to the IVP

Assignment Information

This assignment is worth $\frac{20}{9}$% of your final MAST10005 mark.

Full working should be shown in your solutions to Questions 2 and 3. There will be 1 mark overall for correct mathematical notation.

Full solutions to the assignment will be uploaded to the LMS site approximately 3 days after the assignment is due.