Assignment 1

● Graded

Student

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Total Points

7 / 11 pts

Question 1

Question 2 — **7** / 11 pts

1.1 **2a)** — **1 / 5 pts**

+ 1 pt M: Use bound for cosine to find bounds for sequence

+ 1 pt A: Correctly derived upper and lower bounds

+ 1 pt J: use standard limits to explicitly calculate limits of upper and lower bounds

+ 1 pt J: Stated use of Sandwhich Theorem (or equivalent name)

✔ **+ 1 pt** A: Limit is 0

+ 0 pts Scored zero

1 use sandwich rule, please refer to solutions

1.2 **2b)** — **2** / 2 pts

✔ **+ 1 pt** A: The sequence diverges

✔ **+ 1 pt** J: A valid reason for divergence is provided, such as "oscillates from $\frac{-1}{\sqrt{2}}$ to $\frac{1}{\sqrt{2}}$ ".
Note: Cannot apply limit laws or continuity

+ 0 pts Scored zero

2 Should say oscillates from $\frac{-1}{\sqrt{2}}$ to $\frac{1}{\sqrt{2}}$?

1.3 **2c)** — **3** / 3 pts

✔ **+ 1 pt** J: Changed to a real variable

✔ **+ 1 pt** J: State use of standard limits and continuity to $\tan$ or other appropriate function.

✔ **+ 1 pt** A: Limit is $\tan(1)$

+ 0 pts Scored zero

1.4 **Notation** — **1** / 1 pt

✔ **+ 1 pt** N: Notation correct and unambiguous throughout question 2

+ 0 pts There was at least one notational error in question 2

No questions assigned to the following page.

Calculus 2 Written Assignment 1

1.

a) $\lim_{x\to\pi} \frac{e^x - e^\pi}{\cos(\frac{x}{2})}$, type $(\frac{0}{0})$

$= \lim_{x\to\pi} \frac{e^x}{-\frac{1}{2}\sin(\frac{x}{2})}$, L'Hôpital's Rule

$= \frac{\lim_{x\to\pi}(e^x)}{\lim_{x\to\pi}(-\frac{1}{2}) \cdot \lim_{x\to\pi}(\sin(\frac{x}{2}))}$, limit laws

$= \frac{e^\pi}{-\frac{1}{2}\cdot 1}$; continuity of e^z and $\sin z$

$= -2e^\pi$

b) $f(x) = \begin{cases} \frac{e^x - e^\pi}{\cos(\frac{x}{2})} & x < \pi \\ a\sin\left(\frac{\pi^2}{2x}\right) & x \geq \pi \end{cases}$

A function is continuous if

$\lim_{x\to a} f(x) = f(a)$

$\lim_{x\to a} f(x) = L \iff \lim_{x\to a^-} f(x) = L$ and $\lim_{x\to a^+} f(x) = L$

from (a): $\lim_{x\to\pi^-} f(x) = -2e^\pi$

therefore we need $\lim_{x\to\pi^+} f(x) = -2e^\pi$

Question assigned to the following page: 1.1

(from previous page)

$$\lim_{x \to \pi^+} a \sin\left(\frac{\pi^2}{2x}\right) = -2e^{\pi}$$

$$= \lim_{x \to \pi^+} (a) \cdot \lim_{x \to \pi^+} \left(\sin\left(\frac{\pi^2}{2x}\right)\right) = -2e^{\pi}$$, limit laws

$$= a \sin\left(\frac{\pi^2}{2\pi}\right) = -2e^{\pi}$$, limit laws and continuity of $\sin z$

$$= a \sin\left(\frac{\pi}{2}\right) = -2e^{\pi}$$

$$a = -2e^{\pi}$$

$$\lim_{x \to \pi} f(x) = -2e^{\pi} = f(\pi)$$

Therefore the function is continuous when $a = -2e^{\pi}$

2.

a) $$\lim_{n \to \infty} \frac{n \log(\cos^2(n) + 3)}{2020^n}$$

$$= \lim_{x \to \infty} \frac{x \log(\cos^2(x) + 3)}{2020^x}$$, $x \in \mathbb{R}$

$$= \lim_{x \to \infty} (2020^{-x}) \cdot \lim_{x \to \infty} (x \log(\cos^2(x) + 3))$$, limit laws

$$= \lim_{x \to \infty} \left(\frac{1}{2020}\right)^x \cdot \lim_{x \to \infty} (x \log(\cos^2(x) + 3))$$

$$= 0 \cdot \lim_{x \to \infty} (x \log(\cos^2(x) + 3))$$, standard limit $r^x = 0$

$$= 0$$

Questions assigned to the following page: 1.2 and 1.3

b) $\lim_{n \to \infty} \sin\left(\frac{2(n-1)\pi}{4}\right)$ does not exist as $\sin\left(\frac{2(n-1)\pi}{4}\right)$ oscillates between 1 and -1 as $n \to \infty$ and therefore diverges.

c) $\lim_{n \to \infty} \tan\left((2020n)^{\frac{1}{n}}\right)$

$= \lim_{x \to \infty} \tan\left((2020x)^{\frac{1}{x}}\right)$, $x \in \mathbb{R}$

$= \lim_{x \to \infty} \tan\left(2020^{\frac{1}{x}} \cdot x^{\frac{1}{x}}\right)$

$= \tan\left(\lim_{x \to \infty}\left(2020^{\frac{1}{x}} \cdot x^{\frac{1}{x}}\right)\right)$

$= \tan\left(\lim_{x \to \infty}\left(2020^{1/x}\right) \cdot \lim_{x \to \infty}\left(x^{1/x}\right)\right)$, limit laws

$= \tan(1 \cdot 1)$, standard limits $a^{1/x} = 1$, $x^{1/x} = 1$

$= \tan(1)$, continuity of $\tan z$ at $z = 1$