2. $P = \{\frac{a}{c} \mid a, c \in \mathbb{N} \text{ and } a^2+b^2=c^2 \text{ for some } b \in \mathbb{N}\}$
$S = \{x \in \mathbb{R} \mid \sin(x) \in \mathbb{Q}\}$
$T = \{x \in \mathbb{R} \mid \sin(x) \in P\}$
$C = \{x \in \mathbb{R} \mid \cos(x) \in \mathbb{Q}\}$

(a)(i) $\boxed{\frac{3}{5} \in P}$

(ii) $S \setminus C$
$\Rightarrow \sin(x) \in \mathbb{Q}, \cos(x) \notin \mathbb{Q}$
if $x = \frac{\pi}{6} \Rightarrow \sin(x) = \frac{1}{2}$ and $\cos(x) = \frac{\sqrt{3}}{2}$
$\therefore \boxed{\frac{\pi}{6} \in S \setminus C}$

(iii) $C \setminus S$
$\Rightarrow \cos(x) \in \mathbb{Q}, \sin(x) \notin \mathbb{Q}$
if $x = \frac{\pi}{3} \Rightarrow \cos(x) = \frac{1}{2}$ and $\sin(x) = \frac{\sqrt{3}}{2}$
$\therefore \boxed{\frac{\pi}{3} \in C \setminus S}$

(iv) $C \cap S$
$\Rightarrow \sin(x) \in \mathbb{Q}, \cos(x) \in \mathbb{Q}$
if $x = \frac{\pi}{2} \Rightarrow \cos(x) = 0$ and $\sin(x) = 1$
$\therefore \boxed{\frac{\pi}{2} \in C \cap S}$

(b) $S \not\subseteq T$
$S = \{\ldots, 0, \frac{\pi}{6}, \frac{\pi}{2}, 2\pi, \ldots\}$
Claim $0 \in S$. This means that $\sin(0) = y$ for some $y \in \mathbb{Q}$.
This is true since $\sin(0) = 0$ and $0 \in \mathbb{Q}$.
Claim $0 \notin T$. Suppose for a contradiction that $0 \in T$.
This means that $\sin(0) = \frac{a}{c}$ for some $a, c \in \mathbb{N}$ and $a^2+b^2=c^2$ for some $b \in \mathbb{N}$.
But then $\frac{a}{c} = 0$ and $0 \notin \mathbb{N}$. This is a contradiction.

Therefore $0 \notin T$ and $S \not\subseteq T$. $\square$

(c) $T \subseteq C$

Let $x \in T$. This means that $\sin(x) = \frac{a}{c}$ for some $a, c \in \mathbb{N}$ and $a^2+b^2=c^2$ for some $b \in \mathbb{N}$, hence $\sin(x) \in P$.

This means that a, b, c must form a Pythagorean Triple.

Since $\sin(x) = \frac{\text{opposite}}{\text{hypotenuse}}$ we can consider a as the opposite, b as the adjacent and c as the hypotenuse of a right-angled triangle.

Since $\cos(x) = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\cos(x) = \frac{b}{c}$.

The Pythagorean Identity gives $\cos^2(x)+\sin^2(x)=1$.

Therefore $\frac{b^2}{c^2}+\frac{a^2}{c^2}=1$. $\times c^2$

$\iff a^2+b^2=c^2$

Hence $x \in C$. So $T \subseteq C$. $\square$

3. $D = \{x \in \mathbb{R} \mid \frac{2x^2}{x^2-x-6} > 2\}$

$x \neq 3, -2$ (as we cannot divide by 0)

$\iff \frac{2x^2}{(x-3)(x+2)} > 2$ $(\div 2)$

(divide into cases as the value of x will determine if the operations will be order preserving or order reversing)

$\iff \frac{x^2}{(x-3)(x+2)} > 1$

(i) $x < -2$ or $x > 3$

$\iff x^2 > (x-3)(x+2)$ $(\times(x-3)(x+2))$

$\iff x^2 > x^2 - x - 6$ $(-x^2, +x)$

$\iff x > -6$

$x \in (-6,-2) \cup (3, \infty)$

(ii) $-2 < x < 3$

$\iff x^2 < (x-3)(x+2)$ $(\times(x-3)(x+2))$

$\iff x^2 < x^2 - x - 6$ $(-x^2, +x)$

$\iff x < -6$

no solutions

if $x < -2$ or $x > 3$ then solutions are $(-6,-2) \cup (3, \infty)$

if $-2 < x < 3$ then no solutions

$\boxed{D = (-6,-2) \cup (3, \infty)}$