James La Fontaine
1079860

Wednesday 9:00 AM
José Ayala Hoffmann

(1.)

(a) $\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$, where $a, b, c \in \mathbb{R}$

$$\underset{\substack{R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1}}{=} \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} \underset{\frac{1}{b-a}R_2}{=} (b-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b-a \\ 0 & c-a & c^2-a^2 \end{vmatrix} \underset{\frac{1}{c-a}R_3}{=} (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b-a \\ 0 & 1 & c-a \end{vmatrix}$$

$$\underset{R_3 \to R_3 - R_2}{=} (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b-a \\ 0 & 0 & c-b \end{vmatrix} = (b-a)(c-a) \cdot 1 \cdot 1 \cdot (c-b)$$

$$\boxed{= (b-a)(c-a)(c-b)}$$

(b) $P_1 = (1,0)$ $P_2 = (0,1)$ $P_3 = (2,1)$

$P(x) = \alpha + \beta x + \gamma x^2$
$P(0) = 1 \Rightarrow \alpha = 1$
$P(1) = 0 \Rightarrow \alpha + \beta + \gamma = 0$
$P(2) = 1 \Rightarrow \alpha + 2\beta + 4\gamma = 1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 2 & 4 & 1 \end{array}\right] \underset{\substack{R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1}}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 4 & 0 \end{array}\right]$$

$$\underset{R_3 - 2R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 2 & 2 \end{array}\right] \underset{\frac{1}{2}R_3}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] \underset{R_2 \to R_2 - R_3}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$\therefore \alpha = 1, \beta = -2, \gamma = 1$

$P(1) = 1 - 2 + 1 = 0$ ✓ $P(0) = 1$ ✓ $P(-2) = 1 - 4 + 4 = 1$ ✓

2. $P=(1,-1,-2)$ $\quad Q=(2,1,1)$ $\quad R=(1,2,1)$

(a) $\vec{PQ} = Q-P = (2-1)\,i+(1-(-1))\,j+(1-(-2))\,k$

$= \underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$

$\underset{\sim}{u} = \underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$

$\hat{\underset{\sim}{u}} = \dfrac{\underset{\sim}{u}}{\|\underset{\sim}{u}\|} = \dfrac{1}{\|\underset{\sim}{u}\|}\underset{\sim}{u}$

$= \dfrac{1}{\sqrt{1^2+2^2+3^2}}(\underset{\sim}{i}+2\underset{\sim}{j}+3\underset{\sim}{k}) = \dfrac{1}{\sqrt{14}}(\underset{\sim}{i}+2\underset{\sim}{j}+3\underset{\sim}{k})$

$$\boxed{\begin{aligned} &= \frac{\sqrt{14}}{14}(\underset{\sim}{i}+2\underset{\sim}{j}+3\underset{\sim}{k}) = \frac{\sqrt{14}}{14}\underset{\sim}{i} + \frac{\sqrt{14}}{7}\underset{\sim}{j} + \frac{3\sqrt{14}}{14}\underset{\sim}{k} \\ \text{OR} &= -\frac{\sqrt{14}}{14}(\underset{\sim}{i}+2\underset{\sim}{j}+3\underset{\sim}{k}) = -\frac{\sqrt{14}}{14}\underset{\sim}{i} - \frac{\sqrt{14}}{7}\underset{\sim}{j} - \frac{3\sqrt{14}}{14}\underset{\sim}{k} \end{aligned}}$$

(b) $\vec{QR} = R-Q = (-1,1,0)$ $\qquad \vec{PQ} = (1,2,3)$

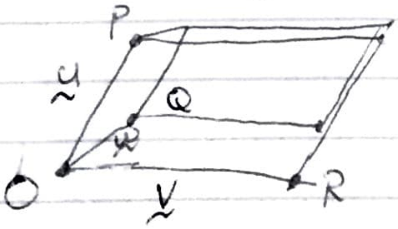
~~$proj_{QR}\, PQ$~~ $\quad \underset{\sim}{u}=(-1,1,0)$ $\quad \underset{\sim}{v}=(1,2,3)$

$proj_{\underset{\sim}{u}}\underset{\sim}{v} = \dfrac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{\|\underset{\sim}{u}\|^2}\underset{\sim}{u} = \dfrac{(-1\cdot1)+(1\cdot2)+(0\cdot3)}{\sqrt{(-1)^2+1^2+0}^{\,2}}(-1,1,0)$

$= \dfrac{-1+2+0}{\sqrt{2}^{\,2}}(-1,1,0)$

$$\boxed{= \frac{1}{2}(-1,1,0) = -\frac{1}{2}\underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j}}$$

(c)



$\underset{\sim}{u} = \vec{OP} = (1,-1,-2)$

$\underset{\sim}{w} = \vec{OQ} = (2,1,1)$

$\underset{\sim}{v} = \vec{OR} = (1,2,1)$

$\underset{\sim}{u}\cdot(\underset{\sim}{v}\times\underset{\sim}{w}) = \begin{vmatrix} 1 & -1 & -2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} \qquad \begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$

$= 1\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} - 2\begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} -1 & -2 \\ 1 & 1 \end{vmatrix}$

$= 1\cdot(-1) - 2\cdot3 + 1\cdot1 = -1-6+1 = -6$

$$\boxed{\text{volume} = |\underset{\sim}{u}\cdot(\underset{\sim}{v}\times\underset{\sim}{w})| = |-6| = 6}$$