

1. [2 marks for submitting, 3 marks for peer reviews] In this question, you should explain your reasoning using *only* the properties discussed on slides 57 to 72 of the lecture slides. Although the marks for this question are purely for submitting a solution and reviewing the work of other students, taking both of these tasks seriously should put you in a better position to answer Question 3, where marks will be awarded for giving a correct argument.

(a) Express the set $A = \{x \in \mathbb{R} \mid x^2 - x - 6 > 0\}$ as a union of intervals.

$$\begin{array}{l}
 x^2 - x - 6 > 0 \\
 \Rightarrow (x-3)(x+2) > 0
 \end{array}
 \quad \left(\begin{array}{l} \text{To solve we divide into cases as} \\ \text{the factors could be positive or negative} \end{array} \right)$$

$ \begin{array}{l} \text{(i) } x-3 > 0 \text{ and } x+2 > 0 \\ \quad \quad \quad \downarrow +3 \quad \quad \quad \downarrow -2 \\ \Rightarrow x > 3 \quad \quad \quad x > -2 \\ \Rightarrow x > 3 \quad \quad \quad \text{(as if } x > 3 \text{ then } x > -2) \\ \quad \quad \quad \text{(transitivity)} \end{array} $	$ \begin{array}{l} \text{(ii) } x-3 < 0 \text{ and } x+2 < 0 \\ \quad \quad \quad \downarrow +3 \quad \quad \quad \downarrow -2 \\ \Rightarrow x < 3 \quad \quad \quad x < -2 \\ \Rightarrow x < -2 \quad \quad \quad \text{(as if } x < -2 \text{ then } x < 3) \\ \quad \quad \quad \text{(transitivity)} \end{array} $
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$$\text{so } A = (-\infty, -2) \cup (3, \infty)$$

(b) Use your answer to (a) to explain briefly why $\{x \in \mathbb{R} \mid x^2 - x - 6 < 0\} = (-2, 3)$.

$x^2 - x - 6 < 0$ means that the answer must be negative.
 As $x^2 - x - 6 > 0$ (is positive) for all values of x in the interval $A = (-\infty, -2) \cup (3, \infty)$ then $x^2 - x - 6$ must be negative for all values of x in the interval $(-2, 3)$.