Melbourne School of Engineering

Engineering Systems Design 2

Mechanics Assignment 2 (due in Workshop 10)

Kinetics of particles

This assignment will contribute towards 2% of your assessment for this subject. Question 1 is not for direct assessment but is used as a basis for Question 3 and is also required for successful completion of the in-class assessment in Workshop 10 (worth 1%). You will get 1% if you can write the program to obtain the graph required by Question 2. You will get another 1% if you can guide Spiderman to land on the roof of building B (see Question 3).

There is no need to submit anything. The demonstrator will check Questions 2 and 3 during the start (Part A) **of Workshop 10.**

1. As a promising team of engineers, you have been chosen by the large engineering company *Ooi Inc.* to lead an assault on rival company *Evans Pty. Ltd.* The *Ooi Inc.* primary weapon, known as the "Workshop Mass Displacer" (WMD for short) will be the spearhead of the assault and is shown in a simplified diagram in Figure 1(a).

   The WMD is a timber, spring powered ball launcher and your final goal in Workshop 10 will be to determine the setup of the WMD required to take out the CEO of *Evans Pty. Ltd* with one shot. The setup of the WMD will be covered in detail in Workshop 10.

   For the moment, the CEO of *Ooi Inc.* has asked you to model the trajectory of a ball as it is fired and leaves the WMD taking into account aerodynamic drag (air resistance). He needs to make sure that the ball he has chosen is a good projectile to use in the assault. He'd do it himself but he's having a caffe latte (one sugar!).

   In Workshop 9, you should have developed an ODE solving function `MyEuler04` that implements the iterative Euler method. You also should have written a function `MyODE04` that contained the ODEs that describe the motion of a projectile in 2-Dimensions neglecting aerodynamic drag. The purpose of this question is to extend the ODEs to handle the more realistic case with drag.

   Consider the projectile in Figure 1(b), fired at an angle of θ from ground level, with an initial velocity of $\vec{V_0}$ with horizontal and vertical components $u_0 := u(t = 0)$ and $v_0 := v(t = 0)$, respectively.

   Senior technicians at *Ooi Inc.* have determined the governing equations of the projectile,

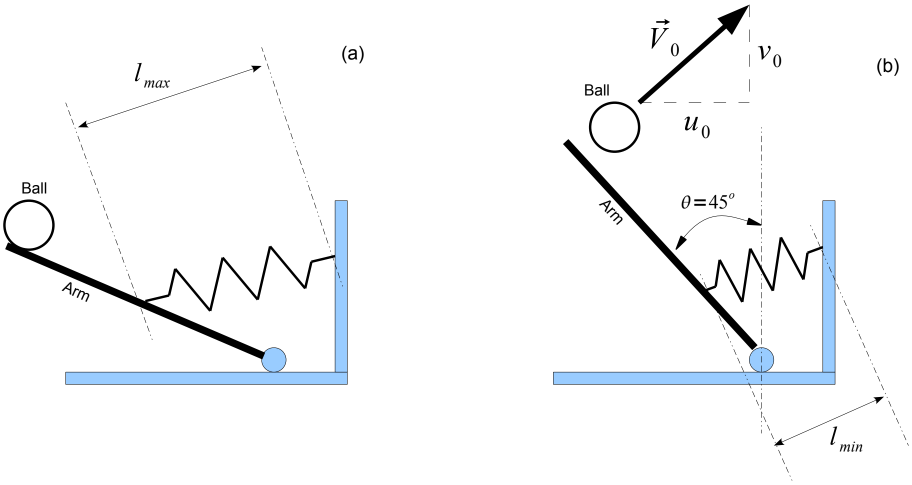

Figure 1: Catapult setup

taking into account the effects of aerodynamic drag, to be given by

$$\frac{dx}{dt} = u \tag{1}$$

$$\frac{du}{dt} = -\frac{u}{m}|V|\frac{C_D}{2}\rho A \tag{2}$$

$$\frac{dy}{dt} = v \tag{3}$$

$$\frac{dv}{dt} = -g - \frac{v}{m}|V|\frac{C_D}{2}\rho A \tag{4}$$

where u and v are the instantaneous horizontal and vertical velocity components, $|V| = \sqrt{(u^2 + v^2)}$, $C_D = 0.5$, $\rho = 1.29$ kg/m^3 and $A = \pi r^2$. The projectile (ball) has mass 12.7 gms and its radius is 2.54 cm. They are depending on you to create MATLAB code that will solve these equations for the trajectory of the projectile.

You need to write three files :

- `Step05.m` - a script that sets up the initial conditions and calls the Euler ODE solver;
- `MyEuler05.m` - a function that performs the Euler method of numerical integration;
- `MyODE05.m` - a function that describes the 2-D projectile ODEs taking into account aerodynamic drag.

It is important that your files have these names as they will be called by a simulation function in Workshop 10.

Note : If you have successfully completed Step 4 from Workshop 9, you can simply

- reuse your `MyEuler04.m` function as `MyEuler05.m` ; and

- modify the ODEs in `MyODE04.m` to now describe the 2-D projectile equations with drag by expanding the vector of variables x to four elements and implementing equations (1)-(4).

Check that your code is correct by plotting the trajectory of the projectile (i.e. the path of the projectile in the x,y axis) and ensuring that you get the same answer with the in-built MATLAB function `ode45` for the initial conditions

$$u(t=0) = 20\cos 45^o$$
$$v(t=0) = 20\sin 45^o$$
$$y(t=0) = x(t=0) = 0$$

2. The governing equation for a pendulum of mass m, on a string which is being shortened at a rate of $V = -dL/dt$ ms^{-1} (see Fig. 2) is given by

$$L\frac{d\omega}{dt} + 2\frac{dL}{dt}\omega + g\sin\theta = 0 \tag{5}$$

where $L(t)$ is the length of the string and

$$\omega = d\theta/dt \tag{6}$$

is the angular velocity. Use the MATLAB program that you wrote in Workshop 9 to solve Eqs. (5) and (6) using the Euler method. You are given that $L(t) = (50 - 5t)$ m, $g = 9.8$ m/s^2, $\omega(t=0) = 0$ and $\theta(t=0) = -30^o$. Plot $\theta(t)$ and $\omega(t)$ for $0 \le t \le 8$. Check your answer using the `ode45()` function in MATLAB. Your plot should look very similar to Fig. 3 (where ω and θ are plotted in radians). If you can produce this graph using your MATLAB program, you will receive 1% of the mark. Note that all you should need to do is to make very slight modifications to the program that you wrote in Part B of Workshop 9.

3. You are a consultant for a engineering company who specialises in superhero problems. One night, while you are having a Caffe Latte (one sugar) and finishing off your dinner, you receive a call from Spiderman (on your spanking new prototype iPhone 5) asking you to help him. Spiderman needs to get to building B from building A (see Fig. 4). The idea is for him to swing from Building A and shortening his web at a constant rate of V m/s and then let go of his web after t_f seconds. After he lets go of his web, his momentum should carry him to Building B. So you can model his trajectory initially as a pendulum on a string that is being shortened (Eqs. (5) and (6)) before he lets go of his web, and as a spherical particle in free flight governed by

$$\frac{du}{dt} = -u\frac{C_D\rho 0.85}{2m}\sqrt{u^2+v^2} \tag{7}$$
$$\frac{dv}{dt} = -g - v\frac{C_D\rho 0.85}{2m}\sqrt{u^2+v^2} \tag{8}$$

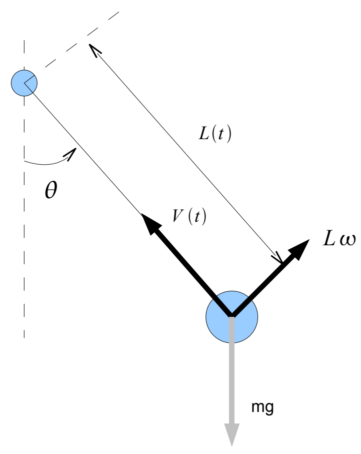

Figure 2: Mass on a string

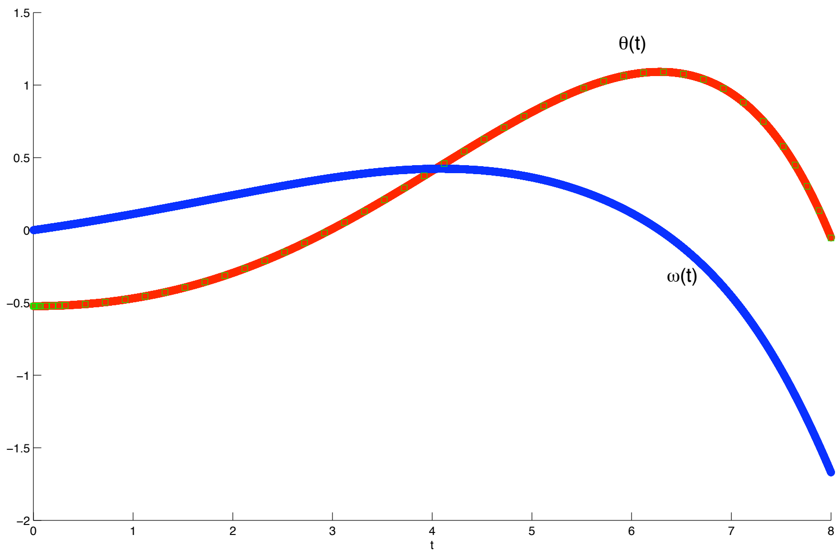

Figure 3: Solution to Question 2

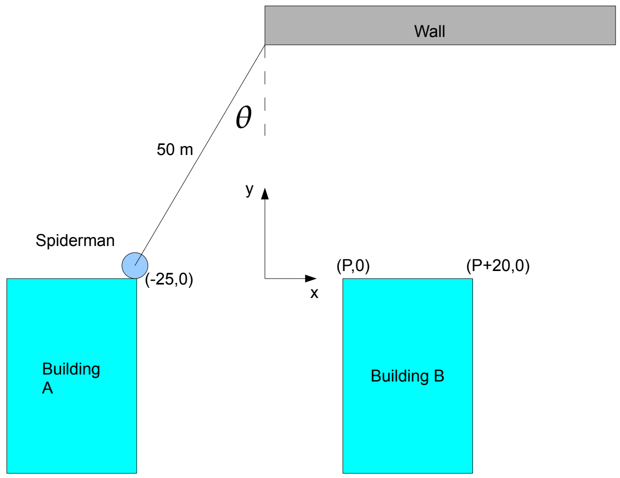

Figure 4: Swinging spiderman

where

$$u = dx/dt \tag{9}$$
$$v = dy/dt \tag{10}$$

after he lets go of his web. Equations (7) and (8) assume that Spiderman has surface area 0.85 m^2. Use $C_D = 0.5$ and $\rho = 1.29$ kg/m^3. You also have the top-secret information that Spiderman weighs 80 kg.

Note that building B is located a distance $(P + 25)$ m from building A and the width of building B is only 20m wide so it is not easy for him to land (he does not have much margin for error). Good ol Spidey has a 50 m web which he can shorten at a constant rate of V m/s and he has attached the web to a wall. So the length of his web at any point in time is given by

$$L(t) = 50 - Vt. \tag{11}$$

At time $t = 0$, the angle $\theta = -30^o$.

Your task is to find (using trial and error) $V(t)$ and how long must he hold on to his web for (t_f seconds) before he lets go so that he will land safely to Building B.

You might like to use the fact that while Spiderman is attached to the web (and just as he lets go of his web), his x and y coordinates are given by

$$x = (50 - Vt)\sin\theta \tag{12}$$
$$y = 50\cos(30^o) - (50 - Vt)\cos\theta. \tag{13}$$

and his velocity components are given by

$$u = -V\sin\theta + \omega(50 - Vt)\cos\theta \tag{14}$$

$$v = V\cos\theta + \omega(50 - Vt)\sin\theta \tag{15}$$

Different groups will be given different value of $20 \leq P \leq 40$ at the start of Workshop 10. You will have 10 minutes to give your demonstrators your value of V and t_f that will get Spiderman safely to Building B. Please note that you should already have the program to solve Eqs. (7)-(10) from Question 1. All you need to do is to change the relevant parameters. The input (initial conditions) would come from the program you wrote for Question 2 (again you have to make the appropriate changes to the relevant parameters).

During this week, please prepare your code with the relevant values of your parameters. You must be ready to run your code once your demonstrator gives you a value of P at the start of Workshop 10. You might also want to do a few "trial runs" and note down what values of V and t_f that will get Spiderman to Building B for values of $20 \leq P \leq 40$.