

Assignment 4

● Graded

21 Hours, 19 Minutes Late

Student

James La Fontaine

Total Points

51 / 51 pts

Question 1

(no title)

7 / 7 pts

1.1 (no title)

7 / 7 pts

 + 7 pts Point adjustment

Question 2

(no title)

44 / 44 pts

2.1 (no title)

10 / 10 pts

 + 10 pts Point adjustment

2.2 (no title)

7 / 7 pts

 + 7 pts Point adjustment

2.3 (no title)

7 / 7 pts

 + 7 pts Point adjustment

2.4 (no title)

10 / 10 pts

 + 10 pts Point adjustment

2.5 (no title)

10 / 10 pts

 + 10 pts Point adjustment

No questions assigned to the following page.

MAST30027 Modern Applied Statistics Assignment 4

Tutorial:Wed 1-2PM, Yidi Deng

James La Fontaine

2023-10-24

No questions assigned to the following page.

Question 1

(a)

1.

$$x_i \sim N(75, \frac{1}{\tau}) \quad \text{for } i=1, \dots, 100$$

$$\Rightarrow f(x_i | \tau) \propto \tau^{1/2} e^{-\frac{\tau}{2}(x_i - 75)^2}$$

$$\tau \sim \text{Gamma}(2, 1)$$

$$\Rightarrow f(\tau) = \frac{1}{\Gamma(2)} \tau e^{-\tau} = \tau e^{-\tau}$$

$$\text{a) } P(\tau | x_1, \dots, x_n) \propto P(x_1, \dots, x_n | \tau) P(\tau)$$

$$\propto \tau e^{-\tau} \prod_{i=1}^{100} \tau^{1/2} e^{-\frac{\tau}{2}(x_i - 75)^2}$$

$$\propto \tau^{50} e^{-\tau} e^{-\frac{\tau}{2} \sum_{i=1}^{100} (x_i - 75)^2}$$

$$\propto \tau^{51} e^{-\tau} \left[1 + \frac{\sum_{i=1}^{100} (x_i - 75)^2}{2} \right]$$

> (posterior_rate = 1 + (sum((X-75)^2)) / 2)

>

> 1805.65

$$\implies (\tau | X_1, \dots, X_n) \sim \text{Gamma}(52, 1805.65)$$

No questions assigned to the following page.

(b)

$$\text{Let } \beta = 1 + \sum_{i=1}^{100} \frac{(x_i - 75)^2}{2}$$

$$\begin{aligned}
b) P(\tilde{x}|x) &= \int p(\tilde{x}|\tau) p(\tau|x) d\tau \\
&= \int \frac{\tau^{1/2}}{\sqrt{2\pi}} e^{-\tau/2} (\tilde{x} - 75)^2 \frac{\beta^{52}}{\Gamma(52)} \tau^{51} e^{-\tau\beta} d\tau \\
&= \frac{\beta^{52}}{\sqrt{2\pi} \Gamma(52)} \int \tau^{\frac{103}{2}} e^{-\frac{\tau}{2}(\tilde{x} - 75)^2 - \tau\beta} d\tau \\
&= \frac{\Gamma(\frac{105}{2}) \beta^{52}}{\sqrt{2\pi} \Gamma(52)} \int \tau^{\frac{105}{2}-1} e^{-\tau \left(\frac{(\tilde{x} - 75)^2}{2} + \beta \right)} \frac{1}{\Gamma(\frac{105}{2})} d\tau \\
&= \frac{\Gamma(\frac{105}{2}) \beta^{52}}{\sqrt{2\pi} \Gamma(52) \left(\frac{(\tilde{x} - 75)^2}{2} + \beta \right)^{\frac{105}{2}}} \int \frac{\left(\frac{(\tilde{x} - 75)^2}{2} + \beta \right)^{\frac{105}{2}}}{\Gamma(\frac{105}{2})} \tau^{\frac{105}{2}-1} e^{-\tau \left(\frac{(\tilde{x} - 75)^2}{2} + \beta \right)} d\tau \\
&= \frac{\Gamma(\frac{105}{2}) \beta^{52}}{\sqrt{2\pi} \Gamma(52) \left(\frac{(\tilde{x} - 75)^2}{2} + \beta \right)^{\frac{105}{2}}} \quad \text{as the integral of the Gamma pdf} = 1 \\
&= \frac{\Gamma(\frac{105}{2}) \beta^{52}}{\Gamma(\frac{104}{2}) \sqrt{2\pi}} \left(\frac{(\tilde{x} - 75)^2}{2} + \beta \right)^{-\frac{105}{2}} \\
&= \frac{\Gamma(\frac{105}{2}) \beta^{52}}{\Gamma(\frac{104}{2}) \sqrt{2\pi}} \left(\beta \left(\frac{(\tilde{x} - 75)^2}{2\beta} + 1 \right) \right)^{-\frac{105}{2}} \\
&= \frac{\Gamma(\frac{105}{2}) \beta^{52}}{\Gamma(\frac{104}{2}) \sqrt{2\pi}} \beta^{-\frac{105}{2}} \left(1 + \frac{1}{104} \frac{52(\tilde{x} - 75)^2}{\beta} \right)^{-\frac{105}{2}} \\
&= \frac{\Gamma(\frac{105}{2})}{\Gamma(\frac{104}{2}) \sqrt{2\pi\beta}} \left(1 + \frac{1}{104} \frac{52(\tilde{x} - 75)^2}{\beta} \right)^{-\frac{105}{2}} \\
&= \frac{\Gamma(\frac{105}{2})}{\Gamma(\frac{104}{2}) \sqrt{104\pi\beta}} \left(1 + \frac{1}{104} \frac{52(\tilde{x} - 75)^2}{\beta} \right)^{-\frac{105}{2}}
\end{aligned}$$

$$\Rightarrow (\tilde{x}|x) \sim t(\nu, \alpha, \beta) \quad \text{where } \nu = 104,$$

$$\alpha = 75,$$

$$\beta = \frac{\beta}{52}$$

from (a)

$$\beta = \frac{105.65}{52}$$

$$= 34.724$$

No questions assigned to the following page.

Question 2

(a)

2. x_1, \dots, x_{100} and y_1, \dots, y_{150} i.i.d

$$x_i \sim N(\mu_1, 1^2)$$

$$y_i \sim N(\mu_2, (\frac{1}{3})^2)$$

$$\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} \sim N(\bar{\mu}, \bar{\Sigma}) \text{ with } \bar{\mu} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ and } \bar{\Sigma} = \begin{pmatrix} \frac{3}{5} & -\frac{2}{5} \\ -\frac{2}{5} & \frac{3}{5} \end{pmatrix}$$

$$f_{\bar{\mu}, \bar{\Sigma}}(\bar{x}) = \frac{1}{2\pi|\bar{\Sigma}|^{1/2}} e^{-\frac{1}{2}(\bar{x}-\bar{\mu})^T \bar{\Sigma}^{-1}(\bar{x}-\bar{\mu})}$$

a) Derive $P(\mu_1, \mu_2 | x_1, \dots, x_{100}, y_1, \dots, y_{150})$

and $P(\mu_2 | \mu_1, x_1, \dots, x_{100}, y_1, \dots, y_{150})$

$$\sum^{-1} = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$$

$$\begin{aligned} P(\mu_1, \mu_2) &\propto e^{(-\frac{1}{2}(\mu_1, \mu_2) \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix})} \\ &= e^{(-\frac{1}{2}(3\mu_1 + 2\mu_2, 2\mu_1 + 3\mu_2) \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix})} \\ &= e^{(-\frac{1}{2}(3\mu_1^2 + 4\mu_1\mu_2 + 3\mu_2^2))} \end{aligned}$$

$$\begin{aligned} P(\mu_1 | \mu_2, X, Y) &\propto P(X, Y | \mu_1, \mu_2) [\text{drop terms without } \mu_1] \\ &\propto e^{-\frac{1}{2} \left(\sum_{i=1}^{100} (x_i - \mu_1)^2 + 3\mu_1^2 + 4\mu_1\mu_2 \right)} \\ &\propto e^{-\frac{1}{2} \left(\sum_{i=1}^{100} (x_i^2 - 2x_i\mu_1 + \mu_1^2) + 3\mu_1^2 + 4\mu_1\mu_2 \right)} \\ &\propto e^{-\frac{1}{2} (100\mu_1^2 - 2\mu_1 \sum_{i=1}^{100} x_i + 3\mu_1^2 + 4\mu_1\mu_2)} \\ &\propto e^{-\frac{1}{2} (103\mu_1^2 - 2\mu_1 (\sum_{i=1}^{100} x_i - 2\mu_2))} \\ &\propto e^{-\frac{103}{2} \left(\mu_1^2 - 2\mu_1 \left(\frac{\sum_{i=1}^{100} x_i - 2\mu_2}{103} \right) \right)} \\ &\propto e^{-\frac{1}{2} \frac{\left(\mu_1 - \frac{\sum_{i=1}^{100} x_i - 2\mu_2}{103} \right)^2}{\frac{1}{103}}} \\ &\Rightarrow (\mu_1 | \mu_2, X, Y) \sim N\left(\frac{\sum_{i=1}^{100} x_i - 2\mu_2}{103}, \frac{1}{103}\right) \end{aligned}$$

$$\begin{aligned} P(\mu_2 | \mu_1, X, Y) &\propto P(X, Y | \mu_1, \mu_2) [\text{drop terms without } \mu_2] \\ &\propto e^{-\left(\sum_{i=1}^{150} (y_i - \mu_2)^2 + 2\mu_1\mu_2 + \frac{3}{2}\mu_2^2 \right)} \\ &\propto e^{-\left(\sum_{i=1}^{150} (y_i^2 - 2y_i\mu_2 + \mu_2^2) + \frac{3}{2}\mu_2^2 + 2\mu_1\mu_2 \right)} \\ &\propto e^{-\left(150\mu_2^2 - 2\mu_2 \sum_{i=1}^{150} y_i + \frac{3}{2}\mu_2^2 + 2\mu_1\mu_2 \right)} \\ &\propto e^{-\left(\frac{303}{2} \mu_2^2 - 2\mu_2 \left(\sum_{i=1}^{150} y_i - \mu_1 \right) \right)} \\ &\propto e^{-\frac{303}{2} \left(\mu_2^2 - 4\mu_2 \left(\frac{\sum_{i=1}^{150} y_i - \mu_1}{303} \right) \right)} \\ &\propto e^{-\frac{1}{2} \frac{\left(\mu_2 - \frac{\sum_{i=1}^{150} y_i - \mu_1}{303} \right)^2}{\frac{1}{303}}} \\ &\Rightarrow (\mu_2 | \mu_1, X, Y) \sim N\left(2 \frac{\sum_{i=1}^{150} y_i - \mu_1}{303}, \frac{1}{303}\right) \end{aligned}$$

No questions assigned to the following page.

```

x = scan(file="assignment4_prob2_x_2023.txt", what=double())
y = scan(file="assignment4_prob2_y_2023.txt", what=double())
length(x)

## [1] 100

length(y)

## [1] 150

mean(x)

## [1] 3.196441

mean(y)

## [1] -1.979781

```

2b

```

GibbsS <- function(mu1, mu2, nreps){

  Gsamples <- matrix(nrow=nreps, ncol=2)
  Gsamples[1,] <- c(mu1, mu2)

  # main loop
  for (i in 2:nreps) {
    mu1 <- rnorm(1, (sum(x) - 2*mu2)/103, sqrt(1/103))
    mu2 <- rnorm(1, 2*(sum(y) - mu1)/303, sqrt(1/303))
    Gsamples[i,] <- c(mu1, mu2)
  }

  return(Gsamples=Gsamples)
}

set.seed(456)

# sample size
nreps <- 500

# initial values
mu1 <- 0
mu2 <- 0

GibbsS1 = GibbsS(mu1, mu2, nreps)

```

No questions assigned to the following page.

```

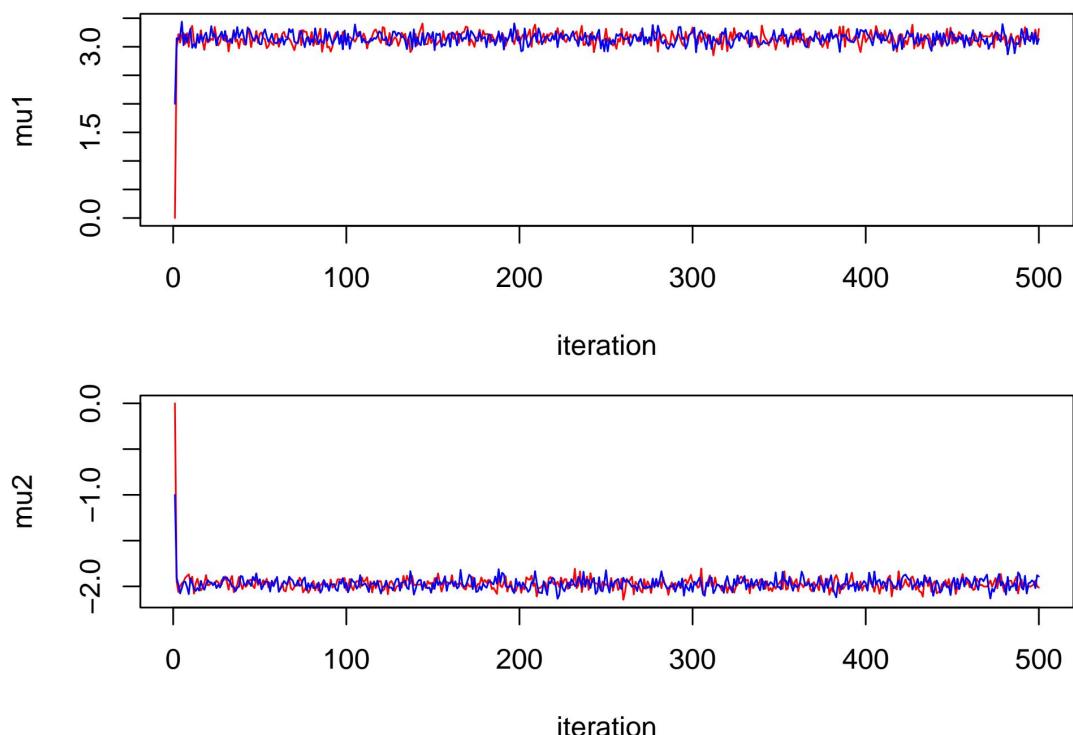
# initial values
mu1 <- 2
mu2 <- -1

GibbsS2 = GibbsS(mu1, mu2, nreps)

par(mfrow=c(2,1), mar=c(4,4,1,1))
plot(1:nreps, GibbsS1[,1], type="l", col="red", ylim = c(min(GibbsS1[,1],GibbsS2[,1]), max(GibbsS1[,1]),
  xlab = "iteration", ylab ="mu1")
points(1:nreps, GibbsS2[,1], type="l", col="blue")

plot(1:nreps, GibbsS1[,2], type="l", col="red", ylim = c(min(GibbsS1[,2],GibbsS2[,2]), max(GibbsS1[,2]),
  xlab = "iteration", ylab ="mu2")
points(1:nreps, GibbsS2[,2], type="l", col="blue")

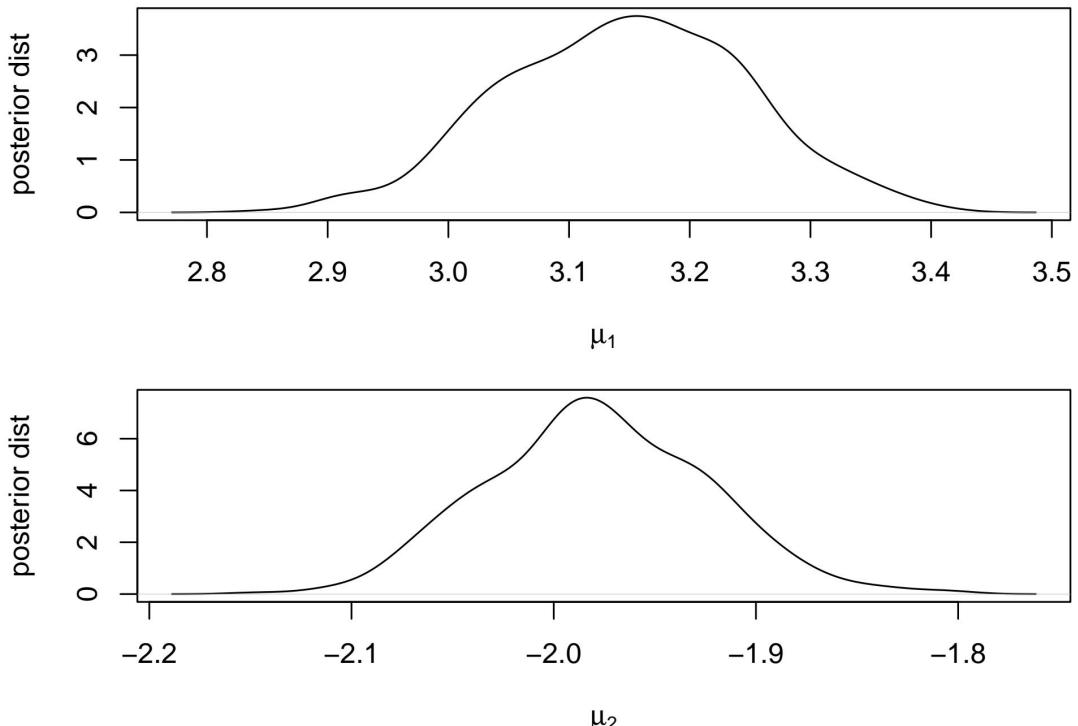
```



No questions assigned to the following page.

2c

```
par(mfrow=c(2,1), mar=c(4,4,1,1))
plot(density(GibbsS1[-(1:50),1]), ylab="posterior dist", xlab=expression(mu[1]), main="")
plot(density(GibbsS1[-(1:50),2]), ylab="posterior dist", xlab=expression(mu[2]), main="")
```



```
mean(GibbsS1[-(1:50),1])
## [1] 3.146983

mean(GibbsS1[-(1:50),2])
## [1] -1.978406

quantile(GibbsS1[-(1:50),1], probs= c(0.05, 0.95))
##      5%      95%
## 2.992497 3.313051

quantile(GibbsS1[-(1:50),2], probs= c(0.05, 0.95))
##      5%      95%
## -2.065780 -1.891366
```

No questions assigned to the following page.

Estimated marginal posterior mean for $\mu_1 = 3.1470$

Estimated marginal posterior mean for $\mu_2 = -1.9784$

90% credible interval for $\mu_1 = (2.9925, 3.3131)$

90% credible interval for $\mu_2 = (-2.0658, -1.8914)$

No questions assigned to the following page.

2d

```

dbinorm <- function(x, mu, Si) {
  # x and mu are vectors length 2 and Si a 2x2 matrix
  # returns the density at x of a bivariate normal (mean mu, var Si)
  exp(-t(x - mu) %*% solve(Si, x - mu)/2)/2/pi/sqrt(det(Si))
}

##### MH algorithm #####
run_metropolis_MCMC <- function(startvalue, iterations){
  chain = array(dim = c(iterations+1,2))
  chain[1,] = startvalue
  for (i in 1:iterations){
    proposal = proposalfunction(chain[i,])

    probab = exp(posterior(proposal) - posterior(chain[i,]))
    if (runif(1) < probab){
      chain[i+1,] = proposal
    }else{
      chain[i+1,] = chain[i,]
    }
  }
  return(chain)
}

# propose new parameter values
proposalfunction <- function(param){
  return(rnorm(2,mean = param, sd= c(0.1,0.1)))
}

# evaluate log posterior at given parameter values
posterior <- function(param){
  return (likelihood(param) + prior(param))
}

# evaluate log prior at given parameter values
prior <- function(param){
  mu_vec = c(0, 0)
  sigma_matrix = matrix(c(3/5, -2/5, -2/5, 3/5), ncol=2)
  return(log(dbinorm(param, mu_vec, sigma_matrix)))
}

# evaluate log likelihood at given parameter values
likelihood <- function(param){
  mu1 = param[1]
  mu2 = param[2]
  ll_x = sum(dnorm(x, mean=mu1, sd=1, log = TRUE))
  ll_y = sum(dnorm(y, mean=mu2, sd=1/sqrt(2), log = TRUE))
  return(ll_x + ll_y)
}

```

No questions assigned to the following page.

```

set.seed(456)

nreps = 10000

# initial values
startvalue = c(0,0)
# simulate 10000 samples
MHchain1 = run_metropolis_MCMC(startvalue, nreps)

# initial values
startvalue = c(2,-1)
# simulate another 10000 samples
MHchain2 = run_metropolis_MCMC(startvalue, nreps)

# remove the first 5000 as burn-in
burnIn = 5000

# computing average acceptance probability
(acceptance1 = 1-mean(duplicated(MHchain1[-(1:burnIn),])))

## [1] 0.4373125

(acceptance2 = 1-mean(duplicated(MHchain2[-(1:burnIn),])))

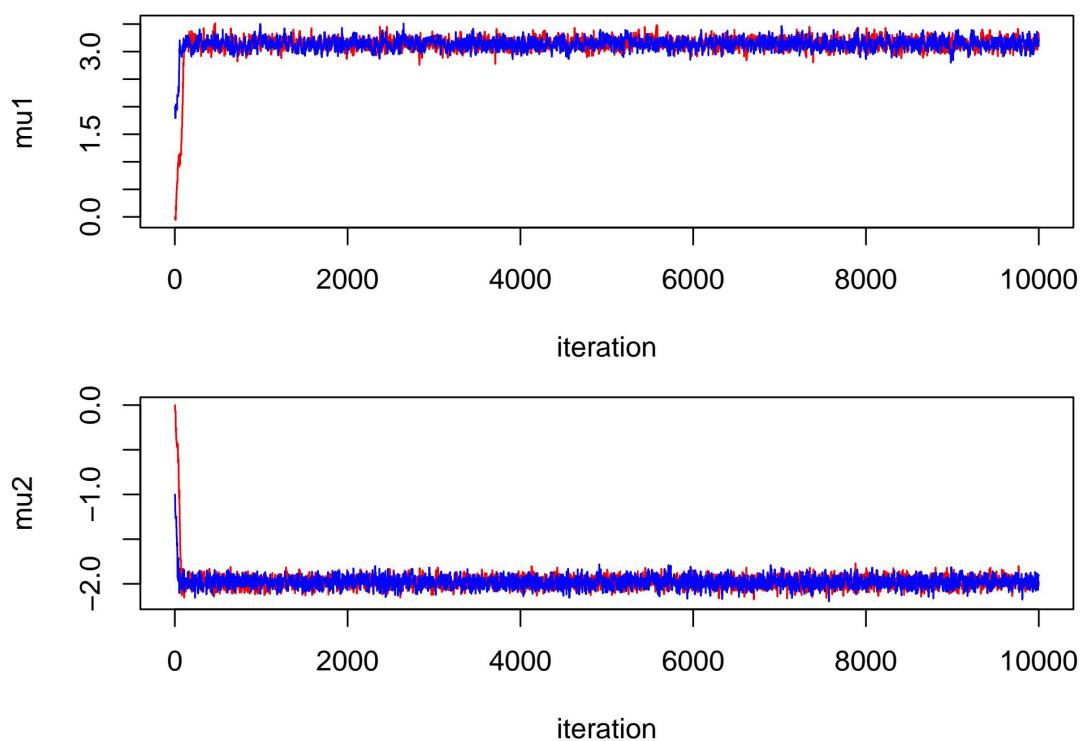
## [1] 0.4435113

par(mfrow=c(2,1), mar=c(4,4,1,1))
plot(1:(nreps+1), MHchain1[,1], type="l", col="red", ylim = c(min(MHchain1[,1], MHchain2[,1]), max(MHchain1[,1], MHchain2[,1])))
  xlab = "iteration", ylab ="mu1")
points(1:(nreps+1), MHchain2[,1], type="l", col="blue")

plot(1:(nreps+1), MHchain1[,2], type="l", col="red", ylim = c(min(MHchain1[,2], MHchain2[,2]), max(MHchain1[,2], MHchain2[,2])))
  xlab = "iteration", ylab ="mu2")
points(1:(nreps+1), MHchain2[,2], type="l", col="blue")

```

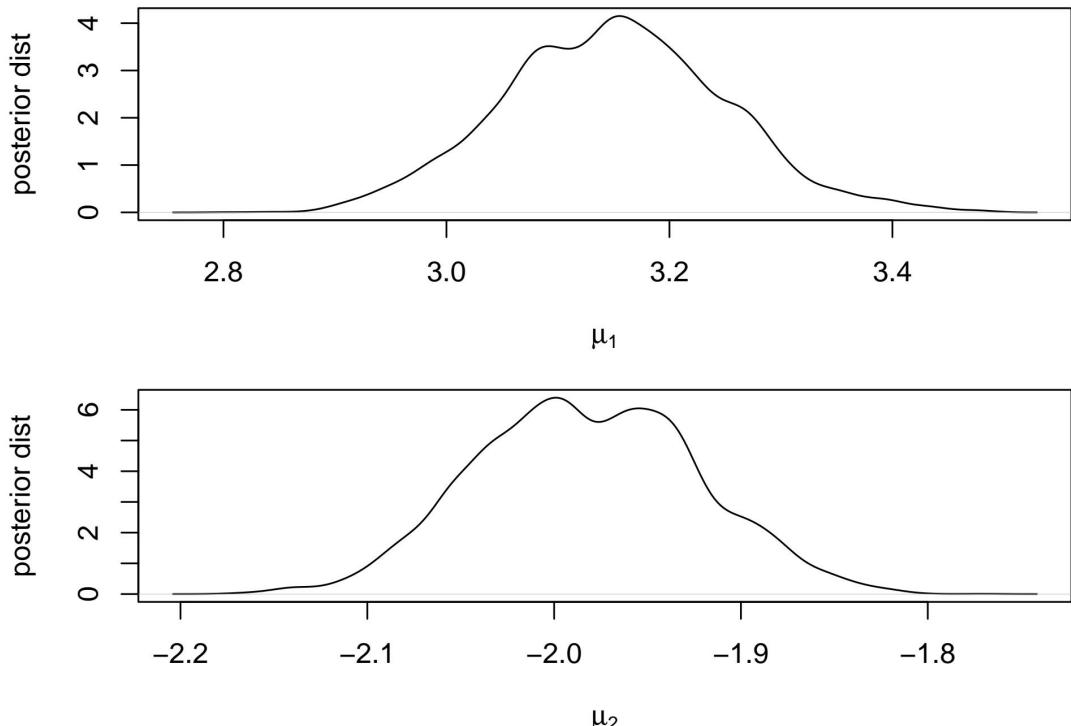
No questions assigned to the following page.



No questions assigned to the following page.

2e

```
par(mfrow=c(2,1), mar=c(4,4,1,1))
plot(density(MHchain1[-(1:burnIn),1]), ylab="posterior dist", xlab=expression(mu[1]), main="")
plot(density(MHchain1[-(1:burnIn),2]), ylab="posterior dist", xlab=expression(mu[2]), main="")
```



```
mean(MHchain1[-(1:burnIn),1])
## [1] 3.150803

mean(MHchain1[-(1:burnIn),2])
## [1] -1.982535

quantile(MHchain1[-(1:burnIn),1], probs= c(0.05, 0.95))
##      5%      95%
## 2.988247 3.312308

quantile(MHchain1[-(1:burnIn),2], probs= c(0.05, 0.95))
##      5%      95%
## -2.077089 -1.883055
```

No questions assigned to the following page.

Estimated marginal posterior mean for $\mu_1 = 3.1508$

Estimated marginal posterior mean for $\mu_2 = -1.9825$

90% credible interval for $\mu_1 = (2.9882, 3.3123)$

90% credible interval for $\mu_2 = (-2.0771, -1.8831)$

No questions assigned to the following page.

(f)

$$f) \quad \text{Log } q_1(\mu_1) \propto E_{\mu_2} \left[-\frac{1}{2} \left(\sum_{i=1}^{100} (x_i - \mu_1)^2 + 3\mu_1^2 + 4\mu_1\mu_2 \right) \right]$$

$$\text{Log } q_2(\mu_2) \propto E_{\mu_1} \left[- \left(\sum_{i=1}^{100} (y_i - \mu_2)^2 + \frac{3}{2}\mu_2^2 + 2\mu_1\mu_2 \right) \right]$$

From (a)

$$\Rightarrow q_1(\mu_1) : \text{pdf of } N(\mu_1^*, \sigma_{\mu_1}^2), \quad \mu_1^* = \frac{\sum_{i=1}^{100} x_i - 2E_{\mu_2}(\mu_2)}{103}, \quad \sigma_{\mu_1}^2 = \frac{1}{103}$$

$$\Rightarrow q_2(\mu_2) : \text{pdf of } N(\mu_2^*, \sigma_{\mu_2}^2), \quad \mu_2^* = 2 \frac{\sum_{i=1}^{100} y_i - E_{\mu_1}(\mu_1)}{303}, \quad \sigma_{\mu_2}^2 = \frac{1}{303}$$

$$E_{\mu_2}(\mu_2) = \mu_2^*$$

$$E_{\mu_1}(\mu_1) = \mu_1^*$$

No questions assigned to the following page.

(g)

2.

$$\begin{aligned}
 g) \text{ E L B O } (q_{\mu_1}^*(\mu_1), q_{\mu_2}^*(\mu_2)) &= E_{\mu_1, \mu_2} [\log p(x, y | \mu_1, \mu_2) - \log (q_{\mu_1}^*(\mu_1) q_{\mu_2}^*(\mu_2))] \\
 &= E_{\mu_1, \mu_2} [\log (p(x|\mu_1) p(y|\mu_2) p(\mu_1, \mu_2)) - \log q_{\mu_1}^*(\mu_1) - \log q_{\mu_2}^*(\mu_2)] \\
 &= E_{\mu_1, \mu_2} [\log (p(x|\mu_1) p(y|\mu_2) p(\mu_1, \mu_2))] - E_{\mu_1, \mu_2} [\log q_{\mu_1}^*(\mu_1)] - E_{\mu_1, \mu_2} [\log q_{\mu_2}^*(\mu_2)] \\
 &\quad \left. \begin{aligned}
 E_{\mu_1, \mu_2} [\log q_{\mu_1}^*(\mu_1)] &\propto -\frac{1}{2} \log \sigma_1^{2*} - \frac{E_{\mu_1}[(\mu_1 - \mu_1^*)^2]}{2 \sigma_1^{2*}} \\
 &\propto -\frac{1}{2} \log \sigma_1^{2*} - \frac{\sigma_1^{2*}}{2 \sigma_1^{2*}} \\
 &\propto -\frac{1}{2} \log \sigma_1^{2*} \\
 \Rightarrow E_{\mu_1, \mu_2} [\log q_{\mu_2}^*(\mu_2)] &\propto -\frac{1}{2} \log \sigma_2^{2*} \\
 \\
 &\propto E_{\mu_1, \mu_2} \left[-\frac{1}{2} \sum_{i=1}^{100} (x_i - \mu_1)^2 - \sum_{i=1}^{150} (y_i - \mu_2)^2 - \frac{3}{2} \mu_1^2 - 2 \mu_1 \mu_2 - \frac{3}{2} \mu_2^2 \right] + \frac{1}{2} \log \sigma_1^{2*} + \log \sigma_2^{2*}
 \end{aligned} \right. \\
 &\text{from (a)} \\
 &\propto -\frac{103}{2} (\sigma_1^{2*} + \mu_1^{2*}) + \mu_1^* \sum_{i=1}^{100} x_i - \frac{303}{2} (\sigma_2^{2*} + \mu_2^{2*}) \\
 &\quad + 2 \mu_2^* \sum_{i=1}^{150} y_i - 2 \mu_1 \mu_2 + \log \sigma_1^{2*} + \log \sigma_2^{2*} + \text{const.} \\
 &\propto -\frac{103}{2} (\mu_1^{2*}) + \mu_1^* \sum_{i=1}^{100} x_i - \frac{303}{2} (\mu_2^{2*}) + 2 \mu_2^* \sum_{i=1}^{150} y_i - 2 \mu_1 \mu_2 + \text{const.}
 \end{aligned}$$

No questions assigned to the following page.

2h

```
# x,y: data
# mu10, mu20 : prior for mu
# initial values for mu1*, mu2*,: mu1.vi.init, mu2.vi.init
# epsilon : If the ELBO has changed by less than epsilon, the CAVI algorithm will stop
# max.iter : maximum number of iteration
cavi.normal <- function(x, y, mu1.vi.init, mu2.vi.init, epsilon=1e-5, max.iter=100) {

  mu1.vi = mu1.vi.init
  mu2.vi = mu2.vi.init

  # store the ELBO for each iteration
  elbo = c()

  # I will store mu*, sigma2*, a*, b* for each iteration
  mu1.vi.list = mu2.vi.list = c()

  # compute the ELBO using initial values of mu*
  elbo = c(elbo, (-103/2) * mu1.vi^2 + mu1.vi*sum(x) - (303/2) * mu2.vi^2 + 2*mu2.vi*sum(y) - 2*mu1.vi*
  mu1.vi.list = c(mu1.vi.list, mu1.vi)
  mu2.vi.list = c(mu2.vi.list, mu2.vi)

  # set the change in the ELBO with 1
  delta.elbo = 1

  # number of iteration
  n.iter = 1

  # If the elbo has changed by less than epsilon, the CAVI will stop.
  while((delta.elbo > epsilon) & (n.iter <= max.iter)){

    # Update mu.vi and sigma2.vi
    mu1.vi = (sum(x) - 2*mu2.vi)/103
    mu2.vi = (2*sum(y)-2*mu1.vi)/303

    # compute the ELBO using the current values of mu*
    elbo = c(elbo, (-103/2) * mu1.vi^2 + mu1.vi*sum(x) - (303/2) * mu2.vi^2 + 2*mu2.vi*sum(y) - 2*mu1.vi*
    mu1.vi.list = c(mu1.vi.list, mu1.vi)
    mu2.vi.list = c(mu2.vi.list, mu2.vi)

    # compute the change in the elbo
    delta.elbo = elbo[length(elbo)] - elbo[length(elbo)-1]

    # increase the number of iteration
    n.iter = n.iter + 1
  }

  return(list(elbo = elbo, mu1.vi.list = mu1.vi.list,
             mu2.vi.list=mu2.vi.list))
}
```

No questions assigned to the following page.

```

}

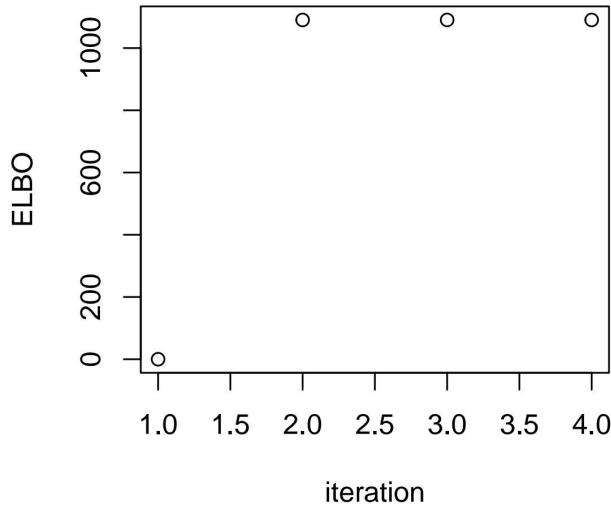
set.seed(456)

cavi1 = cavi.normal(x, y, mu1.vi.init=0, mu2.vi.init=0, epsilon=1e-5, max.iter=100)
cavi.res = cavi1
cavi.res$elbo

## [1] 0.000 1090.321 1090.397 1090.397

plot(cavi.res$elbo, ylab='ELBO', xlab='iteration')

```



```

print(paste("mu1* and mu2* = (",
            round(cavi.res$mu1.vi.list[length(cavi.res$mu1.vi.list)],2), ",",
            round(cavi.res$mu2.vi.list[length(cavi.res$mu2.vi.list)],2), ")",
            sep=""))

## [1] "mu1* and mu2* = (3.14,-1.98)"

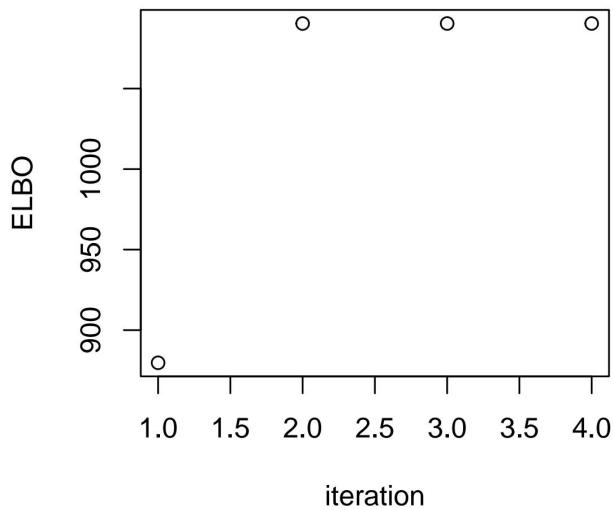
cavi2 = cavi.normal(x, y, mu1.vi.init=2, mu2.vi.init=-1, epsilon=1e-5, max.iter=100)
cavi.res = cavi2
cavi.res$elbo

## [1] 879.7223 1090.3781 1090.3968 1090.3968

```

No questions assigned to the following page.

```
plot(cavi.res$elbo, ylab='ELBO', xlab='iteration')
```



```
print(paste("mu1* and mu2* = (",
            round(cavi.res$mu1.vi.list[length(cavi.res$mu1.vi.list)],2), ",",
            round(cavi.res$mu2.vi.list[length(cavi.res$mu2.vi.list)],2), ")",
            sep=""))
```



```
## [1] "mu1* and mu2* = (3.14,-1.98)"
```