School of Mathematics and Statistics

MAST10007 Linear Algebra, Semester 1 2020

Written assignment 4

Submit your assignment online in Canvas before 12 noon on Monday 11th May.

Name:

Student ID:

- This assignment is worth $1\frac{1}{9}\%$ of your final MAST10007 mark.
- Your solutions should be neatly handwritten in blue or black pen, then uploaded as a single PDF file in **GradeScope**.
- Full explanations and working must be shown in your solutions.
- Marks may be deducted in every question for incomplete working, insufficient justification of steps and incorrect mathematical notation.
- You must use methods taught in MAST10007 Linear Algebra to solve the assignment questions.

New submission guidelines:

- This assignment is being handled using a similar process to that planned for the final exam so you can start to become familiar with it.
- If you have access to a printer, then you should print out this assignment sheet and handwrite your solutions into the answer boxes.
- If you do not have access to a printer, but you can annotate a PDF file using an iPad/Android tablet/Graphics tablet or using Adobe Acrobat, then annotate your answers directly in the boxes on the assignment PDF and save a copy for submission.
- Otherwise, you may handwrite your answers as normal on blank paper and then scan for submission.
- The answer boxes should typically provide sufficient space for your solution, but if you find you need extra space please take a blank sheet of paper and continue your solution there, clearly indicating which question this refers to. Also indicate in the corresponding box that the solution continues at the end.
- Scan your assignment to a PDF file using your mobile phone or scanner, then upload by going to the Assignments menu on Canvas and submit the PDF to the **GradeScope** tool by first selecting your PDF file and then clicking on 'Upload pdf'.

1. Let $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \in \mathbb{R}^2$ and $M_{2,2}$ be the vector space of 2×2 matrices with real entries. Define

$$S = \{A \in M_{2,2} \mid \text{there exists } r \in \mathbb{R} \text{ such that } A\mathbf{v} = r\mathbf{v}\}.$$

(a) Write down an element of S that is not a scalar multiple of the identity matrix I.

(b) Prove that S is a subspace of $M_{2,2}$.

(c) Find a basis for S and calculate the dimension of S.

Hint. there exists $r \in \mathbb{R}$ such that $A\mathbf{v} = r\mathbf{v} \Leftrightarrow \begin{bmatrix} -2 & 1 \end{bmatrix} A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0.$

2. Let $\mathcal{P}_2$ be the real vector space of polynomials of degree at most 2. Define

$$p_0(x) = \tfrac{1}{2}(x-1)(x-2), \quad p_1(x) = -x(x-2), \quad p_2(x) = \tfrac{1}{2}x(x-1).$$

(a) Prove that every polynomial $f(x) \in \mathcal{P}_2$ satisfies

$$f(x) = f(0)p_0(x) + f(1)p_1(x) + f(2)p_2(x).$$

(b) Use (a), or otherwise, to prove that $\{p_0(x), p_1(x), p_2(x)\}$ is a basis of $\mathcal{P}_2$.