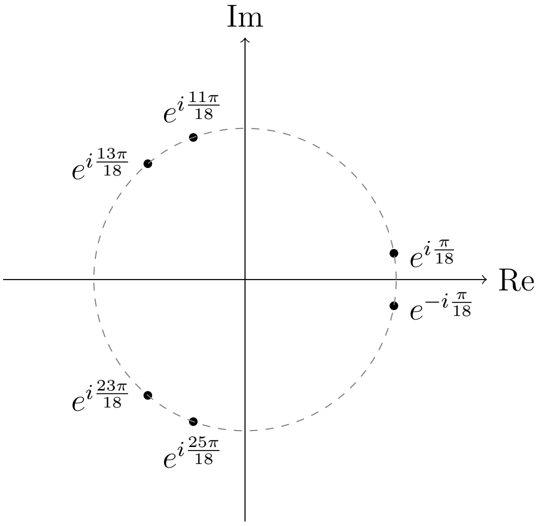
Assignment 2: Solutions and marking scheme

1. Problem 1: As this part varied individually for students, no solutions will be provided. Feedback is provided in WebWork.

   Problem 2: By the root finding formula $w \in \{e^{i\frac{\pi}{18}}, e^{i\frac{13\pi}{18}}, e^{i\frac{25\pi}{18}}\}$.

2. (a) By the root finding formula $B = \{e^{i\frac{-\pi}{18}}, e^{i\frac{11\pi}{18}}, e^{i\frac{23\pi}{18}}\}$. **1A**

   (b) **1A**



   (c) By the quadratic formula

$$z^3 = \frac{\sqrt{3} \pm i}{2}$$

   so $z \in A \cup B = \{e^{i\frac{\pi}{18}}, e^{i\frac{13\pi}{18}}, e^{i\frac{25\pi}{18}}, e^{i\frac{-\pi}{18}}, e^{i\frac{11\pi}{18}}, e^{i\frac{23\pi}{18}}\}$. **2A**

   (d) **1M** *Identify pairs. OK to use the diagram in (b).*

   Yes. The pairs are:

$$e^{-i\frac{\pi}{18}} = \overline{e^{i\frac{\pi}{18}}},$$
$$e^{i\frac{23\pi}{18}} = e^{-i\frac{13\pi}{18}} = \overline{e^{i\frac{13\pi}{18}}}$$
$$e^{i\frac{25\pi}{18}} = e^{-i\frac{11\pi}{18}} = \overline{e^{i\frac{11\pi}{18}}}.$$

   (e) $P(z) = (z - e^{i\frac{\pi}{18}})(z - e^{i\frac{13\pi}{18}})(z - e^{i\frac{25\pi}{18}})(z - e^{-i\frac{\pi}{18}})(z - e^{i\frac{11\pi}{18}})(z - e^{i\frac{23\pi}{18}})$.

   1A *Can still give consequential mark if answers to (a) and (c) are incorrect.*

   (f) **1M** *Proof.* **1M** *Justifying steps in argument.*

$$\begin{aligned}(z - e^{i\theta})(z - e^{-i\theta}) &= z^2 - (e^{i\theta} + e^{-i\theta})z + e^{i\theta}e^{-i\theta} \\ &= z^2 - (e^{i\theta} + \overline{e^{i\theta}})z + e^0 \quad \text{[Example 1.52, Theorem 1.53(2)]} \\ &= z^2 - 2\,\mathrm{Re}(e^{i\theta})z + 1 \quad \text{[Theorems 1.39(1), 1.53(1)]} \\ &= z^2 - 2\cos(\theta)z + 1 \quad \text{[Definition of exponential polar form]}\end{aligned}$$ **1A**

   All coefficients real. Amazing!

   (g) [Not marked.] Using the results of (e) and (f):

$$P(z) = \left(z^2 - 2\cos\left(\frac{\pi}{18}\right)z + 1\right)\left(z^2 - 2\cos\left(\frac{13\pi}{18}\right)z + 1\right)\left(z^2 - 2\cos\left(\frac{11\pi}{18}\right)z + 1\right).$$

1L *For the whole assignment: clear structure, and ALL mathematical notation is correct.*