Assignment 3: Solutions and marking scheme

1. As this question was completed online in WebWork, no marks are shown. Solutions included where possible.

   Problem 1: As this part varied individually for students, no solutions will be provided.

   Problem 2: As this part varied individually for students, no solutions will be provided.

   Problem 3: (i) $f_2(x,y) = (-x,-y)$
   $f_3(x,y) = (y,-x)$
   $f_4(x,y) = (x,y)$

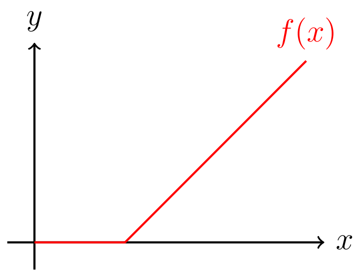
   (ii) $f_4 = f_1 \circ f_3 = f_3 \circ f_1$ is the identity function on $\mathbb{R}^2$, so f_3 is the inverse of f_1.

2. Define $g : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ defined by $f(x,y) = (y,-x)$. Then:

$$f \circ g(x,y) = f(g(x,y)) = f(y,-x) = (-(-x),y) = (x,y), \quad \boxed{\text{1M}}$$
$$g \circ f(x,y) = g(f(x,y)) = g(-y,x) = (x,-(-y)) = (x,y). \quad \boxed{\text{1M}}$$

   which shows that *both* of the equalities in Definition 2.15 hold.

3. (a) [Not marked]



   (b) $g \circ f(x) = g(f(x)) = \max(x-1,0)+1 = \max(x,1)$ or $g(f(x)) = \begin{cases} x, & \text{if } x \geq 1 \\ 1 & \text{if } x < 1. \end{cases}$

   $\boxed{\text{1A}}$

   (c) $f \circ g(x) = f(g(x)) = \max((x+1)-1,0)+1 = \max(x,0) = x$ since $x \geq 0$. $\boxed{\text{1A}}$

   (d) It is not true for all x that $g \circ f(x) \neq x$ since, for example $g(f(0)) = 1 \neq 0$. Hence g cannot be the inverse of f. $\boxed{\text{1M}}$ *Explain why g not inverse of f.*

   Note: This example illustrates why we need to check that *both* of the equalities in Definition 2.15 hold.

4. $\boxed{\text{1M}}$ *For giving reasonable explanations in (c), (d) and (e).*

   (a) [Not marked] $OA = \cos(\theta)$ by definition of cosine.

   (b) [Not marked] $AC = \sin(\theta)$ by definition of sine.

   (c) $\tan(\theta) = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{BD}{OB} = \dfrac{BD}{1} \Rightarrow BD = \tan(\theta)$. $\boxed{\text{1A}}$

   (d) Using the fact that angle $\angle OEF = \theta$:

$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{OF}{EF} = \frac{1}{EF} \Rightarrow EF = \frac{1}{\tan(\theta)} = \cot(\theta) \quad \boxed{\text{1A}} \quad \frac{1}{\tan(\theta)} \textit{ is OK}$$

(e) Using the fact that angle $\angle OEF = \theta$:

$$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{OF}{OE} = \frac{1}{OE} \Rightarrow OE = \frac{1}{\sin(\theta)} = \text{cosec}(\theta) \qquad \boxed{\textbf{1A}} \quad \frac{1}{\sin(\theta)} \textit{ is OK}$$

$\boxed{\textbf{1L}}$ *For the whole question: clear structure, and ALL mathematical notation is correct.*