Melbourne School of Engineering

Engineering Systems Design 2

Mechanics Assignment 1

The maximum number of marks for this assignment is 60. It will contribute towards 2% of the assessment for this subject.

1. **[10 marks in total]**
   Consider the spring system shown in Fig. 1

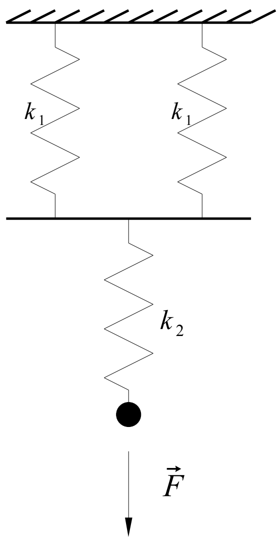

Figure 1: Spring system

(a) Show that the the value of k_1 to ensure that a force F displaces the spring system by a distance x is given by

$$k_1 = \frac{Fk_2}{2k_2x - 2F} \tag{1}$$

(b) If you are given $k_2 = 100$ N/m and $F = 4$ N, use Eq. (1) to evaluate the value of k_1 such that the total extension of the spring system is 6 cm?

(c) Can you use Eq. (1) to find a *realistic* value of k_1 such that the total displacement of the spring system is 6 cm if $k_2 = 10$ N/m and $F = 4$ N? If no *realistic* answer exists, can you explain why?

2. **[10 marks in total]**

(Problem 3.13 in Soustas-Little Inman and Balint)
Figure 2 shows a barge tied to a dock. While drinking a **Caffe Latte one sugar** , the captain holds this barge stationary against the river current by applying a force with the barge engine. The resultant (net) force of the river and the barge engine, $\vec{R}$, has a magnitude of 4000 N and is acting through point C.

(a) if $\theta = 10^o$, calculate the tension in the ropes tied to points A and B.

(b) Use MATLAB to calculate and plot the tensions in the rope for $0^o \leq \theta \leq 20^o$. Is there a value of θ where the tensions in both ropes are equal? If so, what is the value of θ?

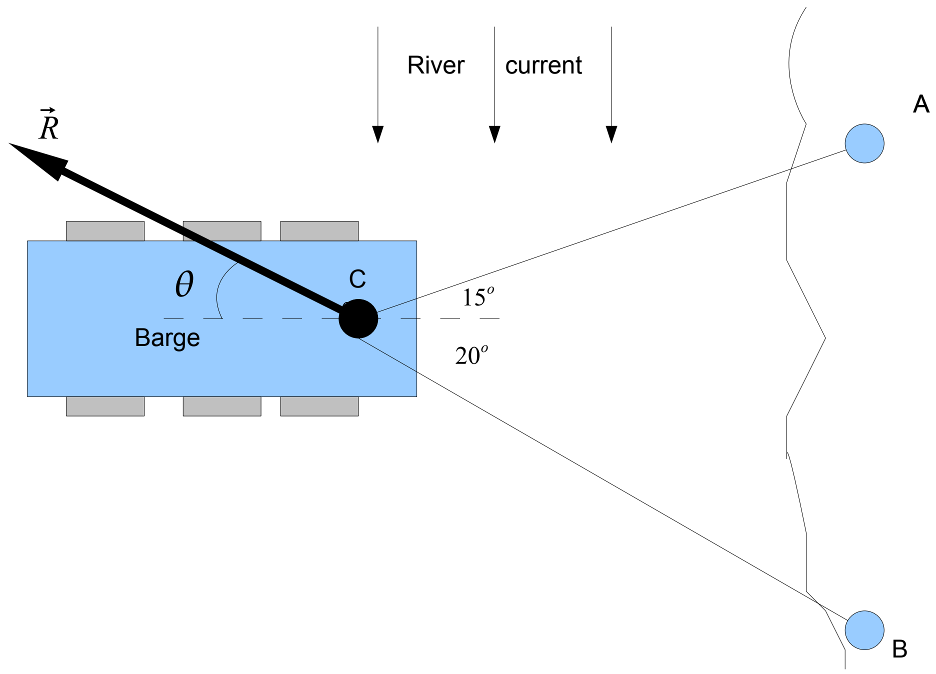

Figure 2: Barge tied to a dock

3. [**18 marks in total**] (Problem 3/122 from Meriam and Kraige)

The mass center a 10 kg arm OC is located at G (see Fig. 3), and the spring constant $k = 1.2$ kN/m is unstretched when $\theta = 0$. The motion of this mechanism occurs in a vertical plane. The value of m_2 is 3kg. The length of point 0 to D, $l_{OD} = 480$ mm. You are also given that l_{0G}=150 mm, l_{0A}=250 mm, l_{0C}=360 mm.

(a) Show that the length of the unstretched spring, $l_0 = 230$ mm.

(b) Show that the length of the spring for any angle θ is given by

$$l_{AD} = \sqrt{(l_{OD} - l_{OA}\cos\theta)^2 + (l_{OA}\sin\theta)^2} \tag{1}$$

(c) Show that

$$\alpha = \arcsin\left(\frac{l_{OA}\sin\theta}{l_{AD}}\right) \tag{2}$$

(d) Explain why the force in the spring, F_s can be written as

$$F_s = k(l_{AD} - l_O) \tag{3}$$

(e) Use the information above and obtain an expression for the applied moment M required for static equilibrium.

(f) Use MATLAB to plot M over the range $0^o \le \theta \le 180^o$.

(g) From your graph, determine the value of θ for which $M = 0$ (if any) and the minimum and maximum values of M along with the corresponding values of θ at which these extremes occur.

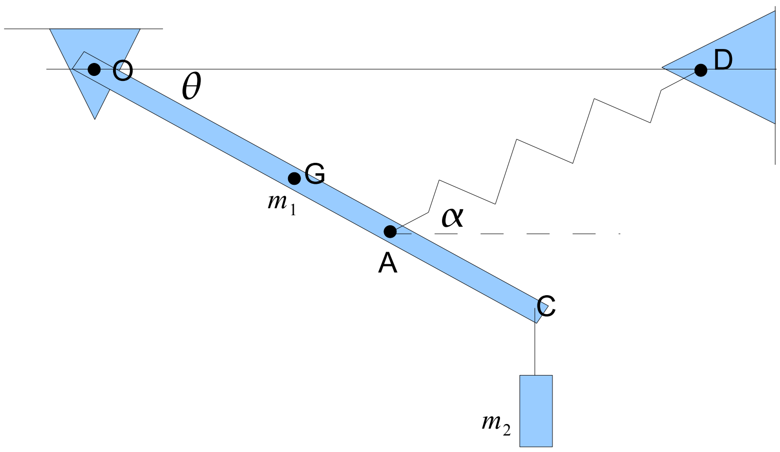

Figure 3: A bar held by a spring

4. **[12 marks in total]**

You have been hired by an engineering consulting company, *Ooi Inc*, to perform analysis over the simplified bridge structure shown in Fig. 4. F_1 is the wind load and F_2 is the combined centre of gravity of all loadings on the bridge. One fine Summer morning, while you are at a coffee shop drinking a **Caffe Latte one sugar** , you are told that $\vec{F}_2$ is usually constant but $\vec{F}_1$ can vary with time. Note that Members AB, BE, CE and CD are all of the same length.

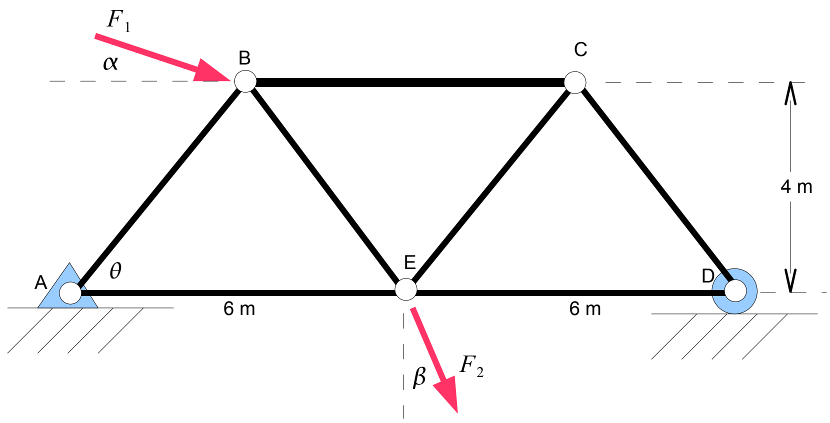

Figure 4: Simplified bridge structure

(a) Perform equilibrium analysis on each joint and show that the reaction forces and the internal forces can be obtained by solving the following matrix equation.

$$\begin{bmatrix} 1 & 0 & \cos\theta & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \sin\theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\cos\theta & 0 & 1 & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sin\theta & 0 & 0 & -\sin\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -\cos\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\sin\theta & -\sin\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\cos\theta & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin\theta & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & -\cos\theta & \cos\theta & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sin\theta & \sin\theta & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} R_{Ax} \\ R_{Ay} \\ N_{AB} \\ N_{AE} \\ N_{BC} \\ N_{BE} \\ N_{CE} \\ N_{CD} \\ N_{DE} \\ R_{Dy} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -F_1\cos\alpha \\ F_1\sin\alpha \\ 0 \\ 0 \\ 0 \\ 0 \\ -F_2\sin\beta \\ F_2\cos\beta \end{Bmatrix}$$

(b) Treat the entire truss as a rigid body and calculate the reaction forces at A and D, R_{Ax}, R_{Ay} and R_{Dy}. Assume that $F_1 = 6$ kN, $F_2 = 8$ kN and $\alpha = \beta = 0$.

(c) Knowing, R_{Ax}, R_{Ay} and R_{Cy} from the previous section, calculate (by hand) the force in each member of the loaded truss.

(d) Use MATLAB to solve the matrix equation given in part (a) for $F_1 = 6$ kN, $F_2 = 8$ kN and $\alpha = \beta = 0$ check that your answer is similar to what you obtained in parts (b) and (c). This part validates your MATLAB program and should give you confidence that everything is fine thus far.

(e) You have been asked to investigate if it is feasible to construct the whole structure out of Aluminium. You know that Young's Modulus for Aluminium is $E = 70 \times 10^9$ N/m^2 and its ultimate stress $\sigma_u = 110 \times 10^6$ N/m^2. What is the minimum dimension of the square beam such that the truss structure will not fail?

5. **[10 marks in total]**

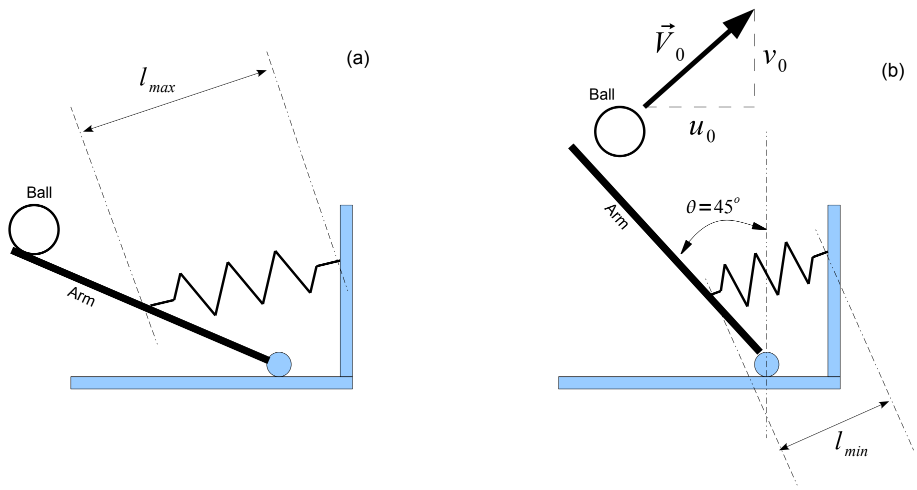

Figure 5: Catapult system. (a) System with spring extended, (b) system at launch point.

In Workshop 10, you will need to model the launch velocity of a catapult system shown in Fig. 5. The way the catapult works is as follows

- The arm is pulled back, putting potential energy into the spring(s) (see Fig. 5(a)). Note that even though only one spring is shown in Fig. 5, there could be more than 1 spring in the actual system.
- When the springs are extended s ufficiently the arm i s released converting the potential energy stored in the springs into kinetic energy by contracting and hence accelerating the arm and ball forward. (Note: Not all the energy is going into the ball)
- The arm is stopped by a bar (not shown in Fig. 5) such that the angle in which the ball ball is released, θ is 45º (Fig. 5(b)).

You are required to write a MATLAB function that predicts the launch component of velocity, u_0 & v_0, given properties of the spring configurations. But before you do that, you will need a bit of theory. Even though the theory given below might seem long, the MATLAB function that you are required to write should not take more than one A4 page.

Theory

In order to predict the values of u_0 & v_0, we will assume that all of the potential energy stored in the springs is converted into kinetic energy. The first step is to determine the energy in a spring. This is a function of the spring constant k, the extension x and the loading built into the spring L

$$E_{\text{spring}} = \frac{1}{2}kx^2 + Lx. \tag{4}$$

To complicate things, the springs do not finish at an extension of 0 (i.e. its rest length, l_{rest}), instead they finish slightly extended still, so you have:

$$E_{\text{spring}} = \frac{1}{2}k(x_{\text{max}}^2 - x_{\text{min}}^2) + L(x_{\text{max}} - x_{\text{min}}) \tag{5}$$

where x_{max} the maximum extension and x_{min} is the minimum extension. If the length of the spring at maximum and minimum extensions are denoted by l_{max} and l_{min} respectively, then

$$x_{\text{max}} = l_{\text{max}} - l_{\text{rest}}, \tag{6}$$

$$x_{\text{min}} = l_{\text{min}} - l_{\text{rest}}. \tag{7}$$

Note that if you have more than one spring in the setup, then you can just add up the energy of each individual spring, i.e. the total energy in the springs is given by

$$E_{\text{total}} = \sum_{i=1}^{N} E_{\text{spring},i} \tag{8}$$

where $E_{\text{spring},i}$ is the energy of each individual spring given by Eq. (5) and N is the number of springs in the system. For example, if $N = 2$, Eq. (8) will expand to

$$\begin{aligned} E_{\text{total}} &= E_{\text{spring},1} + E_{\text{spring},2} \\ &= \frac{1}{2}k_1(x_{\text{max},1}^2 - x_{\text{min},1}^2) + L_1(x_{\text{max},1} - x_{\text{min},1}) \\ &+ \frac{1}{2}k_2(x_{\text{max},2}^2 - x_{\text{min},2}^2) + L_2(x_{\text{max},2} - x_{\text{min},2}) \end{aligned}$$

Since we will assume that the total potential energy, E_{total}, is completely converted into kinetic energy, we will need to express kinetic energy in terms of velocity. Kinetic energy is related to the angular velocity by

$$K = E_{\text{total}} = \frac{I\omega^2}{2} \tag{9}$$

where

$$\omega = \frac{d\theta}{dt} \tag{10}$$

and I is the moment of inertia and ω is the angular velocity. The moment of inertia is a function of the object and the axis of rotation. It is a fairly complex process to determine the formula for I, so we will just give them to you here

$$I_{\text{arm}} = \frac{1}{3} m_{\text{arm}} l_{\text{arm}}^2 \tag{11}$$

and

$$I_{\text{ball}} = m_{\text{ball}} l_{\text{ball}}^2. \tag{12}$$

where m is the mass of the arm or ball, l_{arm} is the length of the arm and l_{ball} is the distance of the ball from the pivotal point. Note that the arm is modeled as an infinitely thin rod of evenly distributed mass rotating about its end, and the ball is modeled as a point mass. You will learn more about moment of inertia in later years of your course. The total moment of inertia for a system is merely the sum of the parts of the system, i.e.

$$I = I_{\text{arm}} + I_{\text{ball}}. \tag{13}$$

The magnitude of the launch velocity ($\vec{V_0}$) of the ball can now be calculated using the following formula

$$V_0 = \omega l_{\text{ball}}. \tag{14}$$

The x and y component of velocity can now be determine by

$$u_0 = V_0 \cos(45^o) \tag{15}$$

$$v_0 = V_0 \sin(45^o) \tag{16}$$

Your task

You are required to write a function that takes as its input, l_{rest}, l_{max}, l_{min}, k and L for each spring. **All these parameters must be arrays** because there could be more than 1 spring in the system. The output of your function should be the u_0 and v_0 component of the launch velocity of the ball. The function declaration and the first few lines of your MATLAB function should look like

```
--------------------------------------------------------------------
function [u0,v0]=GetVelocityFromSetup(lrest,lmax,lmin,k,L)
N=length(k);
xmax = zeros(1,N);
xmin = zeros(1,N);
Etotal=0;

% Fill in the rest of the function here!!!!
--------------------------------------------------------------------
```

Note that the MATLAB function must be called *GetVelocityFromSetup()* to work correctly with the catapult simulator in Workshop 10. The flow chart of your function would look something like Fig. 6. The parameters that you will need to use in your program are given in the last section entitled **Constants**. To ensure that your program is working properly, the solution for one set of parameters is shown below

```
>> lrest(1)=0.149;lmax(1)=0.247;lmin(1)=0.177;k(1)=287.5;L(1)=6.903;
>> lrest(2)=0.149;lmax(2)=0.250;lmin(2)=0.180;k(2)=308.9;L(2)=9.617;
>> [u0,v0]=GetVelocityFromSetup(lrest,lmax,lmin,k,L)

u0 =

    5.3923


v0 =

    5.3923

>>
```

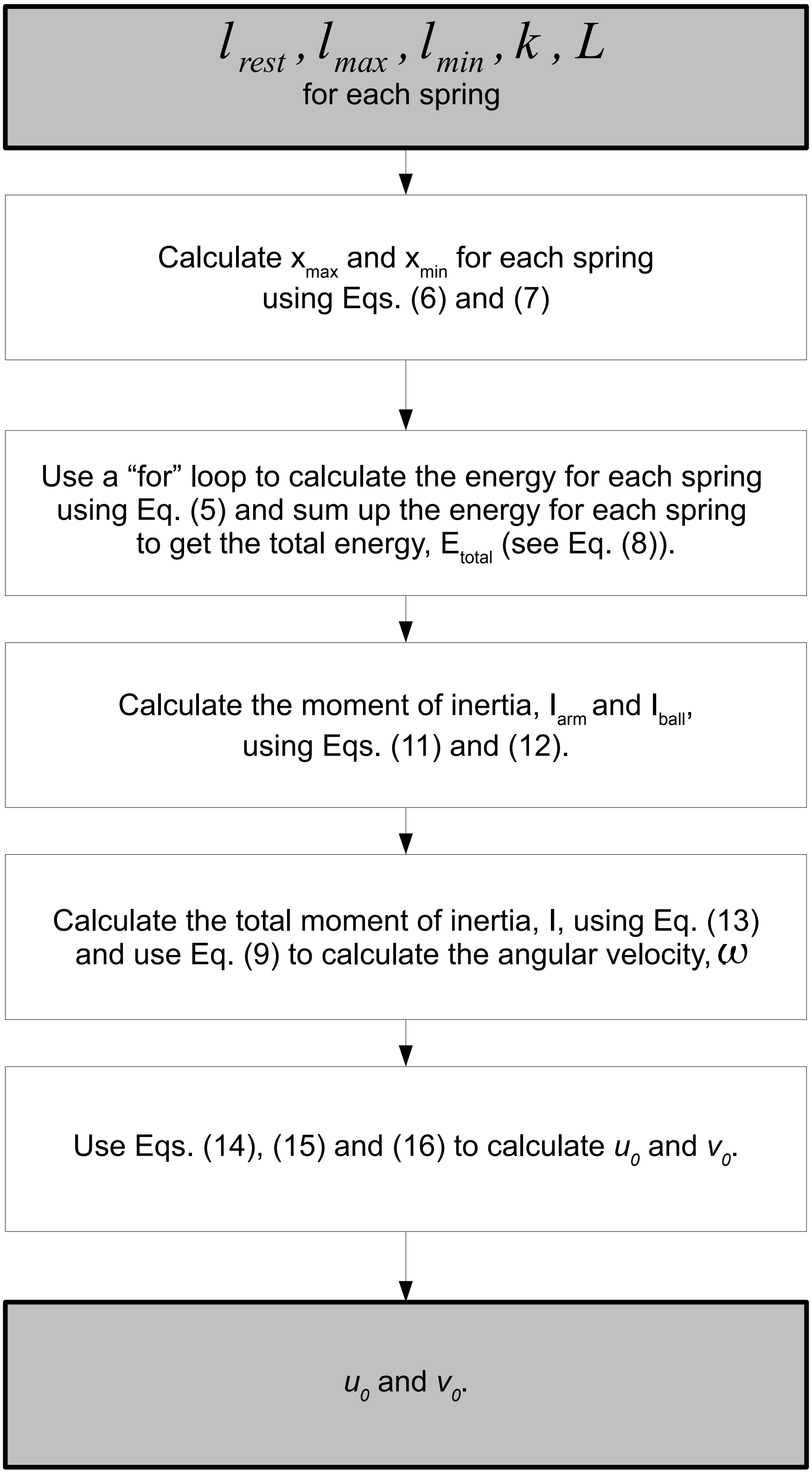

Figure 6: Flowchart

Constants

The parameters of the configuration are given below

Universal

$$g = 9.81 \text{ ms}^{-2} \text{ Gravity}$$

$$\rho = 1.29 \text{ kg/m}^3 \text{ Air Density}$$

Ball

$$r = 2.54 \text{ cm - Radius}$$

$$m_{\text{ball}} = 12.7 \text{ gm} \quad \text{Mass of the ball}$$

$$l_{\text{ball}} = 0.53 \text{ m} \quad \text{Distance of the ball from the pivot}$$

$$h = 0.40 \text{ m} \quad \text{Launch height of the ball}$$

Catapult Data

$$l_{\text{arm}} = 0.51 \text{ m} \quad \text{Length of the arm}$$

$$m_{\text{arm}} = 0.388 \text{ kg} \quad \text{Mass of the arm}$$

Spring 1

$$k_1 = 287.5 \text{ N/m} \quad \text{Spring constant}$$

$$L_1 = 6.903 \text{ N} \quad \text{Loading of the spring}$$

$$l_{\text{rest},1} = 14.9 \text{ cm} \quad \text{Rest length of the spring}$$

Spring 2

$$k_2 = 308.9 \text{ N/m} \quad \text{Spring constant}$$

$$L_2 = 9.617 \text{ N} \quad \text{Loading of the spring}$$

$$l_{\text{rest},2} = 14.9 \text{ cm} \quad \text{Rest length of the spring}$$