ENGR10003 – Engineering Systems Design 2
Assignment Cover Sheet

THE UNIVERSITY OF MELBOURNE

Assignment Title: **Mechanics Assignment 1**

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Question 1

1a) Springs in series and in parallel:

in parallel

$k_{eq(p)} = k_1 + k_1$

$\Rightarrow k_{eq(p)} = 2k_1$

in series

$\frac{1}{k_{total}} = \frac{1}{k_2} + \frac{1}{k_{eq(p)}}$

$F = k_{total}\, x$

$\Rightarrow \frac{1}{k_{total}} = \frac{x}{F}$

$\Rightarrow \frac{x}{F} = \frac{1}{k_2} + \frac{1}{2k_1}$

$K_{eq} = \frac{2k_1k_2}{k_2+2k_1}$

$\Rightarrow \frac{x}{F} = \frac{2k_1 + k_2}{k_2 k_1}$

$\Rightarrow x k_2 k_1 = (2k_1 + k_2)F$

$\Rightarrow x k_2 k_1 = 2k_1F + k_2F$

$\Rightarrow x k_2 k_1 - 2k_1F = k_2F$

$\Rightarrow k_1(k_2x - 2F) = Fk_2$

$\Rightarrow \boxed{k_1 = \frac{Fk_2}{k_2x - 2F}}$

b) $k_1 = \frac{Fk_2}{k_2x - 2F}$

$\Rightarrow k_1 = \frac{(4N)(100N/m)}{2(100N/m)(0.06m) - 2(4N)}$

$\Rightarrow \boxed{k_1 = 100N/m}$

c) $k_1 = \frac{Fk_2}{k_2x - 2F}$

$\Rightarrow k_1 = \frac{(4N)(10N/m)}{2(10N/m)(0.06m) - 2(4N)}$

$\frac{400}{68} = \frac{200}{34} = \frac{100}{17}$

$\Rightarrow k_1 = \frac{40N^2/m}{-6.8N}$

$\Rightarrow k_1 = -\frac{100}{17} N/m$

$\boxed{\text{No realistic answer exists because k values cannot be negative}}$

Question 2

2a)

$$T_{Ax}\vec{i} + T_{Bx}\vec{i} - R_{x}\vec{i} = 0$$
$$T_{Ay}\vec{j} + R_{y}\vec{j} - T_{By}\vec{j} = 0$$

} as the system is in equilibrium

$$\Rightarrow R_x\vec{i} = T_{Ax}\vec{i} + T_{Bx}\vec{i} \quad (1)$$

$$\Rightarrow R_y\vec{j} = -T_{Ay}\vec{j} + T_{By}\vec{j} \quad (2)$$

$$(1)\ 4000\cos(10^\circ) = T_A\cos(15^\circ) + T_B\cos(20^\circ)$$

$$\Rightarrow 4000\cos(10^\circ) - T_A\cos(15^\circ) = T_B\cos(20^\circ)$$

$$\Rightarrow T_B = \frac{4000\cos(10^\circ) - T_A\cos(15^\circ)}{\cos(20^\circ)} \quad (3)$$

sub (3) into (2)

$$(2)\ 4000\sin(10^\circ) = -T_A\sin(15^\circ) + \left(\frac{4000\cos(10^\circ) - T_A\cos(15^\circ)}{\cos(20^\circ)}\right) \times \sin(20^\circ)$$

$$\Rightarrow 694.593 = -0.2588T_A + (4192.05 - 1.0279T_A) \times \sin(20^\circ)$$

$$\Rightarrow 694.593 = -0.2588T_A + 1433.77 - 0.3516T_A$$

$$\Rightarrow -739.177 = -0.6104T_A$$

$$\boxed{\Rightarrow T_A = 1210.97 \approx 1211\text{ N}}$$

$T_B = ?$

2b)
```
function [TA, TB] = CalcRangeOfTensions

theta = 0:0.5:20 ;
A = [cosd(15) cosd(20); sind(15) -sind(20)];

 for i = 1:length(theta)
      C = [4000*cosd(theta(i)); -4000*sind(theta(i))];
     x = inv(A)*C;
     TA(i) = x(1);
    TB(i) = x(2);
end

 plot(theta, TA, 'rs', theta, TB, 'go');
 grid on
 xlabel("theta (degrees)");
 ylabel(" F(N) ");

end
```

Yes, the tensions have an equal magnitude of 2097N with $\boxed{\theta = 2.5^\circ}$

Question 3

3a) length of unstretched spring is l_o

$l_{OD} = l_{OA} + l_o$

$\Rightarrow l_o = l_{OD} - l_{OA}$

$\Rightarrow l_o = 480mm - 250mm$

$\Rightarrow \boxed{l_o = 230mm}$

b) l_{AD} = length of unstretched spring

let l_{ADx} be horizontal component of l_{AD}

let l_{ADy} be vertical component of l_{AD}

$l_{AD} = \sqrt{(l_{ADx})^2 + (l_{ADy})^2}$

l_o = length of unstretched spring

since $l_o = l_{OD} - l_{OA}$,

$l_{ADx} = l_{OD} - l_{OA} \cdot \cos\theta$

since spring attaches from A to D (same vertical level as O), vertical component of l_{OA} is vertical component of spring

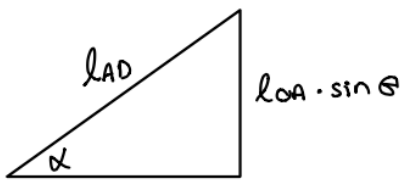
$\Rightarrow l_{ADy} = l_{OA} \cdot \sin\theta$

$\therefore \boxed{l_{AD} = \sqrt{(l_{OD} - l_{OA}\cos\theta)^2 + (l_{OA}\sin\theta)^2}}$

c) l_{AD} = length of stretched spring

$l_{OA} \cdot \sin\theta$ = vertical component of stretched spring

α = angle between stretched spring and horizontal component of spring



$\Rightarrow \sin(\alpha) = \dfrac{l_{OA} \cdot \sin\theta}{l_{AD}}$

$\Rightarrow \boxed{\alpha = \arcsin\left(\dfrac{l_{OA} \cdot \sin\theta}{l_{AD}}\right)}$

d) x = change in length of spring

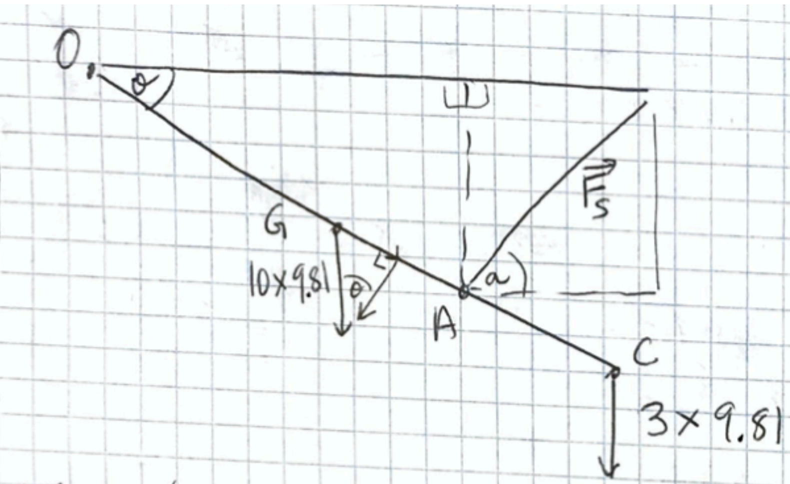
$\Rightarrow x$ = length of stretched spring − length of unstretched spring

$\Rightarrow x = l_{AD} - l_o$

$F_s = kx$

$\Rightarrow \boxed{F_s = k(l_{AD} - l_o)}$

3e.



~~Moment~~ $M_G = 10\times 9.81 \times \cos\theta \times L_{OG}$

$M_C = 3\times 9.81\times \cos\theta \times L_{OC}$

To find moment due to the spring:

Use $M_o = \pm|F_x r_y| \pm |F_y r_x|$ where:

$F_x = F_s \cos a$

$F_y = F_s \sin a$

$r_y = L_{AD}\sin a$

$r_x = L_{OD} - L_{AD}\cos a$

$M_S = L_{AD}\sin a\,(F_s\cos a) - F_s \sin a\,(L_{OD} - L_{AD}\cos a)$

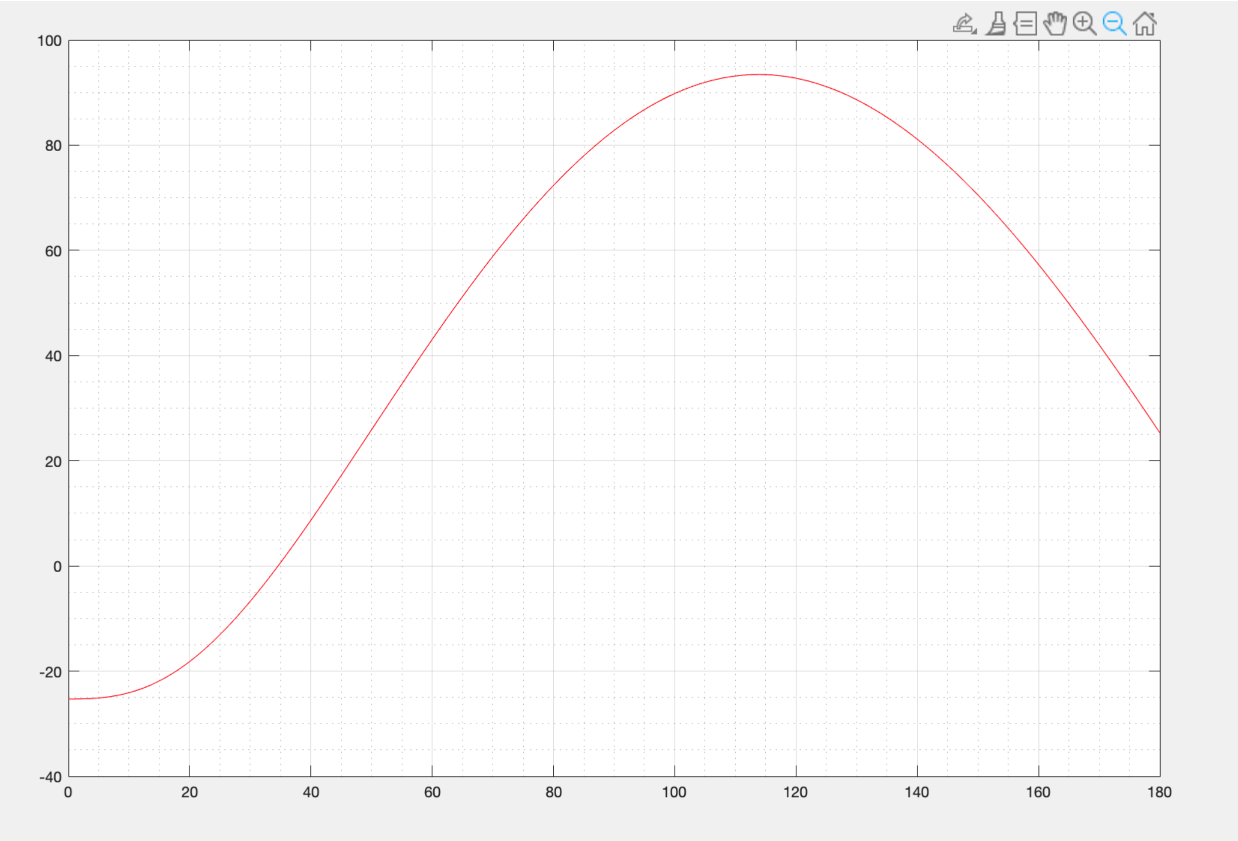
Overall moment: about point O

$M = 10\times 9.81\times\cos\theta\times L_{OG} + 3\times 9.81\times\cos\theta\times L_{OC}$
$+ L_{AD}\sin a\,(F_s\cos a) - F_s\sin a\,(L_{OD} - L_{AD}\cos a)$

For static equilibrium, $M = 0$:

$0 = M_G + M_C + M_S$

3f) Graph of theta vs M



Code:

```
theta=[0:180]
lo=230e-3
lod=480e-3
log=150e-3
loa=250e-3
loc=360e-3
k=1200
lad=sqrt((lod-loa.*cosd(theta)).^2+(loa.*sind(theta)).^2)
a=asind(loa.*sind(theta)./lad)
Fs=k.*(lad-lo)
m1=10
m2=3
g=9.81

Mg=-m1.*g.*cosd(theta).*log
Mc=-m2.*g.*cosd(theta).*loc
Ms=lad.*sind(a).*Fs.*cosd(a)+Fs.*sind(a).*(lod-lad.*cosd(a))

M=Mg+Mc+Ms

plot(theta,M,'r-')
grid on
grid minor
```

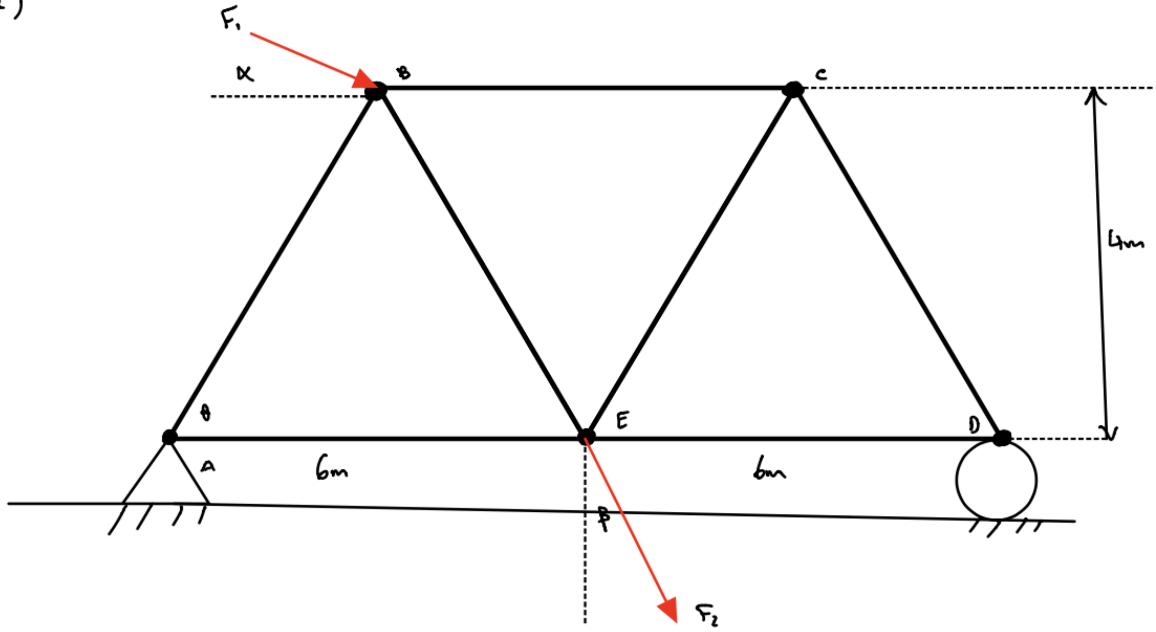
3g) $M=0$ for $\theta = 34.5°$

The minimum moment $= -25$ at $0°$

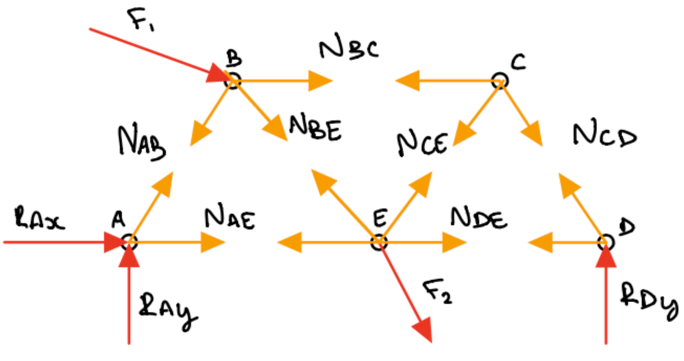
The maximum moment $= 93.5$ at $114°$

Question 4

4a)



Force diagram



Assume bars are in tension

Joint A

Sum of forces in x direction is zero

$\Sigma F_x = R_{Ax} + N_{AB} \cdot \cos\theta + N_{AE}$

Sum of forces in y direction is zero

$\Sigma F_y = R_{Ay} + N_{AB} \sin\theta = 0$

Joint B

Sum of forces in x direction is zero

$$\sum F_x = F_1 \cos\alpha + N_{BC} + N_{BE}\cos\theta - N_{AB}\cos\theta = 0$$

Sum of forces in y direction is zero

$$\sum F_y = -F_1 \sin\alpha - N_{BE}\sin\theta - N_{AB}\sin\theta = 0$$

Joint C

Sum of forces in x direction is zero

$$\sum F_x = -N_{BC} - N_{CE}\cos\theta + N_{CD}\cos\theta$$

Sum of forces in y direction is zero

$$\sum F_y = -N_{CE}\sin\theta - N_{CD}\sin\theta = 0$$

Joint D

Sum of forces in x direction is zero

$$\sum F_x = -N_{DE} - N_{CD}\cos\theta = 0$$

Sum of forces in y direction is zero

$$\sum F_y = R_{Dy} + N_{CD}\sin\theta = 0$$

Joint E

Sum of forces in x direction is zero

$$\sum F_x = -N_{AE} + N_{DE} - N_{BE}\cos\theta + N_{CE}\cos\theta + f_2 \sin\beta = 0$$

Sum of forces in y direction is zero

$$\sum F_y = -f_2\cos\beta + N_{BE}\sin\theta + N_{CE}\sin\theta = 0$$

Matrix form

$$
\begin{array}{cccccccccc|c}
R_{Ax} & R_{Ay} & N_{AB} & N_{AE} & N_{BC} & N_{BE} & N_{CE} & N_{CD} & N_{DE} & R_{Dy} & \\
\end{array}
$$

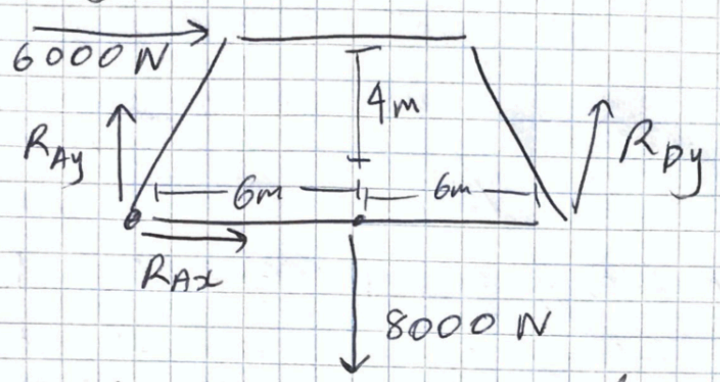
$$
\left[\begin{array}{cccccccccc|c}
1 & 0 & \cos\theta & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & \sin\theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\cos\theta & 0 & 1 & \cos\theta & 0 & 0 & 0 & 0 & -F_1\cos a \\
0 & 0 & -\sin\theta & 0 & 0 & -\sin\theta & 0 & 0 & 0 & 0 & F_1\sin a \\
0 & 0 & 0 & 0 & -1 & 0 & -\cos\theta & \cos\theta & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\sin\theta & -\sin\theta & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\cos\theta & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin\theta & 0 & 1 & 0 \\
0 & 0 & 0 & -1 & 0 & -\cos\theta & \cos\theta & 0 & 1 & 0 & -F_2\sin B \\
0 & 0 & 0 & 0 & 0 & \sin\theta & \sin\theta & 0 & 0 & 0 & F_2\cos B
\end{array}\right]
$$

Therefore:

$$
\begin{bmatrix}
1 & 0 & \cos\theta & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & \sin\theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\cos\theta & 0 & 1 & \cos\theta & 0 & 0 & 0 & 0 \\
0 & 0 & -\sin\theta & 0 & 0 & -\sin\theta & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & -\cos\theta & \cos\theta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\sin\theta & -\sin\theta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\cos\theta & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin\theta & 0 & 1 \\
0 & 0 & 0 & -1 & 0 & -\cos\theta & \cos\theta & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \sin\theta & \sin\theta & 0 & 0 & 0
\end{bmatrix}
\begin{Bmatrix}
R_{Ax} \\ R_{Ay} \\ N_{AB} \\ N_{AE} \\ N_{BC} \\ N_{BE} \\ N_{CE} \\ N_{CD} \\ N_{DE} \\ R_{Dy}
\end{Bmatrix}
=
\begin{Bmatrix}
0 \\ 0 \\ -F_1\cos a \\ F_1\sin a \\ 0 \\ 0 \\ 0 \\ 0 \\ -F_2\sin B \\ F_2\cos B
\end{Bmatrix}
$$

Solving this equation gives all the forces in the ~~st~~ truss structure.

4b) The moment (total) of the truss must be zero, so treating it as a rigid body: $a = B = 0$, $F_1 = 6000N$, $F_2 = 8000$



~~A~~ The horizontal and vertical forces must ~~all~~ also sum to zero, so:

$$0 = -8000 + R_{Dy} + R_{Ay} \qquad (1)$$

$$0 = 6000 + R_{Ax}, \quad R_{Ax} = -6000 \text{ N}. \quad (2)$$

Using point A to calculate the moment:

$$M_A = 6000 \times 4 + 8000 \times 6 + 12 \times R_{Dy}$$

Since the moment must be zero:

$$0 = 6000 \times 4 + 8000 \times 6 \cancel{+} 12 R_{Dy}$$

So $R_{Dy} = 6000$ N

And using eqn (1), $8000 = R_{Dy} + R_{Ay}$

So $R_{Ay} = 2000$ N.

4c) Using the equations from (a) and the values from (b):

☆ $-6000 + N_{AB}\cos\theta + N_{AE} = 0$ (1)

☆ $2000 + N_{AB}\sin\theta = 0$ (2)

$-N_{AB}\cos\theta + N_{BC} + N_{BE}\cos\theta = -6000\cos(0)$ (3)

☆ $-N_{AB}\sin\theta - N_{BE}\sin\theta = 6000\sin(0)$ (4)

$-N_{BC} - N_{CE}\cos\theta + N_{CD}\cos\theta = 0$ (5)

☆ $-N_{CE}\sin\theta - N_{CD}\sin\theta = 0$ (6)

☆ $-N_{CD}\cos\theta - N_{DE} = 0$ (7)

☆ $N_{CD}\sin\theta + 6000 = 0$ (8)

$-N_{AE} - N_{BE}\cos\theta + N_{CE}\cos\theta + N_{DE} = -8000\sin(0)$ (9)

$N_{BE}\sin\theta + N_{CE}\sin\theta = 8000\cos(0)$ (10)

Using (2) and (8): $N_{CD} = \frac{-6000}{\sin\theta}$

$N_{AB} = -\frac{2000}{\sin\theta}$

Using (1): $-6000 - \left(\frac{2000}{\sin\theta}\right)\cos\theta + N_{AE} = 0$

$N_{AE} = 6000 + \frac{2000\cos\theta}{\sin\theta}$

Using (4): $\left(\frac{2000}{\sin\theta}\right)\sin\theta - N_{BE}\sin\theta = 0$

$N_{BE} = \frac{2000}{\sin\theta}$

Using (6): $-N_{CE}\sin\theta - \left(\frac{-6000}{\sin\theta}\right)\sin\theta = 0$

$N_{CE} = \frac{6000}{\sin\theta}$

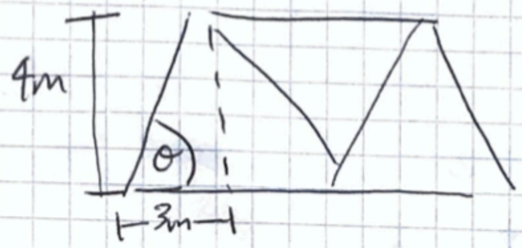
Using (7): $-\left(\frac{-6000}{\sin\theta}\right)\cos\theta - N_{DE} = 0$

$N_{DE} = \frac{6000\cos\theta}{\sin\theta}$

Using (3): $\frac{2000}{\sin\theta}(\cos\theta) + N_{BC} + \left(\frac{2000}{\sin\theta}\right)\cos\theta = -6000$

$N_{BC} = -\frac{4000\cos\theta}{\sin\theta} - 6000$

~~Sin~~ Using the diagram:



So $\tan\theta = \frac{4}{3}$

$\theta = \arctan\left(\frac{4}{3}\right)$

Using θ and the equations find:

$N_{AB} = -2500$

$N_{AE} = 7500$

$N_{BC} = -9000$

$N_{BE} = 2500$

$N_{CE} = 7500$

$N_{CD} = -7500$

$N_{DE} = 4500$

4e) First checking the maximum stretch $= 7500$ N

$\sigma_u = 110 \times 10^6 = \frac{F}{A}$

$110 \times 10^6 = \frac{7500}{r^2}$

$r = 0.00826$ m ~~for~~.

Each side minimum 8.26mm.

But also checking maximum compression $= 9000$ N.

Assuming it will buckle past 9000 N:

$F = \frac{\pi^2 E I}{L^2}$ where L is the longest length $= 6$m.

$9000 = \frac{\pi^2 \times 70 \times 10^9 \times I}{36}$ $I = \frac{d^4}{12}$ for a square cross section

$9000 = \frac{\pi^2 \times 70 \times 10^9 \times d^4}{36 \times 12}$

$d = 0.0487$m so the beam must have dimension min. 4.87cm × 4.87cm

Code for 4d- yielded the same results as calculated in 4c

```
clear all
i=atan(4/3)
F1=6000
F2=8000
a=0
b=0
eqns=[1 0 cos(i) 1 0 0 0 0 0 0;
    0 1 sin(i) 0 0 0 0 0 0 0;
    0 0 -cos(i) 0 1 cos(i) 0 0 0 0;
    0 0 -sin(i) 0 0 -sin(i) 0 0 0 0;
    0 0 0 0 -1 0 -cos(i) cos(i) 0 0;
    0 0 0 0 0 0 -sin(i) -sin(i) 0 0;
    0 0 0 0 0 0 0 -cos(i) -1 0;
    0 0 0 0 0 0 0 sin(i) 0 1;
    0 0 0 -1 0 -cos(i) cos(i) 0 1 0;
    0 0 0 0 0 sin(i) sin(i) 0 0 0]

ans=[0;0;-F1*cos(a);F1*sin(a);0;0;0;0;-F2*sin(b);F2*cos(b)]

res=inv(eqns)*ans

Rax=res(1)
Ray=res(2)
Nab=res(3)
Nae=res(4)
Nbc=res(5)
Nbe=res(6)
Nce=res(7)
Ncd=res(8)
Nde=res(9)
Rdy=res(10)
```

Question 5

```
function [u0, v0] = GetVelocityFromSetup(lrest,lmax,lmin,k,L)
    N = length(k);
    xmax = zeros(1,N);
    xmin = zeros(1,N);
    Espring = zeros(1,N);
    Etotal = 0;
    mball = 0.0127;
    lball = 0.53;
    larm = 0.51;
    marm = 0.388;

    % calculate xmax and xmin for each spring
    for i = 1:N
        xmax(i) = lmax(i) - lrest(i);
        xmin(i) = lmin(i) - lrest(i);
    end

    % calculate the energy for each spring and sum up the energy of each
    % spring to get the total energy
    for i = 1:N
        Espring(i) = 0.5*k(i)*(xmax(i)^2-(xmin(i))^2)+L(i)*(xmax(i)-xmin(i));
        Etotal = Etotal + Espring(i);
    end

    % calculate the moment of inertia
    Iarm = (1/3)*marm*larm^2;
    Iball = mball*lball^2;
```

```
    % calculate the total moment of inertia and use it to calculate the
    % angular velocity
    I = Iarm + Iball;
    ang_vel = sqrt((2*Etotal)/I);

    % calculate u0 and v0
    V0 = ang_vel*lball;
    u0 = V0*cosd(45);
    v0 = V0*sind(45);
end
```