Calculus 2 Written Assignment 1

1.
a) $\lim_{x \to \pi} \frac{e^x - e^\pi}{\cos(\frac{x}{2})}$, type $\left(\frac{0}{0}\right)$

$= \lim_{x \to \pi} \frac{e^x}{-\frac{1}{2}\sin(\frac{x}{2})}$, L'Hôpital's Rule

$= \frac{\lim_{x\to\pi}(e^x)}{\lim_{x\to\pi}(-\frac{1}{2}) \cdot \lim_{x\to\pi}(\sin(\frac{x}{2}))}$, limit laws

$= \frac{e^\pi}{-\frac{1}{2} \cdot 1}$; continuity of e^z and $\sin z$

$= -2e^\pi$

b) $f(x) = \begin{cases} \frac{e^x - e^\pi}{\cos(\frac{x}{2})} & x < \pi \\ a\sin\left(\frac{\pi^2}{2x}\right) & x \geq \pi \end{cases}$

A function is continuous if

$\lim_{x \to a} f(x) = f(a)$

$\lim_{x\to a} f(x) = L \iff \lim_{x \to a^-} f(x) = L$ and $\lim_{x\to a^+} f(x) = L$

from (a): $\lim_{x\to\pi^-} f(x) = -2e^\pi$

therefore we need $\lim_{x\to\pi^+} f(x) = -2e^\pi$

(from previous page)

$$\lim_{x\to\pi^+} a\sin\left(\frac{\pi^2}{2x}\right) = -2e^{\pi}$$

$$= \lim_{x\to\pi^+}(a)\cdot\lim_{x\to\pi^+}\left(\sin\left(\frac{\pi^2}{2x}\right)\right) = -2e^{\pi}$$, limit laws

$$= a\sin\left(\frac{\pi^2}{2\pi}\right) = -2e^{\pi}$$, limit laws and continuity of $\sin z$

$$= a\sin\left(\frac{\pi}{2}\right) = -2e^{\pi}$$

$$a = -2e^{\pi}$$

$$\lim_{x\to\pi} f(x) = -2e^{\pi} = f(\pi)$$

Therefore the function is continuous when $a = -2e^{\pi}$

2.

a) $$\lim_{n\to\infty} \frac{n\log(\cos^2(n)+3)}{2020^n}$$

$$= \lim_{x\to\infty} \frac{x\log(\cos^2(x)+3)}{2020^x}$$, $x\in\mathbb{R}$

$$= \lim_{x\to\infty}(2020^{-x})\cdot\lim_{x\to\infty}(x\log(\cos^2(x)+3))$$, limit laws

$$= \lim_{x\to\infty}\left(\frac{1}{2020}\right)^x\cdot\lim_{x\to\infty}(x\log(\cos^2(x)+3))$$

$$= 0\cdot\lim_{x\to\infty}(x\log(\cos^2(x)+3))$$, standard limit $r^x = 0$

$$= 0$$

b) $\lim_{n\to\infty} \sin\left(\frac{2(n-1)\pi}{4}\right)$ does not exist as $\sin\left(\frac{2(n-1)\pi}{4}\right)$ oscillates between 1 and -1 as $n \to \infty$ and therefore diverges.

c) $\lim_{n\to\infty} \tan\left((2020n)^{\frac{1}{n}}\right)$

$= \lim_{x\to\infty} \tan\left((2020x)^{\frac{1}{x}}\right)$, $x \in \mathbb{R}$

$= \lim_{x\to\infty} \tan\left(2020^{\frac{1}{x}} \cdot x^{\frac{1}{x}}\right)$

$= \tan\left(\lim_{x\to\infty}\left(2020^{\frac{1}{x}} \cdot x^{\frac{1}{x}}\right)\right)$

$= \tan\left(\lim_{x\to\infty}\left(2020^{1/x}\right) \cdot \lim_{x\to\infty}\left(x^{1/x}\right)\right)$, limit laws

$= \tan(1 \cdot 1)$, standard limits $a^{1/x} = 1$
$x^{1/x} = 1$

$= \tan(1)$, continuity of $\tan z$ at $z = 1$