Assignment 9 Due: 6:00PM, Friday 5 June.

Name:

Student ID:

Explainer: Question 1 should be completed in **WebWork** by 6:00PM, Friday 5 June. WebWork should be accessed via Assignment 9 WebWork in the Assignments panel of the MAST10005 LMS Site.

You should upload a scan of neatly presented solutions to Questions 2, 3 and 4 in **Gradescope**, which should be accessed via Assignment 9 Written Part in the Assignments section of the MAST10005 LMS Site. Please do not include your answers to Question 1 in this part.

1. You should complete this question in WebWork by 6:00PM, Friday 5 June. Completing this question first will make the the written part easier because you will have already found and checked the equivalent expression for $\cos^2(x)\sin^4(x)$ needed in Question 2.

2. Use your answer to Problem 3 in WebWork to find the antiderivative

$$\int \cos^2(x)\sin^4(x)\ dx.$$

Solution: From WebWork, we have

$$\cos^2(x)\sin^4(x) = \frac{1}{16} + \frac{1}{32}\cos(6x) - \frac{1}{16}\cos(4x) - \frac{1}{32}\cos(2x).$$

Hence:

$$\int \cos^2(x)\sin^4(x)\ dx$$

$$= \int \frac{1}{16} + \frac{1}{32}\cos(6x) - \frac{1}{16}\cos(4x) - \frac{1}{32}\cos(2x)\, dx$$

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$$= \frac{1}{16}x + \frac{1}{192}\sin(6x) - \frac{1}{64}\sin(4x) - \frac{1}{64}\sin(2x) + C.$$

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3. Consider the following separable differential equation:

$$\frac{dy}{dt} = \cos(t)\cos^2(y).$$

(a) Find all constant solutions of this differential equation (there are infinitely many).

Solution:

Constant solutions occur when

$$\cos^2(y) = 0 \Rightarrow \cos(y) = 0 \Rightarrow y = \frac{\pi}{2} + k\pi.$$

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Hence $y = \frac{\pi}{2} + k\pi$ gives a constant solution for each $k \in \mathbb{Z}$.

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(b) Use your answer to (a) and Theorem 4.44 to explain why range$(y) \subseteq (-\frac{\pi}{2}, \frac{\pi}{2})$ for the solution of this equation satisfying the initial condition $y = 0$ when $t = 0$.

Solution:

Writing $\frac{dy}{dt} = G(y)F(t)$ where $G(y) = \cos^2(y)$ and $F(t) = \cos(t)$,

$$G'(y) = 2\cos(y)\sin(y) \quad \text{and} \quad F(t)$$

are both continuous, so the graph of y cannot intersect the constant solutions $y = -\frac{\pi}{2}$ and $y = \frac{\pi}{2}$ by Theorem 4.44.

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Explain this

This gives range$(y) \subseteq (-\frac{\pi}{2}, \frac{\pi}{2})$.

(c) Solve the initial value problem with initial condition $y = 0$ when $t = 0$. You should explain how you use the result from (b) when solving this problem. [Hint: Review lecture slide 237.]

Solution:

$$\frac{dy}{dt} = \cos(t)\cos^2(y) \Rightarrow \frac{1}{\cos^2(y)}\frac{dy}{dt} = \cos(t)$$

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$$\Rightarrow \int \frac{1}{\cos^2(y)}\frac{dy}{dt}\,\mathrm{d}t = \int \cos(t)\,\mathrm{d}t$$

$$\Rightarrow \int \sec^2(y)\,\mathrm{d}y = \sin(t) + C \quad \text{[Substitution]}$$

$$\Rightarrow \tan(y) = \sin(t) + C$$

$$\Rightarrow y = \arctan(\sin(t) + C)$$

At the previous step we used the fact that

$$y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow \arctan(\tan(y)) = y.$$

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Explain use of (b)

Initial value gives $0 = \arctan(\sin(0) + C) = \arctan(C) \Rightarrow C = 0$, so

$$y = \arctan(\sin(t)).$$

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Whole written part: clear structure, and ALL mathematical notation is correct.

Assignment Information

This assignment is worth $\frac{20}{9}$% of your final MAST10005 mark.

Full working should be shown in your solutions to Questions 2, 3 and 4. There will be 1 mark overall for correct mathematical notation.
Full solutions to the assignment will be uploaded to the LMS site approximately 3 days after the assignment is due.