

MAST20004 Probability  
Semester 1, 2021  
Assignment One: Questions

Due 3 pm, Friday 26 March 2021

<b>Name:</b>
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<b>Student ID:</b>
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**Important instructions:**

- (1) This assignment contains 4 questions, **two** of which will be randomly selected to be marked. Each marked question is worth 10 points and each unmarked question with substantial working is worth 1 point.
- (2) To complete this assignment, you need to write your solutions into the blank answer spaces following each question in this assignment PDF.
  - If you have a printer (or can access one), then you must print out the assignment template and handwrite your solutions into the answer spaces.
  - If you do not have a printer but you can figure out how to annotate a PDF using an iPad/Android tablet/Graphics tablet or using Adobe Acrobat, then annotate your answers directly onto the assignment PDF and save a copy for submission.

Failing both of these methods, you may handwrite your answers as normal on blank paper and then scan for submission (but note that you will thereby miss valuable practice for the exam process). In that case, however, your document should have the same length as the assignment template otherwise Gradescope will reject your submission. So you will need to add as many blank pages as necessary to reach that criterion.

Scan your assignment to a PDF file using your mobile phone (we recommend Cam - Scanner App), then upload by going to the Assignments menu on Canvas and submit the PDF to the GradeScope tool by first selecting your PDF file and then clicking on 'Upload PDF'.

- (3) A poor presentation penalty of 10% of the total available marks will apply unless your submitted assignment meets all of the following requirements:
  - it is a single pdf with all pages in correct template order and the correct way up, and with any blank pages with additional working added only at the end of the template pages;
  - has all pages clearly readable;
  - has all pages cropped to the A4 borders of the original page and is imaged from directly above to avoid excessive 'keystoning'.

These requirements are easy to meet if you use a scanning app on your phone and take some care with your submission - please review it before submitting to double check you have satisfied all of the above requirements.

- (4) Late submission within 20 hours after the deadline will be penalised by 5% of the total available marks for every hour or part thereof after the deadline. After that, the Gradescope submission channel will be closed, and your submission will no longer be accepted. You are strongly encouraged to submit the assignment a few days before the deadline just in case of unexpected technical issues. If you are facing a rather exceptional/extreme situation that prevents you from submitting on time, please contact the tutor coordinator **Robert Maillardet** with formal proofs such as medical certificate.
- (5) Working and reasoning must be given to obtain full credit. Clarity, neatness, and style count.

**Problem 1.** Let  $A_1, \dots, A_n$  be a finite collection of sets.

- (i) Formulate a version of De Morgan's laws for the collection of sets.

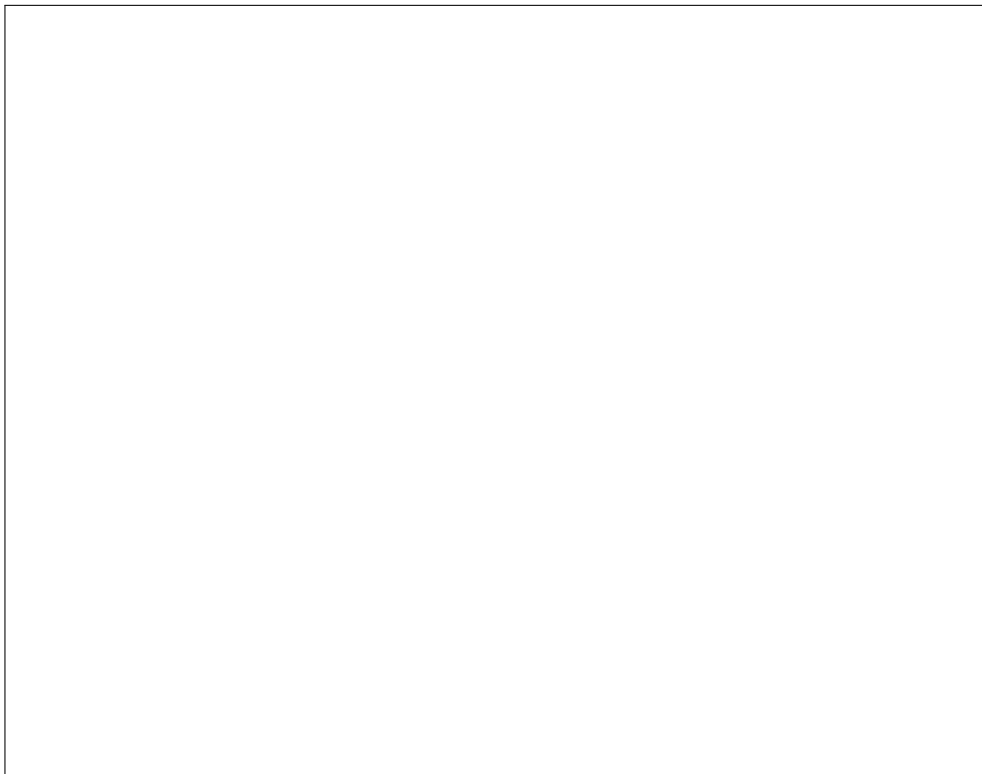
- (ii) Prove the De Morgan's laws in (i) by the elementwise method.

**Problem 2.** Let  $A, B, C$  be three events with  $\mathbb{P}(B) > 0$  and  $\mathbb{P}(B^c) > 0$ . For each of the following statements, determine whether it is true or false. If it is true, give a proof; if it is false, give a counterexample.

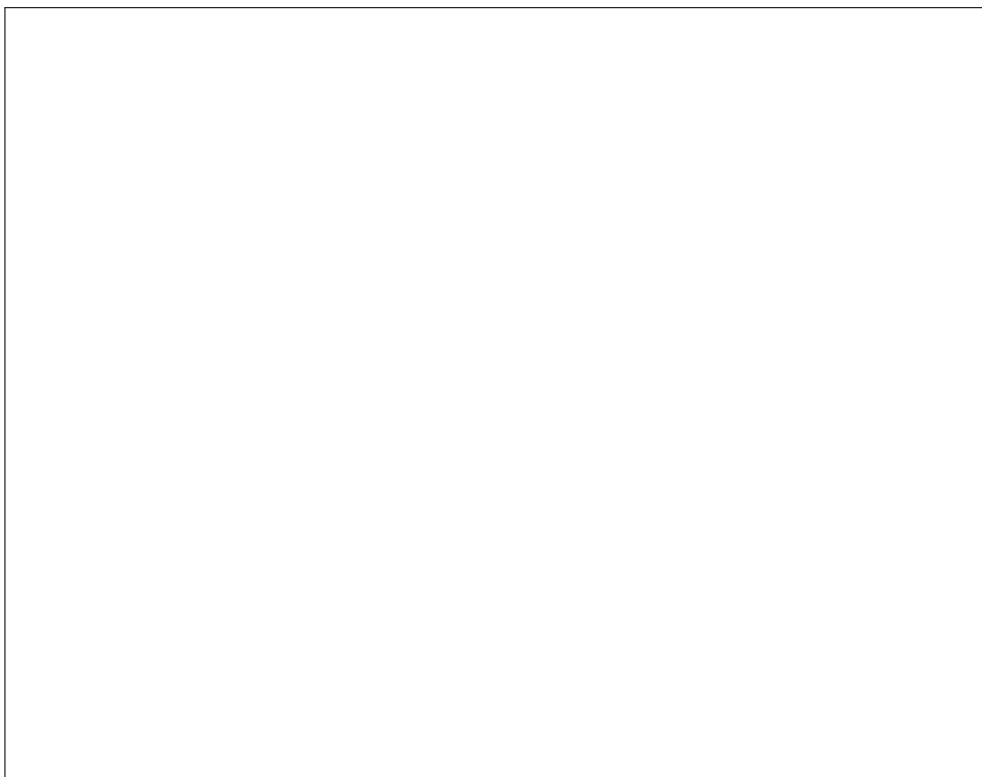
- (i) If  $A$  and  $B$  are independent, then  $\mathbb{P}(A|B) = \mathbb{P}(A|B^c)$ .

- (ii) If  $\mathbb{P}(A|B) = \mathbb{P}(A|B^c)$ , then  $A$  and  $B$  are independent.

(iii) If  $A, B, C$  are independent, then  $A$  and  $B^c \cap C$  are independent.



(iv) If  $A$  and  $B$  are independent,  $A$  and  $C$  are independent, and  $A$  and  $B \cap C$  are independent, then  $A, B, C$  are independent.



**Problem 3.** There are three urns labelled as  $u_1$ ,  $u_2$  and  $u_3$ ; initially  $u_1$  contains 2 black balls and 3 white balls,  $u_2$  has 1 black ball and 4 white balls, and  $u_3$  holds 3 black balls and 2 white balls. We first mix the balls in  $u_1$ , randomly draw a ball from  $u_1$ , place it in  $u_2$ ; next, mix the balls in  $u_2$ , randomly draw a ball from  $u_2$  and place it in  $u_3$ ; finally, mix the balls in  $u_3$  and randomly draw a ball from  $u_3$ .

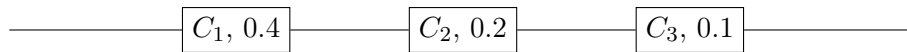
- (i) What is the probability that the ball drawn from  $u_2$  is white?

- (ii) What is the probability that the ball drawn from  $u_3$  is white?

- (iii) Given that the ball drawn from  $u_3$  is white, what is the probability that the ball drawn from  $u_2$  is white?

- (iv) Given that the ball drawn from  $u_3$  is white, what is the probability that the ball drawn from  $u_1$  is white?

**Problem 4.** A circuit contains three mutually independent components  $C_1$ ,  $C_2$ ,  $C_3$  in series as shown in the figure below.



The probability of failure for each component is indicated in the figure respectively.

- (i) What is the probability that the circuit will fail?

- (ii) What is the probability that exactly one component fails?



- (iii) Given that exactly one of the components fails, what are the respective probabilities that the failed component is  $C_1$ ,  $C_2$ ,  $C_3$ ?

- (iv) Assume that exactly one of the components failed, an engineer is assigned to detect the failed component. To minimise the number  $X$  of tests, what is the order of the components they should check? Explain your answer.

- (v) Derive the probability mass function of  $X$  in (iv).