Complexity, 2006-2007
James Lawson

1. (a)
   i. L is NP is there is a nondeterminstic turing machine that decides L in p-time.
   ii. L is NP-complete if it is in NP, and is NP-hard: if $L' \in \mathsf{NP}$ then $L' \leqslant L$.
   iii. Assume L is NP-complete. Assume for contradiction that L is in P. Then take any langauge L' in NP. So $L' \leqslant L$. But given that L is in P, we deduce that L is in P by the downward closure property of reduction. In other words, we have shown that $\mathsf{P} \subseteq \mathsf{NP}$. We already know that $\mathsf{NP} \subseteq \mathsf{P}$, so $\mathsf{P} = \mathsf{NP}$ giving a contradiction.

   (b)
   i. TRI is given sets B, G, H, each with n elements and triples $T \subseteq B \times G \times H$, is there a subset $T' \subseteq T$ such that T' has n elements and no two triples in T' have an element in common.
   ii. Consider a reduction TRI $\leqslant$ SUBSETSUM.
   Take B, G, H of size n and T of size m.
   For each $t_i \in T$, associate string, of $3n$ 0/1s for a $(m+1)$-ary numbers.
   $f(B,G,H,T) = (W,K)$ where $K = \sum_{i=0}^{3n-1}(1+m)^i$ and $W = \{w_{t_i} \mid t_i \in T\}$.
      - ($\Rightarrow$) Assume (B,G,H,T) has a matching, then the sum will have $3n$ 1s because there are no common elements so each n-bit grouping has unique operands in the summation.
      - ($\Leftarrow$) Assume there is a subset sum in $f(B,G,H,T)$.
      There is some $w_{t_1} + w_{t_2} + ... + w_{t_k} = K$. K is a series of $3n$ 1s. Each group of n bits, is a series of 1s. Since each element has a unique n-string each bit is covered by exactly one element. So $k = n$ and each w_{t_i} corresponds to a different tuple, and the tuples sum to K.