Package ‘OptHoldoutSize’

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Title Estimation of Optimal Size for a Holdout Set for Updating a Predictive Score

Version 0.1.0.1

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Description Predictive scores must be updated with care, because actions taken on the basis of existing risk scores causes bias in risk estimates from the updated score. A holdout set is a straightforward way to manage this problem: a proportion of the population is 'held-out' from computation of the previous risk score. This package provides tools to estimate a size for this holdout set and associated errors. Comprehensive vignettes are included. Please see: Haidar-Wehbe S, Emerson SR, Aslett LJM, Liley J (2022) <arXiv:2202.06374> for details of methods.

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Encoding UTF-8

LazyData true

Depends R (>= 3.5.0), matrixStats, mnormt, mvtnorm, ranger, mle.tools

Roxygen list(markdown = TRUE)

Suggests knitr, rmarkdown

VignetteBuilder knitr

RoxygenNote 7.3.2

NeedsCompilation no

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Contents

`add_aspre_interactions` *Add interaction terms corresponding to ASPRE model*

Description

Add various interaction terms to X. Interaction terms correspond to those in ASPRE.

Usage

```
add_aspre_interactions(X)
```

Arguments

`X`	data frame

Value

New data frame containing interaction terms.

Examples

```
# Load ASPRE related data
data(params_aspre)

X=sim_random_aspre(1000,params_aspre)
Xnew=add_aspre_interactions(X)

print(colnames(X))
print(colnames(Xnew))
```

`aspre` *Computes ASPRE score*

Description

Computes ASPRE model prediction on a matrix `X` of covariates

Full ASPRE model from https://www.nejm.org/doi/suppl/10.1056/NEJMoa1704559/suppl_file/nejmoa1704559_append

Model is to predict gestational age at PE; that is, a higher score indicates a lower PE risk, so coefficients are negated for model to predict PE risk.

Usage

```
aspre(X)
```

Arguments

`X`	matrix, assumed to be output of sim_random_aspre with parameter params=params_aspre and transformed using add_aspre_interactions

Value

vector of scores.

Examples

```
# Load ASPRE related data
data(params_aspre)

X=sim_random_aspre(1000,params_aspre)
Xnew=add_aspre_interactions(X)

aspre_score=aspre(Xnew)

plot(density(aspre_score))
```

aspre_emulation *Emulation-based OHS estimation for ASPRE*

Description

This object contains data relating to emulation-based OHS estimation for the ASPRE model. For generation, see hidden code in vignette, or in pipeline at https://github.com/jamesliley/OptHoldoutSize_pipelines

Usage

```
aspre_emulation
```

Format

An object of class `list` of length 4.

aspre_k2 *Cost estimating function in ASPRE simulation*

Description

Estimate cost at a given holdout set size in ASPRE model

Usage

```
aspre_k2(
  n,
  X,
  PRE,
  seed = NULL,
  pi_PRE = 1426/58974,
  pi_intervention = 0.1,
  alpha = 0.37
)
```

Arguments

`n`	Holdout set size at which to estimate k_2 (cost)
`X`	Matrix of predictors
`PRE`	Vector indicating PRE incidence
`seed`	Random seed; set before starting or set to NULL
`pi_PRE`	Population prevalence of PRE if not prophylactically treated. Defaults to empirical value 1426/58974
`pi_intervention`	Proportion of the population on which an intervention will be made. Defaults to 0.1
`alpha`	Reduction in PRE risk with intervention. Defaults to empirical value 0.37

Value

Estimated cost

Examples

```
# Simulate
set.seed(32142)

N=1000; p=15
X=matrix(rnorm(N*p),N,p); PRE=rbinom(N,1,prob=logit(X%*% rnorm(p)))
aspre_k2(1000,X,PRE)
```

---

aspre_parametric *Parametric-based OHS estimation for ASPRE*

Description

This object contains data relating to parametric-based OHS estimation for the ASPRE model. For generation, see hidden code in vignette, or in pipeline at https://github.com/jamesliley/OptHoldoutSize_pipelines

Usage

```
aspre_parametric
```

Format

An object of class `list` of length 4.

---

ci_cover_a_yn *Data for example on asymptotic confidence interval for OHS.*

Description

Data for example for asymptotic confidence interval for OHS. For generation, see example.

Usage

```
ci_cover_a_yn
```

Format

An object of class `matrix` (inherits from `array`) with 11 rows and 5000 columns.

ci_cover_cost_a_yn — *Data for example on asymptotic confidence interval for min cost.*

Description

Data for example for asymptotic confidence interval for min cost. For generation, see example.

Usage

```
ci_cover_cost_a_yn
```

Format

An object of class `matrix` (inherits from `array`) with 11 rows and 5000 columns.

ci_cover_cost_e_yn — *Data for example on empirical confidence interval for min cost.*

Description

Data for example for empirical confidence interval for min cost. For generation, see example.

Usage

```
ci_cover_cost_e_yn
```

Format

An object of class `matrix` (inherits from `array`) with 11 rows and 5000 columns.

ci_cover_e_yn — *Data for example on empirical confidence interval for OHS.*

Description

Data for example for empirical confidence interval for OHS. For generation, see example.

Usage

```
ci_cover_e_yn
```

Format

An object of class `matrix` (inherits from `array`) with 11 rows and 5000 columns.

`ci_mincost` *Confidence interval for minimum total cost, when estimated using parametric method*

Description

Compute confidence interval for cost at optimal holdout size given either a standard error covariance matrix or a set of m estimates of parameters.

This can be done either asymptotically, using a method analogous to the Fisher information matrix, or empirically (using bootstrap resampling)

If sigma (covariance matrix) is specified and method='bootstrap', a confidence interval is generated assuming a Gaussian distribution of (N,k1,theta). To estimate a confidence interval assuming a non-Gaussian distribution, simulate values under the requisite distribution and use then as parameters N,k1, theta, with sigma set to NULL.

Usage

```
ci_mincost(
  N,
  k1,
  theta,
  alpha = 0.05,
  k2form = powerlaw,
  grad_mincost = NULL,
  sigma = NULL,
  n_boot = 1000,
  seed = NULL,
  mode = "empirical",
  ...
)
```

Arguments

`N`	Vector of estimates of total number of samples on which the predictive score will be used/fitted, or single estimate
`k1`	Vector of estimates of cost value in the absence of a predictive score, or single number
`theta`	Matrix of estimates of parameters for function k2form(n) governing expected cost to an individual sample given a predictive score fitted to n samples. Can be a matrix of dimension n x n_par, where n_par is the number of parameters of k2.
`alpha`	Construct 1-alpha confidence interval. Defaults to 0.05
`k2form`	Function governing expected cost to an individual sample given a predictive score fitted to n samples. Must take two arguments: n (number of samples) and theta (parameters). Defaults to a power-law form `k2(n,c(a,b,c))=a n^(-b) + c`.
`grad_mincost`	Function giving partial derivatives of minimum cost, taking three arguments: N, k1, and theta. Only used for asymptotic confidence intervals. F NULL, estimated empirically

sigma	Standard error covariance matrix for (N,k1,theta), in that order. If NULL, will derive as sample covariance matrix of parameters. Must be of the correct size and positive definite.
n_boot	Number of bootstrap resamples for empirical estimate.
seed	Random seed for bootstrap resamples. Defaults to NULL.
mode	One of 'asymptotic' or 'empirical'. Defaults to 'empirical'
...	Passed to function optimize

Value

A vector of length two containing lower and upper limits of confidence interval.

Examples

```
## Set seed
set.seed(574635)

## We will assume that our observations of N, k1, and theta=(a,b,c) are
##  distributed with mean mu_par and variance sigma_par
mu_par=c(N=10000,k1=0.35,A=3,B=1.5,C=0.1)
sigma_par=cbind(
  c(100^2,       1,      0,       0,       0),
  c(    1,  0.07^2,      0,       0,       0),
  c(    0,       0,  0.5^2,    0.05,  -0.001),
  c(    0,       0,   0.05,   0.4^2,  -0.002),
  c(    0,       0, -0.001,  -0.002,  0.02^2)
)

# Firstly, we make 500 observations
par_obs=rmnorm(500,mean=mu_par,varcov=sigma_par)

# Minimum cost and asymptotic and empirical confidence intervals
mincost=optimal_holdout_size(N=mean(par_obs[,1]),k1=mean(par_obs[,2]),
  theta=colMeans(par_obs[,3:5]))$cost
ci_a=ci_mincost(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],alpha=0.05,
  seed=12345,mode="asymptotic")
ci_e=ci_mincost(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],alpha=0.05,
  seed=12345,mode="empirical")


# Assess cover at various m
m_values=c(20,30,50,100,150,200,300,500,750,1000,1500)
ntrial=5000
alpha_trial=0.1 # use 90% confidence intervals
mincost_true=optimal_holdout_size(N=mu_par[1],k1=mu_par[2],
  theta=mu_par[3:5])$cost

## The matrices indicating cover take are included in this package but take
##  around 30 minutes to generate. They are generated using the code below
##  (including random seeds).
data(ci_cover_cost_a_yn)
data(ci_cover_cost_e_yn)

if (!exists("ci_cover_cost_a_yn")) {
  ci_cover_cost_a_yn=matrix(NA,length(m_values),ntrial) # Entry [i,j] is 1
```

```
  #  if ith asymptotic CI for jth value of m covers true mincost
  ci_cover_cost_e_yn=matrix(NA,length(m_values),ntrial) # Entry [i,j] is 1
  #  if ith empirical CI for jth value of m covers true mincost

  for (i in 1:length(m_values)) {
    m=m_values[i]
    for (j in 1:ntrial) {
      # Set seed
      set.seed(j*ntrial + i + 12345)

      # Make m observations
      par_obs=rmnorm(m,mean=mu_par,varcov=sigma_par)
      ci_a=ci_mincost(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],
        alpha=alpha_trial,mode="asymptotic")
      ci_e=ci_mincost(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],
        alpha=alpha_trial,mode="empirical",n_boot=500)

      if (mincost_true>ci_a[1] & mincost_true<ci_a[2])
        ci_cover_cost_a_yn[i,j]=1 else ci_cover_cost_a_yn[i,j]=0
      if (mincost_true>ci_e[1] & mincost_true<ci_e[2])
        ci_cover_cost_e_yn[i,j]=1 else ci_cover_cost_e_yn[i,j]=0
    }
    print(paste0("Completed for m = ",m))
  }

save(ci_cover_cost_a_yn,file="data/ci_cover_cost_a_yn.RData")
save(ci_cover_cost_e_yn,file="data/ci_cover_cost_e_yn.RData")

}

# Cover at each m value and standard error
cover_a=rowMeans(ci_cover_cost_a_yn)
cover_e=rowMeans(ci_cover_cost_e_yn)
zse_a=2*sqrt(cover_a*(1-cover_a)/ntrial)
zse_e=2*sqrt(cover_e*(1-cover_e)/ntrial)


# Draw plot. Convergence to 1-alpha cover is evident. Cover is not far from
#   alpha even at small m.

plot(0,type="n",xlim=range(m_values),ylim=c(0.7,1),xlab=expression("m"),
  ylab="Cover")

# Asymptotic cover and 2*SE pointwise envelope
polygon(c(m_values,rev(m_values)),c(cover_a+zse_a,rev(cover_a-zse_a)),
  col=rgb(0,0,0,alpha=0.3),border=NA)
lines(m_values,cover_a,col="black")

# Empirical cover and 2*SE pointwiseenvelope
polygon(c(m_values,rev(m_values)),c(cover_e+zse_e,rev(cover_e-zse_e)),
  col=rgb(0,0,1,alpha=0.3),border=NA)
lines(m_values,cover_e,col="blue")

abline(h=1-alpha_trial,col="red")
legend("bottomright",c("Asym.","Emp.",expression(paste("1-",alpha))),lty=1,
  col=c("black","blue","red"))
```

`ci_ohs` *Confidence interval for optimal holdout size, when estimated using parametric method*

Description

Compute confidence interval for optimal holdout size given either a standard error covariance matrix or a set of m estimates of parameters.

This can be done either asymptotically, using a method analogous to the Fisher information matrix, or empirically (using bootstrap resampling)

If sigma (covariance matrix) is specified and method='bootstrap', a confidence interval is generated assuming a Gaussian distribution of (N,k1,theta). To estimate a confidence interval assuming a non-Gaussian distribution, simulate values under the requisite distribution and use then as parameters N,k1, theta, with sigma set to NULL.

Usage

```
ci_ohs(
  N,
  k1,
  theta,
  alpha = 0.05,
  k2form = powerlaw,
  grad_nstar = NULL,
  sigma = NULL,
  n_boot = 1000,
  seed = NULL,
  mode = "empirical",
  ...
)
```

Arguments

`N`	Vector of estimates of total number of samples on which the predictive score will be used/fitted, or single estimate
`k1`	Vector of estimates of cost value in the absence of a predictive score, or single number
`theta`	Matrix of estimates of parameters for function k2form(n) governing expected cost to an individual sample given a predictive score fitted to n samples. Can be a matrix of dimension n x n_par, where n_par is the number of parameters of k2.
`alpha`	Construct 1-alpha confidence interval. Defaults to 0.05
`k2form`	Function governing expected cost to an individual sample given a predictive score fitted to n samples. Must take two arguments: n (number of samples) and theta (parameters). Defaults to a power-law form `k2(n,c(a,b,c))=a n^(-b) + c`.
`grad_nstar`	Function giving partial derivatives of optimal holdout set, taking three arguments: N, k1, and theta. Only used for asymptotic confidence intervals. F NULL, estimated empirically

sigma	Standard error covariance matrix for (N,k1,theta), in that order. If NULL, will derive as sample covariance matrix of parameters. Must be of the correct size and positive definite.
n_boot	Number of bootstrap resamples for empirical estimate.
seed	Random seed for bootstrap resamples. Defaults to NULL.
mode	One of 'asymptotic' or 'empirical'. Defaults to 'empirical'
...	Passed to function optimize

Value

A vector of length two containing lower and upper limits of confidence interval.

Examples

```
## Set seed
set.seed(493825)

## We will assume that our observations of N, k1, and theta=(a,b,c) are
##  distributed with mean mu_par and variance sigma_par
mu_par=c(N=10000,k1=0.35,A=3,B=1.5,C=0.1)
sigma_par=cbind(
  c(100^2,       1,      0,       0,       0),
  c(    1,  0.07^2,      0,       0,       0),
  c(    0,       0,  0.5^2,    0.05,  -0.001),
  c(    0,       0,   0.05,   0.4^2,  -0.002),
  c(    0,       0, -0.001,  -0.002,  0.02^2)
)

# Firstly, we make 500 observations
par_obs=rmnorm(500,mean=mu_par,varcov=sigma_par)

# Optimal holdout size and asymptotic and empirical confidence intervals
ohs=optimal_holdout_size(N=mean(par_obs[,1]),k1=mean(par_obs[,2]),
  theta=colMeans(par_obs[,3:5]))$size
ci_a=ci_ohs(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],alpha=0.05,
  seed=12345,mode="asymptotic")
ci_e=ci_ohs(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],alpha=0.05,
  seed=12345,mode="empirical")


# Assess cover at various m
m_values=c(20,30,50,100,150,200,300,500,750,1000,1500)
ntrial=5000
alpha_trial=0.1 # use 90% confidence intervals
nstar_true=optimal_holdout_size(N=mu_par[1],k1=mu_par[2],
  theta=mu_par[3:5])$size

## The matrices indicating cover take are included in this package but take
##  around 30 minutes to generate. They are generated using the code below
##  (including random seeds).
data(ci_cover_a_yn)
data(ci_cover_e_yn)

if (!exists("ci_cover_a_yn")) {
  ci_cover_a_yn=matrix(NA,length(m_values),ntrial) # Entry [i,j] is 1 if ith
```

```
  ##  asymptotic CI for jth value of m covers true nstar
  ci_cover_e_yn=matrix(NA,length(m_values),ntrial) # Entry [i,j] is 1 if ith
  ##  empirical CI for jth value of m covers true nstar

  for (i in 1:length(m_values)) {
    m=m_values[i]
    for (j in 1:ntrial) {
      # Set seed
      set.seed(j*ntrial + i + 12345)

      # Make m observations
      par_obs=rmnorm(m,mean=mu_par,varcov=sigma_par)
      ci_a=ci_ohs(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],
        alpha=alpha_trial,mode="asymptotic")
      ci_e=ci_ohs(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],
        alpha=alpha_trial,mode="empirical",n_boot=500)

      if (nstar_true>ci_a[1] & nstar_true<ci_a[2]) ci_cover_a_yn[i,j]=1 else
         ci_cover_a_yn[i,j]=0
      if (nstar_true>ci_e[1] & nstar_true<ci_e[2]) ci_cover_e_yn[i,j]=1 else
         ci_cover_e_yn[i,j]=0
    }
    print(paste0("Completed for m = ",m))
  }

save(ci_cover_a_yn,file="data/ci_cover_a_yn.RData")
save(ci_cover_e_yn,file="data/ci_cover_e_yn.RData")

}

# Cover at each m value and standard error
cover_a=rowMeans(ci_cover_a_yn)
cover_e=rowMeans(ci_cover_e_yn)
zse_a=2*sqrt(cover_a*(1-cover_a)/ntrial)
zse_e=2*sqrt(cover_e*(1-cover_e)/ntrial)


# Draw plot. Convergence to 1-alpha cover is evident. Cover is not far from
#   alpha even at small m.

plot(0,type="n",xlim=range(m_values),ylim=c(0.7,1),xlab=expression("m"),
  ylab="Cover")

# Asymptotic cover and 2*SE pointwise envelope
polygon(c(m_values,rev(m_values)),c(cover_a+zse_a,rev(cover_a-zse_a)),
  col=rgb(0,0,0,alpha=0.3),border=NA)
lines(m_values,cover_a,col="black")

# Empirical cover and 2*SE pointwiseenvelope
polygon(c(m_values,rev(m_values)),c(cover_e+zse_e,rev(cover_e-zse_e)),
  col=rgb(0,0,1,alpha=0.3),border=NA)
lines(m_values,cover_e,col="blue")

abline(h=1-alpha_trial,col="red")
legend("bottomright",c("Asym.","Emp.",expression(paste("1-",alpha))),lty=1,
  col=c("black","blue","red"))
```

cov_fn *Covariance function for Gaussian process*

Description

Radial kernel covariance function for Gaussian process.

Used for a Gaussian process `GP(m,k(.,.))`, an instance X of which has covariance k(n,n') between X(n) and X(n').

Covariance function is parametrised by `var_u` (general variance) and `k_width` (kernel width)

Usage

```
cov_fn(n, ndash, var_u, k_width)
```

Arguments

`n`	Argument 1: kernel is a function of `ndash-n`
`ndash`	Argument 2: kernel is a function of `ndash-n`
`var_u`	Global variance
`k_width`	Kernel width

Value

Covariance value

Examples

```
# We will sample from Gaussian processes
#  GP(0,k(.,.)=cov_fn(.,.;var_u,theta))
# at these values of n
nvals=seq(1,300,length=100)

# We will consider two theta values
kw1=10; kw2=30

# We will consider two var_u values
var1=1; var2=10

# Covariance matrices
cov11=outer(nvals,nvals,function(n,ndash) cov_fn(n,ndash,var_u=var1,
  k_width=kw1))
cov12=outer(nvals,nvals,function(n,ndash) cov_fn(n,ndash,var_u=var1,
  k_width=kw2))
cov21=outer(nvals,nvals,function(n,ndash) cov_fn(n,ndash,var_u=var2,
  k_width=kw1))
cov22=outer(nvals,nvals,function(n,ndash) cov_fn(n,ndash,var_u=var2,
  k_width=kw2))

# Dampen slightly to ensure positive definiteness
damp=1e-5
cov11=(1-damp)*(1-diag(length(nvals)))*cov11 + diag(length(nvals))*cov11
cov12=(1-damp)*(1-diag(length(nvals)))*cov12 + diag(length(nvals))*cov12
```

```
cov21=(1-damp)*(1-diag(length(nvals)))*cov21 + diag(length(nvals))*cov21
cov22=(1-damp)*(1-diag(length(nvals)))*cov22 + diag(length(nvals))*cov22

# Sample
set.seed(35243)
y11=rmnorm(1,mean=0,varcov=cov11)
y12=rmnorm(1,mean=0,varcov=cov12)
y21=rmnorm(1,mean=0,varcov=cov21)
y22=rmnorm(1,mean=0,varcov=cov22)

# Plot
rr=max(abs(c(y11,y12,y21,y22)))
plot(0,xlim=range(nvals),ylim=c(-rr,rr+10),xlab="n",
  ylab=expression("GP(0,cov_fn(.,.;var_u,theta))"))
lines(nvals,y11,lty=1,col="black")
lines(nvals,y12,lty=2,col="black")
lines(nvals,y21,lty=1,col="red")
lines(nvals,y22,lty=2,col="red")
legend("topright",c("k_width=10, var_u=1", "k_width=30, var_u=1",
  "k_width=10, var_u=10","k_width=30, var_u=10"),
  lty=c(1,2,1,2),col=c("black","black","red","red"))
```

`data_example_simulation` *Data for vignette showing general example*

Description

Data for general vignette. For generation, see hidden code in vignette, or in pipeline at https://github.com/jamesliley/OptH

Usage

```
data_example_simulation
```

Format

An object of class `list` of length 4.

`data_nextpoint_em` *Data for 'next point' demonstration vignette on algorithm comparison using emulation algorithm*

Description

Data containing 'next point selected' information for emulation algorithm in vignette comparing emulation and parametric algorithms. For generation, see hidden code in vignette, or in pipeline at https://github.com/jamesliley/OptHoldoutSize_pipelines

Usage

```
data_nextpoint_em
```

Format

An object of class `list` of length 6.

`data_nextpoint_par` *Data for 'next point' demonstration vignette on algorithm comparison using parametric algorithm*

Description

Data containing 'next point selected' information for parametric algorithm in vignette comparing emulation and parametric algorithms. For generation, see hidden code in vignette, or in pipeline at https://github.com/jamesliley/OptHoldoutSize_pipelines

Usage

```
data_nextpoint_par
```

Format

An object of class `list` of length 6.

`error_ohs_emulation` *Measure of error for emulation-based OHS emulation*

Description

Measure of error for semiparametric (emulation) based estimation of optimal holdout set sizes.

Returns a set of values of n for which a `1-alpha` credible interval for cost at includes a lower value than the cost at the estimated optimal holdout size.

This is not a confidence interval, credible interval or credible set for the OHS, and is prone to misinterpretation.

Usage

```
error_ohs_emulation(
  nset,
  k2,
  var_k2,
  N,
  k1,
  alpha = 0.1,
  var_u = 1e+07,
  k_width = 5000,
  k2form = powerlaw,
  theta = powersolve(nset, k2, y_var = var_k2)$par,
  npoll = 1000
)
```

Arguments

nset	Training set sizes for which k2() has been evaluated
k2	Estimated k2() at training set sizes nset
var_k2	Variance of error in k2() estimate at each training set size.
N	Total number of samples on which the model will be fitted/used
k1	Mean cost per sample with no predictive score in place
alpha	Use 1-alpha credible interval. Defaults to 0.1.
var_u	Marginal variance for Gaussian process kernel. Defaults to 1e7
k_width	Kernel width for Gaussian process kernel. Defaults to 5000
k2form	Functional form governing expected cost per sample given sample size. Should take two parameters: n (sample size) and theta (parameters). Defaults to function powerlaw.
theta	Current estimates of parameter values for k2form. Defaults to the MLE power-law solution corresponding to n,k2, and var_k2.
npoll	Check npoll equally spaced values between 1 and N for minimum. If NULL, check all values (this can be slow). Defaults to 1000

Value

Vector of values n for which 1-alpha credible interval for cost l(n) at n contains mean posterior cost at estimated optimal holdout size.

Examples

```
 # Set seed
set.seed(57365)

# Parameters
N=100000;
k1=0.3
A=8000; B=1.5; C=0.15; theta=c(A,B,C)

# True mean function
k2_true=function(n) powerlaw(n,theta)

# True OHS
nx=1:N
ohs_true=nx[which.min(k1*nx + k2_true(nx)*(N-nx))]

# Values of n for which cost has been estimated
np=50 # this many points
nset=round(runif(np,1,N))
var_k2=runif(np,0.001,0.0015)
k2=rnorm(np,mean=k2_true(nset),sd=sqrt(var_k2))

# Compute OHS
res1=optimal_holdout_size_emulation(nset,k2,var_k2,N,k1)

# Error estimates
ex=error_ohs_emulation(nset,k2,var_k2,N,k1)

# Plot
```

```
plot(res1)

# Add error
abline(v=ohs_true)
abline(v=ex,col=rgb(1,0,0,alpha=0.2))

# Show justification for error
n=seq(1,N,length=1000)
mu=mu_fn(n,nset,k2,var_k2,N,k1); psi=pmax(0,psi_fn(n, nset, var_k2, N)); Z=-qnorm(0.1/2)
lines(n,mu - Z*sqrt(psi),lty=2,lwd=2)
legend("topright",
    c("Err. region",expression(paste(mu(n)- "z"[alpha/2]*sqrt(psi(n))))),
    pch=c(16,NA),lty=c(NA,2),lwd=c(NA,2),col=c("pink","black"),bty="n")
```

`exp_imp_fn` *Expected improvement*

Description

Expected improvement

Essentially chooses the `next point` to add to `n`, called `n*`, in order to minimise the expectation of `cost(n*)`.

Usage

```
exp_imp_fn(
  n,
  nset,
  k2,
  var_k2,
  N,
  k1,
  var_u = 1e+07,
  k_width = 5000,
  k2form = powerlaw,
  theta = powersolve(nset, k2, y_var = var_k2)$par
)
```

Arguments

`n`	Set of training set sizes to evaluate
`nset`	Training set sizes for which a cost has been evaluated
`k2`	Estimates of k2() at training set sizes `nset`
`var_k2`	Variance of error in k2() estimates at each training set size.
`N`	Total number of samples on which the model will be fitted/used
`k1`	Mean vost per sample with no predictive score in place
`var_u`	Marginal variance for Gaussian process kernel. Defaults to 1e7
`k_width`	Kernel width for Gaussian process kernel. Defaults to 5000

`k2form` Functional form governing expected cost per sample given sample size. Should take two parameters: n (sample size) and theta (parameters). Defaults to function `powerlaw`.

`theta` Current estimates of parameter values for k2form. Defaults to the MLE power-law solution corresponding to n,k2, and var_k2.

Value

Value of expected improvement at values n

Examples

```
# Set seed.
set.seed(24015)

# Kernel width and Gaussian process variance
kw0=5000
vu0=1e7

# Include legend on plots or not; inclusion can obscure plot elements on small figures
inc_legend=FALSE

# Suppose we have population size and cost-per-sample without a risk score as follows
N=100000
k1=0.4

# Suppose that true values of a,b,c are given by
theta_true=c(10000,1.2,0.2)
theta_lower=c(1,0.5,0.1) # lower bounds for estimating theta
theta_upper=c(20000,2,0.5) # upper bounds for estimating theta



# We start with five random holdout set sizes (nset0),
#  with corresponding cost-per-individual estimates k2_0 derived
#  with various errors var_k2_0
nstart=4
vwmin=0.001; vwmax=0.005
nset0=round(runif(nstart,1000,N/2))
var_k2_0=runif(nstart,vwmin,vwmax)
k2_0=rnorm(nstart,mean=powerlaw(nset0,theta_true),sd=sqrt(var_k2_0))

# We estimate theta from these three points
theta0=powersolve(nset0,k2_0,y_var=var_k2_0,lower=theta_lower,upper=theta_upper,
  init=theta_true)$par

# We will estimate the posterior at these values of n
n=seq(1000,N,length=1000)

# Mean and variance
p_mu=mu_fn(n,nset=nset0,k2=k2_0,var_k2 = var_k2_0, N=N,k1=k1,theta=theta0,k_width=kw0,
  var_u=vu0)
p_var=psi_fn(n,nset=nset0,N=N,var_k2 = var_k2_0,k_width=kw0,var_u=vu0)

# Plot
yrange=c(-30000,100000)
```

```
plot(0,xlim=range(n),ylim=yrange,type="n",
  xlab="Training/holdout set size",
  ylab="Total cost (= num. cases)")
lines(n,p_mu,col="blue")
lines(n,p_mu - 3*sqrt(p_var),col="red")
lines(n,p_mu + 3*sqrt(p_var),col="red")
points(nset0,k1*nset0 + k2_0*(N-nset0),pch=16,col="purple")
lines(n,k1*n + powerlaw(n,theta0)*(N-n),lty=2)
lines(n,k1*n + powerlaw(n,theta_true)*(N-n),lty=3,lwd=3)
if (inc_legend) {
  legend("topright",
    c(expression(mu(n)),
      expression(mu(n) %+-% 3*sqrt(psi(n))),
      "prior(n)",
      "True",
      "d"),
    lty=c(1,1,2,3,NA),lwd=c(1,1,1,3,NA),pch=c(NA,NA,NA,NA,16),pt.cex=c(NA,NA,NA,NA,1),
    col=c("blue","red","black","purple"),bg="white")
}

## Add line corresponding to recommended new point
exp_imp_em <- exp_imp_fn(n,nset=nset0,k2=k2_0,var_k2 = var_k2_0, N=N,k1=k1,
  theta=theta0,k_width=kw0,var_u=vu0)
abline(v=n[which.max(exp_imp_em)])
```

`gen_base_coefs` *Coefficients for imperfect risk score*

Description

Generate coefficients corresponding to an imperfect risk score, interpretable as 'baseline' behaviour in the absence of a risk score

Usage

```
gen_base_coefs(coefs, noise = TRUE, num_vars = 2, max_base_powers = 1)
```

Arguments

`coefs`	Original coefficients
`noise`	Set to TRUE to add Gaussian noise to coefficients
`num_vars`	Number of variables at hand for baseline calculation
`max_base_powers`	If >1, return a matrix of coefficients, with successively more noise

Value

Vector of coefficients

Examples

```
# See examples for model_predict
```

gen_preds *Generate matrix of random observations*

Description

Generate matrix of random observations. Observations are unit Gaussian-distributed.

Usage

```
gen_preds(nobs, npreds, ninters = 0)
```

Arguments

nobs	Number of observations (samples)
npreds	Number of predictors
ninters	Number of interaction terms, default 0. Up to npreds*(npreds-1)/2

Value

Data frame of observations

Examples

```
# See examples for model_predict
```

gen_resp *Generate response*

Description

Generate random outcome (response) according to a ground-truth logistic model

Usage

```
gen_resp(X, coefs = NA, coefs_sd = 1, retprobs = FALSE)
```

Arguments

X	Matrix of observations
coefs	Vector of coefficients for logistic model. If NA, random coefficients are generated. Defaults to NA
coefs_sd	If random coefficients are generated, use this SD (mean 0)
retprobs	If TRUE, return class probability; otherwise, return classes. Defaults to FALSE

Value

Vector of length as first dimension of dim(X) with outcome classes (if retprobs==FALSE) or outcome probabilities (if retprobs==TRUE)

Examples

```
# See examples for model_predict
```

---

grad_mincost_powerlaw *Gradient of minimum cost (power law)*

Description

Compute gradient of minimum cost assuming a power-law form of k2

Assumes cost function is `l(n;k1,N,theta) = k1 n + k2(n;theta) (N-n)` with `k2(n;theta)=k2(n;a,b,c)= a n^(-b`

Usage

```
grad_mincost_powerlaw(N, k1, theta)
```

Arguments

`N`	Total number of samples on which the predictive score will be used/fitted. Can be a vector.
`k1`	Cost value in the absence of a predictive score. Can be a vector.
`theta`	Parameters for function k2(n) governing expected cost to an individual sample given a predictive score fitted to n samples. Can be a matrix of dimension n x n_par, where n_par is the number of parameters of k2.

Value

List/data frame of dimension (number of evaluations) x 5 containing partial derivatives of nstar (optimal holdout size) with respect to N, k1, a, b, c respectively.

Examples

```
# Evaluate minimum for a range of values of k1, and compute derivative
N=10000;
k1=seq(0.1,0.5,length=20)
A=3; B=1.5; C=0.15; theta=c(A,B,C)

mincost=optimal_holdout_size(N,k1,theta)
grad_mincost=grad_mincost_powerlaw(N,k1,theta)

plot(0,type="n",ylim=c(0,1560),xlim=range(k1),xlab=expression("k"[1]),
  ylab="Optimal holdout set size")
lines(mincost$k1,mincost$cost,col="black")
lines(mincost$k1,grad_mincost[,2],col="red")
legend(0.2,800,c(expression(paste("l(n"["*"],")")),
                      expression(paste(partialdiff[k1],"l(n"["*"],")"))),
    col=c("black","red"),lty=1,bty="n")
```

grad_nstar_powerlaw *Gradient of optimal holdout size (power law)*

Description

Compute gradient of optimal holdout size assuming a power-law form of k2

Assumes cost function is `l(n;k1,N,theta) = k1 n + k2(n;theta) (N-n)` with `k2(n;theta)=k2(n;a,b,c)= a n^(-b`

Usage

```
grad_nstar_powerlaw(N, k1, theta)
```

Arguments

`N`	Total number of samples on which the predictive score will be used/fitted. Can be a vector.
`k1`	Cost value in the absence of a predictive score. Can be a vector.
`theta`	Parameters for function k2(n) governing expected cost to an individual sample given a predictive score fitted to n samples. Can be a matrix of dimension n x n_par, where n_par is the number of parameters of k2.

Value

List/data frame of dimension (number of evaluations) x 5 containing partial derivatives of nstar (optimal holdout size) with respect to N, k1, a, b, c respectively.

Examples

```
# Evaluate optimal holdout set size for a range of values of k1, and compute
#  derivative
N=10000;
k1=seq(0.1,0.5,length=20)
A=3; B=1.5; C=0.15; theta=c(A,B,C)

nstar=optimal_holdout_size(N,k1,theta)
grad_nstar=grad_nstar_powerlaw(N,k1,theta)

plot(0,type="n",ylim=c(-2000,500),xlim=range(k1),xlab=expression("k"[1]),
  ylab="Optimal holdout set size")
lines(nstar$k1,nstar$size,col="black")
lines(nstar$k1,grad_nstar[,2],col="red")
legend("bottomright",c(expression("n"["*"]),
    expression(paste(partialdiff[k1],"n"["*"]))),
    col=c("black","red"),lty=1)
```

`logistic` *Logistic*

Description

Logistic function: -log((1/x)-1)

Usage

```
logistic(x)
```

Arguments

x	argument

Value

value of logit(x); na if x is outside (0,1)

Examples

```
# Plot
x=seq(0,1,length=100)
plot(x,logistic(x),type="l")

# Logit and logistic are inverses
x=seq(-5,5,length=1000)
plot(x,logistic(logit(x)),type="l")
```

`logit` *Logit*

Description

Logit function: 1/(1+exp(-x))

Usage

```
logit(x)
```

Arguments

x	argument

Value

value of logit(x)

Examples

```
# Plot
x=seq(-5,5,length=1000)
plot(x,logit(x),type="l")
```

`model_predict` *Make predictions*

Description

Make predictions according to a given model

Usage

```
model_predict(
  data_test,
  trained_model,
  return_type,
  threshold = NULL,
  model_family = NULL,
  ...
)
```

Arguments

`data_test`	Data for which predictions are to be computed
`trained_model`	Model for which predictions are to be made
`return_type`	??
`threshold`	??
`model_family`	??
`...`	Passed to function `predict.glm()` or `predict.ranger()`

Value

Vector of predictions

Examples

```
## Set seed for reproducibility
seed=1234
set.seed(seed)

# Initialisation of patient data
n_iter <- 500           # Number of point estimates to be calculated
nobs <- 5000            # Number of observations, i.e patients
npreds <- 7             # Number of predictors

# Model family
family="log_reg"

# Baseline behaviour is an oracle Bayes-optimal predictor on only one variable
max_base_powers <- 1
base_vars=1

# Check the following holdout size fractions
frac_ho = 0.1
```

```
# Set ground truth coefficients, and the accuracy at baseline
coefs_general <- rnorm(npreds,sd=1/sqrt(npreds))
coefs_base <- gen_base_coefs(coefs_general, max_base_powers = max_base_powers)

# Generate dataset
X <- gen_preds(nobs, npreds)

# Generate labels
newdata <- gen_resp(X, coefs = coefs_general)
Y <- newdata$classes

# Combined dataset
pat_data <- cbind(X, Y)
pat_data$Y = factor(pat_data$Y)

# For each holdout size, split data into intervention and holdout set
mask <- split_data(pat_data, frac_ho)
data_interv <- pat_data[!mask,]
data_hold <- pat_data[mask,]

# Train model
trained_model <- model_train(data_hold, model_family = family)
thresh <- 0.5

# Make predictions
class_pred <- model_predict(data_interv, trained_model,
                            return_type = "class",
                            threshold = 0.5, model_family = family)


# Simulate baseline predictions
base_pred <- oracle_pred(data_hold,coefs_base[base_vars, ], num_vars = base_vars)


# Contingency table for model-based predictor (on intervention set)
print(table(class_pred,data_interv$Y))

# Contingency table for model-based predictor (on holdout set)
print(table(base_pred,data_hold$Y))
```

`model_train` *Train model (wrapper)*

Description

Train model using either a GLM or a random forest

Usage

```
model_train(train_data, model_family = "log_reg", ...)
```

Arguments

train_data	Data to use for training; assumed to have one binary column called Y
model_family	Either 'log_reg' for logistic regression or 'rand_forest' for random forest
...	Passed to function glm() or ranger()

Value

Fitted model of type GLM or Ranger

Examples

```
# See examples for model_predict
```

---

mu_fn *Updating function for mean.*

Description

Posterior mean for emulator given points n.

Usage

```
mu_fn(
  n,
  nset,
  k2,
  var_k2,
  N,
  k1,
  var_u = 1e+07,
  k_width = 5000,
  k2form = powerlaw,
  theta = powersolve(nset, k2, y_var = var_k2)$par
)
```

Arguments

n	Set of training set sizes to evaluate
nset	Training set sizes for which k2() has been evaluated
k2	Estimated k2() values at training set sizes nset
var_k2	Variance of error in k2() estimate at each training set size.
N	Total number of samples on which the model will be fitted/used
k1	Mean cost per sample with no predictive score in place
var_u	Marginal variance for Gaussian process kernel. Defaults to 1e7
k_width	Kernel width for Gaussian process kernel. Defaults to 5000
k2form	Functional form governing expected cost per sample given sample size. Should take two parameters: n (sample size) and theta (parameters). Defaults to function powerlaw.
theta	Current estimates of parameter values for k2form. Defaults to the MLE powerlaw solution corresponding to n,k2, and var_k2.

Value

Vector Mu of same length of n where Mu_i=mean(posterior(cost(n_i)))

Examples

```
# Suppose we have population size and cost-per-sample without a risk score as follows
N=100000
k1=0.4

# Kernel width and variance for GP
k_width=5000
var_u=8000000

# Suppose we begin with k2() estimates at n-values
nset=c(10000,20000,30000)

# with cost-per-individual estimates
# (note that since empirical k2(n) is non-monotonic, it cannot be perfectly
#  approximated with a power-law function)
k2=c(0.35,0.26,0.28)

# and associated error on those estimates
var_k2=c(0.02^2,0.01^2,0.03^2)

# We estimate theta from these three points
theta=powersolve(nset,k2,y_var=var_k2)$par

# We will estimate the posterior at these values of n
n=seq(1000,50000,length=1000)

# Mean and variance
p_mu=mu_fn(n,nset=nset,k2=k2,var_k2 = var_k2, N=N,k1=k1,theta=theta,
           k_width=k_width,var_u=var_u)
p_var=psi_fn(n,nset=nset,N=N,var_k2 = var_k2,k_width=k_width,var_u=var_u)

# Plot
plot(0,xlim=range(n),ylim=c(20000,60000),type="n",
     xlab="Training/holdout set size",
     ylab="Total cost (= num. cases)")
lines(n,p_mu,col="blue")
lines(n,p_mu - 3*sqrt(p_var),col="red")
lines(n,p_mu + 3*sqrt(p_var),col="red")
points(nset,k1*nset + k2*(N-nset),pch=16,col="purple")
lines(n,k1*n + powerlaw(n,theta)*(N-n),lty=2)
segments(nset,k1*nset + (k2 - 3*sqrt(var_k2))*(N-nset),
         nset,k1*nset + (k2 + 3*sqrt(var_k2))*(N-nset))
legend("topright",
       c(expression(mu(n)),
         expression(mu(n) %+-% 3*sqrt(psi(n))),
         "prior(n)",
         "d",
         "3SD(d|n)"),
       lty=c(1,1,2,NA,NA),lwd=c(1,1,1,NA,NA),pch=c(NA,NA,NA,16,124),
       pt.cex=c(NA,NA,NA,1,1),
       col=c("blue","red","black","purple","black"),bg="white")
```

next_n *Finds best value of n to sample next*

Description

Recommends a value of n at which to next evaluate individual cost in order to most accurately estimate optimal holdout size. Currently only for use with a power-law parametrisation of k2.

Approximately finds a set of n points which, given estimates of cost, minimise width of 95% confidence interval around OHS. Uses a greedy algorithm, so various parameters can be learned along the way.

Given existing training set size/k2 estimates nset and k2, with var_k2[i]=variance(k2[i]), finds, for each candidate point n[i], the median width of the 90% confidence interval for OHS if

nset <- c(nset,n[i]) var_k2 <- c(var_k2,mean(var_k2)) k2 <- c(k2,rnorm(powerlaw(n[i],theta),variance

Usage

```
next_n(
  n,
  nset,
  k2,
  N,
  k1,
  nmed = 100,
  var_k2 = rep(1, length(nset)),
  mode = "asymptotic",
  ...
)
```

Arguments

n	Set of training set sizes to evaluate
nset	Training set sizes for which a loss has been evaluated
k2	Estimated k2() at training set sizes nset
N	Total number of samples on which the model will be fitted/used
k1	Mean loss per sample with no predictive score in place
nmed	number of times to re-evaluate d and confidence interval width.
var_k2	Variance of error in k2() estimate at each training set size.
mode	Mode for calculating OHS CI (passed to ci_ohs): 'asymptotic' or 'empirical'
...	Passed to powersolve and powersolve_se

Value

Vector out of same length as n, where out[i] is the expected width of the 95% confidence interval for OHS should n be added to nset.

Examples

```
# Set seed.
set.seed(24015)

# Kernel width and Gaussian process variance
kw0=5000
vu0=1e7

# Include legend on plots or not; inclusion can obscure plot elements on small figures
inc_legend=FALSE

# Suppose we have population size and cost-per-sample without a risk score as follows
N=100000
k1=0.4

# Suppose that true values of a,b,c are given by
theta_true=c(10000,1.2,0.2)
theta_lower=c(1,0.5,0.1) # lower bounds for estimating theta
theta_upper=c(20000,2,0.5) # upper bounds for estimating theta



# We start with five random holdout set sizes (nset0),
#  with corresponding cost-per-individual estimates k2_0 derived
#  with various errors var_k2_0
nstart=10
vwmin=0.001; vwmax=0.005
nset0=round(runif(nstart,1000,N/2))
var_k2_0=runif(nstart,vwmin,vwmax)
k2_0=rnorm(nstart,mean=powerlaw(nset0,theta_true),sd=sqrt(var_k2_0))

# We estimate theta from these three points
theta0=powersolve(nset0,k2_0,y_var=var_k2_0,lower=theta_lower,upper=theta_upper,init=theta_true)$par

# We will estimate the posterior at these values of n
n=seq(1000,N,length=1000)

# Mean and variance
p_mu=mu_fn(n,nset=nset0,k2=k2_0,var_k2 = var_k2_0, N=N,k1=k1,theta=theta0,k_width=kw0,var_u=vu0)
p_var=psi_fn(n,nset=nset0,N=N,var_k2 = var_k2_0,k_width=kw0,var_u=vu0)

# Plot
yrange=c(-30000,100000)
plot(0,xlim=range(n),ylim=yrange,type="n",
  xlab="Training/holdout set size",
  ylab="Total cost (= num. cases)")
lines(n,p_mu,col="blue")
lines(n,p_mu - 3*sqrt(p_var),col="red")
lines(n,p_mu + 3*sqrt(p_var),col="red")
points(nset0,k1*nset0 + k2_0*(N-nset0),pch=16,col="purple")
lines(n,k1*n + powerlaw(n,theta0)*(N-n),lty=2)
lines(n,k1*n + powerlaw(n,theta_true)*(N-n),lty=3,lwd=3)
if (inc_legend) {
  legend("topright",
    c(expression(mu(n)),
      expression(mu(n) %+-% 3*sqrt(psi(n))),
```

```
        "prior(n)",
        "True",
        "d"),
     lty=c(1,1,2,3,NA),lwd=c(1,1,1,3,NA),pch=c(NA,NA,NA,NA,16),pt.cex=c(NA,NA,NA,NA,1),
     col=c("blue","red","black","purple"),bg="white")
}

## Add line corresponding to recommended new point. This is slow.
nn=seq(1000,N,length=20)
exp_imp <- next_n(nn,nset=nset0,k2=k2_0,var_k2 = var_k2_0, N=N,k1=k1,nmed=10,
                     lower=theta_lower,upper=theta_upper)
abline(v=nn[which.min(exp_imp)])
```

`ohs_array` *Data for vignette on algorithm comparison*

Description

This object contains data relating to the vignette comparing emulation and parametric algorithms. For generation, see hidden code in vignette, or in pipeline at https://github.com/jamesliley/OptHoldoutSize_pipelines

Usage

`ohs_array`

Format

An object of class `array` of dimension 200 x 200 x 2 x 2 x 2.

`ohs_resample` *Data for vignette on algorithm comparison*

Description

This object contains data relating to the first plot in the vignette comparing emulation and parametric algorithms. For generation, see hidden code in vignette, or in pipeline at https://github.com/jamesliley/OptHoldoutSize_p

Usage

`ohs_resample`

Format

An object of class `matrix` (inherits from `array`) with 1000 rows and 4 columns.

optimal_holdout_size *Estimate optimal holdout size under parametric assumptions*

Description

Compute optimal holdout size for updating a predictive score given appropriate parameters of cost function

Evaluates empirical minimisation of cost function `l(n;k1,N,theta) = k1 n + k2form(n;theta) (N-n)`.

The function will return `Inf` if no minimum exists. It does not check if the minimum is unique, but this can be guaranteed using the assumptions for theorem 1 in the manuscript.

This calls the function `optimize` from package `stats`.

Usage

```
optimal_holdout_size(
  N,
  k1,
  theta,
  k2form = powerlaw,
  round_result = FALSE,
  ...
)
```

Arguments

`N`	Total number of samples on which the predictive score will be used/fitted. Can be a vector.
`k1`	Cost value in the absence of a predictive score. Can be a vector.
`theta`	Parameters for function k2form(n) governing expected cost to an individual sample given a predictive score fitted to n samples. Can be a matrix of dimension n x n_par, where n_par is the number of parameters of k2.
`k2form`	Function governing expected cost to an individual sample given a predictive score fitted to n samples. Must take two arguments: n (number of samples) and theta (parameters). Defaults to a power-law form `powerlaw(n,c(a,b,c))=a n^(-b) + c`.
`round_result`	Set to TRUE to solve over integral sizes
`...`	Passed to function `optimize`

Value

List/data frame of dimension (number of evaluations) x (4 + n_par) containing input data and results. Columns size and cost are optimal holdout size and cost at this size respectively. Parameters N, k1, theta.1, theta.2,...,theta.n_par are input data.

Examples

```
# Evaluate optimal holdout set size for a range of values of k1 and two values of
#   N, some of which lead to infinite values
N1=10000; N2=12000
k1=seq(0.1,0.5,length=20)
```

```
A=3; B=1.5; C=0.15; theta=c(A,B,C)

res1=optimal_holdout_size(N1,k1,theta)
res2=optimal_holdout_size(N2,k1,theta)

oldpar=par(mfrow=c(1,2))
plot(0,type="n",ylim=c(0,500),xlim=range(res1$k1),xlab=expression("k"[1]),
  ylab="Optimal holdout set size")
  lines(res1$k1,res1$size,col="black")
  lines(res2$k1,res2$size,col="red")
  legend("topright",as.character(c(N1,N2)),title="N:",col=c("black","red"),lty=1)
plot(0,type="n",ylim=c(1500,1600),xlim=range(res1$k1),xlab=expression("k"[1]),
  ylab="Minimum cost")
  lines(res1$k1,res1$cost,col="black")
  lines(res2$k1,res2$cost,col="red")
  legend("topleft",as.character(c(N1,N2)),title="N:",col=c("black","red"),lty=1)

par(oldpar)
```

`optimal_holdout_size_emulation`
Estimate optimal holdout size under semi-parametric assumptions

---

Description

Compute optimal holdout size for updating a predictive score given a set of training set sizes and estimates of mean cost per sample at those training set sizes.

This is essentially a wrapper for function `mu_fn()`.

Usage

```
optimal_holdout_size_emulation(
  nset,
  k2,
  var_k2,
  N,
  k1,
  var_u = 1e+07,
  k_width = 5000,
  k2form = powerlaw,
  theta = powersolve_general(nset, k2, y_var = var_k2)$par,
  npoll = 1000,
  ...
)
```

Arguments

`nset`	Training set sizes for which a cost has been evaluated
`k2`	Estimated values of k2() at training set sizes `nset`
`var_k2`	Variance of error in k2 estimate at each training set size.
`N`	Total number of samples on which the model will be fitted/used

k1	Mean cost per sample with no predictive score in place
var_u	Marginal variance for Gaussian process kernel. Defaults to 1e7
k_width	Kernel width for Gaussian process kernel. Defaults to 5000
k2form	Functional form governing expected cost per sample given sample size. Should take two parameters: n (sample size) and theta (parameters). Defaults to function powerlaw.
theta	Current estimates of parameter values for k2form. Defaults to the MLE power-law solution corresponding to n,k2, and var_k2.
npoll	Check npoll equally spaced values between 1 and N for minimum. If NULL, check all values (this can be slow). Defaults to 1000
...	Passed to function optimise()

Value

Object of class 'optholdoutsize_emul' with elements "cost" (minimum cost),"size" (OHS),"nset","k2","var_k2","N","k1" (parameters)

Examples

```
# See examples for mu_fn()
```

---

oracle_pred *Generate responses*

Description

Probably for deprecation

Usage

```
oracle_pred(X, coefs, num_vars = 3, noise = TRUE)
```

Arguments

X	Matrix of observations
coefs	Vector of coefficients for logistic model.
num_vars	If noise==FALSE, computes using only first num_vars predictors
noise	If TRUE, uses all predictors

Value

Vector of predictions

Examples

```
# See examples for model_predict
```

params_aspre *Parameters of reported ASPRE dataset*

Description

Distribution of covariates for ASPRE dataset; see Rolnik, 2017, NEJM

Usage

```
params_aspre
```

Format

An object of class `list` of length 16.

plot.optholdoutsize *Plot estimated cost function*

Description

Plot estimated cost function, when parametric method is used for estimation.

Draws cost function as a line and indicates minimum. Assumes a power-law form of k2 unless parameter k2 is set otherwise.

Usage

```
## S3 method for class 'optholdoutsize'
plot(x, ..., k2form = powerlaw)
```

Arguments

`x`	Object of type `optholdoutsize`
`...`	Other arguments passed to `plot()` and `lines()`
`k2form`	Function governing expected cost to an individual sample given a predictive score fitted to n samples. Must take two arguments: `n` (number of samples) and theta (parameters). Defaults to a power-law form `powerlaw(n,c(a,b,c))=a n^(-b) + c`.

Value

No return value; draws a plot only.

Examples

```
# Simple example

N=100000;
k1=0.3
A=8000; B=1.5; C=0.15; theta=c(A,B,C)

res1=optimal_holdout_size(N,k1,theta)

plot(res1)
```

---

plot.optholdoutsize_emul
Plot estimated cost function using emulation (semiparametric)

Description

Plot estimated cost function, when semiparametric (emulation) method is used for estimation.

Draws posterior mean of cost function as a line and indicates minimum. Also draws mean +/- 3 SE.

Assumes a power-law form of k2 unless parameter k2 is set otherwise.

Usage

```
## S3 method for class 'optholdoutsize_emul'
plot(x, ..., k2form = powerlaw)
```

Arguments

x	Object of type `optholdoutsize_emul`
...	Other arguments passed to `plot()`
k2form	Function governing expected cost to an individual sample given a predictive score fitted to n samples. Must take two arguments: n (number of samples) and theta (parameters). Defaults to a power-law form `powerlaw(n,c(a,b,c))=a n^(-b) + c`.

Value

No return value; draws a plot only.

Examples

```
# Simple example

# Parameters
N=100000;
k1=0.3
A=8000; B=1.5; C=0.15; theta=c(A,B,C)

# True mean function
k2_true=function(n) powerlaw(n,theta)
```

```
# Values of n for which cost has been estimated
np=50 # this many points
nset=round(runif(np,1,N))
var_k2=runif(np,0.001,0.002)
k2=rnorm(np,mean=k2_true(nset),sd=sqrt(var_k2))

# Compute OHS
res1=optimal_holdout_size_emulation(nset,k2,var_k2,N,k1)

# Plot
plot(res1)
```

`powerlaw` *Power law function*

Description

Power law function for modelling learning curve (taken to mean change in expected loss per sample with training set size)

Recommended in review of learning curve forms

If `theta=c(a,b,c)` then models as `a n^(-b) + c`. Note `b` is negated.

Note that `powerlaw(n,c(a,b,c))` has limit `c` as `n` tends to infinity, if `a,b > 0`

Usage

```
powerlaw(n, theta)
```

Arguments

`n`	Set of training set sizes to evaluate
`theta`	Parameter of values

Value

Vector of values of same length as `n`

Examples

```
ncheck=seq(1000,10000)
plot(ncheck, powerlaw(ncheck, c(5e3,1.2,0.3)),type="l",xlab="n",ylab="powerlaw(n)")
```

powersolve *Fit power law curve*

Description

Find least-squares solution: MLE of (a,b,c) under model `y_i = a x_i^-b + c + e_i`; `e_i~N(0,y_var_i)`

Usage

```
powersolve(
  x,
  y,
  init = c(20000, 2, 0.1),
  y_var = rep(1, length(y)),
  estimate_s = FALSE,
  ...
)
```

Arguments

`x`	X values
`y`	Y values
`init`	Initial values of (a,b,c) to start. Default c(20000,2,0.1)
`y_var`	Optional parameter giving sampling variance of each y value. Defaults to 1.
`estimate_s`	Parameter specifying whether to also estimate s (as above). Defaults to FALSE (no).
`...`	further parameters passed to optim. We suggest specifying lower and upper bounds for (a,b,c); e.g. lower=c(1,0,0),upper=c(10000,3,1)

Value

List (output from `optim`) containing MLE values of (a,b,c)

Examples

```
# Retrieval of original values
A_true=2000; B_true=1.5; C_true=0.3; sigma=0.002

X=1000*abs(rnorm(10000,mean=4))
Y=A_true*(X^(-B_true)) + C_true + rnorm(length(X),sd=sigma)

c(A_true,B_true,C_true)
powersolve(X[1:10],Y[1:10])$par
powersolve(X[1:100],Y[1:100])$par
powersolve(X[1:1000],Y[1:1000])$par
powersolve(X[1:10000],Y[1:10000])$par
```

powersolve_general *General solver for power law curve*

Description

Find least-squares solution: MLE of (a,b,c) under model `y_i = a x_i^-b + c + e_i`; `e_i~N(0,y_var_i)`

Try a range of starting values and refine estimate.

Slower than a single call to `powersolve()`

Usage

```
powersolve_general(x, y, y_var = rep(1, length(x)), ...)
```

Arguments

x	X values
y	Y values
y_var	Optional parameter giving sampling variance of each y value. Defaults to 1.
...	further parameters passed to optim. We suggest specifying lower and upper bounds for (a,b,c); e.g. lower=c(1,0,0),upper=c(10000,3,1)

Value

List (output from `optim`) containing MLE values of (a,b,c)

Examples

```
# Retrieval of original values
A_true=2000; B_true=1.5; C_true=0.3; sigma=0.002

X=1000*abs(rnorm(10000,mean=4))
Y=A_true*(X^(-B_true)) + C_true + rnorm(length(X),sd=sigma)

c(A_true,B_true,C_true)
powersolve_general(X[1:10],Y[1:10])$par
powersolve_general(X[1:100],Y[1:100])$par
powersolve_general(X[1:1000],Y[1:1000])$par
```

powersolve_se *Standard error matrix for learning curve parameters (power law)*

Description

Find approximate standard error matrix for `(a,b,c)` under power law model for learning curve.

Assumes that
`y_i= a x_i^-b + c + e, e~N(0,s^2 y_var_i^2)`

Standard error can be computed either asymptotically using Fisher information (`method='fisher'`) or boostrapped (`method='bootstrap'`)

These estimate different quantities: the asymptotic method estimates

`Var[MLE(a,b,c)|X,y_var]`

and the boostrap method estimates

`Var[MLE(a,b,c)]`.

Usage

```
powersolve_se(
  x,
  y,
  method = "fisher",
  init = c(20000, 2, 0.1),
  y_var = rep(1, length(y)),
  n_boot = 1000,
  seed = NULL,
  ...
)
```

Arguments

`x`	X values (typically training set sizes)
`y`	Y values (typically observed cost per individual/sample)
`method`	One of 'fisher' (for asymptotic variance via Fisher Information) or 'bootstrap' (for Bootstrap)
`init`	Initial values of (a,b,c) to start when computing MLE. Default c(20000,2,0.1)
`y_var`	Optional parameter giving sampling variance of each y value. Defaults to 1.
`n_boot`	Number of bootstrap resamples. Only used if method='bootstrap'. Defaults to 1000
`seed`	Random seed for bootstrap resamples. Defaults to NULL.
`...`	further parameters passed to optim. We suggest specifying lower and upper bounds; since optim is called on (a*1000^-b,b,c), bounds should be relative to this; for instance, lower=c(0,0,0),upper=c(100,3,1)

Value

Standard error matrix; approximate covariance matrix of MLE(a,b,c)

Examples

```
A_true=10; B_true=1.5; C_true=0.3; sigma=0.1

set.seed(31525)
```

```
X=1+3*rchisq(10000,df=5)
Y=A_true*(X^(-B_true)) + C_true + rnorm(length(X),sd=sigma)

# 'Observations' - 100 samples
obs=sample(length(X),100,rep=FALSE)
Xobs=X[obs]; Yobs=Y[obs]

# True covariance matrix of MLE of a,b,c on these x values
ntest=100
abc_mat_xfix=matrix(0,ntest,3)
abc_mat_xvar=matrix(0,ntest,3)
E1=A_true*(Xobs^(-B_true)) + C_true
for (i in 1:ntest) {
  Y1=E1 + rnorm(length(Xobs),sd=sigma)
  abc_mat_xfix[i,]=powersolve(Xobs,Y1)$par # Estimate (a,b,c) with same X

  X2=1+3*rchisq(length(Xobs),df=5)
  Y2=A_true*(X2^(-B_true)) + C_true + rnorm(length(Xobs),sd=sigma)
  abc_mat_xvar[i,]=powersolve(X2,Y2)$par # Estimate (a,b,c) with variable X
}

Ve1=var(abc_mat_xfix) # empirical variance of MLE(a,b,c)|X
Vf=powersolve_se(Xobs,Yobs,method='fisher') # estimated SE matrix, asymptotic

Ve2=var(abc_mat_xvar) # empirical variance of MLE(a,b,c)
Vb=powersolve_se(Xobs,Yobs,method='bootstrap',n_boot=200) # estimated SE matrix, bootstrap

cat("Empirical variance of MLE(a,b,c)|X\n")
print(Ve1)
cat("\n")
cat("Asymptotic variance of MLE(a,b,c)|X\n")
print(Vf)
cat("\n\n")
cat("Empirical variance of MLE(a,b,c)\n")
print(Ve2)
cat("\n")
cat("Bootstrap-estimated variance of MLE(a,b,c)\n")
print(Vb)
cat("\n\n")
```

---

psi_fn *Updating function for variance.*

Description

Posterior variance for emulator given points n.

Usage

```
psi_fn(n, nset, var_k2, N, var_u = 1e+07, k_width = 5000)
```

Arguments

`n`	Set of training set sizes to evaluate at
`nset`	Training set sizes for which k2() has been evaluated
`var_k2`	Variance of error in k2() estimate at each training set size.
`N`	Total number of samples on which the model will be fitted/used. Only used to rescale var_k2
`var_u`	Marginal variance for Gaussian process kernel. Defaults to 1e7
`k_width`	Kernel width for Gaussian process kernel. Defaults to 5000

Value

Vector Psi of same length of n where Psi_i=var(posterior(cost(n_i)))

Examples

```
# See examples for `mu_fn`
```

`sens10` *Sensitivity at theshold quantile 10%*

Description

Computes sensitivity of a risk score at a threshold at which 10% of samples (or some proportion pi_int) are above the threshold.

Usage

```
sens10(Y, Ypred, pi_int = 0.1)
```

Arguments

`Y`	True labels (1 or 0)
`Ypred`	Predictions (univariate; real numbers)
`pi_int`	Compute sensitivity when a proportion pi_int of samples exceed threshold, default 0.1

Value

Sensitivity at this threshold

Examples

```
# Simulate
set.seed(32142)

N=1000
X=rnorm(N); Y=rbinom(N,1,prob=logit(X/2))

pi_int=0.1
q10=quantile(X,1-pi_int) # 10% of X values are above this threshold

print(length(which(Y==1 & X>q10))/length(which(X>q10)))
print(sens10(Y,X,pi_int))
```

`sim_random_aspre` *Simulate random dataset similar to ASPRE training data*

Description

Generate random population of individuals (e.g., newly pregnant women) with given population parameters

Assumes independence of parameter variation. This is not a realistic assumption, but is satisfactory for our purposes.

Usage

```
sim_random_aspre(n, params = NULL)
```

Arguments

`n`	size of population
`params`	list of parameters

Value

Matrix of samples

Examples

```
# Load ASPRE related data
data(params_aspre)

X=sim_random_aspre(1000,params_aspre)

print(c(median(X$age),params_aspre$age$median))

print(rbind(table(X$parity)/1000,params_aspre$parity$freq))
```

`split_data` *Split data*

Description

Split data into holdout and intervention sets

Usage

```
split_data(X, frac)
```

Arguments

`X`	Matrix of observations
`frac`	Fraction of observations to use for the training set

Value

Vector of TRUE/FALSE values (randomised) with proportion `frac` as TRUE

Examples

```
# See examples for model_predict
```

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