

# Package ‘OptHoldoutSize’

April 8, 2025

**Title** Estimation of Optimal Size for a Holdout Set for Updating a Predictive Score

**Version** 0.1.0.1

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**Description** Predictive scores must be updated with care, because actions taken on the basis of existing risk scores causes bias in risk estimates from the updated score. A holdout set is a straightforward way to manage this problem: a proportion of the population is 'held-out' from computation of the previous risk score. This package provides tools to estimate a size for this holdout set and associated errors. Comprehensive vignettes are included. Please see: Haidar-Wehbe S, Emerson SR, Aslett LJM, Liley J (2022) <[arXiv:2202.06374](https://arxiv.org/abs/2202.06374)> for details of methods.

**License** GPL (>=3)

**Encoding** UTF-8

**LazyData** true

**Depends** R (>= 3.5.0), matrixStats, mnormt, mvtnorm, ranger, mle.tools

**Roxygen** list(markdown = TRUE)

**Suggests** knitr, rmarkdown

**VignetteBuilder** knitr

**RoxygenNote** 7.3.2

**NeedsCompilation** no

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add\_aspre\_interactions

*Add interaction terms corresponding to ASPRE model*

---

### **Description**

Add various interaction terms to X. Interaction terms correspond to those in ASPRE.

### **Usage**

```
add_aspre_interactions(X)
```

### **Arguments**

X                      data frame

**Value**

New data frame containing interaction terms.

**Examples**

```
# Load ASPRE related data
data(params_aspre)

X=sim_random_aspre(1000,params_aspre)
Xnew=add_aspre_interactions(X)

print(colnames(X))
print(colnames(Xnew))
```

---

aspre	<i>Computes ASPRE score</i>
-------	-----------------------------

---

**Description**

Computes ASPRE model prediction on a matrix X of covariates

Full ASPRE model from [https://www.nejm.org/doi/suppl/10.1056/NEJMoa1704559/suppl\\_file/nejmoa1704559\\_append](https://www.nejm.org/doi/suppl/10.1056/NEJMoa1704559/suppl_file/nejmoa1704559_append)

Model is to predict gestational age at PE; that is, a higher score indicates a lower PE risk, so coefficients are negated for model to predict PE risk.

**Usage**

```
aspre(X)
```

**Arguments**

X                      matrix, assumed to be output of `sim_random_aspre` with parameter `params=params_aspre` and transformed using `add_aspre_interactions`

**Value**

vector of scores.

**Examples**

```
# Load ASPRE related data
data(params_aspre)

X=sim_random_aspre(1000,params_aspre)
Xnew=add_aspre_interactions(X)

aspre_score=aspre(Xnew)

plot(density(aspre_score))
```

---

aspre_emulation	<i>Emulation-based OHS estimation for ASPRE</i>
-----------------	---

---

### Description

This object contains data relating to emulation-based OHS estimation for the ASPRE model. For generation, see hidden code in vignette, or in pipeline at [https://github.com/jamesliley/OptHoldoutSize\\_pipelines](https://github.com/jamesliley/OptHoldoutSize_pipelines)

### Usage

```
aspre_emulation
```

### Format

An object of class list of length 4.

---

aspre_k2	<i>Cost estimating function in ASPRE simulation</i>
----------	---

---

### Description

Estimate cost at a given holdout set size in ASPRE model

### Usage

```
aspre_k2(
  n,
  X,
  PRE,
  seed = NULL,
  pi_PRE = 1426/58974,
  pi_intervention = 0.1,
  alpha = 0.37
)
```

### Arguments

n	Holdout set size at which to estimate k_2 (cost)
X	Matrix of predictors
PRE	Vector indicating PRE incidence
seed	Random seed; set before starting or set to NULL
pi_PRE	Population prevalence of PRE if not prophylactically treated. Defaults to empirical value 1426/58974
pi_intervention	Proportion of the population on which an intervention will be made. Defaults to 0.1
alpha	Reduction in PRE risk with intervention. Defaults to empirical value 0.37

**Value**

Estimated cost

**Examples**

```
# Simulate
set.seed(32142)

N=1000; p=15
X=matrix(rnorm(N*p),N,p); PRE=rbinom(N,1,prob=logit(X%% rnorm(p)))
aspre_k2(1000,X,PRE)
```

---

aspre_parametric	<i>Parametric-based OHS estimation for ASPRE</i>
------------------	--

---

**Description**

This object contains data relating to parametric-based OHS estimation for the ASPRE model. For generation, see hidden code in vignette, or in pipeline at [https://github.com/jamesliley/OptHoldoutSize\\_pipelines](https://github.com/jamesliley/OptHoldoutSize_pipelines)

**Usage**

```
aspre_parametric
```

**Format**

An object of class `list` of length 4.

---

ci_cover_a_yn	<i>Data for example on asymptotic confidence interval for OHS.</i>
---------------	--

---

**Description**

Data for example for asymptotic confidence interval for OHS. For generation, see example.

**Usage**

```
ci_cover_a_yn
```

**Format**

An object of class `matrix` (inherits from `array`) with 11 rows and 5000 columns.

---

ci_cover_cost_a_yn	<i>Data for example on asymptotic confidence interval for min cost.</i>
--------------------	---

---

**Description**

Data for example for asymptotic confidence interval for min cost. For generation, see example.

**Usage**

```
ci_cover_cost_a_yn
```

**Format**

An object of class `matrix` (inherits from `array`) with 11 rows and 5000 columns.

---

ci_cover_cost_e_yn	<i>Data for example on empirical confidence interval for min cost.</i>
--------------------	--

---

**Description**

Data for example for empirical confidence interval for min cost. For generation, see example.

**Usage**

```
ci_cover_cost_e_yn
```

**Format**

An object of class `matrix` (inherits from `array`) with 11 rows and 5000 columns.

---

ci_cover_e_yn	<i>Data for example on empirical confidence interval for OHS.</i>
---------------	---

---

**Description**

Data for example for empirical confidence interval for OHS. For generation, see example.

**Usage**

```
ci_cover_e_yn
```

**Format**

An object of class `matrix` (inherits from `array`) with 11 rows and 5000 columns.

---

ci_mincost	<i>Confidence interval for minimum total cost, when estimated using parametric method</i>
------------	---

---

## Description

Compute confidence interval for cost at optimal holdout size given either a standard error covariance matrix or a set of  $m$  estimates of parameters.

This can be done either asymptotically, using a method analogous to the Fisher information matrix, or empirically (using bootstrap resampling)

If sigma (covariance matrix) is specified and method='bootstrap', a confidence interval is generated assuming a Gaussian distribution of  $(N, k1, \theta)$ . To estimate a confidence interval assuming a non-Gaussian distribution, simulate values under the requisite distribution and use then as parameters  $N, k1, \theta$ , with sigma set to NULL.

## Usage

```
ci_mincost(
  N,
  k1,
  theta,
  alpha = 0.05,
  k2form = powerlaw,
  grad_mincost = NULL,
  sigma = NULL,
  n_boot = 1000,
  seed = NULL,
  mode = "empirical",
  ...
)
```

## Arguments

N	Vector of estimates of total number of samples on which the predictive score will be used/fitted, or single estimate
k1	Vector of estimates of cost value in the absence of a predictive score, or single number
theta	Matrix of estimates of parameters for function $k2form(n)$ governing expected cost to an individual sample given a predictive score fitted to $n$ samples. Can be a matrix of dimension $n \times n\_par$ , where $n\_par$ is the number of parameters of $k2$ .
alpha	Construct $1-\alpha$ confidence interval. Defaults to 0.05
k2form	Function governing expected cost to an individual sample given a predictive score fitted to $n$ samples. Must take two arguments: $n$ (number of samples) and $\theta$ (parameters). Defaults to a power-law form $k2(n, c(a, b, c)) = a n^{(-b)} + c$ .
grad_mincost	Function giving partial derivatives of minimum cost, taking three arguments: $N$ , $k1$ , and $\theta$ . Only used for asymptotic confidence intervals. F NULL, estimated empirically

sigma	Standard error covariance matrix for (N,k1,theta), in that order. If NULL, will derive as sample covariance matrix of parameters. Must be of the correct size and positive definite.
n_boot	Number of bootstrap resamples for empirical estimate.
seed	Random seed for bootstrap resamples. Defaults to NULL.
mode	One of 'asymptotic' or 'empirical'. Defaults to 'empirical'
...	Passed to function optimize

### Value

A vector of length two containing lower and upper limits of confidence interval.

### Examples

```
## Set seed
set.seed(574635)

## We will assume that our observations of N, k1, and theta=(a,b,c) are
## distributed with mean mu_par and variance sigma_par
mu_par=c(N=10000,k1=0.35,A=3,B=1.5,C=0.1)
sigma_par=cbind(
  c(100^2,      1,      0,      0,      0),
  c( 1, 0.07^2,      0,      0,      0),
  c( 0,      0, 0.5^2,      0.05, -0.001),
  c( 0,      0, 0.05, 0.4^2, -0.002),
  c( 0,      0, -0.001, -0.002, 0.02^2)
)

# Firstly, we make 500 observations
par_obs=rmnorm(500,mean=mu_par,varcov=sigma_par)

# Minimum cost and asymptotic and empirical confidence intervals
mincost=optimal_holdout_size(N=mean(par_obs[,1]),k1=mean(par_obs[,2]),
  theta=colMeans(par_obs[,3:5]))$cost
ci_a=ci_mincost(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],alpha=0.05,
  seed=12345,mode="asymptotic")
ci_e=ci_mincost(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],alpha=0.05,
  seed=12345,mode="empirical")

# Assess cover at various m
m_values=c(20,30,50,100,150,200,300,500,750,1000,1500)
ntrial=5000
alpha_trial=0.1 # use 90% confidence intervals
mincost_true=optimal_holdout_size(N=mu_par[1],k1=mu_par[2],
  theta=mu_par[3:5])$cost

## The matrices indicating cover take are included in this package but take
## around 30 minutes to generate. They are generated using the code below
## (including random seeds).
data(ci_cover_cost_a_yn)
data(ci_cover_cost_e_yn)

if (!exists("ci_cover_cost_a_yn")) {
  ci_cover_cost_a_yn=matrix(NA,length(m_values),ntrial) # Entry [i,j] is 1
```



```

# if ith asymptotic CI for jth value of m covers true mincost
ci_cover_cost_e_yn=matrix(NA,length(m_values),ntrial) # Entry [i,j] is 1
# if ith empirical CI for jth value of m covers true mincost

for (i in 1:length(m_values)) {
  m=m_values[i]
  for (j in 1:ntrial) {
    # Set seed
    set.seed(j*ntrial + i + 12345)

    # Make m observations
    par_obs=rnorm(m,mean=mu_par,varcov=sigma_par)
    ci_a=ci_mincost(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],
      alpha=alpha_trial,mode="asymptotic")
    ci_e=ci_mincost(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],
      alpha=alpha_trial,mode="empirical",n_boot=500)

    if (mincost_true>ci_a[1] & mincost_true<ci_a[2])
      ci_cover_cost_a_yn[i,j]=1 else ci_cover_cost_a_yn[i,j]=0
    if (mincost_true>ci_e[1] & mincost_true<ci_e[2])
      ci_cover_cost_e_yn[i,j]=1 else ci_cover_cost_e_yn[i,j]=0
  }
  print(paste0("Completed for m = ",m))
}

save(ci_cover_cost_a_yn,file="data/ci_cover_cost_a_yn.RData")
save(ci_cover_cost_e_yn,file="data/ci_cover_cost_e_yn.RData")

}

# Cover at each m value and standard error
cover_a=rowMeans(ci_cover_cost_a_yn)
cover_e=rowMeans(ci_cover_cost_e_yn)
zse_a=2*sqrt(cover_a*(1-cover_a)/ntrial)
zse_e=2*sqrt(cover_e*(1-cover_e)/ntrial)

# Draw plot. Convergence to 1-alpha cover is evident. Cover is not far from
# alpha even at small m.

plot(0,type="n",xlim=range(m_values),ylim=c(0.7,1),xlab=expression("m"),
  ylab="Cover")

# Asymptotic cover and 2*SE pointwise envelope
polygon(c(m_values,rev(m_values)),c(cover_a+zse_a,rev(cover_a-zse_a)),
  col=rgb(0,0,0,alpha=0.3),border=NA)
lines(m_values,cover_a,col="black")

# Empirical cover and 2*SE pointwise envelope
polygon(c(m_values,rev(m_values)),c(cover_e+zse_e,rev(cover_e-zse_e)),
  col=rgb(0,0,1,alpha=0.3),border=NA)
lines(m_values,cover_e,col="blue")

abline(h=1-alpha_trial,col="red")
legend("bottomright",c("Asym.", "Emp."),expression(paste("1-",alpha))),lty=1,
  col=c("black", "blue", "red"))

```

---

ci_ohs	<i>Confidence interval for optimal holdout size, when estimated using parametric method</i>
--------	---

---

### Description

Compute confidence interval for optimal holdout size given either a standard error covariance matrix or a set of  $m$  estimates of parameters.

This can be done either asymptotically, using a method analogous to the Fisher information matrix, or empirically (using bootstrap resampling)

If sigma (covariance matrix) is specified and method='bootstrap', a confidence interval is generated assuming a Gaussian distribution of  $(N, k1, \theta)$ . To estimate a confidence interval assuming a non-Gaussian distribution, simulate values under the requisite distribution and use then as parameters  $N, k1, \theta$ , with sigma set to NULL.

### Usage

```
ci_ohs(
  N,
  k1,
  theta,
  alpha = 0.05,
  k2form = powerlaw,
  grad_nstar = NULL,
  sigma = NULL,
  n_boot = 1000,
  seed = NULL,
  mode = "empirical",
  ...
)
```

### Arguments

N	Vector of estimates of total number of samples on which the predictive score will be used/fitted, or single estimate
k1	Vector of estimates of cost value in the absence of a predictive score, or single number
theta	Matrix of estimates of parameters for function $k2form(n)$ governing expected cost to an individual sample given a predictive score fitted to $n$ samples. Can be a matrix of dimension $n \times n\_par$ , where $n\_par$ is the number of parameters of $k2$ .
alpha	Construct $1-\alpha$ confidence interval. Defaults to 0.05
k2form	Function governing expected cost to an individual sample given a predictive score fitted to $n$ samples. Must take two arguments: $n$ (number of samples) and $\theta$ (parameters). Defaults to a power-law form $k2(n, c(a, b, c)) = a n^{(-b)} + c$ .
grad_nstar	Function giving partial derivatives of optimal holdout set, taking three arguments: $N$ , $k1$ , and $\theta$ . Only used for asymptotic confidence intervals. F NULL, estimated empirically

sigma	Standard error covariance matrix for (N,k1,theta), in that order. If NULL, will derive as sample covariance matrix of parameters. Must be of the correct size and positive definite.
n_boot	Number of bootstrap resamples for empirical estimate.
seed	Random seed for bootstrap resamples. Defaults to NULL.
mode	One of 'asymptotic' or 'empirical'. Defaults to 'empirical'
...	Passed to function optimize

### Value

A vector of length two containing lower and upper limits of confidence interval.

### Examples

```
## Set seed
set.seed(493825)

## We will assume that our observations of N, k1, and theta=(a,b,c) are
## distributed with mean mu_par and variance sigma_par
mu_par=c(N=10000,k1=0.35,A=3,B=1.5,C=0.1)
sigma_par=cbind(
  c(100^2,      1,      0,      0,      0),
  c( 1, 0.07^2,      0,      0,      0),
  c( 0,      0, 0.5^2,      0.05, -0.001),
  c( 0,      0, 0.05, 0.4^2, -0.002),
  c( 0,      0, -0.001, -0.002, 0.02^2)
)

# Firstly, we make 500 observations
par_obs=rmnorm(500,mean=mu_par,varcov=sigma_par)

# Optimal holdout size and asymptotic and empirical confidence intervals
ohs=optimal_holdout_size(N=mean(par_obs[,1]),k1=mean(par_obs[,2]),
  theta=colMeans(par_obs[,3:5]))$size
ci_a=ci_ohs(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],alpha=0.05,
  seed=12345,mode="asymptotic")
ci_e=ci_ohs(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],alpha=0.05,
  seed=12345,mode="empirical")

# Assess cover at various m
m_values=c(20,30,50,100,150,200,300,500,750,1000,1500)
ntrial=5000
alpha_trial=0.1 # use 90% confidence intervals
nstar_true=optimal_holdout_size(N=mu_par[1],k1=mu_par[2],
  theta=mu_par[3:5])$size

## The matrices indicating cover take are included in this package but take
## around 30 minutes to generate. They are generated using the code below
## (including random seeds).
data(ci_cover_a_yn)
data(ci_cover_e_yn)

if (!exists("ci_cover_a_yn")) {
  ci_cover_a_yn=matrix(NA,length(m_values),ntrial) # Entry [i,j] is 1 if ith
```

```

## asymptotic CI for jth value of m covers true nstar
ci_cover_e_yn=matrix(NA,length(m_values),ntrial) # Entry [i,j] is 1 if ith
## empirical CI for jth value of m covers true nstar

for (i in 1:length(m_values)) {
  m=m_values[i]
  for (j in 1:ntrial) {
    # Set seed
    set.seed(j*ntrial + i + 12345)

    # Make m observations
    par_obs=rnorm(m,mean=mu_par,varcov=sigma_par)
    ci_a=ci_ohs(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],
      alpha=alpha_trial,mode="asymptotic")
    ci_e=ci_ohs(N=par_obs[,1],k1=par_obs[,2],theta=par_obs[,3:5],
      alpha=alpha_trial,mode="empirical",n_boot=500)

    if (nstar_true>ci_a[1] & nstar_true<ci_a[2]) ci_cover_a_yn[i,j]=1 else
      ci_cover_a_yn[i,j]=0
    if (nstar_true>ci_e[1] & nstar_true<ci_e[2]) ci_cover_e_yn[i,j]=1 else
      ci_cover_e_yn[i,j]=0
  }
  print(paste0("Completed for m = ",m))
}

save(ci_cover_a_yn,file="data/ci_cover_a_yn.RData")
save(ci_cover_e_yn,file="data/ci_cover_e_yn.RData")

}

# Cover at each m value and standard error
cover_a=rowMeans(ci_cover_a_yn)
cover_e=rowMeans(ci_cover_e_yn)
zse_a=2*sqrt(cover_a*(1-cover_a)/ntrial)
zse_e=2*sqrt(cover_e*(1-cover_e)/ntrial)

# Draw plot. Convergence to 1-alpha cover is evident. Cover is not far from
# alpha even at small m.

plot(0,type="n",xlim=range(m_values),ylim=c(0.7,1),xlab=expression("m"),
  ylab="Cover")

# Asymptotic cover and 2*SE pointwise envelope
polygon(c(m_values,rev(m_values)),c(cover_a+zse_a,rev(cover_a-zse_a)),
  col=rgb(0,0,0,alpha=0.3),border=NA)
lines(m_values,cover_a,col="black")

# Empirical cover and 2*SE pointwise envelope
polygon(c(m_values,rev(m_values)),c(cover_e+zse_e,rev(cover_e-zse_e)),
  col=rgb(0,0,1,alpha=0.3),border=NA)
lines(m_values,cover_e,col="blue")

abline(h=1-alpha_trial,col="red")
legend("bottomright",c("Asym.", "Emp."),expression(paste("1-",alpha))),lty=1,
  col=c("black", "blue", "red"))

```

---

cov_fn	<i>Covariance function for Gaussian process</i>
--------	---

---

### Description

Radial kernel covariance function for Gaussian process.

Used for a Gaussian process  $GP(m, k(\cdot, \cdot))$ , an instance  $X$  of which has covariance  $k(n, n')$  between  $X(n)$  and  $X(n')$ .

Covariance function is parametrised by `var_u` (general variance) and `k_width` (kernel width)

### Usage

```
cov_fn(n, ndash, var_u, k_width)
```

### Arguments

<code>n</code>	Argument 1: kernel is a function of <code>ndash-n</code>
<code>ndash</code>	Argument 2: kernel is a function of <code>ndash-n</code>
<code>var_u</code>	Global variance
<code>k_width</code>	Kernel width

### Value

Covariance value

### Examples

```
# We will sample from Gaussian processes
# GP(0, k(.,.)=cov_fn(.,.;var_u,theta))
# at these values of n
nvals=seq(1,300,length=100)

# We will consider two theta values
kw1=10; kw2=30

# We will consider two var_u values
var1=1; var2=10

# Covariance matrices
cov11=outer(nvals,nvals,function(n,ndash) cov_fn(n,ndash,var_u=var1,
  k_width=kw1))
cov12=outer(nvals,nvals,function(n,ndash) cov_fn(n,ndash,var_u=var1,
  k_width=kw2))
cov21=outer(nvals,nvals,function(n,ndash) cov_fn(n,ndash,var_u=var2,
  k_width=kw1))
cov22=outer(nvals,nvals,function(n,ndash) cov_fn(n,ndash,var_u=var2,
  k_width=kw2))

# Dampen slightly to ensure positive definiteness
damp=1e-5
cov11=(1-damp)*(1-diag(length(nvals)))*cov11 + diag(length(nvals))*cov11
cov12=(1-damp)*(1-diag(length(nvals)))*cov12 + diag(length(nvals))*cov12
```

```

cov21=(1-damp)*(1-diag(length(nvals)))*cov21 + diag(length(nvals))*cov21
cov22=(1-damp)*(1-diag(length(nvals)))*cov22 + diag(length(nvals))*cov22

# Sample
set.seed(35243)
y11=rmnorm(1,mean=0,varcov=cov11)
y12=rmnorm(1,mean=0,varcov=cov12)
y21=rmnorm(1,mean=0,varcov=cov21)
y22=rmnorm(1,mean=0,varcov=cov22)

# Plot
rr=max(abs(c(y11,y12,y21,y22)))
plot(0,xlim=range(nvals),ylim=c(-rr,rr+10),xlab="n",
     ylab=expression("GP(0,cov_fn(.,.;var_u,theta)"))
     lines(nvals,y11,lty=1,col="black")
     lines(nvals,y12,lty=2,col="black")
     lines(nvals,y21,lty=1,col="red")
     lines(nvals,y22,lty=2,col="red")
     legend("topright",c("k_width=10, var_u=1", "k_width=30, var_u=1",
        "k_width=10, var_u=10", "k_width=30, var_u=10"),
        lty=c(1,2,1,2),col=c("black", "black", "red", "red"))

```

---

data\_example\_simulation

*Data for vignette showing general example*

---

### Description

Data for general vignette. For generation, see hidden code in vignette, or in pipeline at <https://github.com/jamesliley/OptH>

### Usage

```
data_example_simulation
```

### Format

An object of class list of length 4.

---

data\_nextpoint\_em

*Data for 'next point' demonstration vignette on algorithm comparison using emulation algorithm*

---

### Description

Data containing 'next point selected' information for emulation algorithm in vignette comparing emulation and parametric algorithms. For generation, see hidden code in vignette, or in pipeline at [https://github.com/jamesliley/OptHoldoutSize\\_pipelines](https://github.com/jamesliley/OptHoldoutSize_pipelines)

### Usage

```
data_nextpoint_em
```

**Format**

An object of class list of length 6.

---

data_nextpoint_par	<i>Data for 'next point' demonstration vignette on algorithm comparison using parametric algorithm</i>
--------------------	--

---

**Description**

Data containing 'next point selected' information for parametric algorithm in vignette comparing emulation and parametric algorithms. For generation, see hidden code in vignette, or in pipeline at [https://github.com/jamesliley/OptHoldoutSize\\_pipelines](https://github.com/jamesliley/OptHoldoutSize_pipelines)

**Usage**

```
data_nextpoint_par
```

**Format**

An object of class list of length 6.

---

error_ohs_emulation	<i>Measure of error for emulation-based OHS emulation</i>
---------------------	---

---

**Description**

Measure of error for semiparametric (emulation) based estimation of optimal holdout set sizes.

Returns a set of values of  $n$  for which a  $1-\alpha$  credible interval for cost at includes a lower value than the cost at the estimated optimal holdout size.

This is not a confidence interval, credible interval or credible set for the OHS, and is prone to misinterpretation.

**Usage**

```
error_ohs_emulation(
  nset,
  k2,
  var_k2,
  N,
  k1,
  alpha = 0.1,
  var_u = 1e+07,
  k_width = 5000,
  k2form = powerlaw,
  theta = powersolve(nset, k2, y_var = var_k2)$par,
  npoll = 1000
)
```

**Arguments**

nset	Training set sizes for which k2() has been evaluated
k2	Estimated k2() at training set sizes nset
var_k2	Variance of error in k2() estimate at each training set size.
N	Total number of samples on which the model will be fitted/used
k1	Mean cost per sample with no predictive score in place
alpha	Use 1-alpha credible interval. Defaults to 0.1.
var_u	Marginal variance for Gaussian process kernel. Defaults to 1e7
k_width	Kernel width for Gaussian process kernel. Defaults to 5000
k2form	Functional form governing expected cost per sample given sample size. Should take two parameters: n (sample size) and theta (parameters). Defaults to function powerlaw.
theta	Current estimates of parameter values for k2form. Defaults to the MLE power-law solution corresponding to n,k2, and var_k2.
npoll	Check npoll equally spaced values between 1 and N for minimum. If NULL, check all values (this can be slow). Defaults to 1000

**Value**

Vector of values n for which 1-alpha credible interval for cost  $l(n)$  at n contains mean posterior cost at estimated optimal holdout size.

**Examples**

```
# Set seed
set.seed(57365)

# Parameters
N=100000;
k1=0.3
A=8000; B=1.5; C=0.15; theta=c(A,B,C)

# True mean function
k2_true=function(n) powerlaw(n,theta)

# True OHS
nx=1:N
ohs_true=nx[which.min(k1*nx + k2_true(nx)*(N-nx))]

# Values of n for which cost has been estimated
np=50 # this many points
nset=round(runif(np,1,N))
var_k2=runif(np,0.001,0.0015)
k2=rnorm(np,mean=k2_true(nset),sd=sqrt(var_k2))

# Compute OHS
res1=optimal_holdout_size_emulation(nset,k2,var_k2,N,k1)

# Error estimates
ex=error_ohs_emulation(nset,k2,var_k2,N,k1)

# Plot
```



```

plot(res1)

# Add error
abline(v=ohs_true)
abline(v=ex,col=rgb(1,0,0,alpha=0.2))

# Show justification for error
n=seq(1,N,length=1000)
mu=mu_fn(n,nset,k2,var_k2,N,k1); psi=pmax(0,psi_fn(n, nset, var_k2, N)); Z=-qnorm(0.1/2)
lines(n,mu - Z*sqrt(psi),lty=2,lwd=2)
legend("topright",
      c("Err. region",expression(paste(mu(n)- "z"[alpha/2]*sqrt(psi(n))))),
      pch=c(16,NA),lty=c(NA,2),lwd=c(NA,2),col=c("pink","black"),bty="n")

```

exp\_imp\_fn

*Expected improvement***Description**

Expected improvement

Essentially chooses the next point to add to  $n$ , called  $n^*$ , in order to minimise the expectation of  $\text{cost}(n^*)$ .

**Usage**

```

exp_imp_fn(
  n,
  nset,
  k2,
  var_k2,
  N,
  k1,
  var_u = 1e+07,
  k_width = 5000,
  k2form = powerlaw,
  theta = powersolve(nset, k2, y_var = var_k2)$par
)

```

**Arguments**

<code>n</code>	Set of training set sizes to evaluate
<code>nset</code>	Training set sizes for which a cost has been evaluated
<code>k2</code>	Estimates of $k2()$ at training set sizes <code>nset</code>
<code>var_k2</code>	Variance of error in $k2()$ estimates at each training set size.
<code>N</code>	Total number of samples on which the model will be fitted/used
<code>k1</code>	Mean cost per sample with no predictive score in place
<code>var_u</code>	Marginal variance for Gaussian process kernel. Defaults to $1e7$
<code>k_width</code>	Kernel width for Gaussian process kernel. Defaults to 5000

k2form	Functional form governing expected cost per sample given sample size. Should take two parameters: n (sample size) and theta (parameters). Defaults to function powerlaw.
theta	Current estimates of parameter values for k2form. Defaults to the MLE power-law solution corresponding to n,k2, and var_k2.

### Value

Value of expected improvement at values n

### Examples

```
# Set seed.
set.seed(24015)

# Kernel width and Gaussian process variance
kw0=5000
vu0=1e7

# Include legend on plots or not; inclusion can obscure plot elements on small figures
inc_legend=FALSE

# Suppose we have population size and cost-per-sample without a risk score as follows
N=100000
k1=0.4

# Suppose that true values of a,b,c are given by
theta_true=c(10000,1.2,0.2)
theta_lower=c(1,0.5,0.1) # lower bounds for estimating theta
theta_upper=c(20000,2,0.5) # upper bounds for estimating theta

# We start with five random holdout set sizes (nset0),
# with corresponding cost-per-individual estimates k2_0 derived
# with various errors var_k2_0
nstart=4
vwmin=0.001; vwmax=0.005
nset0=round(runif(nstart,1000,N/2))
var_k2_0=runif(nstart,vwmin,vwmax)
k2_0=rnorm(nstart,mean=powerlaw(nset0,theta_true),sd=sqrt(var_k2_0))

# We estimate theta from these three points
theta0=powersolve(nset0,k2_0,y_var=var_k2_0,lower=theta_lower,upper=theta_upper,
  init=theta_true)$par

# We will estimate the posterior at these values of n
n=seq(1000,N,length=1000)

# Mean and variance
p_mu=mu_fn(n,nset=nset0,k2=k2_0,var_k2 = var_k2_0, N=N,k1=k1,theta=theta0,k_width=kw0,
  var_u=vu0)
p_var=psi_fn(n,nset=nset0,N=N,var_k2 = var_k2_0,k_width=kw0,var_u=vu0)

# Plot
yrange=c(-30000,100000)
```

```

plot(0,xlim=range(n),ylim=yrange,type="n",
     xlab="Training/holdout set size",
     ylab="Total cost (= num. cases)")
lines(n,p_mu,col="blue")
lines(n,p_mu - 3*sqrt(p_var),col="red")
lines(n,p_mu + 3*sqrt(p_var),col="red")
points(nset0,k1*nset0 + k2_0*(N-nset0),pch=16,col="purple")
lines(n,k1*n + powerlaw(n,theta0)*(N-n),lty=2)
lines(n,k1*n + powerlaw(n,theta_true)*(N-n),lty=3,lwd=3)
if (inc_legend) {
  legend("topright",
        c(expression(mu(n)),
          expression(mu(n) %+-% 3*sqrt(psi(n))),
          "prior(n)",
          "True",
          "d"),
        lty=c(1,1,2,3,NA),lwd=c(1,1,1,3,NA),pch=c(NA,NA,NA,NA,16),pt.cex=c(NA,NA,NA,NA,1),
        col=c("blue","red","black","purple"),bg="white")
}

## Add line corresponding to recommended new point
exp_imp_em <- exp_imp_fn(n,nset=nset0,k2=k2_0,var_k2 = var_k2_0, N=N,k1=k1,
  theta=theta0,k_width=kw0,var_u=vu0)
abline(v=n[which.max(exp_imp_em)])

```

gen\_base\_coefs

*Coefficients for imperfect risk score***Description**

Generate coefficients corresponding to an imperfect risk score, interpretable as 'baseline' behaviour in the absence of a risk score

**Usage**

```
gen_base_coefs(coefs, noise = TRUE, num_vars = 2, max_base_powers = 1)
```

**Arguments**

coefs	Original coefficients
noise	Set to TRUE to add Gaussian noise to coefficients
num_vars	Number of variables at hand for baseline calculation
max_base_powers	If >1, return a matrix of coefficients, with successively more noise

**Value**

Vector of coefficients

**Examples**

```
# See examples for model_predict
```

---

gen_preds	<i>Generate matrix of random observations</i>
-----------	---

---

**Description**

Generate matrix of random observations. Observations are unit Gaussian-distributed.

**Usage**

```
gen_preds(nobs, npreds, ninters = 0)
```

**Arguments**

nobs	Number of observations (samples)
npreds	Number of predictors
niinters	Number of interaction terms, default 0. Up to npreds*(npreds-1)/2

**Value**

Data frame of observations

**Examples**

```
# See examples for model_predict
```

---

gen_resp	<i>Generate response</i>
----------	--------------------------

---

**Description**

Generate random outcome (response) according to a ground-truth logistic model

**Usage**

```
gen_resp(X, coefs = NA, coefs_sd = 1, retprobs = FALSE)
```

**Arguments**

X	Matrix of observations
coefs	Vector of coefficients for logistic model. If NA, random coefficients are generated. Defaults to NA
coefs_sd	If random coefficients are generated, use this SD (mean 0)
retprobs	If TRUE, return class probability; otherwise, return classes. Defaults to FALSE

**Value**

Vector of length as first dimension of dim(X) with outcome classes (if retprobs==FALSE) or outcome probabilities (if retprobs==TRUE)

**Examples**

```
# See examples for model_predict
```

---

grad\_mincost\_powerlaw *Gradient of minimum cost (power law)*


---

## Description

Compute gradient of minimum cost assuming a power-law form of  $k_2$

Assumes cost function is  $l(n; k_1, N, \theta) = k_1 n + k_2(n; \theta) (N-n)$  with  $k_2(n; \theta) = k_2(n; a, b, c) = a n^{-(b+c)}$

## Usage

```
grad_mincost_powerlaw(N, k1, theta)
```

## Arguments

N	Total number of samples on which the predictive score will be used/fitted. Can be a vector.
k1	Cost value in the absence of a predictive score. Can be a vector.
theta	Parameters for function $k_2(n)$ governing expected cost to an individual sample given a predictive score fitted to $n$ samples. Can be a matrix of dimension $n \times n_{\text{par}}$ , where $n_{\text{par}}$ is the number of parameters of $k_2$ .

## Value

List/data frame of dimension (number of evaluations)  $\times$  5 containing partial derivatives of  $n_{\text{star}}$  (optimal holdout size) with respect to  $N$ ,  $k_1$ ,  $a$ ,  $b$ ,  $c$  respectively.

## Examples

```
# Evaluate minimum for a range of values of k1, and compute derivative
N=10000;
k1=seq(0.1,0.5,length=20)
A=3; B=1.5; C=0.15; theta=c(A,B,C)

mincost=optimal_holdout_size(N,k1,theta)
grad_mincost=grad_mincost_powerlaw(N,k1,theta)

plot(0,type="n",ylim=c(0,1560),xlim=range(k1),xlab=expression("k"[1]),
     ylab="Optimal holdout set size")
lines(mincost$k1,mincost$cost,col="black")
lines(mincost$k1,grad_mincost[,2],col="red")
legend(0.2,800,c(expression(paste("l(n"["*"],")")),
                  expression(paste(partialdiff[k1],"l(n"["*"],")"))),
      col=c("black","red"),lty=1,bty="n")
```

---

grad_nstar_powerlaw	<i>Gradient of optimal holdout size (power law)</i>
---------------------	---

---

## Description

Compute gradient of optimal holdout size assuming a power-law form of  $k_2$

Assumes cost function is  $l(n; k_1, N, \theta) = k_1 n + k_2(n; \theta) (N-n)$  with  $k_2(n; \theta) = k_2(n; a, b, c) = a n^b (-b$

## Usage

```
grad_nstar_powerlaw(N, k1, theta)
```

## Arguments

N	Total number of samples on which the predictive score will be used/fitted. Can be a vector.
k1	Cost value in the absence of a predictive score. Can be a vector.
theta	Parameters for function $k_2(n)$ governing expected cost to an individual sample given a predictive score fitted to $n$ samples. Can be a matrix of dimension $n \times n_{\text{par}}$ , where $n_{\text{par}}$ is the number of parameters of $k_2$ .

## Value

List/data frame of dimension (number of evaluations)  $\times$  5 containing partial derivatives of  $nstar$  (optimal holdout size) with respect to  $N$ ,  $k_1$ ,  $a$ ,  $b$ ,  $c$  respectively.

## Examples

```
# Evaluate optimal holdout set size for a range of values of k1, and compute
# derivative
N=10000;
k1=seq(0.1,0.5,length=20)
A=3; B=1.5; C=0.15; theta=c(A,B,C)

nstar=optimal_holdout_size(N,k1,theta)
grad_nstar=grad_nstar_powerlaw(N,k1,theta)

plot(0,type="n",ylim=c(-2000,500),xlim=range(k1),xlab=expression("k"[1]),
     ylab="Optimal holdout set size")
lines(nstar$k1,nstar$size,col="black")
lines(nstar$k1,grad_nstar[,2],col="red")
legend("bottomright",c(expression("n"["*"]),
  expression(paste(partialdiff[k1],"n"["*"]))),
     col=c("black","red"),lty=1)
```

---

`logistic`*Logistic*

---

**Description**

Logistic function:  $-\log((1/x)-1)$

**Usage**

```
logistic(x)
```

**Arguments**

`x`                      argument

**Value**

value of `logit(x)`; `na` if `x` is outside (0,1)

**Examples**

```
# Plot
x=seq(0,1,length=100)
plot(x,logistic(x),type="l")

# Logit and logistic are inverses
x=seq(-5,5,length=1000)
plot(x,logistic(logit(x)),type="l")
```

---

`logit`*Logit*

---

**Description**

Logit function:  $1/(1+\exp(-x))$

**Usage**

```
logit(x)
```

**Arguments**

`x`                      argument

**Value**

value of `logit(x)`

**Examples**

```
# Plot
x=seq(-5,5,length=1000)
plot(x,logit(x),type="l")
```

---

model_predict	<i>Make predictions</i>
---------------	-------------------------

---

## Description

Make predictions according to a given model

## Usage

```
model_predict(
  data_test,
  trained_model,
  return_type,
  threshold = NULL,
  model_family = NULL,
  ...
)
```

## Arguments

data_test	Data for which predictions are to be computed
trained_model	Model for which predictions are to be made
return_type	??
threshold	??
model_family	??
...	Passed to function <code>predict.glm()</code> or <code>predict.ranger()</code>

## Value

Vector of predictions

## Examples

```
## Set seed for reproducibility
seed=1234
set.seed(seed)

# Initialisation of patient data
n_iter <- 500           # Number of point estimates to be calculated
nobs <- 5000           # Number of observations, i.e patients
npreds <- 7            # Number of predictors

# Model family
family="log_reg"

# Baseline behaviour is an oracle Bayes-optimal predictor on only one variable
max_base_powers <- 1
base_vars=1

# Check the following holdout size fractions
frac_ho = 0.1
```



```

# Set ground truth coefficients, and the accuracy at baseline
coefs_general <- rnorm(npreds, sd=1/sqrt(npreds))
coefs_base <- gen_base_coefs(coefs_general, max_base_powers = max_base_powers)

# Generate dataset
X <- gen_preds(nobs, npreds)

# Generate labels
newdata <- gen_resp(X, coefs = coefs_general)
Y <- newdata$classes

# Combined dataset
pat_data <- cbind(X, Y)
pat_data$Y = factor(pat_data$Y)

# For each holdout size, split data into intervention and holdout set
mask <- split_data(pat_data, frac_ho)
data_interv <- pat_data[!mask,]
data_hold <- pat_data[mask,]

# Train model
trained_model <- model_train(data_hold, model_family = family)
thresh <- 0.5

# Make predictions
class_pred <- model_predict(data_interv, trained_model,
                           return_type = "class",
                           threshold = 0.5, model_family = family)

# Simulate baseline predictions
base_pred <- oracle_pred(data_hold, coefs_base[base_vars, ], num_vars = base_vars)

# Contingency table for model-based predictor (on intervention set)
print(table(class_pred, data_interv$Y))

# Contingency table for model-based predictor (on holdout set)
print(table(base_pred, data_hold$Y))

```

---

model\_train

*Train model (wrapper)*


---

## Description

Train model using either a GLM or a random forest

## Usage

```
model_train(train_data, model_family = "log_reg", ...)
```

**Arguments**

train_data	Data to use for training; assumed to have one binary column called Y
model_family	Either 'log_reg' for logistic regression or 'rand_forest' for random forest
...	Passed to function glm() or ranger()

**Value**

Fitted model of type GLM or Ranger

**Examples**

```
# See examples for model_predict
```

---

mu_fn	<i>Updating function for mean.</i>
-------	------------------------------------

---

**Description**

Posterior mean for emulator given points n.

**Usage**

```
mu_fn(
  n,
  nset,
  k2,
  var_k2,
  N,
  k1,
  var_u = 1e+07,
  k_width = 5000,
  k2form = powerlaw,
  theta = powersolve(nset, k2, y_var = var_k2)$par
)
```

**Arguments**

n	Set of training set sizes to evaluate
nset	Training set sizes for which k2() has been evaluated
k2	Estimated k2() values at training set sizes nset
var_k2	Variance of error in k2() estimate at each training set size.
N	Total number of samples on which the model will be fitted/used
k1	Mean cost per sample with no predictive score in place
var_u	Marginal variance for Gaussian process kernel. Defaults to 1e7
k_width	Kernel width for Gaussian process kernel. Defaults to 5000
k2form	Functional form governing expected cost per sample given sample size. Should take two parameters: n (sample size) and theta (parameters). Defaults to function powerlaw.
theta	Current estimates of parameter values for k2form. Defaults to the MLE power-law solution corresponding to n,k2, and var_k2.

**Value**

Vector Mu of same length of n where  $\text{Mu}_i = \text{mean}(\text{posterior}(\text{cost}(n_i)))$

**Examples**

```
# Suppose we have population size and cost-per-sample without a risk score as follows
N=100000
k1=0.4

# Kernel width and variance for GP
k_width=5000
var_u=8000000

# Suppose we begin with k2() estimates at n-values
nset=c(10000,20000,30000)

# with cost-per-individual estimates
# (note that since empirical k2(n) is non-monotonic, it cannot be perfectly
# approximated with a power-law function)
k2=c(0.35,0.26,0.28)

# and associated error on those estimates
var_k2=c(0.02^2,0.01^2,0.03^2)

# We estimate theta from these three points
theta=powersolve(nset,k2,y_var=var_k2)$par

# We will estimate the posterior at these values of n
n=seq(1000,50000,length=1000)

# Mean and variance
p_mu=mu_fn(n,nset=nset,k2=k2,var_k2 = var_k2, N=N,k1=k1,theta=theta,
           k_width=k_width,var_u=var_u)
p_var=psi_fn(n,nset=nset,N=N,var_k2 = var_k2,k_width=k_width,var_u=var_u)

# Plot
plot(0,xlim=range(n),ylim=c(20000,60000),type="n",
     xlab="Training/holdout set size",
     ylab="Total cost (= num. cases)")
lines(n,p_mu,col="blue")
lines(n,p_mu - 3*sqrt(p_var),col="red")
lines(n,p_mu + 3*sqrt(p_var),col="red")
points(nset,k1*nset + k2*(N-nset),pch=16,col="purple")
lines(n,k1*n + powerlaw(n,theta)*(N-n),lty=2)
segments(nset,k1*nset + (k2 - 3*sqrt(var_k2))*(N-nset),
         nset,k1*nset + (k2 + 3*sqrt(var_k2))*(N-nset))
legend("topright",
      c(expression(mu(n)),
        expression(mu(n) %+-% 3*sqrt(psi(n))),
        "prior(n)",
        "d",
        "3SD(d|n)"),
      lty=c(1,1,2,NA,NA),lwd=c(1,1,1,NA,NA),pch=c(NA,NA,NA,16,124),
      pt.cex=c(NA,NA,NA,1,1),
      col=c("blue","red","black","purple","black"),bg="white")
```

next\_n

*Finds best value of n to sample next***Description**

Recommends a value of n at which to next evaluate individual cost in order to most accurately estimate optimal holdout size. Currently only for use with a power-law parametrisation of k2.

Approximately finds a set of n points which, given estimates of cost, minimise width of 95% confidence interval around OHS. Uses a greedy algorithm, so various parameters can be learned along the way.

Given existing training set size/k2 estimates nset and k2, with `var_k2[i]=variance(k2[i])`, finds, for each candidate point `n[i]`, the median width of the 90% confidence interval for OHS if

```
nset <- c(nset,n[i]) var_k2 <- c(var_k2,mean(var_k2)) k2 <- c(k2,rnorm(powerlaw(n[i],theta),variance
```

**Usage**

```
next_n(
  n,
  nset,
  k2,
  N,
  k1,
  nmed = 100,
  var_k2 = rep(1, length(nset)),
  mode = "asymptotic",
  ...
)
```

**Arguments**

n	Set of training set sizes to evaluate
nset	Training set sizes for which a loss has been evaluated
k2	Estimated <code>k2()</code> at training set sizes nset
N	Total number of samples on which the model will be fitted/used
k1	Mean loss per sample with no predictive score in place
nmed	number of times to re-evaluate d and confidence interval width.
var_k2	Variance of error in <code>k2()</code> estimate at each training set size.
mode	Mode for calculating OHS CI (passed to <code>ci_ohs</code> ): 'asymptotic' or 'empirical'
...	Passed to <code>powersolve</code> and <code>powersolve_se</code>

**Value**

Vector out of same length as n, where `out[i]` is the expected width of the 95% confidence interval for OHS should n be added to nset.

## Examples

```
# Set seed.
set.seed(24015)

# Kernel width and Gaussian process variance
kw0=5000
vu0=1e7

# Include legend on plots or not; inclusion can obscure plot elements on small figures
inc_legend=FALSE

# Suppose we have population size and cost-per-sample without a risk score as follows
N=100000
k1=0.4

# Suppose that true values of a,b,c are given by
theta_true=c(10000,1.2,0.2)
theta_lower=c(1,0.5,0.1) # lower bounds for estimating theta
theta_upper=c(20000,2,0.5) # upper bounds for estimating theta

# We start with five random holdout set sizes (nset0),
# with corresponding cost-per-individual estimates k2_0 derived
# with various errors var_k2_0
nstart=10
vwmin=0.001; vwmax=0.005
nset0=round(runif(nstart,1000,N/2))
var_k2_0=runif(nstart,vwmin,vwmax)
k2_0=rnorm(nstart,mean=powerlaw(nset0,theta_true),sd=sqrt(var_k2_0))

# We estimate theta from these three points
theta0=powersolve(nset0,k2_0,y_var=var_k2_0,lower=theta_lower,upper=theta_upper,init=theta_true)$par

# We will estimate the posterior at these values of n
n=seq(1000,N,length=1000)

# Mean and variance
p_mu=mu_fn(n,nset=nset0,k2=k2_0,var_k2 = var_k2_0, N=N,k1=k1,theta=theta0,k_width=kw0,var_u=vu0)
p_var=psi_fn(n,nset=nset0,N=N,var_k2 = var_k2_0,k_width=kw0,var_u=vu0)

# Plot
yrange=c(-30000,100000)
plot(0,xlim=range(n),ylim=yrange,type="n",
     xlab="Training/holdout set size",
     ylab="Total cost (= num. cases)")
lines(n,p_mu,col="blue")
lines(n,p_mu - 3*sqrt(p_var),col="red")
lines(n,p_mu + 3*sqrt(p_var),col="red")
points(nset0,k1*nset0 + k2_0*(N-nset0),pch=16,col="purple")
lines(n,k1*n + powerlaw(n,theta0)*(N-n),lty=2)
lines(n,k1*n + powerlaw(n,theta_true)*(N-n),lty=3,lwd=3)
if (inc_legend) {
  legend("topright",
        c(expression(mu(n)),
          expression(mu(n) %+-% 3*sqrt(psi(n)))),
```

```

      "prior(n)",
      "True",
      "d"),
    lty=c(1,1,2,3,NA),lwd=c(1,1,1,3,NA),pch=c(NA,NA,NA,NA,16),pt.cex=c(NA,NA,NA,NA,1),
    col=c("blue","red","black","purple"),bg="white")
  }

  ## Add line corresponding to recommended new point. This is slow.
  nn=seq(1000,N,length=20)
  exp_imp <- next_n(nn,nset=nset0,k2=k2_0,var_k2 = var_k2_0, N=N,k1=k1,nmed=10,
                    lower=theta_lower,upper=theta_upper)
  abline(v=nn[which.min(exp_imp)])

```

---

ohs\_array

*Data for vignette on algorithm comparison*


---

### Description

This object contains data relating to the vignette comparing emulation and parametric algorithms. For generation, see hidden code in vignette, or in pipeline at [https://github.com/jamesliley/OptHoldoutSize\\_pipelines](https://github.com/jamesliley/OptHoldoutSize_pipelines)

### Usage

```
ohs_array
```

### Format

An object of class array of dimension 200 x 200 x 2 x 2 x 2.

---

ohs\_resample

*Data for vignette on algorithm comparison*


---

### Description

This object contains data relating to the first plot in the vignette comparing emulation and parametric algorithms. For generation, see hidden code in vignette, or in pipeline at [https://github.com/jamesliley/OptHoldoutSize\\_pipelines](https://github.com/jamesliley/OptHoldoutSize_pipelines)

### Usage

```
ohs_resample
```

### Format

An object of class matrix (inherits from array) with 1000 rows and 4 columns.

---

optimal\_holdout\_size    *Estimate optimal holdout size under parametric assumptions*

---

## Description

Compute optimal holdout size for updating a predictive score given appropriate parameters of cost function

Evaluates empirical minimisation of cost function  $l(n; k1, N, \theta) = k1 \cdot n + k2form(n; \theta) \cdot (N-n)$ .

The function will return Inf if no minimum exists. It does not check if the minimum is unique, but this can be guaranteed using the assumptions for theorem 1 in the manuscript.

This calls the function optimize from package stats.

## Usage

```
optimal_holdout_size(
  N,
  k1,
  theta,
  k2form = powerlaw,
  round_result = FALSE,
  ...
)
```

## Arguments

N	Total number of samples on which the predictive score will be used/fitted. Can be a vector.
k1	Cost value in the absence of a predictive score. Can be a vector.
theta	Parameters for function k2form(n) governing expected cost to an individual sample given a predictive score fitted to n samples. Can be a matrix of dimension n x n_par, where n_par is the number of parameters of k2.
k2form	Function governing expected cost to an individual sample given a predictive score fitted to n samples. Must take two arguments: n (number of samples) and theta (parameters). Defaults to a power-law form $powerlaw(n, c(a, b, c)) = a \cdot n^{-b} + c$ .
round_result	Set to TRUE to solve over integral sizes
...	Passed to function optimize

## Value

List/data frame of dimension (number of evaluations) x (4 + n\_par) containing input data and results. Columns size and cost are optimal holdout size and cost at this size respectively. Parameters N, k1, theta.1, theta.2,...,theta.n\_par are input data.

## Examples

```
# Evaluate optimal holdout set size for a range of values of k1 and two values of
# N, some of which lead to infinite values
N1=10000; N2=12000
k1=seq(0.1,0.5,length=20)
```

```

A=3; B=1.5; C=0.15; theta=c(A,B,C)

res1=optimal_holdout_size(N1,k1,theta)
res2=optimal_holdout_size(N2,k1,theta)

oldpar=par(mfrow=c(1,2))
plot(0,type="n",ylim=c(0,500),xlim=range(res1$k1),xlab=expression("k"[1]),
      ylab="Optimal holdout set size")
  lines(res1$k1,res1$size,col="black")
  lines(res2$k1,res2$size,col="red")
  legend("topright",as.character(c(N1,N2)),title="N:",col=c("black","red"),lty=1)
plot(0,type="n",ylim=c(1500,1600),xlim=range(res1$k1),xlab=expression("k"[1]),
      ylab="Minimum cost")
  lines(res1$k1,res1$cost,col="black")
  lines(res2$k1,res2$cost,col="red")
  legend("topleft",as.character(c(N1,N2)),title="N:",col=c("black","red"),lty=1)

par(oldpar)

```

---

optimal\_holdout\_size\_emulation

*Estimate optimal holdout size under semi-parametric assumptions*

---

## Description

Compute optimal holdout size for updating a predictive score given a set of training set sizes and estimates of mean cost per sample at those training set sizes.

This is essentially a wrapper for function `mu_fn()`.

## Usage

```

optimal_holdout_size_emulation(
  nset,
  k2,
  var_k2,
  N,
  k1,
  var_u = 1e+07,
  k_width = 5000,
  k2form = powerlaw,
  theta = powersolve_general(nset, k2, y_var = var_k2)$par,
  npoll = 1000,
  ...
)

```

## Arguments

<code>nset</code>	Training set sizes for which a cost has been evaluated
<code>k2</code>	Estimated values of <code>k2()</code> at training set sizes <code>nset</code>
<code>var_k2</code>	Variance of error in <code>k2</code> estimate at each training set size.
<code>N</code>	Total number of samples on which the model will be fitted/used



k1	Mean cost per sample with no predictive score in place
var_u	Marginal variance for Gaussian process kernel. Defaults to 1e7
k_width	Kernel width for Gaussian process kernel. Defaults to 5000
k2form	Functional form governing expected cost per sample given sample size. Should take two parameters: n (sample size) and theta (parameters). Defaults to function powerlaw.
theta	Current estimates of parameter values for k2form. Defaults to the MLE power-law solution corresponding to n,k2, and var_k2.
npoll	Check npoll equally spaced values between 1 and N for minimum. If NULL, check all values (this can be slow). Defaults to 1000
...	Passed to function optimise()

**Value**

Object of class 'optholdoutsized\_emul' with elements "cost" (minimum cost), "size" (OHS), "nset", "k2", "var\_k2", "N", "k1" (parameters)

**Examples**

```
# See examples for mu_fn()
```

---

oracle_pred	<i>Generate responses</i>
-------------	---------------------------

---

**Description**

Probably for deprecation

**Usage**

```
oracle_pred(X, coefs, num_vars = 3, noise = TRUE)
```

**Arguments**

X	Matrix of observations
coefs	Vector of coefficients for logistic model.
num_vars	If noise==FALSE, computes using only first num_vars predictors
noise	If TRUE, uses all predictors

**Value**

Vector of predictions

**Examples**

```
# See examples for model_predict
```

params_aspre	<i>Parameters of reported ASPRE dataset</i>
--------------	---

### Description

Distribution of covariates for ASPRE dataset; see Rolnik, 2017, NEJM

## Usage

params\_aspre

### Format

An object of class `list` of length 16.

---

plot.optholdoutsizes	<i>Plot estimated cost function</i>
----------------------	-------------------------------------

### Description

Plot estimated cost function, when parametric method is used for estimation.

Draws cost function as a line and indicates minimum. Assumes a power-law form of  $k_2$  unless parameter  $k_2$  is set otherwise.

## Usage

```
## S3 method for class 'optholdoutsizes'
plot(x, ..., k2form = powerlaw)
```

## Arguments

x	Object of type <code>optholdoutsizes</code>
...	Other arguments passed to <code>plot()</code> and <code>lines()</code>
k2form	Function governing expected cost to an individual sample given a predictive score fitted to n samples. Must take two arguments: n (number of samples) and theta (parameters). Defaults to a power-law form $\text{powerlaw}(n, c(a, b, c)) = a n^{(-b)} + c$ .

## Value

No return value; draws a plot only.

**Examples**

```
# Simple example

N=100000;
k1=0.3
A=8000; B=1.5; C=0.15; theta=c(A,B,C)

res1=optimal_holdout_size(N,k1,theta)

plot(res1)
```

---

plot.optholdoutsizes\_emul

*Plot estimated cost function using emulation (semiparametric)*


---

**Description**

Plot estimated cost function, when semiparametric (emulation) method is used for estimation.

Draws posterior mean of cost function as a line and indicates minimum. Also draws mean +/- 3 SE.

Assumes a power-law form of k2 unless parameter k2 is set otherwise.

**Usage**

```
## S3 method for class 'optholdoutsizes_emul'
plot(x, ..., k2form = powerlaw)
```

**Arguments**

x	Object of type optholdoutsizes_emul
...	Other arguments passed to plot()
k2form	Function governing expected cost to an individual sample given a predictive score fitted to n samples. Must take two arguments: n (number of samples) and theta (parameters). Defaults to a power-law form $\text{powerlaw}(n, c(a, b, c)) = a n^{-b} + c$ .

**Value**

No return value; draws a plot only.

**Examples**

```
# Simple example

# Parameters
N=100000;
k1=0.3
A=8000; B=1.5; C=0.15; theta=c(A,B,C)

# True mean function
k2_true=function(n) powerlaw(n,theta)
```

```
# Values of n for which cost has been estimated
np=50 # this many points
nset=round(runif(np,1,N))
var_k2=runif(np,0.001,0.002)
k2=rnorm(np,mean=k2_true(nset),sd=sqrt(var_k2))

# Compute OHS
res1=optimal_holdout_size_emulation(nset,k2,var_k2,N,k1)

# Plot
plot(res1)
```

powerlaw

*Power law function***Description**

Power law function for modelling learning curve (taken to mean change in expected loss per sample with training set size)

Recommended in [review of learning curve forms](#)

If  $\theta = c(a, b, c)$  then models as  $a \cdot n^{-b} + c$ . Note  $b$  is negated.

Note that `powerlaw(n, c(a, b, c))` has limit  $c$  as  $n$  tends to infinity, if  $a, b > 0$

**Usage**

```
powerlaw(n, theta)
```

**Arguments**

<code>n</code>	Set of training set sizes to evaluate
<code>theta</code>	Parameter of values

**Value**

Vector of values of same length as `n`

**Examples**

```
ncheck=seq(1000,10000)
plot(ncheck, powerlaw(ncheck, c(5e3,1.2,0.3)),type="l",xlab="n",ylab="powerlaw(n)")
```

---

powersolve	<i>Fit power law curve</i>
------------	----------------------------

---

## Description

Find least-squares solution: MLE of (a,b,c) under model  $y_i = a x_i^{-b} + c + e_i$ ;  $e_i \sim N(0, y\_var\_i)$

## Usage

```
powersolve(
  x,
  y,
  init = c(20000, 2, 0.1),
  y_var = rep(1, length(y)),
  estimate_s = FALSE,
  ...
)
```

## Arguments

x	X values
y	Y values
init	Initial values of (a,b,c) to start. Default c(20000,2,0.1)
y_var	Optional parameter giving sampling variance of each y value. Defaults to 1.
estimate_s	Parameter specifying whether to also estimate s (as above). Defaults to FALSE (no).
...	further parameters passed to optim. We suggest specifying lower and upper bounds for (a,b,c); e.g. lower=c(1,0,0),upper=c(10000,3,1)

## Value

List (output from optim) containing MLE values of (a,b,c)

## Examples

```
# Retrieval of original values
A_true=2000; B_true=1.5; C_true=0.3; sigma=0.002

X=1000*abs(rnorm(10000,mean=4))
Y=A_true*(X^(-B_true)) + C_true + rnorm(length(X),sd=sigma)

c(A_true,B_true,C_true)
powersolve(X[1:10],Y[1:10])$par
powersolve(X[1:100],Y[1:100])$par
powersolve(X[1:1000],Y[1:1000])$par
powersolve(X[1:10000],Y[1:10000])$par
```

---

powersolve_general	<i>General solver for power law curve</i>
--------------------	---

---

### Description

Find least-squares solution: MLE of (a,b,c) under model  $y_i = a x_i^{-b} + c + e_i$ ;  $e_i \sim N(0, y\_var\_i)$

Try a range of starting values and refine estimate.

Slower than a single call to powersolve()

### Usage

```
powersolve_general(x, y, y_var = rep(1, length(x)), ...)
```

### Arguments

x	X values
y	Y values
y_var	Optional parameter giving sampling variance of each y value. Defaults to 1.
...	further parameters passed to optim. We suggest specifying lower and upper bounds for (a,b,c); e.g. lower=c(1,0,0),upper=c(10000,3,1)

### Value

List (output from optim) containing MLE values of (a,b,c)

### Examples

```
# Retrieval of original values
A_true=2000; B_true=1.5; C_true=0.3; sigma=0.002

X=1000*abs(rnorm(10000,mean=4))
Y=A_true*(X^(-B_true)) + C_true + rnorm(length(X),sd=sigma)

c(A_true,B_true,C_true)
powersolve_general(X[1:10],Y[1:10])$par
powersolve_general(X[1:100],Y[1:100])$par
powersolve_general(X[1:1000],Y[1:1000])$par
```

---

powersolve_se	<i>Standard error matrix for learning curve parameters (power law)</i>
---------------	--

---

## Description

Find approximate standard error matrix for (a,b,c) under power law model for learning curve.

Assumes that

$$y_i = a x_i^{-b} + c + e, \quad e \sim N(0, s^2 y_{\text{var}_i}^2)$$

Standard error can be computed either asymptotically using Fisher information (method='fisher') or bootstrapped (method='bootstrap')

These estimate different quantities: the asymptotic method estimates

$\text{Var}[\text{MLE}(a,b,c) | X, y_{\text{var}}]$

and the bootstrap method estimates

$\text{Var}[\text{MLE}(a,b,c)]$ .

## Usage

```
powersolve_se(
  x,
  y,
  method = "fisher",
  init = c(20000, 2, 0.1),
  y_var = rep(1, length(y)),
  n_boot = 1000,
  seed = NULL,
  ...
)
```

## Arguments

x	X values (typically training set sizes)
y	Y values (typically observed cost per individual/sample)
method	One of 'fisher' (for asymptotic variance via Fisher Information) or 'bootstrap' (for Bootstrap)
init	Initial values of (a,b,c) to start when computing MLE. Default c(20000,2,0.1)
y_var	Optional parameter giving sampling variance of each y value. Defaults to 1.
n_boot	Number of bootstrap resamples. Only used if method='bootstrap'. Defaults to 1000
seed	Random seed for bootstrap resamples. Defaults to NULL.
...	further parameters passed to optim. We suggest specifying lower and upper bounds; since optim is called on (a*1000 <sup>-b</sup> ,b,c), bounds should be relative to this; for instance, lower=c(0,0,0),upper=c(100,3,1)

## Value

Standard error matrix; approximate covariance matrix of MLE(a,b,c)

## Examples

```
A_true=10; B_true=1.5; C_true=0.3; sigma=0.1

set.seed(31525)
```

```

X=1+3*rchisq(10000,df=5)
Y=A_true*(X^(-B_true)) + C_true + rnorm(length(X),sd=sigma)

# 'Observations' - 100 samples
obs=sample(length(X),100,rep=FALSE)
Xobs=X[obs]; Yobs=Y[obs]

# True covariance matrix of MLE of a,b,c on these x values
ntest=100
abc_mat_xfix=matrix(0,ntest,3)
abc_mat_xvar=matrix(0,ntest,3)
E1=A_true*(Xobs^(-B_true)) + C_true
for (i in 1:ntest) {
  Y1=E1 + rnorm(length(Xobs),sd=sigma)
  abc_mat_xfix[i,]=powersolve(Xobs,Y1)$par # Estimate (a,b,c) with same X

  X2=1+3*rchisq(length(Xobs),df=5)
  Y2=A_true*(X2^(-B_true)) + C_true + rnorm(length(Xobs),sd=sigma)
  abc_mat_xvar[i,]=powersolve(X2,Y2)$par # Estimate (a,b,c) with variable X
}

Ve1=var(abc_mat_xfix) # empirical variance of MLE(a,b,c)|X
Vf=powersolve_se(Xobs,Yobs,method='fisher') # estimated SE matrix, asymptotic

Ve2=var(abc_mat_xvar) # empirical variance of MLE(a,b,c)
Vb=powersolve_se(Xobs,Yobs,method='bootstrap',n_boot=200) # estimated SE matrix, bootstrap

cat("Empirical variance of MLE(a,b,c)|X\n")
print(Ve1)
cat("\n")
cat("Asymptotic variance of MLE(a,b,c)|X\n")
print(Vf)
cat("\n\n")
cat("Empirical variance of MLE(a,b,c)\n")
print(Ve2)
cat("\n")
cat("Bootstrap-estimated variance of MLE(a,b,c)\n")
print(Vb)
cat("\n\n")

```

---

psi\_fn

---

Updating function for variance.

---

## Description

Posterior variance for emulator given points n.

## Usage

```
psi_fn(n, nset, var_k2, N, var_u = 1e+07, k_width = 5000)
```



**Arguments**

n	Set of training set sizes to evaluate at
nset	Training set sizes for which k2() has been evaluated
var_k2	Variance of error in k2() estimate at each training set size.
N	Total number of samples on which the model will be fitted/used. Only used to rescale var_k2
var_u	Marginal variance for Gaussian process kernel. Defaults to 1e7
k_width	Kernel width for Gaussian process kernel. Defaults to 5000

**Value**

Vector Psi of same length of n where  $\Psi_i = \text{var}(\text{posterior}(\text{cost}(n_i)))$

**Examples**

```
# See examples for `mu_fn`
```

---

sens10	<i>Sensitivity at theshold quantile 10%</i>
--------	---

---

**Description**

Computes sensitivity of a risk score at a threshold at which 10% of samples (or some proportion pi\_int) are above the threshold.

**Usage**

```
sens10(Y, Ypred, pi_int = 0.1)
```

**Arguments**

Y	True labels (1 or 0)
Ypred	Predictions (univariate; real numbers)
pi_int	Compute sensitivity when a proportion pi_int of samples exceed threshold, default 0.1

**Value**

Sensitivity at this threshold

**Examples**

```
# Simulate
set.seed(32142)

N=1000
X=rnorm(N); Y=rbinom(N,1,prob=logit(X/2))

pi_int=0.1
q10=quantile(X,1-pi_int) # 10% of X values are above this threshold

print(length(which(Y==1 & X>q10))/length(which(X>q10)))
print(sens10(Y,X,pi_int))
```

---

sim_random_aspre	<i>Simulate random dataset similar to ASPRE training data</i>
------------------	---

---

**Description**

Generate random population of individuals (e.g., newly pregnant women) with given population parameters

Assumes independence of parameter variation. This is not a realistic assumption, but is satisfactory for our purposes.

**Usage**

```
sim_random_aspre(n, params = NULL)
```

**Arguments**

n	size of population
params	list of parameters

**Value**

Matrix of samples

**Examples**

```
# Load ASPRE related data
data(params_aspre)

X=sim_random_aspre(1000,params_aspre)

print(c(median(X$age),params_aspre$age$median))

print(rbind(table(X$parity)/1000,params_aspre$parity$freq))
```

---

split\_data

---

*Split data***Description**

Split data into holdout and intervention sets

**Usage**

```
split_data(X, frac)
```

**Arguments**

X                      Matrix of observations

frac                    Fraction of observations to use for the training set

**Value**

Vector of TRUE/FALSE values (randomised) with proportion frac as TRUE

**Examples**

```
# See examples for model_predict
```

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